

Planning a Community Approach to Diabetes Care in Low- and Middle-Income Countries Using Optimization

(Authors' names blinded)

Diabetes is a global health priority, especially in lower-middle-income countries, where over 50% of premature deaths are attributed to high blood glucose. Several studies have demonstrated the feasibility of using Community Health Worker (CHW) programs to provide affordable and culturally tailored solutions for early detection and management of diabetes. We propose an optimization framework to determine personalized CHW visits that maximize glycemic control at a community-level. Our framework explicitly models the trade-off between screening new patients and providing management visits to individuals who are already enrolled in treatment. We account for patients' motivational states, which affect their decisions to enroll or drop out of treatment and, therefore, the effectiveness of the intervention. We incorporate these decisions by modeling patients as utility-maximizing agents within a bi-level provider problem that we solve using approximate dynamic programming. We apply our approach to generate CHW visit plans using real data from a social enterprise serving low-income neighborhoods in urban areas of India. Through extensive simulation experiments, we find that our framework requires up to 73.4% less capacity than the best naive policy to achieve the same performance in terms of health outcomes. Our experiments also show that our solution algorithm can improve upon baseline heuristics by up to 124.5% (in terms of relative performance) using the same CHW capacity.

Key words: Diabetes, Optimization, Dynamic programming, Global health

1. Introduction

The global burden of diabetes has grown rapidly in recent years. As of 2021, more than 460 million people are living with diabetes globally; a number that is projected to increase by 50% in the next 20 years (Murray et al. 2020). Diabetes disproportionately affects low- and middle-income countries (LMICs), which are home to more than 80% of affected individuals (Flood et al. 2021). In addition, it is estimated that almost 90% of individuals with undiagnosed diabetes reside in LMICs (International Diabetes Federation 2021). The lack of a diagnosis often leads to poor glycemic control and, therefore, to increased risks of microvascular and macrovascular complications and mortality (Laiteerapong et al. 2019).

Expanding access to diabetes screening and treatment in LMICs is needed to meet the United Nations Sustainable Development Goal 3.4, which seeks to reduce premature mortality from non-communicable diseases like diabetes by one third before 2030 (United Nations General Assembly 2015). Diabetes is a priority in urban areas, where its prevalence is highest (and growing the fastest) due to diet and lifestyle changes driven by rapid urbanization (Ranasinghe et al. 2021). To tackle this issue, we propose a novel optimization framework for coordinating community health

worker (CHW) visits in the context of a behavioral diabetes intervention for urban environments in LMICs. Our framework is composed of a model for how patients may enroll in the program and a dynamic programming approach for finding the optimal intervention (visit) policy.

Diabetes care is a challenging problem comprised of many different facets, including education and promotion of healthy lifestyle choices; detection, screening, and treatment of the disease; and the integration of patients with the primary healthcare system. Many of these measures require providing health services at the individual level, a challenging issue for healthcare systems in LMICs due to lack of capacity caused by health workforce shortages (Scheffler et al. 2018). The global health community has proposed task shifting, i.e., redistributing tasks from highly qualified professional health workers to workers with shorter training and fewer qualifications – such as CHWs – for managing chronic diseases like diabetes (WHO 2007). CHWs can be sought out from the communities that they will be serving and receive short training from the health system.

There is growing evidence that CHWs can reduce the burden of diabetes through early detection (e.g., screening) and case management (e.g., treatment initiation, education, etc.), especially for populations with low health literacy, limited access to health care, and poor social determinants of health (Jeet et al. 2017, Alaofè et al. 2017, Gyawali et al. 2021). Yet, there is a need for increased research to improve the design and implementation of CHW programs in a manner that integrates workers into existing health systems to meet population needs in the long run (WHO 2018).

Interventions for chronic care typically have high attrition rates that will lead to inefficiencies in planning and capacity allocation if unaccounted for by the provider (Verevkin et al. 2014, Gucciardi et al. 2008). Social stigma is a factor that may lead patients to drop out of treatment, especially when receiving CHW home visits. The use of stigmatizing phrases to describe people with diabetes has been reported in several countries and include terms such as “sick and disabled”, “contagious”, “self-inflicting”, among others (Abdoli et al. 2018) – a depiction of the stigma that can lead patients to refuse CHW home visits (Busza et al. 2018, Nyblade et al. 2017). An additional factor known to cause attrition is the cognitive burden of treatment, which requires learning how to manage their condition and implementing changes in health behavior (Heckman et al. 2015). By modeling patients as utility-maximizing agents that weigh beneficial and adverse treatment factors, providers can predict the impact of their visits plans on patient enrollment and reduce drop out rates.

Although the context of CHWs has been previously studied in the operations literature, prior research has focused on interventions with a single visit type (Cherkesly et al. 2019, Brunskill and Lesh 2010). Due to the high rates of undiagnosed diabetes in LMICs, CHW programs must manage an explicit trade-off between screening visits (i.e., getting new patients on-board) and management visits (i.e., managing ongoing patients). As a result, CHW programs must model patient health

outcomes and account for patient enrollment behavior to determine the optimal timing and type of visit to provide to each individual so that the programs most effectively use their limited resources.

In this paper, we develop an optimization framework for allocating CHW visits to patients in the context of behavioral diabetes interventions in LMICs. Through developing this framework, we provide three major contributions:

1. We extend existing models for chronic care by personalizing visits based on how patients interact with behavioral interventions delivered by CHWs (Section 4). Our modeling approach captures both the disease progression of patients and their individual enrollment decisions based on their physical health state and their motivational state. While this model has several unobserved values, we provide a machine learning method for estimating these quantities using data commonly collected by CHWs that administer these types of interventions.
2. In contrast to approaches that rely on indexability properties, we develop a dynamic programming approach that leverages the patient model to allocate CHW visits such that the number of patients that have their blood glucose levels in control is maximized (Section 5). Our model is designed for resource-constrained settings where the provider may need to trade-off managing patients that are currently enrolled and screening patients to encourage them to join the intervention. We fully characterize the optimal policy for this model in both the single patient case and in the multi-patient resource constrained case. Since the optimal policy in the multi-patient setting is computationally intractable, we provide an approximate dynamic programming approach that can be implemented in practice.
3. We conduct a comprehensive set of experiments that validate our modelling and optimization framework using real-world data obtained from a CHW program that operates in Hyderabad, India. In addition, we assess the generalizability of our approach using artificial patient cohorts (Section 6). Our experiments show that our solution algorithm can improve upon baseline heuristics by up to 124.5% (in terms of relative performance) using the same CHW capacity. Our approach also requires up to 73.4% less CHW capacity than the best baseline heuristic to achieve the same performance in terms of health outcomes.

2. Literature

Our work contributes to three major streams of literature: sequential resource allocation (Section 2.1), personalized healthcare (Section 2.2), and the growing literature on global health operations (Section 2.3).

2.1. Sequential decision-making for resource allocation

Sequential decision-making for resource allocation has been studied outside of health care for applications including retail operations (e.g., Acimovic and Graves 2015, Farias and Madan 2011), product development (e.g., Bhaskaran et al. 2021, Kornish 2001), operations scheduling/reservations (e.g., Keyvanshokoo et al. 2021, Stein et al. 2020), and machine maintenance (e.g., Glazebrook et al. 2006, Ruiz-Hernández et al. 2020). Within healthcare, authors have studied applications such as hepatitis C treatment in prisons (Ayer et al. 2019), HIV screening for veterans (Deo et al. 2013), planning community-based chronic disease care (Deo et al. 2015), hepatocellular carcinoma screening (Lee et al. 2019), breast cancer screening (Ayvaci et al. 2012), and adherence to tuberculosis treatment (Mate et al. 2020, 2021).

Although there are structural similarities between our problem and the machine maintenance problem (e.g., machines deteriorate (disease progression) and require maintenance (management) visits), machine maintenance does not account for individual behavior (i.e., enrollment decisions). Regarding healthcare applications, our work differs from prior literature in two ways. First, we include a behavioral model within our resource allocation problem that accounts for patient decisions using a utility-maximizing framework. This approach accommodates not only heterogeneous patients in terms of disease progression and treatment effectiveness, but also their agency when making treatment decisions. Second, we explicitly model the trade-off between revisiting patients enrolled in treatment and screening new patients who are not enrolled in treatment.

2.2. Personalized healthcare

Personalized healthcare has recently received a significant amount of attention in the operations literature. For example, prior research has applied Markov decision processes to optimize personalized chronic disease screening (e.g., Skandari and Shechter 2021, Hajjar and Alagoz 2022) and multi-armed bandit models for treating multiple myeloma, type 2 diabetes, multiple sclerosis, Warfarin dosing, and physical activity interventions (Negoescu et al. 2018, Keyvanshokoo et al. 2019, Wang et al. 2019, Bastani and Bayati 2020, Mintz et al. 2020). Although we also optimize a personalized chronic disease intervention by accounting for heterogeneous disease progression and treatment effects, our work differs from prior research in two key ways. First, our model solves a personalized health care problem under resource-limited settings, where a provider may need to trade-off offering care for different patients. Second, our approach considers patients' agency, which manifests through their decisions to participate in the intervention (and impacts their outcomes).

Regarding the second point, Aswani et al. (2019) characterized patients' responses in the context of a weight loss intervention. Similar to our approach, their framework used utility functions to

model patients' motivational states, allowing them to predict weight changes as a function of the step goals set for each individual and, therefore, set goals that optimized weight loss. Mintz et al. (2017) extended their behavioral analytics framework for online capacitated multi-agent systems where a single coordinator provides behavioral or financial incentives for each agent using a finite budget. The coordinator's objective depends on the states and decisions of all agents, whose utility functions are initially unknown. In contrast to their work, our setting involves more than one visit type (the equivalent of their incentive), which requires the provider to split their resources between screening and management visits. These visit decisions affect not only patients' motivational states, but also the patient pool size that must be managed by the provider.

2.3. Global health operations

Another related stream of literature applies optimization techniques to global health in resource-limited settings (e.g., Jónasson et al. 2017, Parvin et al. 2018, Boutilier and Chan 2020, Boutilier et al. 2021b). Of particular interest is the stream of literature that applies optimization techniques to improve CHW interventions in LMICs. In this context, most prior research has focused on developing routing models to optimize CHW visits (e.g., Brunskill and Lesh 2010, Cherkesly et al. 2019). Our approach does not include routing decisions because our study setting is urban slums with high population density where CHWs walk over short distances between visits. Additionally, our intervention is planned over medium- to long-term and involves tracking patients over time to optimize their health outcomes, which are explicitly modeled in our framework.

3. Study context

In this section, we provide additional details of our problem setting that are relevant to understand modeling assumptions made in subsequent sections.

3.1. Diabetes

Diabetes is a disease characterized by insufficient insulin production (decline in beta-cell function) or ineffective use of insulin (insulin resistance). These issues affect our ability to metabolize glucose for energy and lead to glucose accumulation in the bloodstream (Wysham and Shubrook 2020). Type 2 diabetes comprises 95% of all diabetes cases and is diagnosed by measuring the blood glucose level with one of several metrics, including random blood glucose, oral glucose tolerance test, post-prandial blood glucose, fasting blood glucose (FBG), and hemoglobin A1c (HbA1c). We use the FBG as our primary metric because it provides a suitable trade-off between diagnostic accuracy and practical feasibility, and is the metric most commonly used in LMICs (Hoyer et al. 2018, Zhao et al. 2013). Measuring the FBG requires a fast (i.e., only water consumption) for 8 to 12 hours, which is typically done overnight.

3.2. Diabetes in India

India is home to 1 in every 7 people living with diabetes globally. It has the second highest burden of diabetes in the world, with an estimated 74.2 million people; a number that is projected to increase by 68% by 2045. The prevalence of diabetes is higher in urban than in rural areas (Ranasinghe et al. 2021) and research estimates that over 53% of individuals living with diabetes are undiagnosed (International Diabetes Federation 2021). While it is estimated that the mean diabetes-related expenditure per person in India is US \$92, the economic burden when accounting for indirect costs such as productivity loss is likely much greater; India alone accounts for 18% of all Disability-Adjusted Life Years attributed to diabetes globally (GBD Results Tool 2019).

3.3. Diabetes care provider – NanoHealth

We collaborated with and obtained data from NanoHealth, a social enterprise and former Hult Prize winner based in Hyderabad, India. Hyderabad is the capital of the state of Telangana and the fourth largest city in India with a population of 7 million (Ministry of Home Affairs 2011), including more than 1.7 million people living in 1400 urban slums (The Times of India 2012, Directorate of Economics and Statistics 2015). The population comprises individuals working as drivers, daily wage earners, domestic helpers, vendors, and self-employed professionals in the unorganized sector of the economy. The average family income of these residents ranges between 15,000 and 30,000 Indian Rupees (INR) per month, which is equivalent to US \$200-US \$400 (US \$1=INR 75), while the median income in Hyderabad is INR 25,000 per month.

NanoHealth operates a diabetes screening and management program in low-income households in urban slums. They employ a team of CHWs, referred to as *Saathis* (Hindi for companion), who are members of the communities they serve and have received basic training on diabetes care. CHWs are assigned densely populated catchment areas with a population of approximately 5,000, which can be covered easily on foot to conduct both screening and management visits. When the data used in this paper were collected (2015-2018), their catchment areas included 52 urban slums and their surrounding communities. On an average day, NanoHealth had 30 active CHWs, who each conducted an average of 144 visits per month (≈ 5 per day). Each CHW was equipped with a “Doc-in-the-Bag” kit that included a weighing scale, measuring tape, blood glucose monitor, and blood pressure/heart rate cuffs. A mobile tablet was used to record patients’ responses to a questionnaire about lifestyle, demographics, symptoms of common ailments, and to record certain anthropomorphic measurements and vitals (Boutilier et al. 2021a). The fact that data entry into mobile devices is already a common practice in CHW programs allows for easier implementation of our proposed solution algorithms.

Our dataset includes longitudinal data (including FBG levels) about a patient cohort treated by NanoHealth, which includes 378 patients that received at least six visits through NanoHealth's intervention in Hyderabad, India.

4. Patient model

In this section, we describe the model of prospective and current patients' decisions to enroll or remain in the CHW-based intervention. We assume that each patient makes these decisions based on the perceived utility (reduction in FBG) and disutility (perceived adverse factors, e.g., social stigma and cognitive burden) of staying enrolled in the intervention. Treatment personalization recognizes patients as individuals with unique health histories and allows providers to achieve improved health outcomes. We leverage our longitudinal data which includes patients' health and visit histories when constructing our patient model. We provide further details as we outline the framework below.

Let \mathcal{P} be the set of patients that can be enrolled or are already enrolled in the CHW intervention and let $i \in \mathcal{P}$ denote the index of a particular patient. Let the time index be given by $t \in \mathcal{T} = \{0, \dots, N\}$. We let $z_{i,t}$ indicate the enrollment status of patient i at time t , where $z_{i,t} = 1$ indicates that the patient is enrolled in the intervention. Screening visits are only conducted for patients where $z_{i,t} = 0$; while management visits are only conducted for patients where $z_{i,t} = 1$. At each period t , the provider's decisions – whether or not to visit a patient – are denoted by the indicator variables $y_{i,t}$, which equal 1 if the provider chooses to send a CHW to visit patient i at time t and 0 otherwise. This variable represents a screening visit for an unenrolled patient and a management visit for an enrolled patient. We assume that the states and actions occur in the following order. The decision to visit a patient in period t , $y_{i,t}$, precedes the patient's enrollment decision in that same period, $z_{i,t}$, implying that the provider can affect this decision and lead the patient to make a different enrollment decision. Updates to patient states occur in the following period and can be impacted by both $y_{i,t}$ and $z_{i,t}$.

We model each patient's disease progression using the FBG. To capture the fact that disease progression is proportional to the current FBG level (Derendorf and Meibohm 1999, Lee et al. 2018), we use the logarithm of patient i 's FBG (denoted as $b_{i,t}$). We assume that $b_{i,t}$ evolves according to a set of linear dynamics of the form:

$$b_{i,t+1} = b_{i,t} + p_i - \mu_i z_{i,t} - \alpha_i y_{i,t} z_{i,t}, \quad (1)$$

where p_i represents the disease progression in one time period, μ_i represents the impact of enrollment, and α_i represents the impact of a management visit (on FBG). To see how to interpret the dynamics equation, let $\tilde{b}_{i,t} = \exp(b_{i,t})$ be the true FBG. Then, we can write the above dynamics as

$\tilde{b}_{i,t+1} = \tilde{b}_{i,t} \exp(p_i - \mu_i z_{i,t} - \alpha_i y_{i,t} z_{i,t})$. The dynamics imply that a patient's FBG increases unless they are enrolled in the program and/or receive a management visit (while they are enrolled). We assume that $p_i < \mu_i + \alpha_i$ to rule out the trivial case of an ineffective intervention, which does not reduce patients' FBG.

Next, we consider the disutility experienced by enrolled patients due to either social stigma of enrollment or the cognitive burden of participation. Let $s_{i,t}$ represent the cumulative disutility level experienced by patient i by period t , and let $s_{i,0}$ represent the baseline level of disutility that a patient will experience from participating in the intervention. We assume that this disutility level evolves according to the following linear dynamic equation:

$$s_{i,t+1} = z_{i,t}(\gamma(s_{i,t} - s_{i,0}) + \beta_i y_{i,t} z_{i,t}), \quad (2)$$

where $\gamma \in (0,1)$ is a discount factor on the previous period's stigma, and $\beta_i > 0$ is the impact on disutility from a management visit. Intuitively, this equation states that the disutility of the patient will increase when they are visited by a CHW, but the disutility impact of a visit will decay exponentially over time. We use this structure because it is reflective of how being visited by a CHW may increase social stigma as perceived by the community, but as time progresses, the memory (and thus the associated stigma) of the visit will fade (Ritchie et al. 2015).

An important aspect of diabetes management is educating patients on disease progression and healthy lifestyle choices (Nazar et al. 2016). As such, we assume that a patient's perception of the adverse effects of enrollment depend on the number of management visits received since enrolling in the intervention. To model this, we introduce the state $\theta_{i,t}$ to represent the patient's perceived importance of disutility and assume that $\theta_{i,t}$ evolves according to the following linear dynamic equation:

$$\theta_{i,t+1} = \rho(\theta_{i,t} - \theta_{i,0}) + \theta_{i,0} - \lambda_i y_{i,t} z_{i,t}. \quad (3)$$

We let $\theta_{i,0}$ denote the steady-state level of disutility perception for patient i , λ_i denote the decrease in the perceived importance of disutility that occurs each time a management visit is received, and $\rho \in (0,1)$ be a discount factor on the previous period's disutility perception. Even though the actual disutility, $s_{i,t}$, may increase when a management visit is received, we assume that the patient's understanding of the importance of treatment and the reassurance provided by the CHWs will decrease the relevance of this factor each time a management visit is received.

4.1. Patient Utility Function and Enrollment Decision

Patients are assumed to have perfect knowledge of their FBG level ($b_{i,t}$), disutility level of enrollment ($s_{i,t}$), and perceived importance of disutility ($\theta_{i,t}$) when making enrollment decisions – an

assumption that serves as an analytical tool to allow us to predict future patient behavior. We note that this assumption is not an attempt to model the true cognitive decision process of a patient, but instead to set up a framework where patients weigh the pros and cons of enrollment based on the relative importance given to a potential FBG improvement (and the corresponding improvement in quality of life) at the cost of increased social stigma and cognitive burden. Patients are also assumed to make decisions based on a myopic utility maximizing framework. While some social scientists may consider patient enrollment behavior “irrational” (Brock and Wartman 1990), others believe that these decisions are rational with respect to heavily discounted future outcomes; an effect that has been previously observed in healthcare decision-making (Cawley 2004). Moreover, previous work (Aswani et al. 2019, Mintz et al. 2017) has shown that despite making these assumptions, resulting models still have strong predictive and prescriptive performance. Therefore, it is reasonable to assume that when patients are deciding whether to enroll/stay enrolled in period t , they only consider their benefit of enrollment in the immediate next period, $t + 1$.

Let $\mathbb{B} := \{0, 1\}$. We denote the utility function of patient i by $U_i : \mathbb{R}^3 \times \mathbb{B} \mapsto \mathbb{R}$. We assume that each patient’s utility function has the structure $U_i = U_{i,b} + U_{i,s}$, where $U_{i,b}$ is the net utility associated with the state $b_{i,t}$ and $U_{i,s}$ is the net utility associated with the state $s_{i,t}$. Specifically, we assume that these functions are of the form:

$$U_{i,b}(b_{i,t}, z_{i,t}) = -b_{i,t+1}(b_{i,t}, z_{i,t}; y_{i,t}), \quad (4)$$

$$U_{i,s}(\theta_{i,t}, s_{i,t+1}, z_{i,t}) = -\theta_{i,t}s_{i,t+1}(s_{i,t}, z_{i,t}; y_{i,t}). \quad (5)$$

Note that $U_{i,b}$ and $U_{i,s}$ are functions of the future states of the patient because the patient’s decisions (and the intervention) can only impact their future states. Intuitively, $U_{i,b}$ implies that patients want to reduce $b_{i,t}$ in the future, while $U_{i,s}$ implies that patients want to reduce the cumulative disutility from future enrollment, scaled by their perception of the disutility. Thus, patients choose whether to enroll (or remain enrolled) by balancing these two parts of the utility function.

Then, the patient’s enrollment decision at time t is given by:

$$z_{i,t} \in (z_{i,t-1} + y_{i,t} - z_{i,t-1}y_{i,t}) \cdot \arg \max_{z \in \mathbb{B}} U_i(b_{i,t+1}, s_{i,t+1}, \theta_{i,t}, z). \quad (6)$$

Intuitively, this approach implies that a patient’s binary enrollment $z_{i,t}$ decision must first consider the decision $z \in \mathbb{B}$ that maximizes their overall utility, where U_i is a function of the utility associated with their future health states ($b_{i,t+1}$), adverse factors ($s_{i,t+1}$), and perception of adverse factors ($\theta_{i,t}$). The inclusion of the factor $(z_{i,t-1} + y_{i,t} - z_{i,t-1}y_{i,t})$ accounts for the situation where a patient’s prior enrollment status may restrict their present enrollment decision (when the factor is zero). Specifically, through the use of a boolean “OR” operator on $z_{i,t-1}$ and $y_{i,t}$, we model the

fact that a patient can only choose to be enrolled in the current period if they were enrolled in the previous period *or* if they were not enrolled in the previous period but received a screening visit.

To characterize patients' decisions we introduce the variable $B_{i,t} := U_i(b_{i,t+1}, s_{i,t+1}, \theta_{i,t}, 1) - U_i(b_{i,t+1}, s_{i,t+1}, \theta_{i,t}, 0)$ representing the net benefit of enrollment to patient i . This leads to the following characterization of the patients' enrollment decisions:

PROPOSITION 1. *Given the patient utility model presented by equations (4),(5),(6), and the patient state dynamic model described in Section 4, at time t , patient enrollment decisions are given by:*

$$z_{i,t} = (z_{i,t-1} + y_{i,t} - z_{i,t-1}y_{i,t}) \cdot \mathbb{1}[B_{i,t} \geq 0], \quad (7)$$

where $\mathbb{1} : \mathbb{R} \mapsto \mathbb{B}$ is an indicator function. Moreover, the benefit $B_{i,t}$ has the following closed form:

$$B_{i,t} = \mu_i - \theta_{i,t}(\gamma(s_{i,t} - s_{i,0}) + s_{i,0}) + (\alpha_i - \theta_{i,t}\beta_i)y_{i,t}. \quad (8)$$

The complete proof of this proposition is found in Appendix EC.1, but we provide a brief sketch here. We can interpret the benefit of enrollment as the sum of the fixed effect of enrollment ($\mu_i - \theta_{i,t}(\gamma(s_{i,t} - s_{i,0}) + s_{i,0})$) and the variable/visit effect ($(\alpha_i - \theta_{i,t}\beta_i)y_{i,t}$). The benefit function shown can be obtained by adding the net utility of enrollment with respect to the FBG state, $b_{i,t+1}$, and adverse factors, $s_{i,t+1}$. The equations for $b_{i,t+1}$ and $s_{i,t+1}$ are then used to obtain a benefit function in terms of the present states, $b_{i,t}$ and $s_{i,t}$. Note that patient enrollment requires not only the utility of enrollment to be equal to or exceed the utility of non-enrollment ($B_{i,t} \geq 0$), but also enrollment in the previous period ($z_{i,t-1} = 1$) or a screening visit in the present period ($y_{i,t} = 1$ if $z_{i,t-1} = 0$).

REMARK 1. According to the assumptions of the model, when $B_{i,t} = 0$ the patient will be ambivalent between enrolling and not enrolling in the intervention. Without loss of generality, we assume that the tie is broken in favor of enrolling in the intervention.

The characterization of patient enrollment decisions in Proposition 1 will later allow us to define when visiting a patient is beneficial for their enrollment and how patient enrollment relates to the optimal visit policy from the provider's perspective.

4.2. Parameter estimation

In this section, we present a model that enables the provider to predict patient enrollment decisions for a proposed visit plan. In practice, the majority of the parameters described in the previous sections cannot be directly observed by the provider. At each time period, the provider observes only the enrollment status of the patient ($z_{i,t}$), a record of whether or not the patient was visited ($y_{i,t}$), and a noisy signal of the patient's FBG. Let $\bar{b}_{i,t}$ be the noisy observation of the log FBG

for patient i in period t . We assume that the relationship between $\bar{b}_{i,t}$ and $b_{i,t}$ is given by: $\bar{b}_{i,t} = b_{i,t} + \epsilon_{i,t}$, where $\epsilon_{i,t}$ are i.i.d. random variables representing the noise in the observations such that $\mathbb{E}[\epsilon_{i,t}] = 0$ and $\mathbb{E}[\epsilon_{i,t}^2] < \infty$, with density function $f_\epsilon: \mathbb{R} \mapsto \mathbb{R}_+$. Note that this is a standard modeling assumption that can be satisfied by various families of distributions including Normal and Laplace distributions (Billingsley 1961). Since each patient's parameters are estimated individually using their longitudinal visit history, we drop the patient index i for the remainder of this section.

Let $\mathcal{K} \subset \mathcal{T}$ be the set of time periods for which there is an observation of the log-FBG for a particular patient. Then, we can use a maximum likelihood estimation (MLE) approach to jointly estimate all unknown model parameters, which include FBG progression (p), visit effect on FBG (α), enrollment effect on FBG (μ), visit effect on disutility (β), visit effect on perception of disutility (λ), two discount factors (γ and ρ). This approach also allows us to obtain FBG estimates for missing periods $t' \in \mathcal{T} \setminus \mathcal{K}$ through the use of our state dynamics equations (1), (2), (3), and (7).

Next, we show that the MLE problem can be formulated as a mixed integer linear program.

PROPOSITION 2. *The MLE problem (EC.1) can be formulated as the following constrained optimization problem:*

$$\underset{\mathbf{b}, \mathbf{s}, \boldsymbol{\theta}, \beta, \alpha, \mu, p, \lambda, \rho, \gamma}{\text{minimize}} \quad \sum_{t \in \mathcal{K}} \log f_\epsilon(\bar{b}_t - b_t) \quad (9a)$$

$$\text{subject to} \quad b_{t+1} = b_t + p - \mu z_t - \alpha y_t z_t, \quad \forall t \in \mathcal{T}, \quad (9b)$$

$$s_{t+1} = \gamma z_t (s_t - s_0) + z_t s_0 + \beta y_t z_t, \quad \forall t \in \mathcal{T}, \quad (9c)$$

$$\theta_{t+1} = \rho(\theta_t - \theta_0) + \theta_0 - \lambda y_t z_t, \quad \forall t \in \mathcal{T}, \quad (9d)$$

$$B_t \geq -M(1 - z_t), \quad \forall t \in \mathcal{T}, \quad (9e)$$

$$B_t \leq M z_t, \quad \forall t \in \mathcal{T}, \quad (9f)$$

$$b_t, s_t, \theta_t \geq 0, \quad \forall t \in \mathcal{T}, \quad (9g)$$

$$\beta, \alpha, \mu, p, \lambda, \rho, \gamma \geq 0, \quad (9h)$$

where $B_t = \mu - \theta_t(\gamma(s_t - s_0) + s_0) + (\alpha - \theta_t \beta)y_t$.

The complete proof is found in Appendix EC.1. The proof begins decomposing the likelihood function in (EC.1) and subsequently applying the log to both sides, allowing the log of the products to be separated as a sum of logarithms. The term including \bar{b}_t is then rewritten using the definition of a noise model and the remaining terms correspond to deterministic system dynamics.

Note that this formulation is nonlinear due to the bi-linear terms in Constraints (9e) and (9f), and cannot be solved directly by commercial solvers. While standard techniques such as McCormick relaxation can be used to remove these terms, we found that using a coarse grid search to perform parameter estimation gave better results. See EC.2 for details on our coarse grid search procedure.

5. CHW Provider Problem

In this section, we describe the dynamic programming framework used to optimize the decisions of the CHW provider (e.g., NanoHealth), while accounting for individual patient enrollment decisions. The goal of the provider is to maximize glycemic control (i.e., the number of individuals whose FBG is less than a given threshold) for the entire community under consideration, i.e., the union of all CHW catchment areas. Throughout this section, we assume that the provider has full knowledge of the patient parameters described in Section 4 for all patients in the targeted community, even for those who have not been screened yet. In practice, pilot trials allow planners to obtain data that can be used as initial estimates for unscreened patients (e.g., Deo and Singh 2018).

In accordance with prevailing clinical guidelines, we define glycemic control through the use of a threshold due to a nonlinear association between FBG and the risk of complications (e.g., coronary heart disease and ischaemic stroke), with substantially higher risks for FBG levels above 125 mg/dL (WHO 2016, Emerging Risk Factors Collaboration 2010). Microvascular complications include end-stage renal disease, retinopathy, and lower-extremity amputation, and macrovascular ones include heart disease/failure, and vascular disease (Laiteerapong et al. 2019).

5.1. Single Patient Problem

To build intuition, we first consider a single-patient problem (Section 5.2 considers the multi-patient problem) and for brevity, we drop the patient index i . Using the dynamics described in Section 4, the state of a patient in period $t \in \mathcal{T}$ is given by $x_t = (b_t, s_t, \theta_t, z_{t-1})$ and the provider's action is given by y_t , which represents the decision of whether or not to visit the patient in period t . The goal of the provider is to maximize the number of periods where the patient's FBG is in control, i.e., less than some threshold representing the maximum tolerable FBG, δ . Hence, we assume that the reward function is given by: $g_t(x_t) = g_t(b_t, s_t, \theta_t, z_{t-1}) = \mathbb{1}(b_t \leq \delta), \forall t \in \mathcal{T}$. The total reward for a planning horizon with N periods is then $\sum_{t=0}^N \mathbb{1}(b_t \leq \delta)$.

Using this reward function and the patient dynamics, the optimization model to determine the optimal timing of CHW visits can be written as:

$$\begin{aligned} & \underset{\mathbf{z}, \mathbf{y}, \mathbf{b}, \mathbf{s}, \boldsymbol{\theta}}{\text{maximize}} && \sum_{t \in \mathcal{T}} \mathbb{1}(b_t \leq \delta) \end{aligned} \tag{10a}$$

$$\text{subject to} \quad (9b) - (9g), \tag{10b}$$

$$z_t = (z_{t-1} + y_t - z_{t-1}y_t) \cdot \mathbb{1}(B_t(y_t) \geq 0), \quad \forall t \in \mathcal{T}, \tag{10c}$$

$$y_t, z_t \in \{0, 1\}, \quad \forall t \in \mathcal{T}, \tag{10d}$$

where $B_t = \mu - \theta_t(\gamma(s_t - s_0) + s_0) + (\alpha - \theta_t\beta)y_t$. The objective function maximizes the total number of periods where the patient's FBG is *in control* (i.e., less than or equal to δ).

Since constraints (9e) and (9f) include bilinear terms between two continuous variables (θ_t and s_t), we leverage dynamic programming rather than standard mixed integer linear programming techniques. Let $V_t(b_t, s_t, \theta_t, z_{t-1})$ for $t \in \mathcal{T}$ represent the optimal value-to-go function, i.e., the optimal number of periods the patient will be in control from period t to period N . Then the dynamic programming equations for our problem can be written as:

$$\begin{aligned} V_t(b_t, s_t, \theta_t, z_{t-1}) = & \mathbb{1}(b_t \leq \delta) + \\ & \max_{y_t \in \{0,1\}} \left\{ V_{t+1}(b_t + p - \mu z_t(y_t) - \alpha y_t z_t(y_t), z_t(y_t)(\gamma(s_t - s_0) + s_0 + \beta y_t), \right. \\ & \left. \rho(\theta_t - \theta_0) + \theta_0 - \lambda y_t z_t(y_t), z_t(y_t)) \right\}, \forall t \in \mathcal{T} = \{0, \dots, N-1\}, \\ V_N(b_N, s_N, \theta_N, z_{N-1}) = & \mathbb{1}(b_N \leq \delta). \end{aligned} \quad (11)$$

5.1.1. Structural results. To obtain structural results for our problem, we first establish that the value function $V_t(b_t, s_t, \theta_t, z_{t-1})$ is nondecreasing in $z_{t-1}, \forall t \in \mathcal{T}$, which is equivalent to $V_t(b_t, s_t, \theta_t, 0) \leq V_t(b_t, s_t, \theta_t, 1), \forall t \in \mathcal{T}$. This result is non-intuitive because it implies that the provider should focus on keeping patients enrolled to achieve better glycemic control, even if it means that some patients should receive less visits to remain enrolled – a situation that may occur when a patient has a large increase in social stigma/cognitive burden for each visit. Before we present the theorem and corresponding proof of this result, we first establish some lemmas.

LEMMA 1. $z_t(z_{t-1}, y_t) = (z_{t-1} + y_t - z_{t-1}y_t) \cdot \mathbb{1}(B_t(y_t) \geq 0)$ is nondecreasing in $z_{t-1}, \forall t \in \mathcal{T}$.

The result is obtained by evaluating the binary variable z_t for the two possible enrollment states z_{t-1} . See Appendix EC.1 for the complete proof. Lemma 1 demonstrates that if a patient is enrolled in the intervention in period $t-1$ then it is more likely they will stay enrolled in future periods irrespective of the CHW visit. Next we consider the structure of the terminal value function.

LEMMA 2. $V_N(b_N, s_N, \theta_N, z_{N-1}) = \mathbb{1}(b_N \leq \delta)$ is nondecreasing in z_{N-1} .

The proof consists of evaluating the threshold function V_N for different values of z_{N-1} and relies on the fact that, in period N , the value function only depends on the FBG state. The complete proof can be found in Appendix EC.1. Lemma 2 demonstrates that if a patient is enrolled in the penultimate time period they are more likely to have their FBG in control than if they are not. With these lemmas, we can establish our primary structural result with the following theorem.

THEOREM 1. Given the dynamic programming equations in (11), $V_t(b_t, s_t, \theta_t, z_{t-1})$ is nondecreasing in $z_{t-1}, \forall t \in \mathcal{T}$. Since $z_{t-1} \in \{0, 1\}$, this is equivalent to $V_t(b_t, s_t, \theta_t, 0) \leq V_t(b_t, s_t, \theta_t, 1), \forall t \in \mathcal{T}$.

The proof is completed by induction and relies on Lemma 1 and Lemma 2. The base case follows from the fact that $V_N(z_{N-2})$ is a composition of nondecreasing functions ($V_N(z_{N-1})$ and $z_{N-1}(z_{N-2})$), and therefore also nondecreasing. The induction step follows a similar process, in addition to evaluating the maximum of monotonically nondecreasing functions. Please see Appendix EC.1 for the complete proof.

5.1.2. Optimal visit policy. We now prove structural results for the optimal policy of the CHW provider. Recall that patient enrollment decisions are given by $z_{i,t} = (z_{i,t-1} + y_{i,t} - z_{i,t-1}y_{i,t}) \cdot \mathbb{1}[B_{i,t} \geq 0]$ and that Theorem 1 implies that a patient will never be worse off if they are enrolled in the CHW intervention.

THEOREM 2. *Given the patient model in Section 4, a policy that solves the dynamic programming equations in (11) must satisfy the following necessary and sufficient conditions:*

- (i) *Visit the patient (i.e., $y_t = 1$) when $z_{t-1} = 1$, $B_t(0) < 0$, and $B_t(1) \geq 0$ or when $z_{t-1} = 0$ and $B_t(1) \geq 0$.*
- (ii) *Never visit the patient (i.e., $y_t = 0$) when $z_{t-1} = 1$, $B_t(0) \geq 0$, and $B_t(1) < 0$.*
- (iii) *Indifferent between visiting and not visiting when $z_{t-1} = 1$, $B_t(0) < 0$, and $B_t(1) < 0$ or when $B_t(0) \geq 0$ and $B_t(1) \geq 0$ or when $z_{t-1} = 0$ and $B_t(1) < 0$.*

REMARK 2. The conditions in Theorem 2 are exhaustive of all possible visit and enrollment states.

The proof of the theorem proceeds by showing that the value function is strictly increasing in y_{t-1} for case (i), strictly decreasing for case (ii), and constant for case (iii). Using the result from Theorem 1, which relates the value function (V_t) to the enrollment status in the previous period (z_{t-1}), the only remaining task is to evaluate the relationship between z_t and y_t . The complete proof can be found in Appendix EC.1.

Theorem 2 establishes the conditions for an optimal policy. Specifically, it determines that a patient should only be visited if the visit will lead to an enrollment or prevent a patient from dropping out of treatment; that a patient should never be visited if the visit will lead to them to drop out of treatment; and that a provider is indifferent if the visit cannot prevent a patient from dropping out, if a patient will remain enrolled regardless of whether a visit is received, or if an unenrolled patient will not enroll in treatment if visited.

Note that because patient decisions are outcomes of thresholding functions, and similarly, the CHW provider value function is a composition of thresholding functions, there may be multiple optimal policies that solve the dynamic programming equations. For example, if all patients' FBG levels are always in control or always out of control for any number of CHW visits received, it would be optimal to visit everyone, visit no one, or any other solution in-between. To choose among

these optimal policies, we look ahead to the multi-patient setting where CHW resources are likely to be constrained. Hence, an advantageous characteristic of an optimal policy is to visit a patient as little as possible, while ensuring they remain enrolled in the program and maintain at least the same level of health effects as when they first enrolled. Thus, we show that the following policy is optimal:

COROLLARY 1. *An optimal policy that solves the CHW provider problem described in Section 5 is given by:*

$$y_t(x_t) = \begin{cases} 1, & \text{if } B_t(y_t = 1) \geq 0 \text{ and } (B_t(y_t = 0) < 0 \\ & \text{or } B_t(y_t = 0) \geq 0 \text{ and } (z_{t-1} = 0 \\ & \text{or } z_{t-1} = 1 \text{ and } B_t(y_t = 1) - B_t(y_t = 0) > 0)) \\ 0, & \text{otherwise.} \end{cases}$$

Clearly, the policy outlined in Corollary 1 meets the conditions in Theorem 2. Intuitively, this policy only assigns a CHW to visit a patient if the visit is necessary for enrollment or to avoid drop out, or if the visit provides a strict improvement in the patient's utility. Figure 1 provides a graphical representation of the policy structure. Overall, the policy visits patients in three situations:

1. $B_t(y_t = 1) \geq 0$ and $B_t(y_t = 0) < 0$. If this patient is not enrolled, the visit results in enrollment, but if this patient is already enrolled, then the visit is required to maintain their enrollment.
2. $B_t(y_t = 1) \geq 0$ and $B_t(y_t = 0) \geq 0$ and $z_{t-1} = 0$. The visit will result in an enrollment.
3. $B_t(y_t = 1) \geq 0$ and $B_t(y_t = 0) \geq 0$ and $z_{t-1} = 1$ and $B_t(y_t = 1) - B_t(y_t = 0) > 0$. The visit is not required to maintain enrollment, but results in a strict improvement in the patient's utility. Visiting a patient in this case can be interpreted as saving up capacity in the future by delaying the need to visit.

5.2. Multi-patient problem

In this section, we extend our results to the multi-patient problem faced by CHW organizations like NanoHealth. CHW organizations are typically resource-limited, especially those operating in global health contexts such as India. As a result, we limit the maximum number of patients that can be visited in each period; in reality, this amount can be derived from the number of CHWs and their total daily working hours. Travel times are minor in the densely populated urban slums where the patients targeted by the intervention reside. Let C represent the number of available CHW visits per period. Recall that \mathcal{P} represents the set of patients and note that we now add back patient indices to our variables. The multi-patient provider problem can be written as:

$$\underset{\mathbf{z}, \mathbf{y}, \mathbf{b}, \mathbf{s}, \boldsymbol{\theta}}{\text{maximize}} \quad \sum_{i \in \mathcal{P}} \sum_{t \in \mathcal{T}} \mathbb{1}(b_{i,t} \leq \delta) \quad (12a)$$

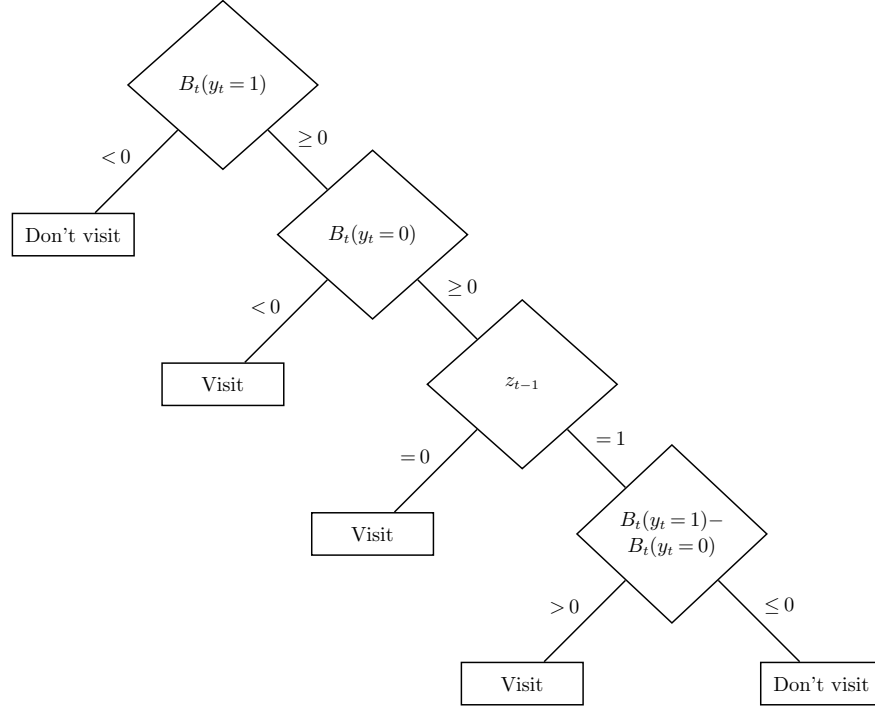


Figure 1 Graphical representation of the structure of the optimal policy given in Corollary 1.

$$\text{subject to } \sum_{i \in \mathcal{P}} y_{i,t} \leq C, \quad \forall t \in \mathcal{T}, \quad (12b)$$

$$b_{i,t+1} = b_{i,t} + p_i - \mu_i z_{i,t} - \alpha_i y_{i,t} z_{i,t}, \quad \forall i \in \mathcal{P}, t \in \mathcal{T}, \quad (12c)$$

$$s_{i,t+1} = \gamma z_{i,t} (s_{i,t} - s_{i,0}) + z_{i,t} s_{i,0} + \beta_i y_{i,t} z_{i,t}, \quad \forall i \in \mathcal{P}, t \in \mathcal{T}, \quad (12d)$$

$$\theta_{i,t+1} = \rho(\theta_{i,t} - \theta_{i,0}) + \theta_{i,0} - \lambda_i y_{i,t} z_{i,t}, \quad \forall i \in \mathcal{P}, t \in \mathcal{T}, \quad (12e)$$

$$z_{i,t} = (z_{i,t-1} + y_{i,t} - z_{i,t-1} y_{i,t}) \cdot \mathbb{1}(B_t(y_t) \geq 0), \quad \forall i \in \mathcal{P}, t \in \mathcal{T}, \quad (12f)$$

$$B_{i,t} \geq -M(1 - z_{i,t}), \quad \forall i \in \mathcal{P}, t \in \mathcal{T}, \quad (12g)$$

$$B_{i,t} \leq M z_{i,t}, \quad \forall i \in \mathcal{P}, t \in \mathcal{T}, \quad (12h)$$

$$y_{i,t}, z_{i,t} \in \{0, 1\}, \quad \forall i \in \mathcal{P}, t \in \mathcal{T}, \quad (12i)$$

$$b_{i,t}, s_{i,t}, \theta_{i,t} \geq 0, \quad \forall i \in \mathcal{P}, t \in \mathcal{T}, \quad (12j)$$

where $B_{i,t} = \mu_i - \theta_{i,t}(\gamma(s_{i,t} - s_{i,0}) + s_{i,0}) + (\alpha_i - \theta_{i,t}\beta_i)y_{i,t}$. The objective function maximizes the total number of patient periods where FBG is in control. Constraint (12b) limits the number of visits in each period to C . The remaining constraints are the same as in the single patient problem, except that there is a set of constraints for each patient. Aside from constraint (12b) the problem is completely separable by patient.

Similar to the single patient problem, we leverage dynamic programming to solve the model. Let $\mathcal{V}_t(\mathbf{b}_t, \mathbf{s}_t, \boldsymbol{\theta}_t, \mathbf{z}_{t-1})$ represent the optimal value-to-go function, where all boldface variables are

vectors of length $|\mathcal{P}|$ with each entry corresponding to the state values for a given patient at time t and \odot represents component-wise multiplication. Using this notation, the dynamic programming equations can be written as:

$$\mathcal{V}_t(\mathbf{b}_t, \mathbf{s}_t, \boldsymbol{\theta}_t, \mathbf{z}_{t-1}) = \sum_{i \in \mathcal{P}} \mathbb{1}(b_{i,t} \leq \delta) + \quad (13)$$

$$\begin{aligned} & \max_{\sum_{i \in \mathcal{P}} y_{i,t} \leq C, y_{i,t} \in \{0,1\}} \left\{ \mathcal{V}_{t+1}(\mathbf{b}_t + \mathbf{p} - \boldsymbol{\mu} \odot \mathbf{z}_t(\mathbf{y}_t) - \boldsymbol{\alpha} \odot \mathbf{y}_t \odot \mathbf{z}_t(\mathbf{y}_t), \mathbf{z}_t(\mathbf{y}_t) \odot (\boldsymbol{\gamma} \odot (\mathbf{s}_t - \mathbf{s}_0) + \mathbf{s}_0 + \boldsymbol{\beta} \odot \mathbf{y}_t), \right. \\ & \left. \boldsymbol{\rho}(\boldsymbol{\theta}_t - \boldsymbol{\theta}_0) + \boldsymbol{\theta}_0 - \boldsymbol{\lambda} \odot \mathbf{y}_t \odot \mathbf{z}_t(\mathbf{y}_t), \mathbf{z}_t(\mathbf{y}_t)) \right\}, \forall t \in \mathcal{T} = \{0, \dots, N-1\}, \\ & \mathcal{V}_N(\mathbf{b}_N, \mathbf{s}_N, \boldsymbol{\theta}_N, \mathbf{z}_{N-1}) = \sum_{i \in \mathcal{P}} \mathbb{1}(b_{i,N} \leq \delta). \end{aligned} \quad (14)$$

5.2.1. Optimal visit policy. To obtain structural results for the optimal visit policy of the multi-patient problem, we first use the optimal policy from the single-patient problem to define a set that represents the patients that would be visited in each period. To build intuition for why this would be of interest, consider the case when the multi-patient problem is uncapacitated, that is $C \geq |\mathcal{P}|$. In this case, the problem is completely separable by patient and can thus be decomposed into $|\mathcal{P}|$ single patient problems that can be solved using the results in Section 5.1. Thus, in this case, we would only visit patients if we would visit them in the single patient setting. We call this group of patients the set of patients of interest.

DEFINITION 1. Let \mathcal{I}_t , the set of patients of interest in period $t \in \mathcal{T}$, be defined as:

$$\mathcal{I}_t := \left\{ i \in \mathcal{P} \mid (B_{i,t}(y_{i,t} = 1) \geq 0) \text{ and } \right. \\ \left. ((B_{i,t}(y_{i,t} = 0) < 0) \text{ or } (z_{i,t-1} = 0) \text{ or } (z_{i,t-1} = 1 \text{ and } B_t(y_t = 1) - B_t(y_t = 0) > 0)) \right\}.$$

In the constrained case (i.e. $C < |\mathcal{P}|$), we prove that an optimal policy to the multi-patient problem can be obtained by appropriately choosing C patients from \mathcal{P} by identifying \mathcal{I}_t . The conditions for inclusion in \mathcal{I}_t are derived directly from Corollary 1 and, for the single patient problem, $y_t = 1$ if \mathcal{I}_t is non-empty. For notational brevity, let $V_{i,t}^j = \mathcal{V}_{i,t}(b_{i,t}, s_{i,t}, \theta_{i,t}, z_{i,t-1}(y_{i,t-1} = j))$ where $j \in \{0,1\}$. Our main result for the structure of the optimal policy in the multi-patient problem is:

THEOREM 3. *In the multi-patient multi-period problem, an optimal policy is given by visiting all patients in the set $\mathcal{S}_t \subset \mathcal{I}_t$, where $\mathcal{S}_t = \{i \in \mathcal{I}_t : y_{i,t}^* = 1\}$ and*

$$\mathbf{y}_t^* \in \left\{ i \in \mathcal{I}_t \mid \sum_{i \in \mathcal{I}_t} y_{i,t} \leq C \right\} \left(\sum_{\{i | y_{i,t} = 1\}} V_{i,t+1}^1 + \sum_{\{i | y_{i,t} = 0\}} V_{i,t+1}^0 \right). \quad (15)$$

Before proving Theorem 3, our intermediate goal is to show that the joint value function $\mathcal{V}_t(\mathbf{b}_t, \mathbf{s}_t, \boldsymbol{\theta}_t, \mathbf{z}_{t-1})$ can always be decomposed additively into patient-wise components. That is $\mathcal{V}_t(\mathbf{b}_t, \mathbf{s}_t, \boldsymbol{\theta}_t, \mathbf{z}_{t-1}) = \sum_{i \in \mathcal{P}} \mathcal{V}_{i,t}(b_{i,t}, s_{i,t}, \theta_{i,t}, z_{i,t})$.

Intuitively, Theorem 3 states that in the multi-patient setting, only patients from the set \mathcal{I}_t should be visited. Moreover, if $|\mathcal{I}_t| > C$, then ties should be broken by selecting patients with larger value-to-go functions when visited. Ties may be present due to the value functions being integer-valued, a consequence of the provider having binary rewards for each patient-period in control. The structure of Theorem 3 is non-trivial to derive and requires the use of several auxiliary results. First, we prove the following lemma regarding the one stage problem (i.e. $t = N - 1$).

LEMMA 3. *For period $N - 1$, an optimal visit policy is given by visiting all patients in set $\mathcal{S}_{N-1} \subset \mathcal{I}_{N-1}$ where $\mathcal{S}_{N-1} := \{i \in \mathcal{I}_{N-1} : y_{i,N-1}^* = 1\}$ where \mathbf{y}_{N-1}^* is defined as:*

$$\mathbf{y}_{N-1}^* \in \arg \max_{\{i \in \mathcal{I}_{N-1} \mid \sum_{i \in \mathcal{I}_{N-1}} y_{i,N-1} \leq C\}} \left(\sum_{\{i \mid y_{i,N-1} = 1\}} V_{i,N}^1 + \sum_{\{i \mid y_{i,N-1} = 0\}} V_{i,N}^0 \right).$$

Note that y_{N-1} can only affect the value function in period N , and that \mathcal{V}_N depends on the FBG and is additively decomposable by patient. The proof is then completed by considering two cases: $|\mathcal{I}_t| \leq C$ and $|\mathcal{I}_t| > C$. The proof of the first case follows directly from Theorem 1 and Corollary 1, and the second case is shown through a proof by contradiction that uses an exchange argument. See Appendix EC.1 for the full proof. Essentially, Lemma 3 establishes that our desired policy structure is optimal in the single-stage problem (and thus the final period of the dynamic programming algorithm). Although this result is trivial, it is not obvious that this structure will hold for all periods in the planning horizon. Our next result shows why this implies that the value functions in the multi-patient multi-period problem also obey the additively decomposable structure.

PROPOSITION 3. *For all $t < N$ the joint value function $\mathcal{V}_t(\mathbf{b}_t, \mathbf{s}_t, \boldsymbol{\theta}_t, \mathbf{z}_{t-1})$ is additively decomposable by patients. That is, at each time t and for each patient $i \in \mathcal{P}$ there exist patient component functions $\mathcal{V}_{i,t}(b_{i,t}, s_{i,t}, \theta_{i,t}, z_{i,t-1})$ such that: $\mathcal{V}_t(\mathbf{b}_t, \mathbf{s}_t, \boldsymbol{\theta}_t, \mathbf{z}_{t-1}) = \sum_{i \in \mathcal{P}} \mathcal{V}_{i,t}(b_{i,t}, s_{i,t}, \theta_{i,t}, z_{i,t-1})$.*

The proof is completed by induction. For the base case (period $N - 1$), the dynamic programming equations in 13 and Lemma 3 yield the desired form. Proving the induction step requires evaluating a maximum. If the optimal solution is to visit no patients or all patients, the proof is trivial. If the optimal solution is to visit a proper subset of patients, the induction step proof follows a similar argument to the base case. See Appendix EC.1 for the full proof.

Lemma 3 and Proposition 3 confirm that \mathcal{V}_t has an additively decomposable structure for all $t \in \mathcal{T}$. These auxiliary results will eventually allow us to interpret Theorem 3 as providing an optimal solution that finds optimal sets of patients belonging to V^0 and V^1 (integer-valued sets) at

each time period, with patients potentially switching between these sets throughout the planning horizon. Proposition 3 holds due to the assumptions that the provider rewards for each patient-period in control and the visit decisions are binary – assumptions that originate directly from the problem setting being studied. To complete the set of intermediate results required to prove Theorem 3, our remaining task is to show that each of the patient component value functions $\mathcal{V}_{i,t}$ is nondecreasing in the enrolment state $z_{i,t-1}$.

PROPOSITION 4. *For all $t < N$ and for each $i \in \mathcal{P}$ the component value functions $\mathcal{V}_{i,t}(b_{i,t}, s_{i,t}, \theta_{i,t}, z_{i,t-1})$ are nondecreasing in $z_{i,t-1}$.*

The proof is completed by induction. The base case relies on defining the component value functions in a manner similar to the proof of Proposition 3 and using Lemma 1 and Lemma 2. The induction step is completed by using Lemma 1 and the induction hypothesis. See Appendix EC.1 for the full proof.

The result of this proposition shows that the structure of the multi-patient problem is similar to that of the single patient, in that we ought to increase the amount of time an individual is enrolled in the program in order to maximize the value-to-go. Now we can proceed to prove the optimal policy for the multi-patient problem.

The proof of Theorem 3 focuses on the constrained case where $|\mathcal{P}| > C$ since the unconstrained case follows directly from Corollary 1. First we consider the case where $|\mathcal{I}_t| \leq C$. Then, by way of contradiction we assume that a visit is allocated for a patient $j \notin \mathcal{I}_t$, and using the definition of \mathcal{I}_t , Proposition 4, and Lemma 1 we reach the contradiction. For the case where $|\mathcal{I}_t| > C$, the proof is completed by contradiction using an exchange argument, following a similar process to the proof of Lemma 3. See Appendix EC.1 for more details.

5.2.2. Implementation of the optimal visit policy. To solve the multi-patient problem, we implement a solution approach that we refer to as the *Enrollment Algorithm (EA)*. We use this approach because it is very challenging (computationally) to exactly evaluate the value function $\mathcal{V}_{i,t}(b_{i,t}, s_{i,t}, \theta_{i,t}, z_{i,t-1})$ for every patient i and time period t , as would be needed to implement Theorem 3 exactly. Instead, because Theorem 3 proved that the optimal set of patients to visit at each period (\mathcal{S}_t) is a subset of the set of patients of interest (\mathcal{I}_t), we estimate \mathcal{S}_t using a subset of \mathcal{I}_t obtained by ranking patients using several criteria.

Intuitively, the EA incorporates the results from Corollary 1 to first identify \mathcal{I}_t for each period t (see Figure 1 for a graphical representation of the conditions defining \mathcal{I}_t). If $|\mathcal{I}_t| \leq C$, then we set $y_{i,t} = 1 \forall i \in \mathcal{I}_t$. If $|\mathcal{I}_t| > C$, then we use one of four tie-breaking rules to rank the patients and select the first C patients to be visited. The four tie-breaking rules used to sort patients $i \in \mathcal{I}_t$ (and the intuition behind them) are:

1. *EA Ascending FBG*: Order patients by ascending $b_{i,t}$. The provider may have more patients with their FBG in-control if they focus on patients with lower FBG (easier to control).
2. *EA Descending FBG*: Order patients by descending $b_{i,t}$. The provider may have a greater impact if they can flip patients with high FBG from out of control to in control.
3. *EA Descending value-to-go*: Estimate the value-to-go of each patient by solving a single-patient problem using the one-step look-ahead policy that is based on Corollary 1. Order patients by descending value-to-go. The value-to-go for each patient is determined based on the number of periods that the patient would have their FBG in control if we were to implement the optimal policy for a single-patient problem, that is, $\tilde{V}_{i,t} = \sum_{t' \in \mathcal{T} | t' \geq t} \mathbb{1}(b_{i,t'} \leq \delta)$. The provider chooses the patients that would have a greater number of periods in control in the single patient problem.
4. *EA Descending value-to-go/visits*: Estimate the value-to-go and the number of visits required in the remaining periods for each patient by solving a single-patient problem. Order patients by descending ratio of value-to-go divided by the number of visits, $\tilde{V}_{i,t}/L_{i,t}$. We calculate the number of visits ($L_{i,t}$) that the patient would require from the present period until the end of the horizon under the optimal single-patient problem policy, that is, $L_{i,t} = \sum_{t' \in \mathcal{T} | t' \geq t} y_{i,t'}$. The provider divides the total number of periods in control for each patient by the number of visits required to obtain a ratio (i.e., return/cost). The ratios are then ranked in descending order.

6. Numerical Experiments

In this section, we describe the data and experimental setup used to evaluate our approach. Section 6.1 describes the overall simulation framework including how we incorporated uncertainty in disease progression, the baseline approaches used for comparison, and the evaluation metrics used. Section 6.2 describes how we employed real-world data from NanoHealth to generate artificial patient cohorts, which we refer to as *expanded scenarios*, to evaluate the generalizability of our approach. Then, Section 6.3 presents results from the simulations performed with the expanded scenarios. In Section 6.4, we present the results from simulations performed with NanoHealth's data. Finally, Section 6.5 describes broad managerial insights obtained by conducting our experiments.

6.1. Simulation framework

In all simulation experiments, we solve the multi-patient CHW visitation problem described in Section 5 using a planning horizon of $N = 60$ periods, corresponding to five years of monthly visits (in line with NanoHealth operations). We conducted simulations for capacity levels ranging from 5% to 100% in 5% increments, where the capacity level is expressed as a percentage of the per period visit capacity (C) divided by the number of patients in the community being served ($|\mathcal{P}|$). All simulations were run using Python 3.7 on a PC with 16 GB of RAM and an Intel Core i5 processor.

6.1.1. Data. As noted in Section 3.3, our dataset includes longitudinal data about a patient cohort treated by NanoHealth, which includes 378 patients that received at least six visits through NanoHealth’s intervention in Hyderabad, India. For each patient, our data includes an FBG reading each time they were visited by a CHW (for both screening and management visits).

6.1.2. Uncertainty in disease progression. We evaluate the performance of our algorithms under uncertain disease progression through the addition of a noise term (ξ) to the FBG update equation: $b_{i,t+1} = b_{i,t} + p_i - \mu_i z_{i,t} - \alpha_i y_{i,t} z_{i,t} + \xi$. We assume that ξ is normally distributed with mean zero and standard deviation σ . The parameter σ was estimated using historical data from NanoHealth by calculating the sample standard deviation of the change in FBG between successive periods across all patients. We solve each instance using 10 different random seeds to generate 10 replications.

6.1.3. Baseline heuristics. We use four naive baseline heuristics as benchmarks:

- *Visit no one*: No patients are visited during the planning horizon.
- *Visit everyone*: Every patient is visited at every period.
- *Descending FBG*: Visit C patients ranked by descending FBG.
- *Ascending FBG*: Visit C patients ranked by ascending FBG.

6.1.4. Evaluation metrics. To compare our solution algorithms, we introduce the concept of patient-periods in control (PPC), i.e., the number of patients with their FBG in control (under the threshold δ) at the end of each period. By computing the total number of PPC across all periods, we can easily compare the relative performance of each heuristic for the full planning horizon. We also compare our algorithms using the number of screening visits, total enrollment, and FBG.

6.2. Expanded Scenarios’ Setup

We evaluate the generalizability of our solution approach through simulation experiments with expanded scenarios (i.e., artificial patient cohorts) that have varying patient parameters (e.g., disease progression, enrollment effect, visit effect, etc.). For each scenario, we evaluate the performance of the four EA implementations (described in Section 5.2.2) and the four baseline heuristics (described in Section 6.1.3).

To generate the scenarios, we first created five patient groups based on the real world data and derived from conversations with our collaborator - specifically, each group is represented by a “patient-type” that includes seven parameters (see Table EC.1). The five groups can be described as (in no particular order):

- Group A: Slow disease progression, high management visit effect, low adverse effect from management visits.

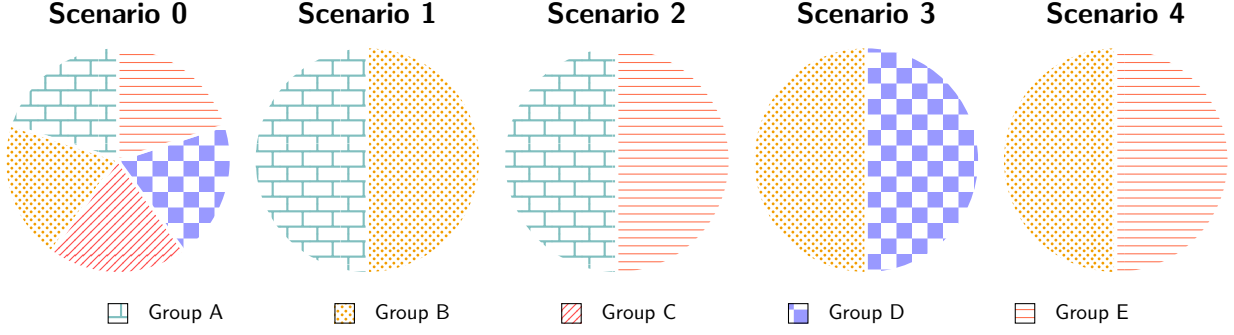


Figure 2 Graphical illustration of the composition of the artificial scenarios.

- Group *B*: Moderate disease progression, high enrollment effect, high adverse effect from management visits.
- Group *C*: Moderate disease progression; high management visit effect; high adverse effect from management visits.
- Group *D*: Fast disease progression; moderate visit and enrollment effects; high adverse effect from management visits.
- Group *E*: Slow disease progression; high management visit effect; high adverse effect from management visits.

Figure 2 provides a graphical representation of the composition of the five scenarios using the patient groups. We build each scenario (i.e., artificial community) in three steps. First, we sample the patient state dynamics parameters from each group described in Table EC.1. Patient parameters are assumed to be independent and to have constant variance across different groups, therefore only the mean vector differs for each group and the parameters can be sampled using truncated normal distributions. We use a lower bound of zero for the truncation based on the assumption that the parameters being sampled are non-negative. Second, we set the discount factors, ρ and γ , to 0.2 for all patients in all scenarios because the majority of real world patients had $\rho = \gamma = 0.2$ after solving the parameter estimation problem in Section 4.2 (over 93% and over 95% for ρ and γ , respectively). Third, we sample initial FBG values (i.e., $b_{i,0}$) using a normal distribution fit to the initial FBG values from the NanoHealth cohort. We confirmed that initial FBG was approximately normally distributed using a Kolmogorov–Smirnov at a significance level of 0.05.

6.3. Expanded Scenarios' Results

This section presents results for the experiments that use artificial patient cohorts. Section 6.3.1 compares PPC at varying capacity levels, Section 6.3.2 explores the breakdown of visit types and patient enrollment for the planning horizon, and Section 6.3.3 compares heuristic approaches with respect to the final log-FBG distribution.

6.3.1. PPC performance. Figure 3 shows five plots (one per scenario) with the percentage of patient-periods in control (PPC) as a function of capacity. Recall that the capacity is expressed as a percentage representing the number of visits per period divided by the total number of people in the community. In all scenarios, the best EA implementation outperforms ranking by Ascending FBG and Descending FBG (baseline heuristics) for capacities up to 65%. In Figure 3e, the EA Descending value-to-go/visits achieves 124.5% greater PPC relative to the Ascending FBG heuristic for the same capacity level (5%). In most scenarios, the EA Descending value-to-go/visits implementation has a superior performance to the others, except for Figures 3b and 3c, where the EA Descending FBG implementation outperforms it at some capacity levels. Alternatively, we can compare the capacity required for the best naive heuristic to perform comparably to the best EA implementation. For example, Figure 3c shows that to achieve a performance of approximately 30% of PPC, the EA with Descending FBG requires a capacity of 13.8%, while a capacity of 36.7% (165.0% greater) is required to achieve the same performance with the best naive heuristic (Ascending FBG).

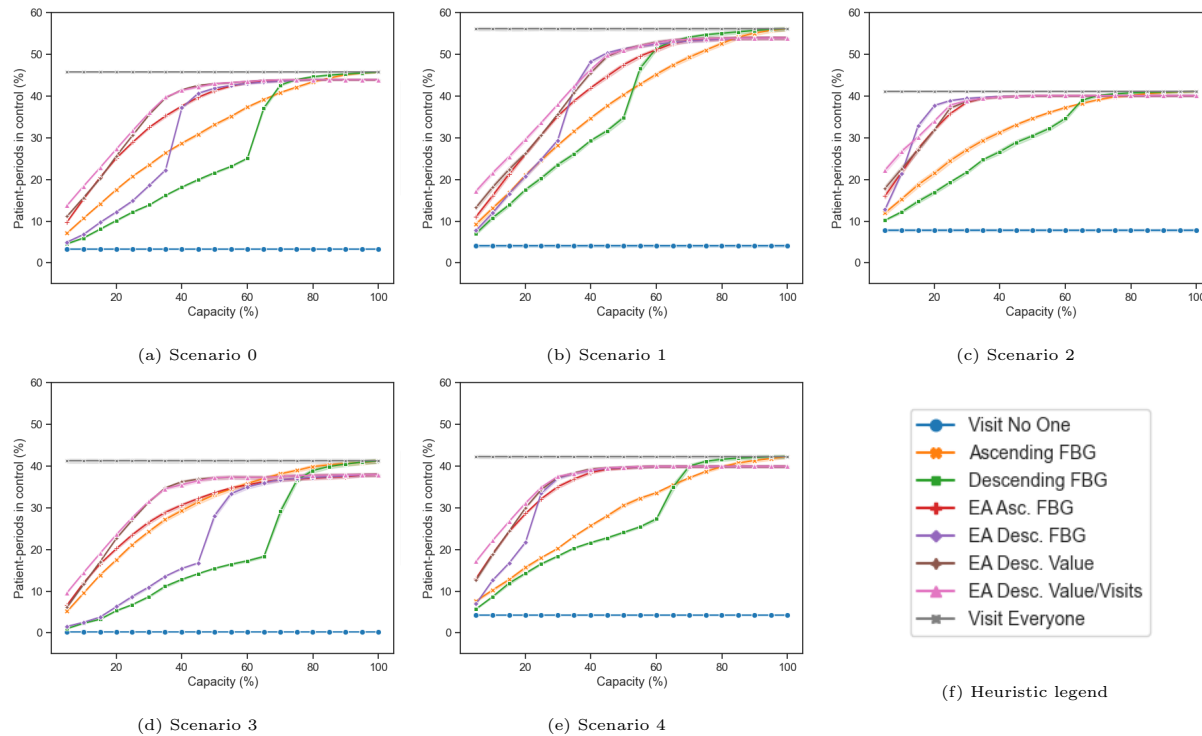


Figure 3 Glycemic control for different scenarios at various capacity levels with shaded 95% confidence intervals.

6.3.2. Visit types and patient enrollment. Figure 4 shows the proportion of CHW visits that were screening visits for different scenarios throughout the planning horizon. We chose a capacity level of 20% for illustrative purposes. As expected, we observe that screening visits are highest at the start of the planning horizon for each scenario and model implementation. For the best performing (in terms of PPC) EA implementations (i.e., Desc. Value/Visits and Desc. Value), we observe that the proportion of screening visits decreases to (nearly) zero after the first five periods. In contrast, the proportion of screening visits for the benchmark heuristics (i.e., Desc. FBG and Asc. FPG) does not decrease as quickly, and in some cases remains high throughout the planning horizon. There are two primary reasons for this. First, the benchmark heuristics (continuously) screen individuals who do not enroll in the program, effectively wasting visit resources. Second, the benchmark heuristics often screen individuals who enroll, but then drop out due to adverse factors of enrollment because they are either visited too frequently or not enough.

Figure 5 displays the proportion of enrolled patients for different scenarios throughout the planning horizon at 20% capacity. In line with the screening patterns observed in Figure 4, enrollment increases rapidly during the first 5 periods. We observe that the best performing implementation (EA Desc. Value/Visits) maintains roughly 30% enrollment across all scenarios, suggesting that there exists an enrollment “sweet spot”. Intuitively, this makes sense for two reasons. First, if we enroll too many patients, we may not have the capacity to conduct management visits causing patients to drop out (and then need to be re-screened). Second, if we do not enroll enough patients, we may not make a large enough impact and we may have excess capacity and over-visit patients, causing them to drop out.

6.3.3. FBG distribution. Figure 6 displays violin plots showing the distribution of log-FBG values for different scenarios at the end of the planning horizon. These figures allow us to analyze not only the performance in terms of glycemic control, but also how they prioritize patients. For example, Figures 6b, 6c, and 6d show a bimodal shape for the EA Descending value-to-go/visits implementation, suggesting that a subset of patients is prioritized. These results highlight that in order to maximize PPC, we need to focus our resources on the subset of patients that benefit most from the intervention and effectively ignore the patients who do not benefit (or whose FBG cannot be controlled). Intuitively, this makes sense because we want to be as efficient as possible with our limited resources and those who do not benefit from the program will need to be cared for using other more effective (for them) interventions.

6.4. NanoHealth Case Study

In this section, we describe the experiments and results using a real patient cohort. To obtain personalized patient parameters (which are needed as input for our experiments), we solved the

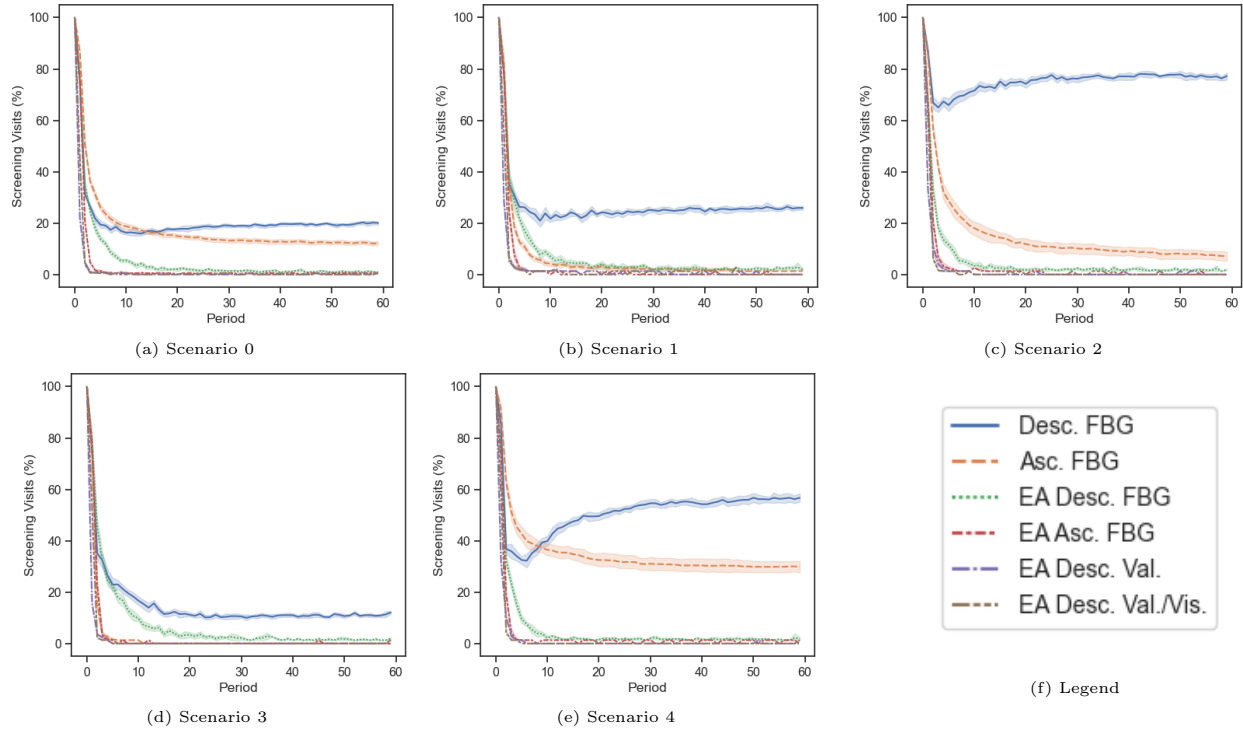


Figure 4 Line graphs for different scenarios showing the proportion of CHW visits that were screening visits at 20% capacity with shaded 95% confidence intervals.

parameter estimation problem described in Section 4.2 for each patient using their actual visit history.

6.4.1. PPC performance. Figure 7a shows box and whisker plots of PPC for each heuristic when the visit capacity per period corresponds to 45% of the total number of patients in the cohort. We chose this capacity level because we observe the largest range in PPC performance between the algorithms. We observe that EA with Descending FBG achieves the best performance with a PPC of 42.1%, followed by EA Ascending FBG with a PPC of 40.6%. These algorithms improve upon the best benchmark heuristic (Descending FBG) by 9.8% and 5.8%, respectively.

Figure 7b shows the PPC at varying capacity levels in 5% increments. We observe that the different algorithms are comparable up until about 20% capacity. As the capacity increases, the EA with Descending FBG has the fastest gain in performance, reaching a 9.8% better performance at 45% capacity with respect to the best benchmark heuristic. Furthermore, to achieve 40% PPC, the EA with Descending FBG requires 34.3% capacity, while the Descending FBG heuristic requires 59.5% capacity, a 25.2% absolute (73.4% relative) difference.

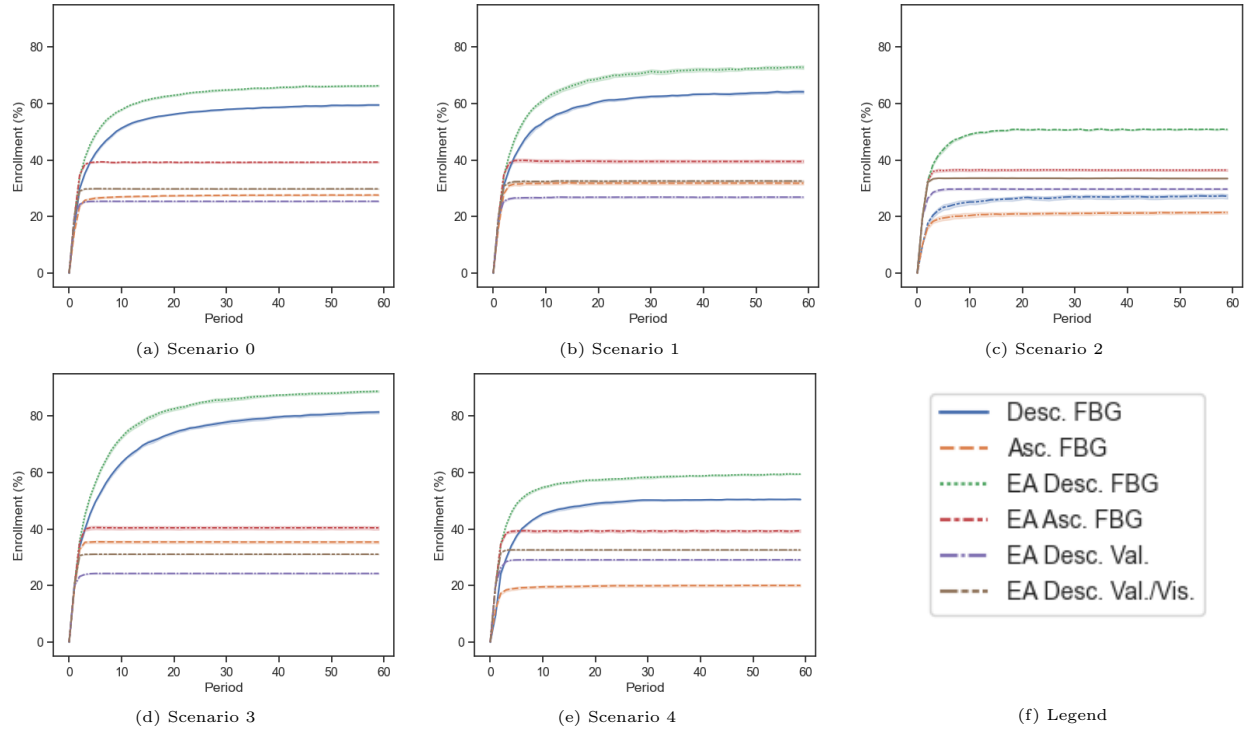


Figure 5 Line graphs for different scenarios showing the proportion of patients enrolled in treatment at 20% capacity with shaded 95% confidence intervals.

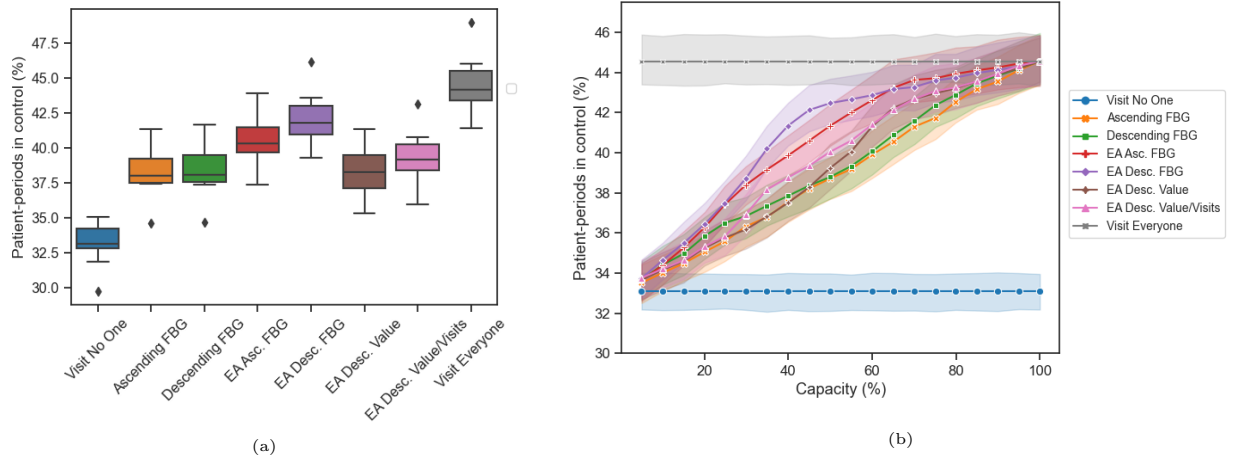


Figure 7 (a) Boxplots showing the interquartile range of PPC for each heuristic at 45% capacity; (b) Line graph of PPC for each heuristic for varying capacity levels (5% increments) with shaded 95% confidence intervals.

6.4.2. Visit types and patient enrollment. Figure 8a displays the proportion of CHW visits that were screening visits across the planning horizon. We observe that the EA implementations have a lower percentage of visits that are dedicated to screening, in addition to a periodic pattern

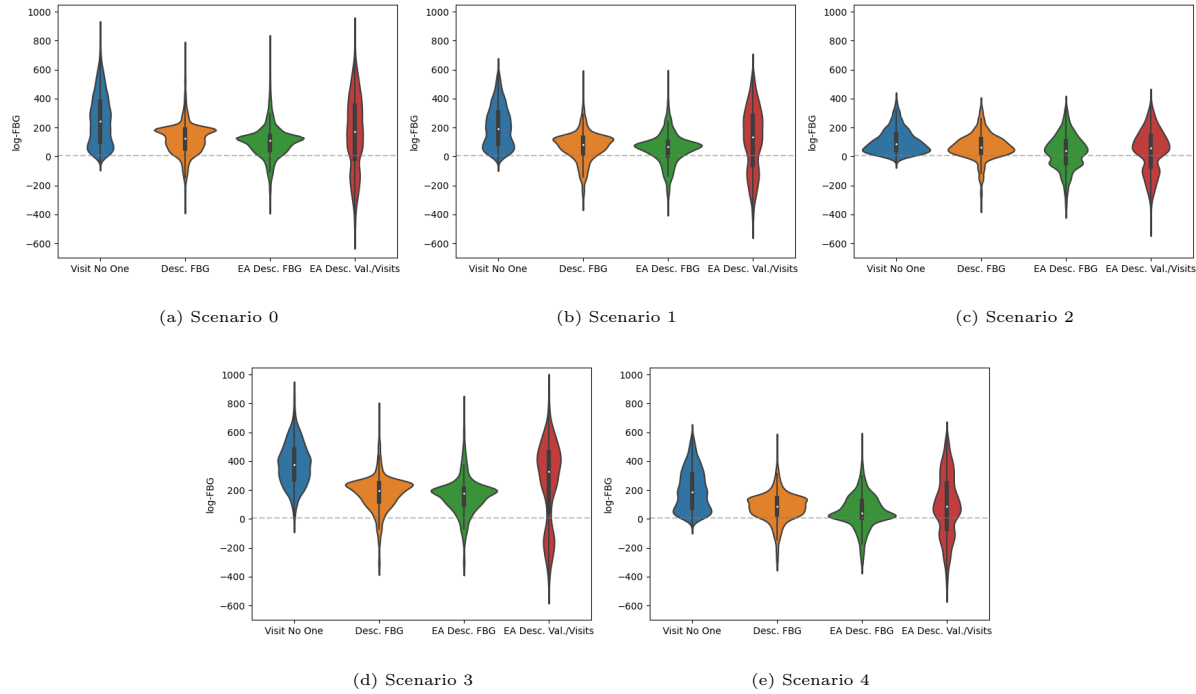


Figure 6 Violin plots showing the distribution of log-FBG values at the end of the planning horizon for different scenarios based on the implementation of four methods: visit no one, rank by descending FBG with 20% capacity, EA with descending FBG with 20% capacity, and EA with value-to-go/visits with 20% capacity. The gray dashed line indicates the FBG threshold δ .

with the screening visits reaching 0% in some periods, then increasing, then returning to 0%. This suggests that some patients may drop out of treatment and require additional screening visits to re-enroll. Furthermore, we observe that the benchmark heuristics (Descending FBG and Ascending FBG) maintain a higher overall percentage of screening visits. However, Figure 8b, which shows the proportion of enrolled patients, demonstrates that these two benchmark heuristics also have the lowest percentage of patients enrolled in treatment during most of the planning horizon. Intuitively, these results suggest that the screening visits conducted by the benchmark heuristics are not allocated as efficiently as the screening visits conducted by the EA implementations. In fact, the enrollment for the Descending FBG and Ascending FBG heuristics never surpasses 81% and 93%, respectively, while all EA implementations maintain enrollment above 90%, with three of them approaching 100%.

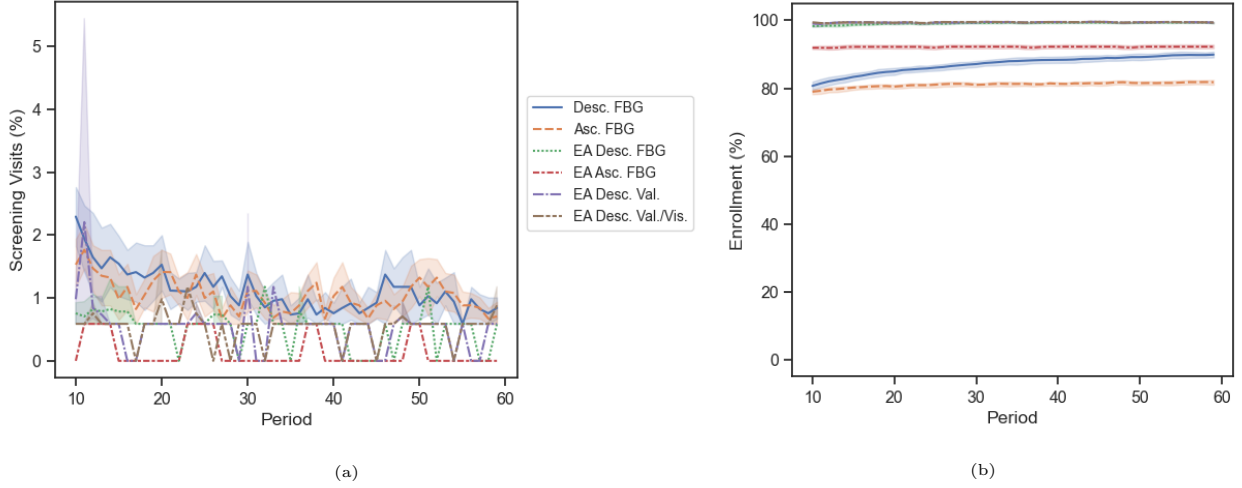


Figure 8 (a) Line graph showing the proportion of CHW visits that were screening visits at 45% capacity with shaded 95% confidence intervals; (b) Line graph showing the proportion of patients enrolled in treatment at 45% capacity with shaded 95% confidence intervals.

6.4.3. FBG distribution. Figure 9 shows violin plots with the distribution of log-FBG levels at the end of the planning horizon. The Visit No One benchmark shows a bimodal distribution due to the fast FBG increase experienced by some patients, and slow increase or stabilization in FBG experienced by other patients, leading to two distinct peaks. Because we modeled disease progression proportionally to current FBG levels, an artifact of our model is that some patients present extremely high FBG levels that would not occur in practice. In these cases, life-threatening conditions would likely occur before those FBG levels are reached. In terms of numerical results, the Visit No One policy achieves a median and 90th percentile log-FBG of 16.6 and 422.1, respectively. Although the EA heuristic with Descending value-to-go/visits has a similar shape with a long tail, the median and 90th percentile are 5.2 and 33.5 (relative improvements of 68.7% and 92.1%), respectively. In contrast, Descending FBG and EA Descending FBG were able to improve the 90th percentile to 29.2 and 28.3 (relative improvements of 93.1% and 93.3%), respectively. Although Descending FBG and EA Descending FBG have similar shape, EA Desc. FBG has a median and 90th percentile FBG that is 32.5% and 3.1% lower than Desc. FBG, respectively.

6.5. Managerial Insights

Our work provides insights not only for NanoHealth's operations, but also other community-based interventions for chronic diseases. We highlight the following key managerial insights:

1. *For NanoHealth's cohort and for all artificial scenarios considered, the best EA implementation consistently outperforms the best benchmark heuristic.* The magnitude of improvement in

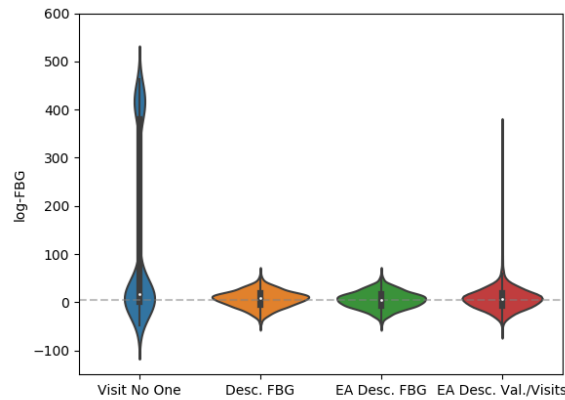


Figure 9 Violin plots showing the distribution of log-FBG values at the end of the planning horizon for NanoHealth’s patient cohort at 45% capacity. The white dots, bold line, and thin line indicate the median, interquartile range (IQR), and 1.5 times the IQR for each violin plot, respectively. The FBG threshold δ is indicated by a horizontal gray dashed line.

PPC varies depending on the patient cohort and capacity level. For example, the EA with Descending FBG improved upon the best naive policy by 9.8% for the NanoHealth cohort at 45% capacity (see Figure 7a) and significantly outperformed the best naive policy for all scenarios at any capacity (see Figure 3). To achieve a PPC of 40% for the NanoHealth cohort, the best EA requires 34.3% capacity, which is 25.2 percentage points (73.4%) lower than the best performing naive policy. For the expanded scenarios, the best performing EA requires 165% less capacity than the best naive policy to achieve a PPC of 30%. These results imply that, for resource-constrained settings in LMICs, our approach allows CHW interventions to effectively treat a larger cohort of patients with limited resources. Unlike the naive approaches, by exploiting the structure in the EA, decision makers are able to efficiently trade off between cohort size and health outcomes.

2. *The EA provides a flexible framework for breaking ties among patients of interest that can be tailored by providers to meet their priorities.* As seen in Figure 6, the EA can be modified with different tie breaking rules to optimize various health outcomes. For example, a provider may implement the EA tie-breaking rule that achieves the lowest median FBG (more efficient) or the lowest 90th percentile FBG (more equitable). The adaptability of our approach allows the provider to make decisions based on their priorities.
3. *Our framework is scalable and can be implemented on mobile tablets already used by CHW programs.* Many CHW programs currently have the infrastructure to implement the EA because they use mobile tablets for data collection and communication. The EA only relies on data typically collected by CHW programs during pilot phases or normal day-to-day operations.

Moreover, our technical methods enable providers to identify patient types and to determine how to best personalize their intervention as opposed to providing a one size fits all treatment.

7. Conclusion

In this paper, we developed a modelling framework to optimize a resource-constrained CHW intervention for diabetes care in urban areas in LMICs. Our framework explicitly models the tradeoff between screening new patients and providing management visits to individuals who are already enrolled in treatment. We account for patients' motivational states, which affect their decisions to enroll or drop out of treatment and, therefore, the effectiveness of the intervention. We incorporate these decisions by modeling patients as utility-maximizing agents within a bi-level provider problem that we solve using approximate dynamic programming. Our scalable heuristics rely on theoretical results from the single-patient problem that are used to estimate the value function in the multi-patient problem. By performing several simulation experiments, we found that our framework's performance in terms of patient-periods in control depends on the composition of the patient cohort targeted by the intervention, and can improve upon baseline heuristics by up to 124.5% (in terms of relative performance) using the same CHW capacity. Finally, we applied our approach to generate CHW visit plans for NanoHealth, a social enterprise in India that provided us with the data for our case study. We found that the best EA implementation for NanoHealth's cohort can achieve the same level of patient-periods in control as the best naive implementation with up to 73.4% less CHW capacity.

Electronic Companion

EC.1. Proofs of Propositions in the Text

Proof of Proposition 1: We can write the benefit function for each component of the utility function separately. To write this function for $B_{i,t}^b$, we first substitute the dynamic equations for $b_{i,t+1}, s_{i,t+1}$ into the utility function components to obtain the following explicit representation in terms of the patient states at time t :

$$\begin{aligned} U_b(b_{i,t}, z_{i,t}) &= -(b_{i,t} + p_i - \mu_i z_{i,t} - \alpha_i y_{i,t} z_{i,t}) \\ U_s(s_{i,t}, \theta_{i,t}, z_{i,t}) &= -\theta_{i,t}(z_{i,t}(\gamma(s_{i,t} - s_{i,0}) + s_{i,0}) + \beta_i y_{i,t} z_{i,t}) \end{aligned}$$

Let $B_{i,t} = B_{i,t}^b + B_{i,t}^s$, where the first component corresponds to the net benefit associated with $b_{i,t}$ and second component corresponds to the net benefit associated with $s_{i,t}$. Then through a similar definition of $B_{i,t}$ by doing the appropriate subtractions we can derive the following forms.

$$\begin{aligned} B_{i,t}^b &= \mu_i + \alpha_i y_{i,t} \\ B_{i,t}^s &= -\theta_{i,t}(\gamma(s_{i,t} - s_{i,0}) + s_{i,0}) - \theta_{i,t} \beta_i y_{i,t} \end{aligned}$$

Combining these expressions gives the first result of the proposition.

To obtain the next result note that if $B_{i,t} \geq 0$ this implies that $U_i(b_{i,t+1}, s_{i,t+1}, \theta_{i,t}, 1) \geq U_i(b_{i,t+1}, s_{i,t+1}, \theta_{i,t}, 0)$ and thus the utility function is maximized when $z_{i,t} = 1$. Likewise, when $B_{i,t} < 0$ this means $U_i(b_{i,t+1}, s_{i,t+1}, \theta_{i,t}, 1) < U_i(b_{i,t+1}, s_{i,t+1}, \theta_{i,t}, 0)$ and therefore $z_{i,t} = 0$. However, $z_{i,t}$ can only be equal to 1 if $y_{i,t}$ or $z_{i,t-1}$ are equal to 1. This gives the second result of the proposition. \square

Proof of Proposition 2: We seek to use a MLE approach to estimate all unknown parameters:

$$(\hat{\mathbf{b}}, \hat{\mathbf{s}}, \hat{\boldsymbol{\theta}}, \hat{\beta}, \hat{\alpha}, \hat{\mu}, \hat{p}, \hat{\lambda}, \hat{\gamma}, \hat{\rho}) \in \arg \min_{\mathbf{b}, \mathbf{s}, \boldsymbol{\theta}, \beta, \alpha, \mu, p, \lambda, \gamma, \rho} p(\{\bar{b}_t\}_{t \in K} | \mathbf{b}, \mathbf{s}, \boldsymbol{\theta}, \beta, \alpha, \mu, p, \lambda, \gamma, \rho, \mathbf{z}, \mathbf{y}) \quad (\text{EC.1})$$

Using the dynamic equations described in Section 4, we can decompose the likelihood function in (EC.1) as follows:

$$\begin{aligned} p(\{\tilde{b}_t\}_{t \in K} | \mathbf{b}, \mathbf{s}, \boldsymbol{\theta}, \beta, \alpha, \mu, p, \lambda, \gamma, \rho, \mathbf{z}, \mathbf{y}) &= \prod_{t \in K} p(\tilde{b}_t | b_t) \prod_{t \in T} p(b_t | b_{t-1}, p, \mu, z_t, \alpha, y_t) p(s_t | s_{t-1}, s_0, \beta, y_t, z_t, \gamma) \\ &\quad p(\theta_t | \theta_{t-1}, \theta_0, \lambda, y_t, z_t, \rho) p(z_t | \mu, \theta_t, s_t, s_0, \alpha, \beta, \gamma, y_t) \quad (\text{EC.2}) \end{aligned}$$

Note that while the product terms related to \tilde{b}_t are only taken over time periods $K \subset T$, the other terms are taken over all time indices. This is due to the definition of the one step dynamics, and

allows us to estimate the values of these parameters during periods for which we do not have direct observations.

Next, we take the log of both sides of the likelihood function in Equation (EC.2) to obtain the log-likelihood:

$$\begin{aligned} \log p(\{\tilde{b}_t\}_{t \in \mathcal{K}} | \mathbf{b}, \mathbf{s}, \boldsymbol{\theta}, \beta, \alpha, \mu, p, \mathbf{z}, \mathbf{y}) &= \sum_{t \in \mathcal{K}} \log p(\tilde{b}_t | b_t) + \sum_{t \in \mathcal{T}} \log p(b_t | b_{t-1}, p, \mu, z_t, \alpha, y_t) \\ &\quad + \log p(s_t | s_{t-1}, s_0, \beta, y_t, z_t, \gamma) + \log p(\theta_t | \theta_{t-1}, \theta_0, \lambda, y_t, z_t, \rho) \\ &\quad + \log p(z_t | \mu, \theta_t, s_t, s_0, \alpha, \gamma, \beta, y_t). \end{aligned}$$

Observe that $p(\tilde{b}_t | b_t) = f_\epsilon(\tilde{b}_t - b_t)$ by definition of the noise model and note that because the remaining terms correspond to deterministic system dynamics, they are degenerate distributions. Thus, the MLE can be expressed as the following constrained optimization problem:

$$\underset{\mathbf{b}, \mathbf{s}, \boldsymbol{\theta}, \beta, \alpha, \mu, p, \lambda, \rho, \gamma}{\text{minimize}} \quad \sum_{t \in \mathcal{K}} \log f_\epsilon(\tilde{b}_t - b_t) \quad (\text{EC.3a})$$

$$\text{subject to} \quad \hat{b}_{t+1} = \hat{b}_t + p - \mu z_t - \alpha y_t z_t, \quad \forall t \in \mathcal{T}, \quad (\text{EC.3b})$$

$$s_{t+1} = \gamma z_t (s_t - s_0) + z_t s_0 + \beta y_t z_t, \quad \forall t \in \mathcal{T}, \quad (\text{EC.3c})$$

$$\theta_{t+1} = \rho(\theta_t - \theta_0) + \theta_0 - \lambda y_t z_t, \quad \forall t \in \mathcal{T}, \quad (\text{EC.3d})$$

$$z_{i,t} = (z_{i,t-1} + y_{i,t} - z_{i,t-1} y_{i,t}) \mathbb{1}[B_{i,t} \geq 0] \quad \forall t \in \mathcal{T}, \quad (\text{EC.3e})$$

$$\hat{b}_t, s_t, \theta_t \geq 0, \quad \forall t \in \mathcal{T}, \quad (\text{EC.3f})$$

$$\beta, \alpha, \mu, p, \lambda, \rho, \gamma \geq 0. \quad (\text{EC.3g})$$

Constraint (EC.3e) is derived from Proposition 1 and the remaining constraints come from the model dynamics. To obtain the final formulation, constraint (EC.3e) is reformulated using disjunctive constraints (Wolsey and Nemhauser 1999). \square

Proof of Lemma 1: Since $z_{t-1} \in \{0, 1\}$, we simply evaluate z_t for $z_{t-1} = 0$ and $z_{t-1} = 1$:

$$z_t(0, y_t) = y_t \cdot \mathbb{1}(B_t(y_t) \geq 0) \leq \mathbb{1}(B_t(y_t) \geq 0) = z_t(1, y_t). \quad \square$$

Proof of Lemma 2: Since V_N is only a function of the FBG value, b_N , and $b_N = b_{N-1} + p - \mu z_{N-1} - \alpha y_{N-1} z_{N-1}$, we have that:

$$V_N(b_N, s_N, \theta_N, z_{N-1}) = \begin{cases} 1, & \text{if } b_{N-1} \leq \delta - p + \mu z_{N-1}(y_{N-1}) + \alpha y_{N-1} z_{N-1}(y_{N-1}), \\ 0, & \text{if } b_{N-1} > \delta - p + \mu z_{N-1}(y_{N-1}) + \alpha y_{N-1} z_{N-1}(y_{N-1}). \end{cases}$$

Since z_{N-1} is a binary variable, we evaluate V_N for $z_{N-1} = 0$ and $z_{N-1} = 1$:

$$V_N(z_{N-1} = 0) = \begin{cases} 1, & \text{if } b_{N-1} \leq \delta - p \\ 0, & \text{if } \delta - p < b_{N-1} \leq \delta - p + \mu + \alpha y_{N-1} \\ 0, & \text{if } b_{N-1} > \delta - p + \mu + \alpha y_{N-1} \end{cases}$$

$$V_N(z_{N-1} = 1) = \begin{cases} 1, & \text{if } b_{N-1} \leq \delta - p \\ 1, & \text{if } \delta - p < b_{N-1} \leq \delta - p + \mu + \alpha y_{N-1} \\ 0, & \text{if } b_{N-1} > \delta - p + \mu + \alpha y_{N-1} \end{cases}$$

Note that $\delta - p \leq \delta - p + \mu + \alpha y_{N-1}$ because $\mu \geq 0$ and $\alpha \geq 0$. Since $V_N(z_{N-1} = 0) \leq V_N(z_{N-1} = 1)$ for all values of b_{N-1} , the proof is complete. \square

Proof of Theorem 1: We proceed by induction. For the base case, we need to show that V_{N-1} is nondecreasing in z_{N-2} , i.e., $V_{N-1}(z_{N-2} = 0) \leq V_{N-1}(z_{N-2} = 1)$, which is equivalent to:

$$\max_{y_{N-1} \in \{0,1\}} \{V_N(z_{N-2} = 0)\} \leq \max_{y_{N-1} \in \{0,1\}} \{V_N(z_{N-2} = 1)\}.$$

Observe that V_N does not depend directly z_{N-2} , but is only affected by it through z_{N-1} . Since V_N is nondecreasing in z_{N-1} (Lemma 2) and z_{N-1} is nondecreasing in z_{N-2} (Lemma 1), it follows that V_N is nondecreasing in z_{N-2} since the composition of nondecreasing functions is also nondecreasing. Lastly, by the same reasoning, we also have that V_{N-1} is nondecreasing in z_{N-2} .

For the inductive hypothesis, we assume that V_{t+1} is nondecreasing in z_t , that is $V_{t+1}(z_t = 0) \leq V_{t+1}(z_t = 1)$. Finally, we show that V_t is nondecreasing in z_{t-1} , which follows the same logic as the base case. First, observe that $V_t(z_{t-1} = 0) \leq V_t(z_{t-1} = 1)$ is equivalent to

$$\max_{y_t \in \{0,1\}} \{V_{t+1}(z_{t-1} = 0)\} \leq \max_{y_t \in \{0,1\}} \{V_{t+1}(z_{t-1} = 1)\}.$$

Similar to the base case, V_{t+1} is a composition function of z_{t-1} through z_t that is a function of z_{t-1} . Since V_{t+1} is nondecreasing in z_t (inductive hypothesis) and z_t is nondecreasing in z_{t-1} (Lemma 1), it follows that V_{t+1} is nondecreasing in z_{t-1} . And, since the maximum of monotonically nondecreasing functions is a monotonically nondecreasing function, we have that V_t is nondecreasing in z_{t-1} . \square

Proof of Theorem 2: It is sufficient to show that $V_t(b_t, s_t, \theta_t, z_{t-1})$ is strictly increasing in y_{t-1} for case (i), $V_t(b_t, s_t, \theta_t, z_{t-1})$ is strictly decreasing in y_{t-1} for case (ii), and $V_t(b_t, s_t, \theta_t, z_{t-1})$ is constant in y_{t-1} for case (iii). Since we have already shown in Theorem 1 that V_t is nondecreasing in z_{t-1} , we only need to show when $z_t(z_{t-1}, y_t) = (z_{t-1} + y_t - z_{t-1}y_t) \cdot \mathbb{1}(B_t(y_t) \geq 0)$ is strictly increasing, strictly decreasing, and constant in y_t . We proceed by enumerating two cases corresponding to $z_{t-1} = 0$ and $z_{t-1} = 1$.

For the case when $z_{t-1} = 1$ (i.e., the patient is enrolled), $z_t(1, 0) > z_t(1, 1)$ (i.e., strictly decreasing) if and only if $B_t(0) \geq 0$ and $B_t(1) < 0$, $z_t(1, 0) < z_t(1, 1)$ if and only if $B_t(0) < 0$ and $B_t(1) \geq 0$, and $z_t(1, 0) = z_t(1, 1)$ if and only if $B_t(0) < 0$ and $B_t(1) < 0$ or $B_t(0) \geq 0$ and $B_t(1) \geq 0$.

For the case when $z_{t-1} = 0$ (i.e., the patient is not enrolled), $z_t(0, 0)$ is never strictly greater than $z_t(0, 1)$ because $z_t(0, 0) = 0$, $z_t(0, 0) < z_t(0, 1)$ if and only if $B_t(1) \geq 0$, and $z_t(0, 0) = z_t(0, 1)$ if and only if $B_t(1) < 0$. \square

Proof of Lemma 3: Note that the argmax is over the sum of value functions with respect to period N because actions taken in period $N - 1$ only impact the FBG (and, therefore, the value function) of the following period, N , which is additively decomposable by Equation (14). Note that, in the single period problem, visiting a patient who is not in set \mathcal{I}_{N-1} would result in a patient who would not benefit in the short term from the visit taking resources instead of one who would by Theorem 1. Thus we must only consider cases based on the size of set \mathcal{I}_{N-1} .

If $|\mathcal{I}_{N-1}| \leq C$, then the proof follows directly from the definition of \mathcal{I}_t , Theorem 1, and Corollary 1 because the problem is equivalent to $|\mathcal{I}_{N-1}|$ single patient problems. To prove the claim for the case where $|\mathcal{I}_{N-1}| > C$, we proceed by contradiction. Without loss of generality, assume $\exists j \in \mathcal{S}_{N-1}, \ell \notin \mathcal{S}_{N-1}$ such that

$$V_{\ell,N}^1 - V_{\ell,N}^0 > V_{j,N}^1 - V_{j,N}^0.$$

Let \mathcal{S}'_{N-1} be the set obtained by swapping the elements j and ℓ , that is, $\mathcal{S}'_{N-1} = \mathcal{S}_{N-1} \cup \{\ell\} \setminus \{j\}$. Let us consider the relationship between the value function of visiting patients in set \mathcal{S}'_{N-1} instead of \mathcal{S}_{N-1} .

$$\sum_{i \notin \mathcal{S}_{N-1}} V_{i,N}^0 + \sum_{i \in \mathcal{S}_{N-1}} V_{i,N}^1 \tag{EC.4}$$

$$= \sum_{i \notin \mathcal{S}_{N-1} \setminus \{\ell\}} V_{i,N}^0 + \sum_{i \in \mathcal{S}_{N-1} \setminus \{j\}} V_{i,N}^1 + V_{j,N}^1 + V_{\ell,N}^0 \tag{EC.5}$$

$$< \sum_{i \notin \mathcal{S}_{N-1} \setminus \{\ell\}} V_{i,N}^0 + \sum_{i \in \mathcal{S}_{N-1} \setminus \{j\}} V_{i,N}^1 + V_{j,N}^0 + V_{\ell,N}^1 \tag{EC.6}$$

$$= \sum_{i \notin \mathcal{S}'_{N-1}} V_{i,N}^0 + \sum_{i \in \mathcal{S}'_{N-1}} V_{i,N}^1 \tag{EC.7}$$

However, \mathcal{S}_{N-1} is defined as the set of patients where $y_{i,N-1} = 1$ that maximizes the sum of value functions for all patients. Therefore, we have reached a contradiction and conclude the proof. \square

Proof of Proposition 3: We will proceed to prove this proposition by induction on t . First let us consider the base case, $t = N - 1$. Using the dynamic programming equations (13),(14), and Lemma 3 we can write the value function at period $N - 1$ as:

$$\mathcal{V}_{N-1}(\mathbf{b}_{N-1}, \mathbf{s}_{N-1}, \boldsymbol{\theta}_{N-1}, \mathbf{z}_{N-2}) = \sum_{i \in \mathcal{P}} \mathbb{1}(b_{i,N-1} \leq \delta) + \sum_{i \in \mathcal{S}_{N-1}} V_{i,N}^1 + \sum_{i \in \mathcal{P} \setminus \mathcal{S}_{N-1}} V_{i,N}^0. \tag{EC.8}$$

If, for each $i \in \mathcal{P}$, we define $\mathcal{V}_{i,N-1}(b_{i,N-1}, s_{i,N-1}, \theta_{i,N-1}, z_{i,N-2}) := \mathbb{1}(b_{i,N-1} \leq \delta) + V_{i,N}^1 \mathbb{1}(i \in \mathcal{S}_{N-1}) + V_{i,N}^0 (1 - \mathbb{1}(i \in \mathcal{S}_{N-1}))$, then it is clear that from equation (EC.8), $\mathcal{V}_{N-1}(\mathbf{b}_{N-1}, \mathbf{s}_{N-1}, \boldsymbol{\theta}_{N-1}, \mathbf{z}_{N-2}) = \sum_{i \in \mathcal{P}} \mathcal{V}_{i,N-1}(b_{i,N-1}, s_{i,N-1}, \theta_{i,N-1}, z_{i,N-2})$.

Next we proceed with the main induction step. Suppose that for some $t + 1 \leq N - 1$ we have that $\mathcal{V}_{t+1}(\mathbf{b}_{t+1}, \mathbf{s}_{t+1}, \boldsymbol{\theta}_{t+1}, \mathbf{z}_t) = \sum_{i \in \mathcal{P}} \mathcal{V}_{i,t+1}(b_{i,t+1}, s_{i,t+1}, \theta_{i,t+1}, z_{i,t})$. Using the dynamic programming equations (13) we can thus rewrite the equation for time t as:

$$\mathcal{V}_t(\mathbf{b}_t, \mathbf{s}_t, \boldsymbol{\theta}_t, \mathbf{z}_{t-1}) = \sum_{i \in \mathcal{P}} \mathbb{1}(b_{i,t} \leq \delta) + \max_{\sum_{i \in \mathcal{P}} y_{i,t} \leq C, y_{i,t} \in \mathbb{B}} \sum_{i \in \mathcal{P}} \mathcal{V}_{i,t+1}(b_{i,t+1}(y_{i,t}), s_{i,t+1}(y_{i,t}), \theta_{i,t+1}(y_{i,t}), z_{i,t}(y_{i,t})) \quad (\text{EC.9})$$

Note that, since $y_{i,t} \in \mathbb{B}$, it can only take on values of either 0 or 1 in the solution to the second term of Equation EC.9. Let $\mathcal{A}_t \subset \mathcal{P}$ be defined as $\mathcal{A}_t := \{i \in \mathcal{P} : y_{i,t}^* = 1\}$ where $\mathbf{y}^* \in \arg \max_{\sum_{i \in \mathcal{P}} y_{i,t} \leq C, y_{i,t} \in \mathbb{B}} \sum_{i \in \mathcal{P}} \mathcal{V}_{i,t+1}(b_{i,t+1}(y_{i,t}), s_{i,t+1}(y_{i,t}), \theta_{i,t+1}(y_{i,t}), z_{i,t}(y_{i,t}))$. If $\mathcal{A}_t = \emptyset$ or $\mathcal{A}_t = \mathcal{P}$ then the result of the proposition trivially follows by the induction hypothesis. If \mathcal{A}_t is a proper non-empty subset of \mathcal{P} then we can form a partition similar to the base case as follows:

$$\mathcal{V}_t(\mathbf{b}_t, \mathbf{s}_t, \boldsymbol{\theta}_t, \mathbf{z}_{t-1}) = \sum_{i \in \mathcal{P}} \mathbb{1}(b_{i,t} \leq \delta) + \sum_{i \in \mathcal{A}_t} V_{i,t+1}^1 + \sum_{i \in \mathcal{P} \setminus \mathcal{A}_t} V_{i,t+1}^0 \quad (\text{EC.10})$$

Defining $\mathcal{V}_{i,t}(b_{i,t}, s_{i,t}, \theta_{i,t}, z_{i,t-1}) := \mathbb{1}(b_{i,t} \leq \delta) + V_{i,t+1}^1 \mathbb{1}(i \in \mathcal{A}_t) + V_{i,t+1}^0 (1 - \mathbb{1}(i \in \mathcal{A}_t))$, and rewriting (EC.10) thus completes the proof. \square

Proof of Proposition 4: Again we will proceed to prove this proposition by induction on t . First we consider the base case of $t = N - 1$. Recall that the component functions at $t = N - 1$ are defined as $\mathcal{V}_{i,N-1}(b_{i,N-1}, s_{i,N-1}, \theta_{i,N-1}, z_{i,N-2}) := \mathbb{1}(b_{i,N-1} \leq \delta) + V_{i,N}^1 \mathbb{1}(i \in \mathcal{S}_{N-1}) + V_{i,N}^0 (1 - \mathbb{1}(i \in \mathcal{S}_{N-1}))$. The first term in this definition doesn't depend on $z_{i,N-2}$ so we only need to focus on the last two terms. Since these last terms are equivalent to the terms of a one-step single patient problem, they are clearly nondecreasing in $z_{i,N-2}$ by Lemma 1 and Lemma 2.

For the main induction step, consider the induction hypothesis that for some $t + 1 \leq N - 1$ for all patients $i \in \mathcal{P}$, $\mathcal{V}_{i,t+1}(b_{i,t+1}, s_{i,t+1}, \theta_{i,t+1}, z_{i,t})$ is nondecreasing in $z_{i,t}$. Now consider $\mathcal{V}_{i,t}(b_{i,t}, s_{i,t}, \theta_{i,t}, z_{i,t-1}) = \mathbb{1}(b_{i,t} \leq \delta) + V_{i,t+1}^1 \mathbb{1}(i \in \mathcal{A}_t) + V_{i,t+1}^0 (1 - \mathbb{1}(i \in \mathcal{A}_t))$, where \mathcal{A}_t are defined similarly as in Proposition 3. Then by direct application of Lemma 1 and using the induction hypothesis, we obtain the desired result. \square

Proof of Theorem 3: First note that if $|\mathcal{P}| \leq C$ then the problem is unconstrained and the result follows trivially from Corollary 1. So for the remainder of the proof we will consider the constrained case when $|\mathcal{P}| > C$.

Now let us consider the case where, at some time t , $|\mathcal{I}_t| \leq C$. Suppose for a contradiction that in this case a visit is allocated to $j \notin \mathcal{I}_t$. To show that $\mathcal{I}_t = \mathcal{S}_t$ is indeed the optimal set of indices to visit it will suffice to show that for j we have $V_{j,t}^1 \leq V_{j,t}^0$ since that would mean we cannot improve the value function by visiting j . By definition of \mathcal{I}_t , since $j \notin \mathcal{I}_t$, one of the following two conditions must hold: either (a) $B_{j,t}(y_{j,t} = 1) < 0$ or (b) $B_{j,t}(y_{j,t} = 1) \geq 0$, $z_{j,t-1} = 1$ and $B_{j,t}(y_{j,t} = 1) \leq B_{j,t}(y_{j,t} = 0)$.

Table EC.1 The table displays the mean value for each parameter for each group and the percentage of patients in each group. The bottom panel displays the percentage of patients from each group in the corresponding artificial cohort for each scenario.

Patient parameter	Group				
	A	B	C	D	E
p	0.05	5	5	7.5	0.05
μ	0.025	4	2	4	0.025
α	0.1	2	4	2	0.35
θ_0	0.7	0.7	0.7	0.7	2
λ	0.5	0.5	0.5	0.5	1.5
s_0	1	0.2	0.2	0.2	0.2
β	0.3	1.5	1.5	1.5	1.5
Simulation cohort composition					
Scenario 0	20%	20%	20%	20%	20%
Scenario 1	50%	50%	0%	0%	0%
Scenario 2	50%	0%	0%	0%	50%
Scenario 3	0%	50%	0%	50%	0%
Scenario 4	0%	50%	0%	0%	50%

First let us consider case (a). Note that, in this case, by Proposition 4 and Lemma 1, it is clear that $V_{j,t}^1 \leq V_{j,t}^0$, with equality holding when $B_{j,t}(y_{j,t} = 0) < 0$. Now consider case (b). By Proposition 4 and Lemma 1, it directly follows that $V_{j,t}^1 = V_{j,t}^0$, so clearly the objective would not be improved by visiting j , thus yielding the desired result. This establishes that at any point in time visiting patients from set \mathcal{I}_t provides an optimal policy.

Next we consider the case where $|\mathcal{I}_t| > C$, meaning \mathcal{S}_t is a strict subset of \mathcal{I}_t . In this case, the result of the theorem follows by a similar argument to that of the proof of Lemma 3. \square

EC.2. Patient parameter estimation

To solve the MLE problem in Proposition 2, we performed grid search on four parameters (s_0 , β , γ and ρ), where we searched over $S = \{0, 1, 2, 3\}$ for s_0 , $B = \{0, 1, 2, 3\}$ for β , $G = \{0.2, 0.5, 0.8, 0.9, 0.99\}$ for γ , and $R = \{0.2, 0.5, 0.8, 0.9, 0.99\}$ for ρ . We define the following variables for the grid search procedure. Let x^* denote the best optimal solution found through grid search and let w^* denote corresponding optimal value. We let \bar{x} be a variable that stores the current best optimal solution, let \bar{w} be a variable that stores the current best optimal value, let w^{UB} denote the upper bound on w^* to initialize the algorithm, and let q represent the optimal value to a particular grid search instance obtained by solving the parameter estimation problem stated in Proposition 2 (denoted below as optProb).

EC.3. Numerical experiments

Table EC.1 provides summary statistics for five patient groups and corresponding scenarios.

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