Probabilistic Graphical Models

Problem Set 2

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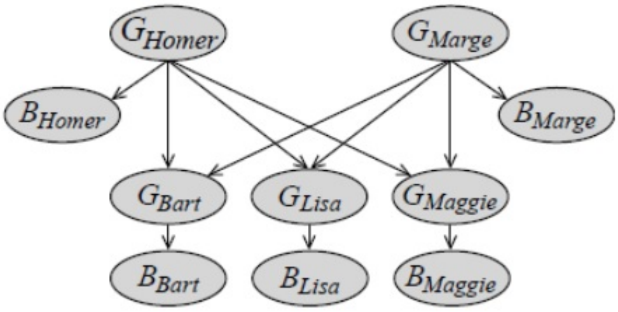
Problem 1: Reading Summary
Write a half-page summary of lecture 2 (max: 1 page). Accompany your report by an audio file (max: 10 minutes) in which you explain in your words important topics of the lecture, particularly:

- For the BN $I \rightarrow S \leftarrow D$, explain how you parameterize $P(I)$, $P(D)$ and $P(S|I,D)$.
- The naive Bayes model
- Factorization and local independencies
- By an example of a simple BN, explain how local independencies results in the BN factorization.
- I-Map, I-Map to factorization, factorization to I-Map
- Reasoning patterns: causal reasoning, evidential reasoning, intercausal reasoning
- Common cause, indirect causal effect, v-structure. Why v-structure is of particular importance?
- D-separation
- I-equivalence; sufficient conditions two BNs are I-equivalent.
- Markov blanket
- Did you learn anything new about representation, inference, learning?

Problem 2: Genetic inheritance
Consider the genetic inheritance Bayesian network G illustrated in the below figure. For simplicity, assume only alleles A and B are possible (hence only genotypes AA, AB, BB and blood types A, AB, B are possible). Additionally, assume genotypes AA, AB, BB are equally probable for G_{Homer} and G_{Marge}.



Factorization:

(a) Write down the joint distribution of the network G in a factorized way.

(b) Let G_p and B_p represent person p's genotype and blood type, respectively. Write down conditional probability table for $P(B_p|G_p)$.

(c) Let m, f be parents of person p. Write down conditional probability table for $P(G_p|G_m, G_f)$.

Independencies:

(d) Compute local independencies of $B_{Bart}, G_{Lisa}, G_{Marge}, B_{Homer}$

(e) Which of the following conditional independence statements are true? Explain why. (Important: just in this part do NOT assume $P(B_p|G_p)$ is deterministic and simply follow d-separation algorithm to answer the questions).

(i) $B_{Homer} \perp B_{Maggie} \mid G_{Marge}$
(ii) $G_{Homer} \perp G_{Marge}$
(iii) $G_{Homer} \perp G_{Marge} \mid B_{Lisa}$
(iv) $G_{Homer} \perp G_{Marge} \mid B_{Homer}, B_{Marge}$
(v) $B_{Bart} \perp G_{Lisa} \mid G_{Homer}, G_{Marge}$
(vi) $B_{Bart} \perp G_{Lisa} \mid B_{Homer}, B_{Marge}$

Inference:

(f) Compute the following probability queries by hand.

(i) $Pr(B_{Bart} = AA)$
(ii) $Pr(G_{Bart} = AA \mid G_{Homer} = AB)$
(iii) $Pr(G_{Bart} = AA \mid G_{Homer} = AB, G_{Lisa} = AB)$
(iv) $Pr(G_{Homer} = AB \mid G_{Marge} = AB)$
(v) $Pr(G_{Homer} = AB \mid G_{Marge} = AB, G_{Bart} = BB)$

(g) Use the *bnlearn* package to compute the above probabilities.

Problem 3: Independence properties of three-node Bayesian Networks

1. For the v-structure $X \to Z \leftarrow Y$, show in general $X \perp Y$ and $X \not\perp Y \mid Z$.
2. For the common cause BN $X \leftarrow Z \to Y$, show in general $X \not\perp Y$ and $X \perp Y \mid Z$.

Submit your solutions to naser.elmi@ut.ac.ir by Esfand 5, 1397.