Counting

- Combinatorics, the study of arrangements of objects
- Counting is used to determine the complexity of algorithms
- Counting is also required to determine whether there are enough telephone numbers or Internet protocol addresses to meet demand

Counting Problems: ordered or unordered arrangements of the objects of a set with or without repetitions (Permutations and Combinations)

The Basics of Counting

Suppose that a password on a computer system consists of six, seven, or eight characters. Each of these characters must be a digit or a letter of the alphabet. Each password must contain at least one digit.

How many such passwords are there?

Basic Counting Principles

- Two basic counting principles, the product rule and the sum rule.

1. Product Rule

Suppose that a procedure can be broken down into a sequence of two tasks.
If there are n1 ways to do the first task and for each of these ways of doing the first task, there are n2 ways to do the second task, then there are (n1*n2) ways to do the procedure.

EXAMPLE 1
A new company with just two employees, Sanchez and Patel, rents a floor of a building with 12 offices.

How many ways are there to assign different offices to these two employees?

Ans:

EXAMPLE 2

The chairs of an auditorium are to be labeled with an uppercase English letter followed by a positive integer not exceeding 100.

What is the largest number of chairs that can be labeled differently?

Ans:

EXAMPLE 3

There are 32 microcomputers in a computer center.
Each microcomputer has 24 ports.

How many different ports to a microcomputer in the center are there?

Ans:

Extended Product Rule

An extended version of the product rule is often useful. Suppose that a procedure is carried out by performing the tasks $T1, T2 \ldots Tm$ in sequence.
If each task Ti , $i = 1, 2, \ldots, n$, can be done in n_i ways, regardless of how the previous tasks were done, then there are $n_1 \cdot n_2 \cdot \cdots \cdot n_m$ ways to carry out the procedure.

EXAMPLE 4

How many different bit strings of length seven are there?

Ans:

EXAMPLE 5

How many different license plates can be made if each plate contains a sequence of three uppercase English letters followed by three digits (and no sequences of letters are prohibited, even if they are obscene)?

Ans:

EXAMPLE 6

What is the value of k after the following code, where n1, n2, . . . , nm are positive integers, has been executed?

$k := 0$
for $i_1 := 1$ **to** n_1
  for $i_2 := 1$ **to** n_2
    .
    .
    .
      for $i_m := 1$ **to** n_m
        $k := k + 1$

Ans:

2. Sum Rule

If a task can be done either in one of n1 ways or in one of n2 ways, where none of the set of n1 ways is the same as any of the set of n2 ways, then there are (n1 + n2) ways to do the task.

EXAMPLE 1

Suppose that either a member of the mathematics faculty or a student who is a mathematics major is chosen as a representative to a university committee.

How many different choices are there for this representative if there are 37 members of the mathematics faculty and 83 mathematics majors and no one is both a faculty member and a student?

Ans:

EXAMPLE 2

A student can choose a computer project from one of three lists. The three lists contain 23, 15, and 19 possible projects, respectively. No project is on more than one list. How many possible projects are there to choose from?

Ans:

EXAMPLE 3

What is the value of k after the following code, where n1, n2, . . . , n_m are positive integers, has been executed?

$k := 0$
for $i_1 := 1$ **to** n_1
    $k := k + 1$
for $i_2 := 1$ **to** n_2
    $k := k + 1$
    .
    .
    .
for $i_m := 1$ **to** n_m
    $k := k + 1$

Ans: