SGT-1

RCP - iTech Supplement no

Name of Student :

Year : FE / SE / TE / BE Branch : Division : Roll No :

* Cardinality of Finite Set

Definition:

Let A be a finite set.
The cardinality of A, denoted by $|A|$ is the no. of elements in set.

- If $A = \phi$, then $|A| = 0$
- If $A \subseteq B$, where B is finite set, then $|A| \le |B|$

Theorem

① $|A \cup B| = |A| + |B|$ where A & B be finite sets

② $|A_1 \cup A_2 \cup \cdots \cup A_n| = |A_1| + |A_2| + \cdots + |A_n|$
where $A_1, A_2, \ldots A_n$ be a finite collection of mutually disjoint finite set.

③ Let A & B two set.
where A is finite set & B be any set (not necessarily finite set)
Then

$$|A - B| = |A| - |A \cap B|$$

④ Principle of Inclusion-Exclusion
Let A & B be finite sets
Then,

$$|A \cup B| = |A| + |B| - |A \cap B|$$

⑤ Mutual Inclusion- Exclusion Principle for Three Sets
Let A, B & C be finite sets
Then,

$$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |B \cap C| - |A \cap C| + |A \cap B \cap C|$$

Ex. ① In a survey, ~~of~~ 2000 people were asked whether they read India Today or Business Times. It was found that 1200 read India Today, 900 read Business Times and 400 read both.

Find how many read atleast one magazine and how many read neither. $A \cup B$ & $\overline{A \cup B}$

$\Rightarrow$

Let, A = Set of people who read India Today.

B = Set of people who read Business Times

Now,

$$|U| = 2000$$
$$|A| = 1200$$
$$|B| = 900$$
$$|A \cap B| = 400$$

we have to find out

$$|U - (A \cup B)| = |U| - |A \cup B|$$

$\therefore$ By the mutual inclusion - exclusion principle

$$|A \cup B| = |A| + |B| - |A \cap B|$$
$$= 1200 + 900 - 400$$
$$|A \cup B| = 1700$$

$$\therefore \ |U - (A \cup B)| = |U| - |A \cup B|$$
$$= 2000 - 1700$$
$$= 300$$

Hence, 1700 read atleast one magazine and 300 read neither.

Ex. ② Among the integers 1 to 300, find how many are not divisible by 3, nor by 5.

Find also, how many are divisible by 5 but not by 7.

Let A = set of integers divisible by 3
B = set of integers divisible by 5
C = set of integers divisible by 7

we have to find

$|\bar{A} \cap \bar{B}|$ and $|A - C|$

By De Morgans Law $|\bar{A} \cap \bar{B}| = \overline{A \cup B}$

$\therefore |\overline{A \cup B}| = |U| - |A \cup B|$

Given

$|U| = 300$

$|A| = \left\lfloor \frac{300}{3} \right\rfloor = 100$, $|B| = \left\lfloor \frac{300}{5} \right\rfloor = 60$ ~~and~~

~~let~~ $\left\lfloor \frac{300}{7} \right\rfloor =$

Now,

$$|A \cup B| = |A| + |B| - |A \cap B|$$

$$= 100 + 60 - \left\lfloor \frac{300}{3 \times 5} \right\rfloor$$

$$= 100 + 60 - \left\lfloor \frac{300}{15} \right\rfloor$$

$$|A \cup B| = 140$$

$$\therefore |\bar{A} \cap \bar{B}| = |\overline{A \cup B}| = |U| - |A \cup B|$$

$$= 300 - 140$$

$$= 160$$

Hence, 160 integers are not divisible by 3 nor by 5

Now $|A - C| = |A| - |A \cap C|$

$$= 100 - \left\lfloor \frac{300}{3 \times 7} \right\rfloor$$

$$= 100 - \left\lfloor \frac{300}{21} \right\rfloor = 100 - 14 = 86$$

Hence,

86 integers are divisible by 3 but not by 7

Ex. (3) In a computer laboratory out of 6 computers

i> 2 have floating point arithmetic unit (FPA)

ii> 5 have magnetic disk memory

iii> 3 have graphic displays

iv> 2 have both floating point arithmetic unit and magnetic disk memory

v> 3 have both magnetic disk memory and graphics display

vi> 1 has both floating point arithmetic unit & graphic display

vii> 1 has FPA, magnetic disk m/m & graphic display

– How many have atleast one specification?

$\Rightarrow$

Let, A = set of computers having FPA unit

B = set of computers having magnetic disk m/m

C = –"– having graphics display

Then,

$|A| = 2$

$|B| = 5$

$|C| = 3$

$|A \cap B| = 2$, $|B \cap C| = 3$, $|A \cap C| = 1$

and $|A \cap B \cap C| = 1$

Now

$|A \cup B \cup C| = ?$

$$\therefore |A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |B \cap C| - |A \cap C| + |A \cap B \cap C|$$

$$= 2 + 5 + 3 - 2 - 3 - 1 + 1$$

$$\therefore |A \cup B \cup C| = 5$$

Hence, 5 computers out of 6, have atleast one specification.

Ex. 4 How many integers between 1 - 1000 are divisible by 2, 3, 5 or 7 ?

$\Rightarrow$

let A, B, C & D denote respectively set of integers from 1 to 1000 divisible by 2, 3, 5 or 7

$\therefore \quad |A| = [1000/2] = 500$

$\therefore \quad |B| = [1000/3] = 333$

$\therefore \quad |C| = [1000/5] = 200$

$\therefore \quad |D| = [1000/7] = 142$

$|A \cap B| = \left[\frac{1000}{6}\right] = 166$

$|A \cap C| = \left[\frac{1000}{10}\right] = 100$

$|A \cap D| = \left[\frac{1000}{14}\right] = 71$

$|B \cap C| = [1000/15] = 66$

$|B \cap D| = [1000/21] = 47$

$|C \cap D| = [1000/35] = 28$

$|A \cap B \cap C| = [1000/30] = 33$

$|B \cap C \cap D| = [1000/105] = 9$

$|A \cap C \cap D| = [1000/70] = 14$

$|A \cap B \cap D| = [1000/42] = 23$

$|A \cap B \cap C \cap D| = [1000/210] = 4$

Now

$$|A \cup B \cup C \cup D| = |A| + |B| + |C| + |D| - |A \cap B| - |A \cap C| - |A \cap D| - |B \cap C| - |B \cap D| - |C \cap D| + |A \cap B \cap C| + |A \cap B \cap D| + |A \cap C \cap D| + |B \cap C \cap D| + |A \cap B \cap C \cap D|$$

$$= 500 + 333 + 200 + 142 - 166 - 100 - 71 - 66 - 47 - 28 + 33 + 23 + 14 + 9 + 4$$

$$= 780$$

Ex. 5 Among the integers 1 to 1000:

i) How many of them are not divisible by 3, nor by 5, nor by 7?

ii) How many are not divisible by 5 and 7 but divisible by 3?

⇒

i) Let, A, B, C denotes set of integers 1 to 1000 divisible by 3, 5 or 7 respectively.

Now,

we have to find $\bar{A} \cap \bar{B} \cap \bar{C}$

i.e integers not divisible by 3, 5 or 7

By Demorgans Law

$\bar{A} \cap \bar{B} \cap \bar{C} = \overline{(A \cup B \cup C)}$

Hence,

$$|\bar{A} \cap \bar{B} \cap \bar{C}| = 1000 - |A \cup B \cup C| \quad \rightarrow (1)$$

$|A| = [1000/3] = 333$

$|B| = [1000/5] = 200$

$|C| = [1000/7] = 142$

$|A \cap B| = [1000/15] = 66$

$|A \cap C| = [1000/21] = 47$

$|B \cap C| = [1000/35] = 28$

$|A \cap B \cap C| = [1000/105] = 9$

Hence

$$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |B \cap C| - |A \cap C| + |A \cap B \cap C|$$

$$= 333 + 200 + 142 - 66 - 28 - 47 + 9$$

$$|A \cup B \cup C| = 543$$

From ①

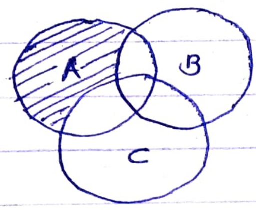
$$|\bar{A} \cap \bar{B} \cap \bar{C}| = 1000 - |A \cup B \cup C|$$

$$= 1000 - 543$$

$$= 457$$

Hence, 457 integers are not divisible by 3, 5 or 7

ii> Consider Venn Diagram



Set of integers not divisible by 5 and 7 but by 3

i.e. $A \cap \bar{B} \cap \bar{C}$

$\therefore A \cap \bar{B} \cap \bar{C} = A \cap (\overline{B \cup C}) = A - (B \cup C)$

from venn diagram

(OR) $|A \cap \bar{B} \cap \bar{C}| = |A| - |A \cap B| - |A \cap C| + |A \cap B \cap C|$

$= 333 - 66 - 47 + 9$

$|A \cap \bar{B} \cap \bar{C}| = 229$

$\therefore |A - (B \cup C)| = |A| - |(A \cap B) \cup (A \cap C)|$

Now

$|(A \cap B) \cup (A \cap C)| = |A \cap B| + |A \cap C| - |A \cap B \cap C|$

$= 66 + 47 - 9$

$= 104$

$\therefore |A - (B \cup C)| = |A| - |(A \cap B) \cup (A \cap C)|$

$= 333 - 104$

$= 229$

Hence, 229 integers are not divisible by 5 & 7 but divisible by 3.

Ex. ⑥ : An investigators interviewed 100 students to determine their preferences for the three drinks - Milk (M), Coffee (C) and Tea (T). He reported following:

10 students had all the three drinks,

20 had 'M' and 'C',

30 had 'C' and 'T'

25 had 'M' and 'T'

12 had 'M' only

5 had 'C' only and 8 had 'T' only.

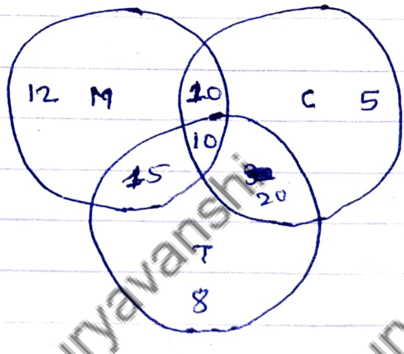
i> How many did not take any of three drinks

ii> How many take milk but not coffee?

iii> How many take tea and coffee but not milk?

⇒

Consider venn diagram



i) $|\bar{M} \cap \bar{C} \cap \bar{T}| = 100 - |M \cup C \cup T|$

$= \cancel{100 - [|M| + |C| + |T| - |M \cap C| - |M \cap T| - |C \cap T|}$
$+ |M \cap C \cap T|]$

$= \cancel{100 - [12 + 5 + 8 - 20 - 25 - 30 + 10]}$

$= \cancel{100 - 40}$

$= 100 - [12 + 10 + 10 + 15 + 20 + 8 + 5]$

$= 100 - 80$

$= 20$

ii) Set of students taking milk but not coffee

$(M - C)$

$\cancel{|M - C| = |M| + |C| - |M \cap C|}$

$\cancel{= 12 + 5 - 10}$

$\cancel{= 7}$

~~OR~~

$\cancel{|M - C| = 12 - 5 = 7}$

$|M - C| = 12 + 15 = 27$

From venn diagram

iii) Set of students taking tea & coffee, but not milk is $(T \cap C) - M$

$|(T \cap C) - M| = |T \cap C| - |T \cap C \cap M|$

$= 30 - 10$

$= 20$

Ex. 7 It was found that in first year of computer science of 80 students : 50 knows 'COBOL', 55 knows 'C', 46 knows 'PASCAL'.

- It was also known that 37 know 'C' & 'COBOL'
- 28 knows 'C' & 'PASCAL'
- 25 knows 'PASCAL' & 'COBOL'.
- 7 students know none of the language.

Find ① How many know all the three languages?
② How many know exactly two languages?
③ How many know exactly one language?

$\Rightarrow$

let, B = set of students who know 'COBOL'
C = —"— know 'C'
P = —"— know 'PASCAL'

Now,

7 students know none of the language

i.e $|\overline{B \cup C \cup P}| = 7$

$\therefore |B \cup C \cup P| = |U| - |\overline{B \cup C \cup P}|$

$= 80 - 7$

$= 73$

$\therefore$ 73 Students know at least one language

① How many know all three language i.e $|B \cap C \cap P| = ?$

Now

$|B \cup C \cup P| = |B| + |C| + |P| - |B \cap C| - |B \cap P| - |C \cap P| + |B \cap C \cap P|$

$\therefore |B \cap C \cap P| = |B \cup C \cup P| - |B| - |C| - |P| + |B \cap C| + |B \cap P| + |C \cap P|$

$= 73 - 50 - 55 - 46 + 37 + 25 + 28$

$= 12$

$\therefore$ 12 Students know all three languages

② How many know exactly two languages?

– Find out : Students who know 'COBOL' & 'C' but not 'PASCAL'

$$|B \cap C \cap \bar{P}| = |B \cap C| - |B \cap C \cap P|$$
$$= 37 - 12 = 25$$

– Find out : Students who know 'COBOL' & 'PASCAL' but not 'C'

$$|B \cap P \cap \bar{C}| = |B \cap P| - |B \cap P \cap C|$$
$$= 25 - 12 = 13$$

– Find out : Students who know 'PASCAL' & 'C' but not 'COBOL'

$$|\bar{B} \cap P \cap C| = |P \cap C| - |B \cap P \cap C|$$
$$= 28 - 12 = 16$$

∴ Hence, the no. of Students who know exactly two languages

$= 25 + 13 + 16$

$= 54$ Students

③ How many knows exactly one language

∴ Find out : Students who know only COBOL & not 'PASCAL' & 'C'

$$\therefore |B \cap \bar{P} \cap \bar{C}| = |B| - |B \cap P| - |B \cap C| + |B \cap P \cap C|$$
$$= 50 - 25 - 37 + 12$$
$$= 0$$

Similarly students who knows only 'C'

$= 55 - 37 - 28 + 12 = 2$

And also, students who know Pascal

$= 46 - 28 - 25 + 12 = 5$

Hence,

No. of students who know exactly one language

$= 0 + 2 + 5 = \cancel{7}$

Ex. (8) A college record gives following information-
119 students enrolled in Introductory Computer science; of these 96 took data structures, 53 took foundations, 39 took Assembly language, 31 took both foundations and Assembly language, 32 took both data structures & Assembly language, 38 took data structures & foundations and 22 took all the three courses.

Is the information correct? Why?

$\Rightarrow$ Let,

D, F & A denote the set of students who took Data Structure, Foundation & Assembly Language.

Given $|D| = 96$, $|F| = 53$, $|A| = 39$

$|F \cap A| = 31$, $|D \cap A| = 32$, $|D \cap F| = 38$

$|F \cap A \cap D| = 22$

$\therefore |F \cup A \cup D| = |D| + |F| + |A| - |F \cap A| - |D \cap A| - |D \cap F| + |F \cap A \cap D|$

$= 96 + 53 + 39 - 31 - 32 - 38 + 22$

$= 109$ which is less than 119

Since, 119 students enrolled for the course, assuming that all these students had taken at least one course, the given info. is NOT correct.

* Power Set

Let A be any set. The power set of A, denoted by P(A) is the set of all subsets of A.

Ex. (1) $A = \{a\}$ $\therefore P(A) = \{\phi, A\}$ i.e $\{\phi, \{a\}\}$

(2) $A = \{a, b\}$ $\therefore P(A) = \{\phi, \{a\}, \{b\}, \{a, b\}\}$

Cardinality of Power Set $|P(A)|$

Let A be a finite set containing n-elements. Then the powerset of A has exactly 2^n elements