② For the difference eqn.

$$a_r - 4a_{r-1} + 4a_{r-2} = (r+1)\cdot 2^r$$

Determine particular solution

$\Rightarrow$

Here $F(r) = (r+1)\cdot 2^r$

and 2 is root of eqn with $m=2$

$\therefore \quad a_r = r^2(P_1 r + P_2)2^r$

$$\therefore \left[r^2(P_1 r + P_2)2^r\right] - 4\left[(r-1)^2\left(P_1(r-1)+P_2\right)2^{r-1}\right]$$

$$+ 4\left[(r-2)^2\left(P_1(r-2)+P_2\right)2^{r-2}\right] = (r+1)\cdot 2^r$$

on simplification we get,

$$6P_1\cdot r\cdot 2^r = r\cdot 2^r$$

$$(-6P_1 + 2P_2)\,2^r = 2^r$$

$\therefore \quad P_1 = 1/6 \quad$ and $\quad P_2 = 1$

$\therefore$ Particular solution is

$$a_r = r^2(P_1 r + P_2)\,2^r$$

$$= r^2\left(\frac{1}{6}\cdot r + 1\right)2^r$$

$$\therefore \quad a_r = r^2\left(\frac{r}{6}+1\right)2^r$$

* Example – where $F(r)$ is nth degree polynomial

① Determine the particular solution for

$$a_r + 5\cdot a_{r-1} + 6\cdot a_{r-2} = 3r^2$$

$\Rightarrow$ Here

$F(r) = 3r^2$ ($n=2$ degree polynomial)

$\therefore$ particular solution is

$$a_r = P_1 r^2 + P_2 r + P_3$$

& $$a_{r-1} = P_1 (r-1)^2 + P_2 (r-1) + P_3$$

$$a_{r-2} = P_1 (r-2)^2 + P_2 (r-2) + P_3$$

So the eqn $a_r + 5a_{r-1} + 6a_{r-2} = 3r^2$ becomes

$$\left[P_1 r^2 + P_2 r + P_3\right] + 5\left[P_1 (r-1)^2 + P_2 (r-1) + P_3\right]$$
$$+ 6\left[P_1 (r-2)^2 + P_2 (r-2) + P_3\right] = 3r^2$$

on solving we get

$$12P_1 r^2 - (34P_1 - 12P_2) r + (29P_1 - 17P_2 + 12P_3) = 3r^2$$

by Comparing L.H.S. & R.H.S.

$$12P_1 = 3$$
$$34P_1 - 12P_2 = 0$$
$$29P_1 - 17P_2 + 12P_3 = 0$$

$\therefore$ $P_1 = 1/4$, $P_2 = 17/24$ & $P_3 = \frac{115}{288}$

particular solution is

$$a_r = P_1 r^2 + P_2 r + P_3$$

$$a_r = \left(\frac{1}{4}\right) r^2 + \left(\frac{17}{24}\right) r + \left(\frac{115}{288}\right)$$

(2) $\Rightarrow$ $a_r + 5a_{r-1} + 6a_{r-2} = 3r^2 - 2r + 1$ — (1)

Here $F(r) = 3r^2 - 2r + 1$

i.e nth degree polynomial

∴ particular solution is

$$a_r = P_1 r^2 + P_2 r + P_3$$

& $$a_{r-1} = P_1 (r-1)^2 + P_2 (r-1) + P_3$$

$$a_{r-2} = P_1 (r-2)^2 + P_2 (r-2) + P_3$$

∴ by putting these values into eqn (1)

$$\left[P_1 r^2 + P_2 r + P_3\right] + 5\left[P_1 (r-1)^2 + P_2 (r-1) + P_3\right]$$

$$+ 6\left[P_1 (r-2)^2 + P_2 (r-2) + P_3\right] = 3r^2 - 2r + 1$$

which ~~simplified~~ simplifies to

$$12P_1 r^2 - (34P_1 - 12P_2) r + (29P_1 - 17P_2 + 12P_3)$$

$$= 3r^2 - 2r + 1$$

On comparing both sides

$$12P_1 = 3$$

$$34P_1 - 12P_2 = 2$$

$$29P_1 - 17P_2 + 12P_3 = 1$$

Which yeilds $P_1 = \frac{1}{4}$, $P_2 = \frac{13}{24}$, & $P_3 = \frac{71}{288}$

∴ particular solution is

$$a_r = P_1 r^2 + P_2 r + P_3$$

$$a_r = \left(\frac{1}{4}\right) r^2 + \left(\frac{13}{24}\right) r + \left(\frac{71}{288}\right)$$

Total Solution

① Find general solution of $a_r - 3a_{r-1} - 4a_{r-2} = 4^r$ —①

⇒

characteristic eqn $\alpha^2 - 3\alpha - 4 = 0$

$\alpha = -1, 4$

∴ Homogeneous solution is

$$a_r^{(h)} = A_1\alpha_1^r + A_2\alpha_2^r = A_1(-1)^r + A_2(4)^r$$

Now

$f(r) = 4^r$ Exponential form & 4 is root of

∴ particular Solution is

$$a_r^{(p)} = P.r.b^r = P.r.4^r$$

Similarlly

$$a_{r-1} = P(r-1)\,4^{(r-1)} \quad \&$$

$$a_{r-2} = P(r-2)\,4^{(r-2)}$$

by putting these values into eqn ①

$$a_r - 3a_{r-1} - 4a_{r-2} = 4^r$$

$$\cancel{P(r-1)\cdot 4^{(r-1)} - 3P(r}$$

$$\therefore\ Pr4^r - 3P(r-1)4^{(r-1)} - 4P(r-2)4^{(r-2)} = 4^r$$

$$\therefore\ Pr - \frac{3P(r-1)}{4} - \frac{4}{4^2}P(r-2) = 1$$

$$4Pr - 3Pr + 3P - Pr + 2P = 4$$

$$\boxed{P = 4/5}$$

∴ particular Solution is $a_r^{p} = P.r.4^r = \left(\frac{4}{5}\right)r\,4^r$

General Solution is

$$a_r = a_r^{(h)} + a_r^{(p)}$$

$$\therefore\ a_r = A_1(-1)^r + A_2(4)^r + \left(\frac{4}{5}\right)r.4^r$$

DSGT - 3 H. E. Suryavanshi (4)

S.E.S

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Marks out of											

Signature of Examiner : Signature of Moderator :

(Start From here only)

(2) Solve $a_n = 2a_{n-1} + 3a_{n-2} + 5^n$, $n \geq 2$

with $a_0 = -2$ & $a_1 = 1$

$\Rightarrow$ $a_n - 2a_{n-1} - 3a_{n-2} = 5^n$ — (1)

Characteristic eq^n is

$$\alpha^2 - 2\alpha - 3 = 0$$

$$(\alpha - 3)(\alpha + 1) = 0$$

$$\alpha = 3, -1$$

$\therefore$ Homogeneous Solution is

$$a_n^{(h)} = A_1 \alpha_1^n + A_2 \alpha_2^n = A_1 (3)^n + A_2 (-1)^n$$

Now

$f(n) = 5^n$ Exponential form & 5 is not characteristic root

$\therefore$ particular Solution is

$$a_n^{(p)} = P \cdot b^n = P \cdot 5^n$$

$a_{n-1} = p \cdot 5^{n-1}$ & $a_{n-2} = p \cdot 5^{n-2}$

by putting these values into eq^n ①

$$a_n - 2a_{n-1} - 3a_{n-2} = 5^n$$

$$p \cdot 5^n - 2 \cdot p \cdot 5^{n-1} - 3 \cdot p\, 5^{n-2} = 5^n$$

$$p - \frac{2p}{5} - \frac{3p}{5^2} = 1$$

$$\boxed{p = \frac{25}{12}}$$

∴ particular soln is

$$a_n^{(p)} = \frac{25}{12}(5^n)$$

∴ Total Solution is

$$a_n = a_n^{(h)} + a_n^{(p)}$$

$$a_n = A_1(3)^n + A_2(-1)^n + \frac{25}{12}(5^n)$$

using initial conditions

$a_0 = -2$ & $a_1 = 1$

we get

$A_1 = \frac{-27}{8}$ & $A_2 = \frac{-17}{24}$

∴ Total soln

$$a_n = \cancel{A_1} \left(\frac{-27}{8}\right)(3^n) + \left(\frac{-17}{24}\right)(-1)^n + \left(\frac{25}{12}\right)(5^n)$$