

TDT4171 Artificial Intelligence Methods

## Exercise 2

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## Part A The Umbrella domain as an HMM

- The set of unobserved variable for a given time-slice  $t$  is the single variable  $Rain_t$ .
- The set of observed variables, or evidence, for a given time-slice is the single variable  $Umbrella_t$ .
- The *dynamic model*  $P(X_t|X_{t-1})$  is:

$$\begin{pmatrix} 0.7 & 0.3 \\ 0.3 & 0.7 \end{pmatrix}$$

where each  $value_{i,j}$  is the probability for changing from state  $i$  to state  $j$ .

- The *observation model*  $P(E_t|X_t)$  is:

$$\begin{pmatrix} 0.9 & 0 \\ 0 & 0.2 \end{pmatrix}$$

where the observation  $Umbrella_t$  is *true*, and

$$\begin{pmatrix} 0.1 & 0 \\ 0 & 0.8 \end{pmatrix}$$

where the observation is *false*. Notice that the matrix for *false* is the *identity*-matrix - the *true*-matrix.

- In this model, we assume that the process is a *stationary process*. A *stationary process* is a process where the laws that transist the state from one to another never changes. In this specific case, this means that we assume that the probability for rain today given rain (or not rain) yesterday is always the same. Yet we also assumes that rain today only depends on rain yesterday, thus making it a *first-order Markov process*.

These assumptions are probably not sufficient for a weather-forecasting-system, but introducing either more sensors or more orders to the Markov process will add much complexity and add little value to this example scenario.

## Part B Filtering with FORWARD

### 1

The calculation is given in the MATLAB-file *partB1.m*. Since MATLAB indexing starts with 1 and not 0, the probabilities for day 2 is in *ans(1, 1, 3)*. Answer is 0.88336.

### 2

These calculations are given in the MATLAB-file *partB2.m*.

Day	<i>Rain</i>	$\neg$ <i>Rain</i>
0	0.50000	0.50000
1	0.81818	0.18182
2	0.88336	0.11664
3	0.19067	0.80933
4	0.73079	0.26921
5	0.86734	0.13266

Figure 1: Normalized forward-messages

## Part C Smoothing using FORWARD-BACKWARD

### 1

The calculation is given in the MATLAB-file *partC1.m*. Since MATLAB indexing starts with 1 and not 0, the probabilities for day 2 is in *ans(:,3)*, and is  $\langle 0.88336, 0.11664 \rangle$ .

### 2

These calculations are given in the MATLAB-file *partC2.m*.

Day	<i>Rain</i>	$\neg$ <i>Rain</i>
0	0.64694	0.35306
1	0.59232	0.40768
2	0.37627	0.62373
3	0.65334	0.34666
4	0.62727	0.37273
5	0.50000	0.50000

Figure 2: Normalized Backward-messages from FORWARD-BACKWARD

Day	<i>Rain</i>	$\neg$ <i>Rain</i>
0	0.044385	0.024223
1	0.066118	0.045508
2	0.090639	0.150251
3	0.45930	0.24370
4	0.69000	0.41000
5	1.00000	1.00000

Figure 3: Backward-messages from FORWARD-BACKWARD

## Part D   Code

All code is written for Octave, but should also work with MATLAB <sup>1</sup>. The files included should be:

- forward.m
- backward.m
- normalize.m
- forwardbackward.m
- partB1.m
- partB2.m
- partC1.m
- partC2.m

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<sup>1</sup>All code runs successfully on MATLAB R2011b x64. Answers given in this report is calculated with Octave 3.2.4.