
Machine Learning

Problem Set 3

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Problem 1: Review part

Write your reviews for the whiteboard notes and the slides of the lectures of this week. Write down all formulas and explain in detail each step of the derivations, if applicable. In addition, write a short summary of ISL Sec 3.1-3 and 6.1.

Problem 2: Conceptual questions

[ISL] chapter 3: questions 1, 3, 5, a written summary of Section 3.6 (Lab). chapter 6: questions 1, 2, 5, 7

Problem 3: Ridge regression-projection matrix

Show that the ridge hat matrix is not a projection matrix.

Problem 4: Weighted linear regression

Derive the optimal solution $\hat{\beta}$ for the *weighted* loss function:

$$\frac{1}{2} \sum_{i=1}^n w[i] \times (y[i] - \beta^T x[i])^2$$

where $w[i]$ is the associated weight for i th data point $(x[i], y[i])$.

Problem 5: Maximum likelihood estimation of multinomial distribution

Suppose X is a multinomial random variable that takes values x^1, x^2, \dots, x^K . The multinomial distribution has K parameters $\Pi = (\pi_1, \dots, \pi_K)$ such that $P(X = x^k) = \pi_k$ subject to $\sum_k \pi_k = 1$. Let the observed data is (n_1, \dots, n_K) where n_k is the number of times the value x^k appears in the data. Prove the ML estimate for π_k is $\frac{n_k}{n}$ where $n = \sum_k n_k$ (hint: use Lagrange multiplier).

Problem 6: Programming: Ridge regression

Implement ridge regression and show the correctness of your code by comparing to the built-in functions in R.

Problem 7: Programming: high-dimensional problem $p \gg n$

Your task is to build a prediction model for the uploaded simulated data (HW3train.RData) with 50 observations and 15000 features. Consider trying different models and choose the best model. Estimate the generalization error of your final model. Write the predictions of your final model for the uploaded test data (HW3test.RData) in a csv file. You may use built-in functions in R. Extra credit for the most accurate predictions.

We encourage discussing the problems with other students, however, similarity between solutions is not allowed. (**Important**) Studying any online or previous solutions, no matter to what extent, is

strictly forbidden and is considered as a violation of the academic honor code. Submit your solutions by Esfand 3, 1398.