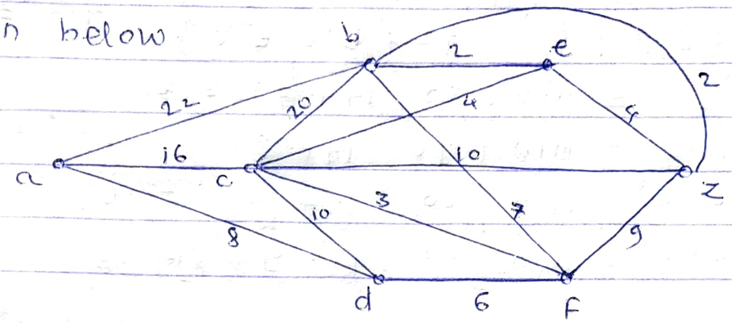
Ex. (2) Determine a shortest path from 'a' to 'z' in the graph shown below



⟹

Dijkstras Algorithm to find shortest path from 'a' to 'z'

① $P = \{\phi\}$ $T = \{a, b, c, d, e, f, z\}$

$L(a) = 0$, $L(x) = \infty$, $\forall x \in T$, $x \neq a$

② $v = a$, permanent label of a is 0 i.e $L(a) = 0$

$P = \{a\}$ $T = \{b, c, d, e, f, z\}$

$L(b) = \min(\text{old\_}L(b) + L(a) + w(a,b))$

$= \min(\infty, 0 + 22)$

$\therefore L(b) = 22$

$L(c) = \min(\infty, 0 + 16) = 16$

$L(d) = \min(\infty, 0 + 8) = 8$

$L(e) = \min(\infty, 0 + \infty) = \infty$

$L(f) = \min(\infty, 0 + \infty) = \infty$

$L(z) = \min(\infty, 0 + \infty) = \infty$.

③ $v = d$, permanent label of d is 8 i.e $L(d) = 8$

$P = \{a, d\}$ $T = \{b, c, e, f, z\}$

$L(b) = \min(\text{old\_}L(b), L(d) + w(d,b))$

$= \min(22, 8 + \infty)$

$L(b) = 22$

$L(c) = \min(16, 8 + 10) = 16$

$L(e) = \min(\infty, 8 + \infty) = \infty$

$L(f) = \min(\infty, 8 + 6) = 14$

$L(z) = \min(\infty, 8 + \infty) = \infty$

$v = f$, — the permanent label of f is 14 i.e $L(f) = 14$

$P = \{a, d, f\}$ $\quad T = \{b, c, e, z\}$

$L(b) = \min(22, 14 + 7) = 21$

$L(c) = \min(16, 14 + 3) = 16$

$L(e) = \min(\infty, 14 + \infty) = \infty$

$L(z) = \min(\infty, 14 + 9) = 23$

④ $v = c$, permanent label of c is 16 i.e $L(c) = 16$

$P = \{a, d, f, c\}$ $\quad T = \{b, e, z\}$

$L(b) = \min(21, 16 + 20) = 21$

$L(e) = \min(\infty, 16 + 4) = 20$

$L(z) = \min(23, 16 + 10) = 23$

⑤ $v = e$, permanent label of e is 20 i.e $L(e) = 20$

$P = \{a, d, f, c, e\}$ $\quad T = \{b, z\}$

$L(b) = \min(21, 20 + 2) = 21$

$L(z) = \min(23, 20 + 4) = 23$

⑥ $v = b$, permanent label of b is 21 i.e $L(b) = 21$

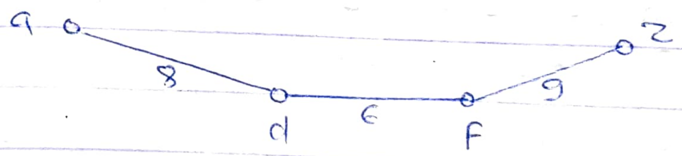
$P = \{a, d, f, c, e, b\}$ $\quad T = \{z\}$

$L(z) = \min(23, 21 + 2) = 23$

⑦ $v = z$, permanent label of z is 23 i.e $L(z) = 23$

$P = \{a, d, f, c, e, b, z\}$ $\quad T = \{\phi\}$

Hence, the length of shortest path from a to z is 23 & path is $a - d - f - z$



DSGT-4 H·E Suryavanshi (5)

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Semester : I / II Name of Subject :

Total Supplements : 1 + =

Signature of Student Signature of Supervisor

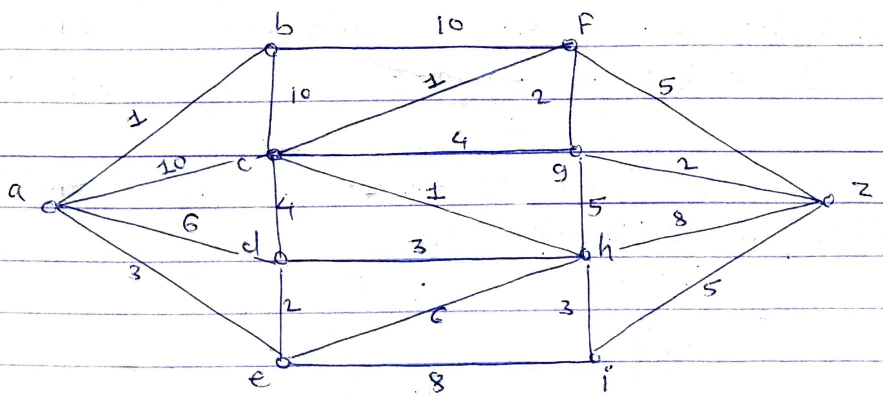
Question Number	1	2	3	4	5	6	7	8	9	10	Total
Marks Obtained											
Marks out of											

Signature of Examiner : Signature of Moderator :

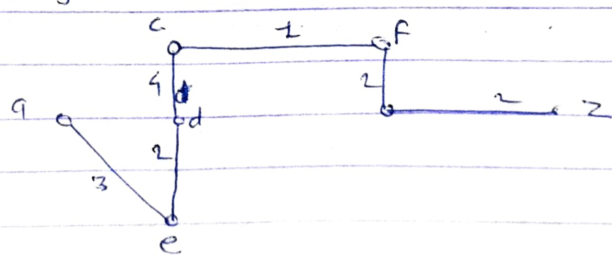
(Start From here only)

H.W.

Ex. (1) Find shortest path from a to z using Dijkstra's algo.



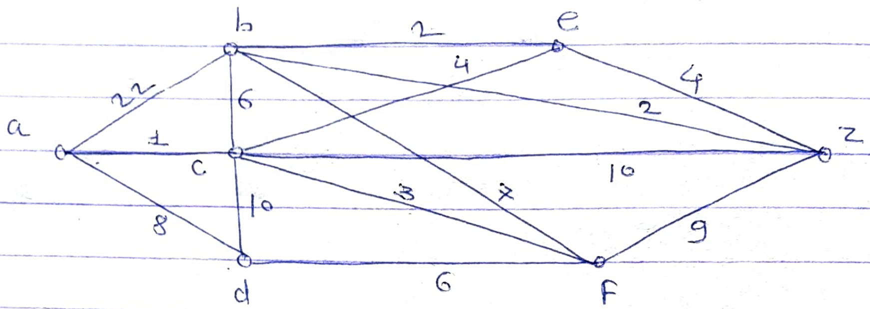
Ans Length of shortest path from a to z is 15 (a-e-d-c-f-g-z)



(33)

H.W.

Ex (2) Apply Dijkstra's shortest path algorithm to find the shortest path between vertices a and z.

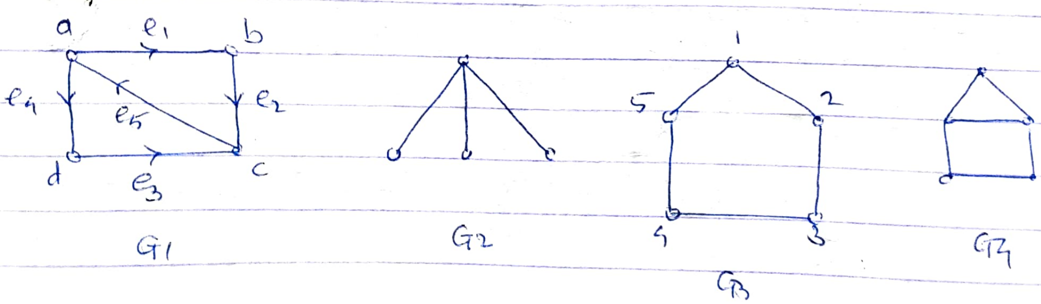


Ans. – length of shortest path is 9
path = a – c – e – z

=x=

* Eulerian Path and Eulerian Circuits
– Swiss Mathematician Leonhard Euler
(1) Eulerian Path
– Every edge of the graph G appears exactly once in the path
(2) Eulerian Circuit
– Circuit which contains every edge of the graph G exactly once
(3) Eulerian Graph
– A graph which has an Eulerian circuit called Eulerian Graph.

– A graph which has an eulerian path may not have an eulerian circuit.



G_1 has eulerian path $(e_1 e_2 e_3 e_4 e_5)$ but G_2 not
G_3 contains eulerian ckt (1 2 3 4 5 1) but G_4 not

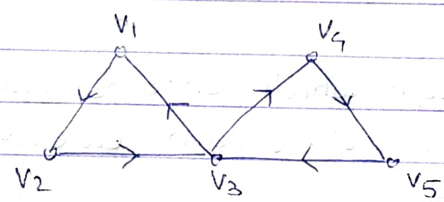
- Existence of eulerian paths/ckts in a graph is related to the degree of vertices as follows

Theorems

(1) Graph possesses eulerian path iff it is connected & has '0' (zero) or two vertices of degree odd (i.e odd degree vertices)

(2) Graph possesses eulerian circuit iff it is connected & its vertices are all of even degree.

(3) Directed graph possesses eulerian circuit if it is connected & incoming & outgoing degree of every vertex is same.



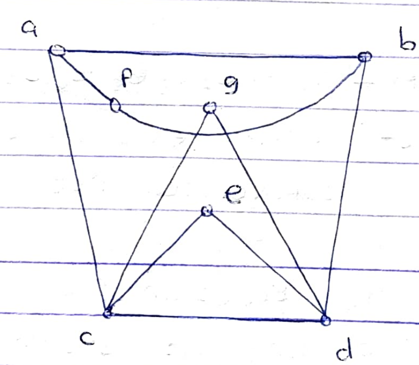
$\overrightarrow{d}(V_1) = \overleftarrow{d}(V_1) = 1$

$\overrightarrow{d}(V_2) = \overleftarrow{d}(V_2) = 1$

$\overrightarrow{d}(V_3) = \overleftarrow{d}(V_3) = 2$

Hence, diagraph has eulerian ckt – $V_1 V_2 V_3 V_4 V_5 V_3 V_1$

Ex (1) Find out the eulerian ckt in the following graph. Also find an euler path from vertex 'a' to 'b'.



⇒ Eulerian circuit -

According to the theorem, graph doesnt contains all vertices of even degree

Hence, graph doesnt have eulerian circuit

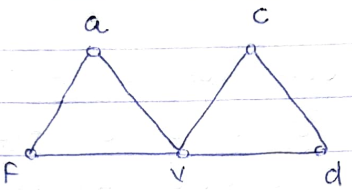
Eulerian Path -

Also, graph has two vertices of odd degree So, graph has eulerian path given as below

⇒ a - c - e - d - c - g - d - b - f - a - b

Ex ② Draw the graph which has eulerian ckt. and cut-vertex also.

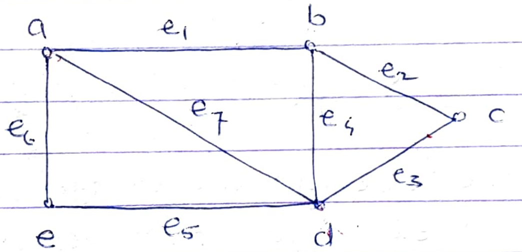
⇒



- Degree of each vertex is even
  Hence, graph has eulerian ckt.
- Cut vertex is 'v'. Its removal forms disconnected graph with two components
- Also, Eulerian path is a-v-c-d-v-f-a

Ex. ③ Draw the graph which contains eulerian path but does not contain eulerian ckt.

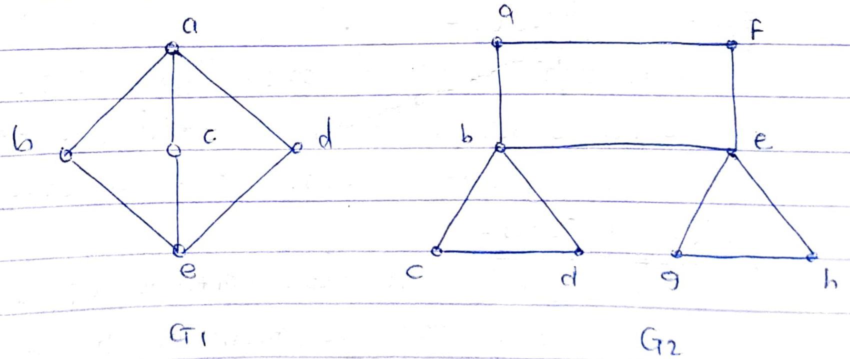
⇒



Eulerian Path – a e_1 e_2 e_3 e_5 e_6 e_7 e_4 b

Eulerian ckt – doesnt exist. bcoz degree of all vertices not even.

Ex. ④ Determine whether Eulerian path & ckt. exits in the graph G_1 and G_2 shown below.

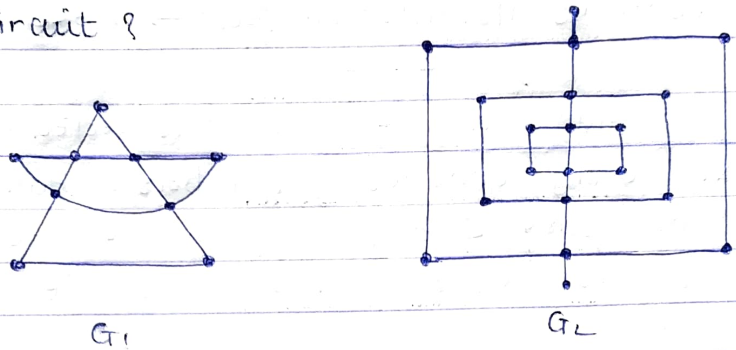


⇒ G_1 contains Eulerian path bcoz exactly two vertices have odd degree (a, e) ⇒ a-d-e-b-a-c-e

G_1 doesnt contains eulerian ckt. bcoz degree of all vertices is not even

In G_2, degree of each vertex is even. Hence G_2 has both eulerian ckt. & path ⇒ a-f-e-h-g-e-b-d-c-b-a

Ex. ⑤ Which of the following graph possess Euler's path or circuit ?



⇒ - In G_1, each vertex is of even degree. Hence it possess eulerian circuit

- In G_2, graph is connected and has exactly two vertices of odd degree. Hence, there exist eulerian path.

* Hamiltonian Path and Hamiltonian Circuit

- Sir William Hamiltonian (1859)

Hamiltonian Circuit

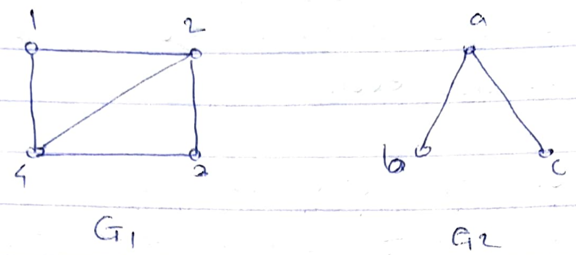
- A circuit in connected graph G is called Hamiltonian Circuit if it contains every vertex of G exactly once (except first & last vertex).

Hamiltonian Path

- A path in connected graph G is a Hamiltonian path if it contains every vertex of G exactly once.

A graph which has Hamiltonian circuit is called a Hamiltonian Graph

A graph that contains the Hamiltonian path may not contain Hamiltonian circuit



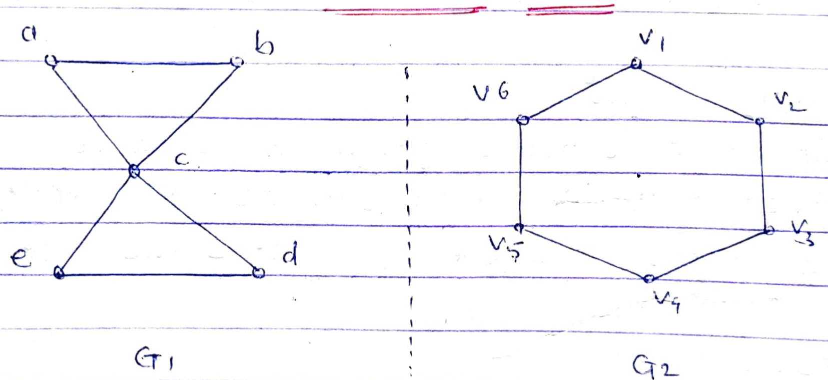
G_1 contains Hamiltonian ckt → (1-2-3-4-1)
G_2 contains Hamiltonian path → (b-a-c)

- Hamiltonian path & ckt's in connected graph
  - Not every connected graph has a Hamiltonian ~~graph~~ path
  - A necessary & sufficient condition for a connected graph to have a Hamiltonian circuit is still unknown.

Theorem ①

Let G be a simple graph with n-vertices. If the sum of degree for each pair of vertices is (n-1) or large, then there exist a Hamiltonian path.

- It is sufficient condition but not necessary condition for the existence of Hamiltonian path.



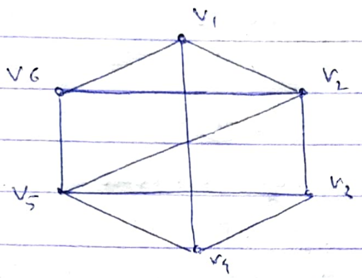
In G_1, $n=5$
∴ Sum of degree for each pair must be >= (n-1) i.e (4)
∴ There exist Hamiltonian path (a-b-c-e-d)

In G_2 $n=6$
∴ sum of degree of pair must be >= (n-1) i.e 5 which is not true
But the graph G_2 has Hamiltonian path- $v_1 v_2 v_3 v_4 v_5 v_6$

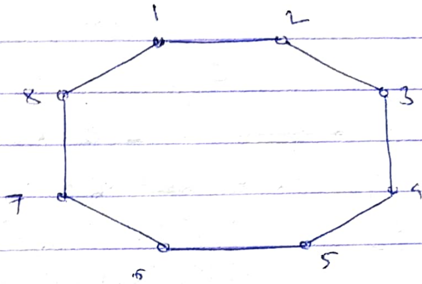
$$\boxed{\text{Total no. of Hamiltonion ckt in a graph} = \frac{(n-1)!}{2}}$$

Theorem ②

Let $G=(V,E)$ be a simple connected graph.
If the degree of each vertex 'v' $>= n/2$
i.e. $d(v) >= (n/2)$ $\forall v \in V$.
then Graph will contain Hamiltonian Circuit.



G_1



G_2

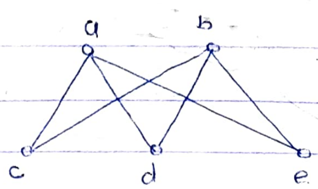
According to theorem ② degree of each vertex is greater than or equal to $n/2$ (i.e 3)
$\therefore$ G_1 contains Hamiltonian circuit - $(v_1 v_2 v_3 v_4 v_5 v_6 v_1)$

In G_2, $n=8$
It contains Hamiltonian ckt - (1 2 3 4 5 6 7 8 1)
But $d(v) = 2 \not\geq 8/2 = 4$
according to theorem ②

Theorem ③

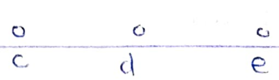
Let G be a connected simple graph. If G has a Hamiltonian circuit then for every proper non-empty subset S of V, the components in graph $(G-S)$ is less than or equal to the number of vertices in S
i.e. $(G-S) \leq |S|$ where $S \subseteq V$

Ex ① consider complete Bipartite graph $K_{2,3}$



$S = V_1 = \{a, b\}$ $|V_1| = m$
$V_2 = \{c, d, e\}$ $|V_2| = n$
i.e $m < n$

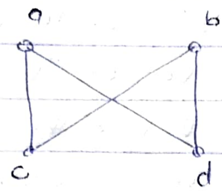
Now $(K_{2,3} - V_1)$ — is null graph with 3-component



let X = components
$\therefore X = 3$

According to theorem ③ No. of components are less than or equal to total vertices in S.
$\therefore$ but here $X \not\leq m$ $\therefore$ It does not contain Hamiltonian Circuit

Ex (2) Let $K_{2,2}$ - complete biperhte graph



Total vertices $= 4$ each vertex v is $d(v) = 2$

$\&\ d(v) \geq n/2$

$2 \geq 4/2$ by theorem (2)

Hence, graph has Hamiltonion circuit

Note : In complete Bipertite graph $(K_{m,n})$, there exist Hamiltonian circuit & path also if $m = n$

* Travelling Salesman Problem

- Problem related to Hamiltonian circuit

Problem -

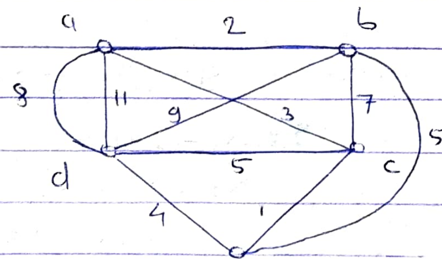
A salesman is required to visit a no. of cities during a trip. Given the distance between the cities, in what order should he travel so as to visit every city precisely once and return home with minimum distance travelled ?

Let $G = (V, E)$ be weighted graph shown in fig.

V = represents the cities

E = represents the road

& $w(E)$ = represents the distance betn two cities



Hamiltonian ckts starts from 'a'

(1) abceda with weight = 22

(2) abecda ⇒ wt = 21

(3) acebda ⇒ wt = 26 &

so on

If G is K_n with n-nodes, then there are $\frac{(n-1)!}{2}$ possible Hamiltonian ckts. However for large value of n, it is highly inefficient algorithm

- Nearest - Neighbour Method

- provides good results for salesman problem.