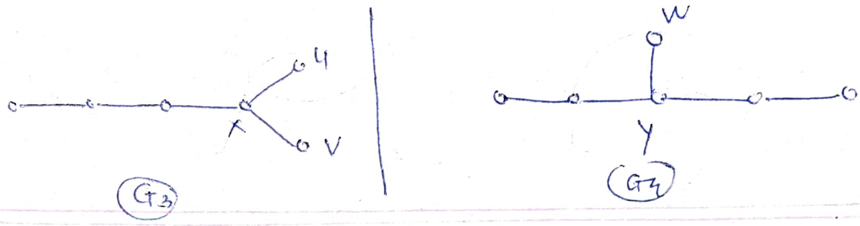



$G_3 \not\cong G_4$ — Not isomorphic

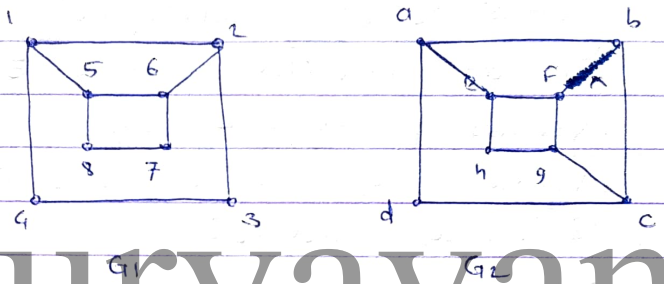
Since

$G_3$ has vertex $x$ (degree-3) connected to two pendent vertices

$G_4$ has vertex $y$ (degree-3) but not connected to pendent vertices

$\therefore$ Adjacency not preserved.

Ex.① Determine whether following graphs are isomorphic or not.






$\Rightarrow$

- Both G1 & G2 contains 8-vertices & 10-edges.
- No. of vertices with degree 2 in G1 & G2 $\Rightarrow$ 4
- No. of vertices with degree 3 in G1 & G2 $\Rightarrow$ 4
- For Adjacency,

vertex 1 in $G_1$ (degree = 3) connected / adjacent to two vertices with degree 3 (i.e 2, 5) and one vertex with degree 2 (i.e 4)

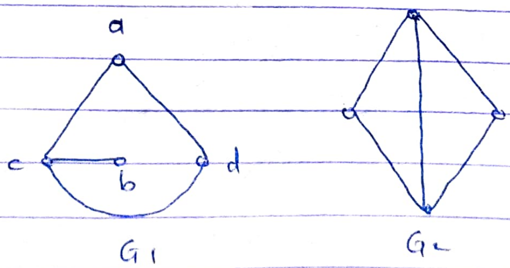
There is no such a vertex in $G_2$

Hence, adjacency is not presered

So $G_1$ & $G_2$ are <u>not</u> <u>isomorphic</u> $(G_1 \not\cong G_2)$

Ex. ② Find whether following pairs of graph are isomorphic or not.

Ⓐ

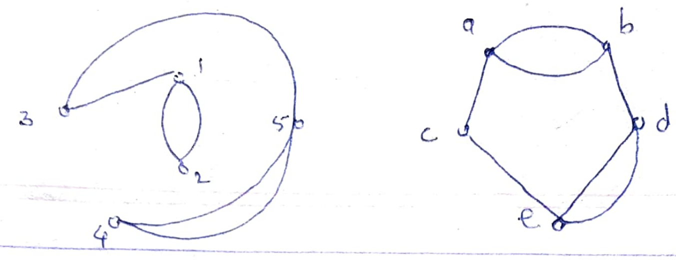



$\Rightarrow$ $G_1 \not\cong G_2$

Not isomorphic

Since, No. of edges in $G_1 = 4$ while in $G_2 = 5$

(B)

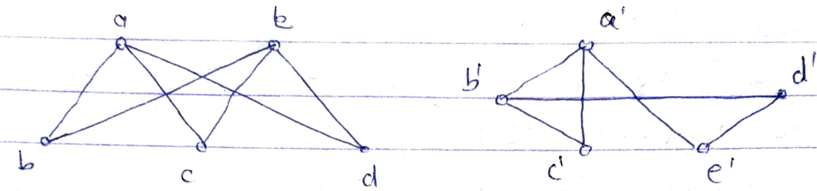



$G_3$ $G_4$

$\Rightarrow$

$\therefore$ $\underline{G_3 \not\cong G_4}$ No. of edges in $G_3 = 6$ while $G_4$ has 7

(C)




$G_5$ $G_6$

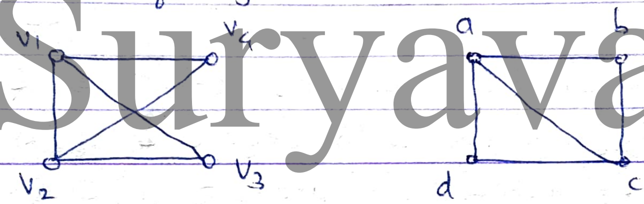
$\Rightarrow$

$\therefore$ $G_5 \not\cong G_6$ (Not isomorphic)

In $G_5$, a-vertex of degree 3 is adjacent to 3 vertices of degree 2. But in $G_6$, both the vertices a' & b' of degree 3 are not adjacent to 3. vertices of degree 2



(D)



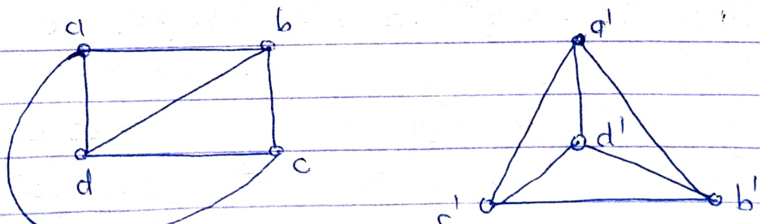

$G_7$ $G_8$

$\Rightarrow$

- $\underline{\text{Yes}}$ $G_7 \cong G_8$ (isomorphic)
- In $G_7$ & $G_8$, there are 2 vertices of degree 2 and 2 vertices of degree 3
- Also adjacency is preserved

$\therefore$ one-one correspondence betn vertices is given by $V_1 \rightarrow a$, $V_2 \rightarrow c$, $V_3 \rightarrow d$ & $V_4 \rightarrow b$

(E)




$G_9$ $G_{10}$

$\Rightarrow$ Both contains same no. of vertices (4) & same no. of edges (6). Also adjacency is preserved $G_9 \cong G_{10}$ (i.e. isomorphic)

Ex. (3) Find whether $K_6$ and $K_{3,3}$ are isomorphic or not?

$\Rightarrow$

Both contains 6 - vertices

But, no. of edges in $K_6 \Rightarrow \frac{6 \times 5}{2} = 15$ $\left(\frac{n(n-1)}{2}\right)$

while no. of edges in $K_{3,3} \Rightarrow 3 \times 3 = 9$ $(m * n)$

$15 \neq 9$

Hence $K_6$ & $K_{3,3}$ are not isomorphic

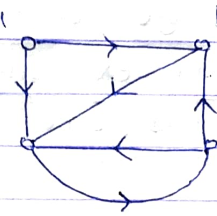
Ex. (4) Determine whether the following graphs $G = (V, E)$ and $G^* = (V^*, E^*)$ are isomorphic or not

$G = (\{a, b, c, d\}, \{(a,b) (a,d) (b,d) (c,d) (c,b) (d,c)\})$

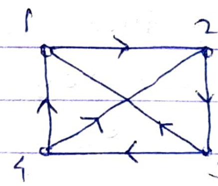
$G^* = (\{1,2,3,4\}, \{(1,2) (2,3) (3,1) (3,4) (4,1) (4,2)\})$

$\Rightarrow$

Graphs are




G




G*

$G \neq G^*$

Not isomorphic

- No. of vertices & edges are same in $G$ & $G^*$ but degree of vertices do not match.

G contains 1 - vertex of degree 2 & 3 of degree 3

while $G^*$ contains 4 - vertex & all of degree 3

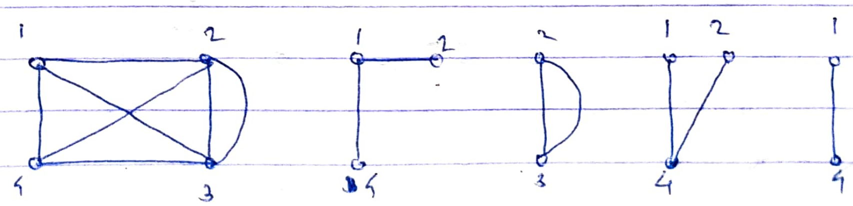
=x=

## * New Graphs from old ones

### (1) Subgraph

Let $G = (V, E)$ be any given graph.

Then $G' = (V', E')$ is called subgraph of G if $V' \subseteq V$ and $E' \subseteq E$




G H1 H2 H3 H4

Here $H_1, H_2, H_3$ and $H_4$ are subgraphs of G

Properties

① Each graph is subgraph of itself.

② A single vertex of a graph G is a subgraph of G

③ A single edge together with its end vertices is also a subgraph of a graph G

④ A subgraph of a subgraph of a graph G is a subgraph of G.

—x—

② Edge Disjoint Subgraphs

Two subgraphs $H_1$ and $H_2$ of graph G are said to be edge disjoint subgraphs of G if there is no edge common betn $H_1$ and $H_2$ (but may have vertex common)

e.g. $H_1$ and $H_2$ are disjoint subgraphs of G

③ Vertex Disjoint Subgraphs

Two subgraphs $H_1$ and $H_2$ of graph G are said to be vertex disjoint subgraphs of a graph G if there is no vertex common between them (i.e they do not have ~~ver~~ common edge also)

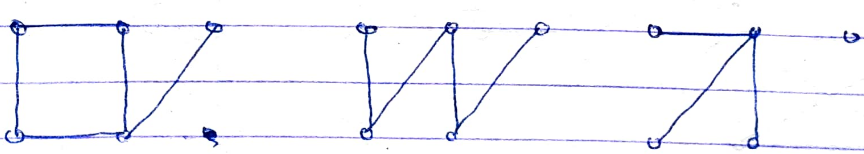
eg. $H_2$ and $H_4$ – vertex disjoint subgraphs



④ Spanning Subgraphs

Let G = (V, E) be any graph.

Then G' is said to be the spanning subgraph of G if its vertex set V' is equal to the vertex set V of G

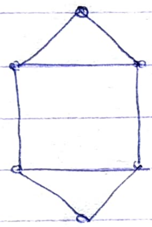
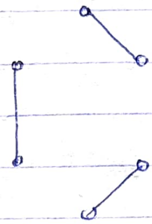
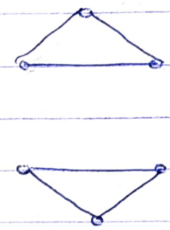


G G' G''

In fig. G' and G'' are spanning subgraphs of the graph G.

⑤ **Factors of Graph**

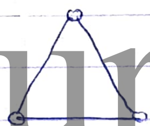
A K-Factor of a graph is defined to be a spanning subgraph of the graph with the degree of each of its vertex is being K.

G      1-Factor graph      2-Factor graph

- A graph might have many different K-Factors or might not have any K-Factor at all for some K.
- Fig. shown below depicts a graph which does not have any 1-Factor graph.

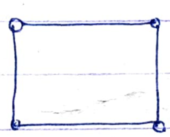




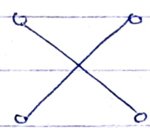
⑥ **Complement of a graph**

Let G be a simple graph.

Then the complement of G denoted by $\overline{G}$ is the graph whose vertex set is the same as the vertex set of G and in which two vertices are adjacent if and only if they are not adjacent in G.

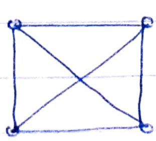


$\Rightarrow$

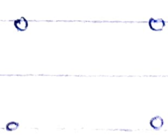


G      $\overline{G}$

Complement of a complete graph is a null graph and vice-versa.



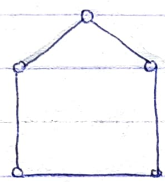
$\Rightarrow$



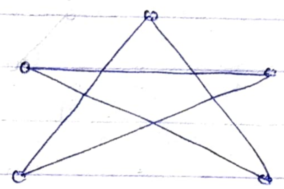
G      $\overline{G}$

A graph is said to be <u>self complementory</u> if it is isomorphic to its complement.

A graph G and its self complementary graph are shown in fig. below.



graph G.



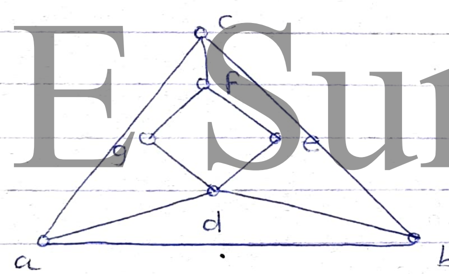
G'

Self complementary graph of G

$G \cong G'$

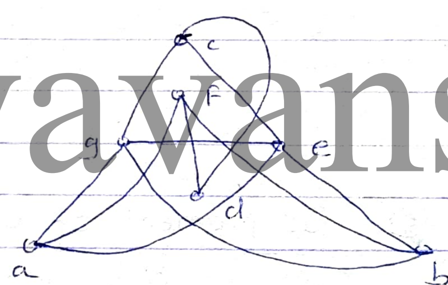
Ex. ① Find complement of graph shown below. Is it self-complementory?

⇒




G

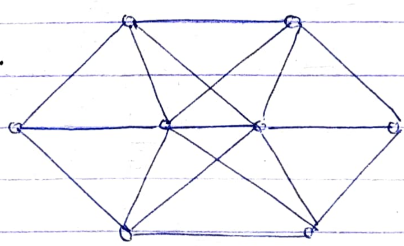
⇒




$\overline{G}$

Graph G is not self-complementory because it is not isomorphic to its complement $\overline{G}$.

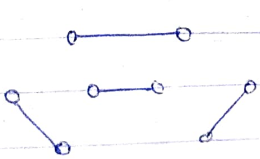
Ex. ② Find 1-factor and 2-factor graphs of G graph shown below.



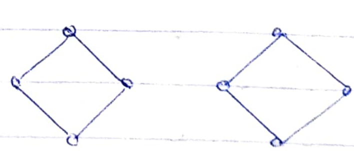
G

⇒

1-factor graph



2-factor graph



DSGT-4 H.E Suryavanshi ③

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***TEST (I / II) / PRELIMINARY EXAMINATION***

Name of Candidate : ____________________

(IN BLOCK LETTERS) (Surname) (First Name) (Middle Name)

Year : FE / SE / TE / BE Branch : ________ Division : ________ Roll No. : ________

Semester : I / II Name of Subject : ____________________

Total Supplements : 1 + ______ = ______

Signature of Student Signature of Supervisor

| Question Number | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | Total |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Marks Obtained | | | | | | | | | | | |
| Marks out of | | | | | | | | | | | |

Signature of Examiner : Signature of Moderator :

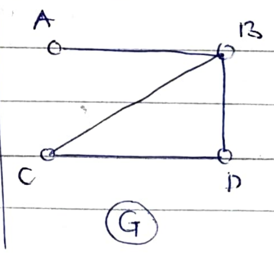


**(Start From here only)**

Ex. ③ For the graph G show in fig determine whether $H = (V', E')$ is subgraph of G, where

i> $V' = \{A, B, F\}$ & $E' = \{\{A,B\}, \{A,F\}\}$

ii> $V' = \{B, C, D\}$ & $E' = \{(B,C)\ (B,D)\}$



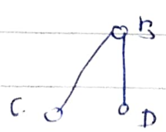

⇒

i> Given $V' = \{A, B, F\}$ & $E' = \{\{A,B\}\ \{A,F\}\}$

<u>No</u>, H is not subgraph of G. because it contains vertex 'F' which doesn't belongs to G

ii> Given $V' = \{B, C, D\}$ & $E' = \{\{B,C\}\ \{B,D\}\}$

<u>Yes</u>, All the vertices & edges of H belongs to G




(7)

## * Operations on Graphs

① Union of two graphs
② Intersection of two graphs
③ Ring Sum of two graphs
④ Removal of an Edge
⑤ Removal of a Vertex

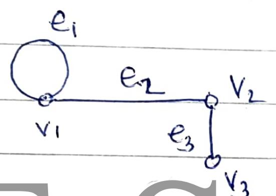
⇒ ① **Union of two graphs**

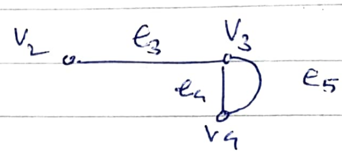
Let $G_1 = (V_1\ E_1)$ and $G_2 = (V_2\ E_2)$.

The union of two graphs is denoted by

$\boxed{G_1 \cup G_2}$

where, $V_1 \cup V_2$ and $E_1 \cup E_2$.



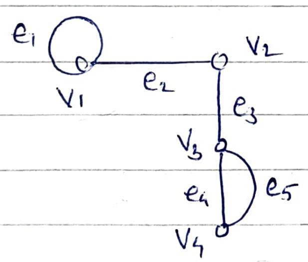




$G_1$ : $V_1 = \{V_1, V_2, V_3\}$ ; $G_2$ : $V_2 = \{V_2, V_3, V_4\}$
$E_1 = \{e_1, e_2, e_3\}$ ; $E_2 = \{e_3, e_4, e_5\}$



Then,

$G_1 \cup G_2$ is shown below




$G_1 \cup G_2 \Rightarrow V_1 \cup V_2 = \{V_1, V_2, V_3, V_4\}$
$E_1 \cup E_2 = \{e_1, e_2, e_3, e_4, e_5\}$

⇒ ② **Intersection of Two graphs**

Let $G_1 = (V_1\ E_1)$ and $G_2 = (V_2\ E_2)$

$\boxed{G_1 \cap G_2}$

where $V_1 \cap V_2$ and $E_1 \cap E_2$

In Graph $G_1$ & $G_2$ shown above

$G_1 \cap G_2 \Rightarrow$ 


$V_1 \cap V_2 = \{V_2\ V_3\}$
$E_1 \cap E_2 = \{e_3\}$

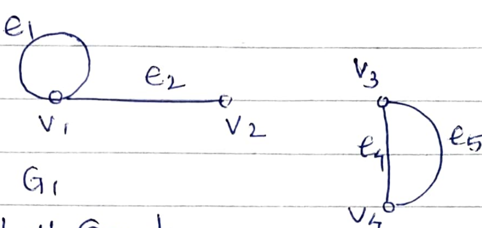
- If $G_1$ & $G_2$ are Edge disjoint graph then, $G_1 \cap G_2 \Rightarrow$ Null Graph
- If $G_1$ & $G_2$ are Vertex disjoint graph then, $G_1 \cap G_2 \Rightarrow$ Empty set

⇒ ③ **Ring Sum of Two Graphs**

Let $G_1 = (V_1\ E_1)$ and $G_2 = (V_2\ E_2)$

Ring sum of two graphs consisting of the vertex set $V_1 \cup V_2$ and edges that in either $G_1$ or $G_2$ but not in both.

$G_1 \oplus G_2$




For any graph $G_1$

$G \oplus G$ = Null Graph

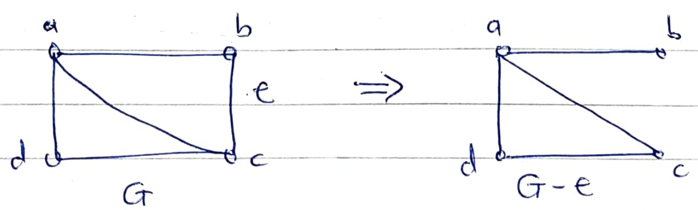


⇒ ④ **Removal of an Edge**

Let $G = (V, E)$ be any graph.

Let $e \in E$. Then the graph $(G-e)$ can be obtained by removing the edge 'e' from the graph.

– Removal of any edge 'e' from graph G doesn't mean the removal of its end vertices

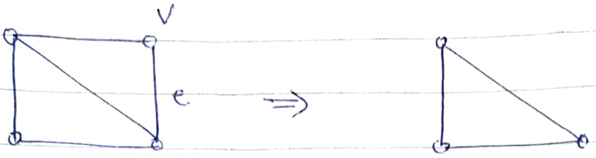



⇒ ⑤ **Removal of vertex**

Let $G = (V, E)$ be any graph.

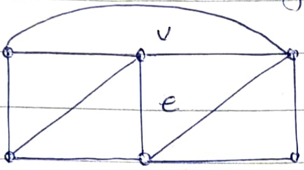
Let $v \in V$. The graph $(G-v)$ can be obtained by removing vertex 'v' from graph G.

– Removal of v means, removal of all these edges also which are incident on v



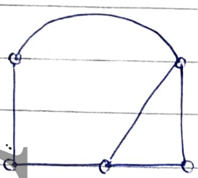

G $\Rightarrow$ $(G - v)$

Ex ① Draw the graphs (i) $G - v$ ② $G - e$, where the graph G is shown in fig.

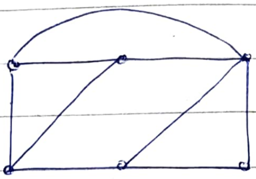



G

$\Rightarrow$ ① $G - v$



② $G - e$



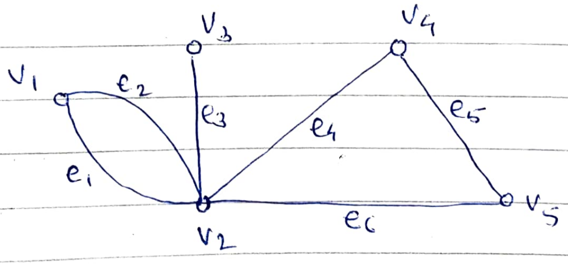


## * Paths and Circuits

Let $G = (V, E)$ be any graph.
Let $V_0$ and $V_n$ be any two vertices in V.

A path 'p' of length 'n' from $V_0$ to $V_n$ is a sequence of vertices and edges of the form $V_0 e_1 V_1 e_2 \cdots e_n V_n$. where each edge $e_j$ is an edge betn $V_{j-1}$ and $V_j$.

The vertices $V_0$ and $V_n$ called end-points of path and other vertices $V_1 V_2 \cdots V_{n-1}$ called interior vertices




path I = $V_1 e_2 V_2 e_4 V_4$

path II = $V_4 e_4 V_2 e_2 V_1 e_1 V_2 e_3 V_3$

path III = $V_3 e_3 V_2 e_2 V_1 e_1 V_2 e_3 V_3$