

$\alpha = 2\theta$ とおく。変形をすると $\theta = \frac{\alpha}{2}$

$\cos 2\theta = 1 - 2\sin^2 \theta$ より

$$\cos 2\theta = \cos \alpha \quad (1)$$

$$1 - 2\sin^2 \theta = 1 - 2\sin^2 \frac{\alpha}{2} \quad (2)$$

この為

$$\cos \alpha = 1 - 2\sin^2 \frac{\alpha}{2} \quad (3)$$

$$2\sin^2 \frac{\alpha}{2} = 1 - \cos \alpha \quad (4)$$

$$\sin^2 \frac{\alpha}{2} = \frac{1 - \cos \alpha}{2} \quad (5)$$

同様にして、 $\cos 2\theta = 2\cos^2 \theta - 1$ より

$$\cos 2\theta = \cos \alpha \quad (6)$$

$$2\cos^2 \theta - 1 = 2\sin^2 \frac{\alpha}{2} - 1 \quad (7)$$

この為

$$\cos \alpha = 2\sin^2 \frac{\alpha}{2} - 1 \quad (8)$$

$$2\cos^2 \frac{\alpha}{2} = 1 + \cos \alpha \quad (9)$$

$$\cos^2 \frac{\alpha}{2} = \frac{1 + \cos \alpha}{2} \quad (10)$$

$$\sin^2 \frac{\alpha}{2} = \frac{1 - \cos \alpha}{2} \quad \cos^2 \frac{\alpha}{2} = \frac{1 + \cos \alpha}{2} \quad (11)$$

この 2 つの式を利用し

$$\tan^2 \frac{\alpha}{2} = \frac{\sin^2 \frac{\alpha}{2}}{\cos^2 \frac{\alpha}{2}} \quad (12)$$

$$= \frac{\frac{1 - \cos \alpha}{2}}{\frac{1 + \cos \alpha}{2}} = \frac{1 - \cos \alpha}{1 + \cos \alpha} \quad (13)$$