## * Recurrence Relation

A recurrence relation for a sequence $\{a_n\}$ is an equation that expresses $a_n$ in terms of one or more of the previous terms of the sequence, namely $a_0, a_1, a_2 \cdots a_{n-1}$ $\forall n$ with $n \geq n_0$ where $n_0$ is non-negative integer.

A sequence is called Solution of a recurrence relⁿ if its terms satisfys the recurrence relⁿ.

Ex. ① $a_n = a_{n-1} + 3$, $n \geq 1$ with $a_0 = 2$

$\rightarrow$

$a_0 = 2$

$a_1 = a_0 + 3 = 2+3 = 5$

$a_2 = a_1 + 3 = 5+3 = 8$

$a_3 = a_2 + 3 = 8+3 = 11$

....

Numeric function / solution $\{2, 5, 8, 11, \cdots\}$

Ex. ② Fibonacci Sequence

$a_n = a_{n-2} + a_{n-1}$, $n \geq 2$ with $a_0 = 1$ & $a_1 = 1$

$\rightarrow$

$a_0 = 1$

$a_1 = 1$

$a_2 = a_0 + a_1 = 1+1 = 2$

$a_3 = a_1 + a_2 = 1+2 = 3$

$a_4 = a_2 + a_3 = 2+3 = 5$

.....

Solution $\{1, 1, 2, 3, 5, \cdots\}$

Ex. ③ $a_n = a_{n-1} - a_{n-2}$ for $n = 2, 3, 4, \dots$
Let $a_0 = 3$, $a_1 = 5$ find $a_2$ & $a_3$?

→

$a_0 = 3$

$a_1 = 5$

$a_2 = a_1 - a_0 = 5 - 3 = 2$

$a_3 = a_2 - a_1 = 2 - 5 = -3$

Ex. ④ Find first five terms

— i> $a_n = 6_{n-1}$ with $a_0 = 2$

$a_0 = 2$

$a_1 = 6 \cdot a_0 = 6 \cdot 2 = 12$

$a_2 = 6 \cdot a_1 = 6 \cdot 12 = 72$

.......

$\{2, 12, 72, \dots\}$

— ii> $a_n = a_{n-1}^2$, $a_1 = 2$

$a_1 = 2$

$a_2 = a_1^2 = 2^2 = 4$

$a_3 = a_2^2 = 4^2 = 16$

.....

$\{2, 4, 16, \dots\}$

— iii> $a_n = a_{n-1} + 3a_{n-2}$, with $a_0 = 1$ & $a_1 = 2$

$a_0 = 1$

$a_1 = 2$

$a_2 = a_1 + 3 \cdot a_0 = 2 + 3 \cdot 1 = 5$

$a_3 = a_2 + 3 \cdot a_1 = 5 + 3 \cdot 2 = 11$

$\{1, 2, 5, 11, \dots\}$

Ex. ⑤ $a_n = 2^n + 5 \cdot 3^n$ for $n = 0, 1, 2, \dots$

i> Find $a_0$, $a_1$, $a_2$, $a_3$ & $a_4$

ii> Show that $a_2 = 5a_1 - 6a_0$

→ $a_0 = 2^0 + 5 \cdot 3^0 = 1 + 5 \cdot 1 = 6$

$a_1 = 2^1 + 5 \cdot 3^1 = 2 + 15 = 17$

$a_2 = 2^2 + 5 \cdot 3^2 = 4 + 5 \cdot 9 = 49$

ii> $a_2 = 5a_1 - 6a_0$

LHS $a_2 = 2^2 + 5 \cdot 3^2 = 4 + 5 \cdot 9 = 4 + 45 = 49$

RHS $5a_1 - 6a_0 = 5(17) - 6(6)$

$= 85 - 36$

$= 49$

$\therefore$ LHS = RHS

Ex. ⑥ Determine whether seq. $\{a_n\}$, where $a_n = 3n$ for every non-negative integer $n$, is a solution of the recurrence relation
$$a_n = 2a_{n-1} - a_{n-2} \quad \text{for } n = 2, 3, 4 \ldots$$

$\rightarrow$ i> let $a_n = 3n$ for every non-negative int $n$.

for $n \geq 2$

$$2a_{n-1} - a_{n-2} = 2[3(n-1)] - [3(n-2)]$$
$$= 2[3n-3] - [3n-6]$$
$$= 6n - 6 - 3n + 6$$
$$= 3n$$

$\therefore$ $a_n = 3n$ is solution for $2a_{n-1} - a_{n-2}$

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ii> $a_n = 5$, $a_n = 2a_{n-1} - a_{n-2}$

$$2a_{n-1} - a_{n-2} = 2(5) - 5$$
$$= 10 - 5$$
$$= 5$$
$$= a_n$$

$\therefore$ $a_n = 5$ is solution for $2a_{n-1} - a_{n-2}$

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iii> $a_n = 2^n$, $a_n = 2a_{n-1} - a_{n-2}$

$$2a_{n-2} - a_{n-2} = 2 \cdot 2^{n-1} - 2^{n-2}$$
$$= 2 \cdot 2^n \cdot 2^{-1} - 2^n \cdot 2^{-2}$$
$$= \frac{2}{2} \cdot 2^n - \frac{2^n}{2^2}$$
$$= 2^n\left[1 - \frac{1}{4}\right]$$
$$= 2^n\left(\frac{3}{4}\right) \neq a_n$$

$\therefore$ $a_n = 2^n$ is not sol$^n$ for $2a_{n-1} - a_{n-2}$