DSGT-2 H. E. Suryavanshi





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***TEST (I / II ) / PRELIMINARY EXAMINATION***

Name of Candidate : ______________________

(IN BLOCK LETTERS ) (Surname) (First Name) (Middle Name)

Year : FE / SE / TE / BE Branch : ________ Division : ________ Roll No. : ________

Semester : I / II Name of Subject : ______________________

Total Supplements : 1 + ______ = ______

Signature of Student Signature of Supervisor

| Question Number | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | Total |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Marks Obtained | | | | | | | | | | | |
| Marks out of | | | | | | | | | | | |

Signature of Examiner : Signature of Moderator :

**(Start From here only)**

* DSGT $\boxed{\text{Unit-2}}$

* **Basic concept of Relation -**

Let $\{A_1, A_2, \cdots A_n\}$ be a finite collection of sets. R is subset of $A_1 \times A_2 \times \cdots \times A_n$ is called n-ary relation on $\{A_1, A_2, \cdots A_n\}$

If $R = \phi$, $R \rightarrow$ void / empty relation

If $R = A_1 \times A_2 \times \cdots \times A_n$, $R \rightarrow$ universal relation

If $A_i = A$ for all $i$, $R \rightarrow$ n-ary relation

If $n = 1, 2,$ or $3$ $\rightarrow$ Unary, Binary, Ternary rel$^n$

Ex. ① Let $A = \{1, 2, 3\}$ R = "x is less than y"

$\therefore$ $R = \{(1,2)\ (1,3)\ (2,3)\}$

Then R = Binary Relation

## * Binary Relation

Let A & B non-empty sets. Then binary Relation R from A to B is subset of $A \times B$

i.e $R \subseteq A \times B$

**Domain : D(R)**

- It is a set of elements in A that are related to some elements in B i.e

$$D(R) = \{ a \in A \mid \text{for some } b \in B, (a,b) \in R \}$$

**Range : Rn (R)**

- It is the set of elements in B, that are related to some elements in A i.e

$$Rn(R) = \{ b \in B \mid \text{for some } a \in A, (a,b) \in R \}$$

**Ex. ①** : Let $A = \{2, 3, 4, 5\}$

let R be the relation on A ($aRb$ iff $a<b$)

Find D(R) & Rn(R)

⇒

$A = \{2, 3, 4, 5\}$

$R = \{(2,3)\ (2,4)\ (2,5)\ (3,4)\ (3,5)\ (4,5)\}$

$\therefore D(R) = \{2, 3, 4\}$

$\therefore Rn(R) = \{3, 4, 5\}$

## * Complement of a Relation

The complement of relation R, denoted by $\bar{R}$ is as

$$\bar{R} = \{ (a,b) \mid (a,b) \notin R \}$$

i.e $a\bar{R}b$ iff $a \not{R} b$

Ex ① Let $A = \{1, 2, 3, 4\}$ & $B = \{a, b, c\}$

$R = \{(1, a)\ (1, b)\ (2, c)\ (3, a)\ (4, b)\}$

and

$S = \{(1, b)\ (1, c)\ (2, a)\ (3, b)\ (4, b)\}$

Find i> $\bar{R}$ & $\bar{S}$

ii> verify De Morgan's laws for R & S

$\Rightarrow$

i> $A \times B = \{(1, a)\ (1, b)\ (1, c)\ (2, a)\ (2, b)\ (2, c)\ (3, a)\ (3, b)\ (3, c)\ (4, a)\ (4, b)\ (4, c)\}$

$(A\times B) - R \rightarrow \therefore \bar{R} = \{(1, c)\ (2, a)\ (2, b)\ (3, b)\ (3, c)\ (4, a)\ (4, c)\}$

&

$(A\times B) - S \rightarrow \therefore \bar{S} = \{(1, a)\ (2, b)\ (2, c)\ (3, a)\ (3, c)\ (4, a)\ (4, c)\}$

ii> De Morgans Law

$$\overline{R \cup S} = \bar{R} \cap \bar{S} \quad \&$$
$$\overline{R \cap S} = \bar{R} \cup \bar{S}$$

$\therefore R \cup S = \{(1, a)\ (1, b)\ (1, c)\ (2, a)\ (2, c)\ (3, a)\ (3, b)\ (4, b)\}$

$(A\times B) - R \cup S \hookrightarrow \overline{R \cup S} = \{(2, b)\ (3, c)\ (4, a)\ (4, c)\}$

$\bar{R} \cap \bar{S} = \{(2, b)\ (3, c)\ (4, a)\ (4, c)\}$

Hence $\overline{R \cup S} = \bar{R} \cap \bar{S}$

Now $R \cap S = \{(1, b)\ (4, b)\}$

$\overline{R \cap S} = \{(1, a)\ (1, c)\ (2, a)\ (2, b)\ (2, c)\ (3, a)\ (3, b)\ (3, c)\ (4, a)\ (4, c)\}$

$\bar{R} \cup \bar{S} = \{(1, a)\ (1, c)\ (2, a)\ (2, b)\ (2, c)\ (3, a)\ (3, b)\ (3, c)\ (4, a)\ (4, c)\}$

Hence, $\overline{R \cap S} = \bar{R} \cup \bar{S}$

## * Converse of a Relation:

Given relation from A to B, is R
The converse relation of R, denoted by $R^c$ is the relation from B to A, defined as

$$R^c = \{(b, a) \mid (a, b) \in R\}$$

clearly

$$R^c \subseteq B \times A$$

### Theorem

Let R, S be relations from A to B. Then

(i) $(R^c)^c = R$

(ii) $(R \cup S)^c = R^c \cup S^c$

(iii) $(R \cap S)^c = R^c \cap S^c$

**Ex. (1)**

Let $A = \{1, 2, 3, 4\}$ & $B = \{a, b, c\}$

Given $R = \{(1, a) (3, a) (3, c)\}$

Find (i) $R^c$ (ii) $D(R^c)$ (iii) $R_n(R^c)$

$\Rightarrow$ (i) $R = \{(1, a) (3, a) (3, c)\}$

$R^c = \{(a, 1) (a, 3) (c, 3)\}$

(ii) $D(R^c) = \{a, c\} = R_n(R)$

(iii) $R_n(R^c) = \{1, 3\} = D(R)$

**Ex (2)**

Let $A = \{2, 3, 4, 6\}$. Let R & S be relations on A that

$R = \{(a, b) \mid a = b + 1 \text{ or } b = 2a\}$

and

$S = \{(a, b) \mid a \text{ divides } b\}$

find $(R \cap S)^c$

$\Rightarrow \quad A = \{2, 3, 4, 6\}$

$R = \{(3,2)\ (4,3)\ (2,4)\ (3,6)\}$

&

$S = \{(2,2)\ (2,4)\ (2,6)\ (3,3)\ (3,6)\ (4,4)\ (6,6)\}$

$\therefore\ R \cap S = \{(2,4)\ (3,6)\}$

Then

$(R \cap S)^c = \{(4,2)\ (6,3)\}$

=x=

* **Composition of Binary Relations**

**Composit Relation**

— Relations that are formed from an existing sequence of relations.

**Ex.**

Relationship of grand father who is fathers / mothers father

**Definition**

Let $R_1$ be relation from A to B

$R_2$ be relation from B to C

Then,

composit relation from A to C, denoted by $R_1 \cdot R_2$ (or $R_1 R_2$)

**Theorems**

(1) $R_1$, $R_2$ & $R_3$ be relations from A to B, B to C and C to D.

Then $(R_1 \cdot R_2) \cdot R_3 = R_1 \cdot (R_2 \cdot R_3)$

(2) $R_1$, $R_2$ be relations from A to B & B to A C

Then

$(R_1 \cdot R_2)^c = R_2^c \cdot R_1^c$

Ex. ①  $A = \{a, b, c, d\}$

$R_1 = \{(a,a)\ (a,b)\ (b,d)$ &

$R_2 = \{(a,d)\ (b,c)\ (b,d)\ (c,b)\}$

Find $R_1 \cdot R_2$, $R_2 \cdot R_1$, $R_2^2$ & $R_2^3$

$\Rightarrow$

$R_1 \cdot R_2 = \{(a,d)\ (a,c)\ (a,d)\}$

$R_2 \cdot R_1 = \{(c,d)\}$

$R_1^2 = \{(a,a)\ (a,b)\ (a,d)\}$

$R_2^2 = \{(b,b)\ (c,c)\ (c,d)\}$

$R_2^3 = \{(b,c)\ (b,d)\ (c,b)\}$

Ex. ②

$A = \{1, 2, 3, 4\}$ Let $R_1$ defined as

$R_1 = \{(x,y) \mid x+y=5\}$ let $R_2$ defined as

$R_2 = \{(x,y) \mid y-x=2\}$

verify that $(R_1 \cdot R_2)^c = R_2^c \cdot R_1^c$

$\Rightarrow$

$A = \{1, 2, 3, 4\}$

$R_1 = \{(1,4)\ (2,3)\ (3,2)\ (4,1)\}$

$R_2 = \{(1,3)\ (2,4)\}$

$R_1 \cdot R_2 = \{(3,4)\ (4,3)\}$

$(R_1 \cdot R_2)^c = \{(4,3)\ (3,4)\}$

$R_1^c = \{(4,1)\ (3,2)\ (2,3)\ (1,4)\}$

$R_2^c = \{(3,1)\ (4,2)\}$

$R_2^c \cdot R_1^c = \{(3,4)\ (4,3)\}$ Hence $(R_1 \cdot R_2)^c = R_2^c \cdot R_1^c$

**Ex. ③** $A = \{2, 3, 4, 5, 6\}$

$R_1 = \{ (a, b) \mid a - b = 2 \}$

$R_2 = \{ (a, b) \mid a + 1 = b \text{ or } a = 2b \}$

$\Rightarrow$ Find ① $R_1 \cdot R_2$ ② $R_2 \cdot R_1$ ③ $R_1 \cdot R_2 \cdot R_1$

④ $R_1^2$ ⑤ $R_1 \cdot R_2^2$

$\Rightarrow$

$A = \{2, 3, 4, 5, 6\}$

$R_1 = \{ (4,2)\ (5,3)\ (6,4) \}$

$R_2 = \{ (2,3)\ (3,4)\ (4,5)\ (5,6)\ (4,2)\ (6,3) \}$

① $R_1 \cdot R_2 = \{ (4,3)\ (5,4)\ (6,2)\ (6,5) \}$

② $R_2 \cdot R_1 = \{ (3,2)\ (5,4)\ (4,3) \}$

③ $R_1 \cdot R_2 \cdot R_1 = R_1 \cdot (R_2 \cdot R_1)$

$= \{ (5,2)\ (6,3) \}$

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* **Matrix Relation ($M_R$)** — matrix Representation of Relation

$R \subseteq A \times B$

$\therefore$ $M_R$ (A − Rows, B − columns)

**Ex. ①** $A = \{ a, b, c, d \}$

$B = \{ 1, 2, 3 \}$

and

$R = \{ (a,1)\ (a,2)\ (b,1)\ (c,2)\ (d,1) \}$

Find the relation matrix

$\Rightarrow$

$M_R$ : (4 rows, 3 columns)

$$M_R = \begin{array}{c} \\ a \\ b \\ c \\ d \end{array} \begin{array}{c} \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} \end{array}$$

**Ex. 2** $A = \{1, 2, 3, 4, 8\}$

$B = \{1, 4, 6, 9\}$

Let $aRb$ iff $a \mid b$ (a divides b)

Find the relation matrix

$\Rightarrow$

$R = \{(1,1)\ (1,4)\ (1,6)\ (1,9)\ (2,4)\ (2,6)\ (3,6)\ (3,9)\ (4,4)\}$

$$M_R = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 8 \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 4 & 6 & 9 \end{array} \\ \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

**Ex. 3** Let $A = \{a, b, c, d\}$ and

$$M_R = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix}$$ Find $R$

$\Rightarrow$

$R = \{(a,a)\ (a,b)\ (b,c)\ (b,d)\ (c,c)\ (c,d)\ (d,a)\ (d,c)\}$

**Ex. 4** $A = \{1, 2, 3, 4, 8\} = B$ ; $aRb$ iff $a+b <= 9$

Find $M_R$

$\Rightarrow$

$R = \{(1,1)\ (1,2)\ (1,3)\ (1,4)\ (1,8)\ (2,1)\ (2,2)\ (2,3)$
$(2,4)\ (3,1)\ (3,2)\ (3,3)\ (3,4)\ (4,1)\ (4,2)$
$(4,3)\ (4,4)\ (8,1)\}$

$$M_R = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 8 \end{array} \begin{array}{c} \begin{array}{ccccc} 1 & 2 & 3 & 4 & 8 \end{array} \\ \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

DSGT-2 H. E. Suryavanshi (3)



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Signature of Student Signature of Supervisor

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Signature of Examiner : Signature of Moderator :

**(Start From here only)**

* Relation Matrix Operation

Boolean matrix → having entry either 0 or 1.

$A = [a_{ij}]$

$B = [b_{ij}]$ be $m \times n$ boolean matrix

① $A + B = [c_{ij}]$ where,

$c_{ij} = 1$ if $a_{ij} = 1$ or $b_{ij} = 1$

$= 0$ if $a_{ij}$ and $b_{ij}$ — both zero

② $A \cdot B = [d_{ij}]$ where,

$d_{ij} = 1$ if $a_{ij} = b_{ij} = 1$

$= 0$ if $a_{ij} = 0$ or $b_{ij} = 0$

i.e (Rows × Columns)

**Ex ①** $A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix}$ & $B = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$

$A + B = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$ & $A \cdot B = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 1 \end{bmatrix}$

## * Properties of Relation of Matrix

Let $R_1$ : relation from $A$ to $B$

$R_2$ : $B \to C$

1. $M_{R_1 \cdot R_2} = M_{R_1} \cdot M_{R_2}$

2. $M_{R^c} =$ transpose of $M_R$

3. $M_{(R_1 \cdot R_2)^c} = M_{R_2^c \cdot R_1^c} = M_{R_2^c} \cdot M_{R_1^c}$

**Ex. ①**

$A = \{1, 2, 3, 4\}$

$R_1 = \{(1,1)\ (1,2)\ (2,3)\ (2,4)\ (3,4)\ (4,1)\ (4,2)\}$

and

$R_2 = \{(3,1)\ (4,4)\ (2,3)\ (2,4)\ (1,1)\ (1,4)\}$

verify ① $M_{R_1 \cdot R_2} = M_{R_1} \cdot M_{R_2}$

② $M_{R_1^c} =$ transpose of $M_{R_1}$

③ $M_{(R_1 \cdot R_2)^c} = M_{R_2^c} \cdot M_{R_1^c}$

$\Rightarrow$

① $M_{R_1} = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 \end{bmatrix}$ & $M_{R_2} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

$R_1 \cdot \cancel{R_2} = \{(1,1)\ (1,2)\ (2,3)\ (2,4)\ (3,4)\ (4,1)\ (4,2)$

$R_2 = \{(1,1)\ (1,4)\ (2,3)\ (2,4)\ (3,1)\ (4,4)\}$