on Solving $A_1 + A_2 = -7$ &

$2A_1 + 5A_2 = -8$

we get

$A_1 = -9$ & $A_2 = 2$

so total solution (i.e. eqⁿ ①) becomes

$$\boxed{a_r = -9(2)^r + 2(5)^r + 8 + 2r}$$

—x—

* **Problems on Homogeneous solutions only**

Ex ① Consider difference eqⁿ

$$a_r + 6a_{r-1} + 12a_{r-2} + 8a_{r-3} = 0$$

$\Rightarrow$

$\therefore$ characteristic eqⁿ is

$$\alpha^3 + 6\alpha^2 + 12\alpha + 8 = 0$$

$$\alpha = -2, -2, -2 \quad (m=3)$$

$\therefore$ Homogeneous solution is

$$a_r = (A_1 r^2 + A_2 r + A_3)\alpha_1^r$$

$$a_r = (A_1 r^2 + A_2 r + A_3)(-2)^r$$

Ex ② Consider the difference eqⁿ

$$4a_r - 20a_{r-1} + 17a_{r-2} - 4a_{r-3} = 0$$

$\Rightarrow$

$\therefore$ characteristic eqⁿ is

$$4\alpha^3 - 20\alpha^2 + 17\alpha - 4 = 0$$

$\therefore$ characteristic roots are

$$\alpha = \frac{1}{2}, \frac{1}{2}, 4 \quad (m=2)$$

$\therefore$ So Homogeneous solution is

$$a_r = (A_1 r + A_2)\alpha_1^r + A_3\alpha_2^r$$

$$a_r = (A_1 r + A_2)(\tfrac{1}{2})^r + A_3(4)^r$$

* Problems on particular solutions only

Ex. ① consider the difference eq^n

$$a_r - 5 \cdot a_{r-1} + 6a_{r-2} = 1$$

⇒

$F(r) = 1$ constant, so particular sol^n

$a_r = P$ for all r

∴ we obtain,

$$P - 5P + 6P = 1$$

$$2P = 1$$

$$P = 1/2$$

∴ Particular solution is

$$a_r = \frac{1}{2}$$

Ex. ② consider the difference eq^n

$$a_r + 5a_{r-1} + 6a_{r-2} = 42 \cdot 4^r \quad \text{— ①}$$

⇒

$F(r) = 42 \cdot 4^r$ (Exponential function) — $p \cdot b^r$ & b — is not chara. root

∴ The form of particular sol^n is

$$a_r = P \cdot 4^r$$

so $a_{r-1} = P \cdot 4^{r-1}$ &

$a_{r-2} = P \cdot 4^{r-2}$

by putting these values into eq^n ①

we get

$$P \cdot 4^r + 5 \cdot P 4^{r-1} + 6 \cdot P 4^{r-2} = 42 \cdot 4^r$$

by solving it

$$\boxed{P = 16}$$

∴ particular solution $a_r^{(P)} = P \cdot 4^r$

$$a_r^{(P)} = 16 \cdot 4^r$$

$$p \cdot 4^r + 5 \cdot p4^{r-1} + 6 \cdot p4^{r-2} = 42 \cdot 4^r$$

$$\frac{p \cdot 4^r}{4^r} + \frac{5 \cdot p \cdot 4^{r-1}}{4^r} + \frac{6 \cdot p \cdot 4^{r-2}}{4^r} = 42$$

$$p + \frac{5p}{4} + \frac{6p}{4^2} = 42$$

$$p + \frac{5p}{4} + \frac{6p}{16} = 42$$

$$\frac{16p}{16} + \frac{20p}{16} + \frac{6p}{16} = 42$$

$$\frac{(16+20+6)p}{16} = 42$$

$$\frac{42p}{16} = 42$$

$$\boxed{p = 16}$$

Ex 3 consider the difference eqn

$$a_r + 5a_{r-1} + 6a_{r-2} = 3r^2 \quad —(1)$$

$\Rightarrow$ Nth degree polynomial

~~The characteristic roots of the eqn (2,3)~~

$\therefore$ ~~$f(r) = 3r^2$ where 3 is root~~

so, particular solution is of the form

$$a_r = P_1 r^2 + P_2 r + P_3 \quad —(2)$$

$$\& \quad a_{r-1} = P_1(r-1)^2 + P_2(r-1) + P_3$$

$$a_{r-2} = P_1(r-2)^2 + P_2(r-2) + P_3$$

by putting these values into eqn (1)

$$(P_1 r^2 + P_2 r + P_3) + 5(P_1(r-1)^2 + P_2(r-1) + P_3) + 6(P_1(r-2)^2 + P_2(r-2) + P_3) = 3r^2$$

on solving this, we get

$$12P_1 r^2 - (34P_1 - 12P_2)r + (29P_1 - 17P_2 + 12P_3) = 3r^2$$

on comparing L.H.S. with R.H.S.

$12 P_1 r^2 = 3r^2 \Rightarrow 12P_1 = 3$

$P_1 = 3/12 = 1/4$

$\therefore \boxed{P_1 = 1/4}$

also,

$34 P_1 - 12 P_2 = 0$

$\therefore P_2 = \frac{34}{12} P_1 = \frac{34}{12}\left(\frac{1}{4}\right) = \frac{17}{24}$

$\therefore \boxed{P_2 = 17/24}$

and

$29 P_1 - 17 P_2 + 12 P_3 = 0$

$\therefore \boxed{P_3 = \frac{115}{288}}$

$\therefore$ particular solution is $a_r = P_1 r^2 + P_2 r + P_3$

$$a_r^{(P)} = \left(\frac{1}{4}\right) r^2 + \left(\frac{17}{24}\right) r + \left(\frac{115}{288}\right)$$

Ex. 4 Consider the difference eqn

$$a_r + 5a_{r-1} + 6a_{r-2} = 3r^2 - 2r + 1 \quad -(1)$$

$\Rightarrow$

$F(r) = 3r^2 - 2r + 1$

i.e $n = 2$ degree polynomial

so, particular solution is of the form

$a_r = P_1 r^2 + P_2 r + P_3$

and $a_{r-1} = P_1 (r-1)^2 + P_2 (r-1) + P_3$

$a_{r-2} = P_1 (r-2)^2 + P_2 (r-2) + P_3$

by putting these values into eqn (1)

$$(P_1 r^2 + P_2 r + P_3) + 5\left(P_1 (r-1)^2 + P_2 (r-1) + P_3\right) + 6\left(P_1 (r-2)^2 + P_2 (r-2) + P_3\right) = 3r^2 - 2r + 1$$

which simplifies to

$$12P_1r^2 - (34P_1 - 12P_2)r + (29P_1 - 17P_2 + 12P_3) = 3r^2 - 2r + 1$$

Comparing two sides

$$12P_1 = 3$$

$$34P_1 - 12P_2 = 2$$

$$29P_1 - 17P_2 + 12P_3 = 1$$

which yeild

$$P_1 = \frac{1}{4}, \quad P_2 = \frac{13}{24} \quad \& \quad P_3 = \frac{71}{288}$$

Therefore,
the particular solution is

$$a_r^{(p)} = P_1r^2 + P_2r + P_3$$

$$a_r^{(p)} = \left(\frac{1}{4}\right)r^2 + \left(\frac{13}{24}\right)r + \left(\frac{71}{288}\right)$$

① $a_r - 7a_{r-1} + 10a_{r-2} = 0$ Given $a_0 = 0$ & $a_1 = 3$

② $a_r - 4a_{r-1} + 4a_{r-2} = 0$ given $a_0 = 1$ & $a_1 = 6$

① $a_r - 7a_{r-1} + 10a_{r-2} = 3^r$ $a_0 = 0$ & $a_1 = 1$

② $a_r + 6a_{r-1} + 9a_{r-2} = 3$ $a_0 = 0$ & $a_1 = 1$

③ $a_r + a_{r-1} + a_{r-2} = 0$ $a_0 = 0$ & $a_1 = 2$

① $a_r - a_{r-1} - a_{r-2} = 0$ $a_0 = 1$ $a_1 = 1$

② $\cancel{a_r - 2a_{r-1} + 2a_{r-3} = 0}$ $a_0 = 2,\ a_1 = 1,\ a_2 = 1$

$a_r - 2a_{r-1} + 2a_{r-2} - a_{r-3} = 0$

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* Particular solution

* Example — when $F(r)$ = constant

① $a_r - 5a_{r-1} + 6a_{r-2} = 1$
Determine particular solution

⇒

$F(r) = 1$. constant

so, particular solution is of the form

$a_r = P$ for all r

i.e $a_{r-1} = P$, $a_{r-2} = P$

∴ $a_r - 5a_{r-1} + 6a_{r-2} = 1$ becomes

$P - 5P + 6P = 1$

$2P = 1$ ∴ $\boxed{P = 1/2}$

$\therefore$ particular solution is

$$a_r = P$$

$$a_r = 1/2$$

② $a_r - a_{r-1} - 6a_{r-2} = -30$

Determine particular solution

$\Rightarrow$

Here, $F(r) = -30$ i.e constant

$\therefore$ particular solution is

$a_r = P$ for all r

$\therefore$ $a_r - a_{r-1} - 6a_{r-2} = -30$ becomes

$$P - P - 6P = -30$$

$$-6P = -30$$

$$\boxed{P = 5}$$

$\therefore$ particular solution is

$$a_r = P$$

$$\therefore a_r = 5$$

* Example – when $F(r)$ = linear polynomial

① $a_r - 7a_{r-1} + 10a_{r-2} = 8r + 6$

Determine particular solution

$\Rightarrow$

Here, $F(r) = 8r + 6$ i.e linear polynomial

$\therefore$ particular solution is

$$a_r = P_1 r + P_2$$

$$a_{r-1} = P_1(r-1) + P_2$$

$$a_{r-2} = P_2(r-2) + P_2$$

$\therefore$ $a_r - 7a_{r-1} + 10a_{r-2} = 8r + 6$ becomes

$$(P_1 r + P_2) - 7(P_1(r-1) + P_2) + 10(P_1(r-2) + P_2) = 8r + 6$$

$P_1 r + P_2 - 7P_1 r + 7P_1 - 7P_2$
$+ 10P_1 r - 20P_1 + 10P_2 = 8r + 6$

$\therefore 4P_1 r - 13P_1 + 4P_2 = 8r + 6$ $\therefore$ from equating

$4P_1 r = 8r$

& $-13P_1 + 4P_2 = 6$

$\therefore \boxed{P_1 = 2}$ and $4P_2 = 6 + 13P_1$

$= 6 + 26$

$4P_2 = 32$

$\boxed{P_2 = 8}$

$\therefore$ particular solution is

$a_r = P_1 r + P_2$

$a_r = 2r + 8$

*** Example** – when $f(r)$ = Exponential form $= d \cdot b^r$ and b is not root of equation

(1) $a_r + 5a_{r-1} + 6a_{r-2} = 42 \cdot 4^r$

Determine particular solution

$\Rightarrow$

Here $f(r) = 42 \cdot 4^r$ i.e Exponential form and 4 is not characteristic root of eqn

$\therefore$ particular solution is of the form

$a_r = P \cdot 4^r$

Similarly, $a_{r-1} = P \cdot 4^{r-1}$ & $a_{r-2} = P \cdot 4^{r-2}$

$\therefore a_r + 5a_{r-1} + 6a_{r-2} = 42 \cdot 4^r$ becomes

$P \cdot 4^r + 5P \cdot 4^{r-1} + 6P \cdot 4^{r-2} = 42 \cdot 4^r$

$P + 5P4^{-1} + 6P4^{-2} = 42$

$P + \frac{5P}{4} + \frac{6P}{(4)^2} = 42$

$\frac{16P + 20P + 6P}{16} = 42$ $\therefore \boxed{P = 16}$

$\therefore$ particular solution $a_r = p.4^r$

$a_r = 16 \cdot 4^r$

* Example - $F(r) =$ Exponential form $= d \cdot b^r$ and 'b' is characteristic root of eq^n. repeated m-times

① Consider the difference eq^n.

$$a_r - 2 \cdot a_{r-1} = 3 \cdot 2^r$$

Determine particular solution

$\Rightarrow$

Here $F(r) = 3 \cdot 2^r$ i.e Exponential and 2 is root of eq^n repeated once (i.e $m=1$)

$\therefore$ particular solution is of the form

$$a_r = P \cdot r \cdot 2^r$$

also $a_{r-1} = P \cdot (r-1) \cdot 2^{r-1}$

$\therefore$ $a_r - 2 \cdot a_{r-1} = 3 \cdot 2^r$ becomes

$$P \cdot r \cdot 2^r - 2\left[P(r-1) \cdot 2^{r-1}\right] = 3 \cdot 2^r$$

$$Pr - \frac{2P(r-1)}{2} = 3$$

$$Pr - Pr + P = 3$$

$$\boxed{P=3}$$

$\therefore$ particular solution is

$$a_r = P \cdot r \cdot 2^r$$

$\therefore$ $a_r = (3) \cdot r \cdot 2^r$