* Chains & Anti-chains

Let $(A, \le)$ be a poset. A subset of A is called a **chain** if every pair of elements in the subset are related.

In any chain with finite no. of elements $\{a_1, a_2 \cdots a_k\}$, there is an element a_1 that is less than every element in the chain. & there is element a_2 which is less than every element except a_1

∴ So we have sequence $a_1 \le a_2 \le a_3 \le \cdots \le a_k$

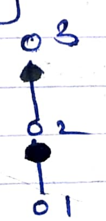
∴ The no. of elements in the chain is called **length of chain**.

If A itself is chain, then the poset $(A, \le)$ called a totally ordered set or linearly ordered set.

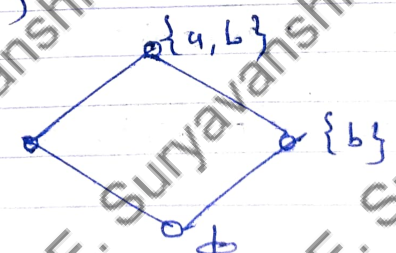
Anti-chain

A subset of A is called anti-chain if no two distinct elements in the subset are related.

Ex. ① Let $A = \{1, 2, 3\}$ & let the partial order $\le$ mean "less than or equal to". Then $(A, \le)$ is a chain & its Hasse diagram is



Ex. ② Let $A = \{a, b\}$ & consider its poset $(P(A), \subseteq)$



Chains

1. $\{\phi, \{a\}\}$
2. $\{\{a\}, \{a, b\}\}$
3. $\{\phi, \{a\}, \{a,b\}\}$
4. $\{\phi, \{b\}\}$
5. $\{\{b\} \{a, b\}\}$
6. $\{\phi, \{b\}, \{a, b\}\}$

Anti-chains

$\{\{a\}, \{b\}\}$

length of longest element is $\underline{3}$

* Lattice

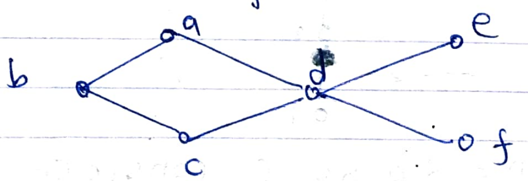
Let $(A, \leq)$ be a poset with partial order $\leq$.

① Maximal & Minimum elements

An element $a \in A$ is called maximal element if there is no element $b \in A$ such that $b \neq a$ & $a \leq b$.

An element $c \in A$ is called minimal element if there is no element $d \in A$ such that $d \neq c$ & $d \leq a$

Ex. ① consider the poset whose Hasse digram



Maximal elements = a, e

Minimal elements = c, f

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Year : FE / SE / TE / BE Branch : ________ Division : ________ Roll No. : ________

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Signature of Student Signature of Supervisor

Question Number	1	2	3	4	5	6	7	8	9	10	Total
Marks Obtained											
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Signature of Examiner : Signature of Moderator :

(Start From here only)

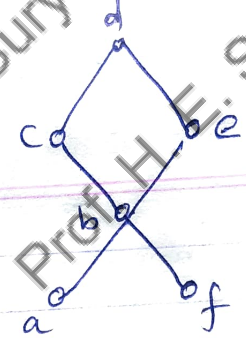
(2) Upper Bound & Lower Bound

Let a, b be elements in a poset $(A, \leq)$. An element c is said to be an upper bound of a & b if $a \leq c$ & $b \leq c$.

An element c is said to be least upper bound (lub) of a & b if c is upper bound of a & b and if there is no other upper bound 'd' of a & b such that $d \leq c$.

Similarly an element e is said to be a lower bound of a & b if $e \leq a$ & $e \leq b$; & e is called greatest lower bound (glb) of a & b if there is no other lower bound f of a, b such that $e \leq f$.

Ex.



① Upper bounds for $\{c, e\} \rightarrow d$

$lub \{c, e\} \rightarrow d$

lower bounds for $\{c, e\}$ are $\rightarrow b, a, \& f$

$glb \{c, e\} = b$

② Upper bounds for $\{a, f\} \rightarrow b, c, e, d$

$lub \{a, f\} = b$

lower bounds for (a, f) — doesn't exist.

③ Upper bounds for $\{b, d\} \rightarrow d$

lower bound for $\{b, d\} \rightarrow b, a \& f$

$glb \{b, d\} = b$

---

* Functions

let A & B be non-empty sets.

A function f from A to B, denoted by $f : A \rightarrow B$, is a relation from A to B. Such that for every $a \in A$, there exists a unique $b \in B$ such that $(a, b) \in f$

Normally, if $(a, b) \in f$, we write $f(a) = b$

If $f(a) = b$ &

$f(a) = c$ then $b = c$

In general, Function is only many-to-one or one-to-one relation

A one-to-many relation is not function.

① **Domain** $D(f)$

Set A is called as domain of f.

② **Codomain**

Set B is called as co-domain of F

③ **Range** $R(f)$

set $\{f(a) \mid a \in A\}$ which is subset of B called as range of f.

a – element 'a' is called argument of 'f'

$f(a)$ – is called the value of function

Functions are also called as mappings or transformations, since they can be thought of as rules for assigning to each element $a \in A$, the unique element $f(a) \in B$.

$\therefore$ $f(a)$ is ~~pr~~ image of 'a' & a is pre-image of ~~$f(a)$~~ b i.e. $f(a)$

Ex. ① $A = \{1, 2, 3\}$ & $B = \{a, b, c, d\}$

let $F : A \to B$ be defined as

$F(1) = a$

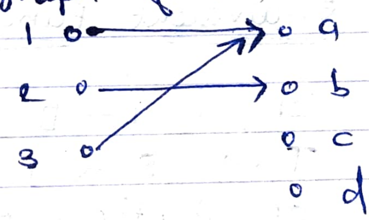
$F(2) = b$

$F(3) = a$

F is function

& $R(f) = \{a, b\}$

Graph of f

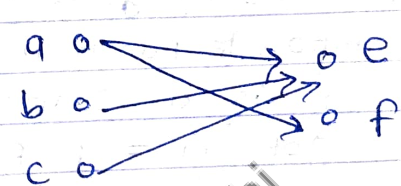


Ex ② let

$A = \{a, b, c\}$ & $B = \{e, f\}$

let $R = \{(a, e)\ (b, e)\ (a, f)\ (c, e)\}$

Graph of R



$\therefore$ R is not a function

since $f(a) = e$ & $f(a) = f$

Note = Function is one-to-one or many-to-one

* Partial Functions

- consider domain of a function as subset of another set known as source & codomain as target.
  - ∴ The function has set A as its domain but is not defined for some arguments.
- Let, A & B be two sets. A partial function f with domain A & codomain B is any function from A' to B where $A' \subseteq A$.
  - for any element $x \in A - A'$, the value of $f(x)$ is undefined
- A function which is not partial is sometimes called as total function.

Ex ① $f : R \rightarrow R$ & $f(x) = 1/x$

It is partial function because it is undefined for $x = 0$ value

Ex. ② $f : R \rightarrow R$ & $f(x) = \sqrt{x}$

It is partial function, as $\sqrt{x}$ is not defined for $x < 0$ in R

* Equivalent Functions (identical)

Let, $f : A \rightarrow B$ & $g : C \rightarrow D$ be functions. Then f & g are said to be equivalent or identical only if $A = C$, $B = D$ & $f(a) = g(a)$ for all $a \in A$

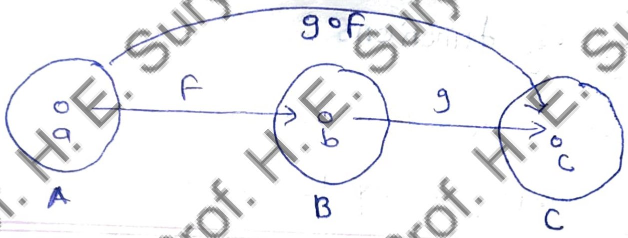
* Composite function

Let, $f : A \rightarrow B$ & $g : B \rightarrow C$ be two functions. Then composite function of f & g denoted as gof is a relation from $A \rightarrow C$ where $gof(a) = g(f(a))$.

& $gof : A \rightarrow C$ is also a function.

Note -

gof is defined only when the range of f is a subset of the domain of g.



Ex. ① let $f: \mathbb{Z} \to \mathbb{Z}$ defined as
$f(x) = x^2 + 2x + 2$ &
$g: \mathbb{Z} \to \mathbb{Z}$ defined as
$g(x) = x - 1$

Then

$g \circ f : \mathbb{Z} \to \mathbb{Z}$ defined as

$$g \circ f(x) = g(f(x)) = (x^2 + 2x + 2) - 1$$
$$= (x+1)^2$$

and $f \circ g : \mathbb{Z} \to \mathbb{Z}$ defined as

$$f \circ g(x) = f(g(x)) = (x-1)^2 + 2(x-1) + 2$$
$$= x^2 + 1$$

$f \circ f : \mathbb{Z} \to \mathbb{Z}$ defined as

$$f \circ f(x) = f(f(x)) = f(x^2 + 2x + 2)$$
$$= (x^2 + 2x + 2)^2 + 2(x^2 + 2x + 2) + 2$$

$g \circ g : \mathbb{Z} \to \mathbb{Z}$ defined as

$$g \circ g(x) = g(g(x)) = (x-1) - 1 = x - 2$$

Ex. ② let $f: \mathbb{Z} \to \mathbb{R}$ defined as $f(x) = \frac{(x+1)}{2}$ &

$g: \mathbb{R} \to \mathbb{R}$ be defined as $g(x) = x^2$

Then

$g \circ f : \mathbb{Z} \to \mathbb{R}$ defined as

$$g \circ f(x) = g(f(x)) = g\left(\frac{x+1}{2}\right) = \left(\frac{x+1}{2}\right)^2$$

$$g \circ f(x) = \frac{(x+1)^2}{4}$$

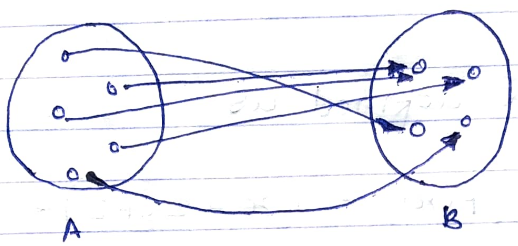
* Special types of functions

Let $f: A \to B$ be function

① **Surjective** (onto / many-to-one)

f is called Surjective function if $f(A) = B$

i.e. Range of f is equal to codomain of f

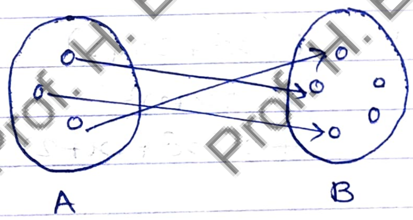


1. many-to-one
2. Range <= Codomain
3. Every "B" has one matching "A"
   - There won't be a "B" left out

② **Injective** (one-to-one)

f is called injective function if for elements $a, a' \in A$, $a \neq a'$ implies $f(a) \neq f(a')$

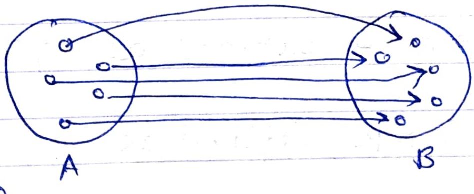
or if $f(a) = f(a')$ then $a = a'$



1. one-to-one
2.

③ **Bijection** (one-to-one / many-to-one)

f is called bijection if f is both Surjective & injective.



Theorem ①

Let $f: A \to B$ & $g: B \to C$ be functions.

Then i> If f & g Surjective, then $gof \to$ Surjective

ii> If f & g injective, then $gof \to$ injective

iii> If f & g bijective, then $gof \to$ bijective

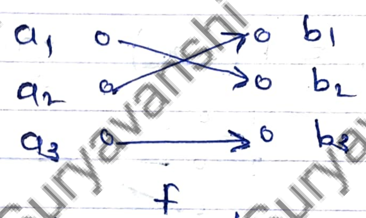
Theorem ② Let $f: A \to B$ & $g: B \to C$ functions.
Then i> If $g \circ f$ surjective, then $g \to$ surjective
ii> If $g \circ f$ injective, then $f \to$ injective
iii> If $g \circ f$ bijective, then $g \to$ surjective &
$f \to$ injective

* **Inverse function** (f^{-1})

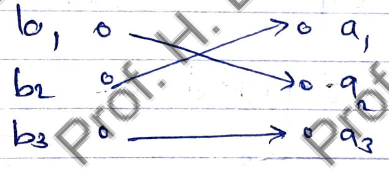
The concept of inverse function is analogous to that of the converse of relation
Let,
$f: A \to B$ be bijection from A to B.
$f^{-1}: B \to A$

Ex ① $f: \{a_1, a_2, a_3\} \to \{b_1, b_2, b_3\}$

defined as



Then
f^{-1} is given by graph



properties of inverse function

① $(f^{-1})^{-1} = f$

② If f & g - bijective functions $A \to B$, $B \to C$
then
$(g \circ f)^{-1} = f^{-1} \circ g^{-1}$

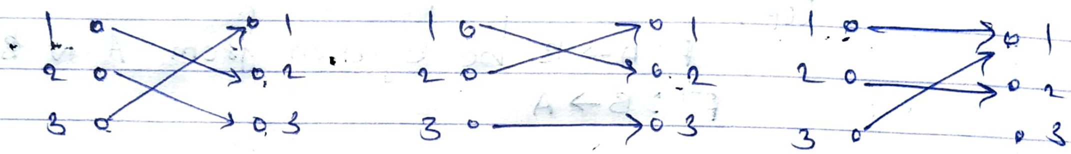
Ex. ① functions f, g, h are defined on a set $X = \{1,2,3\}$
as $f = \{ \cancel{(1,2)}\ (1,2)\ (2,3)\ (3,1) \}$

$g = \{ (1,2)\ (2,1)\ (3,3) \}$

$h = \{ (1,1)\ (2,2)\ (3,1) \}$

① Find $f\circ g$, $g\circ f$. Are they equal?
② Find $f\circ g\circ h$ and $f\circ h\circ g$.

$\Rightarrow$ Graphically f, g & h depicted as follows

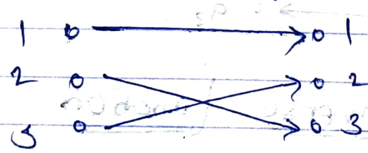


$\underline{\underline{f}}$ $\qquad$ $\underline{\underline{g}}$ $\qquad$ $\underline{\underline{h}}$

(i) $f\circ g = f(g(x))$ graphically depicted as

$$f\circ g = \{ (1,3)\ (2,2)\ (3,1) \}$$

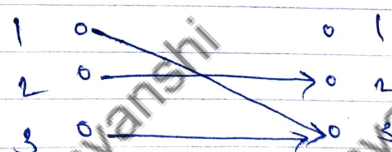
$g\circ f = g(f(x))$ depicted as



$$g\circ f = \{ (1,1)\ (2,3)\ (3,2) \}$$

$\therefore$ $f\circ g \neq g\circ f$

(ii) $f\circ g\circ h = (f\circ g)\circ h$ depicted as
or
$$f(g(h(x))) = f\circ g\circ h$$



$$f\circ g\circ h = \{ (1,3)\ (2,2)\ (3,3) \}$$

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Signature of Student Signature of Supervisor

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Marks Obtained											
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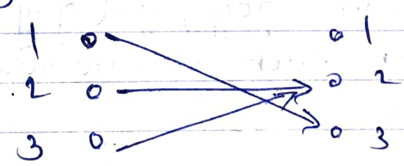
$$f \circ h \circ g = f(h(g(x))) \quad \text{or} \quad f \circ (h \circ g)$$



$$f \circ h \circ g = \{(1,3)(2,2)(3,2)\}$$

Ex (2) $A = \{a, b, c, d\}$

$B = \{s, t, u\}$

$C = \{l, m, n\}$

obtain the composition of following functions

$f : A \to B$ & $g : B \to C$

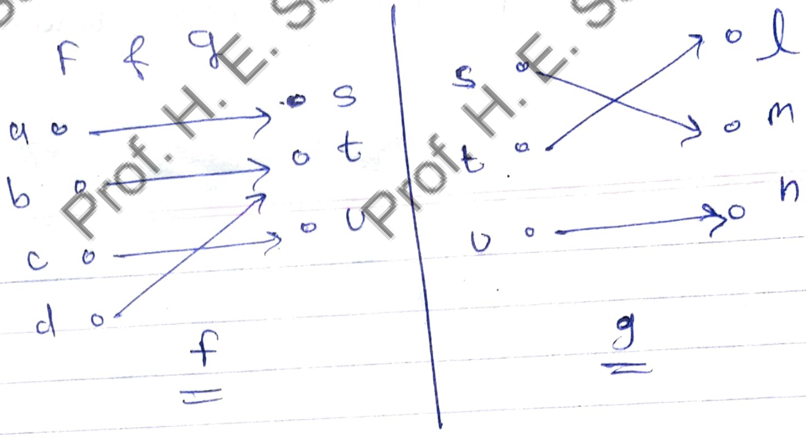
Where,

$f = \{(a,s)(b,t)(c,u)(d,t)\}$

$g = \{(s,m)(t,l)(u,n)\}$

Find $g \circ f$

E ⇒ Given f & g



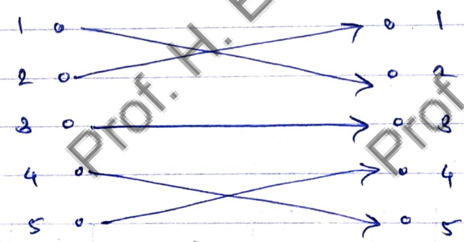
Now, composition of f & g i.e gof

i.e $g(f(x))$

$gof = \{(a,m)(b,l)(c,n)(d,l)\}$

Ex (4) Let $A = \{1,2,3,4,5\}$

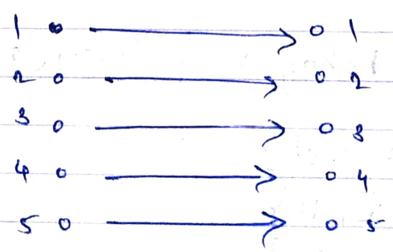
$g: A \to A$ is as shown in the fig.



Find the composition gog, $go(gog)$. Determine whether each is one-to-one or onto function

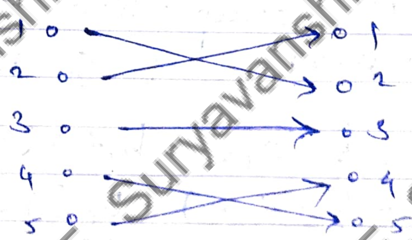
⇒

gog



gog is one-to-one function

go(gog)



one-one & onto function

Ex. ⑤ let $f(x) = x+2$
$g(x) = x-2$ &
$h(x) = 3x$ for $x \in R$
where R = Set of Real no.
Find gof, fog, fof, gog, foh, hog, hof, fohog.

$\Rightarrow$

$f(x) = x+2$
$g(x) = x-2$
$h(x) = 3x$

$gof = g(f(x)) = (x+2)-2 = x$

$fog = f(g(x)) = (x-2)+2 = x$

$fof = f(f(x)) = (x+2)+2 = x+4$

$gog = g(g(x)) = (x-2)-2 = x-4$

$foh = f(h(x)) = (3x)+2 = 3x+2$

$hog = h(g(x)) = 3(x-2) = 3x-6$

$hof = h(f(x)) = 3(x+2) = 3x+6$

$fohog = f(h(g(x)))$
$\therefore h(g(x)) = 3(x-2) = 3x-6$

$fohog = f(3x-6)$
$= (3x-6)+2$
$= 3x-4$

=====x=====

Ex. ⑥ If $f(x) = x^2+1$ and $g(x) = x+2$ are functions from R to R, where R is the set of real numbers.
Find fog & gof.

$\Rightarrow$

$f(x) = x^2+1$
$g(x) = x+2$

Now, $fog = f(g(x)) = (x+2)^2+1 = x^2+4x+5$
& $gof = g(f(x)) = (x^2+1)+2 = x^2+3$

Ex. (7) let $f(x) = 2x+3$

$g(x) = 3x+4$,

$h(x) = 4x$ for $x \in R$, where

R = set of real numbers.

Find $g \circ f$, $f \circ g$, $f \circ h$, $h \circ f$, $g \circ h$

$\Rightarrow$

$f(x) = 2x+3$

$g(x) = 3x+4$

$h(x) = 4x$

$$g \circ f = g(f(x)) = 3\cdot(2x+3)+4$$
$$= 6x+13$$

$$f \circ g = f(g(x)) = 2\cdot(3x+4)+3$$
$$= 6x+11$$

$$f \circ h = f(h(x)) = 2(4x)+3$$
$$= 8x+3$$

$$h \circ f = h(f(x)) = 4(2x+3)$$
$$= 8x+12$$

$$g \circ h = g(h(x)) = 3(4x)+4$$
$$= 12x+4$$

---

* Pigeonhole Principle

If A & B are finite sets & bijection exists from A to B.

then their cardinalities are same.

Hence, if A & B are any two sets such that $|A| > |B|$,

then no bijection can exists from A to B.

This fact is stated as 'Pigeon hole principle'

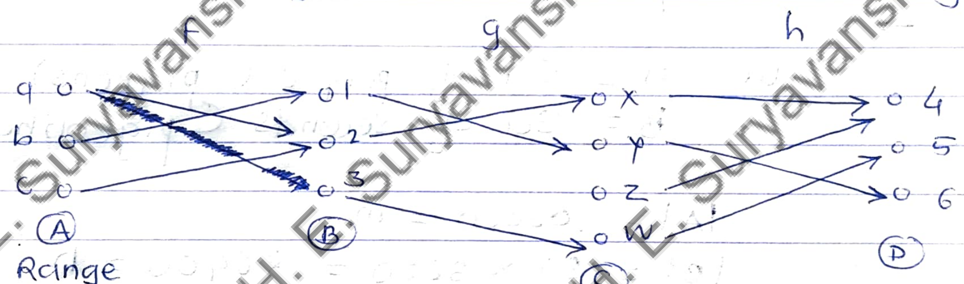
The principle states that if there are $n+1$ pigeons & only n-pigeonholes then two pigeons will share the same hole.

[Theorem: If $(n+1)$ objects are put into n boxes, then at least one box contains two or more objects.

Ex. Among 13 people there are two who have their birthdays in the same month.]

Ex. 8 The functions $f: A \to B$, $g: B \to C$, $h: C \to D$ are defined in the following diagram. Determine the range of each function. State which functions are into and which are onto. Draw the diagram of composite function $(h \circ g \circ f)$

—



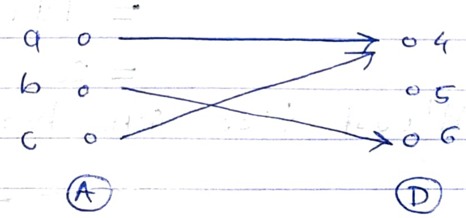
$\Rightarrow$ Range

$R(f) = \{1, 2\}$ f is into-function

$R(g) = \{x, y, w\}$ g is into-function

$R(h) = \{4, 5, 6\}$ h is onto-function

$(h \circ g \circ f)$ is



hogof

* Functions-

Binary relation $R: A \to B$, is said to be a function if for every element a in A, there is unique element b in B so that $(a, b) \in R$.

notation $R(a) = b$ | b is image of a

Set A is called domain &
B is called Rang.

The notion of function is a formalizati of the notion of associating or assigning an element in the range to each of the element in domain.

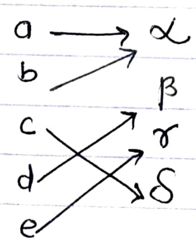
Function Types

① **Onto / Surjection-**
If every element of B is the image of one or more elements of A.

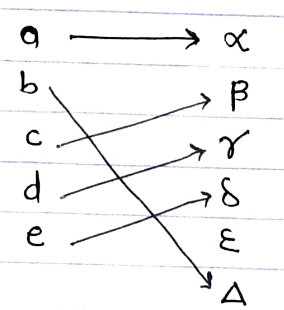
② **One-to-one / Injection-**
If no two elements of A having same image

③ **One-to-one onto / Bijection-**
If it is both an onto and one to one

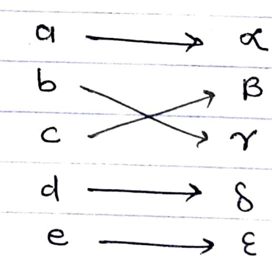
Example



① onto / Surjection



② one-to-one / Injection



③ one-to-one onto / Bijection