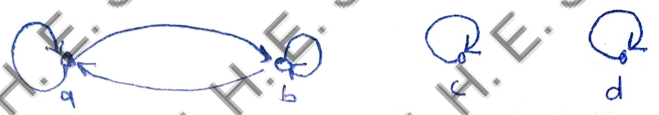
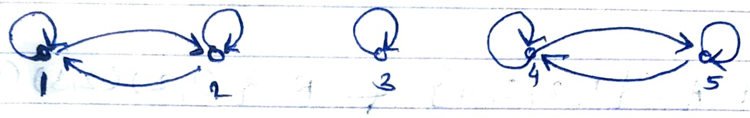


Ex. ② Let $A = \{1, 2, 3, 4, 5\}$

$\pi = \{ \{1,2\} \; \{3\} \; \{4,5\} \}$

Find the equivalence Relation determined by π & draw its diagraph.

$\Rightarrow$ $R = \{ (1,1) (2,2) (1,2) (2,1) (3,3) (4,4) (5,5) (4,5) (5,4) \}$



—x—

Let R be an equivalence Relation on A. we denote by A/R the partition induced by R. Hence a partition of A is called as a quotient set of A.

—x—

Ex. ① Let $A = \{1, 2, 3\}$ &

$R = \{ (1,1) (2,2) (1,3) (3,1) (3,3) \}$

Find A/R (partition)

$\Rightarrow$

$R = \{ (1,1) (2,2) (1,3) (3,1) (3,3) \}$

$[1] = \{1, 3\}$

$[2] = \{2\}$

$[3] = \{1, 3\} = [1]$

from Theorem ① i.e $\{ [a]_R \mid a \in A \}$

$A/R = \{ \{1,3\} \; \{2\} \}$

—x—

* Compatible Relation

- A Relation R on a set A is said to be compatible if it is reflexive & symmetric.

Ex. ① All equivalence relations are compatible Relations

Ex. ② The Relation of "being friend of" is a compatible Relation.

* Transitive Closure

- The transitive closure of a relⁿ R is the smallest transitive relation containing R, & denoted by R^*.

Method to find the transitive closure

- let A be set $|A| = n$, & R be relation on A.

Then

$$R^* = R \cup R^2 \cup R^3 \cup \cdots \cup R^n$$

Ex. ① let $A = \{1,2,3,4\}$ & $R = \{(1,2)(2,3)(3,4)\}$ R be a relation on A.
Find R^* & draw its diagraph.

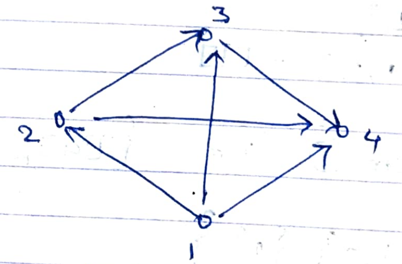
$\Rightarrow$ $R = \{(1,2)(2,3)(3,4)\}$

$R^2 = R \cdot R = \{(1,3)(2,4)\}$
$R^3 = R \cdot R^2 = \{(1,4)\}$
$R^4 = \phi$

$|A| = 4$ ∴ upto R^4 only

Hence $R^* = R \cup R^2 \cup R^3$
$= \{(1,2)(2,3)(3,4)(1,3)(2,4)(1,4)\}$

Diagraph of R^*



Ex. ② let $A = \{a,b,c,d\}$,
$R_1 = \{(a,a)(b,b)(a,c)(a,b)\}$ &
$R_2 = \{(a,a)(b,d)(d,c)\}$

Find $(R_1 \cup R_2)^*$ & draw its diagraph.

$\Rightarrow$ $R = R_1 \cup R_2 = \{(a,a)(b,b)(c,c)(a,b)(b,d)(d,c)\}$

Note $|A| = 4$ so go upto R^4

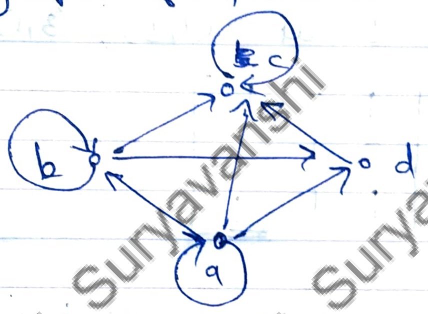
$R^2 = R \cdot R = \{(a,b)(b,d)(a,d)(b,c)(a,a)(b,b)(c,c)$
$(d,c)\}$

$R^3 = R \cdot R^2 = \{(a,a)(a,b)(a,c)(a,d)(b,b)(b,c)$
$(b,d)(d,c)(c,c)\}$

$R^4 = R \cdot R^3 = \{(a,a)(a,b)(a,c)(a,d)(b,b)(b,c)(b,d)$
$(c,c)(d,c)\}$

$R^* = R \cup R^2 \cup R^3$
$= \{(a,a)(b,b)(c,c)(a,b)(b,d)(d,c)$
$(a,d)(b,c)(a,c)\}$

Diagraph of R^*



* **Warshall Algorithm**

For large sets of relations, finding transitive closure of a relation, by computing various powers of R or products of the relation matrix MR, is quite impractical.

Ex. ① Use Warshall Algo. to find the transitive closure of R, where $M_R = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix}$ & A=

$\Rightarrow$ From matrix M_R,

$R = \{ (1,1)\ (1,3)\ (2,2)\ (3,1)\ (3,2) \}$

Now, $W_0 = \cancel{M_R}\ M_R$

For W_1, 1 is interior-vertex for path from (1,1) to (1,3), (3,1) to (1,1) & (3,1) to (1,3)

So so set 1 at (1,3), (3,1) & (3,3)

$\therefore W_1 = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$

For W_2, 2 is interior-vertex for path from (3,2) to (2,2) so set 1 at (3 which is already present $\therefore W_2 = W$

For W_3, 3 is interior-vertex for path from (1,3) to (3,1) & (1,3) to (3,2)

So set 1 at (1,1) & (1,2)

$W_3 = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$

$\therefore \cancel{W_R}\ M_{R^*} = W_3$

$R^* = \{ (1,1)\ (1,2)\ (1,3)\ (2,2)\ (3,1)\ (3,2)\ (3$

DSGT-2 H.E. Suryawanshi

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Warshall Algorithm

Ex. ① let $A = \{1,2,3,4\}$ & $R = \{(1,2)\ (2,4)\ (1,3)\ (3,2)\}$
Find transitive closure of R (i.e R^*) using warshall algorithm.

⇒

$$W_0 = M_R = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

For W_1 (i.e find in R, whether 1 is interior vertex or not)

∴ 1 is not interior vertex in R

$\therefore W_1 = W_0$

For ω_2, 2 is interior-vertex for path from (1,2) (2,4) & (3,2) (2,4)
So we have 1 in position (1,4), (3,4)

hence, $\omega_2 = \begin{bmatrix} 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

For ω_3, 3 is interior-vertex in path (1,3) (3,2). So set 1 at (1,2)
but, it already exist.
Hence $\omega_3 = \omega_2$

For ω_4, 4 is not interior-vertex in R
hence, $\omega_4 = \omega_3$

& So $M_{R^*} = \omega_4 = \begin{bmatrix} 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

Ex. ② Let $A = \{a_1, a_2, a_3, a_4, a_5\}$ &
let R be a relation on A whose matrix is

$M_R = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 \end{bmatrix}$

⟹

from M_R

$R = \{(a_1\ a_1) (a_1\ a_4) (a_2\ a_2) (a_3\ a_4)$
$(a_3\ a_5) (a_4\ a_1) (a_5\ a_2) (a_5\ a_5)\}$

Now

$\omega_0 = M_R$

For ω_1, a_1 is interior-vertex
for path $(a_1\ a_1)$ to $(a_1\ a_4)$
& $(a_4\ a_1)$ to $(a_1\ a_4)$
So set 1 at $(a_1\ a_4)$ & $(a_4\ a_4)$
where 1 already exist at $(a_1\ a_4)$
So set 1 at only $(a_4\ a_4)$

$$\therefore W_1 = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 \end{bmatrix}$$

For W_2, a_2 is interior vertex (a_5, a_2) to $(a_2 a_2)$
Set 1 at (a_5, a_2) which is already exist
so $W_2 = W_1$

For W_3, a_3 is not-interior vertex
so $W_3 = W_2$

For W_4, a_4 is interior vertex for path $(a_1\ a_4)$ to $(a_4\ a_1)$ &
$(a_3\ a_4)$ to $(a_4\ a_1)$
So set 1 at $(a_1\ a_1)$ & $(a_3\ a_1)$

$$W_4 = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 \end{bmatrix}$$

For W_5, a_5 interior vertex $(a_3 a_5)$ to $(a_5 a_2)$
so set 1 at $(a_3\ a_2)$

$$W_5 = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 \end{bmatrix}$$

Hence

$$M_{R^*} = W_5$$

Home Work

Ex ① Find transitive closure of R by Warshall algo, when $A = \{1, 2, 3, 4, 5, 6\}$
& $R = \{(x, y) \mid |x - y| = 2\}$

Hint

$R = \{(1,3)\ (3,1)\ (2,4)\ (4,2)\ (3,5)\ (5,3)\ (4,6)\ (6,4)\}$

* Partial Ordering Relation's

Defn. A binary relation R on set A is a partial order if R is
- reflexive,
- anti-symmetric &
- transitive

The ordered pair (A, R) is called partially-ordered set or poset.

Ex. ① Reln "<" is partial order on the set of real numbers.

* Hasse Diagrams

A poset can be depicted by Hasse Diagram which includes following rules
① All arrow heads on edges are omitted.
② Loops are omitted as reflexivity is implied by defn. of partial order
③ Arc/edge not present for transitivity

—x—

From Dictionary
A digraph which represents finite poset, in which nodes are elements of the poset & arrows represents the order relation between elements called Hasse Dig.

—x—

Ex. ① Let $A = \{2, 3, 4, 6\}$ & $R = \{(a,b) \mid a \text{ divides } b\}$. Show that R is partial order & draw its Hasse diagram.

$\Rightarrow$

$R = \{(2,2)(2,4)(2,6)(3,3)(3,6)(4,4)(6,6)\}$

$\therefore$ R — Reflexive relation
Since $(2,2)(3,3)(4,4)(6,6) \in R$

R : anti-Symmetric

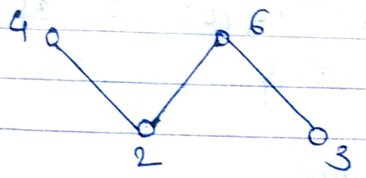
Since, if $a|b$, $b \nmid a$ unless $a=b$

R : transitive

Since, $a|b$ & $b|c$ implies $a|c$

Hence, R — partial order relation

∴ Hasse Diagram for R



Ex. ② $A = \{1,2,3,4\}$ and

$R = \{(1,1)(1,2)(2,2)(2,4)(1,3)(3,3)(3,4)$
$(1,4)(4,4)\}$

Show that R — partial order relation & draw its Hasse diagram

⟹

R — Reflexive

Since $(1,1)(2,2)(3,3)(4,4) \in R$

R — Anti-Symmetric

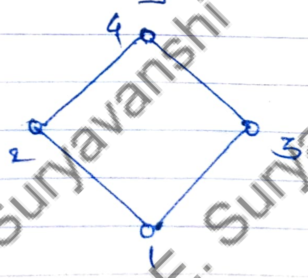
Since,

$(1,1) \in R \rightarrow 1=1$	~~(1,2) ∈ R but (2,1)~~ ∉ R
$(2,2) \in R \rightarrow 2=2$	~~(1,3) ∈ R but (3,1)~~ ∉ R
$(3,3) \in R \rightarrow 3=3$	~~(1,4) ∈ R but (4,1)~~ ∉ R
$(4,4) \in R \rightarrow 4=4$	~~(2,4) ∈ R but (4,2)~~ ∉ R
	~~(3,4) ∈ R but (4,3)~~ ∉ R

R — transitive

Since, $(1,1)(1,2) \in R$ & $(1,2) \in R$
$(2,2)(2,4) \in R$ & $(2,4) \in R$
$(1,3)(3,3) \in R$ & $(1,3) \in R$
$(3,3)(3,4) \in R$ & $(3,4) \in R$
$(1,4)(4,4) \in R$ & $(1,4) \in R$

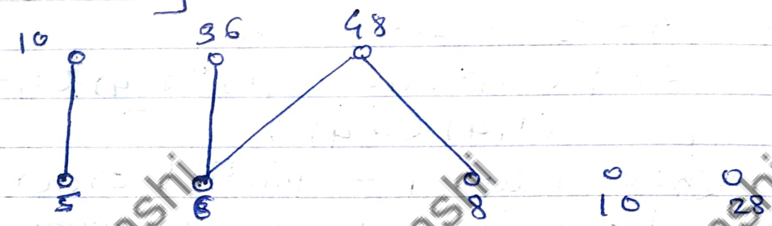
Now, Hasse diagram of R



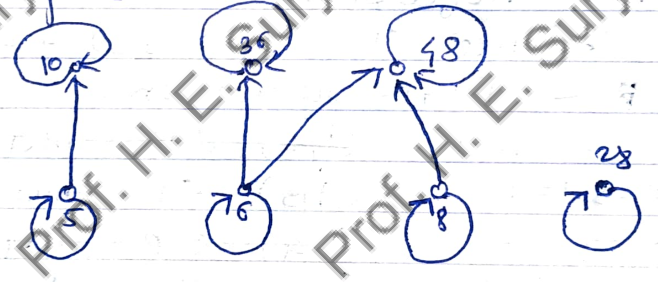
Ex. ⑧ Let R be relation on set
$A = \{5, 6, 8, 10, 28, 36, 48\}$
& $R = \{(a, b) \mid a \text{ is divisor of } b\}$
Draw Hasse diagram & compare it with diagraph. Determine whether R is reflexive, transitive & symmetric

$\Rightarrow$ $R = \{(5,5)\ (5,10)\ (6,6)\ (6,36)\ (6,48)\ (8,8)$
$(8,48)\ (10,10)\ (28,28)\ (36,36)$
$(48,48)\}$

Hasse Diagram



Diagraph



R – Reflexive
– Since $(5,5)\ (6,6)\ (8,8)\ (10,10)\ (28,28) \in R$
Similarly $(36,36)\ (48,48) \in R$

R – Not Symmetric
– Since $(5,10) \in R$ but $(10,5) \notin R$
$(6,36) \in R$ but $(36,6) \notin R$
& so on for $(6,48)\ (8,48)$