2 Engineering Mathematics

1. Probability and Statistics

01. Company X shipped 5 computer chips, 1 of which was defective and Company Y shipped 4 computers chips, 2 of which were defective. One computer chip is to be chosen uniformly at random from the 9 chips shipped by the companies. If the chosen chip is found to be defective, what is the probability that the chip came from Company Y ?
(a) 2/9 (b) 4/9
(c) 2/3 (d) 1/2 [ISRO - 2007]

02. A program consists of two modules executed sequentially. Let $f_1(t)$ and $f_2(t)$ respectively denote the probability density functions of time taken to execute the two modules. The probability density function of the overall time taken to execute the program given by
(a) $f_1(t) + f_2(t)$ (b) $\int_0^t f_1(x)f_2(x)dx$
(c) $\int_0^t f_1(x)f_2(t-x)dx$ (d) $\max(f_1(t) + f_2(t))$
[ISRO - 2007]

03. Let f(x) be the continuous probability density function of a random variable x. The probability then $a < x \leq b$, is
(a) $f(b - a)$ (b) $f(b) - f(a)$
(c) $\int_a^b f(x)dx$ (d) $\int_a^b xf(x)dx$
[ISRO - 2009]

04. If the mean of a normal frequency distribution of 1000 times is 25 and its standard deviation is 2.5, then its maximum ordinate is
(a) $\frac{1000}{\sqrt{2\pi}}e^{-25}$ (b) $\frac{1000}{\sqrt{2\pi}}$
(c) $\frac{1000}{\sqrt{2\pi}}e^{-2.5}$ (d) $\frac{400}{\sqrt{2\pi}}$ [ISRO - 2009]

05. If the pdf of a Poisson distribution is given by
$f(x) = \frac{e^{-2}2^x}{x!}$, then its mean is
(a) 2^x (b) 2 (c) -2 (d) 1
[ISRO - 2009]

06. Three coins are tossed simultaneously. The probability that they will fall two heads and one tail is
(a) 5/8 (b) 1/8 (c) 2/3 (d) 3/8
[ISRO - 2011]

07. If a program P calls two subprograms P1 and P2 and P1 can fail 50% of the time and P2 can fail 40% of the time, what is the failure rate of program P
90−20
(a) 50% (b) 60% (c) 70% (d) 10%
[ISRO - 2013]

08. Let P(E) denote the probability of the occurrence of event E. If P(A) = 0.5 and P(B) = 1, then the values of P(A/B) and P(B/A) respectively are
(a) 0.5, 0.25 (b) 0.25, 0.5
(c) ~~0.5, 1~~ (d) 1, 0.5
$\frac{0.5}{1}, \frac{0.5}{0.5}$
[ISRO - 2013]

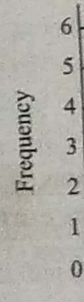
09. The probability that two friends are born in the same month is ____?
(a) 1/6 (b) 1/12 (c) 1/144 (d) 1/24
[ISRO - 2014]

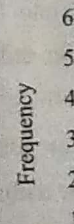
10. If A and B be two arbitrary events, then
(a) $P(A \cap B) = P(A)P(B)$
(b) $P(A \cup B) = P(A)+P(B)$
(c) $P(A / B) = P(A \cap B)+P(B)$
(d) $P(A \cup B) \leq P(A) + P(B)$
[ISRO - 2017(May)]

11. A bag contains
Two balls are
discarded if the
are different
is returned to
process will te
(a) 1
(c) 0

12. A class of
containing 5
row. If the st
probability th
be empty is
(a) 1/5
(c) 1/4

13. For the distr





Which o
distributi

R - 1/2 R̄ - 1/2 R - 19
B - 1/2 B̄ - 1/2 B = 18.

11. A bag contains 19 red balls and 19 black balls. Two balls are removed at a time repeatedly and discarded if they are of the same colour, but if they are different black ball is discarded and red ball is returned to the bag. The probability that this process will terminate with one red ball is

(a) 1 (b) 1/21
(c) 0 (d) 0.5

[ISRO - 2017(Dec)]

12. A class of 30 students occupy a classroom containing 5 rows of seats, with 8 seats in each row. If the students seat themselves at random, the probability that the sixth seat in the fifth row will be empty is

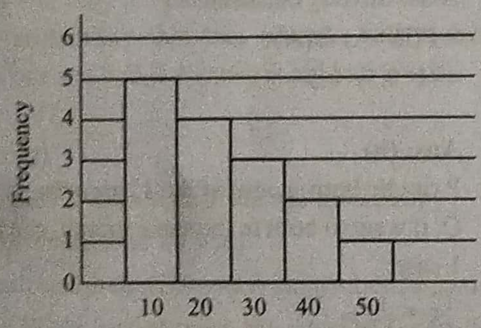
(a) 1/5 (b) 1/3
(c) 1/4 (d) 2/5

[ISRO - 2018]

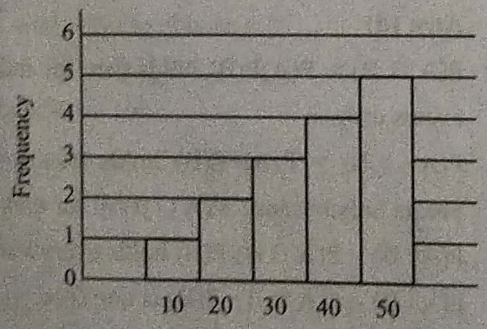
30/40 1/4 8 8 8 8 5 38th 39 seat

13. For the distributions given below:

Distribution A



Distribution B



Which of the following is correct for the above distributions?

(a) Standard deviation of A is significantly lower than standard deviation of B
(b) Standard deviation of A is slightly lower than standard deviation of B
(c) Standard deviation of A is same as standard deviation of B
(d) Standard deviation of A is significantly higher than standard deviation of B

[ISRO - 2020]

KEY & Detailed Solutions				
01. (c)	02. (c)	03. (c)	04. (d)	05. (b)
06. (d)	07. (c)	08. (c)	09. (b)	10. (d)
11. (a)	12. (c)	13. (c)		

01. Ans: (c)

Sol: Probability that chip is chosen from company X = 5/9

Probability that chip is chosen from company Y = 4/9

Number of Defective chips from company X= 1

Number of Defective chips from company Y = 2

Probability that chip is defective from company X = 5/9 * 1/5

Probability that chip is defective from company Y = 4/9 * 2/4

Probability that chip is defective = 5/9 * 1/5 + 4/9 *2/4

Given chip is defective, probability that it is from the company

Y = P(Defective Company Y) /P(Defective)

= (4/9 * 2/4)/(5/9*1/5 + 4/9 *2/4)

= 2/3

02. Ans: (c)

Sol: We assume the total time to be 't' units and f1 executes for 'x' units.

Since f1(t) and f2(t) are executed sequentially,

So, f2 is executed for 't – x' units.

We apply convolution on the sum of two independent random variables to get probability density function of the overall time taken to execute the program.

f1(x) * f2 (t–x)

03. Ans: (c)

Sol: (c) should be used if probability density function is given

(b) should be used if probability distribution function is given

(d) must be used to calculate expectation when pdf is given

04. Ans: (d)

Sol: We know that pdf of normally distributed

RV $f(x) = \dfrac{1}{\sigma\sqrt{2\pi}} e^{\frac{-(x-\mu)}{2a^2}}$

Therefore $x = \mu$, therefore our pdf $= f(x) = \dfrac{1}{\sigma\sqrt{2\pi}} e^0$

Therefore max probability $= \dfrac{1}{2.5\sqrt{2\pi}}$

Therefore for 1000 items $= \dfrac{1000}{2.5\sqrt{2\pi}} = \dfrac{400}{\sqrt{2\pi}}$

05. Ans: (b)

Sol: $f(x) = (e^{-2}2^x)/x!$

In the poison distribution expression,

$(e^{-\lambda}\lambda^k)/k!$

Here, k = number of events

λ = expected value /average/mean over 'k' number of values

So, mean = 2, option (b) is correct.

06. Ans: (d)

Sol: ${}^nC_k\,(p)^k.\,(1-p)^{n-k}$

Here, n = 3, k = 2, prob(head) = p = 1/2, prob(tail) = (1–p) = 1/2

$= {}^3C_2\,(1/2)^2.\,(1/2)^1$

$= 3 * (1/2)^3$

$= 3/8$

07. Ans: (c)

Sol: $P1 \cup P2 = P1 + P2 - (P1 \cap P2)$

$= \left(\frac{50}{100}\right) + \left(\frac{40}{100}\right) - \left(\frac{50}{100} * \frac{40}{100}\right)$

$= \left(\frac{90}{100}\right) - \left(\frac{2000}{10000}\right)$

$= \left(\frac{90}{100}\right) - \left(\frac{20}{100}\right) = \left(\frac{70}{100}\right)$

= 70%

08. Ans: (c)

Sol: P(A) = 0.5

P(B) = 1

Here there is no dependency in event A and B. S

$P(A \cap B) = P(A)*P(B)$

P(A/B) = probability of occurrence of event

when B has already occured

$= P(A \cap B) / P(B)$

= (0.5 *1)/1 = 0.5

P(B/A) = probability of occurence of event B whe

A has already occurred

$= P(B \cap A) / P(A)$

= (1*0.5) /0.5 = 1

09. Ans: (b)

Sol: P can be born in any of the 12 months.

Q, if want to born in the same month, can have onl

1 case:

$\frac{12}{12} * \frac{1}{12} = \frac{1}{12}$

10. Ans: (d)

Sol: $P(A \cap B) = P(A)P(B)$ holds true for independen events only.

$P(A \cup B) = P(A)+P(B)$ holds true for disjoin events only,because $P(A \cap B)=0$ for them.

$P(A / B) = P(A \cap B)/P(B)$ holds everywhere.

$P(A / B) = P(A \cap B)+P(B)$ is not True.

$P(A \cup B) \leq P(A) + P(B)$ is always True.

Because, $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

11. Ans: (a)

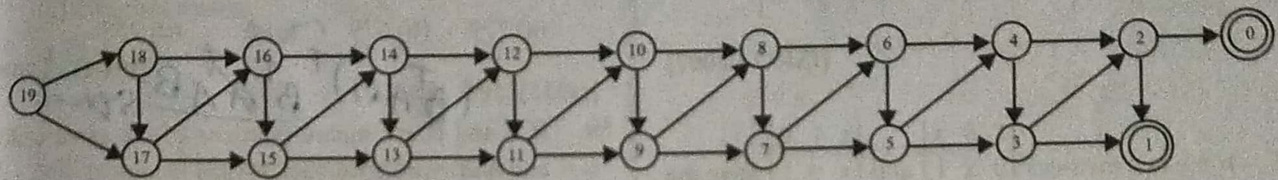
Sol: Number of Red Balls:

count changes from n to (n-2), only when both balls are of red colour, otherwise no change in the count.

Number of black balls:

If both balls are black, count changes from n to (n–2)

If both balls are different, count changes from n to (n–1)



So the following cases are possible:

Final count of red balls = 1 & final count of black balls = 1.

The next iteration gives, two different balls, black is discarded, red is left.

Final count of red balls = 1 & final count of black balls = 0, only red balls is left.

The probability that this process will terminate with one red ball is 1 (always red ball is left).

12. Ans: (c)

Sol: There are 5 rows with 8 seats in each row.

So, there are total 40 seats

If sixth seat in the fifth row is empty then 30 students have 39 choices of seats. So, ways to choose from given choices = ${}^{39}C_{30}$

But, total ways to choose = ${}^{40}C_{30}$

Probability = ${}^{39}C_{30}/{}^{40}C_{30}$

Probability = 1/4

13. Ans: (c)

Sol: Distribution A:

x_i	f_i	$x_i f_i$	$x_i^2 f_i$
10	5	50	500
20	4	80	1600
30	3	90	2700
40	2	80	3200
50	1	50	2500
	15	350	10500

$$\sigma_1 = \sqrt{\frac{n\Sigma x_i^2 f_i - (\Sigma x_i f_i)^2}{n}}$$

where, $n = \sum f_i = 15$

$$\sigma_1 = \sqrt{\frac{15(10500) - (350)^2}{(15)^2}} = 12.472$$

Distribution B:

x_i	f_i	$x_i f_i$	$x_i^2 f_i$
10	1	10	100
20	2	40	800
30	3	90	2700
40	4	160	6400
50	5	250	12500
	15	550	22500

$$\text{So, } \sigma_2 = \sqrt{\frac{15(22500) - (550)^2}{(15)^2}} = 12.472$$

$$\Rightarrow \sigma_1 = \sigma_2$$

2. Linear Algebra

$1+\cos\theta$

$(1-\lambda)^2=\cos^2\theta$

$1-\lambda=\pm\cos\theta$ $\lambda=1-\cos\theta$

01. Eigen vectors of $\begin{bmatrix}1-\lambda & \cos\theta\\ \cos\theta & 1\end{bmatrix}$ are

(a) $\begin{bmatrix}a^n & 1\\ 0 & a^n\end{bmatrix}$ (b) $\begin{bmatrix}a^n & n\\ 0 & a^n\end{bmatrix}$

(c) $\begin{bmatrix}a^n & na^{n-1}\\ 0 & a^n\end{bmatrix}$ (d) $\begin{bmatrix}a^n & na^{n-1}\\ -n & a^n\end{bmatrix}$

[ISRO - 2007]

02. If the two matrices $\begin{bmatrix}1 & 0 & x\\ 0 & x & 1\\ 0 & 1 & x\end{bmatrix}$ and $\begin{bmatrix}x & 1 & 0\\ x & 0 & 1\\ 0 & x & 1\end{bmatrix}$ have the same determinant, then the value of X is

(a) 1/2 (b) $\sqrt{2}$

(c) $\pm 1/2$ (d) $\pm 1/\sqrt{2}$

[ISRO - 2008]

03. If a square matrix A satisfies $A^TA = I$, then the matrix A is

(a) Idempotent (b) Symmetrical

(c) Orthogonal (d) Hermitian

[ISRO - 2008]

04. A square matrix A is called orthogonal if A'A =

(a) I (b) A (c) –A (d) –I

[ISRO - 2009]

05. If two adjacent rows of a determinant are interchanged, the value of the determinant

(a) becomes zero

(b) remains unaltered

(c) becomes infinite

(d) becomes negative of its original value

[ISRO - 2009]

06. If $\begin{vmatrix}3 & 3\\ x & 5\end{vmatrix}=3$. The values of x is

(a) 2 (b) 3 (c) 4 (d) 5

[ISRO - 2009]

07. If A, B, C are any three matrices, then A'+B'+C' is equal to

(a) a null matrix (b) A+B+C

(c) (A+B+C)' (d) –(A+B+C)

[ISRO - 2009]

08. $\begin{vmatrix}265 & 240 & 219\\ 240 & 225 & 198\\ 219 & 198 & 181\end{vmatrix}=$

(a) 779 (b) 679 (c) 0 (d) 256

[ISRO - 2009]

$(B^TAB)^T$ B^TAB sym

09. If A and B are square matrices of the same order and A is symmetric, then B^TAB is

(a) Skew symmetric (b) Symmetric

(c) Orthogonal (d) Idempotent

$\begin{bmatrix}1&2\\3&4\end{bmatrix}\begin{bmatrix}1&2\\2&1\end{bmatrix}\begin{bmatrix}1&3\\2&4\end{bmatrix}$

[ISRO - 2011]

10. What is the matrix that represents rotation of an object by θ° about the origin in 2D?

(a) cos θ –sin θ sin θ cos θ

(b) sin θ –cos θ cos θ sin θ

(c) cos θ –sin θ cos θ sinθ

(d) sin θ –cos θ cos θ sin θ

[ISRO - 2011]

11. What is the matrix transformation which takes the independent vectors $\begin{bmatrix}1\\2\end{bmatrix}$ and $\begin{bmatrix}2\\5\end{bmatrix}$ and transforms them to $\begin{bmatrix}1\\1\end{bmatrix}$ and $\begin{bmatrix}3\\2\end{bmatrix}$ respectively?

(a) $\begin{bmatrix}1 & -1\\ 1 & 0\end{bmatrix}$ (b) $\begin{bmatrix}0 & 0\\ 0.5 & 0.5\end{bmatrix}$

(c) $\begin{bmatrix}-1 & 0\\ 1 & 1\end{bmatrix}$ (d) $\begin{bmatrix}-1 & 1\\ 1 & 0\end{bmatrix}$ $\begin{bmatrix}1\\2\end{bmatrix}=\begin{bmatrix}1\\1\end{bmatrix}$

[ISRO - 2013]

12. The Guass-Seidal iterative method can be used to solve which of the following sets?

(a) Linear algebraic equations

(b) Linear and non-linear algebraic equations

(c) Linear differential equations

(d) Linear and non-linear differential equations

[ISRO - 2013]

5

13. The rank of the matrix $A = \begin{bmatrix} 1 & 2 & 1 & -1 \\ 9 & 5 & 2 & 2 \\ 7 & 1 & 0 & 4 \end{bmatrix}$ is

(a) 0 (b) 1
(c) 2 (d) 3

[ISRO - 2014]

14. If A is a skew symmetric matrix, then A^t

(a) diagonal matrix (b) A
(c) 0 (d) –A

[ISRO - 2017(May)]

15. If C is a skew-symmetric matrix of order n and X is $n \times 1$ column matrix, then $X^T CX$ is a

(a) scalar matrix
(b) null matrix
(c) unit matrix
(d) matrix will all elements 1

[ISRO - 2017(Dec)]

$-8-16+24$

0

KEY & Detailed Solutions				
01. (*)	02. (a)	03. (c)	04. (a)	05. (d)
06. (c)	07. (c)	08. (c)	09. (b)	10. (a)
11. (d)	12. (a)	13. (c)	14. (d)	15. (b)

01. Ans: (*)

Sol: $A = \begin{bmatrix} 1 & \cos\theta \\ \cos\theta & 1 \end{bmatrix}$

For eigen λ

$|A - \lambda I| = 0$

$$\begin{vmatrix} 1-\lambda & \cos\theta \\ \cos\theta & 1-\lambda \end{vmatrix} = 0$$

$(1-\lambda)^2 - \cos^2\theta = 0$

$(1-\lambda+\cos\theta)(1-\lambda-\cos\theta) = 0$

$\lambda = (1+\cos\theta)$ or $\lambda = 1-\cos\theta$

Eigen vectors for $\lambda = (1+\cos\theta)$

$AX = \lambda X$

$AX - \lambda X = 0$

$(A - \lambda I)X = 0$

$$\begin{bmatrix} 1-(1+\cos\theta) & \cos\theta \\ \cos\theta & 1-(1+\cos\theta) \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} -\cos\theta & \cos\theta \\ \cos\theta & -\cos\theta \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$$

$\Rightarrow -x_1\cos\theta + x_2\cos\theta = 0$

$x_1\cos\theta - x_2\cos\theta = 0$

$\Rightarrow x_1\cos\theta = x_2\cos\theta$

$x_1 = x_2$

$\begin{bmatrix} 1 \\ 1 \end{bmatrix}$ is the eigen vector

For $\lambda = 1-\cos\theta$, The eigen vectors are

$$\begin{bmatrix} 1-(1-\cos\theta) & \cos\theta \\ \cos\theta & 1-(1-\cos\theta) \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} \cos\theta & \cos\theta \\ \cos\theta & \cos\theta \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$x_1\cos\theta + x_2\cos\theta = 0$

$x_1\cos\theta = -x_2\cos\theta$

$\begin{bmatrix} 1 \\ -1 \end{bmatrix}$ is another eigen vector.

02. Ans: (a)

Sol: Determinant of 1st Matrix = $X^2 - 1$

Determinant of 2nd Matrix = $-X^2 - X$

Since the determinants are equal:

$X^2 - 1 = -X^2 - X$

$2X^2 + X - 1 = 0$

$X = 1/2$

03. Ans: (c)

Sol: A square matrix A is an orthogonal matrix if its transpose is equal to its inverse. An orthogonal matrix A is necessarily invertible and unitary. Conditions for an orthogonal matrix.

$A^T = A^{-1}$ and

$A.A^T = A^T.A = I$,

Where I is an Identity matrix

04. Ans: (a)

Sol: An orthogonal matrix is one in which $A'A = I$, i.e., identity matrix.

Orthogonal matrix: When the product of a matrix to its transpose gives identity matrix.

Suppose A is a square matrix with real elements and of $n \times n$ order and AT or A' is the transpose of A.

$AA^T = I$

05. Ans: (d)

Sol: Let us take a determinant $A = \begin{vmatrix} m & n \\ p & q \end{vmatrix}$

By interchanging the column, we get

$B = \begin{vmatrix} n & m \\ q & p \end{vmatrix}$

So $A = -B$

Now by interchanging the rows, we get

$C = \begin{vmatrix} p & q \\ m & n \end{vmatrix}$

So $A = -C$

06. Ans: (c)

Sol: $15 - 3x = 3$

$x = 4$

07. Ans: (c)

Sol: 1. $(A^T)^T = A$

2. $(A + B)^T = A^T + B^T$ and $(A - B)^T = A^T - B^T$

3. $(kA)^T = kA^T$

4. $(AB)^T = B^T A^T$

08. Ans: (c)

Sol: $265((225*181) - (198 - 198)) - 240((240*181) - (198*219)) + 219((240*198) - (219*225))$

$= 403065 - 18720 - 384345 = 0$

09. Ans: (b)

Sol: For a Symmetric matrix, $A^T = A$

So, $B^TAB = B^T. A.B$

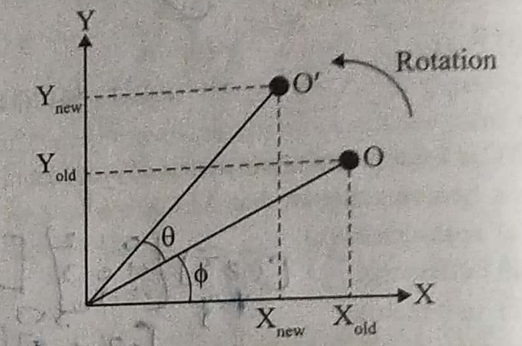
Taking transpose of $B^T. A, B$

$(B^T. A. B)^T = B^T. A^T. (B^T)^T = B^T. A. B$ // $(B^T)^T = B$

So, it is a symmetric matrix.

10. Ans: (a)

Sol:
- Initial coordinates of the object $O = (X_{old}, Y_{old})$
- Initial angle of the object O with respect to origin $= \phi$
- Rotation angle $= \theta$
- New coordinates of the object O after rotation $= (X_{new}, Y_{new})$



This rotation is achieved by using the following rotation equations

- $X_{new} = X_{old} \times \cos\theta - Y_{old} \times \sin\theta$
- $Y_{new} = X_{old} \times \sin\theta - Y_{old} \times \cos\theta$

$$R = \begin{bmatrix} \cos(-\theta) & \sin(-\theta) \\ -\sin(-\theta) & \cos(-\theta) \end{bmatrix}$$

$$R = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

The given answer is given for negative sense rotation.

11. Ans: (d)

Sol: option(a):

$$\begin{bmatrix} 1 & -1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 1-2 \\ 1+0 \end{bmatrix} = \begin{bmatrix} -1 \\ 1 \end{bmatrix} = 1\begin{bmatrix} -1 \\ 1 \end{bmatrix} = \lambda X$$

Which form is not of any of the transformed vectors given i.e $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$ & $\begin{bmatrix} 3 \\ 2 \end{bmatrix}$

Similarly option (b):

$$\begin{bmatrix} 0 & 0 \\ 0.5 & 0.5 \end{bmatrix}\begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 0+0 \\ 0.5+1 \end{bmatrix} = \begin{bmatrix} 0 \\ 1.5 \end{bmatrix} = \lambda X \text{ form} \neq \begin{bmatrix} 1 \\ 1 \end{bmatrix} \text{or} \begin{bmatrix} 3 \\ 2 \end{bmatrix}$$

Similarly option (c):

$$\begin{bmatrix}-1 & 0\\ 1 & 1\end{bmatrix}\begin{bmatrix}1\\2\end{bmatrix}=\begin{bmatrix}-1+0\\1+2\end{bmatrix}=\begin{bmatrix}-1\\3\end{bmatrix}=\lambda X \text{ form} \neq \begin{bmatrix}1\\1\end{bmatrix}\text{or}\begin{bmatrix}3\\2\end{bmatrix}$$

Option (d):

Now, option (d): Only satisfies this:

$$\begin{bmatrix}-1 & 1\\ 1 & 0\end{bmatrix}\begin{bmatrix}1\\2\end{bmatrix}=\begin{bmatrix}-1+2\\1+0\end{bmatrix}=\begin{bmatrix}1\\1\end{bmatrix}$$

1st transformed vector & 2nd $\begin{bmatrix}-1 & 1\\ 1 & 0\end{bmatrix}\begin{bmatrix}2\\5\end{bmatrix}$

$$=\begin{bmatrix}-2+5\\2+0\end{bmatrix}=\begin{bmatrix}3\\2\end{bmatrix}$$

12. Ans: (a)

Sol: The Gauss-Seidel method, also known as the Liebmann method or the method of successive displacement, is an iterative method used to solve a linear system of equations.

13. Ans: (c)

Sol: After performing

$R2 \to R2 - 9R1$

$R3 \to R3 - 7R1$

And then $R3 \to R3 - R2$

We have to

Calculate the number of linearly independent rows

No	A_1	A_2	A_3	A_4
1	1	2	1	–1
2	0	–13	–17	11
3	0	0	0	0

i.e; 2

14. Ans: (d)

Sol: If A is skew Symmetric Matrix $A^T = -A$

15. Ans: (b)

Sol: Let $C=\begin{bmatrix}0 & B & C\\ -B & 0 & D\\ -C & -D & 0\end{bmatrix}$, and $X=\begin{bmatrix}P\\Q\\R\end{bmatrix}$, then

$$CX=\begin{bmatrix}BQ+CR\\-BP+DR\\-CP-DQ\end{bmatrix}$$

$$X^TCX=[P\ \ Q\ \ R]\times\begin{bmatrix}BQ+CR\\-BP+DR\\-CP-DQ\end{bmatrix}$$

$\to$ [PBQ +PCR–PBQ+QDR–PCR –QDR]

$\to$ [PBQ – PBQ – PCR – PCR+ QDR – QDR]

$\to$ [0]

3. Calculus

01. A root α of equation $f(x)=0$ can be computed to any degree of accuracy if a 'good' initial approximation x_0 is chosen for which
(a) $f(x_0)>0$ (b) $f(x_0)\, f''(x_0)>0$
(c) $f(x)_0\, f''(x_0)<0$ (d) $f''(x_0)>0$
[ISRO - 2009]

02. Which of the following statement is correct
(a) $\Delta(U_k V_k) = U_k \Delta V_k + V_k \Delta U_k$
(b) $\Delta(U_k V_k) = U_{k+1} \Delta V_k + V_{k+1} \Delta U_k$
(c) $\Delta(U_k V_k) = V_{k+1} \Delta U_k + U_k \Delta V_k$
(d) $\Delta(U_k V_k) = U_{k+1} \Delta V_k + V_k \Delta U_k$
[ISRO - 2009]

03. The formula
$\int_{x0}^{xn} y(n)dx \simeq h/2(y_0+2y_1+\ldots\ldots+2y_{n-1}+y_n)$
$-\ h/12\ (\nabla y_n - \Delta y_0) - h/24\ (\nabla^2 y_n + \Delta^2 y_0) - 19h/720$
$(\nabla^3 y_n - \Delta^3 y_0)$ .... is called
(a) Simpson rule (b) Trapezoidal rule
(c) Romberg's rule (d) Gregory's formula
[ISRO - 2009]

04. The value of x at which y is minimum for $y = x^2 - 3x + 1$ is
(a) $-3/2$ (b) $3/2$
(c) 0 (d) $-5/4$
[ISRO - 2009]

05. n^{th} derivative of x^n is
(a) nx^{n-1} (b) $n^n.\, n!$
(c) $nx^n!$ (d) $n!$
[ISRO - 2011]

nx^{n-1}
$n.n-1x^{n-2}$
$n.n-1.n-2$

06. What is the least value of the function $f(x)=2x^2-8x-3$ in the interval $[0, 5]$?
(a) -15 (b) 7 (c) -11 (d) -3
[ISRO - 2013]

$4x-8$

07. $\lim_{x\to 0} \frac{\sqrt{1+x}-\sqrt{1-x}}{x}$ is given by
(a) 0 (b) -1 (c) 1
[IS

$-f'(x)=0$

08. If $x=-1$ and $x=2$ are extreme points ... $\log|x| + \beta x^2 + x$ then
(a) $\alpha=-6, \beta=-1/2$ (b) $\alpha=2, \beta=-$
(c) $\alpha=2, \beta=1/2$ (d) $\alpha=-6, \beta=$
[ISRO - 2

09. Let $f(x)=\log|x|$ and $g(x)=\sin x$. If A is t... $f(g(x))$ and B is the range of $g(f(x))$ the...
(a) $[-1, 0]$ (b) $[-1, 0)$
(c) $[-\infty, 0]$ (d) $[-\infty, 1]$
[ISRO - 2

10. The domain of the function log (log sin(...
(a) $0<x<\pi$
(b) $2n\pi<x<(2n+1)\pi$, for n in N
(c) Empty set
(d) None of the above
[ISR

KEY & Detailed Solutions

01. (a)	02. (b)	03. (b)	04. (b)
06. (c)	07. (c)	08. (b)	09. (a)

01. Ans: (a)
Sol: $f(x_0)>0$ is appropriate answer

02. Ans: (b)
Sol: $\Delta(U_k V_k) = U_{k+1} \Delta V_k + V_{k+1} \Delta U_k$

03. Ans: (b)
Sol: Trapezoidal rule is the answer.
Simpson's rule:
In numerical integration, Simpson's r... several approximations for definite i... named after Thomas Simpson.

The most basic of these rules, called Simpson's 1/3 rule, or just Simpson's rule, reads

$$\int_a^b f(x)dx \approx \frac{b-a}{6}\left[f(a)+4f\left(\frac{a+b}{2}\right)+f(b)\right]$$

Trapezoidal rule:

The trapezoidal rule works by approximating the region under the graph of the function f(x) as a trapezoid and calculating its area. It follows that

$$\int_a^b f(x)dz = \frac{\Delta x}{2}\begin{pmatrix} f(x_0)+2f(x_1)+2f(x_2)+2f(x_3)+2f(x_4) \\ +\ldots\ 2f(x_{N-1})+f(x_N)\end{pmatrix}$$

Replacing the f(x) with y(n), with some modifications, we get the series given in the question.

Romberg's Rule:

In numerical analysis, Romberg's method (Romberg 1955) is used to estimate the deinite integral $\int_a^b f(x)dx$

by applying Richardson extrapolation repeatedly on the trapezium rule

Using,

$$h_n = \frac{1}{2^n}(b-a)$$

The method can be inductively defined by

$R(0,0) = h_1(f(a)+f(b))$

$$R(n,0) = \frac{1}{2}R(n-1,0)+h_n\sum_{k=1}^{2^{n-1}} f(a+(2k-1)h_n)$$

$R(n,m) = R(n,m-1) + \frac{1}{4^m-1}(R(n,m-1)$
$-R(n-1,m-1))$

Where $n \geq m$ & $m \geq 1$

Gregory's series:

Gregory's series is an infinite Taylor series expansion of the inverse tangent function.

$$\int_0^x \frac{du}{1+u^2} = \arctan x = x - \frac{x^3}{3}+\frac{x^5}{5}-\frac{x^7}{7}+\ldots\ldots$$

04. Ans: (b)

Sol: $y = x^2 - 3x + 1$

$f(x) = 2x - 3 = 0$

$\Rightarrow x = \frac{3}{2}$

$f''(x) = 2 > 0$

Therefore point x = 3/2 is point of minima

$f(x = 3/2) = 9/4 - 9/2 + 1 = -5/4$

If we look at the other options

$f(x = -3/2) = 9/4 + 9/2 + 1 = 31/4$

$f(x = 0) = 0 - 0 + 1 = 1$

$\therefore$ 3/2 is the correct choice.

05. Ans: (d)

Sol: $f(x) = x^n$

$f'(x) = n.\ x^{n-1}$

$f''(x) = n(n-1).\ x^{n-2}$ and so on

So, n^{th} derivative of $f(x) = n(n-1)(n-2)\ldots\ldots 1.x^{n-n}$

$= n(n-1)(n-2)\ldots\ldots 1.x^0$

$= n!$

06. Ans: (c)

Sol: $f(X) = 2x^2 - 8x - 3$

$f'(X) = 4x - 8$

$f'(X) = 0 \Rightarrow X = 2$

$f''(X) = 4 > 0$, hence it has minimal value at the point $X = 2$

Minimum value is $2 \times (2)^2 - 8 \times (2) - 3 = -11$

07. Ans: (c)

Sol: $\lim_{x\to 0} \frac{1}{2\sqrt{1+x}} + \frac{1}{2\sqrt{1-x}} = \frac{1}{2}+\frac{1}{2} = 1$

08. Ans: (b)

Sol: **Step 1: Finding derivative of the given function**

Let the given function be $f(x) = \alpha \log|x| + \beta x^2 + x$

$f'(x) = \frac{\alpha}{x} + 2\beta x + 1$

To find the extreme points we equate f '(x) to zero and find the roots

$\frac{\alpha}{x} + 2\beta x + 1 = 0$

$\Rightarrow 2\beta x^2 + x + \alpha = 0$

Step 2: Comparing the roots of this equation with the extreme points given

$x = -1$ and $x = 2$ are the roots of this equation

Putting $x = -1$

$2\beta - 1 + \alpha = 0$

$\Rightarrow 2\beta + \alpha = 1$ ...... (1)

$\Rightarrow 2\beta + \alpha = 1$ ...... (1)

Putting $x = 2$

$8\beta + 2 + \alpha = 0$

$\Rightarrow 8\beta + \alpha = -2$...... (2)

Step 3: Solving (1) and (2) to find α and β

Subtracting (1) from (2)

$6\beta = -3$

$\Rightarrow \beta = \frac{-1}{2}$

substituting β in (1)

$-1 + \alpha = 1$

$\Rightarrow \alpha = 2$

Hence, correct answer is (C) $\alpha = 2, \beta = \frac{-1}{2}$

09. Ans: (a)

Sol: $f(x) = \log|x|$

$g(x) = \sin x$

$f(g(x)) = \log|\sin x|$

We know

$-1 \le \sin x \le 1$

$0 \le |\sin x| \le 1$

log (0) is undefined

log (1) is "0" (maximum value)

$-\infty \le \log|\sin x| \le 0$

A = Range of $\log|\sin x|$ is $(-\infty, 0]$

$g(f(x)) = \sin[\log|x|]$

$-1 \le \sin x \le 1$

B = Range of $\sin(\log|x|)$ is $[-1\ 0]$

$A \cap B = (-\infty, 0] \cap [-1\ 0] = [-1\ 0]$

10. Ans: (c)

Sol: log(log sin(x))

$-1 <= \sin x <= +1$

log a is defined for positive values of a,

log sin(x) is defined for sin(x) = (0,1]

Possible values for log sin(x) = $(-\infty, 0]$

Domain to log(log sin(x)) = Not defined

Miscellaneous

01. The cubic polynomial y(x) which takes the following values: y(0) = 1, y(1) = 0, y(2) = 1 and y(3) = 10 is,

(a) x^3+2x^2+1 (b) x^3+3x^2-1
(c) x^3+1 (d) x^3-2x^2+1

[ISRO - 2009]

02. $x = a\cos(t)$, $y = b\sin(t)$ is the parametric form of

(a) Ellipse (b) Hyperbola
(c) Circle (d) Parabola

[ISRO - 2009]

03. The formula

$P_k = y_0 + k\nabla y_0 + \frac{k(k+1)}{2}\nabla^2 y_0 + ... + \frac{k...(k+n-1)}{n!}\nabla^n y_0$ is

(a) Newton's backward formula
(b) Gauss forward formula
(c) Gauss backward formula
(d) Stirling's formula

[ISRO - 2009]

04. Given

X :	0	4	10	16
Y :	6	10	16	28

The interpolated value at X = 4 using piecewise linear interpolation is

(a) 11 (b) 4 (c) 22 (d) 10

[ISRO - 2011]

05. The arithmetic mean of attendance of 49 students of class A is 40% and that of 53 students of class B is 35%. Then the % of arithmetic mean of attendance of class A and B is

(a) 27.2% (b) 50.25%
(c) 51.13% (d) 37.4%

[ISRO - 2011]

06. Let R be the radius of a circle. What is the angle subtended by an arc of length R at the centre of the circle?

(a) 1 degree (b) 1 radian
(c) 90 degrees (d) π radians
[ISRO - 2014]

07. The conic section this is obtained when a right circular cone is cut through a plane that is parallel to the side of the cone is called _________?
(a) Parabola (b) Hyperbola
(c) Circle (d) Ellipse
[ISRO - 2014]

08. What is the median of the data if its mode is 15 and the mean is 30?
(a) 20 (b) 25
(c) 22.5 (d) 27.5
[ISRO - 2014]

09. Which of the following is true?
(a) $\sqrt{3}+\sqrt{7}=\sqrt{10}$ (b) $\sqrt{3}+\sqrt{7}\le\sqrt{10}$
(c) $\sqrt{3}+\sqrt{7}<\sqrt{10}$ (d) $\sqrt{3}+\sqrt{7}>\sqrt{10}$
[ISRO - 2016]

10. Using Newton-Raphson method a root correct to 3 decimal places of the equation $x^3-3x-5=0$
(a) 2.222 (b) 2.275
(c) 2.279 (d) None of the above
[ISRO - 2017(May)]

11. If vector $\vec{a}=2\hat{i}+\lambda\hat{j}+\hat{k}$ and $\vec{b}=\hat{i}-2\hat{j}+3\hat{k}$ are perpendicular to each other, then value of λ is
(a) 2/5 (b) 2 (c) 3 (d) 5/2
[ISRO - 2017(Dec)]

12. If x+2y = 30, then
$\left(\frac{2y}{5}+\frac{x}{3}\right)+\left(\frac{x}{5}+\frac{2y}{3}\right)$ will be equal to
(a) 8 (b) 16
(c) 18 (d) 20 **[ISRO - 2020]**

KEY & Detailed Solutions

01. (d)	02. (a)	03. (a)	04. (d)
05. (d)	06. (b)	07. (a)	08. (b)
09. (d)	10. (c)	11. (d)	12. (b)

01. Ans: (d)

Sol: $y(x) = x^3-2x^2+1$
Now, putting x = 0 we get y(x) = 0 – 0 + 1 = 1
Similarly x = 1 we get y(x) = 0
x = 2 then y = 1
x = 3 then y = 10

02. Ans: (a)

Sol: x = a cos t
or, $x^2 = a^2\cos^2 t$
or, $\frac{x^2}{a^2}=\cos^2 t$ .... (1)
y = b sin t
or, $y^2 = b^2\sin^2 t$
or, $\frac{y^2}{b^2}=\sin^2 t$ .... (2)
from (1) and (2)
$\frac{x^2}{a^2}+\frac{y^2}{b^2}=\cos^2 t+\sin^2 t=1$

03. Ans: (a)

Sol: • Formula of Newton's Backward Interpolation
$$y_n(x)=y_n+p\nabla y_n+\frac{p(p+1)}{2!}\nabla^2 y_n+\ldots\ldots+\frac{p(p-1)\ldots(p+n-1)}{n!}\nabla^n y_n$$
Here p = k

04. Ans: (d)

Sol: **Piecewise linear interpolation** is a simple way of connecting points through straight lines
So equation Of line joining first 2 points is
y – x = 6
So, at x = 4, y is 10

05. Ans: (d)

Sol: AM of attendance of 49 students of class A = 40%

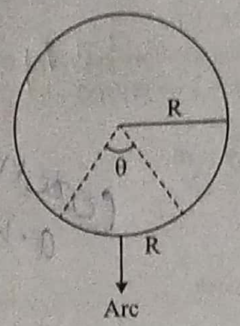
AM of attendance of 53 students of class B = 35%

AM of attendance of class A and B

$$\frac{49\times 40+35\times 53}{(49+53)}=\frac{3815}{102}=37.4019\%$$

06. Ans: (b)

Sol:



$$\text{Length of arc} = 2\pi r\frac{\theta}{360^\circ}$$

$$R = 2\pi R\frac{\theta}{360^\circ}$$

$$\theta = \frac{360^\circ}{2\pi}$$

$$\theta = \frac{180^\circ}{\pi}$$

$\therefore \pi$ Radian = 180°

$$\theta = \frac{\pi \text{ Radion}}{\pi} = 1 \text{ Radian}$$

07. Ans: (a)

Sol: If the cutting plane is exactly the same as the one line that forms the lump, the conic cannot be fixed and is called a parabola.

08. Ans: (b)

Sol: **Mode = 3 × Median − 2 × Mean**

Calculation:

$15 = 3\times \text{Median} - 2\times 30$

$3\times\text{Median} = 15 + 2\times 30$

$\text{Median} = \frac{75}{3}$

$\text{Median} = 25$

09. Ans: (d)

Sol: $(\sqrt{3}+\sqrt{7})^2=(\sqrt{3})^2+2.\sqrt{3}.\sqrt{7}+(\sqrt{7})^2=3+2\sqrt{21}+$

$= 10+2\sqrt{21} > (\sqrt{10})^2$

So $\sqrt{3}+\sqrt{7} > \sqrt{10}$

10. Ans: (c)

Sol: $f(x)=x^3-3x-5$

$f'(x)=3x^2-3$

$x_{n+1}=x_n-[f(x_n)/f'(x_n)]$

Iteration no.	x_n	$f(x_n)$	$f'(x_n)$	x_{n+1}
1	$x_0 = 3$	13	24	2.45833
2	2.45833	2.48165	15.13016	2.29431
3	2.29431	0.19399	12.79158	2.27914
4	2.27914	0.00153	12.58344	2.27902
5	2.27902	0.000015	12.581796	2.27902

11. Ans: (d)

Sol: Vectors are perpendicular i.e. θ is 90, cos 90 = 0

$\Rightarrow \vec{a}.\vec{b}\cos\theta = 0$

$(2\hat{i}+\lambda\hat{j}+\hat{k}).(\hat{i}-2\hat{j}+3\hat{k})$

$\Rightarrow 2-2\lambda+3=0$

$\Rightarrow 5-2\lambda=0$

$\lambda=\frac{5}{2}$

Hence, the value of λ is $\frac{5}{2}$

12. Ans: (b)

Sol: $\left(\frac{2y}{5}+\frac{x}{3}\right)+\left(\frac{x}{5}+\frac{2y}{3}\right)$

Taking LCM we get:

$$\left(\frac{6y+5x+3x+10y}{15}\right)$$

$$\left(\frac{8x+16y}{15}\right)$$

$$\left(\frac{8(x+2y)}{15}\right)=\frac{8\times 30}{15}=16$$