Theory of SET

* SETS

- A set is collection of objects
- An object is called an element or member of the set.
- The term class is also used to denote a set.
- A set may contain finite or infinite no. of elements
- A set is called empty or null set if it contains no element.
  - An empty set is denoted by ϕ.

* Notations

- A set is denoted by capital letters eg. $A, B, C \cdots Z$
- Elements of the set are denoted by small letters eg a, b, c

  If x is element of set A

  $\therefore$ "$x \in A$" // $\in$: belongs to

  If x is not element of set A

  $\therefore$ "$x \notin A$" //

Various ways of describing a set

① **Listing Method**

eg. $A = \{ \text{pencil, byte, } 5 \}$

$B = \{ 2, 4, 6, 8 \cdots \}$

② **Statement Form**

eg. A = The set of all equilateral triangles

B = The set of all prime ministers of India

③ **Set builder notation**

$A = \{ x \mid P(x) \}$

eg. $A = \{ x \mid x > 10 \}$

$B = \{ x \mid x \text{ is real and } x^8 - 5x^4 + 4 = 0 \}$

* Some Special Sets (Numbers sets)

N - Set of all natural no. $\{1, 2, 3 \cdots\}$

Z → Set of all integers $\{\cdots -2, -1, 0, 1, 2 \cdots\}$

Z^+ - Set of all positive integers $\{0, 1, 2, \cdots\}$

Q - Set of Rational no.

Q^+ - Set of non-negative Rational no.

IR - Set of Real no.

IR^+ - Set of complex no.

* Subsets

- If every element of a set A is also an element of set B, then A is subset of B. or A is contained in B. $(A \subseteq B)$
  - If A is not subset of B, then $(A \nsubseteq B)$

Ex. ①

$A = \{1, 3, 6\}$ and $B = \{-1, 1, 2, 3, 4, 6\}$

$C = \{1, 2, 3\}$

Then $A \subseteq B$

$A \nsubseteq C$

Note: i> Every set is a subset of itself.

ii> Empty set is a subset of any set.

* Universal Set

- If all sets, considered during a specific discussion are subsets of a given set, then this set is called as the Universal Set & denoted by U.

* Equality of Sets

- Two sets A & B are equal if $A \subseteq B$ & $B \subseteq A$ implies $A = B$

Ex. $A = \{$ BASIC, COBOL, FORTRAN $\}$

$B = \{$ FORTRAN, COBOL, BASIC $\}$

then

$A = B$

Ex. ① Let $A = \{ a, b, \{a,b\}, \{\{a,b\}\} \}$
Identify each of the following statements as true or false.

i> $a \in A$ → TRUE — a is element of A

ii> $\{a\} \in A$ → FALSE — $\{a\}$ is not element but subset of A

iii> $\{a,b\} \in A$ → TRUE — $\{a,b\}$ is element of A & listed third in the set

iv> $\{\{a,b\}\} \subseteq A$ → TRUE

v> $\{a,b\} \subseteq A$ → TRUE

* SET OPERATIONS

① Complement of a Set

Let, A be the set.
Complement of A, denoted by $\bar{A}$ is defined as
$$\bar{A} = \{ x \mid x \notin A \}$$

Ex.
i> If $A = \{ x \mid x$ is a real no. and $x \le 7 \}$,
then $\bar{A} = \{ x \mid x$ is real no. and $x > 7 \}$

② Union of two sets

The union of two sets A and B is the set consisting of all elements which are in A, or in B or in both sets A and B. It is denoted by $A \cup B$.
$$A \cup B = \{ x \mid x \in A \text{ or } x \in B \}$$

Ex. i> $A = \{2, 4, 6, 8, 10\}$
$B = \{1, 2, 6, 8, 12, 15\}$
$\therefore A \cup B = \{1, 2, 4, 6, 8, 10, 12, 15\}$

Note:
$A \cup \phi = A$
$A \cup U = U$ (U : universal set)
$A \cup \bar{A} = U$

③ Intersection of sets.

The intersection of two sets A and B, denoted by $A \cap B$ is the set consisting of elements which are in A as well as in B.

$$\therefore \quad A \cap B = \{x \mid x \in A \text{ and } x \in B\}$$

if $A \cap B = \phi$, disjoint set.

Ex. i> If $A = \{a, b, c, g\}$ & $B = \{d, e, f, g\}$

then $A \cap B = \{g\}$

Ex. ii> If $A = \{n \mid n \in N, \ 4 < n < 12\}$ &

$B = \{n \mid n \in N, \ 5 < n < 10\}$

then

$$A \cap B = \{6, 7, 8, 9\} = B$$

Note :

$$A \cap \phi = \phi$$
$$A \cap \bar{A} = \phi$$
$$A \cap U = A$$

④ Difference of Sets (Relative Complement)

Let, A & B be any two sets

The difference A − B is the set defined as

$$A - B = \{x \mid x \in A \text{ and } x \notin B\}$$

is the (relative) complement of B in A

Similarly,

$$B - A = \{x \mid x \in B \text{ and } x \notin A\}$$

is the complement of A in B.

Ex. i> If $A = \{1, 2, 3, \ldots 10\}$

$B = \{1, 3, 5, \ldots 9\}$ i.e odd no.

then

$$A - B = \{2, 4, 6, 8, 10\}$$
$$B - A = \{\phi\}$$

* properties of difference

let A and B be any two sets, then

i> $\overline{A} = U - A$

ii> $A - A = \phi$

iii> $A - \overline{A} = A$,

$\overline{A} - A = \overline{A}$

iv> $A - \phi = A$

v> $A - B = A \cap \overline{B}$

vi> $A - B = B - A$ iff $A = B$

vii> $A - B = A$ iff $A \cap B = \phi$

viii> $A - B = \phi$ iff $A \subseteq B$

⑤ Symmetric Differences

Symmetric difference of two sets A & B, denoted by $A \oplus B$, is defined as

$$A \oplus B = \{x / x \in A - B \text{ or } x \in B - A\}$$

In other words

$$A \oplus B = (A - B) \cup (B - A) \quad // \quad \text{OR} \quad (A \cup B) - (A \cap B)$$

Ex. i> If $A = \{a, b, e, g\}$

$B = \{d, e, f, g\}$

then $A \oplus B = \{a, b, d, f\}$

ii> If $A = \{2, 4, 5, 9\}$

$B = \{x \in Z^+ \mid x^2 \leq 16\}$

then

$$A \oplus B = \{0, 1, 3, 5, 9\}$$

* properties of Symmetric Difference

i> $A \oplus A = \phi$

ii> $A \oplus \phi = A$

iii> $A \oplus U = \overline{A}$

iv> $A \oplus \overline{A} = U$

v> $A \oplus B = (A \cup B) - (A \cap B)$

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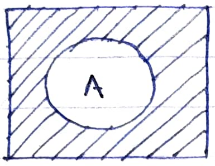
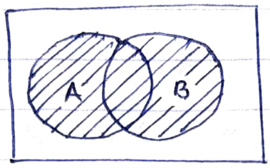
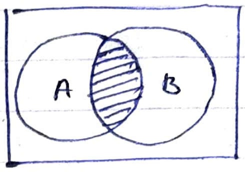
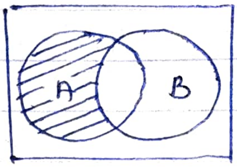
Year : FE / SE / TE / BE Branch : Division : Roll No :

* Representation of Set operations on Venn Diagrams

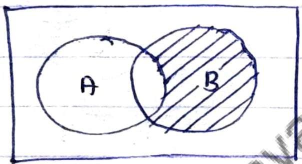


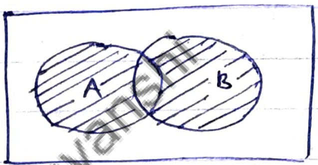






① $\bar{A}$ =   ② $A \cup B$ =   ③ $A \cap B$ =   ④ $A - B$ =





⑤ $B - A$ =   ⑥ $A \oplus B$ =

* Algebra of Set Operations

① Commutativity
i> $A \cup B = B \cup A$
ii> $A \cap B = B \cap A$

② Associativity
i> $A \cup (B \cup C) = (A \cup B) \cup C \Rightarrow A \cup B \cup C$
ii> $A \cap (B \cap C) = (A \cap B) \cap C \Rightarrow A \cap B \cap C$

③ Distributivity
i> $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$
ii> $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

④ Idempotent Laws
i> $A \cup A = A$
ii> $A \cap A = A$

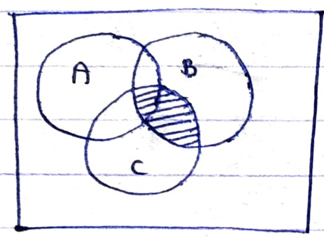
⑤ Absorptions Laws
i> $A \cup (A \cap B) = A$
ii> $A \cap (A \cup B) = A$

⑥ De Morgans Laws i> $\overline{A \cup B} = \overline{A} \cap \overline{B}$
ii> $\overline{A \cap B} = \overline{A} \cup \overline{B}$

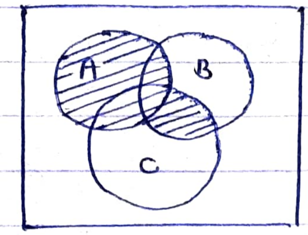
⑦ Double Complement $\overline{\overline{A}} = A$

For distributive Laws

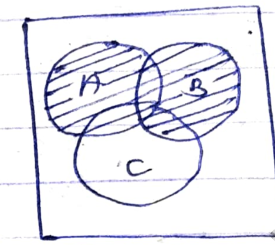
$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$



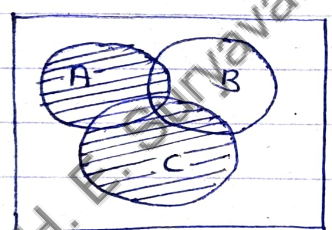
$B \cap C$



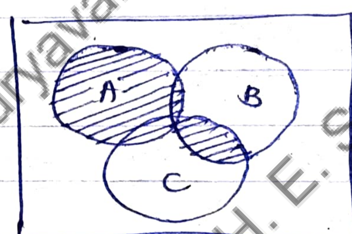
$A \cup (B \cap C)$



$A \cup B$



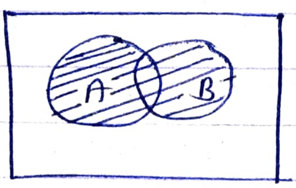
$A \cup C$



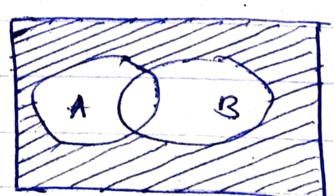
$(A \cup B) \cap (A \cup C)$

De Morgans Laws

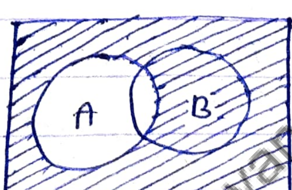
$$\overline{A \cup B} = \overline{A} \cap \overline{B}$$



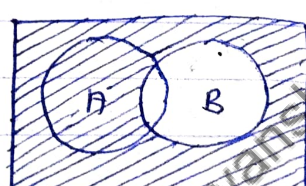
$A \cup B$



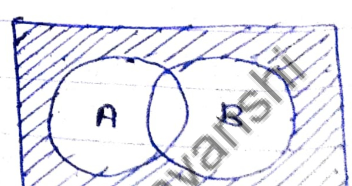
$\overline{A \cup B}$



$\overline{A}$



$\overline{B}$



$\overline{A} \cap \overline{B}$

Ex. ① Let $U = \{ n \mid n \in N,\ n \leq 15 \}$

$A = \{ n \mid n \in N,\ 4 < n < 12 \}$

$B = \{ n \mid n \in N,\ 8 < n < 15 \}$

$C = \{ n \mid n \in N,\ 5 < n < 10 \}$

find

$\bar{A} - \bar{B}$ and $\bar{C} - \bar{A}$

$\Rightarrow$

$U = \{ 1, 2, 3, 4, 5 \ldots\ldots 14, 15 \}$

$A = \{ 5, 6, 7, 8, 9, 10, 11 \}$

$B = \{ 9, 10, 11, 12, 13, 14 \}$

$C = \{ 6, 7, 8, 9 \}$

$\therefore\ \bar{A} = \{ 1, 2, 3, 4, 12, 13, 14, 15 \}$

$\bar{B} = \{ 1, 2, 3, 4, 5, 6, 7, 8, 15 \}$

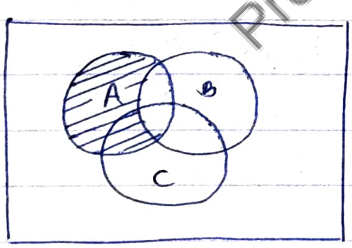
& $\bar{C} = \{ 1, 2, 3, 4, 5, 10, 11, 12, 13, 14, 15 \}$

$\therefore\ \bar{A} - \bar{B} = \{ 12, 13, 14 \}$

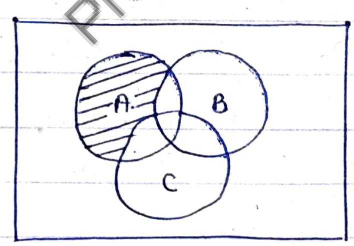
& $\bar{C} - \bar{A} = \{ 5, 10, 11 \}$

Ex. ② show that $(A - B) - C = A - (B \cup C)$ using Venn diagram.

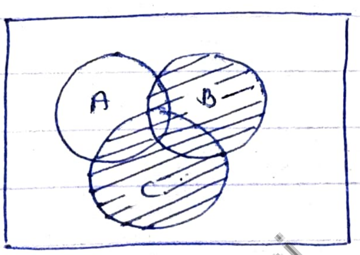
$\Rightarrow$



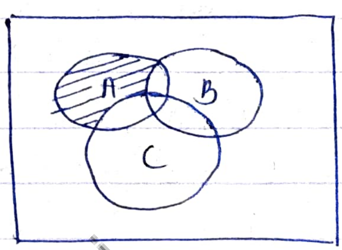
$A - B$



$(A - B) - C$



$B \cup C$

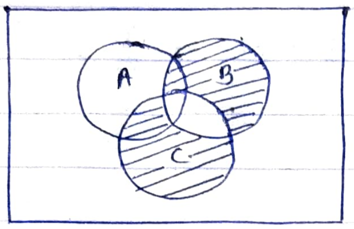


$A - (B \cup C)$

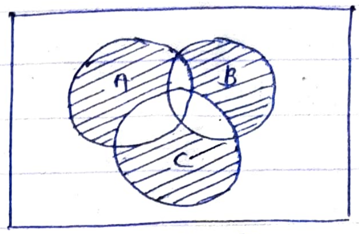
Ex. ③ using Venn diagram, prove or disprove

$$A \oplus (B \oplus C) = (A \oplus B) \oplus C$$

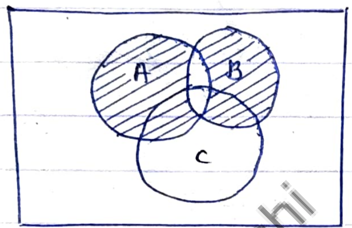
$\Rightarrow$



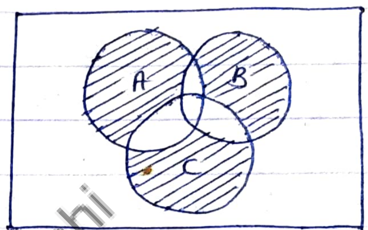
$B \oplus C$



$A \oplus (B \oplus C)$



$A \oplus B$



$(A \oplus B) \oplus C$

Ex. ④ i> Given that $A \cup B = A \cup C$, is it necessary that $B = C$?

ii> Given that $A \cap B = A \cap C$, is it necessary that $B = C$?

$\Rightarrow$

i> No,

let $A = \{1, 2, 3\}$ and $B = \{1\}$, $C = \{3\}$

$\therefore A \cup B = \{1, 2, 3\} = A \cup C.$

but

$B \neq C$

ii> No,

let $A = \{1, 2\}$, $B = \{2, 3, 4, 5\}$ and $C = \{2, 6, 7\}$

then

$A \cap B = \{2\} = A \cap C.$

but

$B \neq C.$