

Machine Learning

Problem Set 6

Hesam Montazeri
Fereshteh Fallah
Mozhgan Mozaffari Legha
Ordibehesht 19, 1399
(May 8, 2020)

Problem 1: Review part

Write your reviews for the whiteboard notes and the slides of the lectures of this week. Write down all formulas and explain in detail each step of the derivations, if applicable.

Problem 2: Convex functions/sets

Solve the following problems related to convex functions/sets:

- (a) Prove the nonnegative weighted average of two convex functions is convex.
- (b) Show that if $a, b \geq 0$ and $0 \leq \theta \leq 1$, then $a^\theta b^{1-\theta} \leq \theta a + (1 - \theta)b$.
- (c) Prove $f(x) = x \log x$ is a convex function. Plot the function $f(x)$ and its tangent at $x = 2$.
- (d) Prove the intersection of two convex sets is a convex set.
- (e) Is the union of convex sets convex? If not provide a counterexample.

Problem 3: KKT conditions

Using the Karush-Kuhn-Tucker (KKT) conditions, solve the following nonlinear programming problem:

$$\begin{aligned} \max \quad & f(x, y) = xy \\ \text{s.t.} \quad & x + y^2 \leq 2 \\ & x, y \geq 0 \end{aligned}$$

Problem 4: Dual problem of quadratic programming

Derive the dual optimization problem of a quadratic programming.

Problem 5: Complementary slackness (Optional)

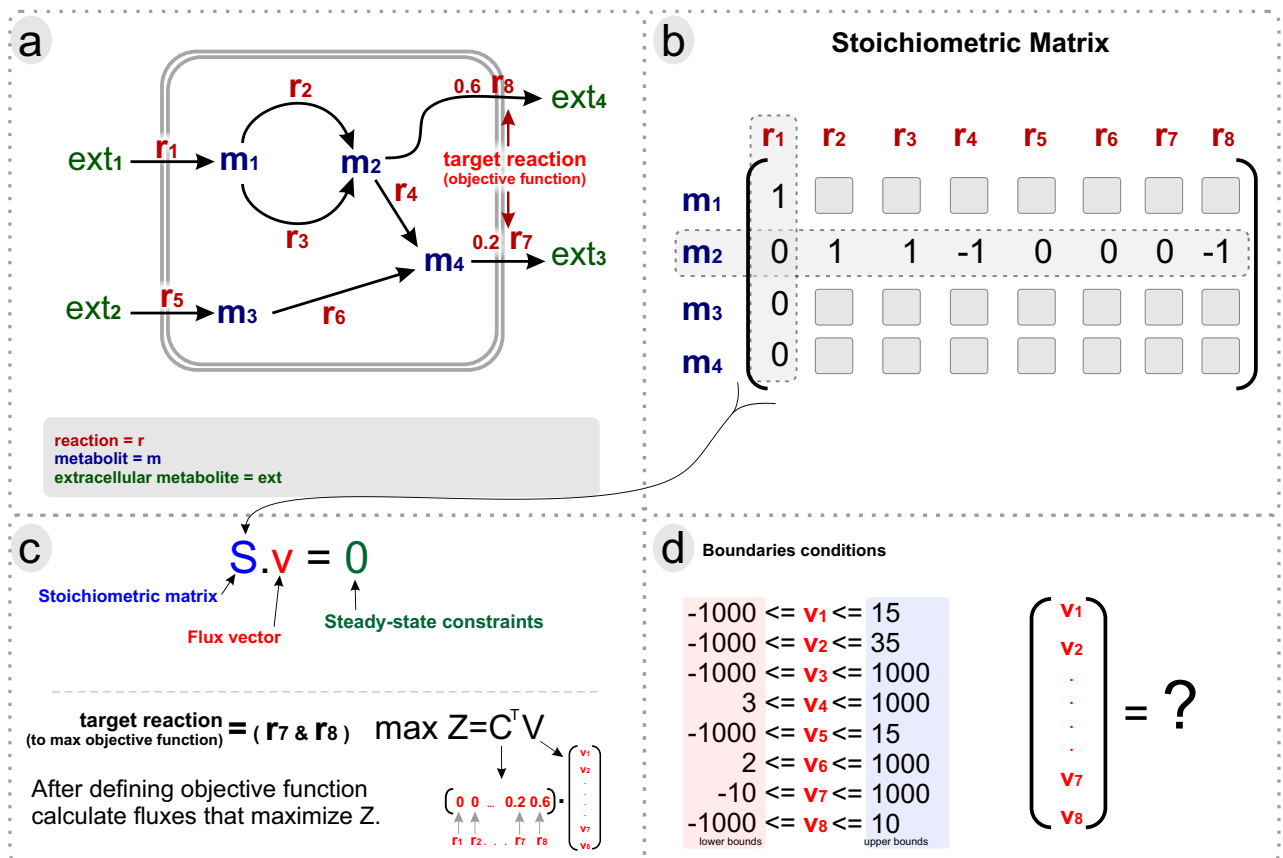
Prove if strong duality holds, the optimal primal and dual solutions satisfy complementary slackness conditions.

Problem 6: Programming: metabolic networks

(Thanks to Ehsan Zangene for his kind contributions in this problem!) Given a toy metabolic network

(panel *a* of the following figure), corresponding stoichiometric matrix (*b*), steady-state constraint (*c*), and boundary constraints (*d*), your task is to

- Complete the stoichiometric matrix in part *b*.
- Write down the linear program for maximization of the objective function specified in part *a* under the above-mentioned constraints.
- Write the dual optimization problem.
- Solve the primal and dual optimization problems using the built-in LP solvers in R.



We encourage discussing the problems with other students, however, similarity between solutions is not allowed. (**Important**) Studying any online or previous solutions, no matter to what extent, is strictly forbidden and is considered as a violation of the academic honor code. Submit your solutions by Ordibehesht 24, 1399.