1 Discrete Mathematics

1. Logic

01. Identify the correct translation into logical notation of the following assertion.
Some boys in the class are taller than all the girls
Note: taller (x,y) is true if x is taller than y.
(a) (∃x) (boy(x) → (∀x) (girl(y) ∧ taller (x, y)))
(b) (∃x) (boy(x) ∧ (∀x) (girl(y) ∧ taller (x, y)))
(c) (∃x) (boy(x) → (∀x) (girl(y) → taller (x, y)))
(d) (∃x) (boy(x) ∧ (∀x) (girl(y) → taller (x, y)))
[ISRO - 2007]

02. Which one of the following Boolean expression is NOT a tautology?
(a) ((a → b) ∧ (b → c))→ (a → c)
(b) (a ↔ c) → (~ b → (a∧c))
(c) (a ∧ b ∧ c) → (c ∨ a)
(d) a → (b → a)
[ISRO - 2017(May)]

03. The proposition (P⇒Q)∧ (Q⇒P) is a
(a) tautology (b) contradiction
(c) contingency (d) absurdity
[ISRO - 2017(Dec)]

04. If T(x) denotes x is a trigonometric function, P(x) denotes x is a periodic function and C(x) denotes x is a continuous function then the statement "It is not the case that some trigonometric functions are not periodic" can be logically represented as
(a) ~∃x[T(x) ∧ P(x)] (b) ∃x[T(x) ∨ P(x)]
(c) ∃x[¬T(x) ∧ P(x)] (d) ∃x[T(x) ∧ P(x)]
[ISRO - 2017(Dec)]

y eat x → y is bear

x is fish

that's bear

all y eat x

05. Given that
B(a) means "a is a bear"
F(a) means "a is a fish" and
E(a, b) means "a eats b"
Then what is the best meaning of
∀x [F(x) →∀y(E(y,x)→b(y))]
(a) Every fish is eaten by some bear
(b) Bears eat only fish
(c) Every bear eats fish
(d) Only bears eat fish
[ISRO - 2020]

KEY & Detailed Solutions

01. (d)	02. (b)	03. (c)	04. (a)	05. (d)

01. **Ans: (d)**

Sol: **(∃x) (boy(x) → (∀y) (girl(y) ∧ taller(x,y))) is** equivalent to
~((∃x)(boy(x))∨ ((∀y) (girl(y) ∧ taller(x,y)))
The above expression returns true, If there does not exist a boy in the class or All the students are girls (i.e No boy in the class) and **taller(girl,boy)**
But **No boy in the class** implies there is no chance for having a boy who is taller than all the girls, which contradicts the given statement.
(∃x) (boy(x) ∧ (∀y) (girl(y) ∧ taller(x,y)))
The above expression returns true,If there exists a boy in the class and All the students are girls (i.e No boy in the class) and **taller(girl,boy)**
But **All the students are girls & there exists a boy in the class** cannot be TRUE simultaneously, The statement is inconsistent.
(∃x) (boy(x) → (∀y) (girl(y) → taller(x,y))) is equivalent to
~ ((∃x) (boy(x)) ∨ (∀y) (girl(y) → taller(x,y)))
The above expression returns true, If there does not exist a boy in the class or if there exists a boy taller than all the girls.
But **there does not exist a boy** implies there is no chance for having a boy who is taller than all the girls, which contradicts the given statement.
(∃x) (boy(x) ∧ (∀y) (girl(y) → taller(x,y))) means There is a boy and he is taller than all the girls.

02. Ans: (b)

Sol: $((a\to b)\wedge(b\to c))\to(a\to c)$

$\equiv((\sim a\vee b)\wedge(\sim b\vee c))\to(\sim a\vee c)$

$\equiv\sim((\sim a\vee b)\wedge(\sim b\vee c))\vee(\sim a\vee c)$

$\equiv((a\wedge\sim b)\vee(b\vee\sim c))\vee(\sim a\vee c)$

$\equiv(\sim a\vee(a\wedge\sim b))\vee((b\wedge\sim c)\vee c)$

$\equiv((\sim a\vee a)\wedge(\sim a\vee\sim b))\vee((b\vee c)\wedge(\sim c\vee c))$

$\equiv(T\wedge(\sim a\vee\sim b))\vee((b\vee c)\wedge T)$

$\equiv\sim a\vee(\sim b\vee b)\vee c$

$\equiv\sim a\vee T\vee c$

$\equiv T$

Hence, $((a\to b)\wedge(b\to c))\to(a\to c)$ is tautology.

$\mathbf{(a\leftrightarrow c)\to(\sim b\to(a\wedge c))}$

$\equiv((a\to c)\wedge(c\to a))\to((\sim b\to(a\wedge c))$

$\equiv\sim((\sim a\vee c)\wedge(\sim c\vee a))\vee((b\vee(a\wedge c))$

$\equiv\sim((a\wedge c)(\sim a\wedge\sim c))\vee((b\vee(a\wedge c))$

$\equiv((a\wedge\sim c)\vee(\sim a\wedge c))\vee((b\vee(a\wedge c))$

$\equiv((a\wedge\sim c)\vee c(\sim a\wedge a))\vee b$

$\equiv((a\wedge\sim c)\vee c\vee b$

$\equiv a\vee b\vee c$

Hence, $(a\leftrightarrow c)\to(\sim b\to(a\wedge c))$ is not a tautology. (can be a contingency)

$\mathbf{(a\wedge b\wedge c)\to(c\vee a)}$

$\equiv\sim(a\wedge b\wedge c)\vee(c\vee a)$

$\equiv\sim a\sim b\sim c\vee c\vee a$

$\equiv(a\vee\sim a)\vee\sim b\vee(\sim c\vee c)$

$\equiv T\vee\sim b\vee T$

$\equiv T$

Hence, $(a\wedge b\wedge c)\to(c\vee a)$ is a tautology.

$\mathbf{a\to(b\to a)}$

$\equiv\sim a\vee(\sim b\vee a)$

$\equiv(\sim a\vee a)\vee\sim b$

$\equiv T\vee\sim b$

$\equiv T$

Hence, $a\to(b\to a)$ is a tautology.

03. Ans: (c)

The proposition $(P\Rightarrow Q)\wedge(Q\Rightarrow P)$ is a contingency.

P	Q	$P\Rightarrow Q$	$Q\Rightarrow P$	$(P\Rightarrow Q)\wedge(Q\Rightarrow P)$
T	T	T	T	T
T	F	F	T	F
F	T	T	F	F
F	F	T	T	T

04. Ans: (a)

Sol: $\sim\exists x[T(x)\wedge\sim P(x)]$

It is not the case that some trigonometric functions are not periodic

$\sim\exists x[T(x)\vee\sim P(x)]$

It is not the case that some functions are trigonometric functions or non-periodic

$\sim\exists x[\neg T(x)\wedge\sim P(x)]$

It is not the case that some non-trigonometric functions are not periodic

$\sim\exists x[T(x)\wedge P(x)]$

It is not the case that some trigonometric functions are periodic

05. Ans: (d)

Sol:

Statement	Expression
Every fish is eaten by some bear	$\forall x(F(x)\to\exists(B(y)\wedge E(y,x))$
Bears eat only fish	$\forall x(B(x)\to\forall y(E(x,y)\to F(y))$
Every bear eats fish	$\forall x(B(x)\to\exists y(F(y)\wedge E(x,y))$
Only bears eat fish	$\forall x(F(x)\to\forall y(E(y,x)\to B(y))]$

$\forall x(F(x)\to\forall y(E(y,x)\to B(y))]$ states that If every fish(x) is eaten by some y, that eater y should be a bear. In other words, Only bears eat fish.

2. Combinatorics

01. The shift operator E is defined as
$E[f(x_i)] = f(x_i+h)$ and $E^{-1}[f(x_i)] = f(x_i - h)$
Then Δ (forward difference) in terms of E is
(a) E − 1 (b) E
(c) $1 - E^{-1}$ (d) 1 − E
[ISRO - 2009]

02. How many diagonals can be drawn by joining the angular points of an octagon?
(a) 14 (b) 20 (c) 21 (d) 28
[ISRO - 2013]

03. What is the sum to infinity of the series, $3+6x^2+9x^4+12x^6+.....$ given $|x| < 1$?
(a) $\frac{3}{(1+x^2)}$ (b) $\frac{3}{(1+x^2)^2}$
(c) $\frac{3}{(1-x^2)^2}$ (d) $\frac{3}{(1-x^2)}$
[ISRO - 2016]

KEY & Detailed Solutions		
01. (a)	02. (b)	03. (c)

01. Ans: (a)
Sol: Forward difference operator is Δ
$\Delta(f(x)) = f(x+h) - f(x)$
$f(x+h) = f(x) + \Delta(f(x))$
Shift operator
$E(f(x)) = f(x+h)$
Hence,
$E(f(x)) = f(x) + \Delta(f(x))$
so $E = 1 + \Delta$
gives $\Delta = E - 1$

02. Ans: (b)
Sol: If we have a polygon having n vertices, then each vertex can give a total of (n−3) diagonals. (By joining a vertex to every other vertices except its 2 neighbours).
So total number of diagonal $= n*(n-3)/2$
(Dividing by 2, because we are counting each diagonal twice, for example the diagonal connecting 3[rd] vertex and 5[th] vertex is counted with respect to both the vertices).
Here n = 8 for octagon,
No. of diagonals $= (8-3)*8/2 = 20$

03. Ans: (c)
Sol: $3+6x^2+9x^4+12x^6+....$
$= 3(1+2x^2+3x^4+4x^6+......)$
Let us substitute $x^2 = y$
$= 3(1+2y+3y^2+4y^3+......)$
$= 3[1/(1-y)^2]$
Replace y with x^2
$= 3[1/(1-x^2)^2]$

3. Graph Theory

01. A graph with "n" vertices and n-1 edges that is not a tree, is
(a) Connected (b) Disconnected
(c) Euler (d) A circuit
[ISRO - 2007]

02. If a graph requires k different colour for its proper colouring, then the chromatic number of the graph is
(a) 1 (b) k (c) k–1 (d) k/2
[ISRO - 2007]

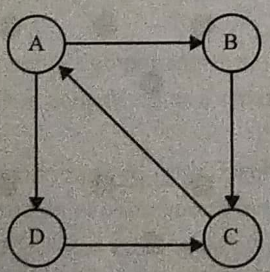
03. Let X be the adjacency matrix of a graph G with no self loops. The entries along the principal diagonal of X are
(a) all zeros (b) all ones
(c) both zeros and ones (d) different
[ISRO - 2007]

04. The number of distinct simple graphs with up to three nodes is
(a) 15 (b) 10 (c) 7 (d) 9
[ISRO - 2008]

05. Maximum number of edges in a n-node undirected graph without self loops is
(a) n^2 (b) n(n–1) /2
(c) n – 1 (d) n(n+1) /2 **[ISRO - 2008]**

$\binom{n}{2}$ Max

06. Consider the graph shown in the figure below.



Min $\binom{n-1}{2}$

(Path exists to ele to ele)

Which of the following is a valid strong component?
(a) A, C, D (b) A, B, D
(c) B, C, D (d) A, B, C
[ISRO - 2008]

07. If G is a graph with e edges and n vertices the sum of the degrees of all vertices in G is
(a) e (b) e/2
(c) e^2 (d) 2 e **[ISRO - 2009]**

08. Let G be an arbitrary graph with n nodes and k components. If a vertex is removed from G, the number of components in the resultant graph must necessarily lie between.
(a) k and n (b) k–1 and k+1
(c) k–1 and n–1 (d) k+1 and n–k
[ISRO - 2009]

09. A graph in which all nodes are of equal degree, is known as
(a) Multigraph (b) Non regular graph
(c) Regular graph (d) Complete graph
[ISRO - 2009]

10. If in a graph G there is one and only one path between every pair of vertices then G is a
(a) Path (b) Walk
(c) Tree (d) Circuit
[ISRO - 2009]

11. A simple graph (a graph without parallel edge or loops) with n vertices and k components can have at most
(a) n edges
(b) (n–k) edges
(c) (n–k) (n–k+1) edges
(d) (n–k) (n–k+1)/2 edges. **[ISRO - 2009]**

12. How many edges are there in a forest with v vertices and k components?
(a) (v+1) – k (b) (v+1)/2 – k
(c) v – k (d) v + k
[ISRO - 2011]

13. The number of edges in a 'n' vertex complete graph is

(a) $n * (n-1)/2$ (b) n^2
(c) $n * (n+1)/2$ (d) $n * (n+1)$

[ISRO - 2013]

14. A cube of side 1 unit is placed in such a way that the origin coincides with one of its top vertices and the three axes run along three of its edges. What are the co-ordinates of the vertex which is diagonally opposite to the vertex whose coordinates are (1, 0, 1) ?

(a) (0, 0, 0) (b) (0, −1, 0)
(c) (0, 1, 0) (d) (1, 1, 1)

[ISRO - 2014]

15. A given connected graph G is a Euler Graph if and only if all vertices of G are of

(a) Same degree (b) Even degree
(c) Odd degree (d) Different degree

[ISRO - 2016]

16. Maximum number of edges in a n-node undirected graph without self-loops is

(a) n^2 (b) $\frac{n(n-1)}{2}$
(c) $n-1$ (d) $\frac{n(n+1)}{2}$

[ISRO - 2016]

17. The number of edges in a regular graph of degree d and n vertices is

(a) maximum of n and d (b) n + d
(c) nd (d) nd/2

[ISRO - 2018]

KEY & Detailed Solutions

01. (b)	02. (b)	03. (a)	04. (c)	05. (b)
06. (d)	07. (d)	08. (c)	09. (c)	10. (c)
11. (d)	12. (c)	13. (a)	14. (b)	15. (b)
16. (b)	17. (d)			

01. Ans: (b)

Sol: A graph with "n" vertices and n-1 edges that is not a tree, is disconnected and contains a cycle as well. Because, If the given graph is a tree, neither disconnectedness nor cycles are allowed.
With the help of n-1 edges, we can create a tree of n nodes. If we don't want the resultant to be a tree, we have to isolate one node at least, which leads to disconnectivity.
For that, we need to create a cycle in the remaining component.

02. Ans: (b)

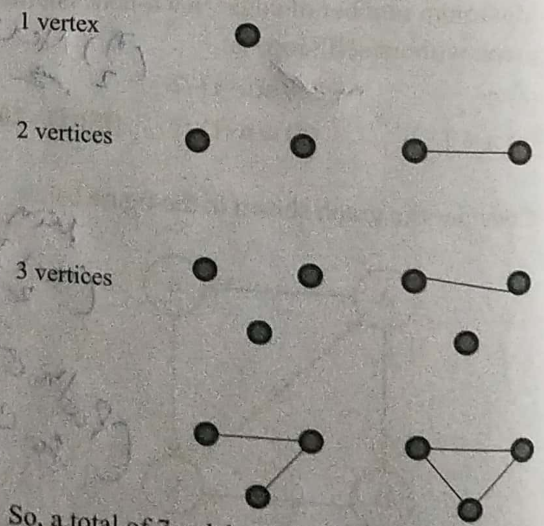
Sol: The chromatic number of a graph is the smallest number of colors needed to color the vertices of it, so that no two adjacent vertices share the same color and if a graph requires k different colors for its proper coloring, then k is the chromatic number of the graph.

03. Ans: (a)

Sol: In an adjacency matrix of a graph G the diagonal entries show the connectivity with themselves.
Since the Graph G has no self loops, all the entries should be 0.

04. Ans: (c)

Sol:



So, a total of 7 unlabelled graphs are possible.

05. Ans: (b)

Sol: If there are n nodes, we can pick any of the two nodes to draw a line So, Maximum number of edges in a n-node undirected graph without self loops is equal to number of different pair of nodes that can be chosen from n nodes

$= {}^nC_2 = (n*(n-1)/2)$

06. Ans: (d)

Sol: (a) A, C, D is not a valid strong component, there is no path between A,C

(b) A, B, D is not a valid strong component, there is no path between B,D

(c) B, C, D is not a valid strong component, there is no path between B,D

(d) A, B, C is a valid strong component, there is a path between every pair of vertices.

07. Ans: (d)

Sol: Since the given graph is undirected, every edge contributes 2 to the sum of degrees.

So the sum of degrees is 2E.

08. Ans: (c)

Sol: If the number of components is k,

If a vertex is removed from Graph, the lower bound on the number of components in the resultant graph is **k–1**.

It happens when one of the components is an isolated vertex (i.e. one vertex which is equivalent to one component is removed).

If the number of nodes is n

The upper bound of the number of components(k) is n.

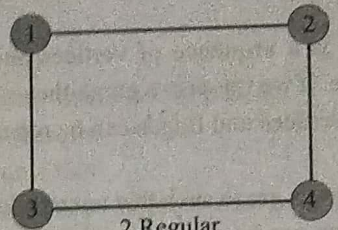
The number of components is n, when each node in the graph is considered as a separate component.

$k \le n$

If a vertex is removed from Graph, the upper bound on the number of components in the resultant graph is **n–1**.(i.e. one vertex which is equivalent to one component is removed).

09. Ans: (c)

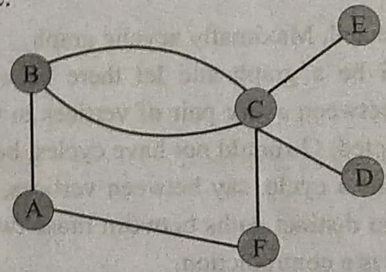
Sol: A graph is called a **regular graph**, if the degree of each vertex is equal. A graph is called K-regular if the degree of each vertex in the graph is K.



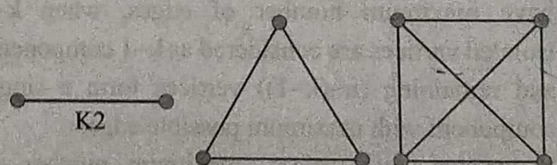
2 Regular

A graph is called a **non-regular graph** if the degrees of vertices are not equal.

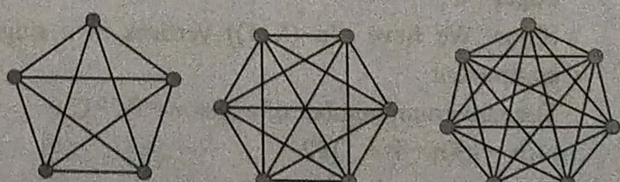
In mathematics and more specifically in graph theory, a **multigraph** is a graph which is permitted to have multiple edges (also called parallel edges), that is, edges that have the same end nodes. Thus two vertices may be connected by more than one edge.



A simple graph of n vertices having exactly one edge between each pair of vertices is called a complete graph. A complete graph of n vertices is denoted by K_n. Total number of edges are $n*(n-1)/2$ with n vertices in the complete graph.



K2 K3 K4



K5 K6 K7

Every K_n is (n–1) regular graph but every (n–1) regular graph may or may not be a K_n

10. Ans: (c)

Sol: Walk

A walk is a sequence of vertices and edges of a graph i.e. if we traverse a graph then we get a walk.

Note: Vertices and Edges can be repeated.

Circuit

Traversing a graph such that not an edge is repeated but vertex can be repeated and it is closed also i.e. it is a closed trail.

Vertex can be repeated.

Edge can not be repeated.

Path

It is a trail in which neither vertices nor edges are repeated, Vertex not repeated

Edge not repeated.

Tree

Connected, Maximally acyclic graph

Let G be a graph and let there be exactly one path between every pair of vertices in G. So G is connected. G should not have cycles, because if G contains a cycle, say between vertices, then there are two distinct paths between those two vertices, which is a contradiction.

Thus G is connected and is without cycles, therefore it is a tree.

11. Ans: (d)

Sol: The graph with n nodes and k components can have maximum number of edges, when k–1 isolated vertices are considered as k–1 components and remaining (n–(k–1)) vertices form a single component with maximum possible edges.

If we have v vertices, maximum number of edges$={}^{v}C_2$

Since, We have (n–(k–1)) vertices in a single component

The maximum number of edges $= {}^{(n-(k-1))}C_2$

$=((n-k)(n-k+1))/2$

12. Ans: (c)

Sol: A forest is a disjoint Union of Trees.

If the forest has V vertices, K components

If the number of vertices in each component is V_i

If the number of edges in the component $= V_i - 1$

The number of edges in a tree with "n" vertices

because, number of edges in a tree with "n" vertices is "n–1".

The Sum of edges of all the components

$$=\sum_{i=1}^{k}(V_i-1)$$

$$=\sum_{i=1}^{k}V_i-\sum_{i=1}^{k}1$$

Sum of vertices of all components = V

So, $\sum_{i=1}^{k}V_i = V$

We know $\sum_{i=1}^{k}1 = k$

Hence

Number of edges

$$=\sum_{i=1}^{k}V_i-\sum_{i=1}^{k}1$$

$= V–K$

13. Ans: (a)

Sol: Complete graph is a simple undirected graph with the maximum number of edges possible.

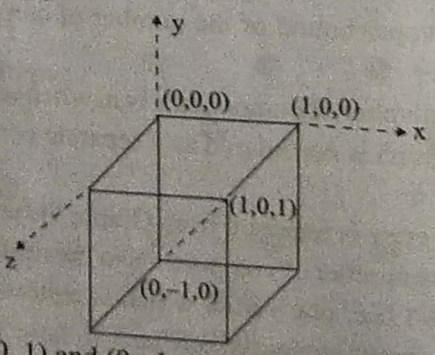
If there are n nodes, we can pick any of the two nodes to draw a line

So, Maximum number of edges in a n-node undirected graph without self loops is equal number of different pair of nodes that can be chosen from n nodes

$= {}^{n}C_2 = (n*(n-1))/2$

14. Ans: (b)

Sol:



(1, 0, 1) and (0,–1, 0) are diagonally opposite.

15. Ans: (b)

Sol: A graph containing the Eulerian cycle in it is called a Euler graph.

A given connected graph G is a Euler Graph if and only if all vertices of G are of even degree.

Eulerian Cycle

An undirected graph has an Eulerian cycle, if the following two conditions are met.

1. All vertices with non-zero degree are connected. We don't care about vertices with zero degree because they don't belong to the Eulerian Cycle or Path.
2. All vertices have even degree.

Eulerian Path

An undirected graph has Eulerian Path,if the following two conditions are met.

1. All vertices with non-zero degree are connected. We don't care about vertices with zero degree because they don't belong to the Eulerian Cycle or Path
2. If zero or two vertices have odd degrees and all other vertices have even degree.

   Note that only one vertex with odd degree is not possible in an undirected graph (sum of all degrees is always even in an undirected graph)

   A graph with no edges is considered Eulerian because there are no edges to traverse.

   In the Eulerian path, each time we visit a vertex v, we walk through two unvisited edges with one endpoint as v.

   Therefore, all middle vertices in the Eulerian Path must have even degree. For the Eulerian Cycle, any vertex can be a middle vertex, therefore all vertices must have even degree.

16. Ans: (b)

Sol: Complete graph is a simple undirected graph with the maximum number of edges possible.

If there are n nodes, we can pick any of the two nodes to draw a line

So, Maximum number of edges in a n-node undirected graph without self loops is equal to number of different pair of nodes that can be chosen from n nodes

$= {}^{n}C_{2} = (n*(n-1))/2$

17. Ans: (d)

Sol: As every vertex has degree d,

sum of degrees = n*d.

we know that

2* (number of edges) = sum of degrees

So,

2* number of edges = nd

number of edges = (nd/2)

4. Set Theory

01. The set of all Equivalence Classes of a set A of Cardinality C
(a) is of cardinality 2^c
(b) has the same cardinality as A
(c) forms a partition of A
(d) is of cardinality C^2
[ISRO - 2007]

02. Which one of the following is 'true'
(a) $R \cap S = (R \cup S) - [(R - S \cup (S - R)]$
(b) $R \cup S = (R \cap S) - [(R - S \cup (S - R)]$
(c) $R \cap S = (R \cup S) - [(R - S \cap (S - R)]$
(d) $R \cap S = (R \cup S) \cup (R - S)$
[ISRO - 2011]

03. The number of elements in the power set of the set {{A,B},C} is
(a) 7 (b) 8 (c) 3 (d) 4
[ISRO - 2013]

04. The number of bit strings of length 8 that will either start with 1 or end with 00 is _____?
(a) 32 (b) 128 (c) 160 (d) 192
[ISRO - 2014]

96

05. Let A be a finite set having x elements and let B be a finite set having y elements. What is the number of distinct functions mapping B into A.
(a) x^y (b) $2^{(x+y)}$
(c) y^x (d) $y! / (y-x)!$
[ISRO - 2014]

06. If (G,.) is a group such that $(ab)^{-1} = a^{-1} b^{-1}$, $\forall$ a,b $\in$ G then G is a/an
(a) Commutative semi group
(b) Abelian group
(c) Non-abelian group
(d) None of these
[ISRO - 2016]

07. The symmetric difference of sets A = {1, 2, 3, 4, 5, 6, 7, 8} and B = {1, 3, 5, 6, 7, 8, 9} is
(a) {1, 3, 5, 6, 7, 8}
(b) {2, 4, 9}
(c) {2, 4}
(d) {1, 2, 3, 4, 5, 6, 7, 8, 9}
[ISRO - 2017(May)]

A-B∪B-A
(A∪B)-(A∩B)

08. Suppose A is a finite set with n elements. The number of elements and the rank of the largest equivalence relation on A are
(a) {n, 1} (b) {n, n}
(c) $\{n^2, 1\}$ (d) $\{1, n^2\}$
[ISRO - 2017(Dec)]

09. Consider the set of integers I. Let D denote "divide with an integer quotient" (e.g. 4D8 but 4D7). Then D is
(a) Reflexive, not symmetric, transitive
(b) Not reflexive, not antisymmetric, transitive
(c) Reflexive, antisymmetric, transitive
(d) Not reflexive, not antisymmetric, not transitive
[ISRO - 2017(Dec)]

10. The number of elements in the power set of {{1,2, {2,1,1}, {2,1,1,2}} is
(a) 3 (b) 8 (c) 4 (d) 2
[ISRO - 2017(Dec)]

11. The function f: [0, 3] → [1, 29] defined by $f(x) = 2x^3 - 15x^2 + 36x + 1$ is
(a) injective and surjective
(b) surjective but not injective
(c) injective but not surjective
(d) neither injective nor surjective
[ISRO - 2017(Dec)]

12. (G, *) is an abelian group. Then
(a) $x = x^{-1}$, for any x belonging to G
(b) $x = x^2$, for any x belonging to G
(c) $(x * y)^2 = x^2 * y^2$, for any x, y belonging to G
(d) G is of finite order
[ISRO - 2018]

$(xs+yz)\cap x$

13. If A = {x, y, z} and B = {u, v, w, x}, and the universe is {s, t, u, v, w, x, y, z}. Then $(A\cup B)\cap(A\cap B)$ is equal to

{x}

(a) {u, v, w, x}
(b) { }
(c) {u, v, w, x, y, z}
(d) {u, v, w}

[ISRO - 2020]

KEY & Detailed Solutions				
01. (c)	02. (a)	03. (d)	04. (c)	05. (a)
06. (b)	07. (b)	08. (c)	09. (b)	10. (d)
11. (b) & (c)		12. (c)	13. (*)	

01. Ans: (c)

Sol: Partition: Partition of a set is a collection of n disjoint subsets, say P1, P2, ... Pn that satisfies the following three conditions

Pi does not contain the empty set.

$[P_i \neq \{\phi\}$ for all $0 < i \leq n]$

The union of the subsets must be equal to the entire original set.

$[P1 \cup P2 \cup ... \cup Pn = S]$

The intersection of any two distinct sets is empty.

$[Pa \cap Pb = \{\phi\}$, for $a \neq b$ where $n \geq a, b \geq 0]$

Let S = {a, b, c, d, e, f, g, h}

One probable partitioning is {a}, {b, c, d}, {e, f, g, h}

Another probable partitioning is {a, b}, {c, d}, {e, f, g, h}

Let S = { 1, 2, 3}, n = |S| = 3

The partitions are

{{1}, {2, 3}}

{{1,2}, {3}}

{{1, 3}, {2}}

{{1}, {2}, {3}}

Equivalence Class: Every element x of set X is a member of the equivalence class [x]

Every Two equivalence classes [x] and [y] are either equal or disjoint.

If x~y is a equivalence relation between elements x, y then [x] = [y] and vice versa.

$x \sim y \Leftrightarrow [x] = [y] \Leftrightarrow [x] \cap [y] = \phi$

By the definitions of equivalence class & partition we can conclude that,

"Set of all equivalence classes of x form a partition of X" (or)

"Every element of x belongs to one and only equivalence class.

(or)

Every partition of X comes from an equivalence relation in this way, according to which x ~ y. If and only If x & y belongs to the same set of the partition.

Example:

Set A = { 1, 2, 3}

Let us consider a Equivalence Relation R on Set A = {(1,1), (2,2), (3,3), (1,2), (2,1)}

The above relation is equivalent, because it is reflexive, symmetric, transistive.

1 is related to 2

Hence

{1,2} is a equivalence class

{3} is a equivalence class

{{1,2}, {3}} is partition of A

02. Ans: (a)

Sol: Let us take

R = {1,2}

S = {2,3}

Then

$R\cap S = \{2\}$

$(R\cup S) = \{1,2,3\}$

$(R-S) = \{1\}$

$(S-R) = \{3\}$

$(R\cup S)-[(R-S)\cup(S-R)]$

$= \{1, 2, 3\} - [\{1\} \cup \{3\}]$

$= \{2\} = R\cap S$

$\mathbf{(R\cup S)-[(R-S)\cup(S-R)]=R\cap S}$

$(R\cap S)-[(R-S)\cup(S-R)]$

$= \{2\} - [\{1\} \cup \{3\}]$

$= \{2\} \neq R\cup S$

$(R\cap S)-[(R-S)\cup(S-R)]$ **is not equal to** $(R\cup S)$

$(R\cup S)-[(R-S)\cap(S-R)]$

$=\{1,2,3\}-[\{1\}\cap\{3\}]$

$=\{1,2,3\}\neq R\cap S$

$(R\cup S)-[(R-S)\cup(S-R)]$ **is not equal to** $R\cap S$

$(R\cup S)\cup(R-S)$

$=\{1,2,3\}\cup\{1\}$

$=\{1,2,3\}\neq R\cap S$

$(R\cup S)\cup(R-S)$ **is not equal to** $R\cap S$

Hence. Option(a) $R\cap S=(R\cup S)-[(R-S)\cup(S-R)]$ is the correct choice.

03. Ans: (d)

Sol: A set containing n elements has 2^n elements in its power set.

Given set $\{\{A, B\},C\}$ has 2 elements.

element1 $=\{A, B\}$

element2 $= C$

Hence, a set containing 2 elements has $2^2 = 4$ elements in its power set.

04. Ans: (c)

Sol: The number of bit strings of length 8 that will either start with 1 or end with 00 is

R = The number of bit strings of length 8 that start with 1

Out of 8 positions, first position can be filled in one way, remaining 7 positions can be filled in 2 ways (either or 1)

$|R| = 2^7 = 128$

S = The number of bit strings of length 8 that ends with 00

Out of 8 positions, last two positions can be filled in one way with 00, remaining 6 positions can be filled in 2 ways(either or 1)

$|S| = 2^6 = 64$

$R\cap S$ = The number of bit strings of length 8 that start with 1 and end with 00

Out of 8 positions,first position can be filled in one way, last two positions can be filled in one way with 00, remaining 5 positions can be filled in 2 ways (either or 1)

$|R\cap S| = 2^5 = 32$

The number of bit strings of length 8 that will either start with 1 or end with 00 is

$|R\cup S| = |R| + |S| - |R\cap S|$

$= 128 + 64 - 32$

$= 160$

05. Ans: (a)

Sol: Set A has x elements and set B has y elements.

Each element in B has x choices to be mapped to and being a function it must map to some element

Since each element has exactly x choices.

The total number of functions from B to A

$= x \times x \times x \times \ldots \times x$ (y times) $= x^y$.

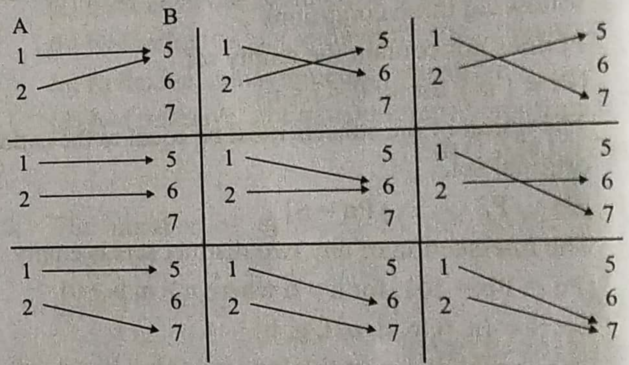
Let us take

$A=\{1,2\}\rightarrow |A| = 2$

$B=\{5,6,7\}\rightarrow |B| = 3$

The number of functions from A to B

is $|B|^{|A|} = 3^2 = 9$



06. Ans: (b)

Sol: We know that, If a, b are two elements of group G

$(ab)^{-1} = (b^{-1}a^{-1})$

But, Given $(ab)^{-1} = a^{-1}b^{-1}$

Hence,we can conclude

$a^{-1}b^{-1} = b^{-1}a^{-1}$

Take the inverse again

$(a^{-1}b^{-1})^{-1} = (b^{-1}a^{-1})^{-1}$

$((b^{-1})^{-1}(a^{-1})^{-1}) = ((a^{-1})^{-1}(b^{-1})^{-1})$

$ba = ab$

In a group (G, *) is said to be abelian if

$(a*b) = (b*a)\ \forall a, b \in G$

Hence, Option(b) Abelian Group is the correct choice.

07. Ans: (b)

Sol: $A = \{1, 2, 3, 4, 5, 6, 7, 8\}$

$B = \{1, 3, 5, 6, 7, 8, 9\}$

$A - B = \{2, 4\}$

$B - A = \{9\}$

Symmetric difference $= A \Delta B$

$= (A - B) \cup (B - A)$

$= \{2, 4\} \cup \{9\}$

$= \{2, 4, 9\}$

08. Ans: (c)

Sol: Let us consider Set $A = \{1, 2, 3\}$

$|A| = 3$

The largest equivalence relation is when each element is related to every other element.

Hence $n \times n = 3^2$ ordered pairs are possible.

$\{(1,1),(2,2),(3,3),(1,2),(1,3),(2,3),(3,2),(2,1),(3,1)\}$ is the largest equivalence relation.

Matrix of largest equivalence relation

$$E = \begin{array}{c} \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{c} \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \end{array}$$

$R_2 \leftarrow R_2 - R_1$

$R_3 \leftarrow R_3 - R_1$

$$\begin{array}{c} \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{c} \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \end{array}$$

Rank (E) = 1 (one non-zero row)

So, the number of elements $= n^2$

Rank = 1s

09. Ans: (b)

Sol: Given relation is not reflexive

For reflexivity, for all $x \in I$, $R(x, x)$.

0 cannot divide 0.

So, this is violated for $x = 0$. So, the given relation is not reflexive.

Given relation is not symmetric

1 is a divisor of 2

But 2 is not divisor of 1

For symmetry, for all $x, y \in I$, $R(x, y) \Rightarrow R(y, x)$. This is violated for $x = 2$, $y = 1$.

Given relation is not anti symmetric

For anti-symmetry, for all $x, y \in I$, $(R(x, y) \wedge R(y, x)) \Rightarrow x = y$.

1 is a divisor of -1

-1 is a divisor of 1

and 1 and -1 are not equal.

We have $R(-1, 1)$ and $R(1, -1)$, so R is not anti-symmetric.

Given relation is transitive.

For transitivity, for all $x, y, z \in I$, $R(x, y) \wedge R(y, z) \Rightarrow R(x, z)$.

As per the rule of division this holds.

$R(x, y) \rightarrow y = k_1 x$ (for some integer k_1)

$R(y, z) \rightarrow z = k_2 y$ (for some integer k_2)

Hence, we can write $z = k_2 k_1 x \rightarrow z = kx$ (substitute $k_2 k_1 = k$)

Hence, $R(x, z)$ holds.

10. Ans: (d)

Sol: Set does not allow repetition of elements. (where as multi set/bag allows it)

$\{1, 1\} = \{1\}$

Set is not bothered about the order of the elements.

$\{1, 2\} = \{2, 1\}$

Given Set

$A = \{\{1,2\}, \{2,1,1\}, \{2,1,1,2\}\}$

But all the 3 elements are the same.

$\{1,2\} = \{2,1,1\} = \{2,1,1,2\}$

Hence

$A = \{\{1,2\}\}$

$|A| = 1$

A set containing n elements has 2^n elements in its power set.

Hence, a set containing 1 element has $2^1 = 2$ elements in its power set.

11. Ans: Insufficient data (b) or (c)

Sol: If f is continuous

Answer is (b)

If the function is continuous, the derivative is ≥ 0, but never 0 on an interval of the domain, we conclude the function is monotonically increasing. Notice monotonically increasing $\Rightarrow$ monotonically non-decreasing. Monotonically non-decreasing is a weaker condition and monotonically increasing is a stronger one.

If the derivative is ≤ 0 but never 0 on an interval: That is called monotonically decreasing. And monotonically decreasing $\Rightarrow$ monotonically non-increasing.

If the function is decreasing on some interval but increasing on another,it must be going over the same points at least twice. (Which means function is not one-one).

An one-one function should be either monotonically increasing or monotonically decreasing, but not both.

$f(x) = 2x^3 - 15x^2 + 36x + 1$

$f'(x) = 6x^2 - 30x + 36 = -6(x^2 - 5x + 6)$

$= 6(x-2)(x-3)$

f(x) is increasing in [0, 2) and decreasing in (2, 3].

Hence, f (x) is not one-one(injective).

in the given domain [0, 3], f is increasing in [0, 2) and decreasing in (2, 3]

Considering the domain [0, 2)

$f(0) = 1$

$f(2) = 29$

Since f is increasing in this domain,

$1 <= f(x) <= 29$

Range includes [1 29]

Considering the domain [2, 3)

$f(2) = 29$

$f(3) = 28$

Since f is decreasing in this domain,

$29 >= f(x) >= 28$ In other words,

$28 <= f(x) <= 29$

Range includes [28 29]

Range of f is $[1\ 29] \cup [28\ 29] = [1\ 29]$

Given Codomain is [1 29]

so,Co-domain=range,Hence **f is onto(surjective).**

If f is discrete & Integer domain is considered

Answer is (c).

Domain = {0, 1, 2, 3}

Co-domain = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29}

$f(0) = 1$

$f(1) = 24$

$f(2) = 29$

$f(3) = 28$

Range = {1, 24, 28, 29}

No two elements of the domain are mapped to the same element, Hence **One-one(injective).**

Co-domain is not equal to the Range, Hence not onto **(not surjective)**

12. Ans: (c)

Sol: If G is an abelian group, **$x = x^{-1}$, for any x belonging to G** need to be true.

(Z, +) is an infinite abelian group but the following are not true for it:

Identity element (e) = 0

1 is element in Z, but

Inverse(1)= −1 (because 1+(−1) =0)

But If **$x = x^{-1}$, for any x belonging to G** holds G has to be an abelian group.

$x = x^2$, for any x belonging to G is just another way of expressing **$x=x^{-1}$, for any x belonging to G**

So the same explanation holds here.

$(x*y)^2 = x^{2}*y^2$, for any x, y belonging to G

$(x*y)^2 = x^{2}*y^2$

$xyxy = xxyy$

Multiply with x^{-1} on both sides

$yxy = xyy$

Multiply with y^{-1} on both sides

$yx = xy$

In a group (G , *) is said to be abelian if $(x*y) = (y*x)\ \forall x,y \in G$, Hence C is the answer.

G is of finite order

A group having finite number of elements may or may not be an abelian group.

Consider finite set of Matrices under Multiplication (M, *)

Even though the Group is finite, It is not abelian.

13. Ans: None of the options

Sol: Given

$A = \{x, y, z\}$

$B = \{u, v, w, x\}$

$\mu = \{s, t, u, v, w, x\ y, z\}$

$\overline{B} = \mu - A = \{s, t, y, z\}$

$A \cup \overline{B} = \{s, t, x, y, z\}$

$A \cap B = \{x\}$

$(A \cup \overline{B}) \cap (A \cap B) = \{x\}$