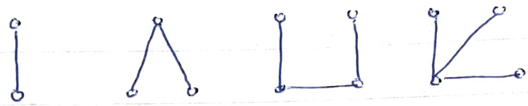
TREES

Trees were discovered by Kirchoff in 1847 while investigating the electrical networks.

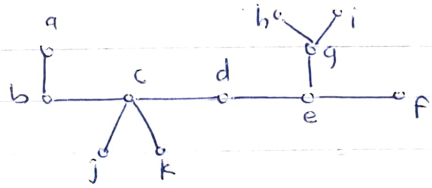
In computer science, trees are useful in organizing and relating data in data base and analysis of algorithm.

* **Definition & properties of trees**

- A tree is simple & connected graph without any circuits.
- Ex.



- A collection of disjoint trees is known as forest.
- A vertex of degree 1 in a tree is called a leaf or terminal node.
- A vertex of degree greater than one is called a branch node or internal node.



b, c, d, e ⇒ branch nodes
a, h, i etc ⇒ leaf nodes

- A tree which is defined as non-cyclic connected graph, can be defined in terms of no. of edges & vertices in the given graph.

Theorems:

1. G is a tree iff there exists a unique path between every pair of vertices of G
2. G is a tree iff if G is connected & has exactly $(n-1)$ edges, where n is the no. of vertices in G.

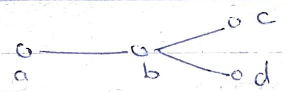
Summary -
1. G is connected & circuitless graph
2. G is connected & has $(n-1)$ edges.
3. G is circuitless & has $(n-1)$ edges
4. There is exactly one path betn every pair of vertices in G.

- In any tree, there are at least two pendant vertices.
- No vertex can be zero degree, at least two vertices of one degree in a tree.

* Eccentricity of a vertex $E(V)$

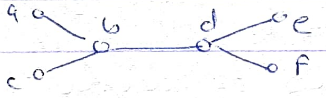
The eccentricity $E(V)$ of a vertex v in a graph G is the distance from v to the farthest from v in G

i.e $E(V) = \max\_dist(V_i, v)$



$E(a) = 2$, $E(b) = 1$, $E(c) = 2$, $E(d) = 2$

- A vertex with minimum eccentricity is called a center of G. (Ex. vertex b is center)
- Every tree has either one or two centers.

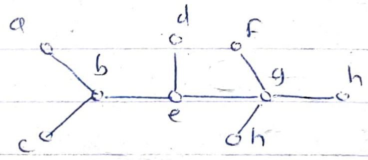


Here $E(a) = E(c) = E(e) = E(f) = 3$
$E(b) = E(d) = 2$

$\therefore$ b & d – centers

* Cut-points

In any tree, all the vertices except pendant vertices (degree-1) are cut-points/vertices.



b, e, g, = cut-points.

Ex. (1) Is it possible to draw a tree with five vertices having degree 1, 1, 2, 2, 4 ?

$\Rightarrow$ Given $n=5$ $\therefore$ $e = (n-1) = 4$ edges $\boxed{e=4}$ Tree

Now, By handshaking lemma

$$\sum_{i=1}^{n} d(v_i) = 2e$$

$1+1+2+2+4 = 2 \times e$ $\Rightarrow$ $\boxed{e=5}$ Graph

$\therefore$ It is not possible to draw such a tree.

Ex. (2) Show that it is possible to draw a tree with 10 vertices which has vertices either of degree 1 or 3. Draw the tree.

Is it possible to draw the same type of tree with 11 vertices.

$\Rightarrow$ Given

$n = 10 \quad \therefore e = (n-1) = 9$ edges

Let x be no. of vertices of degree 1 &

y be no. of vertices of degree 3

$\therefore x + y = 10$ —— (1)

Now,

By handshaking lemma

$$\sum_{i=1}^{n} d(v_i) = 2e$$

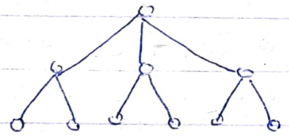
$$1x + 3y = 2 \times 9$$

$$x + 3y = 18 \quad \text{—— (2)}$$

on solving (1) & (2)

$$x = 6 \quad \& \quad y = 4$$

$\therefore$ there are 6-vertices of degree 1 & 4-vertices of degree 3 in a tree with 10 vertices



Now, $x + y = 11$ —— (3) $\quad e = (n-1) = 10$

& By handshaking lemma

$x + 3y = 20$ —— (4)

on solving (3) & (4)

$x = \frac{13}{2} \quad \& \quad y = \frac{9}{2}$ (which is impossible

$\therefore$ There is no tree with 11 vertices which has vertices of degree 1 or 3

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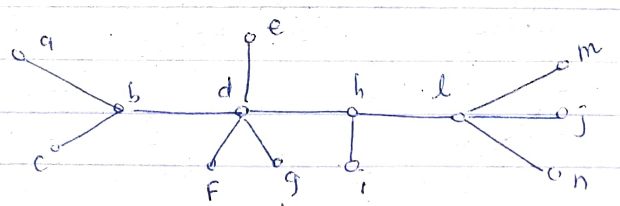
Ex. (6) Find the center of the following tree.



⇒ Center of tree = vertices with minimum eccentricity

E(a) = 5, E(b) = 4, E(c) = 5, E(d) = 3

E(e) = 4, E(f) = 4, E(g) = 4, E(h) = 3

E(l) = 4, E(m) = E(n) = E(j) = 5

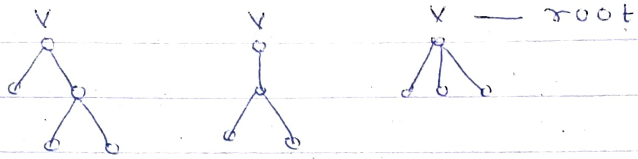
Here, E(d) = E(h) = 3 (minimum eccentricity)

Hence d & h are the centers of the tree

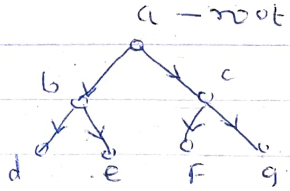
(57)

* Rooted & Binary Trees

- **Rooted Trees**
  - A tree in which one vertex (called root) is distinguished from all other vertices is known as rooted tree.
  - Trees without any root are called free trees or simply trees.
  - A vertex of degree-1 is called a leaf &
  - All the vertices (including roots) that are not leaves are called interior nodes.



- A directed tree (a tree with directions) is called a rooted tree if there is exactly one vertex whose incoming degree is zero & all other vertices having incoming degree one.
  - The vertex with incoming degree-0 zero is known as root of the tree.



a – root
b, c – branch node
d, e, f, g – leaf nodes

- In directed tree, vertex with outgoing degree 0 (zero) is called leaf (pendant) and vertex whose outgoing degree is non-zero is called branch node or an internal node.

① vertex - x in rooted tree is said to be at level - n if there is path of length - n from root to vertex - x

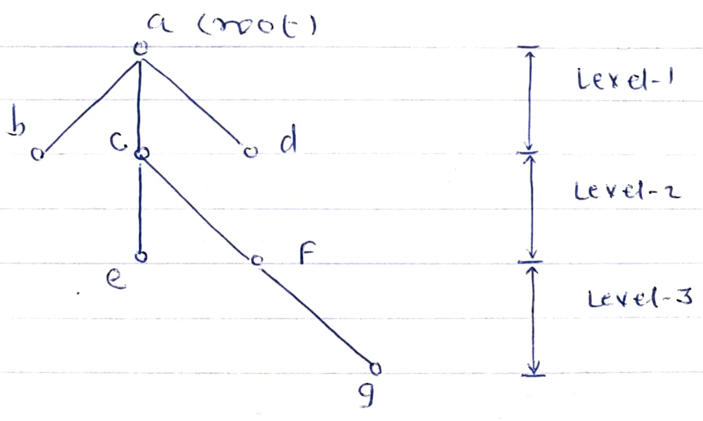
② The height of the tree is the maximum of the levels of its vertices.

③ In rooted tree, level of vertex y is greater than x $\Rightarrow$ y is below x.

- If there is edge joing x to y then
- x is Father of y &
- y is son of x
- Two vertices are called brothers if they are sons of same vertex.

- y is called descendent of x if there is path $P = (x\ v_1\ v_2\ v_3 \cdots v_{n-1}\ y)$ from x to y and x is called ancestor of y

④ The degree of tree is the maximum degree of the nodes in tree.



G

degree of tree (G) = 3

a = root

(b, c, d) & (e, f) = brothers

a - Father of (b, c, d)

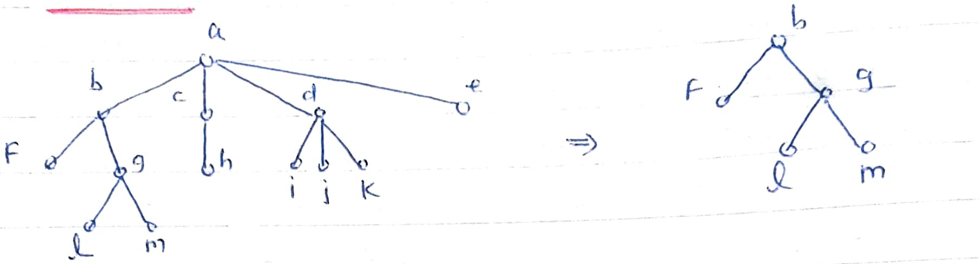
c - Father of (e, f)

g - descendent of a

a - ancestor of g, e etc

⑤ A Forest is a set of disjoint trees. If the root is removed, then forest can be formed.

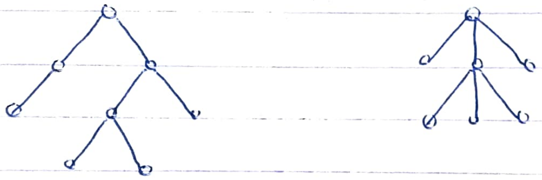
⑥ In a rooted tree T, a vertex-x, together with all its descendants, is called the <u>subtree</u> of T rooted at x



T   T' (subtree)

⑦ A rooted tree in which every interior node has atmost m-sons is called <u>m-ary tree</u>

- A m-ary tree is called <u>regular tree</u> or Full m-ary tree if every branch node has exactly m-sons.



G_1 G_2

G_1 = 2-ary tree but not regular

G_2 = 3-ary tree & also regular tree

<u>Theorem</u>

A regular m-ary tree with i-interior nodes has $(mi+1)$ nodes at all

i.e

$$\boxed{n = (mi+1)}$$

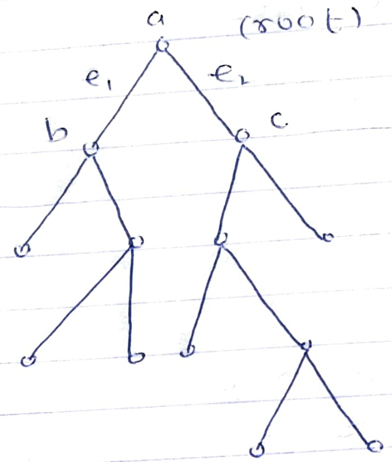
m - ary

i - intermediate nodes

- <u>Binary Tree</u> -

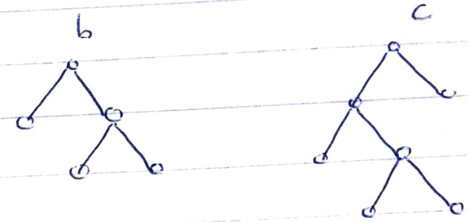
- In Binary Tree, every internal vertex has at most 2 sons.
- A binary tree is a Full or regular binary tree if each internal vertex has exactly 2 sons.

Let T be a full binary tree with height greater than zero & a- as root.
- Deleting root a along with its edges produces two disjoint trees.
- These disjoint binary trees are called left-subtree & Right-subtree of the root 'a'



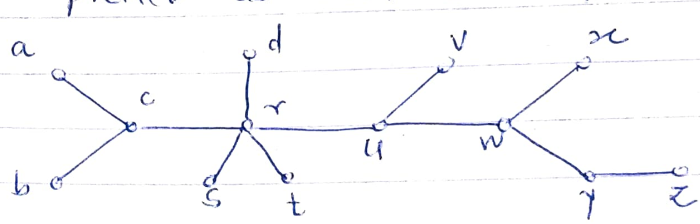
Full-binary tree contains $\left(\frac{n+1}{2}\right)$ number of leaves

(T)



left-subtree (T_1)   Right subtree (T_2)

Ex(i) Consider the tree as shown in fig.
i> which of the vertices if any are cut-points?
ii> find all the vertices at level three, if the vertex picked as root is U & W.



-i> c, r, u, w, y - cut-points
ii> if U = root then a, b & z
if W = root then d, s, t & c

Ex (2) What is the total no. of nodes in full binary tree with 20 leaves?

⇒ Case - 1

$n = mi + 1$

Let n = total nodes $n = 20 + i \Rightarrow i = n - 20$

then no. of internal nodes $i = (n-20)$

In full binary tree $m=2$

$n = 2i + 1$

$n = 2(n-20) + 1$

$n = 2n - 40 + 1$

$n = 39$

∴ 39 nodes

Case - 2

In full binary tree,

no. of leaves $= \left(\frac{n+1}{2}\right)$

∴ $20 = \frac{n+1}{2}$

$40 = n+1$

∴ $n = 39$

∴ total 39 nodes

Ex (3) Does there exist a ternary tree with exactly 21 nodes?

⇒

Here $m=3$ (ternary tree)

$n = 21$

Now $n = mi + 1$

$21 = 3i + 1$

$20 = 3i$

Solution to above eq^n is not integer

Thus, there is no such a tree exists

Ex (4) If there are 60 contestants in a single elimination tournament, how many matches are played?

⇒

- Single elimination tournament represented by full binary tree ($m=2$)

- According to the problem, there are 60 contenstants (leaves)
- Let, i be the no. of internal vertices.
  ∴ total vertices $n = 60 + i$

Now,

$$n = mi + 1$$
$$(60 + i) = 2i + 1$$
$$i = 59$$

∴ 59 mathches

Ex (5) 19 lamps are to be connected to a single electrical outlet, using extension chords, each of which has 4 outlets.
Find the no. of extension chords needed and draw the corresponding tree.

⇒

Tree with 4 outlets (i.e $m=4$)
19 lamps ∴ $n = 19 + i$

Now, by theorem

$$n = mi + 1$$
$$(19 + i) = 4i + 1$$
$$i = 6$$

∴ Hence, six extension chords are required to connect 19 lamps with a single outlet.