Thin Disc Flight Dynamics with Applications to Ballochore Seed Dispersal

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Final Thesis Presentation

Outline

- Motivations
- Methods
- Theory
- Results
- Conclusions

Motivations

Biological & Physical

Example Seed Dispersal Mechanisms

Anemochory (Wind-Aided Seed Dispersal)

Credit: [Tenor]

Ballochory (Ballistic Seed Dispersal)

Credit: Smithsonian Institute

Seed Dispersal Characteristic Ranges

	Dispersal Mode	Description	Example Species	Characteristic Range
Independent (autochory)	Barochory	Dispersal by gravity	Apple, Coconut	0.1 - 1 m
	Ballochory	Ballistic launching of seeds	Squirting cucumber	1 - 10 m
Vector-dependent (allochory)	Anemochory	Wind-aided dispersal	Dandelion	10 - 1000 m
	Zoochory	Animal-aided dispersal	Apple, Acorn (Oak)	10 - 1000 m
	Hydrochory	Water-aided dispersal	Apple, Coconut	10 - 100,000 m

Far more effective…

Many plants rely on a combination of dispersal mechanisms (diplochory), both in “series” and “parallel”

Credit: Vittoz and Engler, *Seed Dispersal Distances*

Guiding Questions

Adaptive advantages of ballochory over other dispersal methods?

What seed dispersal parameters are optimized by evolution?

How effective are seed dispersal mechanisms in certain ballochores?

Methods

Modelling & Simulation

Case Study: *Ruellia Ciliatiflora*

- *R. ciliatiflora:*
  - Perennial herb
  - South America, SE U.S.
- Seed dispersal:
  - Ballochore
  - Thin, disc-like seeds
  - High spin
- Well-studied in literature
  - Eric Cooper '18 Senior Thesis

Ruellia Ciliatiflora - Overview

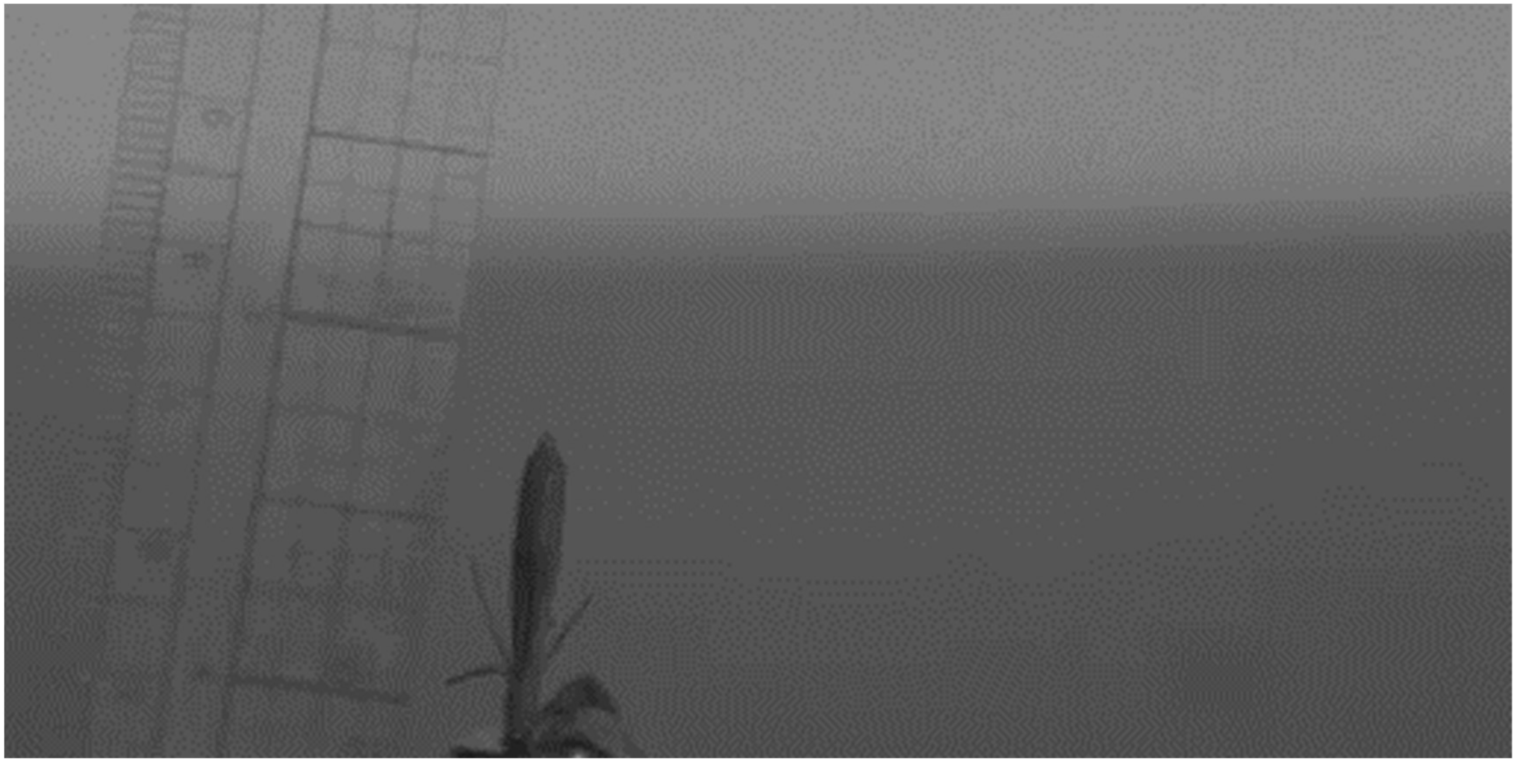
Credit: Erin Tripp & Eric Cooper

a) Fruits of *R. ciliatiflora*

b) Close-up of a single fruit shows arrangement of seeds inside

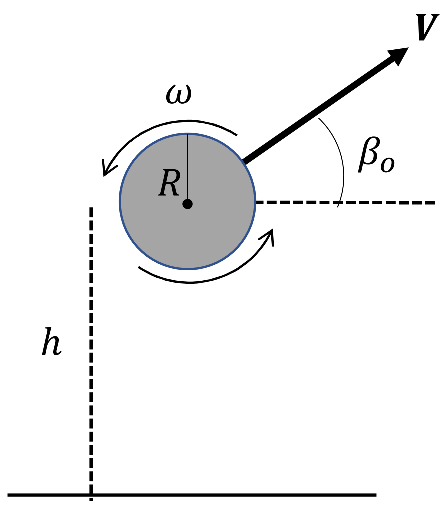
c) Individual seeds of *R. ciliatiflora* with 1mm scale bar

Ruellia Ciliatiflora – Seed Dispersal

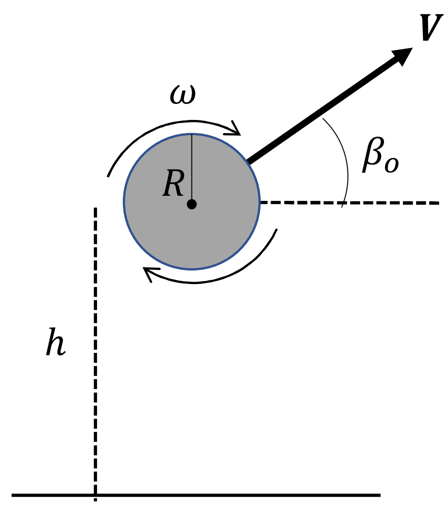


Credit: Eric Cooper, Dwight Whitaker

Seed Orientation & Stability



Backspin Orientation – **stable**



Topspin Orientation – **unstable**

Approach

- Model seed as thin disc, derive equation of motion for a thin, spinning disc moving in air
- Simulate dispersal events under a variety of initial conditions, search for conditions that yield optimal cases (dispersal range, others?)
- Focus on initial launch orientation ϕ_o (Backspin, sidespin, topspin)

Theory

Forces

- Gravity
- Drag
- Lift
- Magnus

Gravity

- $\vec{F}_g = -mg\hat{y}$

Lift & Drag

- Result from displacement of fluid by a moving body
- Expressed as $|\mathrm{F}| = C\,\frac{1}{2}\rho|V|^2 A$

Lift

- Acts perpendicular to flow
- $C_L = 2\pi\sin\alpha$ (inviscid result)
- $A = \pi R^2\cos\alpha$ --- planform area
- High pressure below wing, low pressure above – Effectively, the wing "sucks" itself upward

Drag

- Acts antiparallel to flow
- C_D determined experimentally
- $A = \pi R^2\cos\alpha + 2Rd\sin\theta$ --- frontal area
- Also, lift-induced drag, scales with ${C_L}^2$

Lift

Direction Constraints

$$\mathbf{F_L} \cdot \mathbf{V} = 0$$

$$\mathbf{F_L} \cdot (\mathbf{V} \times \mathbf{L}) = 0$$

$$|\mathbf{F_L}| = \frac{\pi^2 r^2 \rho}{2} \sin(2\alpha)|\mathbf{V}|^2$$

System of 3 Equations

Component-wise Result

$$F_{Lx} = \frac{\frac{\pi^2 r^2 \rho}{2} \sin(2\alpha)|\mathbf{V}|^2 (L_y V_x V_y + L_z V_x V_z - L_x(V_y^2 + V_z^2))}{\sqrt{\mathbf{V}^2 (L_x^2(V_y^2+V_z^2) + L_y^2(V_x^2+V_z^2) + L_z^2(V_x^2+V_y^2) - 2L_x L_z V_x V_z - 2L_y V_y(L_x V_x + L_z V_z))}}$$

$$F_{Ly} = -\frac{\frac{\pi^2 r^2 \rho}{2} \sin(2\alpha)|\mathbf{V}|^2 (L_y(V_x^2 + V_z^2) - V_y(L_x V_x + L_z V_z))}{\sqrt{\mathbf{V}^2 (L_x^2(V_y^2+V_z^2) + L_y^2(V_x^2+V_z^2) + L_z^2(V_x^2+V_y^2) - 2L_x L_z V_x V_z - 2L_y V_y(L_x V_x + L_z V_z))}}$$

$$F_{Lz} = -\frac{\frac{\pi^2 r^2 \rho}{2} \sin(2\alpha)|\mathbf{V}|^2 (L_z(V_x^2 + V_y^2) - V_z(L_x V_x + L_y V_y))}{\sqrt{\mathbf{V}^2 (L_x^2(V_y^2+V_z^2) + L_y^2(V_x^2+V_z^2) + L_z^2(V_x^2+V_y^2) - 2L_x L_z V_x V_z - 2L_y V_y(L_x V_x + L_z V_z))}}$$

But remember, L components are represented as

$$L_x = \frac{V_x}{|\mathbf{V}|}\sin(\alpha) + \frac{V_y}{|\mathbf{V}|}\cos(\alpha)\cos(\phi) + \frac{V_z}{|\mathbf{V}|}\cos(\alpha)\sin(\phi) \tag{2.17}$$

$$L_y = \frac{V_y}{|\mathbf{V}|}\frac{V_x}{\sqrt{V_x^2+V_z^2}}\sin(\alpha) - \frac{\sqrt{V_x^2+V_z^2}}{|\mathbf{V}|}\cos(\alpha)\cos(\phi) + \frac{V_y}{|\mathbf{V}|}\frac{V_z}{\sqrt{V_x^2+V_z^2}}\cos(\alpha)\sin(\phi) \tag{2.18}$$

$$L_z = \frac{V_z}{\sqrt{V_x^2+V_z^2}}\sin(\alpha) - \frac{V_x}{\sqrt{V_x^2+V_z^2}}\cos(\alpha)\sin(\phi) \tag{2.19}$$

Drag

- C_D determined experimentally via video analysis of *R. ciliatiflora* seeds
- Lift-induced drag adds to C_D, 'penalty' associated with lift production

$$C_{D,induced} = \frac{{C_L}^2}{\pi * AR * 0.5} = 2\pi^2 \, (\sin\alpha)^2$$

Credit: NASA Langley

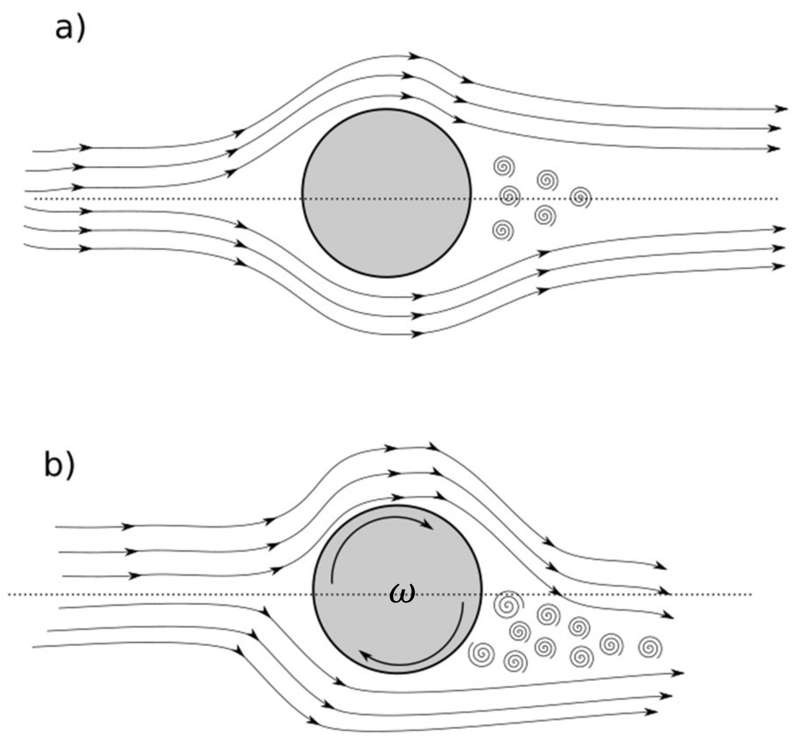
$$F_{Dx} = |\mathbf{F_D}| \sin(\alpha)\cos(\beta)\cos(\gamma) = -\frac{(0.301 + 2\pi^2 \sin(\alpha)^2)}{2}(\pi r^2 |\sin(\alpha)| + 2dr|\cos(\alpha)|)\rho|\mathbf{V}|\mathbf{V_x})$$

$$F_{Dy} = |\mathbf{F_D}| \sin(\alpha)\cos(\beta)\cos(\gamma) = -\frac{(0.301 + 2\pi^2 \sin(\alpha)^2)}{2}(\pi r^2 |\sin(\alpha)| + 2dr|\cos(\alpha)|)\rho|\mathbf{V}|\mathbf{V_y})$$

$$F_{Dz} = |\mathbf{F_D}| \sin(\alpha)\cos(\beta)\cos(\gamma) = -\frac{(0.301 + 2\pi^2 \sin(\alpha)^2)}{2}(\pi r^2 |\sin(\alpha)| + 2dr|\cos(\alpha)|)\rho|\mathbf{V}|\mathbf{V_z})$$

Magnus Effect

- Only experienced by spinning bodies
- Viscous, spin-induced deflection of wake turbulence behind bluff body
- $F_M = C_M \frac{1}{2} \rho |V|^2 A(\widehat{\omega} \times \hat{V})$
- Complex dependence of C_M on cylinder shape and material, and flow properties
- Determine C_M through video analysis of *R. ciliatiflora* seeds



Magnus Effect

Direction: $\vec{\omega} \times \vec{\boldsymbol{V}}$

$$F_{Mx} = C_M \rho \frac{|\mathbf{V}|^2 \omega}{|\mathbf{L}|}(2rd)(L_y V_z - L_z V_y)$$

$$F_{My} = C_M \rho \frac{|\mathbf{V}|^2 \omega}{|\mathbf{L}|}(2rd)(L_z V_x - L_x V_z)$$

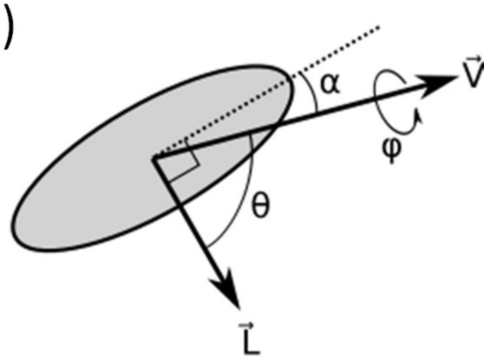
$$F_{Mz} = C_M \rho \frac{|\mathbf{V}|^2 \omega}{|\mathbf{L}|}(2rd)(L_x V_y - L_y V_x)$$

Torques

- Lift Torque
- Lift Pseudo-torque
- Gravity Pseudo-torque

Lift Torque

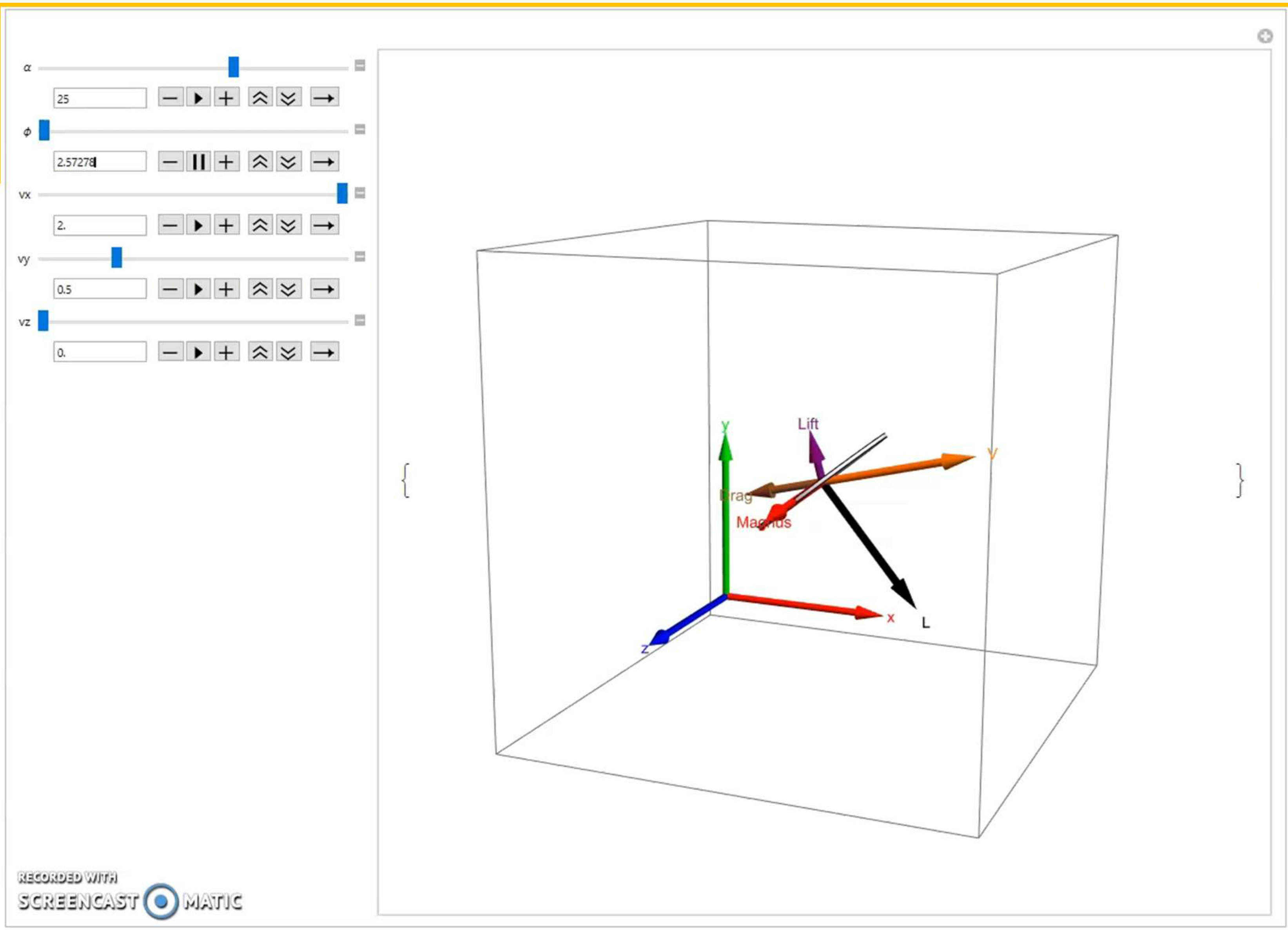
- Aerodynamic center: Point where lifting torque is independent of α
- Thin airfoil: a.c. is at 25% chord length (quarter-chord point)
- Acts in direction to increase α – but remember, disc is spinning rapidly
  - Torque instead induces a change in ϕ, (precession of $\vec{L}$ about $\vec{V}$)



$$|\boldsymbol{\tau}_{\boldsymbol{lift}}| = \left(\vec{r}_{a.c.} \times \vec{F}_{lift}\right) = \left(\frac{\pi r}{8}\right)\left(\frac{\pi^2}{2} r^2 \rho |\boldsymbol{V}|^2 \sin 2\alpha\right) \sin\alpha$$

$$= \frac{\pi^3}{16} r^3 \rho |\boldsymbol{V}|^2 \sin 2\alpha \sin\theta$$

$$\boxed{\frac{d\phi}{dt} = -\frac{|\boldsymbol{\tau}_{\boldsymbol{lift}}|}{|\boldsymbol{L}| \sin\theta} = -\frac{\pi^3}{16|\boldsymbol{L}|} r^3 \rho |\boldsymbol{V}|^2 \sin 2\alpha}$$



α
25
ϕ
2.57278
vx
2.
vy
0.5
vz
0.
Lift
V
Drag
Magnus
y
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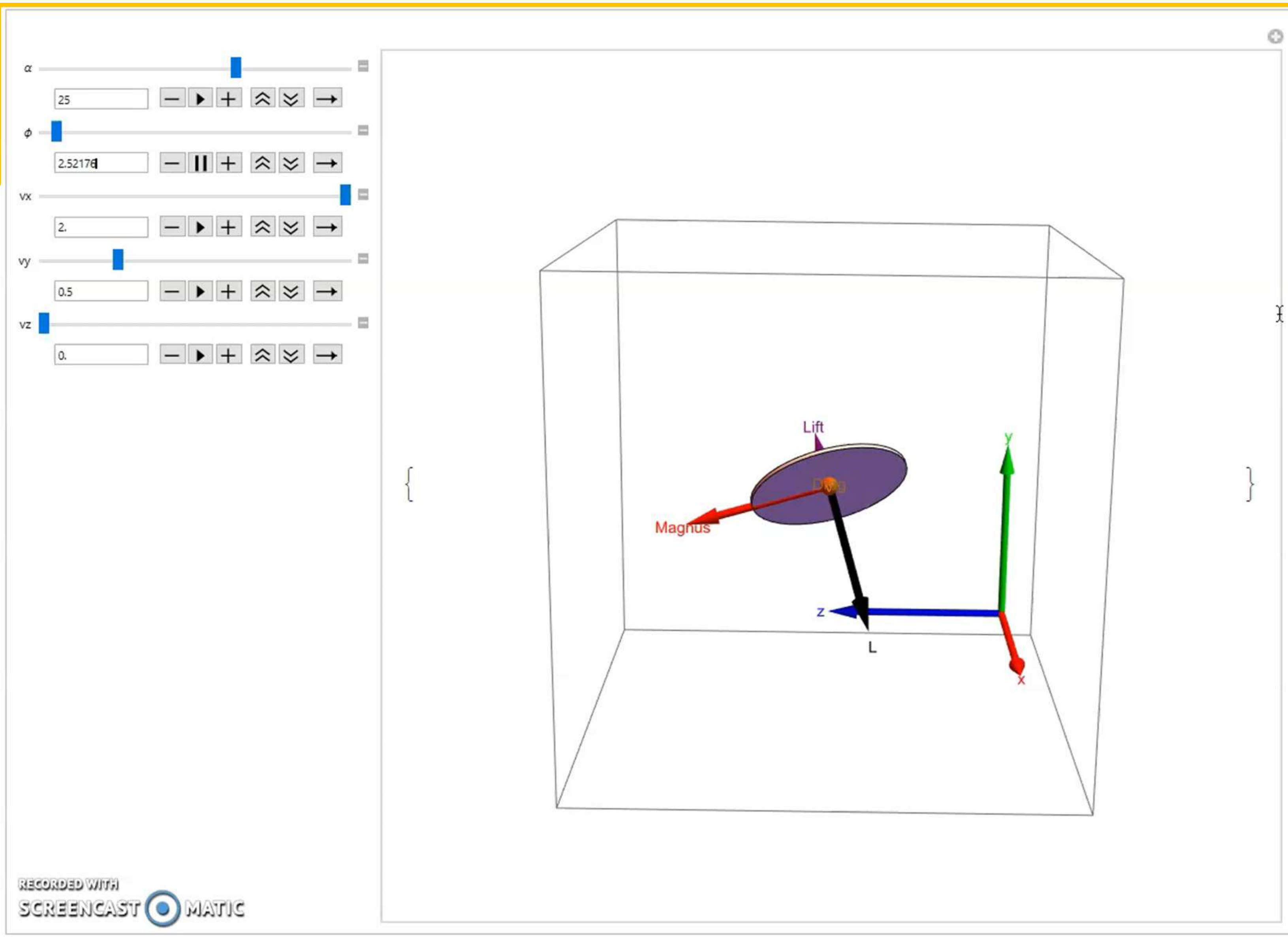
z
L
RECORDED WITH
SCREENCAST MATIC



α
25
ϕ
2.52176
vx
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vy
0.5
vz
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Lift
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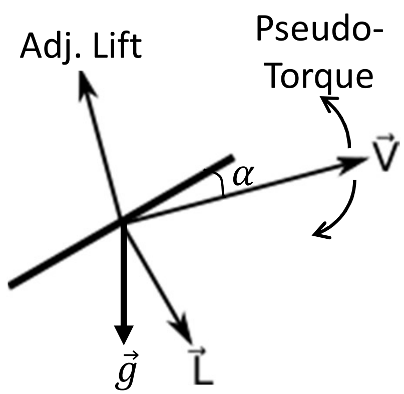
RECORDED WITH
SCREENCAST MATIC

Pseudo-Torques

- Fictional, think Coriolis Force – result of choice of coordinate frame
- Result of defining body frame with respect to $\vec{V}$ - changes in $\vec{V}$ changes the disc orientation as viewed from the body frame
- Only impact α, not ϕ

$$\boxed{\frac{d\alpha}{dt} = \frac{|\boldsymbol{F}_\perp|}{m|\boldsymbol{V}|} = \frac{\pi^2 r^2}{2m}\rho|\boldsymbol{V}|\sin 2\alpha - \frac{g}{|\boldsymbol{V}|}\cos\beta\cos\phi}$$



Equation of Motion

Translational

$$\ddot{x} = \frac{1}{m}(F_{Lx} + F_{Mx} - F_{Dx})$$

$$\ddot{y} = \frac{1}{m}(F_{Ly} + F_{My} - F_{Dy} - mg)$$

$$\ddot{z} = \frac{1}{m}(F_{Lz} + F_{Mz} - F_{Dz})$$

Rotational

$$\dot{\alpha} = g\frac{\sqrt{V_x^2 + V_z^2}}{|\mathbf{V}|^2}\cos(\phi) - \frac{\rho|\mathbf{V}|\pi^2 r^2}{2m}\sin(2\alpha)$$

$$\dot{\phi} = -\frac{\pi^3 r^3 \rho |\mathbf{V}|^2}{16|\mathbf{L}|}\sin(2\alpha)$$

System of 5 coupled nonlinear differential equations

Stability

- Small angle approximation yields

$$\frac{d\alpha}{dt} = \frac{\sqrt{V_x{}^2 + V_z{}^2}}{|\boldsymbol{V}|^2} g\phi - \frac{\pi^2 r^2}{m} \rho |\boldsymbol{V}| \alpha$$

$$\frac{d\phi}{dt} = \frac{\pi^3}{16|\boldsymbol{L}|} r^3 \rho |\boldsymbol{V}|^2 \alpha$$

- Combine equations, solve for ϕ

$$\frac{d^2\phi}{dt^2} + 2\xi \frac{d\phi}{dt} + \omega_o{}^2 = 0 \quad \text{where}$$

$$\omega_o = \sqrt{\frac{\pi^3 r^3 \rho g}{16\boldsymbol{L}||} \sqrt{V_x{}^2 + V_z{}^2}} \approx 15s$$

$$\xi = \frac{4\pi r \rho \, |\boldsymbol{L}| \, |\boldsymbol{V}|^2}{m^2 g \sqrt{V_x{}^2 + V_z{}^2}} \approx 4$$

Overdamped harmonic oscillator

Backspin - only stable orientation

Convergence $\sim 0.25\ s$

Coordinate Systems

Laboratory Frame

- Fixed, inertial reference frame
- Define disc velocity with respect to laboratory frame
- Describes disc position with respect to launch point
- Disc translational dynamics

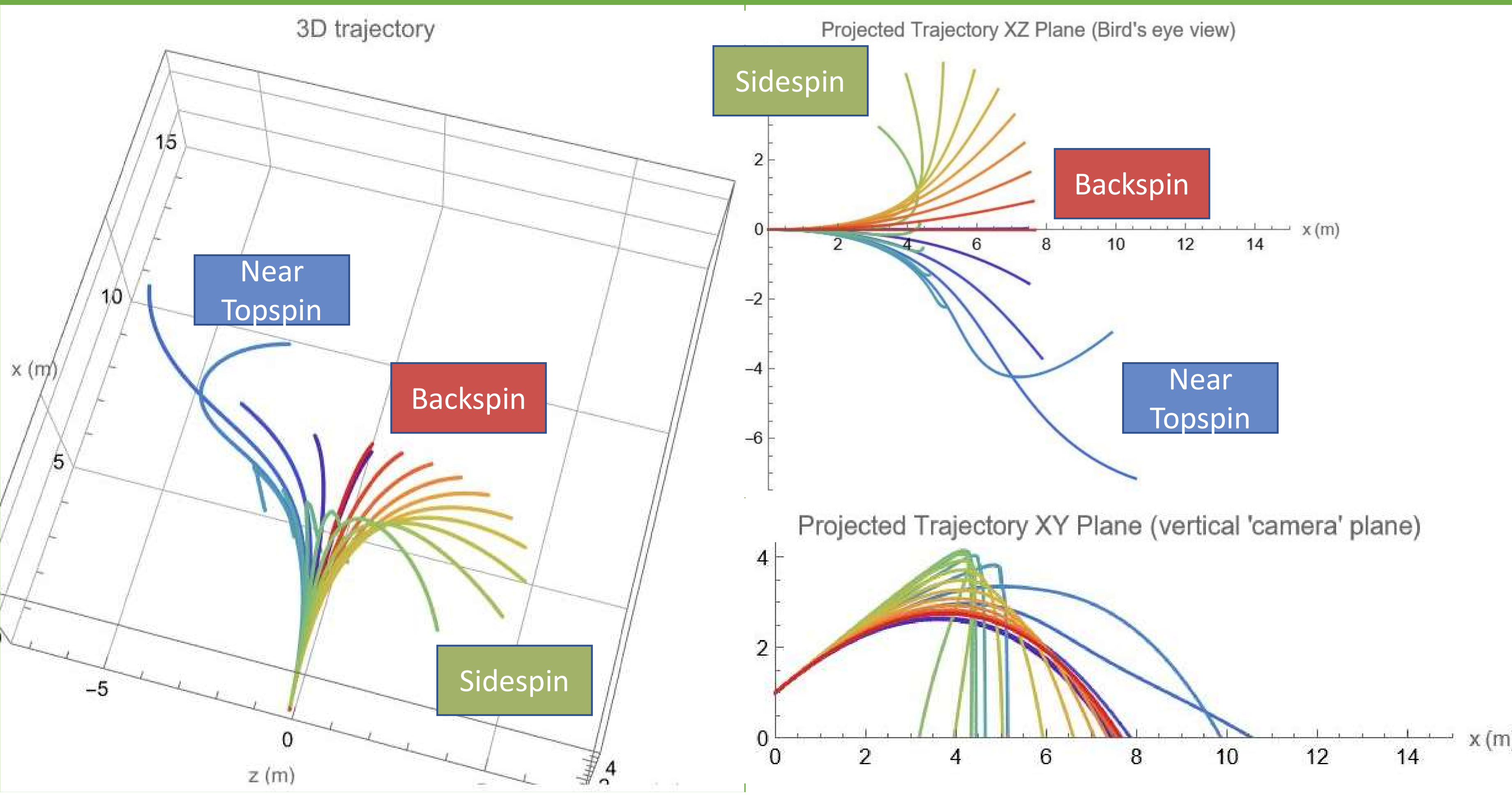
Body Frame

- Fixed to center of disc
- Orientation defined with respect to disc’s velocity vector
- Describes disc orientation with respect to airflow
- Disc angular dynamics

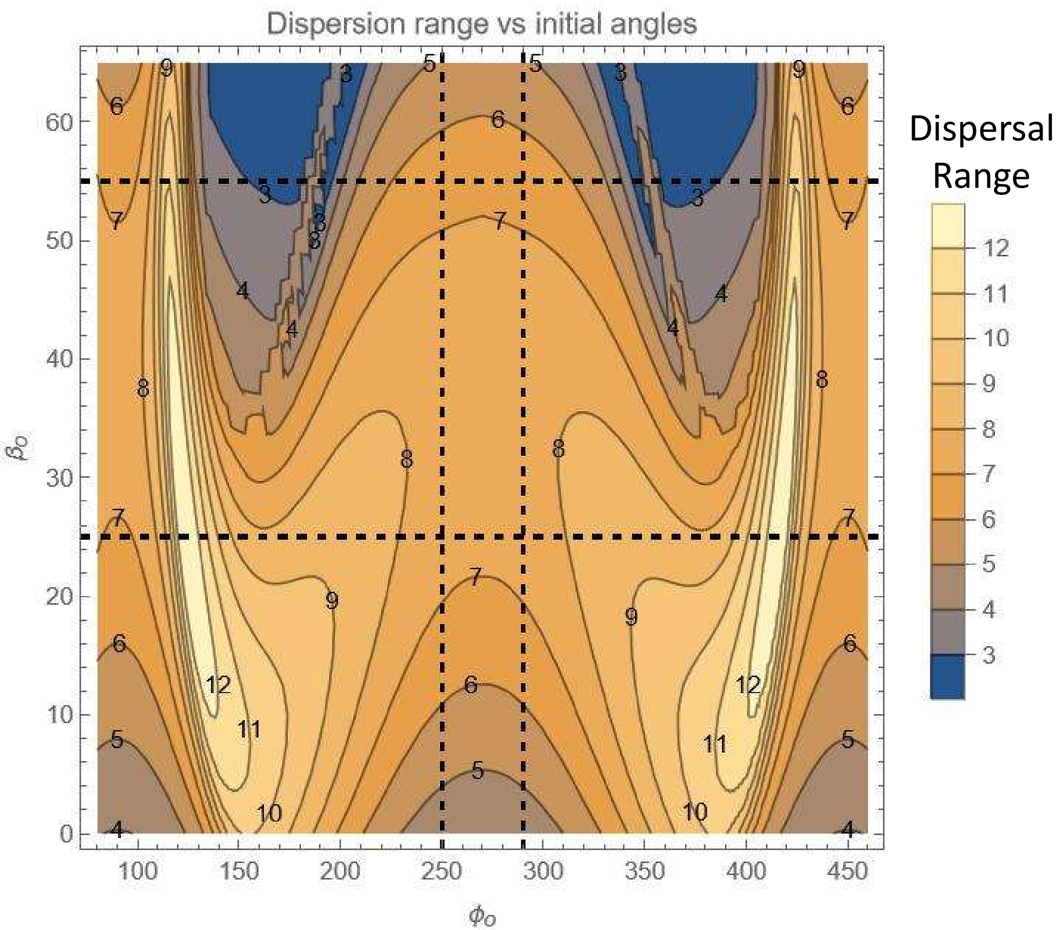
Results

Numerical Simulation of E.o.M.

Seed Flights – ϕ 10^o increments ($90^o - 270^o$)



Seed Flights – Varied ϕ & β

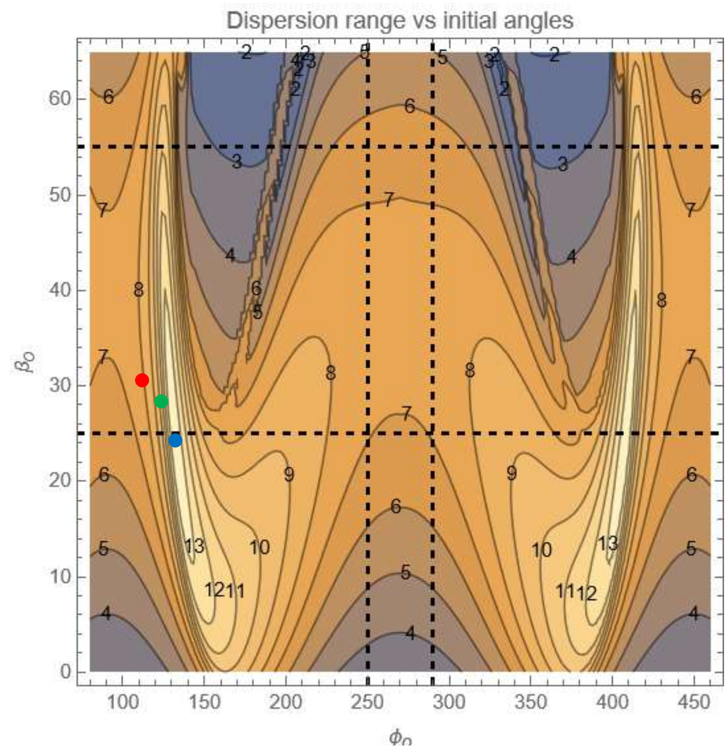


Parameter	Initial Value
Spin rate ω	1500 Hz
Launch height h	1m

Graphic:
X: Launch Orientation ϕ
Y: Launch Angle β
Contour: Dispersal Range (m)
Lines: Range of conditions observed for *R. ciliatiflora*

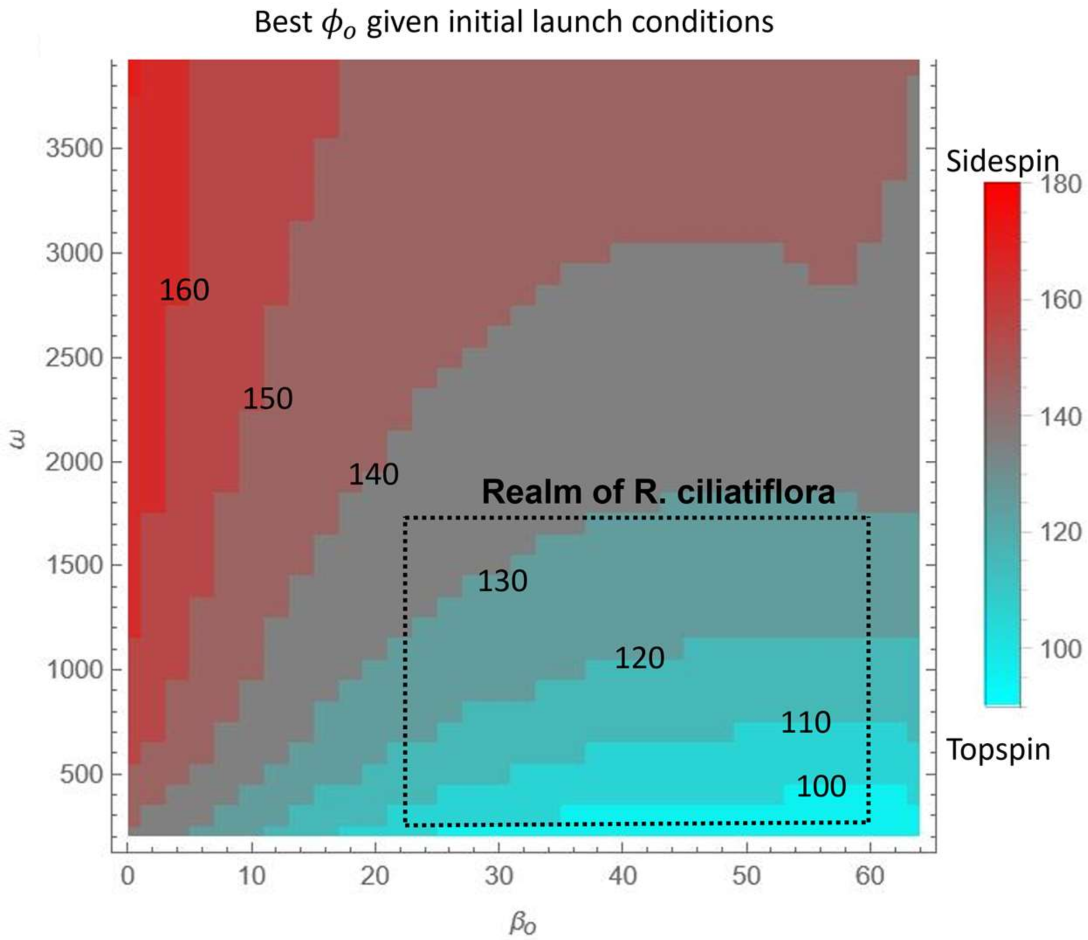
Observations:
- Maximum dispersal range near topspin ~130^o
  - Narrow ridge. Deep valley adjacent.
  - Perturbation in ϕ_o of ~20^o → failed launch
- Good dispersal range at backspin
  - Not as good as best range, but more robust
  - Consistent with experimental observations (avg observed range ~7m)

Seed Flights – Varied ϕ, β, ω



Spin rate $\omega = 1500 Hz$
Launch height h = 0.6m

Seed Flights – Optimal ϕ given β, ω



<u>Graphic</u>:
X: Launch Angle (degree)
Y: Spin Rate (Hz)
Contour: Range-maximizing ϕ_o
Box: Approximate conditions observed for *R. ciliatiflora*

Note: ~90,000 runs to generate this plot

<u>Observations</u>:
- Ideal ϕ_o varies by ~40 degrees in the realm of *R. ciliatiflora* launch conditions
- Recall that ~20 degrees change in ϕ_o can put you in the valley of failed launches

Conclusions

Range vs Robustness

Conclusions

- Seed dispersal ranges on the order of 12m are possible, but difficult to achieve
  - Optimal launch orientation varies by ~40 degrees in the parameter space occupied by *R. ciliatiflora*
  - Optimal depends on height, launch angle, spin rate, seed location in fruit
  - Proximity to danger zone – failed launches of only ~3m
- Practically, backspin is more accessible, consistent range of ~7m
  - Robustness – Independent of other launch parameters, far away from danger zone
  - Simplicity – Obtainable with simple fruit, works for all seeds in fruit
  - Efficiency – Requires less information to encode in DNA
  - Structural – Fruits can grow vertically, avoid a cantilever stress

Future Work

- Parameter tuning:
  - Induced drag efficiency factor – lit review and/or wind tunnel to determine e for disc
  - Magnus & drag coefficients – more video analysis, wind tunnel?
- Code optimization:
  - Improve my functions –inefficient
  - Check numerical solver used by Mathematica – validate results
- Other species:
  - Different seed morphologies & launch parameters – *Hura crepitans*
  - Comparative evolution/biology

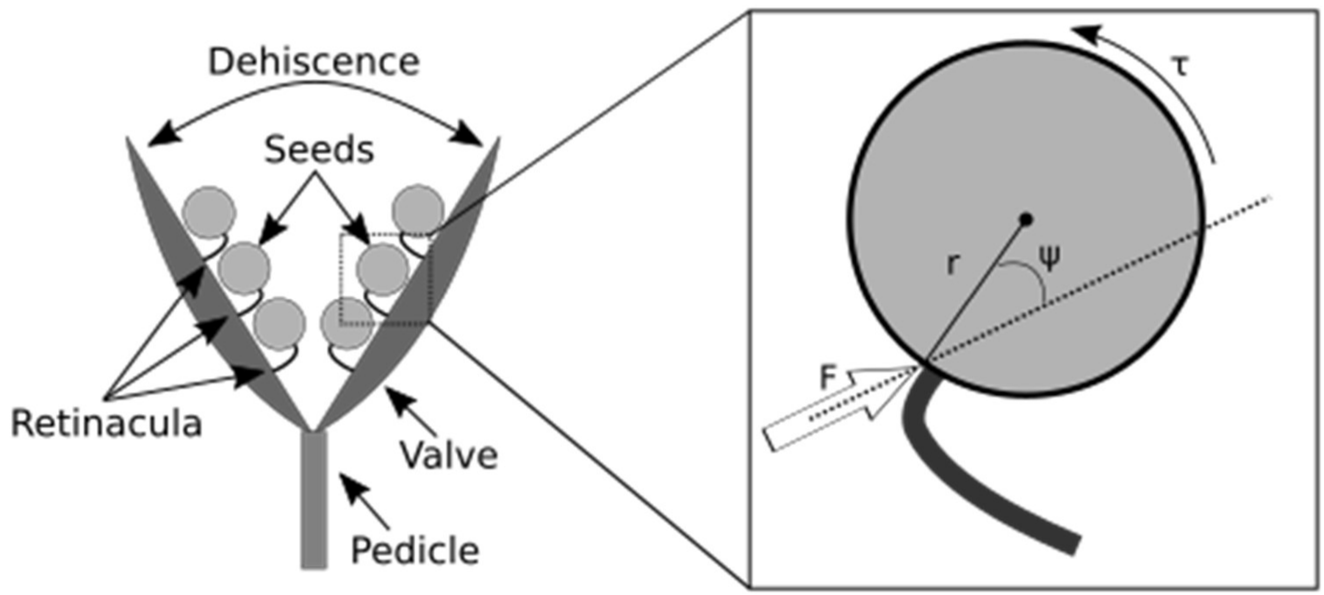
Acknowledgements

- Eric Cooper ('18) & Prof. Whitaker
- Phys/Astro Class of '22 / Entering '18
- PP XC/TF + Friends
- Entire Phys/Astro Department

Questions?

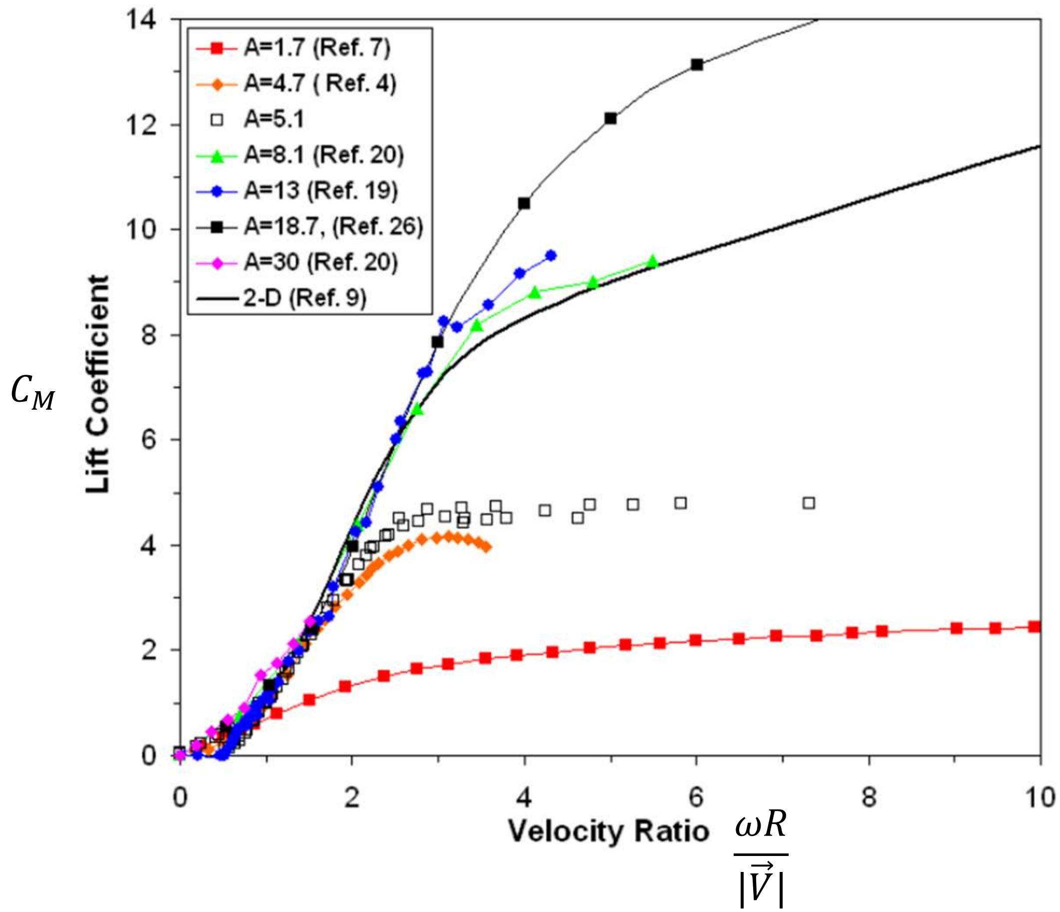
Unused Slides

R. Ciliatfilora Launch Mechanism



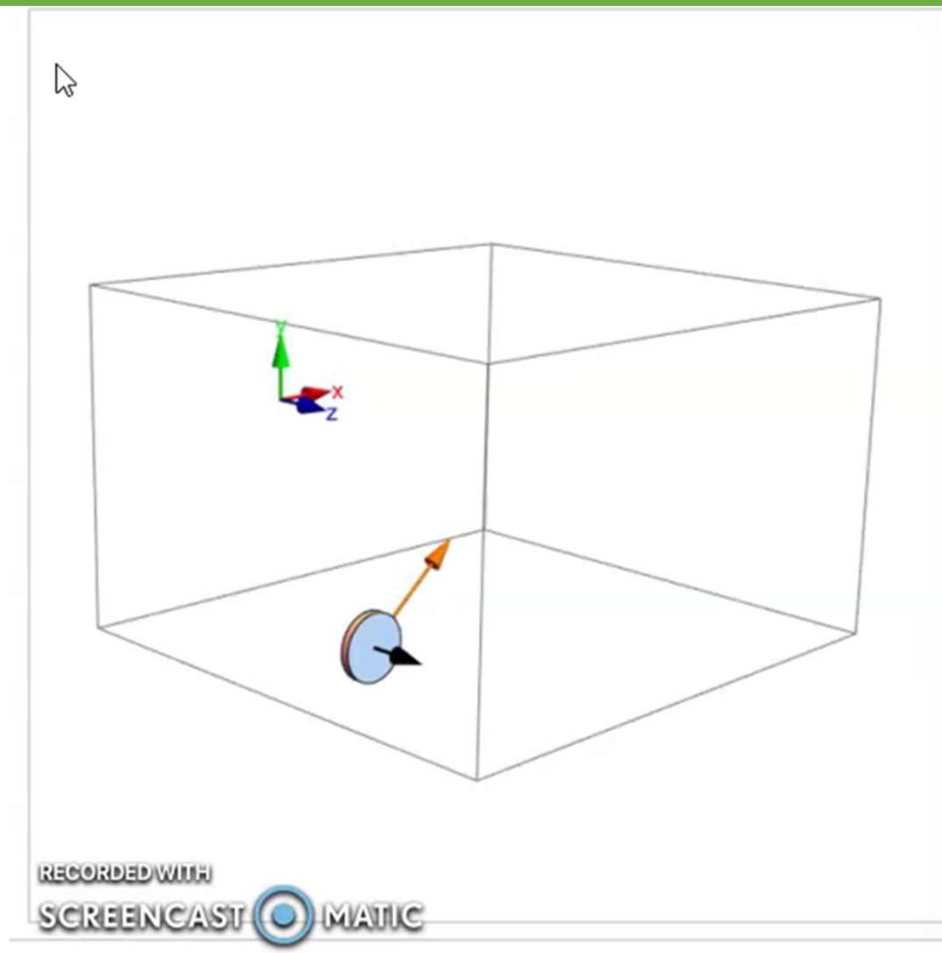
Maximum attainable rotation rate ~3200 rot/sec = ~200k rpm
Observed rotation rates ~1600 rot/sec = ~100k rpm – Largest naturally occurring rotation rate
(For reference, jet engine rotates ~20k rpm, small prop plane ~2k rpm)

Magnus Effect

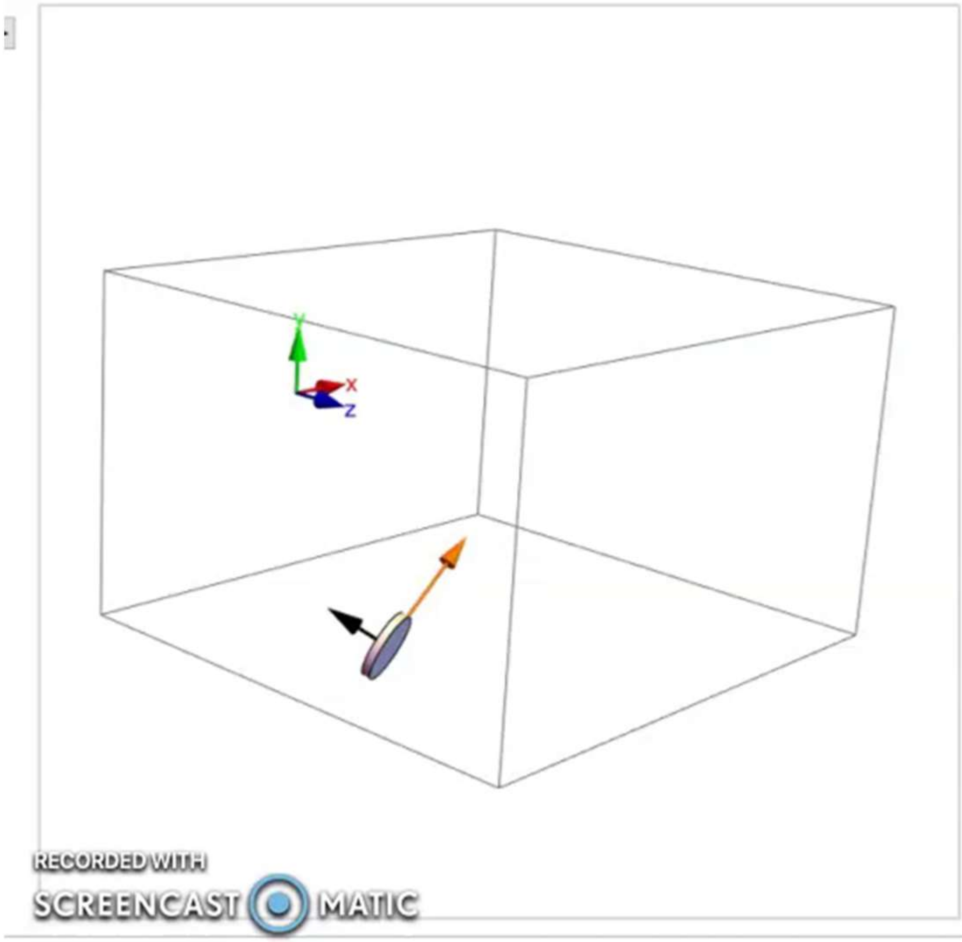


Credit: C. Badalamenti, *The Effects of Endplates on a Rotating Cylinder in Crossflow*

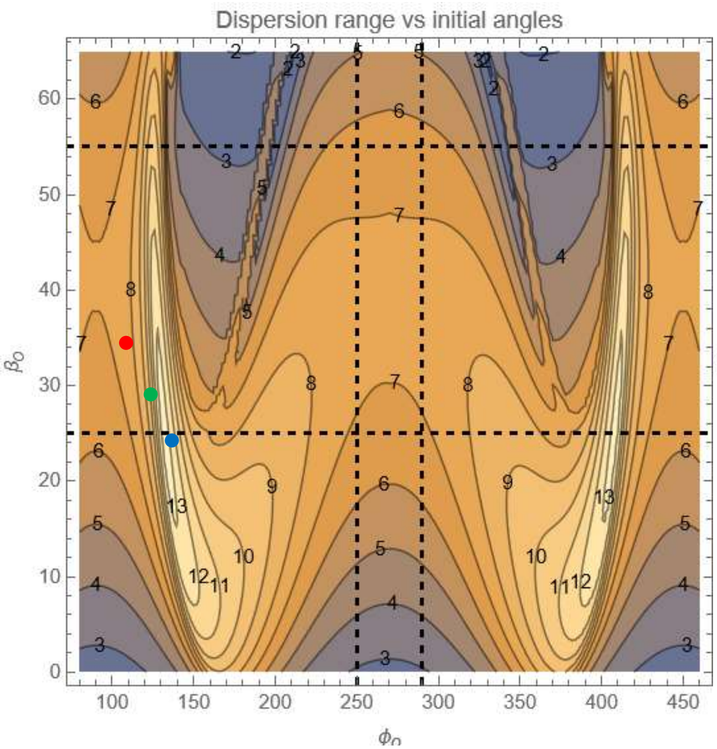
Example Flights – Backspin $\phi = 270^o$



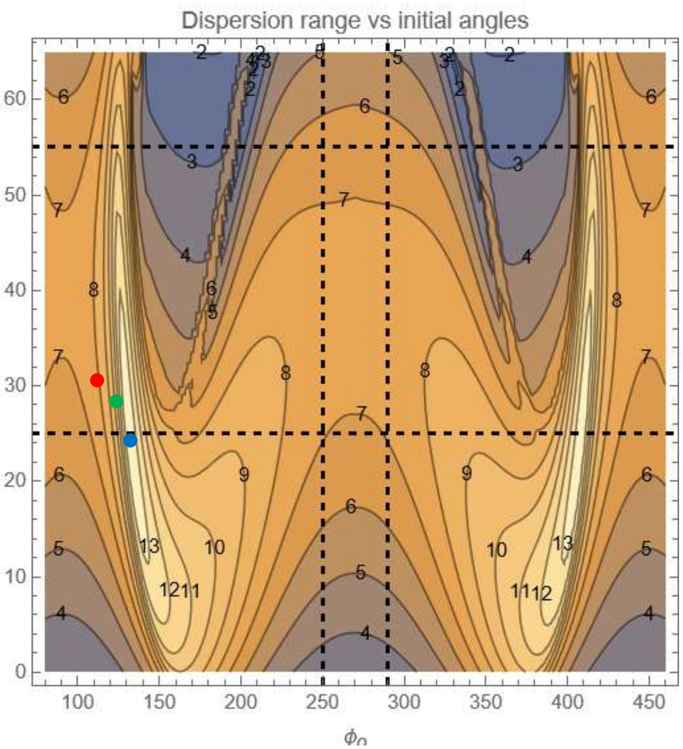
Example Flights – Near topspin $\phi = 120^o$



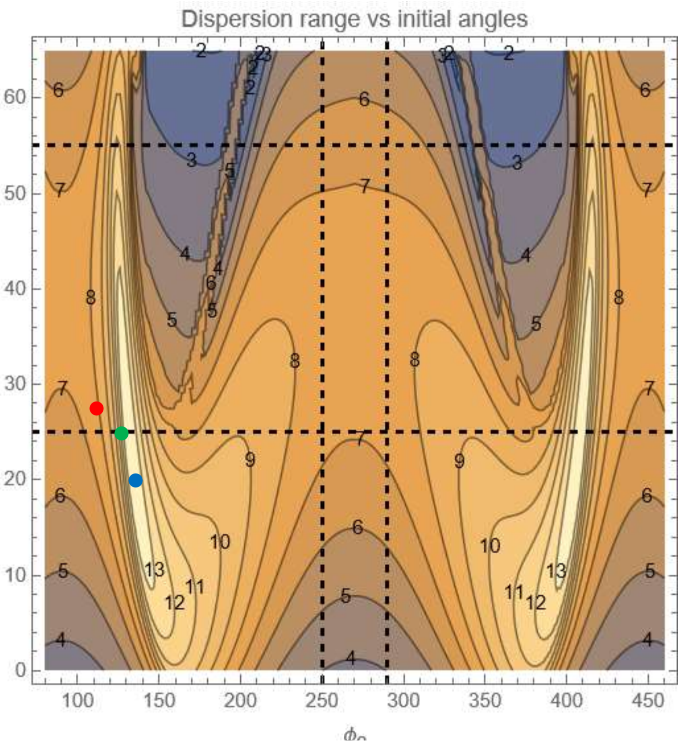
Seed Flights – Varied ϕ, β, ω, h



Spin rate $\omega = 1500Hz$
Launch height h = 0.4m



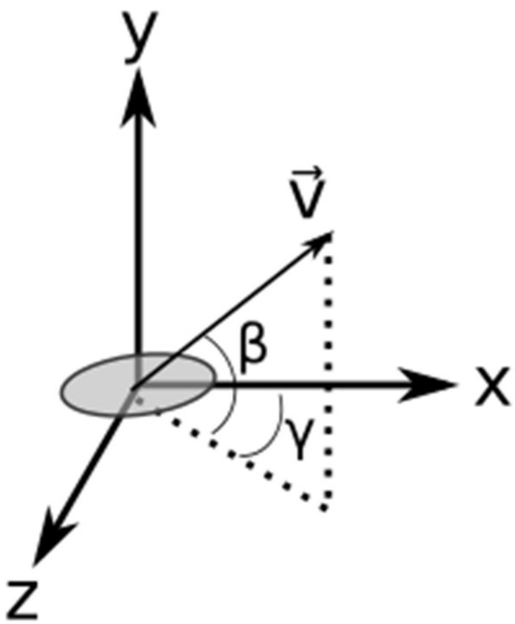
Spin rate $\omega = 1500Hz$
Launch height h = 0.6m



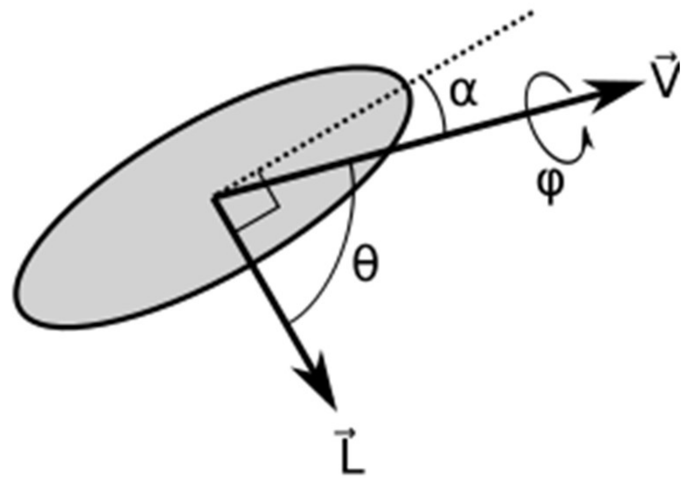
Spin rate $\omega = 1500Hz$
Launch height h = 0.8m

Coordinate Systems

Laboratory Frame



Body Frame



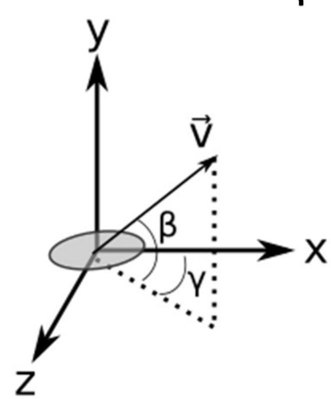
Coordinate System Variables

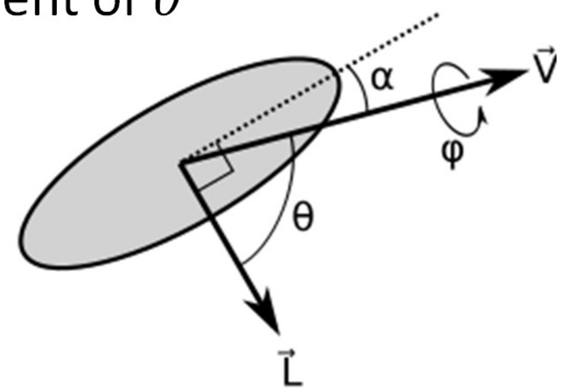
Laboratory Frame

- $\vec{X} = [x, y, z]$: Position vector of disc center of mass
- $\vec{V} = \dot{\vec{X}} = [\dot{x}, \dot{y}, \dot{z}]$: Velocity vector of disc center of mass
- β: Polar angle of velocity – Euler angle 1 for body frame construction
- γ: Azimuthal angle of velocity – Euler angle 2 for body frame construction

Body Frame

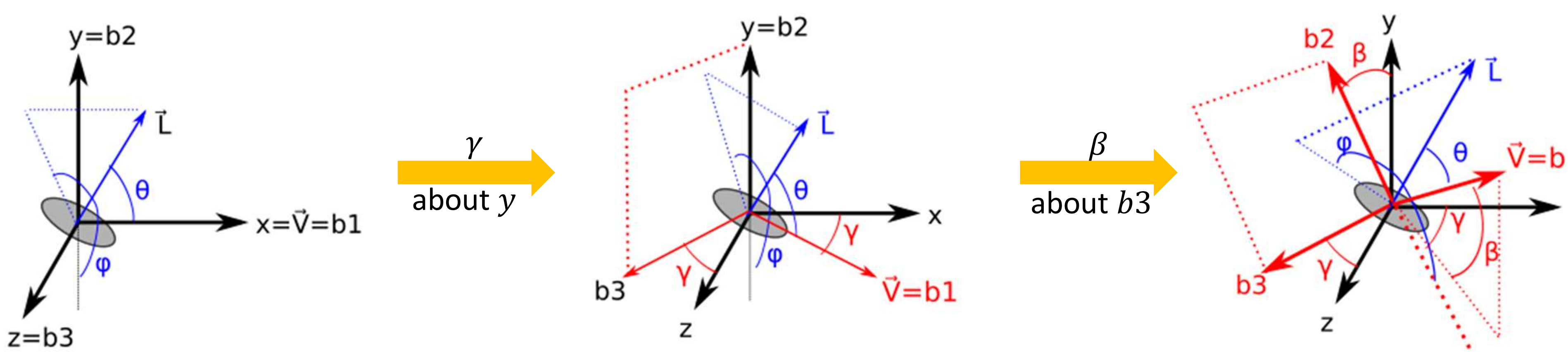
- $\vec{L}$: Disc angular momentum vector. Uniquely describes orientation
- θ: Polar angle of L w.r.t. $\vec{V}$
- ϕ: Azimuthal/Precession angle of L w.r.t. $\vec{V}$
- α: Complement of θ





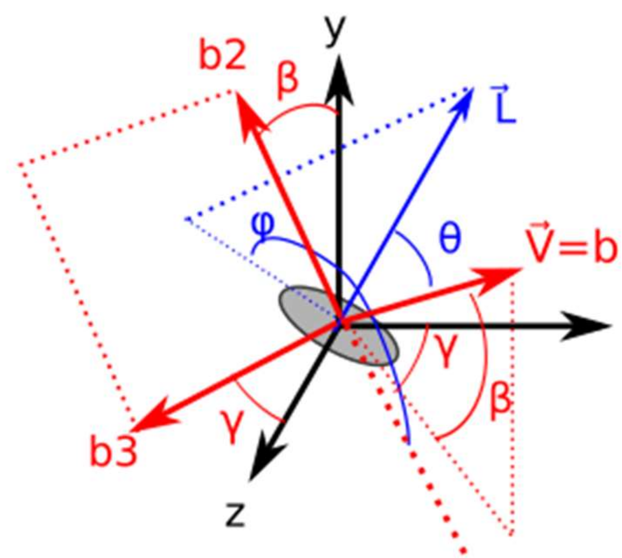
Linking the Coordinate Systems

- Start with $\vec{V}$ aligned with x axis. Then body frame axes $[b1, b2, b3]$ are aligned with lab frame axes $[x, y, z]$.
- Rotate $\vec{V}$ about y axis by angle γ
- Rotate $\vec{V}$ about the new $b3$ axis by angle β



Linking the Coordinate Systems

- $\boldsymbol{L}_{lab} = \boldsymbol{M}_{rot} * \boldsymbol{L}_{body}$
- $\boldsymbol{L}_{body} = |\vec{L}| \begin{pmatrix} \cos\theta \\ -\sin\theta\cos\phi \\ -\sin\theta\sin\phi \end{pmatrix}$
- $\boldsymbol{M}_{rot} = \begin{pmatrix} \cos\beta\cos\gamma & -\sin\beta & -\cos\beta\sin\gamma \\ \sin\beta\cos\gamma & \cos\beta & -\sin\beta\cos\gamma \\ \sin\gamma & 0 & \cos\gamma \end{pmatrix}$



Linking the Coordinate Systems

$$L_x = \frac{V_x}{|\mathbf{V}|}\sin(\alpha) + \frac{V_y}{|\mathbf{V}|}\cos(\alpha)\cos(\phi) + \frac{V_z}{|\mathbf{V}|}\cos(\alpha)\sin(\phi) \tag{2.17}$$

$$L_y = \frac{V_y}{|\mathbf{V}|}\frac{V_x}{\sqrt{V_x^2 + V_z^2}}\sin(\alpha) - \frac{\sqrt{V_x^2 + V_z^2}}{|\mathbf{V}|}\cos(\alpha)\cos(\phi) + \frac{V_y}{|\mathbf{V}|}\frac{V_z}{\sqrt{V_x^2 + V_z^2}}\cos(\alpha)\sin(\phi) \tag{2.18}$$

$$L_z = \frac{V_z}{\sqrt{V_x^2 + V_z^2}}\sin(\alpha) - \frac{V_x}{\sqrt{V_x^2 + V_z^2}}\cos(\alpha)\sin(\phi) \tag{2.19}$$