C2-001_Practice 03

Date	@2022년 8월 15일 오전 9:30
Lecture Note	C2-001_Lecture_03-linear transform.pdf
Practice (pdf)	
Solution (pdf)	
Topics	Lecture 03: Linear Transformation II
Week	3

- Please mark what you think is the correct answer (O or X) and show why you choose it.
  1. Show that $\langle \cdot, \cdot \rangle$ defined for all $\mathbf{x} = [x_1, x_2]^\top \in \mathbb{R}^2$ and $\mathbf{y} = [y_1, y_2]^\top \in \mathbb{R}^2$ by $\langle \mathbf{x}, \mathbf{y} \rangle := x_1 y_1 - (x_1 y_2 + x_2 y_1) + 2(x_2 y_2)$ is an inner product.
  2. Consider $\mathbb{R}^2$ with $\langle \cdot, \cdot \rangle$ defined for all $\mathbf{x}$ and $\mathbf{y}$ in $\mathbb{R}^2$ as $\langle \mathbf{x}, \mathbf{y} \rangle := \mathbf{x}^\top \mathbf{A} \mathbf{y} = \mathbf{x}^\top \begin{bmatrix} 2 & 0 \\ 1 & 2 \end{bmatrix} \mathbf{y}$.
     Is $\langle \cdot, \cdot \rangle$ an inner product?
  3. Compute the distance between $\mathbf{x} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ and $\mathbf{y} = \begin{bmatrix} -1 \\ -1 \\ 0 \end{bmatrix}$ using
     a. $\langle \mathbf{x}, \mathbf{y} \rangle := \mathbf{x}^\top \mathbf{y}$
     b. $\langle \mathbf{x}, \mathbf{y} \rangle := \mathbf{x}^\top \mathbf{A} \mathbf{y} = \mathbf{x}^\top \begin{bmatrix} 2 & 1 & 0 \\ 1 & 3 & -1 \\ 0 & -1 & 2 \end{bmatrix} \mathbf{y}$

4. Compute the angle between $\mathbf{x} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$ and $\mathbf{y} = \begin{bmatrix} -1 \\ -1 \end{bmatrix}$ using

   a. $\langle \mathbf{x}, \mathbf{y} \rangle := \mathbf{x}^\top \mathbf{y}$

   b. $\langle \mathbf{x}, \mathbf{y} \rangle := \mathbf{x}^\top \mathbf{B} \mathbf{y} = \mathbf{x}^\top \begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix} \mathbf{y}$

5. Find the projection matrix P_π on to the line through the origin spanned by $\mathbf{b} = \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}$. $\mathbf{b}$ is a direction and a basis of the one-dimensional subspace (i.e., line through origin).

6. rotate the vectors $\mathbf{x} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$, $\mathbf{y} = \begin{bmatrix} 0 \\ -1 \end{bmatrix}$ by 30°.