

DSQT-1 Tutorial

Ex. ① P : Gopal is tall q : Gopal is handsome

- i> Gopal is tall & handsome.
- ii> Gopal is tall but not handsome.
- iii> It is false that Gopal is short or handsome.
- iv> Gopal is neither tall nor handsome
- v> It is not true that Gopal is short or not handsome.

Ex. ② P : High speed driving is dangerous
 q : Rajesh was a wise man

- i> $P \wedge q$ ii> $\neg P \wedge q$ iii> $\neg(P \wedge q)$ iv> $(P \wedge q) \vee (\neg P \wedge \neg q)$
- v> $(P \vee q) \wedge \neg(P \wedge q)$

Ex. ③ P : Today is Monday q : The grass is wet
 r : The dish ran away with the spoon

- i> $\neg r \wedge q$ ii> $\neg q \vee r$ iii> $\neg(P \vee q)$ iv> $P \vee \neg r$

Ex. ④ construct truth table & determine the truth value

- i> $(P \wedge (P \rightarrow q)) \rightarrow q$ ii> $(P \leftrightarrow q) \leftrightarrow ((P \wedge q) \vee (\neg P \wedge \neg q))$

Ex. ⑤ determine: Tautology, Contradiction

- i> $(P \rightarrow \neg P) \rightarrow \neg P$ ii> $(P \vee q) \wedge (\neg P \rightarrow q)$
- iii> $(P \wedge q) \leftrightarrow P$ iv> $(P \rightarrow q) \wedge (q \rightarrow r) \rightarrow (P \rightarrow r)$

Ex. ⑥ Show that following statements are tautological

- i> $(P \wedge (P \rightarrow q)) \rightarrow q$ ii> $(P \rightarrow q) \leftrightarrow (q \vee \neg P)$

Ex. ⑦ Show that following statements are logically equivalent

- i> $P \rightarrow (q \rightarrow r)$ and $P \rightarrow (\neg q \vee r)$
- ii> $P \leftrightarrow q$ and $(P \wedge q) \vee (\neg P \wedge \neg q)$
- iii> $P \rightarrow (q \rightarrow r)$ and $(P \rightarrow q) \rightarrow (P \rightarrow r)$

Ex. ⑧ Show that following statements are tautology without T.T.

- i> $(P \wedge (P \rightarrow q)) \rightarrow q$ ii> $(P \rightarrow q) \wedge \neg q \rightarrow \neg p$

Ex ⑨ For $A = \{a, b, \{b, c\}, \phi\}$ determine the following set

- i> $A - \{a\}$ ii> $A - \{b, c\}$ iii> $\{\{b, c\}\} - A$
 iv> $A - \{c, \phi\}$ v> $\{a\} - \{A\}$

Ex ⑩ If $U = \{n \in \mathbb{N} \mid 1 \leq n \leq 9\}$

$$A = \{1, 2, 4, 6, 8\} \quad B = \{2, 4, 5, 9\}$$

$$C = \{x \in \mathbb{Z}^+ \mid x^2 \leq 16\} \quad \text{and} \quad D = \{7, 8\}$$

Find i> $A \oplus B$, $B \oplus C$, $C \oplus D$

ii> $A - B$, $B - A$, $C - D$

iii> $\overline{A \cup B}$, $\overline{A \cap B}$

iv> $A \cap (\overline{C \cup D})$

• Ex ⑪ Using Venn diagrams, prove or disprove the followings

i> $(A - B) - C = (A - C) - B$

ii> $(A - B) - C = (A - C) - (B - C)$

iii> $(A - B) \cap (A - C) = A - (B \cup C)$

iv> $(A - C) \cup (B - C) = (A \cup B) - C$

• Ex ⑫ Consider set of integers from 1 to 250. Find how many of these are divisible by 3, 5, & 7?

Also indicate how many are divisible by 3 or 7 but not by 5.

→ i> $|A \cup B \cup C| = 136$ ← 3 and 7 not 5 3 or 7 not 5

ii> $|A \cap B \cap \overline{C}| = |A \cap C| - |A \cap B \cap C| = 9$ ↓

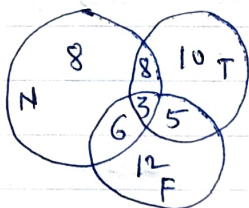
iii> $|A \cup B \cup C| - |B|$ or $|A \cup C| - |B| = |A| + |C| - |A \cap C| - |B| = 57$

• Ex ⑬ How many integers between 1 to 2000 are divisible by 2, 3, 5 or 7?

→ $|A \cup B \cup C \cup D| = ?$ → 1449 1560

• Ex ⑭ In a Survey of 60 people, it was found that 25 read Newsweek Magazine, 26 read Time, 26 read Fortune. Also 9 read both Newsweek & Fortune, 11 read both Newsweek & Time, 8 read both Time & Fortune, 8 read no magazine

- i> Find out the no. of people who read all three magazine i.e. $|N \cap T \cap F| = 3$
- ii> Determine the no. of people who read exactly one magazine i.e. $x+y+z = 8+10+12 = 30$
- iii> Find in correct no. in all regions of Venn diagram



iv> Exactly two $\rightarrow 19$

- Ex. (15) A survey of 500, television watchers produced the following info. - 285 watch Football, 195 watch hockey, 115 watch basket ball, 45 watch Football & basket ball, 70 watch Football & hockey, 50 watch hockey & basket ball and 50 do not watch any of three games

$190 + 95 + 40$ - i> How many watch all 3 games? $\rightarrow 20$
 $70 + 50 + 30$ - ii> How many watch exactly one game? $\rightarrow 325$
 $70 + 50 + 30$ - iii> ——— exactly two games $\rightarrow 105$

- Ex. (16) i> Among 50 students in a class, 26 got an A in the First examination and 21 got A in second examination. If 17 students did not get an A in either examination, how many students got A in both examination? $\rightarrow 14$

- Ex. (17) 100 of the 120 engineering students in a college take part in atleast one of the activities - group discussion, debate and quiz. Also 65 participate in group discussion, 45 participate in debate, 42 participate in quiz, 20 participate in group discussion & debate, 25 participate in group discussion & quiz, 15 participate in debate & quiz.

Find (i) who participate in all activities! $\rightarrow 8$

(ii) Who participate in exactly one activities? $\rightarrow (28+18+10=56)$

Ex. (18) In a survey of new cars, it is found that 60 had Air Conditioner (AC), 48 had Power Steering (PS), 44 had Power Windows (PW), 36 had AC+PW, 20 had AC+PS, 16 had PW+PS, 12 had all three. Find the no. of cars that had

- i) only PW $\rightarrow 4$
- ii) PS & PW but not AC $\rightarrow 4$
- iii) AC & PS but not PW $\rightarrow 8$

Ex. (19) Show that $5^n - 4n - 1$ is divisible by 16 for all $n \geq 2$

$\rightarrow 5(5^{k-1}) - 4k$

$n \geq 2$

or $(5^k - 4k - 1) + 4(5^k - 1)$ or $5(5^k - 1) - 4k$

Ex. (20) Prove mathematical induction for $n \geq 1$

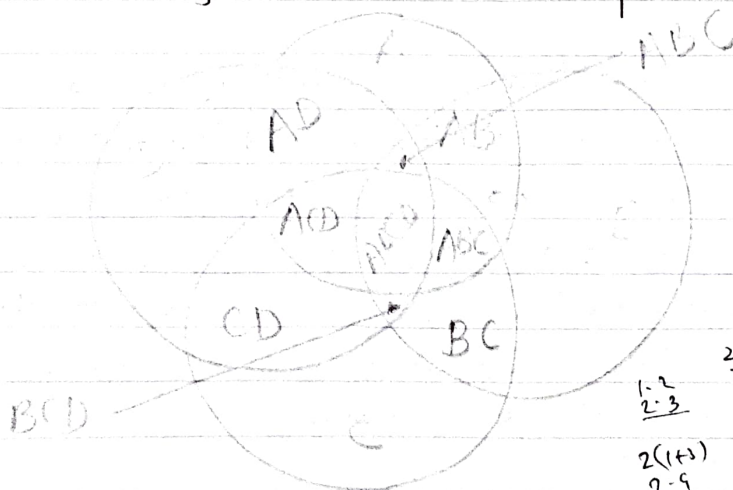
$1 \cdot 2 + 2 \cdot 3 + \dots + n(n+1) = \frac{n(n+1)(n+2)}{3}$

How Actually 375^*

16, 30, 44, 58, 72, 86, 100, 114, 128, 142, 156, 170, 184, 198, 212, 226, 240, 254, 268, 282, 296, 310, 324, 338, 352, 366, 380, 394, 408, 422, 436, 450, 464, 478, 492, 506, 520, 534, 548, 562, 576, 590, 604, 618, 632, 646, 660, 674, 688, 702, 716, 730, 744, 758, 772, 786, 800, 814, 828, 842, 856, 870, 884, 898, 912, 926, 940, 954, 968, 982, 996, 1010, 1024, 1038, 1052, 1066, 1080, 1094, 1108, 1122, 1136, 1150, 1164, 1178, 1192, 1206, 1220, 1234, 1248, 1262, 1276, 1290, 1304, 1318, 1332, 1346, 1360, 1374, 1388, 1402, 1416, 1430, 1444, 1458, 1472, 1486, 1500, 1514, 1528, 1542, 1556, 1570, 1584, 1598, 1612, 1626, 1640, 1654, 1668, 1682, 1696, 1710, 1724, 1738, 1752, 1766, 1780, 1794, 1808, 1822, 1836, 1850, 1864, 1878, 1892, 1906, 1920, 1934, 1948, 1962, 1976, 1990, 2004, 2018, 2032, 2046, 2060, 2074, 2088, 2102, 2116, 2130, 2144, 2158, 2172, 2186, 2200, 2214, 2228, 2242, 2256, 2270, 2284, 2298, 2312, 2326, 2340, 2354, 2368, 2382, 2396, 2410, 2424, 2438, 2452, 2466, 2480, 2494, 2508, 2522, 2536, 2550, 2564, 2578, 2592, 2606, 2620, 2634, 2648, 2662, 2676, 2690, 2704, 2718, 2732, 2746, 2760, 2774, 2788, 2802, 2816, 2830, 2844, 2858, 2872, 2886, 2900, 2914, 2928, 2942, 2956, 2970, 2984, 2998, 3012, 3026, 3040, 3054, 3068, 3082, 3096, 3110, 3124, 3138, 3152, 3166, 3180, 3194, 3208, 3222, 3236, 3250, 3264, 3278, 3292, 3306, 3320, 3334, 3348, 3362, 3376, 3390, 3404, 3418, 3432, 3446, 3460, 3474, 3488, 3502, 3516, 3530, 3544, 3558, 3572, 3586, 3600, 3614, 3628, 3642, 3656, 3670, 3684, 3698, 3712, 3726, 3740, 3754, 3768, 3782, 3796, 3810, 3824, 3838, 3852, 3866, 3880, 3894, 3908, 3922, 3936, 3950, 3964, 3978, 3992, 4006, 4020, 4034, 4048, 4062, 4076, 4090, 4104, 4118, 4132, 4146, 4160, 4174, 4188, 4202, 4216, 4230, 4244, 4258, 4272, 4286, 4300, 4314, 4328, 4342, 4356, 4370, 4384, 4398, 4412, 4426, 4440, 4454, 4468, 4482, 4496, 4510, 4524, 4538, 4552, 4566, 4580, 4594, 4608, 4622, 4636, 4650, 4664, 4678, 4692, 4706, 4720, 4734, 4748, 4762, 4776, 4790, 4804, 4818, 4832, 4846, 4860, 4874, 4888, 4902, 4916, 4930, 4944, 4958, 4972, 4986, 5000, 5014, 5028, 5042, 5056, 5070, 5084, 5098, 5112, 5126, 5140, 5154, 5168, 5182, 5196, 5210, 5224, 5238, 5252, 5266, 5280, 5294, 5308, 5322, 5336, 5350, 5364, 5378, 5392, 5406, 5420, 5434, 5448, 5462, 5476, 5490, 5504, 5518, 5532, 5546, 5560, 5574, 5588, 5602, 5616, 5630, 5644, 5658, 5672, 5686, 5700, 5714, 5728, 5742, 5756, 5770, 5784, 5798, 5812, 5826, 5840, 5854, 5868, 5882, 5896, 5910, 5924, 5938, 5952, 5966, 5980, 5994, 6008, 6022, 6036, 6050, 6064, 6078, 6092, 6106, 6120, 6134, 6148, 6162, 6176, 6190, 6204, 6218, 6232, 6246, 6260, 6274, 6288, 6302, 6316, 6330, 6344, 6358, 6372, 6386, 6400, 6414, 6428, 6442, 6456, 6470, 6484, 6498, 6512, 6526, 6540, 6554, 6568, 6582, 6596, 6610, 6624, 6638, 6652, 6666, 6680, 6694, 6708, 6722, 6736, 6750, 6764, 6778, 6792, 6806, 6820, 6834, 6848, 6862, 6876, 6890, 6904, 6918, 6932, 6946, 6960, 6974, 6988, 7002, 7016, 7030, 7044, 7058, 7072, 7086, 7100, 7114, 7128, 7142, 7156, 7170, 7184, 7198, 7212, 7226, 7240, 7254, 7268, 7282, 7296, 7310, 7324, 7338, 7352, 7366, 7380, 7394, 7408, 7422, 7436, 7450, 7464, 7478, 7492, 7506, 7520, 7534, 7548, 7562, 7576, 7590, 7604, 7618, 7632, 7646, 7660, 7674, 7688, 7702, 7716, 7730, 7744, 7758, 7772, 7786, 7800, 7814, 7828, 7842, 7856, 7870, 7884, 7898, 7912, 7926, 7940, 7954, 7968, 7982, 7996, 8010, 8024, 8038, 8052, 8066, 8080, 8094, 8108, 8122, 8136, 8150, 8164, 8178, 8192, 8206, 8220, 8234, 8248, 8262, 8276, 8290, 8304, 8318, 8332, 8346, 8360, 8374, 8388, 8402, 8416, 8430, 8444, 8458, 8472, 8486, 8500, 8514, 8528, 8542, 8556, 8570, 8584, 8598, 8612, 8626, 8640, 8654, 8668, 8682, 8696, 8710, 8724, 8738, 8752, 8766, 8780, 8794, 8808, 8822, 8836, 8850, 8864, 8878, 8892, 8906, 8920, 8934, 8948, 8962, 8976, 8990, 9004, 9018, 9032, 9046, 9060, 9074, 9088, 9102, 9116, 9130, 9144, 9158, 9172, 9186, 9200, 9214, 9228, 9242, 9256, 9270, 9284, 9298, 9312, 9326, 9340, 9354, 9368, 9382, 9396, 9410, 9424, 9438, 9452, 9466, 9480, 9494, 9508, 9522, 9536, 9550, 9564, 9578, 9592, 9606, 9620, 9634, 9648, 9662, 9676, 9690, 9704, 9718, 9732, 9746, 9760, 9774, 9788, 9802, 9816, 9830, 9844, 9858, 9872, 9886, 9900, 9914, 9928, 9942, 9956, 9970, 9984, 10000

Ex (21) $p(n) = 1 + 4 + 7 + \dots + (3n-2) = \frac{n(3n-1)}{2}$

Ex. (22) There are 21 cricket players in Indian team. out of these, 6 players taking part in one day match, 7 players in t20 and 5 in both. How many players not taking part in onday and t20. Ans = 13



$$\frac{1 \cdot 2}{2 \cdot 3} = \frac{2(1+1)}{2 \cdot 3}$$

$$\frac{2 \cdot 3 \cdot 4}{3 \cdot 4 \cdot 5} = \frac{2 \cdot 3 \cdot 4}{3 \cdot 4 \cdot 5}$$

$$\frac{3 \cdot 4 \cdot 5}{4 \cdot 5 \cdot 6} = \frac{3 \cdot 4 \cdot 5}{4 \cdot 5 \cdot 6}$$

$$\frac{4 \cdot 5 \cdot 6}{5 \cdot 6 \cdot 7} = \frac{4 \cdot 5 \cdot 6}{5 \cdot 6 \cdot 7}$$

$$\frac{5 \cdot 6 \cdot 7}{6 \cdot 7 \cdot 8} = \frac{5 \cdot 6 \cdot 7}{6 \cdot 7 \cdot 8}$$

$$\frac{6 \cdot 7 \cdot 8}{7 \cdot 8 \cdot 9} = \frac{6 \cdot 7 \cdot 8}{7 \cdot 8 \cdot 9}$$

$$\frac{7 \cdot 8 \cdot 9}{8 \cdot 9 \cdot 10} = \frac{7 \cdot 8 \cdot 9}{8 \cdot 9 \cdot 10}$$

$$\frac{8 \cdot 9 \cdot 10}{9 \cdot 10 \cdot 11} = \frac{8 \cdot 9 \cdot 10}{9 \cdot 10 \cdot 11}$$

$$\frac{9 \cdot 10 \cdot 11}{10 \cdot 11 \cdot 12} = \frac{9 \cdot 10 \cdot 11}{10 \cdot 11 \cdot 12}$$

$$\frac{10 \cdot 11 \cdot 12}{11 \cdot 12 \cdot 13} = \frac{10 \cdot 11 \cdot 12}{11 \cdot 12 \cdot 13}$$

$$\frac{11 \cdot 12 \cdot 13}{12 \cdot 13 \cdot 14} = \frac{11 \cdot 12 \cdot 13}{12 \cdot 13 \cdot 14}$$

$$\frac{12 \cdot 13 \cdot 14}{13 \cdot 14 \cdot 15} = \frac{12 \cdot 13 \cdot 14}{13 \cdot 14 \cdot 15}$$

$$\frac{13 \cdot 14 \cdot 15}{14 \cdot 15 \cdot 16} = \frac{13 \cdot 14 \cdot 15}{14 \cdot 15 \cdot 16}$$

$$\frac{14 \cdot 15 \cdot 16}{15 \cdot 16 \cdot 17} = \frac{14 \cdot 15 \cdot 16}{15 \cdot 16 \cdot 17}$$

$$\frac{15 \cdot 16 \cdot 17}{16 \cdot 17 \cdot 18} = \frac{15 \cdot 16 \cdot 17}{16 \cdot 17 \cdot 18}$$

$$\frac{16 \cdot 17 \cdot 18}{17 \cdot 18 \cdot 19} = \frac{16 \cdot 17 \cdot 18}{17 \cdot 18 \cdot 19}$$

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$$\frac{30 \cdot 31 \cdot 32}{31 \cdot 32 \cdot 33} = \frac{30 \cdot 31 \cdot 32}{31 \cdot 32 \cdot 33}$$

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$$\frac{40 \cdot 41 \cdot 42}{41 \cdot 42 \cdot 43} = \frac{40 \cdot 41 \cdot 42}{41 \cdot 42 \cdot 43}$$

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$$\frac{43 \cdot 44 \cdot 45}{44 \cdot 45 \cdot 46} = \frac{43 \cdot 44 \cdot 45}{44 \cdot 45 \cdot 46}$$

$$\frac{44 \cdot 45 \cdot 46}{45 \cdot 46 \cdot 47} = \frac{44 \cdot 45 \cdot 46}{45 \cdot 46 \cdot 47}$$

$$\frac{45 \cdot 46 \cdot 47}{46 \cdot 47 \cdot 48} = \frac{45 \cdot 46 \cdot 47}{46 \cdot 47 \cdot 48}$$

$$\frac{46 \cdot 47 \cdot 48}{47 \cdot 48 \cdot 49} = \frac{46 \cdot 47 \cdot 48}{47 \cdot 48 \cdot 49}$$

$$\frac{47 \cdot 48 \cdot 49}{48 \cdot 49 \cdot 50} = \frac{47 \cdot 48 \cdot 49}{48 \cdot 49 \cdot 50}$$

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$$\frac{49 \cdot 50 \cdot 51}{50 \cdot 51 \cdot 52} = \frac{49 \cdot 50 \cdot 51}{50 \cdot 51 \cdot 52}$$

$$\frac{50 \cdot 51 \cdot 52}{51 \cdot 52 \cdot 53} = \frac{50 \cdot 51 \cdot 52}{51 \cdot 52 \cdot 53}$$

$$\frac{51 \cdot 52 \cdot 53}{52 \cdot 53 \cdot 54} = \frac{51 \cdot 52 \cdot 53}{52 \cdot 53 \cdot 54}$$

$$\frac{52 \cdot 53 \cdot 54}{53 \cdot 54 \cdot 55} = \frac{52 \cdot 53 \cdot 54}{53 \cdot 54 \cdot 55}$$

$$\frac{53 \cdot 54 \cdot 55}{54 \cdot 55 \cdot 56} = \frac{53 \cdot 54 \cdot 55}{54 \cdot 55 \cdot 56}$$

$$\frac{54 \cdot 55 \cdot 56}{55 \cdot 56 \cdot 57} = \frac{54 \cdot 55 \cdot 56}{55 \cdot 56 \cdot 57}$$

$$\frac{55 \cdot 56 \cdot 57}{56 \cdot 57 \cdot 58} = \frac{55 \cdot 56 \cdot 57}{56 \cdot 57 \cdot 58}$$

$$\frac{56 \cdot 57 \cdot 58}{57 \cdot 58 \cdot 59} = \frac{56 \cdot 57 \cdot 58}{57 \cdot 58 \cdot 59}$$

$$\frac{57 \cdot 58 \cdot 59}{58 \cdot 59 \cdot 60} = \frac{57 \cdot 58 \cdot 59}{58 \cdot 59 \cdot 60}$$

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$$\frac{59 \cdot 60 \cdot 61}{60 \cdot 61 \cdot 62} = \frac{59 \cdot 60 \cdot 61}{60 \cdot 61 \cdot 62}$$

$$\frac{60 \cdot 61 \cdot 62}{61 \cdot 62 \cdot 63} = \frac{60 \cdot 61 \cdot 62}{61 \cdot 62 \cdot 63}$$

$$\frac{61 \cdot 62 \cdot 63}{62 \cdot 63 \cdot 64} = \frac{61 \cdot 62 \cdot 63}{62 \cdot 63 \cdot 64}$$

$$\frac{62 \cdot 63 \cdot 64}{63 \cdot 64 \cdot 65} = \frac{62 \cdot 63 \cdot 64}{63 \cdot 64 \cdot 65}$$

$$\frac{63 \cdot 64 \cdot 65}{64 \cdot 65 \cdot 66} = \frac{63 \cdot 64 \cdot 65}{64 \cdot 65 \cdot 66}$$

$$\frac{64 \cdot 65 \cdot 66}{65 \cdot 66 \cdot 67} = \frac{64 \cdot 65 \cdot 66}{65 \cdot 66 \cdot 67}$$

$$\frac{65 \cdot 66 \cdot 67}{66 \cdot 67 \cdot 68} = \frac{65 \cdot 66 \cdot 67}{66 \cdot 67 \cdot 68}$$

$$\frac{66 \cdot 67 \cdot 68}{67 \cdot 68 \cdot 69} = \frac{66 \cdot 67 \cdot 68}{67 \cdot 68 \cdot 69}$$

$$\frac{67 \cdot 68 \cdot 69}{68 \cdot 69 \cdot 70} = \frac{67 \cdot 68 \cdot 69}{68 \cdot 69 \cdot 70}$$

$$\frac{68 \cdot 69 \cdot 70}{69 \cdot 70 \cdot 71} = \frac{68 \cdot 69 \cdot 70}{69 \cdot 70 \cdot 71}$$

$$\frac{69 \cdot 70 \cdot 71}{70 \cdot 71 \cdot 72} = \frac{69 \cdot 70 \cdot 71}{70 \cdot 71 \cdot 72}$$

$$\frac{70 \cdot 71 \cdot 72}{71 \cdot 72 \cdot 73} = \frac{70 \cdot 71 \cdot 72}{71 \cdot 72 \cdot 73}$$

$$\frac{71 \cdot 72 \cdot 73}{72 \cdot 73 \cdot 74} = \frac{71 \cdot 72 \cdot 73}{72 \cdot 73 \cdot 74}$$

$$\frac{72 \cdot 73 \cdot 74}{73 \cdot 74 \cdot 75} = \frac{72 \cdot 73 \cdot 74}{73 \cdot 74 \cdot 75}$$

$$\frac{73 \cdot 74 \cdot 75}{74 \cdot 75 \cdot 76} = \frac{73 \cdot 74 \cdot 75}{74 \cdot 75 \cdot 76}$$

$$\frac{74 \cdot 75 \cdot 76}{75 \cdot 76 \cdot 77} = \frac{74 \cdot 75 \cdot 76}{75 \cdot 76 \cdot 77}$$

$$\frac{75 \cdot 76 \cdot 77}{76 \cdot 77 \cdot 78} = \frac{75 \cdot 76 \cdot 77}{76 \cdot 77 \cdot 78}$$

$$\frac{76 \cdot 77 \cdot 78}{77 \cdot 78 \cdot 79} = \frac{76 \cdot 77 \cdot 78}{77 \cdot 78 \cdot 79}$$

$$\frac{77 \cdot 78 \cdot 79}{78 \cdot 79 \cdot 80} = \frac{77 \cdot 78 \cdot 79}{78 \cdot 79 \cdot 80}$$

$$\frac{78 \cdot 79 \cdot 80}{79 \cdot 80 \cdot 81} = \frac{78 \cdot 79 \cdot 80}{79 \cdot 80 \cdot 81}$$

$$\frac{79 \cdot 80 \cdot 81}{80 \cdot 81 \cdot 82} = \frac{79 \cdot 80 \cdot 81}{80 \cdot 81 \cdot 82}$$

$$\frac{80 \cdot 81 \cdot 82}{81 \cdot 82 \cdot 83} = \frac{80 \cdot 81 \cdot 82}{81 \cdot 82 \cdot 83}$$

$$\frac{81 \cdot 82 \cdot 83}{82 \cdot 83 \cdot 84} = \frac{81 \cdot 82 \cdot 83}{82 \cdot 83 \cdot 84}$$

$$\frac{82 \cdot 83 \cdot 84}{83 \cdot 84 \cdot 85} = \frac{82 \cdot 83 \cdot 84}{83 \cdot 84 \cdot 85}$$

$$\frac{83 \cdot 84 \cdot 85}{84 \cdot 85 \cdot 86} = \frac{83 \cdot 84 \cdot 85}{84 \cdot 85 \cdot 86}$$

$$\frac{84 \cdot 85 \cdot 86}{85 \cdot 86 \cdot 87} = \frac{84 \cdot 85 \cdot 86}{85 \cdot 86 \cdot 87}$$

$$\frac{85 \cdot 86 \cdot 87}{86 \cdot 87 \cdot 88} = \frac{85 \cdot 86 \cdot 87}{86 \cdot 87 \cdot 88}$$

$$\frac{86 \cdot 87 \cdot 88}{87 \cdot 88 \cdot 89} = \frac{86 \cdot 87 \cdot 88}{87 \cdot 88 \cdot 89}$$

$$\frac{87 \cdot 88 \cdot 89}{88 \cdot 89 \cdot 90} = \frac{87 \cdot 88 \cdot 89}{88 \cdot 89 \cdot 90}$$

$$\frac{88 \cdot 89 \cdot 90}{89 \cdot 90 \cdot 91} = \frac{88 \cdot 89 \cdot 90}{89 \cdot 90 \cdot 91}$$

$$\frac{89 \cdot 90 \cdot 91}{90 \cdot 91 \cdot 92} = \frac{89 \cdot 90 \cdot 91}{90 \cdot 91 \cdot 92}$$

$$\frac{90 \cdot 91 \cdot 92}{91 \cdot 92 \cdot 93} = \frac{90 \cdot 91 \cdot 92}{91 \cdot 92 \cdot 93}$$

$$\frac{91 \cdot 92 \cdot 93}{92 \cdot 93 \cdot 94} = \frac{91 \cdot 92 \cdot 93}{92 \cdot 93 \cdot 94}$$

$$\frac{92 \cdot 93 \cdot 94}{93 \cdot 94 \cdot 95} = \frac{92 \cdot 93 \cdot 94}{93 \cdot 94 \cdot 95}$$

$$\frac{93 \cdot 94 \cdot 95}{94 \cdot 95 \cdot 96} = \frac{93 \cdot 94 \cdot 95}{94 \cdot 95 \cdot 96}$$

$$\frac{94 \cdot 95 \cdot 96}{95 \cdot 96 \cdot 97} = \frac{94 \cdot 95 \cdot 96}{95 \cdot 96 \cdot 97}$$

$$\frac{95 \cdot 96 \cdot 97}{96 \cdot 97 \cdot 98} = \frac{95 \cdot 96 \cdot 97}{96 \cdot 97 \cdot 98}$$

$$\frac{96 \cdot 97 \cdot 98}{97 \cdot 98 \cdot 99} = \frac{96 \cdot 97 \cdot 98}{97 \cdot 98 \cdot 99}$$

$$\frac{97 \cdot 98 \cdot 99}{98 \cdot 99 \cdot 100} = \frac{97 \cdot 98 \cdot 99}{98 \cdot 99 \cdot 100}$$

$$\frac{98 \cdot 99 \cdot 100}{99 \cdot 100 \cdot 101} = \frac{98 \cdot 99 \cdot 100}{99 \cdot 100 \cdot 101}$$

$$\frac{99 \cdot 100 \cdot 101}{100 \cdot 101 \cdot 102} = \frac{99 \cdot 100 \cdot 101}{100 \cdot 101 \cdot 102}$$

$$\frac{100 \cdot 101 \cdot 102}{101 \cdot 102 \cdot 103} = \frac{100 \cdot 101 \cdot 102}{101 \cdot 102 \cdot 103}$$

$$\frac{101 \cdot 102 \cdot 103}{102 \cdot 103 \cdot 104} = \frac{101 \cdot 102 \cdot 103}{102 \cdot 103 \cdot 104}$$

$$\frac{102 \cdot 103 \cdot 104}{103 \cdot 104 \cdot 105} = \frac{102 \cdot 103 \cdot 104}{103 \cdot 104 \cdot 105}$$

$$\frac{103 \cdot 104 \cdot 105}{104 \cdot 105 \cdot 106} = \frac{103 \cdot 104 \cdot 105}{104 \cdot 105 \cdot 106}$$

$$\frac{104 \cdot 105 \cdot 106}{105 \cdot 106 \cdot 107} = \frac{104 \cdot 105 \cdot 106}{105 \cdot 106 \cdot 107}$$

$$\frac{105 \cdot 106 \cdot 107}{106 \cdot 107 \cdot 108} = \frac{105 \cdot 106 \cdot 107}{106 \cdot 107 \cdot 108}$$

$$\frac{106 \cdot 107 \cdot 108}{107 \cdot 108 \cdot 109} = \frac{106 \cdot 107 \$$