

---

# Probabilistic Graphical Models

## Problem Set 6

---

Hesam Montazeri  
Fahimeh Palizban  
Naser Elmi

Farvardin 21, 1398  
(April 10, 2019)

### Problem 1: Reading Summary

Write a summary of the previous lecture. Accompany your report by an audio file (max: 10 minutes) in which you explain in your words important topics of the lecture, particularly:

- Limitations of MLE
- Joint probabilistic model over parameters and data
- Bayesian parameter estimation (slides 4-5)
- Prediction using ML framework; Bayesian prediction; Bayesian prediction for thumbtack example
- Conjugate prior; Beta distribution
- Posterior and Bayesian prediction for Beta prior
- Dirichlet prior; conjugate prior for multinomial distribution
- Bayesian prediction for Dirichlet prior

\* Write down all formulas in your written summary and explain in detail each step of the derivation. In your audio file, only mention the main points of the derivations.

### Problem 2: Conjugate prior for Poisson distribution

Poisson distribution is a commonly used distribution for modeling count data, as an example, it can represent the number of mutations in a given stretch of DNA after a particular cancer therapy. The Poisson distribution has a single parameter  $\lambda$  representing event rate. Suppose that we have  $M$  IID samples  $x[1], \dots, x[M]$  from  $X \sim \text{Poisson}(\lambda)$  where  $P(x) = \frac{\lambda^x e^{-\lambda}}{x!}$  is the probability mass function.

- Find the maximum likelihood estimate for  $\lambda$ .
- Assume the prior for  $\lambda$  is a Gamma distribution, i.e.  $\lambda \sim \text{Gamma}(\alpha, \beta)$  with  $P(\lambda) = \frac{\beta^\alpha}{\Gamma(\alpha)} \lambda^{\alpha-1} e^{-\beta\lambda}$ . Find the posterior of  $\lambda$ .
- Compare the mean of posterior distribution with the MLE.

### Problem 3: Bayesian analysis of WNT signalling pathway in colorectal cancer

JAGS (Just another Gibbs sampler) is a program for Bayesian analysis of hierarchical models. In this

exercise, you should perform Bayesian analysis of HW4, problem 3 parts b and c using the package `rjags`, an R interface the JAGS (deadline for this problem: Ordibehesht 2, 1398).

Submit your solutions to [naser.elmi@ut.ac.ir](mailto:naser.elmi@ut.ac.ir) by Farvardin 26, 1398.