$R_1 \cdot R_2 = \{(1,1)\ (1,4)\ (1,3)\ (2,1)\ (2,4)\ (3,4)\ (4,4)$
$(4,1)\ (4,3)\}$

$$M_{R_1 \cdot R_2} = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 \end{bmatrix} \quad \text{—— (1)}$$

$$M_{R1} \cdot M_{R2} = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 \end{bmatrix} \quad \text{—— (2)}$$

from (1) & (2)

$$M_{R1 \cdot R2} = M_{R1} \cdot M_{R2}$$

(2) $R_1^c = \{(1,1)\ (2,1)\ (3,2)\ (4,2)\ (4,3)\ (1,4)\ (2,4)\}$

$$M_{R_1^c} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \end{bmatrix} = \text{transpose of } M_{R_1}$$

(3) $(R_1 \cdot R_2)^c = \{(1,1)\ (4,1)\ (3,1)\ (1,2)\ (4,2)\ (4,3)\ (4,4)$
$(1,4)\ (3,4)\}$

$$M_{(R_1 \cdot R_2)^c} = \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

$$M_{R_2^c} \cdot M_{R_1^c} = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

$$= M_{R_2^c} \cdot M_{R_1^c}$$

* Graphical Representation of a Relation -

If A is finite set & R is relation on A, then R is represented by graph using points / circles, called nodes or vertices

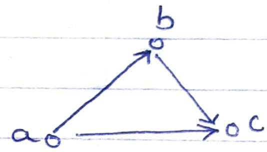
Ex. ① aRb

② aRa

③ $aRb \wedge bRa$

④ $aRb \wedge bRb$

⑤ $aRb \wedge bRc \wedge aRc$



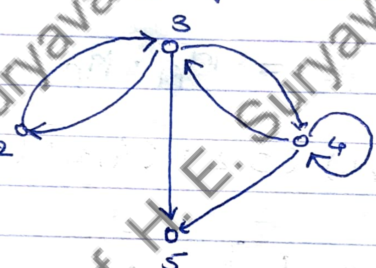
Examples

① $A = \{2, 3, 4, 5\}$

$R = \{(2,3)(3,2)(3,4)(3,5)(4,3)(4,4)(4,5)\}$

draw its digraph.

$\Rightarrow$

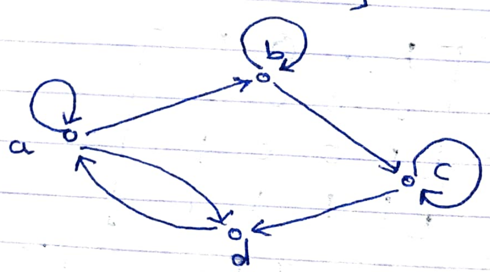


② $A = \{a, b, c, d\}$ and $M_R = \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix}$

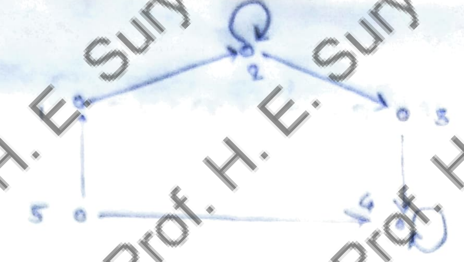
draw the digraph of R

$\Rightarrow$

$R = \{(a,a)(a,b)(a,d)(b,b)(b,c)(c,c)$
$(c,d)(d,a)\}$



③ Find the relation determined by the digraph and give its matrix.



$A = \{1, 2, 3, 4, 5\}$

$R = \{(1,2)(2,2)(2,3)(3,4)(4,4)(5,4)(5,1)\}$

$$M_R = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 \end{bmatrix}$$

* Special Properties of Binary Relations -

Let, R be a Relation on a set A

① Reflexive Relation (Every element related to itself)

- R is reflexive if for every element $a \in A$, aRa i.e $(a,a) \in R$
- R is not reflexive if for some element $a \in A$, $a \not R a$ i.e $(a,a) \in R$

Ex. ① $A = \{a, b\}$ & $R = \{(a,a)(a,b)(b,b)\}$

$\therefore$ R is reflexive

Ex. ② $A = \{1, 2\}$ & $R = \{(1,1)(1,2)\}$

$\therefore$ R is not-reflexive since $(2,2) \notin R$

② Irreflexive Relation (Elements cannot related to itself)

- R is irreflexive if for every element $a \in A$, $a \not R a$ i.e $(a, a) \notin R$.

Ex. ① $A = \{1, 2\}$ & $R = \{(1,2)(2,1)\}$

$\therefore$ R irreflexive since $(1,1)(2,2) \notin R$

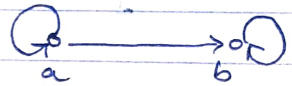
Ex. ② $A = \{1, 2\}$ & $R = \{(1,2), (2,2)\}$

$\therefore$ R is not-irreflexive since $(2,2) \in R$
also,

R is not-reflexive since $(1,1) \notin R$

Note - If R - Reflexive
then M_R - have digonal entries
- If R - Irreflexive:
then M_R - digonal elements zero

Diagraph of Reflexive Relation



③ Symmetric Relation

- R is symmetric if whenever aRb then bRa
- R is not-symmetric if for some a & b ∈ A, aRb but $b \not R a$

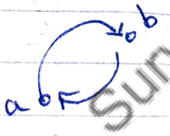
Note

R - symmetric then M_R is symmetric matrix
i.e. If $A_{ij} = 1$ then $A_{ji} = 1$
If $A_{ji} = 0$ then $A_{ij} = 0$ (For $i \neq j$)

Ex. ① A = Set of people.
If aRb - a is friend of b
then obviously b is related to a
$\therefore$ " friend " - symmetric Relation

Ex. ② A = set of people
aRb if a is brother of b
$\therefore$ Not-symmetric
since, b can be sister of a
It is symmetric if A = set of males

④ Diagraph of symmetric Relation



OR

$(a,b) \in R$ but $(b,a) \notin R$

④ **Asymmetric Relation** { R is asymmetric if it is both anti-symmetric & irreflexive }

- R is asymmetric if whenever aRb, then $b \not R a$
- R not-asymmetric if for some a and b, we have both aRb, & bRa

Ex.① $A = \{2, 4, 5\}$ R – "is a divisor of"
Then
$R = \{(2,2)(2,4)(4,4)(5,5)\}$

$\therefore$ R is not-asymmetric since $\{(2,2), (4,4), (5,5)\} \in R$

Ex.② A = R set of Real no.
let R = relation '<'
Then $a < b \rightarrow b \not< a$.

⑤ **Anti-Symmetric Relation** –

- R is Anti-Symmetric if whenever aRb & bRa then $a = b$
- R not Anti-Symmetric if we have elements $a, b \in A$ such that $a \neq b$ but both aRb and bRa

Ex.① A = R, let R be the relation '$\leq$'.
Then $a \leq b$ and $b \leq a \rightarrow a = b$
Hence '$\leq$' is Anti-Symmetric relation

Ex.② $A = \{1, 2, 3\}$ and $R = \{(1,2)(2,1)(2,3)\}$

R – Not-Anti-Symmetric since $(1,2)(2,1) \in R$
R – Not-Symmetric since $(2,3) \in R$ but $(3,2) \notin R$
R – Not-Asymmetric since $(1,2)$ & $(2,1) \in R$

⑥ **Transitive Relation**

R is transitive if aRb, bRc then aRc
R – Not-transitive if aRb, bRc but $a \not R c$

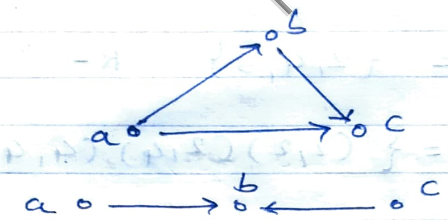
Ex. ① A → Set of people & R = "brother of"

Then 'a' → brother of b, b → brother of c

then a → brother of c

∴ transitive relation

Diagraph



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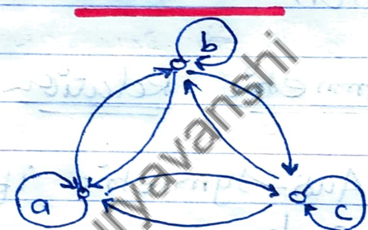
* Equivalence Relation

A binary relation R on set A is called equivalence relation if it is reflexive, symmetric and transitive.

digraph



Ex. ①

$A = \{a, b, c, d\}$

$R = \{(a,a)\ (b,a)\ (b,b)\ (c,c)\ (d,d)\ (d,c)\}$

⇒ Determine whether R – Equivalence relⁿ.

R – reflexive since (a,a) (b,b) (c,c) & (d,d)

R – Not-symmetric since $(b,a) \in R$
but $(a,b) \notin R$

∴ R – Not-Equivalence relation

Ex. ②

Let $A = \{a, b, c\}$ and let $M_R = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$

Determine wheather R is an equivalence relation.

⇒

$R = \{(a,a)\ (b,b)\ (b,c)\ (\cancel{c,d})\ (c,b)\ (c,c)\}$

R : Reflexive since (a,a) (b,b) (c,c) ∈ R

R : Symmetric since (b,c) ∈ R & (c,b) ∈ R

R : transitive since

(b,b) & (b,c) ∈ R implies (b,c) ∈ R

(b,c) & (c,b) ∈ R implies (b,b) ∈ R &

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Marks Obtained											
Marks out of											

Signature of Examiner : Signature of Moderator :

(Start From here only)

* Properties of Equivalence Relations

(1) If R_1 & R_2 are equivalence relations on set A then $R_1 \cap R_2$ is an equivalence relations

(2) If R_1 & R_2 are equivalence Relations, it is not necessary that $R_1 \cup R_2$ is also an equivalence relation.

= x =

* Equivalence Classes –

Let, R be an equivalence Relation on a set A. For every $a \in A$, let $[a]_R$ denote that the set $\{x \in A \mid x R a\}$.

Then, $[a]_R$ is called equivalence class of a with respect to R.

$=$ $[a]_R \neq \phi$ Since $a \in [a]_R$

Rank of R is the no. of distinct equivalence classes of R if the no. of classes is finite; otherwise the rank is said to be finite.

Theorems

Let R be an equivalence relⁿ. on set A. Then following hold:

① For all $a, b \in A$, either $[a] = [b]$ or $[a] \cap [b] = \phi$

② $A = \bigcup_{a \in A} [a]$

Ex. ①

Let $A = \{a, b, c\}$ & let $R = \{(a,a) (b,b) (c,c) (a,b) (b,a)\}$

where R is equivalence relation

$\Rightarrow$

Equivalence classes of elements of A

$[a] = \{a, b\}$

$[b] = \{b, a\} = [a]$

$[c] = \{c\}$

$\therefore$ Then Rank of R is $\underline{2}$

Ex. ② $A = \{1, 2, 3, 4\}$ &

$R = \{(1,1) (1,2) (1,3) (2,1) (2,2) (3,1) (2,3) (3,2) (3,3) (4,4)\}$

- Show that R is an equivalence relation & determine the equivalence classes & hence find the rank of R

$\Rightarrow$

R: reflexive since $(1,1) (2,2) (3,3) (4,4) \in R$

R: Symmetric $(1,2), (2,1) \in R$

similarly, $(2,3), (3,2), (1,3)(3,1) \in R$

R: transitive

since $(1,2)$ & $(2,1) \in R$

implies $(1,1) \in R$

Similarly,

$(1,3)(3,1) \in R \rightarrow (1,1) \in R$
$(2,3)(3,2) \in R \rightarrow (2,2) \in R$
$(3,1)(1,3) \in R \rightarrow (3,3) \in R$
$(3,2)(2,1) \in R \rightarrow (3,1) \in R$

hence,

R is equivalence Relation.

$\therefore$ Equivalence classes of A are

$[1] = \{1,2,3\}$
$[2] = \{1,2,3\} = [1]$
$[3] = \{1,2,3\} = [1]$
$[4] = \{4\}$

$\therefore$ there are two distinct ~~ive~~ equivalence classes. Hence Rank of R is 2

* **Partitions**

- The concept of partition is closely Related to the equivalence Reln.

Defn:

- A partition of non-empty set A is collection of sets $\{A1, A2, \dots An\}$ such that

i> $A = \bigcup_{i=1}^{n} A_i$

ii> $A_i \cap A_j = \phi$, for $i \neq j$ (i.e sets A_i are mutually disjoint)

- The partition of A is denoted by π. (pi)
- An element of partition is called a block.
- The Rank of π is the no. of blocks of π.

Ex.① $A = \{1,2,3\}$

Then $\pi_1 = \{\{1,2\}, \{3\}\}$ partition of A

& $\pi_2 = \{\{1,3\}, \{2\}\}$

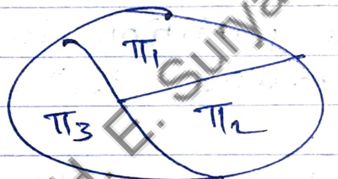
$\pi_3 = \{\{1\} \{2\} \{3\}\}$ & so on

Ex. ② Let Z = Set of all integers
E = Set of all even integers
O = Set of all odd integers
Then
$\{E, O\}$ is a partition of Z.

Ex. ③ The rooms (Flats) in a building block form a partition.

Ex. ④ The main m/m of multi-programmed computer System is partitioned & a seperate prog. is stored in each block of the partition.

Following diagram represents partition of se



Theorem

① Let, A be non-empty set & R an equivalence relations & ~~partitions~~ ~~one~~ on A., then the set of equivalence classes $\{[a]_R \mid a \in A\}$ constitutes a partition of A. i.e (A/R)

② let, A be non-empty set, & let π be a partition of A.. Then π includes an equivalence relation on A.

Ex. ① Let $A = \{a, b, c, d\}$
$\pi = \{\{a, b\}, \{c\}, \{d\}\}$

Find the equivalence relation induced by π & construct its graph

$\Rightarrow$ $R = \{(a,a) (b,b) (a,b) (b,a) (c,c) (d,d)\}$