$X_1, X_2, X_3 \sim$ iid uniform in $[-1, 1]$

$X = (X_1, X_2, X_3)$

$$f(x) = f_0 + f_1(x_1) + f_2(x_2) + f_3(x_3)$$
$$+ f_{12}(x_1, x_2) + f_{13}(x_1, x_3) + f_{23}(x_2, x_3)$$
$$+ f_{123}(\underbrace{x_1, x_2, x_3}_{x}).$$

where:

$$f_0 = \int_{-1}^{1}\int_{-1}^{1}\int_{-1}^{1} f(x) \cdot \left(\frac{1}{2}\right)^3 dx_1 dx_2 dx_3 \equiv \text{const}$$

↓ mean of f.

$$f_1^{(x_1)} = \int_{-1}^{1}\int_{-1}^{1} f(x) \cdot \left(\frac{1}{2}\right)^2 dx_2 dx_3 - f_0$$

$$f_2^{(x_2)} = \int_{-1}^{1}\int_{-1}^{1} f(x) \cdot \left(\frac{1}{2}\right)^2 dx_1 dx_3 - f_0$$

$$f_3^{(x_3)} = \int_{-1}^{1}\int_{-1}^{1} f(x) \cdot \left(\frac{1}{2}\right)^2 dx_1 dx_2 - f_0$$

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$$f_{12}(x_1,x_2) = \int_{-1}^{1} f(x)\cdot\left(\frac{1}{2}\right)^1 dx_3 - f_1(x_1) - f_2(x_2) - f_0$$

$$f_{13}(x_1,x_3) = \int_{-1}^{1} f(x)\cdot\left(\frac{1}{2}\right)^1 dx_2 - f_1(x_1) - f_3(x_3) - f_0$$

$$f_{23}(x_2,x_3) = \int_{-1}^{1} f(x)\cdot\left(\frac{1}{2}\right)^1 dx_1 - f_2(x_2) - f_3(x_3) - f_0$$

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$$f_{123}(x_1,x_2,x_3) = f(x) - f_0 - f_1(x_1) - f_2(x_2) - f_3(x_3) - f_{12}(x_1,x_2) - f_{13}(x_1,x_3) - f_{23}(x_2,x_3)$$

one can show that:

$$\mathrm{Var}[f] = \int_{-1}^{1} f_1^2 \left(\frac{1}{2}\right)^1 dx_1 + \int_{-1}^{1} f_2^2 \left(\frac{1}{2}\right)^1 dx_2$$

$$+ \int_{-1}^{1} f_3^2 \left(\frac{1}{2}\right)^1 dx_3 + \int_{-1}^{1}\int_{-1}^{1} f_{12}^2 \left(\frac{1}{2}\right)^2 dx_1 dx_2$$

$$+ \int_{-1}^{1}\int_{-1}^{1} f_{13}^2 \left(\frac{1}{2}\right)^2 dx_1 dx_3$$

$$+ \int_{-1}^{1}\int_{-1}^{1} f_{23}^2 \left(\frac{1}{2}\right)^2 dx_2 dx_3 .$$

$$+ \int_{-1}^{1}\int_{-1}^{1}\int_{-1}^{1} f_{123}^2 \left(\frac{1}{2}\right)^3 dx_1 dx_2 dx_3 . \;//$$

orthogonality check:

$$\int_{-1}^{1} f_1(x_1) \cdot \left(\frac{1}{2}\right)^1 dx_1 \equiv 0$$

$$\int_{-1}^{1} f_2(x_2) \cdot \left(\frac{1}{2}\right)^1 dx_2 \equiv 0$$

$$\int_{-1}^{1} f_3(x_3) \cdot \left(\frac{1}{2}\right)^1 dx_3 \equiv 0$$

$$\int_{-1}^{1}\int_{-1}^{1} f_{12}(x_1,x_2) \cdot \left(\frac{1}{2}\right)^2 dx_1 dx_2 \equiv 0$$

$$\int_{-1}^{1}\int_{-1}^{1} f_{13}(x_1,x_3) \cdot \left(\frac{1}{2}\right)^2 dx_1 dx_3 \equiv 0$$

$$\int_{-1}^{1}\int_{-1}^{1} f_{23}(x_2,x_3) \cdot \left(\frac{1}{2}\right)^2 dx_2 dx_3 \equiv 0$$

$$\int_{-1}^{1}\int_{-1}^{1}\int_{-1}^{1} f_{123}(x_1,x_2,x_3) \left(\frac{1}{2}\right)^3 dx_1 dx_2 dx_3 \equiv 0$$