University of TEHRAN

Probabilistic Graphical Models

Problem Set 3

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Esfand 6, 1397
(Feb 25, 2019)

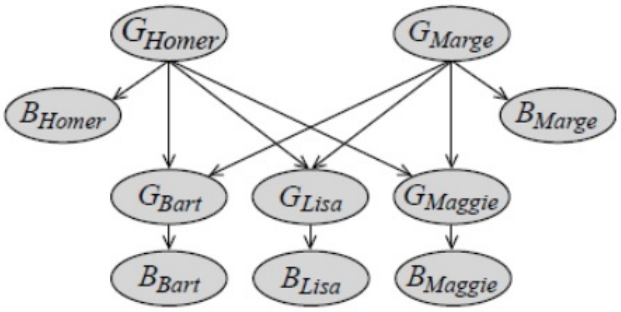
Problem 1: Reading Summary

Write a half-page summary of lecture 2 (max: 1 page). Accompany your report by an audio file (max: 10 minutes) in which you explain in your words important topics of the lecture, particularly:

- Common cause, indirect causal effect, v-structure. Why v-structure is of particular importance?
- D-separation
- I-equivalence; conditions for I-equivalency of two BNs.
- Markov blanket
- Main question of Section "From Distribution to Graphs"
- minimal I-map; its uniqueness property
- Perfect map

Problem 2: Genetic Inheritance-Independence Properties

Consider the genetic inheritance Bayesian network G illustrated in the below figure. For simplicity, assume only alleles A and B are possible (hence only genotypes AA, AB, and BB and blood types A, AB, and B are possible). Additionally, assume genotypes AA, AB, and BB are equally probable for G_{Homer} and G_{Marge}.



(a) Which of the following conditional independence statements are true? Explain why. (Important: in this part do NOT assume $P(B_p|G_p)$ is deterministic and simply follow d-separation algorithm to answer the questions).

(i) $B_{Homer} \perp B_{Maggie} \mid G_{Marge}$

(ii) $G_{Homer} \perp G_{Marge}$

(iii) $G_{Homer} \perp G_{Marge} \mid B_{Lisa}$

(iv) $G_{Homer} \perp G_{Marge} \mid B_{Homer}, B_{Marge}$

(v) $B_{Bart} \perp G_{Lisa} \mid G_{Homer}, G_{Marge}$

(vi) $B_{Bart} \perp G_{Lisa} \mid B_{Homer}, B_{Marge}$

(b) Use the *bnlearn* package to assess the above independence statements.

(c) Write the Markov blankets of G_{Lisa} and G_{Homer}.

(d) Find an I-equivalent BN for the above network.

Problem 3: Independence properties of three-node Bayesian Networks

1. For the v-structure $X \rightarrow Z \leftarrow Y$, show in general $X \perp Y$ and $X \not\perp Y \mid Z$.
2. For the common cause BN $X \leftarrow Z \rightarrow Y$, show in general $X \not\perp Y$ and $X \perp Y \mid Z$.

Problem 4: Minimal I-maps
Given $I = \{A \perp B,\ D \perp A, C \mid B\}$, find minimal I-maps for I for the following orderings:

(a) C, D, B, A

(b) A, B, C, D

(c) D, B, C, A

(d) Which one is a perfect map for I?

Submit your solutions to naser.elmi@ut.ac.ir by Esfand 14, 1397.