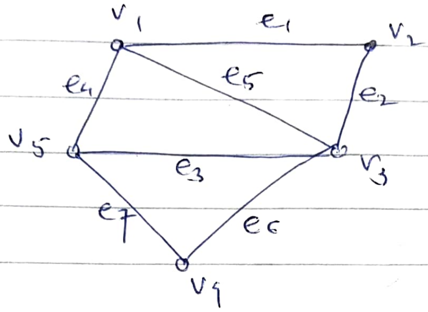
In simple graph (no loops & parallel edges), a path may be described by giving only the sequence of vertices traversed in the path.

- For ex. the path $(v_5\, e_4\, v_1\, e_1\, v_2\, e_2\, v_3)$ can be written as $(v_5\, v_1\, v_2\, v_3)$



① Simple Path

- Path is called simple if edges do not repeat in the path

ex. path I and II — Simple path
path III - Not simple bcoz e_3 repeated twice

② Elementary Path

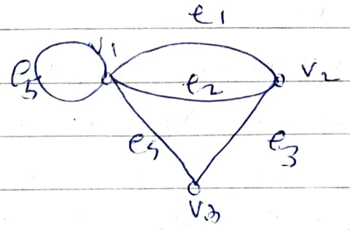
- vertices do not repeat in the path

ex. path I - elementary

③ Circuit

- If the end vertices of the path are same then it is called circuit.

ex. path-III — circuit



$C_1 = (v_1\, e_1\, v_2\, e_2\, v_1)$

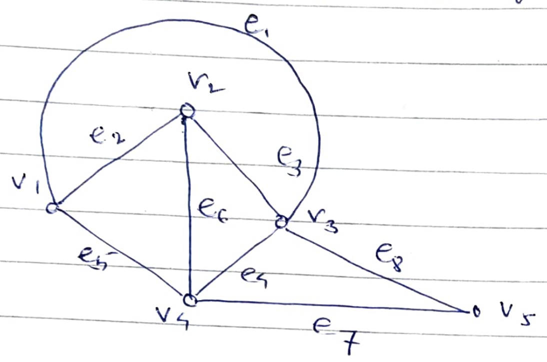
$C_2 = (v_3\, e_4\, v_1\, e_1\, v_2\, e_3\, v_3)$

④ Simple and Elementary Circuit

A circuit in a graph G is called simple circuit if it does not include the same edge twice

- A circuit is called elementary circuit if it does not meet the same vertex twice (except for first and last vertex)

- Length of circuit = No. of edges



G

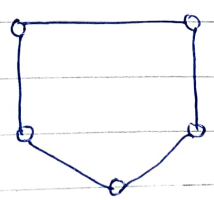
$C_1 = (v_1\, e_1\, v_3\, e_3\, v_2\, e_2\, v_1)$ — simple & elementary)

$C_2 = (v_1\, e_2 v_2\, e_6\, \underline{v_4}\, e_7\, v_5\, e_8\, v_3\, e_4 \underline{v_4}\, e_5\, v_1)$ — elementary

* Connected and Disconnected Graphs

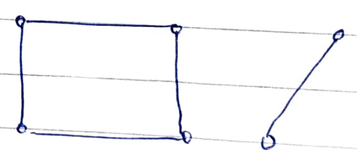
- A graph is connected graph if there exists a path between every pair of vertices, otherwise the graph is disconnected.

- Disconnected graph consists of two or more parts called components
- Each component is connected graph but there is no path between two vertices if they belongs to two different components.
- Connected graph has only one component.



G_1

Connected Graph



G_2

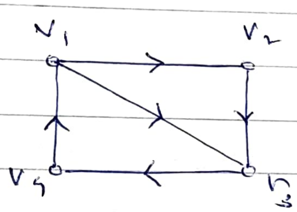
Disconnected Graph

Diagraph

- A directed graph (digraph) is called **strongly connected** if for every pair of vertices 'a' & 'b' in the diagraph, there is a path from 'a' to 'b' as well as path from 'b' to 'a'.

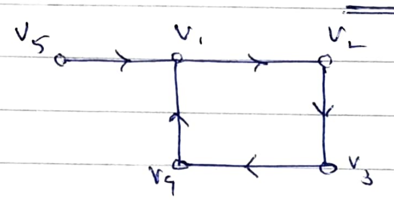
- A diagraph is **weakly connected** if it is not strongly connected and its underlying graph is connected.

- A diagraph which is neither strongly connected nor weakly connected is known as **disconnected diagraph**.



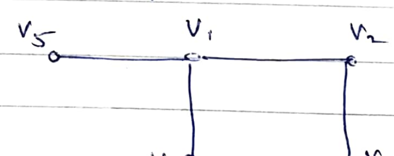
D_1

(Strongly connected)



D_2

(Weakly connected)



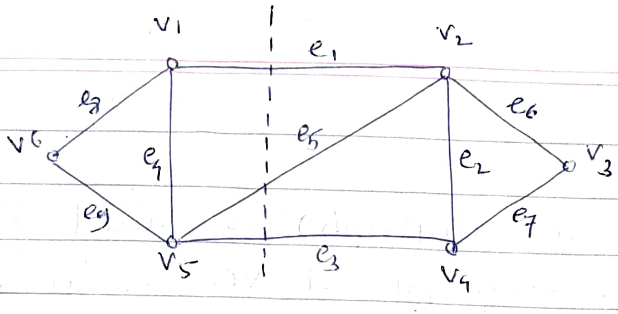
(Underlying graph of D_2)

* Edge and vertex Connectivity

① Edge Connectivity

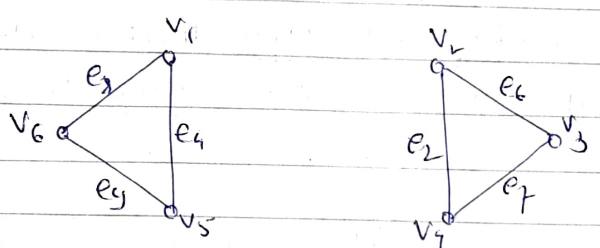
- In connected graph, **cut-set** is minimal set of edges whose removal disconnects the graph and increases the components of graph by one

- i.e cut-set in connected graph G is set of edges whose removal from G leaves G disconnected



G

For cut-set $\{e_1, e_3, e_5\}$ graph G will have 2 components G_1 and G_2

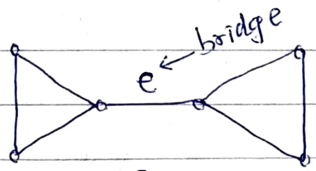


G_1 G_2

Hence, if 'S' is cut-set then $(G-S)$ has exactly two components

Bridge or Isthmus

- If cut-set contains only one edge then that edge is called an isthmus or bridge



$\lambda(G_3) = 1$

G_3

Edge Connectivity $\lambda(G)$

- No. of edges in smallest cut-set of a connected simple graph is called edge connectivity
- i.e smallest no. of edges whose removal disconnects the graph G.
- In above graph G_3, $\lambda(G_3) = 1$

(2) Vertex Connectivity $k(G)$

- Smallest no. of vertices whose removal disconnects the graph

DSGT- 4   H. E. Suryavanshi   (4)

The Shirpur Education Society's

R. C. PATEL INSTITUTE OF TECHNOLOGY

Nimzari Naka, Shirpur, Dist - Dhule (MS)

Ph No. : (02563) 259600,259801 Telefax : (02563) 259801

Website : http://www.rcpit.ac.in

High Caliber Technical Education in an Environment that Promotes Excellence

TEST (I / II) / PRELIMINARY EXAMINATION

Name of Candidate : ____________________

(IN BLOCK LETTERS)   (Surname)   (First Name)   (Middle Name)

Year : FE / SE / TE / BE   Branch : ________ Division : ________ Roll No. : ________

Semester : I / II Name of Subject : ____________________

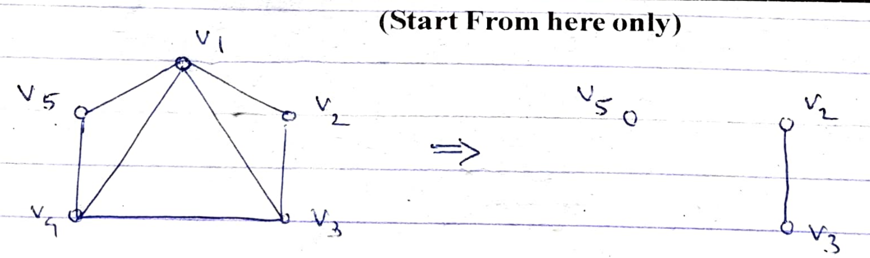
Total Supplements : 1 + ______ = ______

Signature of Student   Signature of Supervisor

Question Number	1	2	3	4	5	6	7	8	9	10	Total
Marks Obtained											
Marks out of											

Signature of Examiner :   Signature of Moderator :

(Start From here only)



G

Removal of $\{v_1 v_4\}$ disconnects G

$K(G) = 2$ i.e $\{v_1 v_4\}$

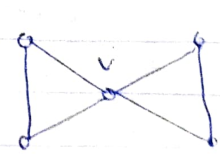
<u>K- connected graph</u>

- graph whose vertex connectivity is 'k'

<u>Seperable Graph</u>

- vertex connectivity is one i.e $K(G) = 1$
- cut-vertex or cut-point

e.g. 'v' is cut-point



- Edge and vertex connectivity are related to minimum degree of vertex in graph.

i.e $K(G) \le \lambda(G) \le \delta$

where,

$K(G)$ - Vertex connectivity

$\lambda(G)$ - Edge connectivity

δ - minimum degree of vertex in G

- Edge connectivity is also related to no. of edges & vertices

$\therefore \quad \lambda(G) \le \left\lfloor \frac{2e}{n} \right\rfloor$

Ex ① Find edge connectivity for complete graph K_5

$\Rightarrow$ Here $n=5$ and $e = \frac{n(n-1)}{2} = \frac{5\times4}{2} = 10$

$\therefore$ edge connectivity $\le \frac{2e}{n}$

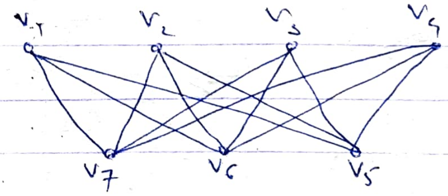
$\frac{2\times10}{5} = 4$

$\therefore \lambda(G) = 4$

Ex. ② Find $\lambda(G)$, $K(G)$ For $K_{4,3}$ graph

$\Rightarrow$ $K_{4,3}$

$\therefore$



Here $n = 7$

$e = 12$

$\therefore$ edge connectivity $(\lambda) = \frac{2e}{n} = \frac{2\times12}{7}$

$= \frac{24}{7}$

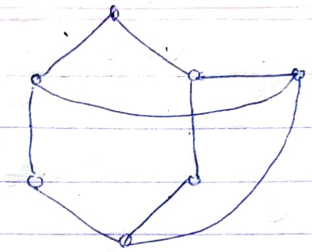
$\lambda(G) = 3$

Now,

if vertex $(v_5, v_6 \,\&\, v_7)$ removed then graph will be disconnected

$\therefore K(G) = 3$ i.e vertex connectivity

Ex. ③ Find edge connectivity of Graph given in fig.



=> Here $n = 7$
and $e = 9$

∴ Edge connectivity $\lambda(G) \leq \frac{2e}{n}$

$= \frac{2 \times 9}{7}$

∴ $\lambda(G) = 2$ i.e. edge connectivity

* Shortest Path Algorithm

The algorithm was found by Dijkstra in 1959. and is known as Dijkstra's shortest path algo.
This algorithm gives the shortest length of the path from the vertex 'a' to vertex 'z' but it does not give the actual path for the shortest distance from vertex 'a' to vertex 'z'.

Algorithm -

$G = (V, E)$ ⇒ be a simple graph
a & z ⇒ any two vertices of the graph.
$L(x)$ ⇒ Label - length of shortest path from 'a' to 'x'
W_{ij} ⇒ Weight of edge $e_{ij} = (v_i, v_j)$

Step ① $P = \{\phi\}$
$T = \{$ All vertices of Graph G $\}$

where
P = Set of vertices having permanent label.

Set $L(a) = 0$
$L(x) = \infty \ \forall x \in T$ and $x \neq a$

Step ②
Select vertex 'v' from T which has smallest label
This label is called permanent label of 'v'.
$P = P \cup \{v\}$ & $T = T - \{v\}$

IF $v = z$ then $L(z)$ is the length of Shortest path from a to z & stop.

Step ③

IF $v \neq z$ revise the labels of vertices of T i.e vertices that do not have permanent label

$L(x) = \min \{ \text{old-}L(x), L(v) + w(v, x) \}$

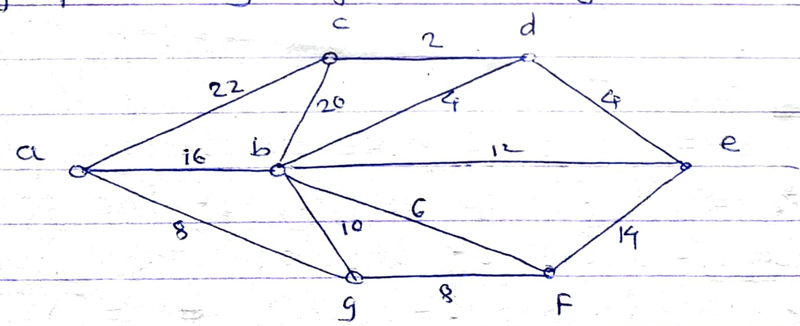
where

$w(v, x)$ – weight of edge joining 'v' to 'x'

IF there is no direct edge from 'v' to 'x' then ~~x∈~~ $w(v, x) = \infty$

Step ④ Repeate Step ② and ③ until 'z' gets permanent label.

Ex ① Find the shortest path for the following graph using Dijkstra's Algorithm. (from a to e)



⟹ According to the Dijkstras shortest path algorithm

① $P = \{\phi\}$ $T = \{a, b, c, d, e, F, g\}$

$L(a) = 0$ $L(x) = \infty$ $\forall x \in T, x \neq a$

② $v = a$, the permanent label of $a = 0$ i.e $L(a) = 0$

$P = \{a\}$ $T = \{b, c, d, e, f, g\}$

$L(b) = \min(\text{old-}L(b), L(a) + w(a, b))$

$= \min(\infty, 0 + 16) = \min(\infty, 16)$

$\therefore L(b) = 16$

$L(c) = \min\left(\text{old\_}L(c),\ L(a) + w(a,c)\right)$

$= \min(\infty,\ 0 + 22) = \min(\infty, 22)$

$L(c) = 22$

Similarly,

$L(d) = \min(\infty,\ 0 + \infty) = \infty$

$L(e) = \min(\infty,\ 0 + \infty) = \infty$

$L(f) = \min(\infty,\ 0 + \infty) = \infty$

$L(g) = \min(\infty,\ 0 + 8) = 8$

③ $v = g$, permanent label of $g = 8$ i.e $L(g) = 8$

$P = \{a, g\}$ $T = \{b, c, d, e, f\}$

$L(b) = \min\left(\text{old\_}L(b),\ L(g) + w(g,b)\right)$

$= \min(16,\ 8 + 10) = \min(16, 18)$

$\therefore L(b) = 16$

Similarly,

$L(c) = \min\left(\text{old\_}L(c),\ L(g) + w(g,c)\right)$

$= \min(22,\ 8 + \infty) = \min(22, \infty)$

$\therefore L(c) = 22$

$L(d) = \min(\infty,\ 8 + \infty) = \infty$

$L(e) = \min(\infty,\ 8 + \infty) = \infty$

$L(f) = \min(\infty,\ 8 + 8) = 16$

④ $v = b$, permanent label of $b = 16$ i.e $L(b) = 16$

$P = \{a, g, b\}$ $T = \{c, d, e, f\}$

$L(c) = \min\left(\text{old\_}L(c),\ L(b) + w(b,c)\right)$

$= \min(22,\ 16 + 20)$

$\therefore L(c) = 22$

$L(d) = \min(\infty,\ 16 + 4) = 20$

$L(e) = \min(\infty,\ 16 + 12) = 28$

$L(f) = \min(\overset{16}{\cancel{\infty}},\ 16 + 6) = \cancel{22}\ 16$

⑤ $v = f$, the permanent label of $f = 16$ i.e $L(f) = 16$

$P = \{a, g, b, f\}$ $T = \{c, d, e\}$

$L(c) = \min(\text{old\_}L(c),\ L(f) + w(f, c))$

$= \min(22,\ 16 + \infty)$

$\therefore L(c) = 22$

$L(d) = \min(20,\ 16 + \infty) = 20$

$L(e) = \min(28,\ 16 + 14) = 28$

(6) $v = d$, the permanent label of d is 20 i.e $L(d) = 20$

$P = \{a, g, b, f, d\}$ $\quad T = \{c, e\}$

$L(c) = \min(\text{old\_}L(c),\ L(d) + w(d, c))$

$= \min(22,\ 20 + 2)$

$\therefore L(c) = 22$

$L(e) = \min(28,\ 20 + 4) = 24$

(7) $v = c$, the permanent label of $c = 22$ i.e $L(c) = 22$

$P = \{a, g, b, f, d, c\}$ $\quad T = \{e\}$

$L(e) = \min(\text{old\_}L(e),\ L(c) + w(c, e))$

$= \min(24,\ 22 + \infty)$

$L(e) = 24$

(8) $v = e$, the permanent label of $e = 24$ i.e $L(e) = 24$

$P = \{a, g, b, f, d, c\}$ $\quad T = \{\phi\}$

Hence,

the length of shortest path from 'a' to 'e' is 24

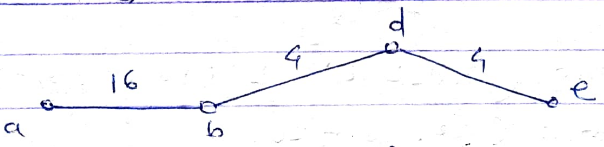


Fig. shortest path from a to e i.e (a-b-d-e)