

Probabilistic Graphical Models

Problem Set 10

Hesam Montazeri
Fahimeh Palizban
Naser Elmi

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Problem 1: Reading Summary

Write a summary of the previous lecture. Accompany your report by an audio file (max: 10 minutes) in which you explain in your words important topics of the lecture, particularly:

- How hidden variables can simplify the structure of a Bayesian networks?
- Observation mechanism
- Expectation maximization; sufficient statistics (whole section)
- Structure learning for incomplete data
- Regulatory motif finding; motivate how the EM can be used for this problem.

* Write down all formulas in your written summary and explain in detail each step of the derivation. In your audio file, only mention the main points of the derivations.

Problem 2: Mixtures of Gaussians

Linear combination of Gaussians can give rise to complex multimodal distributions, useful for modeling many real data sets. The probability density function of a mixture of K one-dimensional Gaussian distributions with their own mean μ_k and variance σ_k^2 can be written as

$$p(x) = \sum_{k=1}^K \lambda_k \mathcal{N}(x | \mu_k, \sigma_k^2)$$

where parameters λ_k are called *mixing coefficients*.

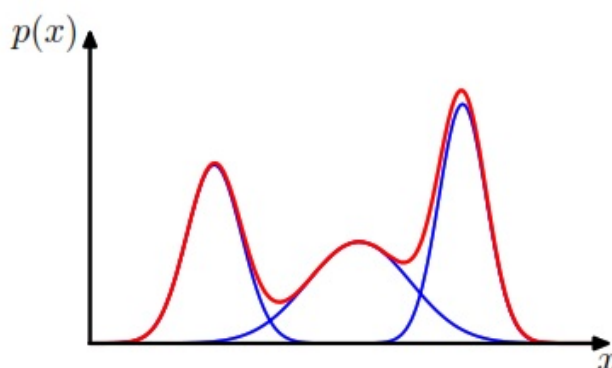


Figure 1: Mixture of three Gaussians in blue and their sum in red [Bishop, 2006]

Suppose a set of N i.i.d data points $x_i, i = 1, \dots, N$, without observing their corresponding components, are given. In addition, assume $K = 2$ for the sake of simplicity. In this exercise, you need to

- (a) give a graphical representation for modeling the Gaussian mixture model.
- (b) use the EM algorithm for parameter estimation. Derive the E and M steps (explain in detail all steps).
- (c) simulate a mixture of two Gaussians in R and implement the EM algorithm, obtained in the previous part, to perform parameter estimation. Plot the simulated data, true and learned components.

Submit your solutions to naser.elmi@ut.ac.ir and fahimehpalizban@ut.ac.ir by Ordibehesht 23, 1398.