Engineering Mathematics PYQ's

Q) Consider the system of linear equations

$x + 2y + z = 5$

$2x + ay + 4z = 12$

$2x + 4y + 6z = b$

$$\left[\begin{array}{ccc|c} 1 & 2 & 1 & 5 \\ 2 & a & 4 & 12 \\ 2 & 4 & 6 & b \end{array}\right]$$

$a = 4$

$\longrightarrow AX = 0$

The values of a and b such that there exists a non-trivial null space and the system admits infinite solutions are

$R_2 \to R_2 - 2R_1,\ R_3 \to R_3 - 2R_1$

$$\left[\begin{array}{ccc|c} 1 & 2 & 1 & 5 \\ 0 & a-4 & 2 & 2 \\ 0 & 0 & 4 & b-10 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & 2 & 1 & 5 \\ 0 & 0 & 2 & 2 \\ 0 & 0 & 4 & b-10 \end{array}\right]$$

$R_2 \longleftrightarrow R_3$

$$A \quad \begin{bmatrix} 1 & 2 & 1 \\ 0 & 0 & 4 \\ 0 & a-4 & 2 \end{bmatrix} \quad \begin{bmatrix} 1 & 2 & 1 \\ 0 & 0 & 4 \\ 0 & 0 & 2 \end{bmatrix}$$

$R_3 \to R_3 - \frac{1}{2}R_2$

$$\begin{bmatrix} 1 & 2 & 1 \\ 0 & 0 & 4 \\ 0 & 0 & 0 \end{bmatrix}$$

$R_3 \to R_3 - 2R_2$

$$\left[\begin{array}{ccc|c} 1 & 2 & 1 & 5 \\ 0 & 0 & 2 & 2 \\ 0 & 0 & 0 & b-14 \end{array}\right]$$

$b - 14 = 0$

$b = 14$

A) a = 8, b=14
B) a=4, b=12
C) a=8, b=12
D) a=4, b=14

Q) If $A = \begin{bmatrix} 10 & 2k+5 \\ 3k-3 & k+5 \end{bmatrix}$

is a symmetric matrix, the value of k is ________.

A) 8

B) 5

C) -0.4

D) $\frac{1+\sqrt{1561}}{12}$

$2k+5 = 3k-3$

$8 = k$

$A = A^T$

$\begin{bmatrix} 10 & 2k+5 \\ 3k-3 & k+5 \end{bmatrix} = \begin{bmatrix} 10 & 3k-3 \\ 2k+5 & k+5 \end{bmatrix}$

Q) The product of eigenvalue of the matrix P is

$$P = \begin{bmatrix} 2 & 0 & 1 \\ 4 & -3 & 3 \\ 0 & 2 & -1 \end{bmatrix}$$

$2(3-6)+1(8)$

$-6+8=2$

A) -6
B) 2
C) 6
D) -2

Trace = sum of eigen values

Product of e v = Determinant

= 2

Q) The matrix $\left|\begin{bmatrix} 1 & a \\ 8 & 3 \end{bmatrix}\right| = 3-8a < 0$

where a >0, has a negative eigenvalue if a is greater than

$3 < 8a$

$a > \frac{3}{8}$

A) ⅜
B) ⅛
C) ¼
D) ⅕

Q) When six unbiased dice are rolled simultaneously, the probability of getting all distinct numbers (i.e., 1, 2, 3, 4, 5, and 6) is

A) 1/324
B) 5/324
C) 7/324
D) 11/324

$$\underset{6}{\downarrow}\quad\underset{5}{\downarrow}\quad\underset{4}{\downarrow}\quad\underset{3}{\downarrow}\quad\underset{2}{\downarrow}\quad\underset{1}{\downarrow}$$

(1, 1)

$$\frac{6!}{6^6} = \frac{\cancel{6}\times 5\times \cancel{4}^{\,2}\times \cancel{3\times 2}\times 1}{\cancel{6}\times\cancel{6}\times\underset{3}{\cancel{6}}\times\underset{3}{\cancel{6}}\times 6\times 6}$$

$$= \frac{5}{324}$$

Q) Consider a permutation sampled uniformly at random from the set of all permutations of {1, 2, 3, ..., n} for some $n \geq 4$. Let X be the event that 1 occurs before 2 in the permutation, and Y the event that 3 occurs before 4. Which one of the following statements is TRUE?

A) The event X and Y are mutually exclusive
B) The event X and Y are independent
C) Either event X or Y must occur
D) Event X is more likely than event Y

$X \cap Y = \phi$

X	1	2
Y	3	4

$\{1, 2, 3, 4, 5\}$

$\{2, 1, 4, 3, 5\}$

Q) For a given biased coin, the probability that the outcome of a toss is a head is 0.4. This coin is tossed 1,000 times. Let X denote the random variable whose value is the number of times that head appeared in these 1,000 tosses. The standard deviation of X (rounded to 2 decimal places) is _____.

$n = 1000$

$p = 0.4$

Binomial Distribution

$$\text{St dev} = \sqrt{np(1-p)}$$

$$= \sqrt{1000 \times 0.4(1-0.4)}$$

$$= \sqrt{240} = 15.49$$

The lifetime of a component of a certain type is a random variable whose probability density function is exponentially distributed with parameter 2. For a randomly picked component of this type, the probability that, its lifetime exceeds the expected lifetime (rounded to 2 decimal places) is ___________.

$\lambda = 2$

$E(X) = \frac{1}{\lambda} = \frac{1}{2} = 0.5$

$P[X > E(X)]$

$f(x) = \lambda e^{-\lambda x}, \quad x > 0$

$= 2e^{-2x}$

$= P[X > 0.5]$

$= \int_{0.5}^{\infty} 2e^{-2x}\, dx$

$= \cancel{2}\left[\frac{e^{-2x}}{-\cancel{2}}\right]_{0.5}^{\infty} = e^{-1} =$

Q) Let x and y be random variables, not necessarily independent, that take real values in the interval [0,1]. Let z=xy and let the mean values of x,y,z be x’, y’, z’, respectively. Which one of the following statements is TRUE?

A) z’ = x’y’
B) z’ < = x’y’
C) Z’ >= x’y’
D) Z’ <= x’

cov = −ve

$0 \le y \le 1$

$0 \le xy \le x$

$E(0) \le E(xy) \le E(x)$

$0 \le z' \le x'$

$\mathrm{cov}(x,y) = E(xy) - E(x)E(y)$

$\mathrm{cov}(x,y) = E(z) - E(x)E(y)$

$\mathrm{cov}(x,y) = z' - x'y'$

$z' = x'y' + \mathrm{cov}(x,y)$

X & Y

$\mathrm{cov}(X,Y)$

Q) A bag contains 10 red balls and 15 blue balls. Two balls are drawn randomly without replacement. Given that the first ball drawn is red, the probability (rounded off to 3 decimal places) that both balls drawn are red is ________

25

10R

15B

1 → Red

↓

9R

15B

$$\text{Prob} = \frac{9}{24}$$

$$= 0.375$$

~~BR~~, RB, ~~BR~~, RR

Q) Consider a random experiment where two fair coins are tossed. Let A be the event that denotes HEAD on both the throws, B be the event that denotes HEAD on the first throw, and C be the event that denotes HEAD on the second throw. Which of the following statements is/are TRUE?

A) A and B are independent
B) A and C are independent
C) B an C are independent
D) prob(B|C) = prob(B) $\Rightarrow \frac{P(B\cap C)}{P(C)} = P(B)$

$\{HH, HT, TH, TT\}$

$A \to HH$

$B \to HT, HH$

$C \to TH, HH$

$P(A|B) = P(A)$

$\frac{P(A\cap B)}{P(B)} = P(A)$

$\frac{1/4}{2/4} \neq \frac{1}{4}$

$\frac{1/4}{\cancel{2}/4_2} = \frac{\cancel{2}}{\cancel{4}_2}$

$\frac{1}{2} = \frac{1}{2}$

Q) The product of all eigenvalues of the matrix $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$

is ______________________,

A) -1
B) 0
C) 1
D) 2

Q) Let

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 4 & 1 & 2 & 3 \\ 3 & 4 & 1 & 2 \\ 2 & 3 & 4 & 1 \end{bmatrix} \text{ and } B = \begin{bmatrix} 3 & 4 & 1 & 2 \\ 4 & 1 & 2 & 3 \\ 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 1 \end{bmatrix}$$

Let det(A) and det(B)

denote the determinants of the matrices A and B, respectively.

Which one of the options given below is TRUE?

A) det(A) = det(B)
B) det(B) = -det(A)
C) det(A) = 0
D) det(AB) = det(A) + det(B)

Q) Let A be an n × n matrix over the set of all real numbers ℝ. Let B be a matrix obtained from A by swapping two rows. Which of the following statements is/are TRUE?

A) The determinant of B is the negative of the determinant of A
B) if A is invertible, then B is also invertible
C) If A is symmetric, then B is also symmetric
D) If the trace of A is zero, then trace of B is also zero

$$A = \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix} \qquad \begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}$$

$$B = \begin{bmatrix} 2 & 4 \\ 1 & 2 \end{bmatrix} \qquad \begin{bmatrix} 2 & -1 \\ 1 & 2 \end{bmatrix}$$

Q) Let A be any n x m matrix, where m > n. Which of the following statements is/are TRUE about the system of linear equations Ax = 0?

7 var 5 eqⁿ → 3

2

A) There exists at least m-n linearly independent solutions to this system

B) There exists m-n linear independent vector such that every solution is a linear combination of these vector

C) There a non-zero solution in which at least m-n variables are 0

D) There exists a solution in which at least n variable are non-zero

$m-n=2$

$x+y+z=0$

$\begin{bmatrix}-1\\1\\0\end{bmatrix}, \begin{bmatrix}-1\\0\\1\end{bmatrix}$

$AX=0 \quad m-n=1$

$$\begin{cases} x+y+z=0 \\ 2x+2y+2z=0 \end{cases}$$

$x+y+z=0$

$y=1, z=0$ $\begin{bmatrix}-1\\1\\0\end{bmatrix}$ | $y=0, z=1$ $\begin{bmatrix}-1\\0\\1\end{bmatrix}$

3 var, 2 eqⁿ

$m-n=1$

$x+y+z=0$

$x+y+3z=0$

$z=0$

$x+y=0$

$x=1$

$y=-1$

$\begin{bmatrix}1\\-1\\0\end{bmatrix}$

$x+y+z+4w=0$

$x+y+z+3w=0$

$x+y+2z+4w=0$

$w=0$

$z=0$

$\begin{bmatrix}x\\y\\0\\0\end{bmatrix}$

Q) Let f(x) be a continuous function from R to R such that f(x)=1−f(2−x)

Which one of the following options is the CORRECT value of $\int_0^2 f(x)dx$? ?

A) 0
B) 1
C) 2
D) -1

$$\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx$$

$$f(x) = 1 - f(2-x)$$

$$\int_0^2 f(x) = \int_0^2 1\,dx - \int_0^2 f(2-x)\,dx$$

$$\int_0^2 f(x)dx = 2 - \int_0^2 f(x)\,dx$$

$$\Rightarrow \cancel{2}\int_0^2 f(x)dx = \cancel{2}$$

$$\int_0^2 f(x)\,dx = 1$$

Q) Let f(x)=x3+15x2−33x−36 be a real-valued function. Which of the following statements is/are TRUE?

A) f(x) does not have a local maximum
B) f(x) has a local maximum
C) f(x) does not have a local minimum
D) f(x) has a local minimum

$f(x) = x^3 + 15x^2 - 33x - 36$

$f'(x) = 3x^2 + 30x - 33 = 0$

$x^2 + 10x - 11 = 0$

$x^2 + 11x - x - 11 = 0$

$(x+11)(x-1) = 0$

$x = 1, -11$

$f''(x) = 6x + 30$

$f''(1) = 36 > 0$ minima

$f''(-11) = -36 < 0$ maxima

12:40pm

Q) The value of the definite integral

$$\int_{-3}^{3}\int_{-2}^{2}\int_{-1}^{1}(4x^2y - z^3)\,dz\,dy\,dx$$

is ____0____. (Rounded off to the nearest integer)

$$\int_{-3}^{3}\int_{-2}^{2}\left[4x^2yz - \frac{z^4}{4}\right]_{-1}^{1} dy\,dx$$

$$\int_{-3}^{3}\int_{-2}^{2}\left[4x^2y\times 2 - 0\right]$$

$$\int_{-3}^{3}\int_{-2}^{2} 8x^2y \,dy\,dx$$

$$= \int_{-3}^{3}\left[\frac{8x^2y^2}{2}\right]_{-2}^{2} dx$$

$$= \int_{-3}^{3} 0\,dx$$

$$= 0$$

Q) The value of the following limit is ______________.

$$\lim_{x\to 0^+} \frac{\sqrt{x}}{1-e^{2\sqrt{x}}}$$

$$\lim_{x\to 0^+} \frac{\frac{1}{2\cancel{\sqrt{x}}}}{0-\cancel{2}e^{2\sqrt{x}} \times \frac{\cancel{1}}{\cancel{2}\cancel{\sqrt{x}}}}$$

$$= -\frac{1}{2}$$

$$= -0.5$$

Q) Suppose that f : R → R is a continuous function on the interval [-3, 3] and a differentiable function in the interval (-3, 3) such that for every x in the interval, f'(x) ≤ 2. If f(-3) = 7, then f(3) is at most ________.

Lagrange's Mean value thm

f → cont [a, b]

f → diff (a, b)

$\exists\, c \in (a, b)$

$$\boxed{f'(c) = \frac{f(b) - f(a)}{b - a}}$$

$$g'(c) = \frac{g(b) - g(a)}{b - a}$$

Rolle's thm

$f(a) = f(b)$

$\Rightarrow \boxed{f'(c) = 0}$

Cauchy MVT

f & g cont [a, b]

diff (a, b)

$$\frac{f'(c)}{g'(c)} = \frac{f(b) - f(a)}{g(b) - g(a)}$$

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

$$f'(c) = \frac{f(3) - f(-3)}{3 + (+3)}$$

$$\frac{f(3) - 7}{6} = f'(c) \le 2$$

$f(3) - 7 \le 12$

$f(3) \le 19$

Q) Consider the following expression

$$\lim_{x \to -3} \frac{\sqrt{2x+22}-4}{x+3} \quad \text{———} \quad \frac{0}{0}$$

The value of the above expression (rounded to 2 decimal places) is

________________.

$$\lim_{x \to -3} \frac{\frac{1 \times \cancel{2}}{\cancel{2}\sqrt{2x+22}} - 0}{1}$$

$$= \frac{1}{4} = 0.25$$

Let $P = \begin{bmatrix} 1 & 1 & -1 \\ 2 & -3 & 4 \\ 3 & -2 & 3 \end{bmatrix}$ and $Q = \begin{bmatrix} -1 & -2 & -1 \\ 6 & 12 & 6 \\ 5 & 10 & 5 \end{bmatrix}$ be two matrices.

Then the rank of $P + Q$ is ___________ .

$$\begin{bmatrix} 0 & -1 & -2 \\ 8 & 9 & 10 \\ 8 & 8 & 8 \end{bmatrix}$$

$R_1 \leftrightarrow R_3$

$$\begin{bmatrix} 8 & 8 & 8 \\ 8 & 9 & 10 \\ 0 & -1 & -2 \end{bmatrix}$$

$R_2 \rightarrow R_2 - R_1$

$$\begin{bmatrix} 8 & 8 & 8 \\ 0 & 1 & 2 \\ 0 & -1 & -2 \end{bmatrix}$$

$R_3 \rightarrow R_3 + R_2$

$$\begin{bmatrix} 8 & 8 & 8 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix} \rightarrow 2$$

GATE-CS-2015

If the following system has non-trivial solution,

$$px + qy + rz = 0$$
$$qx + ry + pz = 0$$
$$rx + py + qz = 0$$

then which one of the following options is True?

(A) $p - q + r = 0$ or $p = q = -r$

(B) $p + q - r = 0$ or $p = -q = r$

(C) $p + q + r = 0$ or $p = q = r$ ✓

(D) $p - q + r = 0$ or $p = -q = -r$

$$AX = 0$$
$$|A| = 0$$

$$\begin{vmatrix} p & q & r \\ q & r & p \\ r & p & q \end{vmatrix} = 0$$

$$R_1 \to R_1 + R_2 + R_3$$

$$\begin{vmatrix} p+q+r & p+q+r & p+q+r \\ q & r & p \\ r & p & q \end{vmatrix} = 0$$

$$(p+q+r)\begin{vmatrix} 1 & 1 & 1 \\ q & r & p \\ r & p & q \end{vmatrix} = 0$$

$$p+q+r = 0$$

$m = \text{Rank}(R)$

Suppose that P is a 4×5 matrix such that every solution of the equation Px=0 is a scalar multiple of [2 5 4 3 1]^T. The rank of P is ___________

4×5

A → no of eqⁿ 4

5 no of var

~~Ax~~ $Px = 0$

1 free variable

Rank P + free variable = no of variables

Rank P = 5 − 1

= 4

Let $c_1 \ldots . . c_n$ be scalars, not all zero, such that $\sum_{i=1}^{n} c_i a_i$ = 0 where a_i are column vectors in R^n.

Consider the set of linear equations

$Ax = b$

where $A = [a_1 \ldots . . a_n]$ and $b = \sum_{i=1}^{n} a_i$. The set of equations has

A. a unique solution at $x = J_n$ where J_n denotes a n-dimensional vector of all 1.
B. no solution
C. infinitely many solutions
D. finitely many solutions

$AX = \lambda X$

Let M be a 2×2 matrix with the property that the sum of each of the rows and also the sum of each of the columns is the same constant c. Which (if any) any of the vectors must be an eigenvector of M

(a) U = $\begin{bmatrix}1\\0\end{bmatrix}$

(b) V = $\begin{bmatrix}0\\1\end{bmatrix}$

(c) W = $\begin{bmatrix}1\\1\end{bmatrix}$

(d) None of these

$$\begin{bmatrix}1 & 2\\2 & 1\end{bmatrix}\begin{bmatrix}1\\0\end{bmatrix} = \begin{bmatrix}1\\2\end{bmatrix} \alpha$$

$$\begin{bmatrix}1 & 2\\2 & 1\end{bmatrix}\begin{bmatrix}0\\1\end{bmatrix} = \begin{bmatrix}2\\1\end{bmatrix} \alpha$$

$$\begin{bmatrix}1 & 2\\2 & 1\end{bmatrix}\begin{bmatrix}1\\1\end{bmatrix} = \begin{bmatrix}3\\3\end{bmatrix} = 3\begin{bmatrix}1\\1\end{bmatrix}$$

$AX = 3X$

Find the value of

$$\lim_{x \to 0} \frac{\tan x - x}{x^3}$$

(A) 1/3
(B) -1/6
(C) 1/2
(D) None of these

Answer: (A)

$$\lim_{x \to 0} \frac{\frac{\tan x}{x} - 1}{x^2}$$

$$\lim_{x \to 0} \frac{\sin x}{x} = 1$$

$$\lim_{x \to 0} \frac{\sec^2 x - 1}{3x^2}$$

$$\lim_{x \to 0} \frac{2\sec^2 x \tan x}{6x}$$

$$\lim_{x \to 0} \frac{1}{3} \frac{\sec^2 x}{\cos x} \frac{\sin x}{x}$$

$$\frac{1}{3} \times 1 \times 1$$

$$= 1/3$$

If $g(x) = 1 - x$ and $h(x) = \frac{x}{x-1}$, then $\frac{g(h(x))}{h(g(x))}$ is

(A) $\frac{h(x)}{g(x)}$

(B) $\frac{-1}{x}$

(C) $\frac{g(x)}{h(x)}$

(D) $\frac{x}{(1-x)^2}$

$\frac{\frac{x}{x-1}}{1-x} = \frac{x}{(x-1)(1-x)} = \frac{-x}{(x-1)^2}$

$g(h(x)) = g\left(\frac{x}{x-1}\right)$

$= 1 - \frac{x}{x-1}$

$= \frac{\cancel{x} - 1 - \cancel{x}}{x-1}$

$= \frac{-1}{x-1}$

$h(g(x)) = h(1-x)$

$= \frac{1-x}{1-x-1} = \frac{-1+x}{x}$

$= \frac{x-1}{x}$

$\frac{g(h(x))}{h(g(x))} = \frac{\frac{-1}{x-1}}{\frac{x-1}{x}} = \frac{-x}{(x-1)^2}$

If $\int_0^{2\pi} |x\sin x|\,dx = k\pi$ then the value of k is equal to ________.

(A) 2

(B) 3

(C) 4

(D) 5

$$\int_0^{2\pi} f(x)\,dx = 2\int_0^{\pi} f(x)\,dx \quad \boxed{f(2\pi - x) = f(x)}$$

S	A
T	C

$$\int_0^{\pi} x\sin x\,dx + \int_{\pi}^{2\pi} -x\sin x\,dx = k\pi$$

$$\left[-x\cos x + \sin x\right]_0^{\pi} - \left[-x\cos x + \sin x\right]_{\pi}^{2\pi}$$

$$\left[-\pi(-1) + 0 + 0\right] - \left[-2\pi + 0 + \pi(-1) - 0\right]$$

$$= 4\pi$$

The value of $\int_0^{\pi/4} x\cos(x^2)dx$ correct to three decimal places is _______ .

L A

Determinant

Eigen value & EiV

LU decomp

Rank & types of matrix

Calculus

Limit

Definite Integral

Prob & Stat

Probability

Prob distribution

Let f(x) = $x^{-(1/3)}$ and A denote the area of the region bounded by f(x) and the X-axis, when x varies from –1 to 1. Which of the following statements is/are True?

1. f is continuous in [–1, 1]
2. f is not bounded in [–1, 1]
3. A is nonzero and finite

(A) 2 only
(B) 3 only
(C) 2 and 3 only
(D) 1, 2 and 3

$$f(x) = \frac{1}{x^{1/3}}$$

$$A = \int_{-1}^{1} f(x)\, dx$$

$$= \int_{-1}^{0} \frac{1}{x^{1/3}}\, dx + \int_{0}^{1} \frac{1}{x^{1/3}}\, dx$$

$$= \boxed{\quad}$$

If for non-zero x, $af(x)+bf\left(\frac{1}{x}\right)=\frac{1}{x}-25$ where $a \neq b$ then $\int_1^2 f(x)\,dx$ is

(a) $\frac{1}{a^2-b^2}\left[a(\ln 2-25)+\frac{47b}{2}\right]$

(b) $\frac{1}{a^2-b^2}\left[a(2\ln 2-25)-\frac{47b}{2}\right]$

(c) $\frac{1}{a^2-b^2}\left[a(2\ln 2-25)+\frac{47b}{2}\right]$

(d) $\frac{1}{a^2-b^2}\left[a(\ln 2-25)-\frac{47b}{2}\right]$

$$a\int_1^2 f(x)\,dx + b\int_1^2 f(1/x)\,dx = \ln 2 - 25 \qquad \times a$$

$$a\int_1^2 f(1/x)\,dx + b\int_1^2 f(x)\,dx = \frac{47}{2} \qquad \times b$$

$$a^2\int_1^2 f(x)\,dx - b^2\int_1^2 f(x)\,dx = a(\ln 2 - 25) - \frac{47b}{2}$$

$x_1, x_2 - - x_n \longrightarrow \mu$
$\longrightarrow \sigma$

Which of the following assertions are CORRECT?

P: Adding 7 to each entry in a list adds 7 to the mean of the list

Q: Adding 7 to each entry in a list adds 7 to the standard deviation of the list

R: Doubling each entry in a list doubles the mean of the list

S: Doubling each entry in a list leaves the standard deviation of the list unchanged

(A) P, Q

(B) Q, R

(C) P, R

(D) R, S

$$\text{Mean} = \frac{x_1 + x_2 + - - - + x_n}{n} = \mu$$

$$\text{St dev} = \sqrt{\frac{\Sigma (x_i - \mu)^2}{n}}$$

P $\quad x_1+7, x_2+7, - - -, x_n+7$

$$\mu_2 = \frac{x_1+7+x_2+7+ - - - + x_n+7}{n}$$

$$= \frac{x_1 + - - + x_n}{n} + \frac{7\cancel{n}}{\cancel{n}}$$

$$\mu_2 = \mu + 7$$

Q $\quad \text{St dev} \sqrt{\frac{\Sigma \left(x_i + \cancel{7} - (\mu + \cancel{7})\right)}{n}}$

$= \sigma$

R $\quad 2x_1 - - - 2x_n$

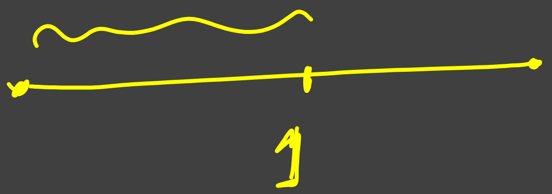
$$2\left(\frac{x_1 + x_2 - + x_n}{n}\right)$$

2μ

S $\quad \text{st dev} = \sqrt{\frac{\Sigma (2x_i - 2\mu)^2}{n}}$

Suppose you break a stick of unit length at a point chosen uniformly at random. Then the expected length of the shorter stick is _________ .

5 min



$$0 \le X \le 1/2$$

$$f(x) = \begin{cases} \frac{1}{1/2 - 0} = 2, & 0 \le x \le 1/2 \\ 0, & \text{otherwise} \end{cases}$$

$$E(x) = \frac{b+a}{2} = \frac{1/2 + 0}{2} = \frac{1}{4} = 0.25$$

Let X be a random variable following normal distribution with mean +1 and variance 4. Let Y be another normal variable with mean -1 and variance unknown If P(X <=-1) = P(Y >=2). the standard deviation of Y is

(A) 3
(B) 2
(C) sqrt(2)
(D) 1

$$P\left(\frac{X-\mu_x}{\sigma_x} \le \frac{-1-1}{\sqrt{4}}\right) = P\left(\frac{Y-\mu_Y}{\sigma_Y} \ge \frac{2-(-1)}{\sigma_Y}\right)$$

$$P\left(Z \le -\frac{\cancel{2}}{\cancel{2}}\right) = P\left(Z \ge 3/\sigma_Y\right)$$

$$P(Z \ge 1) = P(Z \ge 3/\sigma_Y)$$

$$1 = \frac{3}{\sigma_Y} \Rightarrow \sigma_Y = 3$$

Two people, P and Q, decide to independently roll two identical dice, each with 6 faces, numbered 1 to 6. The person with the lower number wins, In case of a tie, they roll the dice repeatedly until there is no tie. Define a trial as a throw of the dice by P and Q. Assume that all 6 numbers on each dice are equi-probable and that all trials are independent. The probability (rounded to 3 decimal places) that one of them wins on the third trial is

________ .

Prob of no one wining

$= P(\text{Tie}) =$

$= \frac{1}{6} \times \frac{1}{6} \times 6$

$= \frac{1}{6}$

$P(\text{not tie}) = \frac{5}{6}$

$\frac{1}{6} \times \frac{1}{6} \times \frac{5}{6}$

$= 0.023$