Probabilistic Graphical Models

Problem Set 11

Hesam Montazeri
Fahimeh Palizban
Naser Elmi

Ordibehesht 25, 1398
(May 15, 2019)

Problem 1: Reading Summary

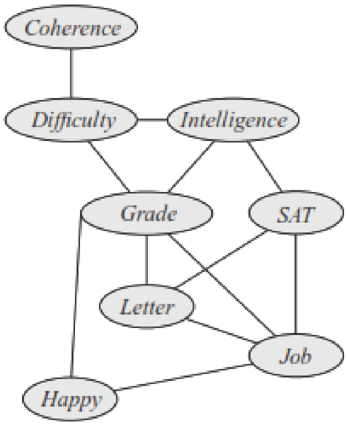
Write a summary of the previous lecture. Accompany your report by an audio file (max: 10 minutes) in which you explain in your words important topics of the lecture, particularly:

- Undirected graphical models versus Bayesian networks
- Factors
- Independencies in Markov networks; Markov blanket
- Explain why associating factors directly with edges is insufficient to parameterize an arbitrary distribution.
- Factor product; reduced Markov networks; factor marginalization
- Gibbs Distribution; Markov network factorization
- Motivate why factors are not equivalent to marginal probabilities of the variables.
- Pairwise Markov networks
- Variable elimination
- Read section A.3.3 of the textbook. Implement algorithms A.3 and A.4 and compare their performance.

* Write down all formulas in your written summary and explain in detail each step of the derivation. In your audio file, only mention the main points of the derivations.

Problem 2: Markov networks vs. Bayesian Networks

Consider the following Markov network:



(a) Which of the following independence statements hold? (1) $H \perp D \mid G$, (2) $C \perp J \mid G, I$.

(b) Write the Markov blanket of node G.

(c) Write down the factorized form of the joint distribution over all variables. Assume the model is parameterized by factors over each single node and each edge in the network.

Now consider the Bayesian network B_1: $A \rightarrow C \leftarrow B; C \rightarrow D$.

(d) Write the join distribution of the B_1 in a factorized form.

(e) Suppose the Markov network M_1 is defined as the skeleton of B_1. Write the joint distribution of the M_1 in a factorized form. Can you find a mapping between factors of M_1 and CPDs of B_1?

(f) Compare sets of independencies encoded by M_1 and B_1. Find a Markov network that is a minimal I-map for $I(B_1)$.

(g) Draw the reduced Markov network to the context $S = s^0$ and $L = l^1$.

Problem 3: Variable Elimination

In this exercise, you are required to implement different operators on factors and variable elimination algorithm. In particular

(a) **Factor product:** write a function that gets two factors and returns the product.

(b) **Factor reduction:** write a function that gets a factor ϕ and a context $U = u$ and returns the reduction of factor ϕ to the context $U = u$.

(c) **Factor marginalization:** write a function that gets a factor ϕ and the variable X. The function needs to sum out the variable X in ϕ and returns the reduced factor.

(d) **Variable elimination:** implement the sum-product variable elimination algorithm (algorithm 9.1). Compare the performance of your code with the *cpquery* function in *bnlearn* in an example of your choice (with at least five nodes).

(e) **Sum-Product-VE for computing conditional probabilities:** implement algorithm 9.2 of the textbook.

Apply the implemented functions to some simple examples and write the outputs in your report. Optionally, you may use *R markdown*, a powerful framework in R for report generation, for this part.

Submit your solutions to naser.elmi@ut.ac.ir and fahimehpalizban@ut.ac.ir by Ordibehesht 30, 1398. The deadline for 3-d and 3-e is on Khordad 6.