

Unit-3

* Pigeonhole Principle -

If 'k' is a positive integer and $k+1$ or more objects are placed into k boxes, then there is at least one box containing two or more of the objects.

Also called 'Dirichlet drawer principle'.

4-Boxes 5-pigeons (A, B, C, D, E)

A	B
C	D, E

Corollary - A function f from a set with $(k+1)$ or more elements to a set with k elements is not one-to-one.

* Extended Pigeonhole Principle -

If 'k' objects are placed in 'n' boxes then at least one box must hold at least $\left\lceil \frac{k}{n} \right\rceil$ objects.

OR $N = \text{holes}$
 $kN + 1 = \text{pigeons}$

Ex. ① Among any group of 367 people, there must be at least two with same birthday, bcoz there are only 366 possible birthdays

Ex. ② In any group of 27 english words, there must be at least two that begins with same letters bcoz there are only 26 alphabets in english.

Ex. ③ How many students must be in a class to guarantee that at least two students receive the same score on the final exam, if the exam is graded on the scale 0 to 100?

→ scale = 0 to 100 = 101 marks

two students with same grade

∴ Total students = 102

Ex. ④ If 7 colors are used to paint 50 bicycles, show that at least 8 of them will be of same color

→ Extended Pigeonhole Principle - $\left\lceil \frac{N}{k} \right\rceil$
 $N = 50$ (pigeons)
 $k = 7$ (holes)

$$\left\lceil \frac{50}{7} \right\rceil = \left\lceil 7.14 \right\rceil = 8 \text{ bicycles}$$

Ex. ⑤ Among 100 people there are at least $\left\lceil \frac{100}{12} \right\rceil = 9$ who are born in same month.

Ex. ⑥ show that in a group of 50 students at least 5 are born in same month

→ $\left\lceil \frac{50}{12} \right\rceil = \left\lceil 4.16 \right\rceil = 5$

or $N = 12$ (months)
 $kN + 1 = 50$ (Pigeons)

$$k \cdot 12 + 1 = 50$$

$$k \cdot 12 = 50 - 1$$

$$k = \frac{49}{12} = 4 \text{ } 1 \text{ remainder}$$

∴ At least $4 + 1 = 5$ Students are borned in same month.

Ex. ⑦ What is the minimum no. of students required in DM subject in class to be sure that at least six will receive the same grade (A, B, C, D, E)?

→ Grades $N = 5$ $\left\lceil \frac{N}{5} \right\rceil = 6$
 ∴ $N = 5 \cdot 5 + 1 = 26$ Students