The Shirpur Education Society's
R. C. PATEL INSTITUTE OF TECHNOLOGY
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High Caliber Technical Education in an Environment that Promotes Excellence

TEST (I / II) / PRELIMINARY EXAMINATION

Name of Candidate : ______________________________
(IN BLOCK LETTERS) (Surname) (First Name) (Middle Name)

Year : FE / SE / TE / BE Branch : ________ Division : ________ Roll No. : ________

Semester : I / II Name of Subject : ______________________________

Total Supplements : 1 + _____ = _____

Signature of Student Signature of Supervisor

Question Number	1	2	3	4	5	6	7	8	9	10	Total
Marks Obtained											
Marks out of											

Signature of Examiner : Signature of Moderator :

(Start From here only)

* Nearest- Neighbour Method

① Start with any vertex (v_1) & choose the vertex closest to v_1 to form initial path of edge.

② Let v_n denotes latest vertex added to path.
Select v_{n+1} closest to v_n & not in path. Then add it to path

③ Repeat Step ② until all vertices of G are included in path.

④ At last, complete the ckt by adding the edge which connects starting vertex to the last vertex.

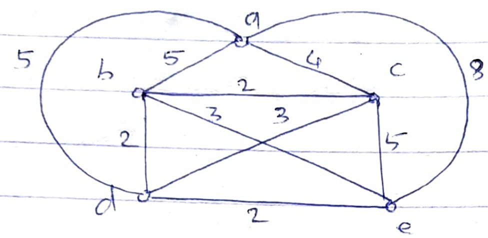
The circuit obtained by this method will be the required Hamiltonian circuit.

Ex. ① Use nearest-neighbour method to find out Hamiltonian ckt.

① Starting at vertex 'a'

② Starting at vertex 'd'

③ Determine the minimum Hamiltonian ckt.

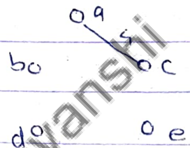


⟹

① Start with 'a'

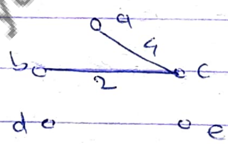
Select vertex with minimum wt.

i> Path = {a, c}

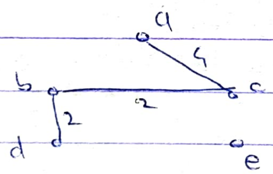


ii> Select b wt (2)

Path = {a, c, b}

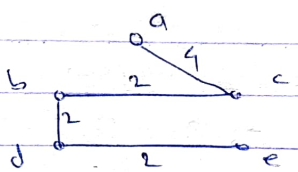


iii> select d wt (2)



path = {a, c, b, d}

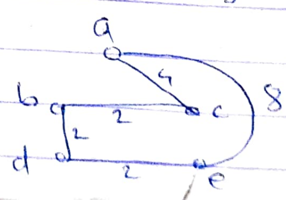
iv> path = {a, c, b, d, e}



v) Since all vertices are traversed, to complete Hamiltonian ckt, connect a to e

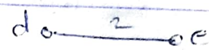
∴ Hamiltonian ckt = {a, c, b, d, e, a}

Total wt. of ckt = 18

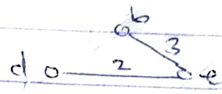


(2) Start from 'd'

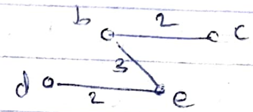
i> path = { d, e }



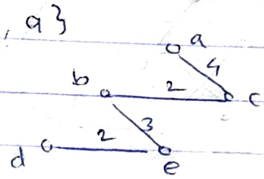
ii> path = { d, e, b }



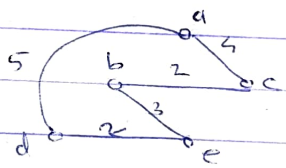
iii> Path = { d, e, b, c }



iv> Path = { d, e, b, c, a }

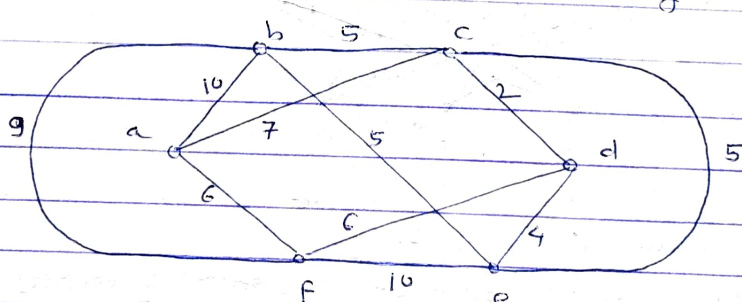


v> Hamiltonian
ckt = { d, e, b, c, a, d } & weight = 16



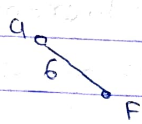
(3) The minimum Hamiltonian ckt is "debcad" with weight = 16

Ex. (2) Use nearest-neighbour method to find the Hamiltonian ckt. starting from 'a' in the following graph. Find its weight

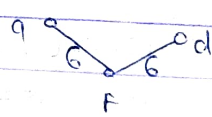


⇒ starting from 'a'

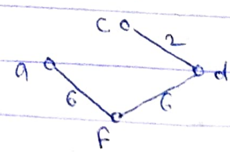
i> path = { a, f }



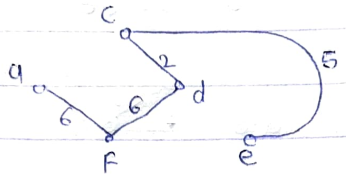
ii> path = { a, F, d }



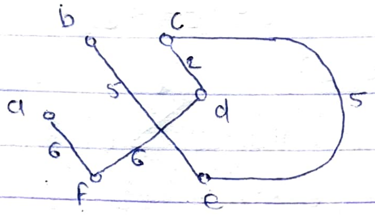
iii> path = { a, F, d, c }



iv> path = {a, F, d, c, e}

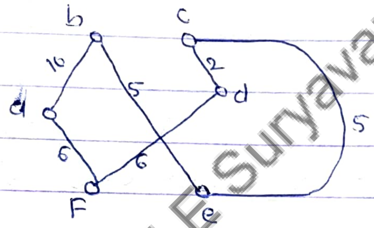


v> path = {a, F, d, c, e, b}



vi> Hamiltonian ckt = {a, F, d, c, e, b, a}

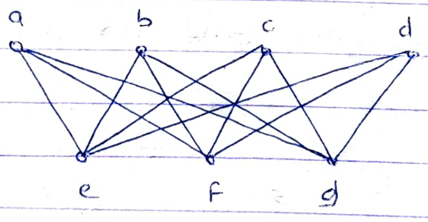
weight = 34



Ex. (3) Find Hamiltonian path and ckt in $K_{4,3}$

⇒

The complete Bipartite Graph $K_{4,3}$ given by



Here,

Total vertices = 7

also, degree sum of any pair of vertices $\geq (7-1)$

Hence, by theorem (1) graph $K_{4,3}$ has Hamiltonian path & it is ⇒ a - g - b - F - c - e - d

Also, $K_{4,3}$ doesn't contain a Hamiltonian circuit because $m \neq n$ i.e $(4 \neq 3)$

Ex ④ Is there a Hamiltonian path in complete Bipartite graph $K_{4,4}$ and $K_{4,5}$?

⇒

① In $K_{4,4}$

Total vertices = 8

Degree of each vertex = 4

Hence, degree sum of any pair is 8 (i.e 4+4)

By Theorem ①

$$\text{degree sum} \geq (n-1)$$

$$8 \geq (8-1)$$

Hence, $K_{4,4}$ contains Hamiltonian path

② In $K_{4,5}$

Total vertices = 9

4-vertices with degree = 5

& 5-vertices with degree = 4

Therefore, degree sum of any pair

$(4+4) = 8 \geq (9-1)$ i.e $(n-1)$ & $n=9$ total vertices

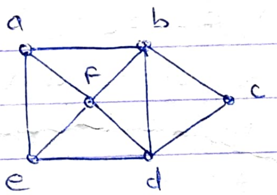
$(5+5) = 10 \geq (9-1)$

$(4+5) = 9 \geq (9-1)$

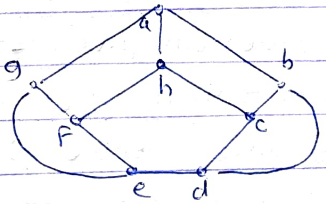
Hence, by theorem ①,

- $K_{4,5}$ contains Hamiltonian path.

Ex. ⑤ Is there a Hamiltonian circuit in the graphs G_1 & G_2



G_1



G_2

⇒ In G_1,

Hamiltonian path = a-b-c-d-f-e

Hamiltonian ckt = a-b-c-d-f-e-a

In G_2, Hamiltonian path = a-b-d-c-h-f-e-g

Hamiltonian ckt = a-b-d-c-h-f-e-g-a

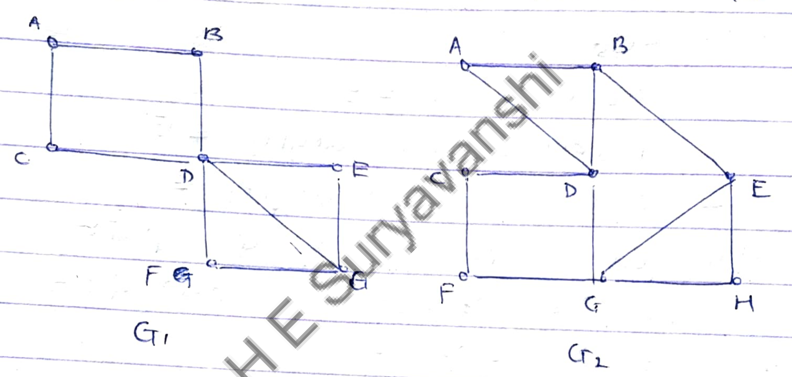
Ex ⑥ Is there a Hamiltonian ckt in a complete Bipartite Graph $K_{4,4}$, $K_{4,5}$ & $K_{4,6}$?

⇒

In complete Bipartite graph $K_{m,n}$ Hamiltonian ckt exist iff $m = n$

In $K_{4,4}$, Hamiltonian ckt exist But In $K_{4,5}$ & $K_{4,6}$ (i.e $m \neq n$) Hamiltonian ckt. doesn't exist.

Ex. ⑦ Determine which of the following graphs G_1 & G_2 represents Eulerian path, Eulerian ckt, Hamiltonian path & Hamiltonian ckt.



⇒ In G_1.

① No Eulerian ckt. bcoz degree of each vertex is not even.

② Eulerian path exist bcoz exactly two vertices of odd degree (D & F) are present in G_1

∴ Eulerian path ~~G~~ GEDBACDFGD

③ No Hamiltonian ckt. bcoz to cover all vertices exactly once, vertex D has to be traversed twice

④ Hamiltonian path = EGFDBAC

In G_2

① No eulerian ckt. (degree not even)

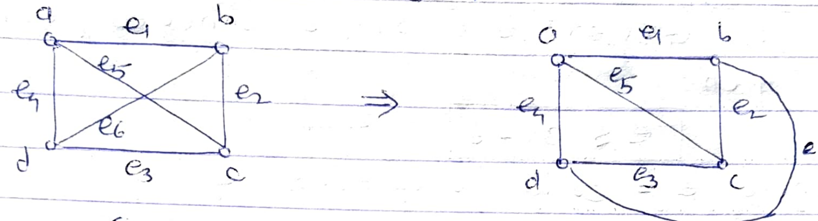
② Eulerian path = BADCFGHEGDBE

③ It has Hamiltonian path & ckt. Both It is ABEHGFCDA

* Planner Graph -

A graph is a planner graph if it can be drawn on the plane with no intersecting edges i.e no edge can cross each other.

A graph which is non-planner in one representation may become a planner graph after redrawing it



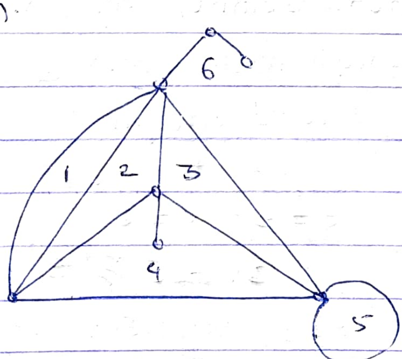
G1 (Non-planner graph) $\Rightarrow$ G2 (Planner Graph)

Regions -

A planner representation of graph divides the plane into regions (also called windows, faces & meshes). A region is characterized by the set of edges forming its boundary.

A region is finite, if its area is finite & it is said to be infinite if its area is infinite.

A planner ~~has~~ graph has exactly one infinite region.



Regions - 1, 2, 3, 4 & 5 finite

Region - 6 infinite

Eulers Formula -

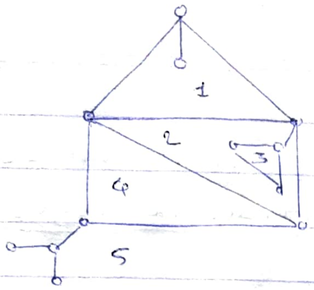
A planner graph may have no. of regions. The no. of regions in graph depends on no. of vertices & no. of edges.

The relation betn vertices, edges & regions is given by Eulers Formula

$$\boxed{v - e + r = 2}$$

v - vertices, e - edges

r - regions



G

Here $v = 12$, $e = 15$

$\therefore r = \cancel{v - e + 2} = 2 - v + e$

$= \cancel{12 - 15 +} = 2 - 12 + 15$

$\boxed{r = 5}$

Corollary

If $G(V,E)$ is a simple connected planner graph then

$$\boxed{e \leq 3v - 6} \quad \text{——} \ (1)$$

where e = total no. of edges &

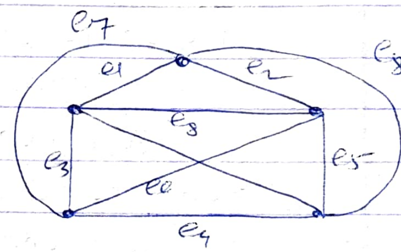
v = total no. of vertices in graph G.

- The most important application of this corollary is to show that the complete graph K_5 on 5 vertices are non-planner.

K_5 is known as Kuratowski's First Graph.

In K_5 $v = \cancel{10}\ 5$

$e = \frac{n(n-1)}{2} = \frac{5(4)}{2} = 10$



$\therefore e \leq 3v - 6$

$10 \leq 3(5) - 6$

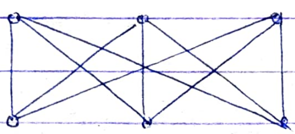
$10 \nleq 9$

Hence K_5 is non-planner graph

G (K_5)

- Every simple planner graph must satisfy (1) but converse need not be true

- $K_{3,3}$ Kuratowski's second Graph



$K_{3,3}$

$e = 9$, $v = 6$

$\therefore 3v - 6 \geq e$

$K_{3,3}$ satisfys eq^n (1) but still its not planner graph.

- Kuratowski enables us to determine the planarity of a graph.

The planarity of graph is clearly not affected if an edge is divided into two edges by the insertion of a new vertex of degree 2 or if two edges are incident with vertex of degree 2 are combined as single edge by the removal of vertex.

DSCIT-4 H'E-Suryawanshi (7)

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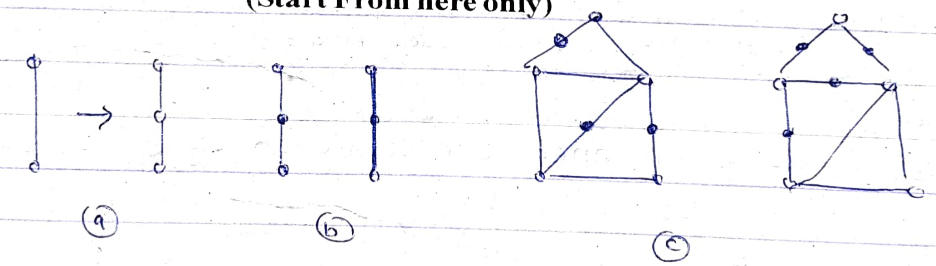
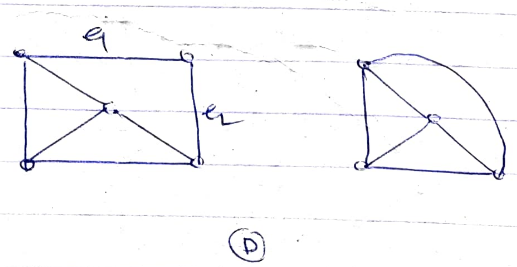
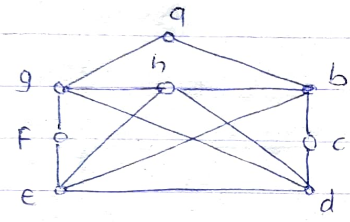




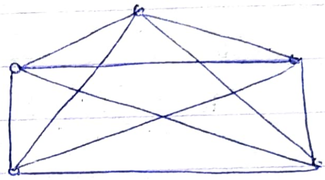
* Kuratowski Theorem

A graph is planner graph iff it does not contain any subgraph that is isomorphic to within vertices of degree 2 to either K_5 or $K_{3,3}$

consider the graph G

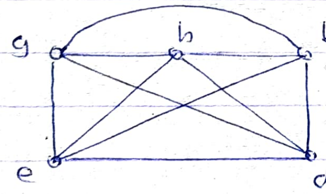


G



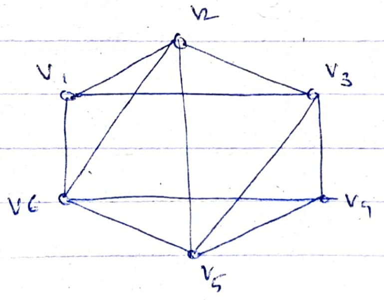
K_5 (Kuratowski's First Graph)

- Merge the edges which are incident on the vertices a, f, c.
- After merging the graph G becomes isomorphic to Graph K_5

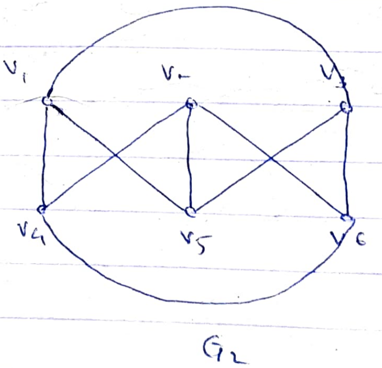


G (After merging edges of a, f & c)

Ex. ① Draw the planner representation of graph shown in Fig. below.



G_1



G_2