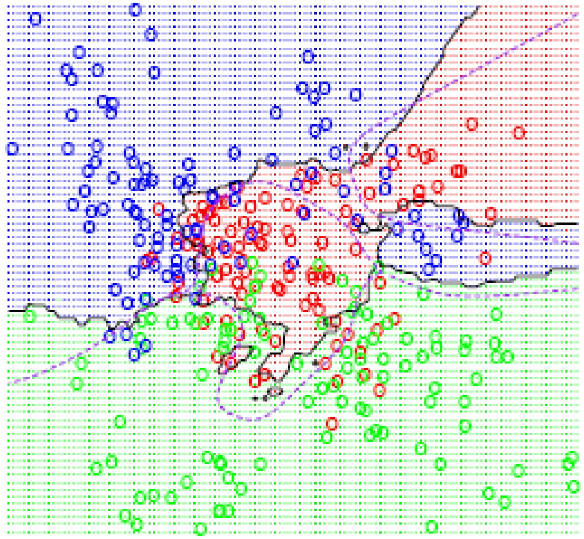
Machine Learning

Lecture 4: Regularization; Bias-Variance tradeoff

The lectures are mainly offered on white board accompanied by some slides.



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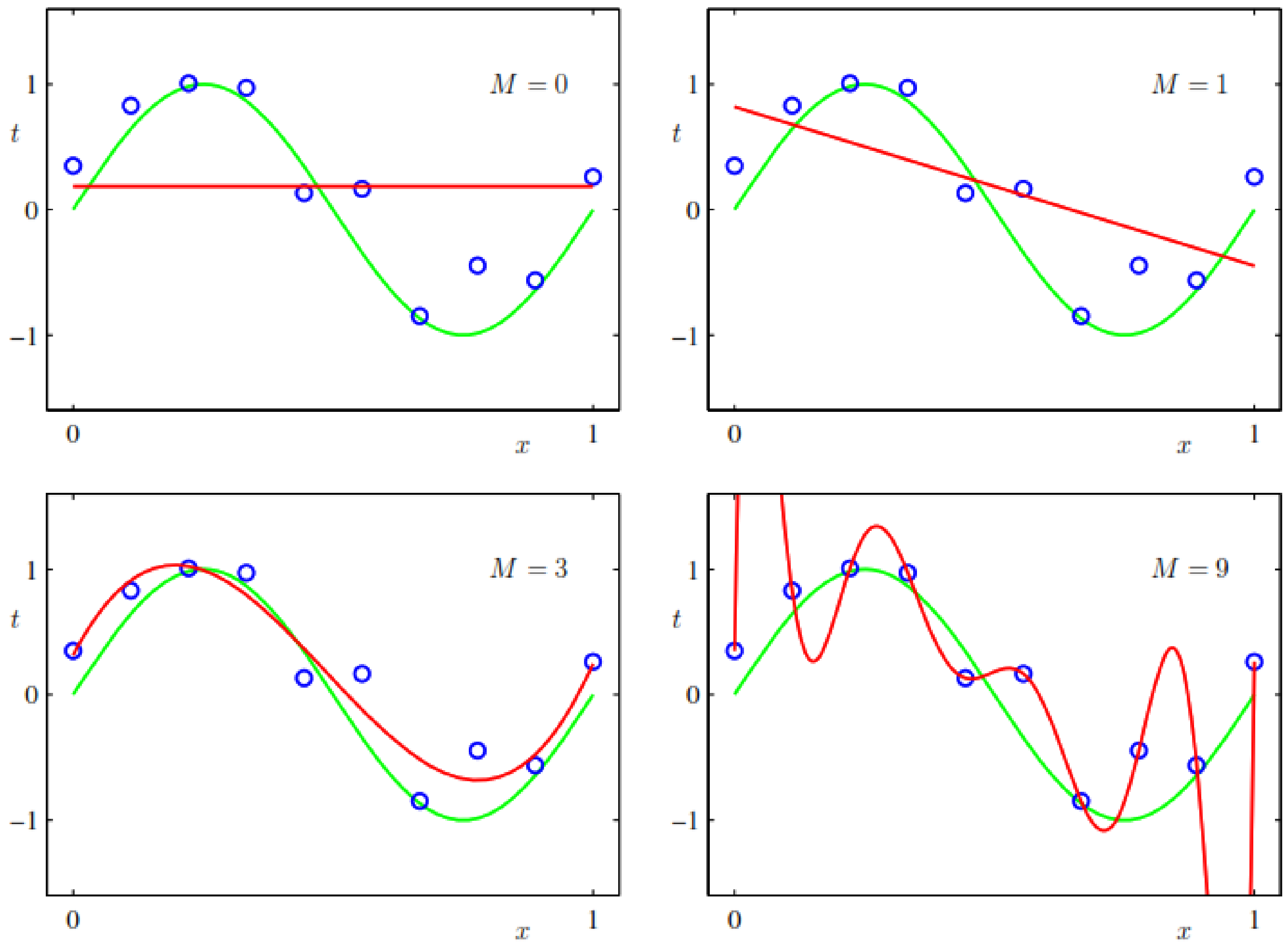
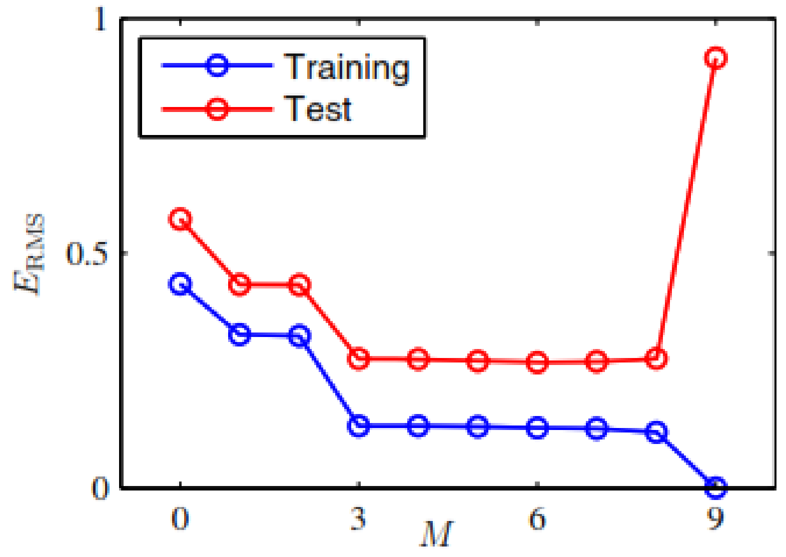
Bahman 21, 1398

Polynomials having various orders M





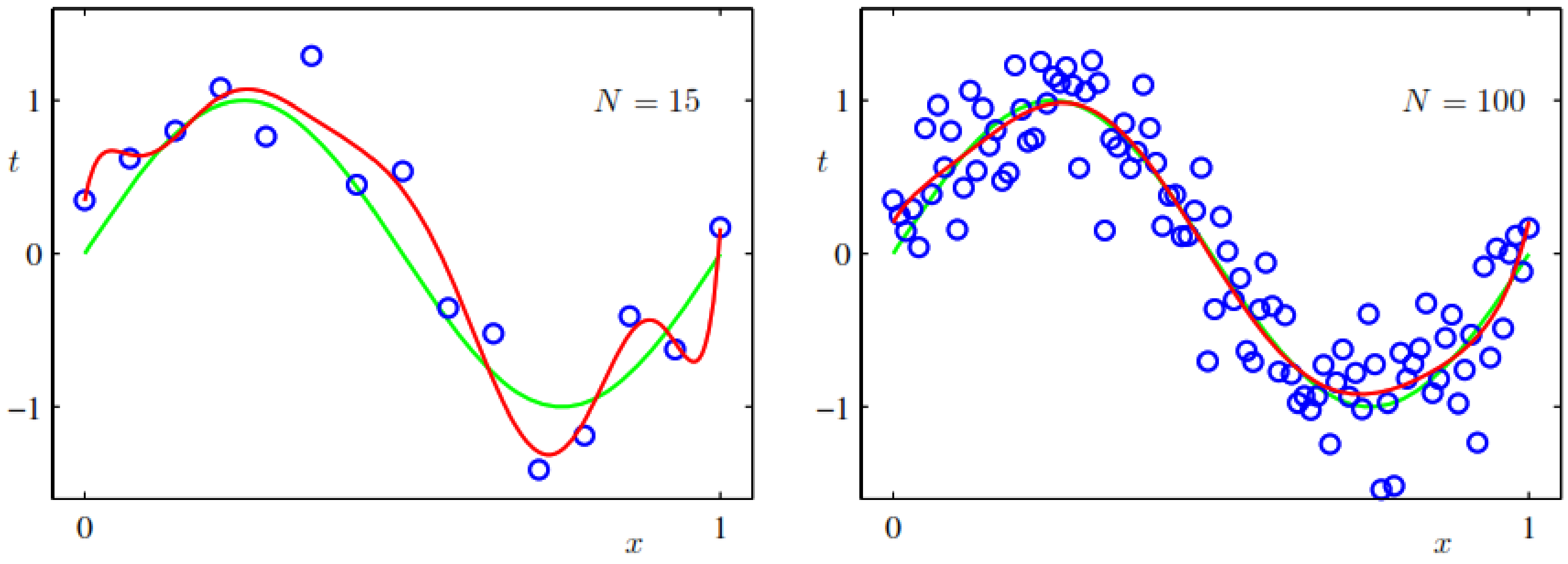
[Bishop]

Magnitude of the coefficients increases with p

	$M = 0$	$M = 1$	$M = 6$	$M = 9$
$w_0^\star$	0.19	0.82	0.31	0.35
$w_1^\star$		-1.27	7.99	232.37
$w_2^\star$			-25.43	-5321.83
$w_3^\star$			17.37	48568.31
$w_4^\star$				-231639.30
$w_5^\star$				640042.26
$w_6^\star$				-1061800.52
$w_7^\star$				1042400.18
$w_8^\star$				-557682.99
$w_9^\star$				125201.43

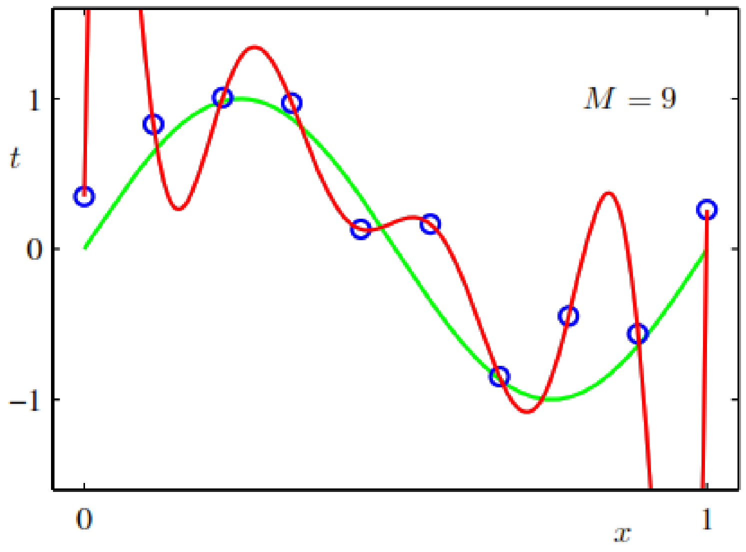
[Bishop]

The increasing size of the data set reduces the over-fitting problem

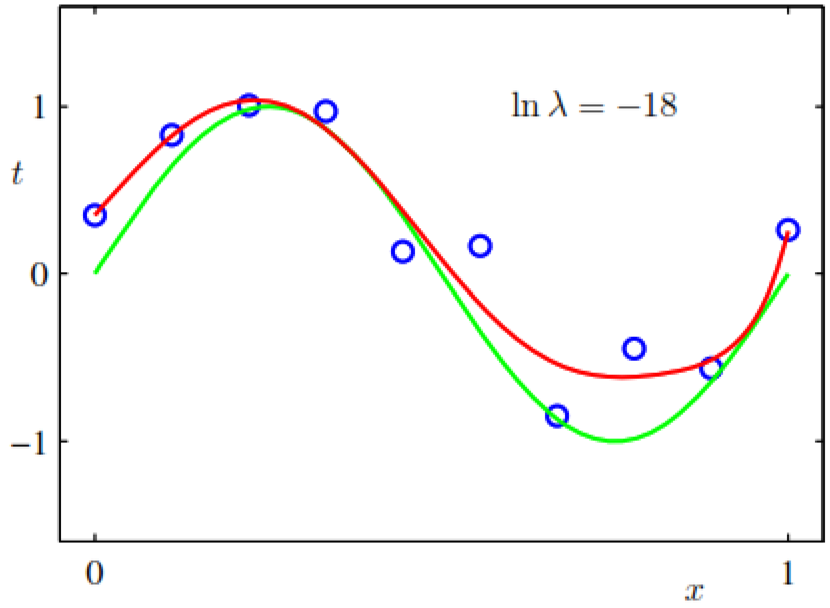


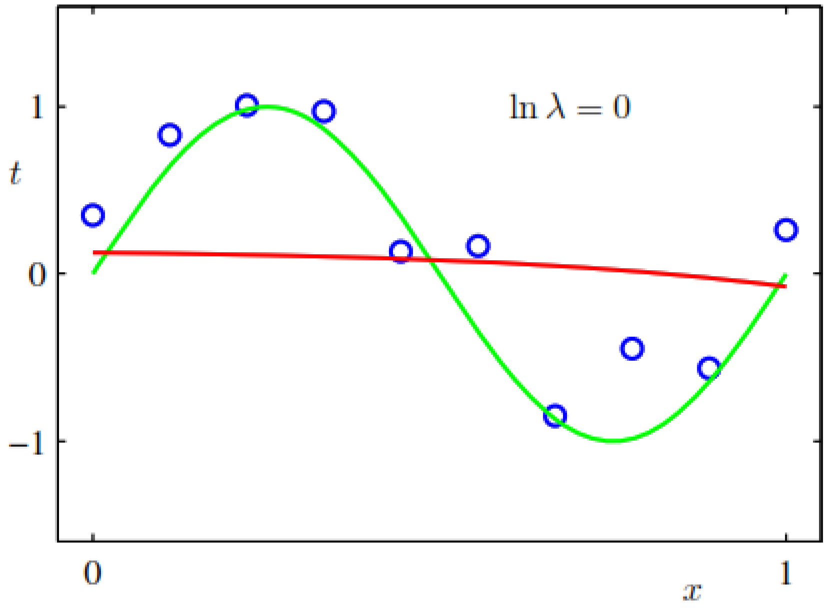
[Bishop]

Regularized error function



	$\ln \lambda = -\infty$	$\ln \lambda = -18$	$\ln \lambda = 0$
$w_0^\star$	0.35	0.35	0.13
$w_1^\star$	232.37	4.74	-0.05
$w_2^\star$	-5321.83	-0.77	-0.06
$w_3^\star$	48568.31	-31.97	-0.05
$w_4^\star$	-231639.30	-3.89	-0.03
$w_5^\star$	640042.26	55.28	-0.02
$w_6^\star$	-1061800.52	41.32	-0.01
$w_7^\star$	1042400.18	-45.95	-0.00
$w_8^\star$	-557682.99	-91.53	0.00
$w_9^\star$	125201.43	72.68	0.01





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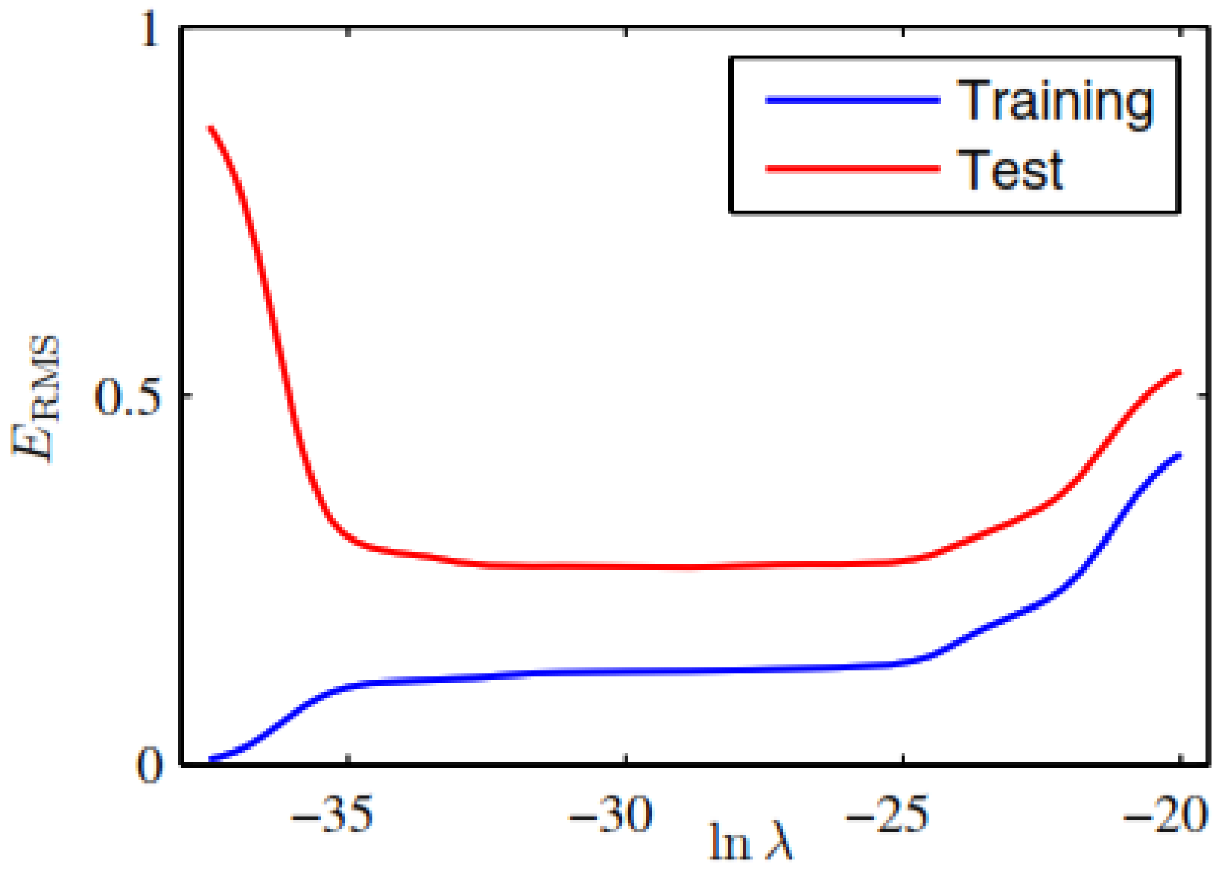


[Bishop]

Example: sine target

$f : [-1, 1] \to \mathbb{R} \qquad f(x) = \sin(\pi x)$

Only two training examples! $\quad N = 2$

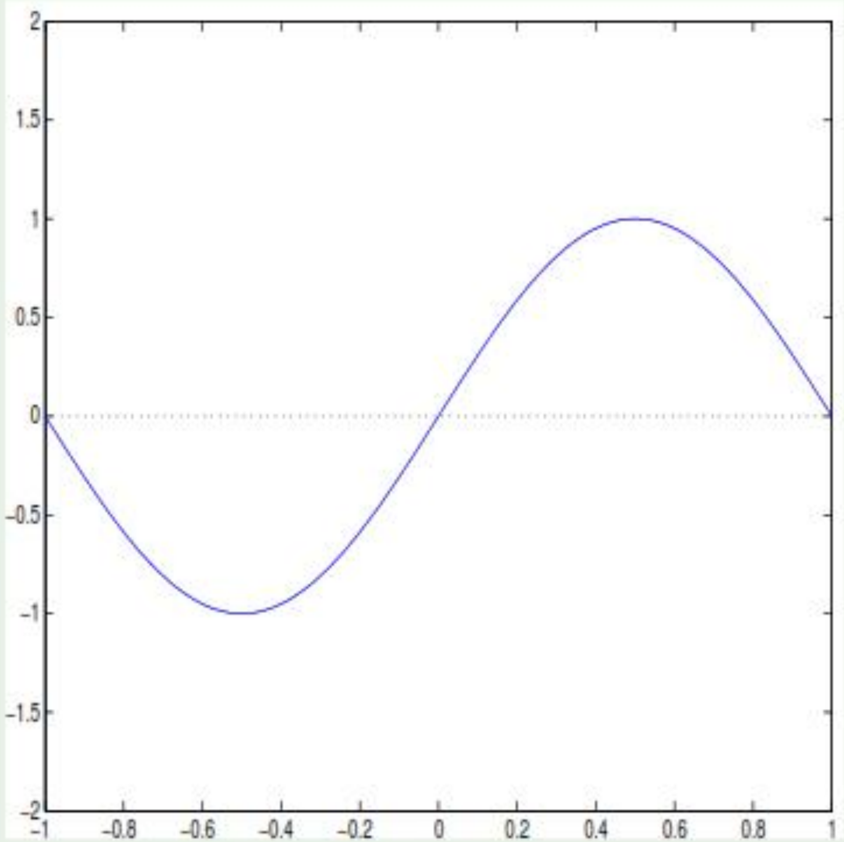
Two models used for learning:

$$\mathcal{H}_0: \quad h(x) = b$$

$$\mathcal{H}_1: \quad h(x) = ax + b$$

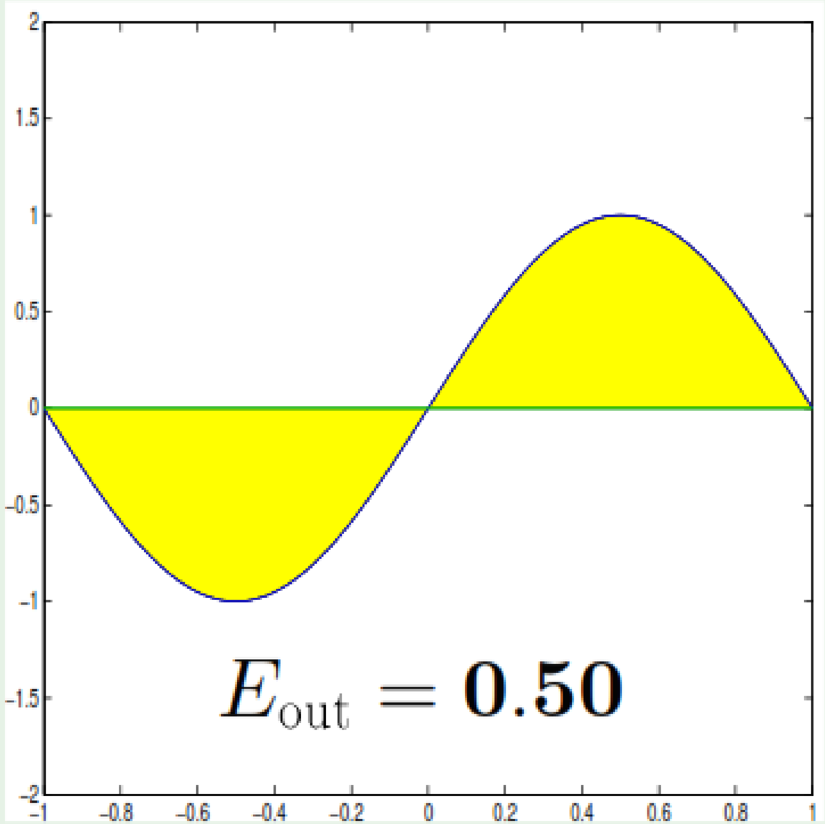
Which is better, $\mathcal{H}_0$ or $\mathcal{H}_1$?

f

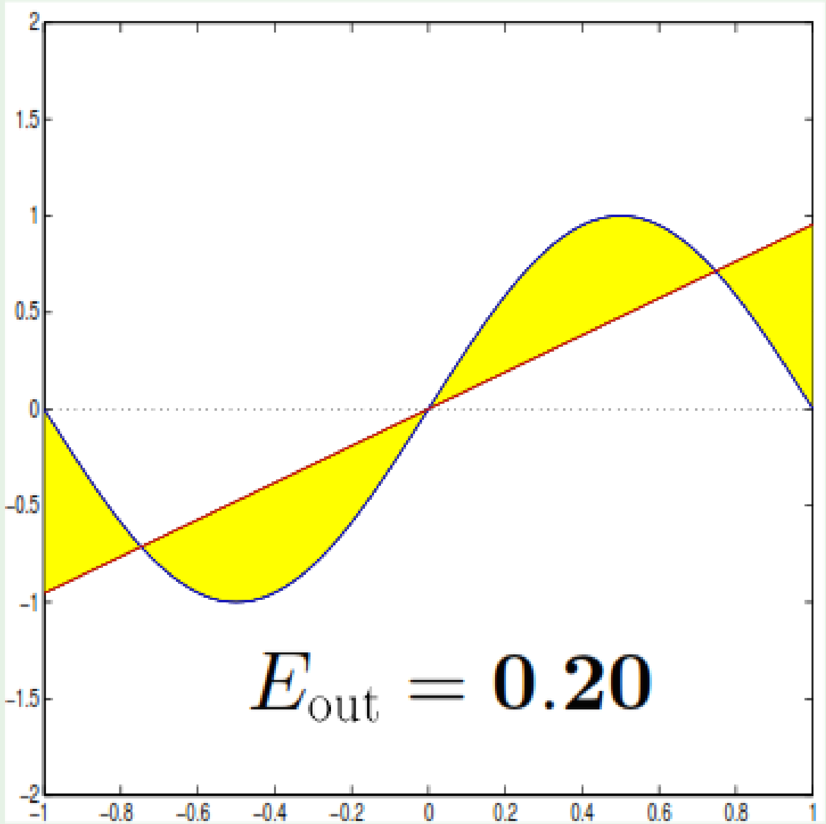


Approximation - $\mathcal{H}_0$ versus $\mathcal{H}_1$

$\mathcal{H}_0$



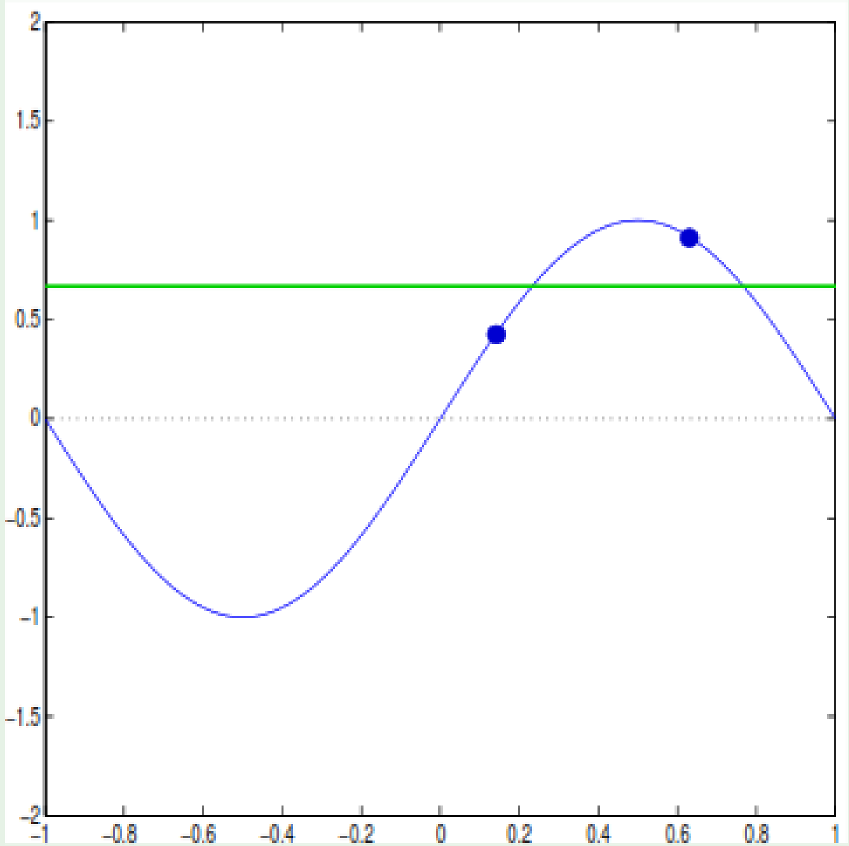
$\mathcal{H}_1$



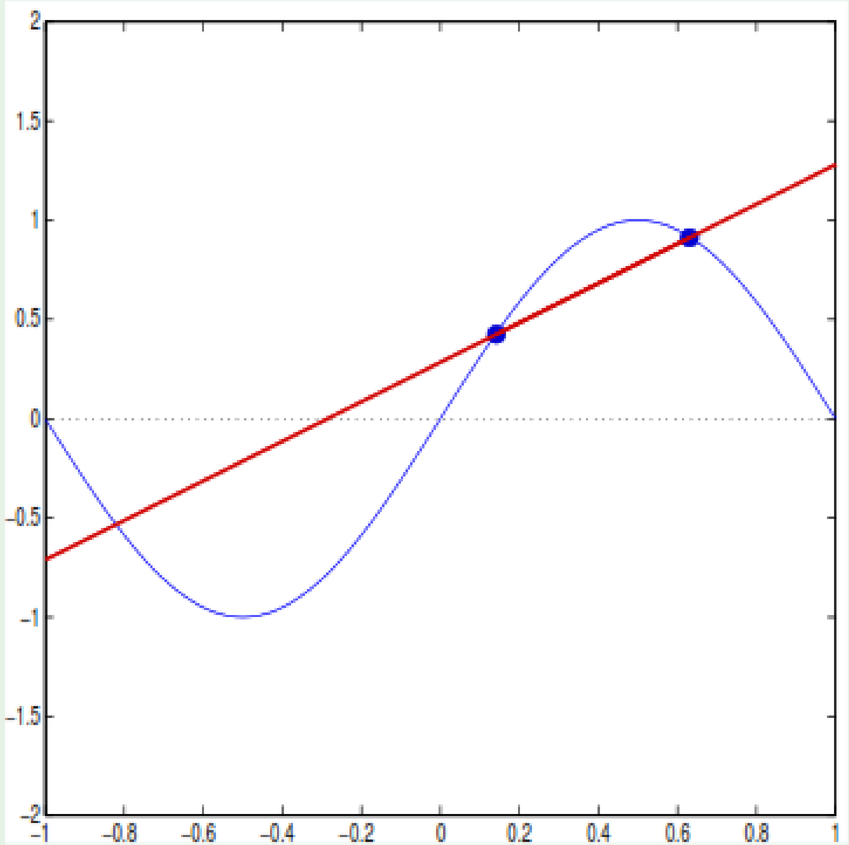
Slides from https://work.caltech.edu/lectures.html

Learning - $\mathcal{H}_0$ versus $\mathcal{H}_1$

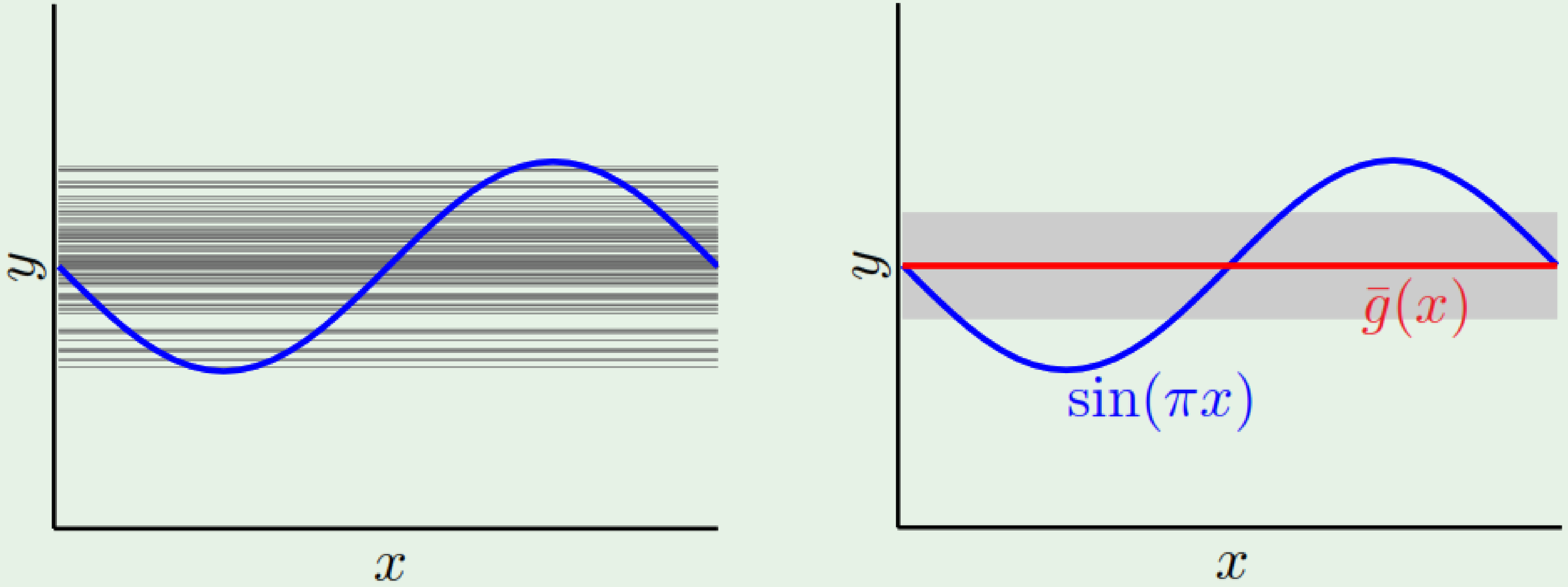
$\mathcal{H}_0$



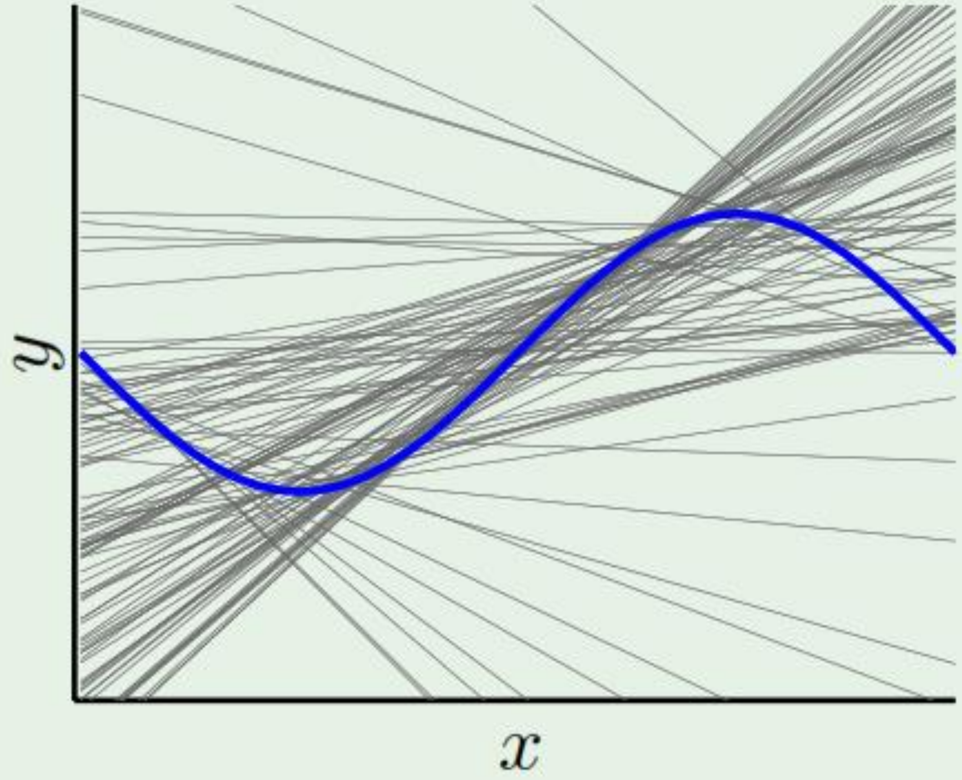
$\mathcal{H}_1$

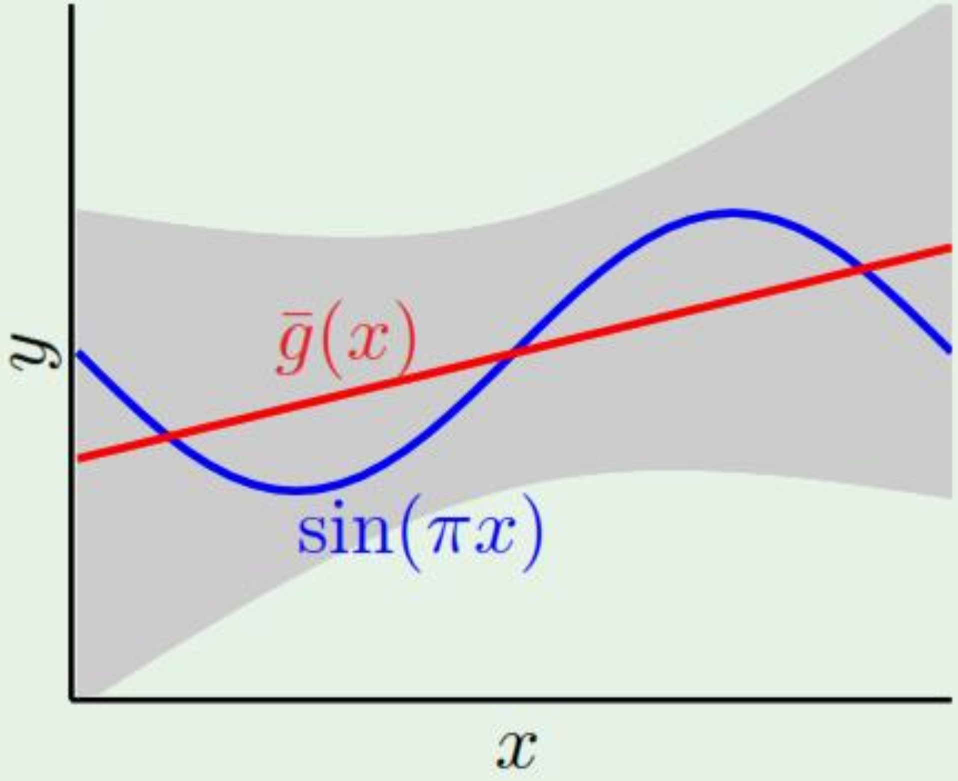


Bias and variance - $\mathcal{H}_0$

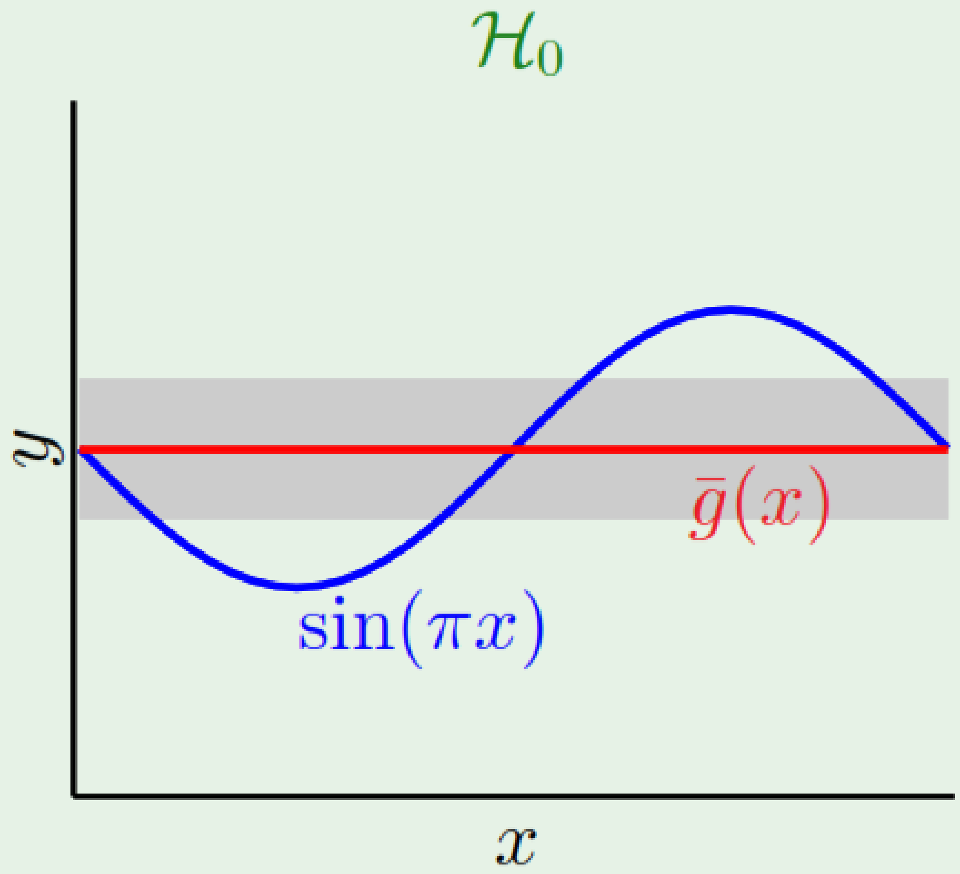


Bias and variance - $\mathcal{H}_1$

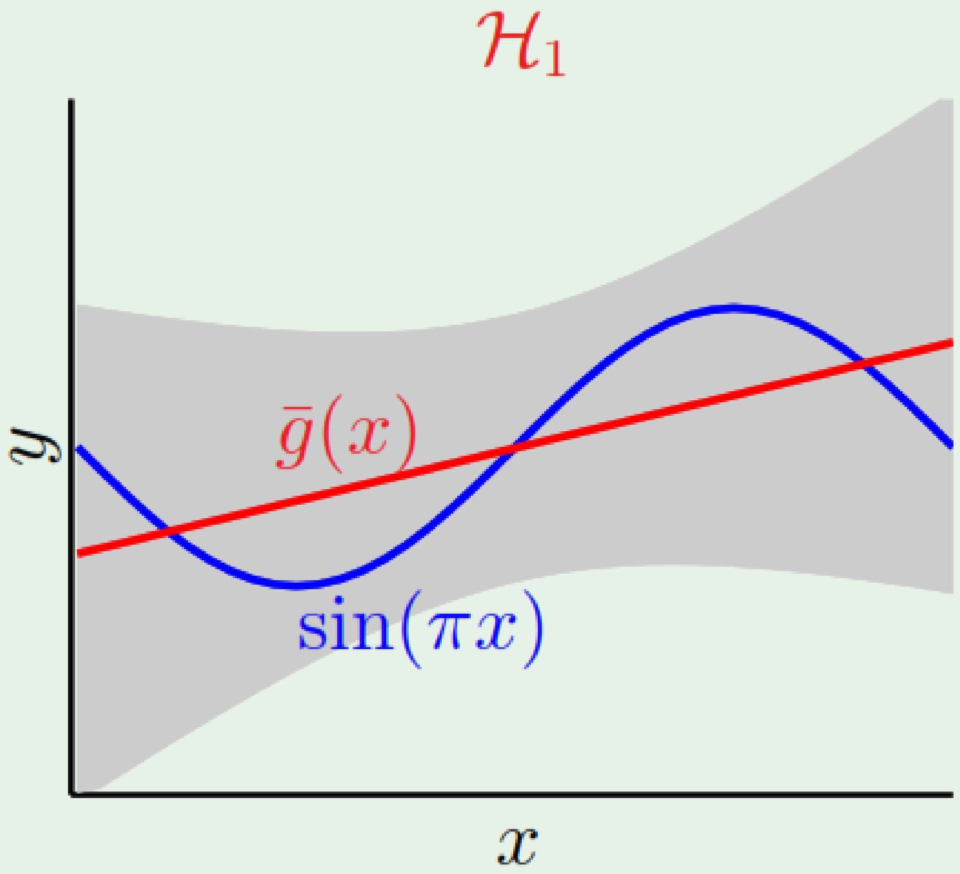




and the winner is ...



bias = **0.50** var = **0.25**



bias = **0.21** var = **1.69**

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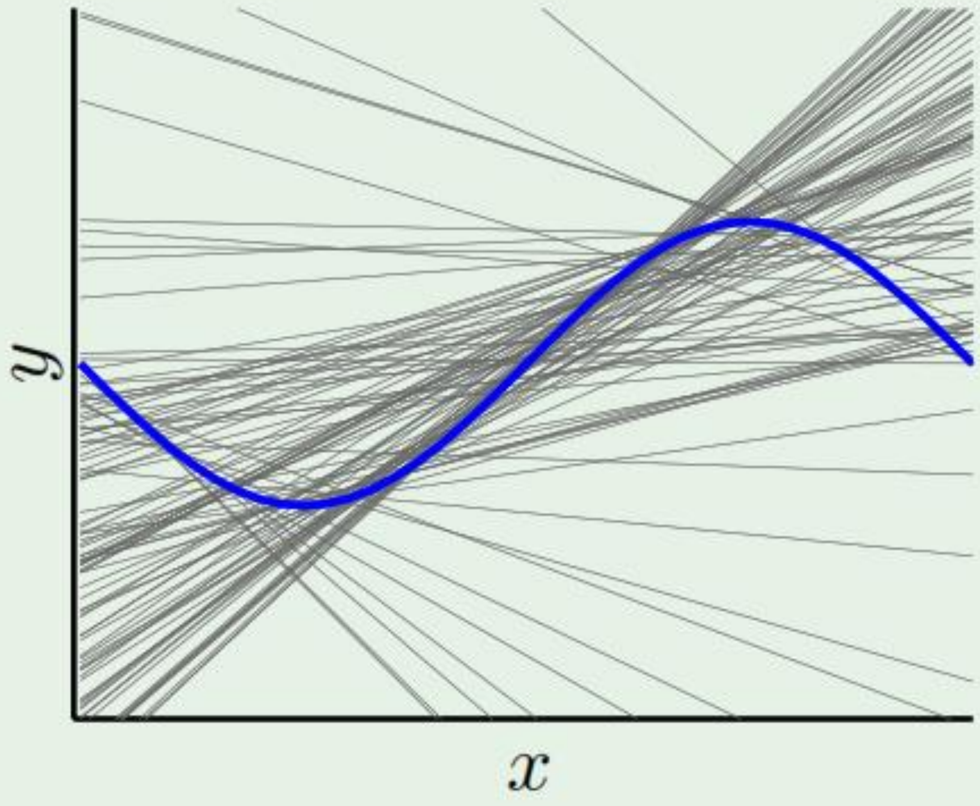
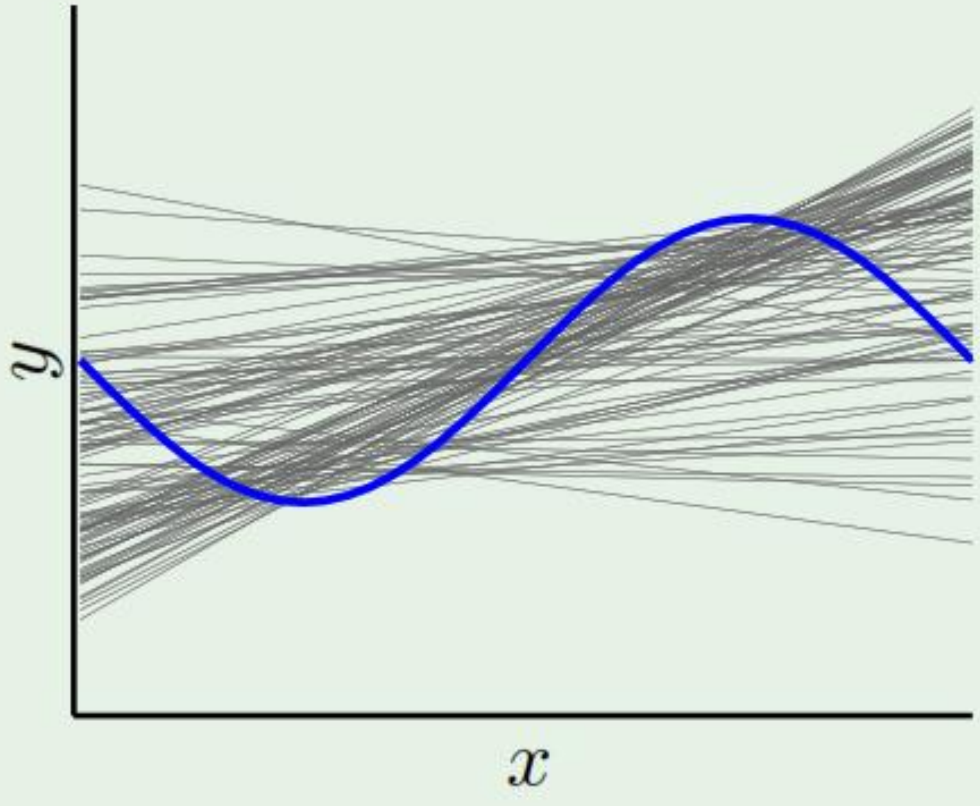
A familiar example



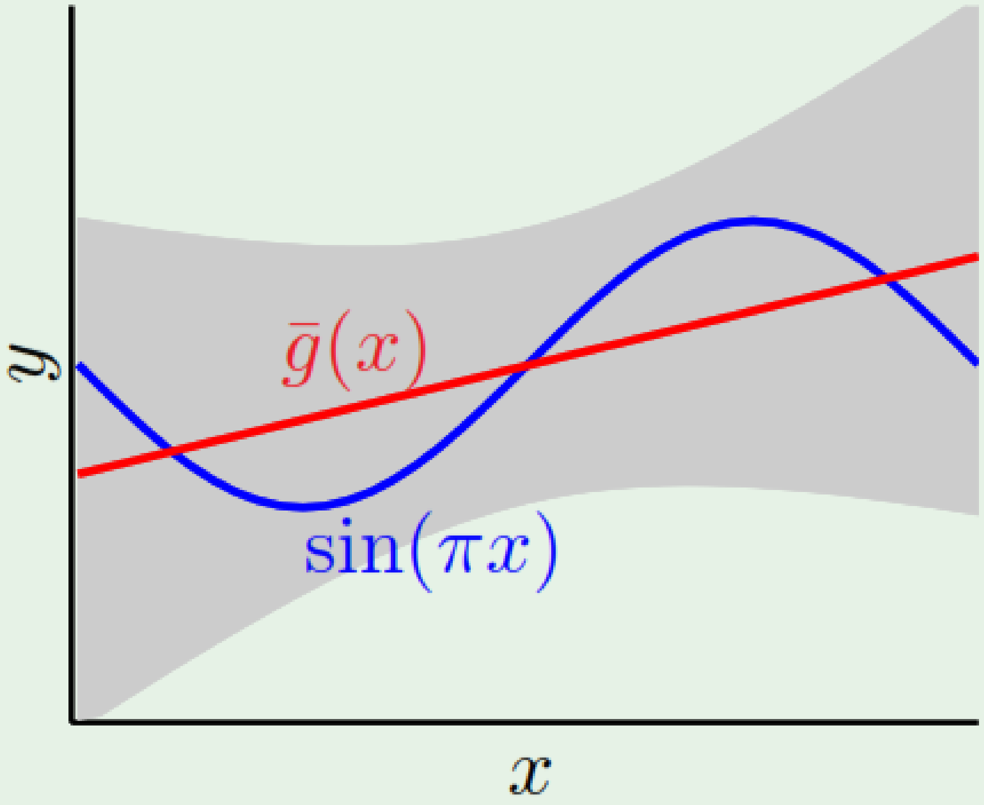
without regularization



with regularization

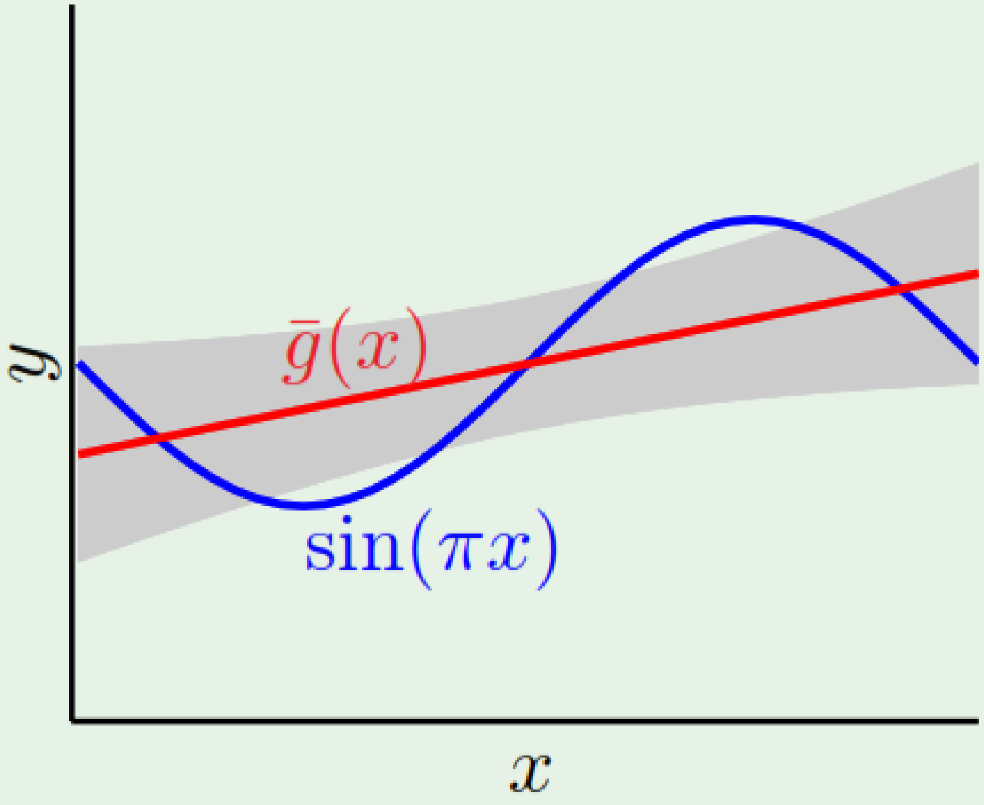
and the winner is . . .

without regularization



bias $= \mathbf{0.21}$ var $= \mathbf{1.69}$

with regularization



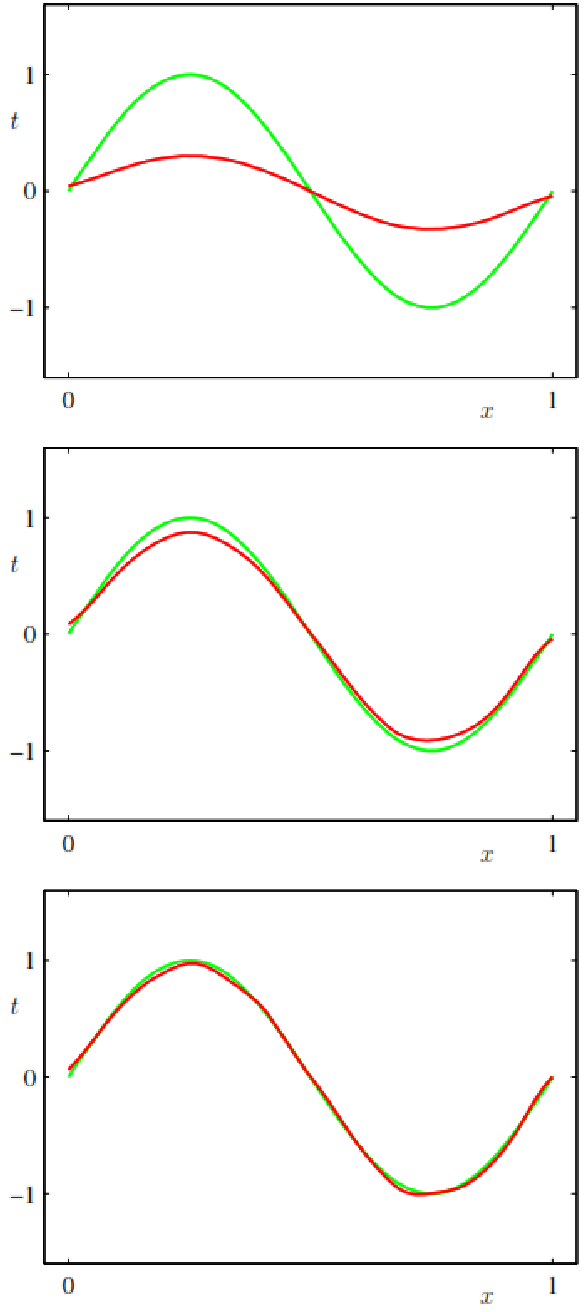
bias $= \mathbf{0.23}$ var $= \mathbf{0.33}$

Slides from https://work.caltech.edu/lectures.html

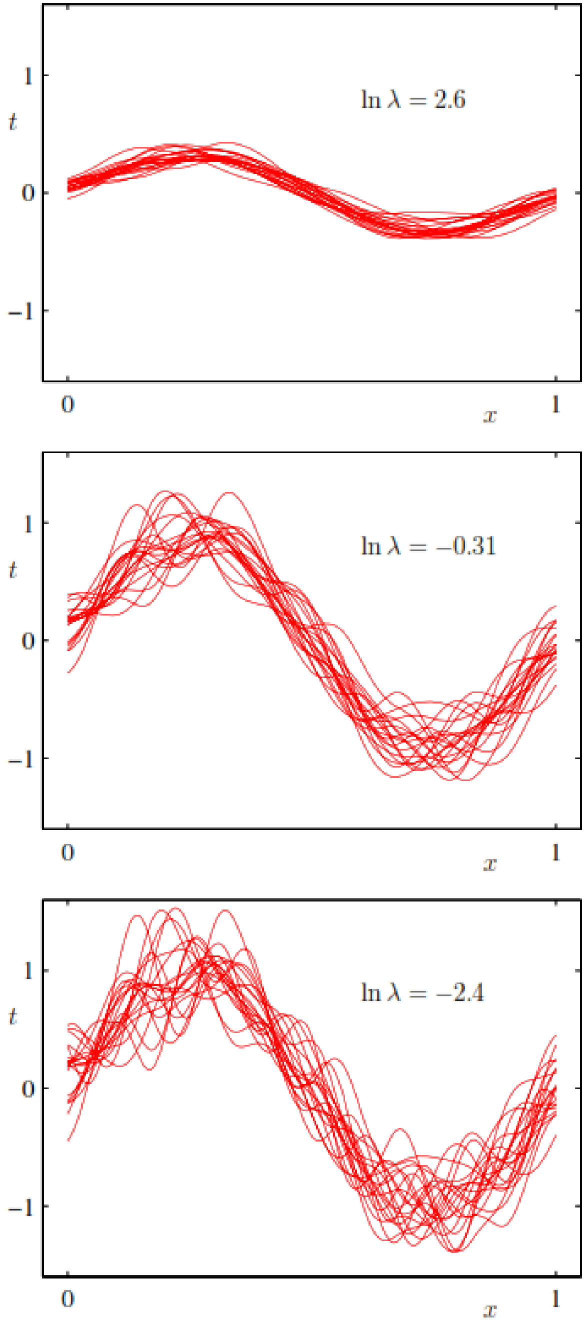
100 datasets

n=100

P=25



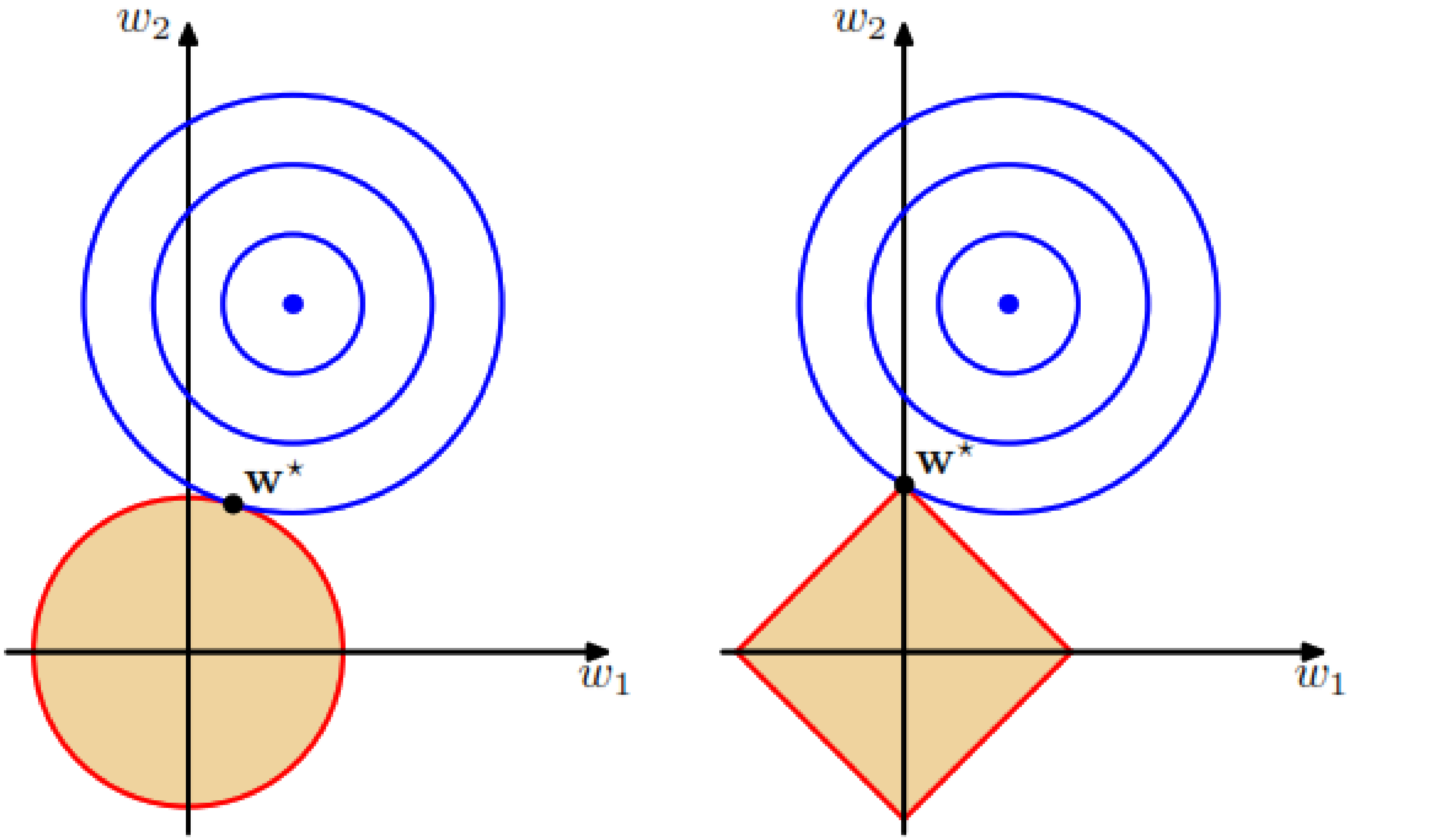
Average of 100 fits



Results of all fits

[Bishop]

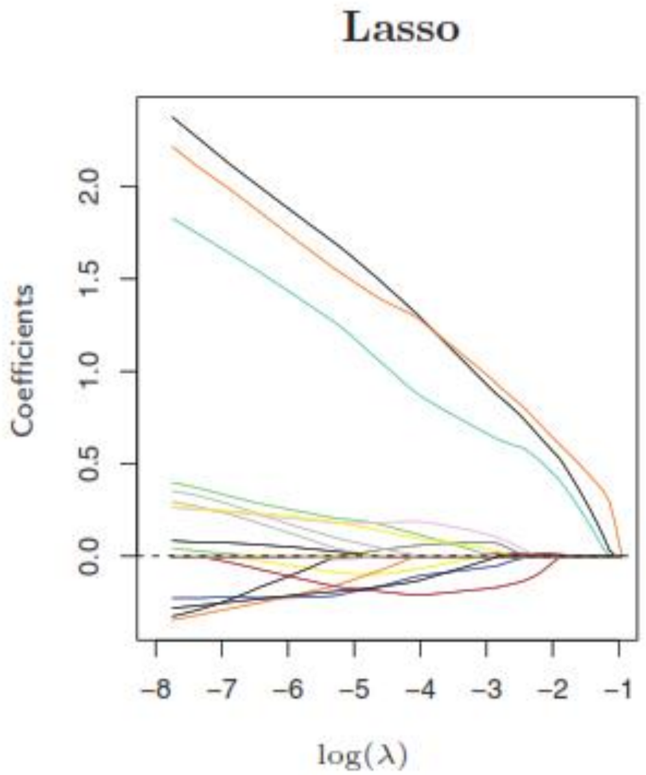
Contours of unregularized error function



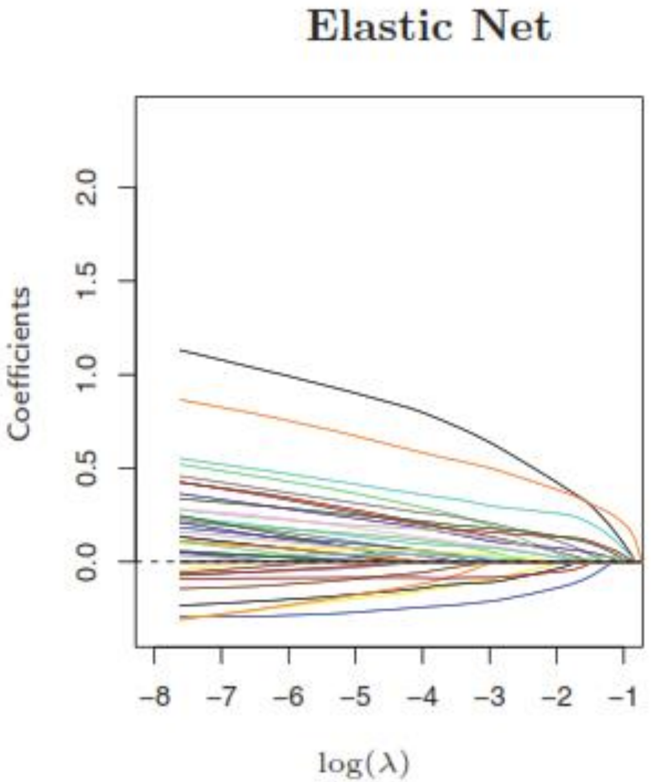
[Bishop]

Elastic net

$$\hat{\beta} = \arg\min_{\beta} (Y - X\beta)^T (Y - XB) + \lambda(\alpha\|\beta\|_1 + (1-\alpha)\|\beta\|_2^2)$$



19 non-zero coefficients



39 non-zero coefficients, but with smaller magnitudes

ESL, fig.18.5

References

- Pattern Recognition and Machine Learning by Christopher Bishop
- Learning from data by Abu-Mostafa, Y.S., Magdon-Ismail, M. and Lin, H.T
    - Slides 7-14 are from the lectures 8 and 12 of *Learning from data* course at Caltech
    - https://work.caltech.edu/lectures.html