DSGT-1 Tutorial

Ex. ① p: Gopal is tall q: Gopal is handsome

i) Gopal is tall & handsome.

ii) Gopal is tall but not handsome.

iii) It is false that Gopal is short or handsome.

iv) Gopal is neither tall nor handsome

v) It is not true that Gopal is short or not handsome.

Ex. ② p: High speed driving is dangerous

q: Rajesh was a wise man

i) $p \wedge q$ ii) $\sim p \wedge q$ iii) $\sim(p \wedge q)$ iv) $(p \wedge q) \vee (\sim p \wedge \sim q)$

v) $(p \vee q) \wedge \sim(p \wedge q)$

Ex. ③ p: Today is Monday q: The grass is wet

r: The dish ran away with the spoon

i) $\sim r \wedge q$ ii) $\sim q \vee r$ iii) $\sim(p \vee q)$ iv) $p \vee \sim r$

Ex. ④ construct truth table & determine the truth value

i) $(p \wedge (p \to q)) \to q$ ii) $(p \leftrightarrow q) \leftrightarrow ((p \wedge q) \vee (\sim p \wedge \sim q))$

Ex. ⑤ determine : Tautology, Contradiction

i) $(p \to \sim p) \to \sim p$ ii) $(p \vee q) \wedge (\sim p \to q)$

iii) $(p \wedge q) \leftrightarrow p$ iv) $(p \to q) \wedge (q \to r) \to (p \to r)$

Ex. ⑥ Show that following statements are tautological

i) $(p \wedge (p \to q)) \to q$ ii) $(p \to q) \leftrightarrow (q \vee \sim p)$

- Ex. ⑦ Show that following statements are logically equivalent

i) $p \to (q \to r)$ and $p \to (\sim q \vee r)$

ii) $p \leftrightarrow q$ and $(p \wedge q) \vee (\sim p \wedge \sim q)$

iii) $p \to (q \to r)$ and $(p \to q) \to (p \to r)$

Ex. ⑧ Show that following statements are tautology without T.T.

i) $(p \wedge (p \to q)) \to q$ ii) $(p \to q) \wedge \sim q \to \sim p$

Ex ⑨ For $A = \{a, b, \{b,c\}, \phi\}$ determine the following set

i> $A - \{a\}$ ii> $A - \{b,c\}$ iii> $\{\{b,c\}\} - A$

iv> $A - \{c, \phi\}$ v> $\{a\} - \{A\}$

Ex ⑩ If $U = \{ n \in N \mid 1 \leq n \leq 9 \}$

$A = \{1, 2, 4, 6, 8\}$ $B = \{2, 4, 5, 9\}$

$C = \{ x \in Z^+ \mid x^2 \leq 16 \}$ and $D = \{7, 8\}$

Find i> $A \oplus B$, $B \oplus C$, $C \oplus D$

ii> $A - B$, $B - A$, $C - D$

iii> $\overline{A \cup B}$, $\overline{A \cap B}$

iv> $A \cap (\bar{C} \cup D)$

- Ex ⑪ Using Venn diagrams, prove or disprove the followings

i> $(A - B) - C = (A - C) - B$

ii> $(A - B) - C = (A - C) - (B - C)$

iii> $(A - B) \cap (A - C) = A - (B \cup C)$

iv> $(A - C) \cup (B - C) = (A \cup B) - C$

- Ex ⑫ Consider set of integers from 1 to 250. Find how many of these are divisible by 3, 5, & 7? Also indicate how many are divisible by 3 or 7 but not by 5.

→ i> $|A \cup B \cup C| = 136$ ← 3 and 7 not 5

ii> $|A \cap C \cap \bar{B}| = |A \cap C| - |A \cap B \cap C| = 9$ ↓ 3 or 7 not 5

iii> ~~$A \cup B$~~ $(A \cup C) - B$ or $|(A \cup C) \cap \bar{B}| = |A| + |C| - |A \cap C| - |B| = 57$

- Ex ⑬ How many integers between 1 to 2000 are divisible by 2, 3, 5 or 7?

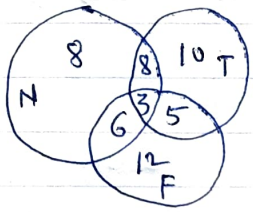
→ $|A \cup B \cup C \cup D| = ?$ → ~~1499~~ 1560 1561, 89

- Ex ⑭ In a survey of 60 people, it was found that 25 read Newsweek Magazine, 26 read Time, 26 read Fortune. Also 9 read both Newsweek & Fortune, 11 read both Newsweek & Time, 8 read both Time & Fortune, & read no magazine

i> Find out the no. of people who read all three magazine i.e $|N \cap T \cap F| = 3$

ii> Determine the no. of people who read exactly one magazine ie $x+y+z = 8+10+12 = 30$

iii> Find in correct no. in all regions of Venn diagram



iv> Exactly two → 19

- Ex. (15) A survey of 5000, television watchers produced the following info. - 285 watch Football, 195 watch hockey 115 watch basket ball, 45 watch Football & basketball, 70 watch Football & hockey, 50 watch hockey & basket ball and 50 do not watch any of three games

  —i> How many watch all 3 games? → 20

  190 + 95 + 40 – ii> How many watch exactly one game? → 525

  ~~700~~ 50+25+30 - iii> —"— exactly two games → 105

- Ex. (16) i> Among 50 students in a class, 26 got an A in the first examination and 21 got A in second examination. If 17 students did not get an A in either examination, how many students got A in both examination? → 14

Ex. (17) 100 of the 120 engineering students in a college take part in atleast one of the activities - group discussion, debate and quiz. Also 65 participates in group discussion, 45 participates in debate, 42 participate in quiz, 20 participate in group discussion & debate, 25 participate in group discussion & quiz, 15 participate in debate & quiz.

Find (i) who participate in all activities ! → 8

(ii) who participate in exactly one activities ? → (28+18+10=56)

Ex. (18) In a survey of new cars, it is found that 60 had Air Conditioner (AC), 48 had Power Steering (PS), 44 had Power Windows (PW), 36 had AC+PW, 20 had AC+PS, 16 had PW + PS, 12 had all three. Find the no. of cars that had

-i> only PW → 4

-ii> PS & PW but not AC → 4

-iii> AC & PS but not PW → 8

Ex. (19) Show that $5^n - 4n - 1$ is divisible by 16 for all ~~n≥1~~ $n \geq 2$

→ ~~$5^{k+1} - 4k$~~

or $\underbrace{(5^k - 4k - 1)}_{P(k)} + 4(5^k - 1)$ OR $5(5^k - 1) - 4k$

Ex. (20) Prove mathematical induction for $n \geq 1$

$$1\cdot2 + 2\cdot3 + \cdots + n(n+1) = \frac{n(n+1)(n+2)}{3}$$

How Actually 375*

~~6~~, ~~30-35~~, ~~50-54~~, ~~55~~, 86, ~~104~~, 105, ~~172~~, 174-75, ~~195~~, ~~201~~, 206, 207

Ex (21) $P(n) = 1 + 4 + 7 + \cdots + (3n-2) = \frac{n(3n-1)}{2}$

Ex. (22) There are 21 cricket players in Indian team. Out of these, 6 players taking part in one day match, 7 playes in t20 and 5 in both. How many players not taking part in onday and t20. Ans = 13