DSGT-4 H·E Suryavanshi ①

The Shirpur Education Society's

R. C. PATEL INSTITUTE OF TECHNOLOGY

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High Caliber Technical Education in an Environment that Promotes Excellence

TEST (I / II) / PRELIMINARY EXAMINATION

Name of Candidate : ____________________

(IN BLOCK LETTERS) (Surname) (First Name) (Middle Name)

Year : FE / SE / TE / BE Branch : ________ Division : ________ Roll No. : ________

Semester : I / II Name of Subject : ____________________

Total Supplements : 1 + ________ = ________

Signature of Student Signature of Supervisor

Question Number	1	2	3	4	5	6	7	8	9	10	Total
Marks Obtained											
Marks out of											

Signature of Examiner : Signature of Moderator :

(Start From here only)

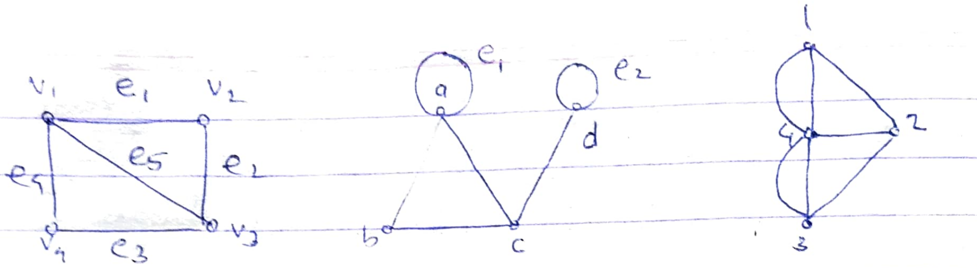
* Basic terminology of Graph-

A graph is collection of points (vertices) and collection of lines (edges).

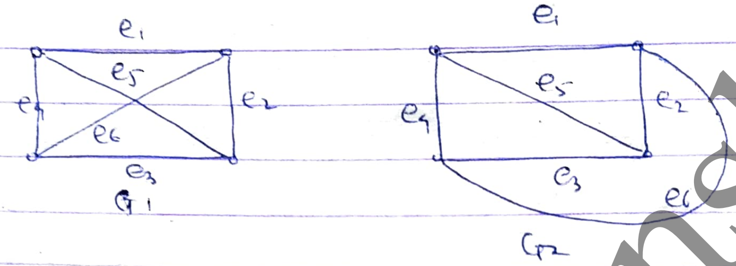
mathematically, graph G is an ordered pair (V, E) where V- set of vertices & E- set of edges.

Let e_{ij} is associated with (v_i, v_j). Then v_i and v_j called end vertices or terminal vertices.

Vertex is also refered to as node, junction or point and edge is also called as line, element or arc.



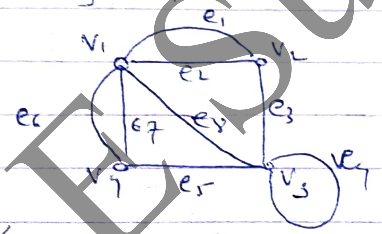
Representation of graph is not unique. Following graphs G_1 & G_2 represents the same path.



* Self-loops and parallel edges

- For any edge e_{ij}, if the end-points v_i & v_j are same, then e_{ij} is called as self loop or loop
- If there are more than one edge associated with given pair of vertices then those edges are called parallel / multiple edges.

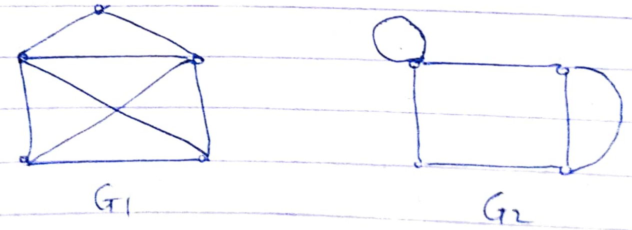
eg $e_1 = (v_1, v_2)$ & $e_2 = (v_1, v_2)$



Similarly, e_6 & e_7 are parallel edges

* Simple and Multiple Graphs

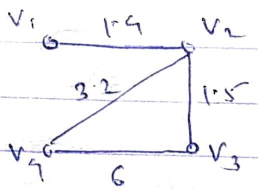
A graph that has neither self-loops nor parallel edges is called Simple graph otherwise it is called multiple graph.



G_1 — Simple Graph
G_2 — Multiple Graph

* Weighted Graph

Let G be a graph with vertex set V & edge set E. If each edge or each vertex or both are associated with some positive real no. then the graph is called a weighted graph.



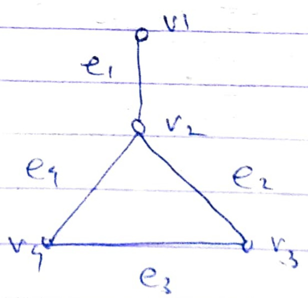
* Finite and Infinite Graphs

A graph with finite no. of vertices as well as finite no. of edges is called a finite graph, otherwise it is infinite graph.

A graph shown above is finite graph.

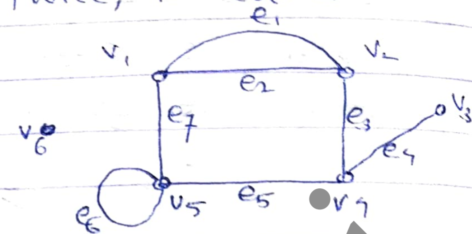
* Adjacency and Incidence

- If two vertices are joined directly by at least one edge then these vertices are called adjacent vertices. eg. v_1 & v_2 - adjacent vertices but v_1 & v_4 are not adjacent
- Two non-parallel edges are said to be adjacent if they are incident on a common vertex.
  eg. e_1 & e_2 are adjacent
  e_3 & e_4 also adjacent
- For incidence, if the vertex v_i is the end vertex of edge $e_{ij} = (v_i, v_j)$ then the edge e_{ij} is said to be incident on v_i. Similarly, e_{ij} is said to be incident on v_j. Eg. e_1 incident on v_1 & v_2



* Degree of vertex

- The no. of edges incident on a vertex v_i with self-loop counted as twice, is called the degree of vertex v_i.
  - denoted by $d(v_i)$



$d(v_1) = 3$, $d(v_2) = 3$

$d(v_3) = 1$, $d(v_4) = 3$

$d(v_5) = 4$ $d(v_6) = 0$

* Isolated vertex and Pendant vertex

- vertex with degree zero called Isolated
- vertex of degree 1 is called pendant

eg. In above Fig.

v_6 i.e $d(v_6) = 0$ → Isolated vertex

v_3 i.e $d(v_3) = 1$ → Pendant vertex

* Handshaking Lemma

Let G be the graph with e - no. of edges & n - no. of vertices.

Since each edge contributes two degree the sum of the degrees of all vertices in G is twice the no. of edges in G.

i.e $$\sum_{i=1}^{n} d(v_i) = 2e$$

① Show that maximum degree of any vertex in simple graph with n- vertices is $\boxed{(n-1)}$

⇒

Simple Graph

$G = (E, V) = (8, 5)$

Now $n = 5$

∴ maximum degree $d(v_i) = (n-1) = 4$

∴ eg $d(v_2) = d(v_5) = 4$

② Show that maximum no. of edges in simple graph with n- vertices is $\boxed{\frac{n(n-1)}{2}}$

⇒

By handshaking Lemma

$$\sum_{i=1}^{n} d(v_i) = 2e \quad (e = \text{no. of edges in } G)$$

$\therefore \quad d(v_1) + d(v_2) + \cdots + d(v_n) = 2e$

Since maximum degree of each vertex $= (n-1)$

$$\therefore \quad \underbrace{(n-1) + (n-1) + \cdots + (n-1)}_{n\text{-time}} = 2e$$

$$\therefore \quad n(n-1) = 2e$$

$$\therefore \quad \boxed{e = \frac{n(n-1)}{2}}$$

③ How many nodes are necessary to construct a graph with 6-edges in which each node is of degree 2

$\Rightarrow$

$$\therefore \quad \sum_{i=1}^{n} d(v_i) = 2e = 2 \times 6 = 12$$

$$\therefore \quad d(v_1) + d(v_2) + \cdots + d(v_n) = 12$$

$$\underbrace{2 + 2 + \cdots + 2}_{n \text{ times}} = 12$$

$$2n = 12$$

$\boxed{n = 6}$ $\therefore$ 6 nodes Required

④ Determine no. of edges in graph with 6-nodes, 2 of degree 4 and 4 of degree 2. Draw graph

$\Rightarrow$

$n = 6$ $\therefore$ by handshaking lemma $\sum_{i=1}^{6} d(v_i) = 2e$

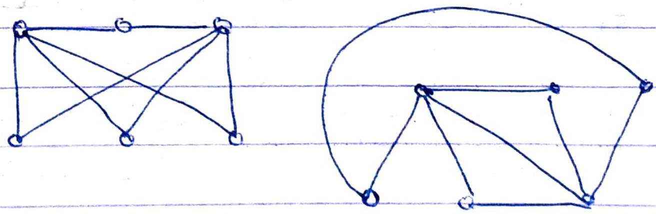
$$\therefore \quad d(v_1) + d(v_2) + \cdots + d(v_6) = 2e$$

Now $2v$ of degree 4 & $4v$ of degree 2

$$\therefore \quad (4+4) + (2+2+2+2) = 2e$$

$$16 = 2e$$

$\boxed{e = 8}$ $\therefore$ 8 edges Required



⑤ Is it possible to construct graph with 12 nodes such that 2 nodes have degree 3 & remaining nodes have degree 4

$\Rightarrow$ $n = 12$ (nodes / vertices)

$\therefore$ by handshaking lemma

$$\sum_{i=1}^{12} d(v_i) = 2e$$

$$\therefore (2 \times 3) + (10 \times 4) = 2e$$

$$6 + 40 = 2e$$

$$46 = 2e$$

$$\boxed{e = 23}$$

$\therefore$ Yes it is possible to construct such graph.

⑥ Is it possible to draw a simple graph with 4 vertices & 7 edges ? Justify

$\Rightarrow$ In simple graph

maximum ~~degree~~ edges $= \dfrac{n(n-1)}{2}$ (n = vertices)

$\therefore$ Graph with 4 vertices $= \dfrac{4 \times 3}{2} = 6$ edges

Therefore, graph with 4 vertices & cannot have 7 edges. Hence the graph doesn't exist

=x=

* **Some Important and Useful Graphs**

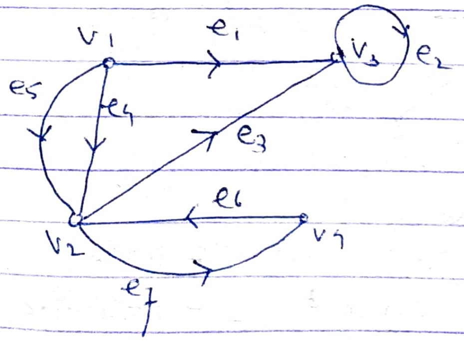
① Directed Graph (Diagraph)

- If each edge of graph has direction, then the graph is called directed graph.

$D = (V, A)$ D - Graph

V - set of vertices

A - directed edges / Arcs



$e_1 = (v_1, v_3)$

$e_2 = (v_3, v_3)$

$e_3 = (v_2, v_3)$

$e_4 = (v_1, v_2)$

$e_5 = (v_1, v_2)$

$e_6 = (v_4, v_2)$

$e_7 = (v_2, v_4)$

v_1 & v_2 – joined by more than one arce with same direction. Such arcs are called multiple arcs.

Arcs e_6 & e_7 – Not multiple arc because direction different.

Diagraph without loops & multiple arcs are known as simple digraphs.

Incidence

- **Indegree and Outdegree**
  - indegree of vertex u of digraph D is defined as no. of arcs which are incident into u.

    denoted by $\overrightarrow{d}(u)$
  - Similarly, outdegree = no. of arcs which are incident of out u & denoted by $\overleftarrow{d}(u)$

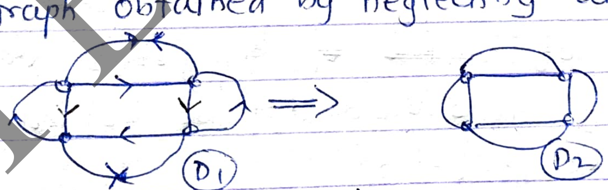
* // Self-loop considered in both indegree & outdegree

Indegree : $\overrightarrow{d}(v_1) = 0$, $\overrightarrow{d}(v_2) = 3$, $\overrightarrow{d}(v_3) = 3$, $\overrightarrow{d}(v_4) = 1$

outdegree : $\overleftarrow{d}(v_1) = 3$, $\overleftarrow{d}(v_2) = 2$, $\overleftarrow{d}(v_3) = 1$, $\overleftarrow{d}(v_4) = 1$

Underlying Graph of Digraph

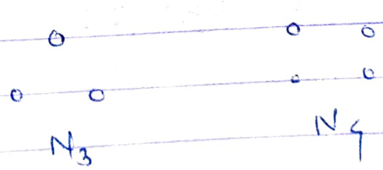
Graph obtained by neglecting directions of arcs



D_1 – simple graph but D_2 is not simple graph & contains parallel edges

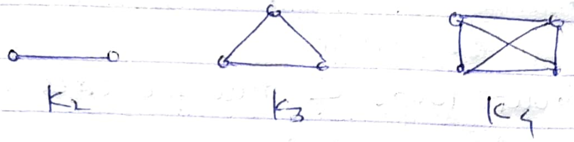
② Null Graph (N_n)

If Edge set of graph with n-vertices is empty set then graph is null graph.



③ **Complete Graph (K_n)**

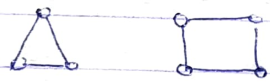
If in a graph G, degree of each vertex is $(n-1)$ then G is called complete graph.



In complete graph K_n, no. of edges $= \frac{n(n-1)}{2}$

④ **Regular Graph**

Degree of each vertex is same



Complete graph is also a regular graph but regular graph need not be a complete graph.

⑤ **Bipartite Graph** -

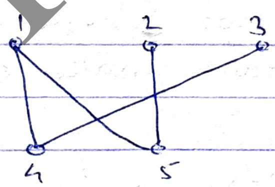
$G = (V, E)$ V - vertices, E - edges & G - Graph

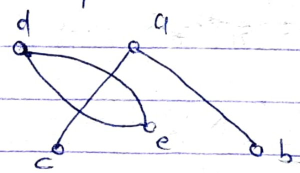
G is bipartite if its vertex set V can be partitioned into two disjoint subsets V_1 & V_2

where $V_1 \cup V_2 = V$ &

$V_1 \cap V_2 = \phi$

- Each edge of ~~vertex~~ V_1 G joins vertex V_1 to V_2
- Vertices of V_1 should not be joined, similarly, V_2
- Do not have self-loop.





$V_1 = \{1, 2, 3\}$ $V_1 = \{a, d\}$

$V_2 = \{4, 5\}$ $V_2 = \{b, c, e\}$

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⑥ Complete Bipartite Graph ($K_{m,n}$)

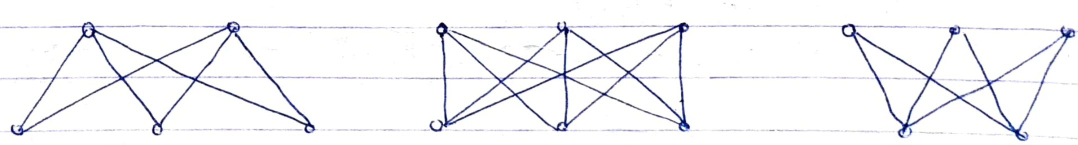
A Bipartite Graph is called complete bipartite graph if each vertex of V_1 is joined to every vertex of V_2 by an unique edge.

– denoted by

$K_{m,n}$ m = no. of vertices in V_1 &

n = no. of vertices in V_2

∴ total no. of edges in complete Bip. Graph $m*n$

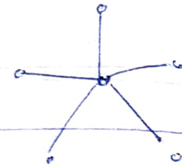


$K_{2,3}$ $m=2$, $n=3$ $m<n$

$K_{3,3}$ $m=n=3$

$K_{3,2}$ $m=3$, $n=2$ $m>n$

Graph $K_{1,n}$ known as star eg $K_{1,5}$



- Every complete bipartite graph is regular if $m=n$

$\therefore$ $K_{3,3}$ — regular

Ex. ① How many edges has each of the following graph

i> K_{10} ii> $K_{5,7}$

— (i) K_{10} — bipartite graph edges $= \frac{n(n-1)}{2} = \frac{10 \times 9}{2} = 45$

— (ii) $K_{5,7}$ — complete bi-graph edges $= m \times n = 5 \times 7 = 35$

* Isomorphism -

Two graphs are thought of as equivalent (called isomorphic) if they have identical behaviour interms of graph-theoritic properties.

Two graphs $G_1(V_1 E_1)$ & $G'(V' E')$ are said to be isomorphic to each other if there is one-one correspondence betn vertices and edges such that incidence relationship is preserved.

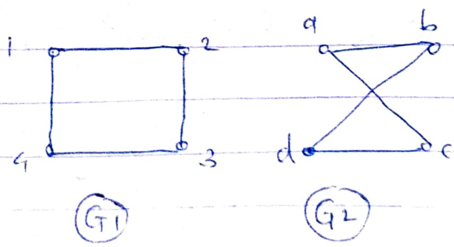
- eg. e - incident edge on vertices V_1 & V_2 of G then e' must be incident on V_1' & V_2' of G'
- Adjacency betn vertices is preserved
- denoted by $G_1 \cong G_2$

Isomorphic graphs must have

① Same no. of vertices

② Same no. of edges

③ An equal no. of vertices with given degree



G_1 is isomorphic to G_2

$\therefore$ one-one correspondence betn vertices

1-a, 2-b, 3-d, 4-c