TDT4136 Logic And Reasoning Systems

Exercise 2

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Innhold

1 Propositional Logic

1.1 Truth and proofs

1.1.1 Truth table for $((p \Rightarrow q) \wedge (\neg q \Rightarrow r) \wedge r) \Rightarrow p$

p	q	r	$\neg q$	(p $\Rightarrow q$)	($\neg q \Rightarrow r$)	((p $\Rightarrow q) \wedge (-\neg q \Rightarrow r) \wedge r$)	((p $\Rightarrow q) \wedge (\neg q \Rightarrow r) \wedge r) \Rightarrow p$
0	0	0	1	1	0	0	1
0	0	1	1	1	1	1	0
0	1	0	0	1	1	0	1
0	1	1	0	1	1	1	0
1	0	0	1	0	0	0	1
1	0	1	1	0	1	0	1
1	1	0	0	1	1	0	1
1	1	1	0	1	1	1	1

1.1.2 How many models can you find for the formula: $((p \Rightarrow q) \wedge (-q \Rightarrow r) \wedge r) \Rightarrow p$?

Det er 6 modeller som tilfredsstiller formelen.

1.1.3 Is the formula $((p \Rightarrow q) \wedge (\neg q \Rightarrow r) \wedge r) \Rightarrow p$ satisfiable?

Ja, den er vist tilfredsstilt i tabellen over.

1.1.4 The iacme-algorithm

Given the $KB = (p \Rightarrow q) \wedge (\neg q \Rightarrow r) \wedge r$. ACME Soft wants to sell you its fantastic inference algorithm: iacme. They claim that the following derivation is good: $KB \underset{iacme}{\vdash} p$. Explain if iacme is any good or not.

ACME Soft påstår at de har en algoritme *iacme* slik at p kan bevises ut ifra KB, eller at p medfølger fra KB. Problemet er at KB er sann også i tilfeller der p ikke er det. Dermed kan ikke algoritmen *iacme* være noe bra.

1.1.5 Express the following sentences in Propositional Logic:

"If John drives to the airport then he catches the plane." $D \Rightarrow P$

"If John's car is working then he drives to the airport." $C \Rightarrow D$

"John did not catch the plane." $\neg P$

1.1.6 Convert the above formulas to Conjunctive Normal Form and prove with resolution that: "John's car is not working."

$D \Rightarrow P$ blir $\neg D \vee P$

$C \Rightarrow D$ blir $\neg C \vee D$

$-P$ er alt på denne formen

Settningen blir da: $(\neg D \vee P) \wedge (\neg C \vee D) \wedge (\neg P)$

Tabell for denne setningen:

D	P	C	$\neg D$	$\neg P$	$\neg C$	$\neg D \vee P$	$\neg C \vee D$	$(\neg D \vee P) \wedge (\neg C \vee D) \wedge (\neg P)$
0	0	0	1	1	1	1	1	1
0	0	1	1	1	0	1	0	0
0	1	0	1	0	1	1	1	0
0	1	1	1	0	0	1	0	0
1	0	0	0	1	1	0	1	0
1	0	1	0	1	0	0	1	0
1	1	0	0	0	1	1	1	0
1	1	1	0	0	0	1	1	0

Vi ser at den eneste modellen der kunnskapsbasen er sann, er en modell der bilen ikke fungerer (-D), altså kan vi konkludere med at bilen ikke fungerer.

1.1.7 The Deduction Theorem

Given $a_1, a_2, \ldots, a_n \models a$. Prove that $a_1, a_2, \ldots, a_{n-1} \models (a_n \Rightarrow a)$ by using the Deduction Theorem and standard logical equivalences?

Vi bruker deduksjonsteoremet, $a \models b \Leftrightarrow a \Rightarrow b$, og får følgende utfall:

$$\frac{a \vee (b \Rightarrow a) \equiv (b \Rightarrow a) \qquad \dfrac{(a \models b) \Leftrightarrow (a \Rightarrow b) \qquad \dfrac{a_1, a_2, \ldots, a_n \models a}{a_1, a_2, \ldots, a_{n-1} \models a \vee a_n \models a}}{a_1, a_2, \ldots, a_{n-1} \models a \vee a_n \Rightarrow a}}{a_1, a_2, \ldots, a_{n-1} \models (a_n \Rightarrow a)}$$

(Jeg ønsker gjerne en kommentar på denne, da jeg føler meg litt på gyngende grunn…)

2 First Order Predicate Logic

2.1 Express the following sentences in first order predicate logic

2.1.1 A boy kisses a girl

$\exists x\ \exists y\ boy(x) \wedge girl(y) \wedge kiss(x, y)$

2.1.2 Francine talked to every person in the room

$\forall x\ person(x) \wedge in(x, Room) \Rightarrow talk\_to(Francine, x)$

2.1.3 All paid jobs absorb and degrade the mind

$\forall x\ paid(x) \wedge job(x) \Rightarrow absorbs(x, Mind) \wedge degrades(x, Mind)$

2.2 Express the following sentences in first order predicate logic

2.2.1 Jack loves a girl who doesn't love him

$\exists x\ girl(x) \wedge loves(Jack, x) \wedge \neg loves(x, Jack)$

2.2.2 Jack was given a rose by all the girls, except by the one he loves

$\forall x\ \exists y\ \exists z\ girl(x) \wedge \neg loves(Jack, x) \wedge girl(z) \wedge loves(Jack, z) \wedge rose(y) \Rightarrow gives(x, y, Jack) \wedge \neg gives(z, y, Jack)$

2.2.3 Not everything that can be counted counts, and not everything that counts can be counted

$\exists x\ \exists y\ (countable(x) \wedge \neg counts(x)) \wedge (\neg countable(y) \wedge counts(y))$