University of TEHRAN

Probabilistic Graphical Models

Problem Set 6

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Problem 1: Reading Summary
Write a summary of the previous lecture. Accompany your report by an audio file (max: 10 minutes) in which you explain in your words important topics of the lecture, particularly:

- Limitations of MLE
- Joint probabilistic model over parameters and data
- Bayesian parameter estimation (slides 4-5)
- Prediction using ML framework; Bayesian prediction; Bayesian prediction for thumbtack example
- Conjugate prior; Beta distribution
- Posterior and Bayesian prediction for Beta prior
- Dirichlet prior; conjugate prior for multinomial distribution
- Bayesian prediction for Dirichlet prior

* Write down all formulas in your written summary and explain in detail each step of the derivation. In your audio file, only mention the main points of the derivations.

Problem 2: Conjugate prior for Poisson distribution
Poisson distribution is a commonly used distribution for modeling count data, as an example, it can represent the number of mutations in a given stretch of DNA after a particular cancer therapy. The Poisson distribution has a single parameter λ representing event rate. Suppose that we have M IID samples $x[1], \dots x[M]$ from $X \sim \textit{Poisson}(\lambda)$ where $P(x) = \frac{\lambda^x e^{\lambda}}{x!}$ is the probability mass function.

(a) Find the maximum likelihood estimate for λ.

(b) Assume the prior for λ is a Gamma distribution, i.e. $\lambda \sim \textit{Gamma}(\alpha, \beta)$ with $P(\lambda) = \frac{\beta^{\alpha}}{\Gamma(\alpha)} \lambda^{\alpha-1} e^{-\beta\lambda}$. Find the posterior of λ.

(c) Compute the Bayesian prediction over the next observation $X[M+1]$. Compare it with the MLE.

Problem 3: Bayesian analysis of WNT signalling pathway in coloretal cancer
JAGS (Just another Gibbs sampler) is a program for Bayesian analysis of hierarchical models. In this

exercise, you should perform Bayesian analysis of HW4, problem 3 parts b and c using the package rjags, an R interface the JAGS (deadline for this problem: Ordibehesht 2, 1398).

Submit your solutions to naser.elmi@ut.ac.ir by Farvardin 26, 1398.