$$\Gamma = \sqrt{\frac{\left(\frac{p}{p_0}\right)^{\frac{2}{\gamma}} - \left(\frac{p}{p_0}\right)^{\frac{\gamma+1}{\gamma}}}{\frac{\gamma-1}{\gamma+1}\left(\frac{2}{\gamma+1}\right)^{\frac{2}{\gamma-1}}}}, \qquad \frac{p_0}{p} = \left(1 + \frac{\gamma-1}{2}M^2\right)^{\frac{\gamma}{\gamma-1}} \tag{1}$$

上記 Γ に $\frac{p_0}{p}$ を代入すると次の式が得られる。

$$\Gamma = M\left[\frac{2+(\gamma-1)M^2}{\gamma+1}\right]^{-\frac{\gamma+1}{2(\gamma-1)}} \tag{2}$$

根号の中の分子を計算する。

$$\left(\frac{p}{p_0}\right)^{\frac{2}{\gamma}} - \left(\frac{p}{p_0}\right)^{\frac{\gamma+1}{\gamma}} = \left(\frac{p_0}{p}\right)^{-\frac{2}{\gamma}} - \left(\frac{p_0}{p}\right)^{-\frac{\gamma+1}{\gamma}} = \left(\left(\frac{p_0}{p}\right)^{\frac{\gamma-1}{\gamma}} - 1\right)\left(\frac{p_0}{p}\right)^{-\frac{\gamma+1}{\gamma}} \tag{3}$$

$$= \left(\left(1 + \frac{\gamma-1}{2}M^2\right) - 1\right)\left(1 + \frac{\gamma-1}{2}M^2\right)^{-\frac{\gamma+1}{\gamma-1}} \tag{4}$$

$$= \left(\frac{\gamma-1}{2}M^2\right) \cdot 2^{\frac{\gamma+1}{\gamma-1}}\left(2+(\gamma-1)M^2\right)^{-\frac{\gamma+1}{\gamma-1}} \tag{5}$$

$$= 2^{\frac{2}{\gamma-1}}(\gamma-1)M^2\left(2+(\gamma-1)M^2\right)^{-\frac{\gamma+1}{\gamma-1}} \tag{6}$$

分母の変形を行う。

$$\frac{\gamma-1}{\gamma+1}\left(\frac{2}{\gamma+1}\right)^{\frac{2}{\gamma-1}} = (\gamma-1)\frac{1}{\gamma+1} \cdot 2^{\frac{2}{\gamma-1}}\left(\frac{1}{\gamma+1}\right)^{\frac{2}{\gamma-1}} \tag{7}$$

$$= 2^{\frac{2}{\gamma-1}}(\gamma-1)\left(\frac{1}{\gamma+1}\right)^{\frac{\gamma+1}{\gamma-1}} = 2^{\frac{2}{\gamma-1}}(\gamma-1)(\gamma+1)^{-\frac{\gamma+1}{\gamma-1}} \tag{8}$$

これらを Γ に当てはめる。

$$\Gamma = \sqrt{\frac{\left(\frac{p}{p_0}\right)^{\frac{2}{\gamma}} - \left(\frac{p}{p_0}\right)^{\frac{\gamma+1}{\gamma}}}{\frac{\gamma-1}{\gamma+1}\left(\frac{2}{\gamma+1}\right)^{\frac{2}{\gamma-1}}}} = \sqrt{\frac{2^{\frac{2}{\gamma-1}}(\gamma-1)M^2\left(2+(\gamma-1)M^2\right)^{-\frac{\gamma+1}{\gamma-1}}}{2^{\frac{2}{\gamma-1}}(\gamma-1)(\gamma+1)^{-\frac{\gamma+1}{\gamma-1}}}} \tag{9}$$

$$= \sqrt{\frac{M^2\left(2+(\gamma-1)M^2\right)^{-\frac{\gamma+1}{\gamma-1}}}{(\gamma+1)^{-\frac{\gamma+1}{\gamma-1}}}} = |M|\sqrt{\left(\frac{2+(\gamma-1)M^2}{\gamma+1}\right)^{-\frac{\gamma+1}{\gamma-1}}} \tag{10}$$

$$= |M|\left(\frac{2+(\gamma-1)M^2}{\gamma+1}\right)^{-\frac{\gamma+1}{2(\gamma-1)}} \tag{11}$$

$M \geq 0$ であれば、$\Gamma = M\left(\frac{2+(\gamma-1)M^2}{\gamma+1}\right)^{-\frac{\gamma+1}{2(\gamma-1)}}$ である。