RRR Robotic Arm Design

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Het Patel

ECE 573: Advanced Robotics and Computer Vision

Professor: Dr. Sudhir Shrestha

Sonoma State University

Overview:

The project demonstrates a design and working model of RRR robotic arm which has 3 degrees of freedom. The arm consists of three rotary joints which are controlled by three servo motors. These servo motors are responsible for the movement. These robotic arms can be used in a variety of applications, including manufacturing, assembly, and research. The goal of this project is to implement the Forward and Inverse Kinematics using equations and programming the same into microcontroller.

Required Components:

The project consists of below parts:

1. 3x Servo Motors
2. 3x Links
3. 1x ESP8266 Node MCU
4. Jumper wires for connection
5. Power Supply of 5V

The project can be divided into two parts: Forward Kinematics and Inverse Kinematics.

Forward Kinematics:

Forward kinematics is a method used in robotics to determine the position and orientation of the end-effector, such as a robotic arm or gripper, based on the joint angles and other parameters of the robotic system. It is an essential component of robot motion planning and control.

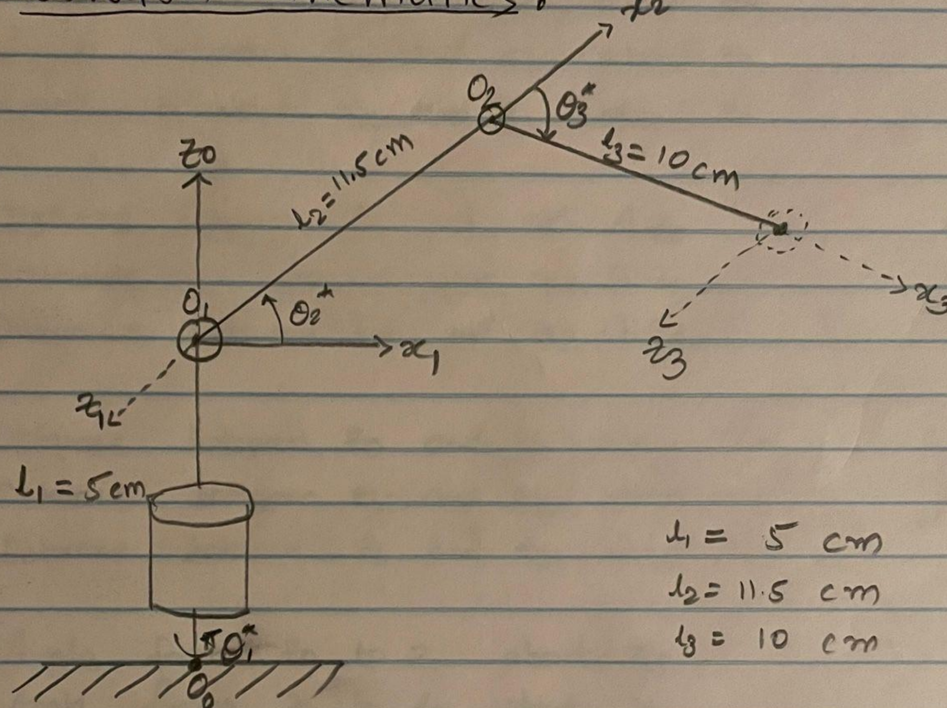
The forward kinematics of a robotic arm involve calculating the position and orientation of the end-effector relative to the base of the robot. This is done by breaking down the robot arm into a series of joints, each of which can be rotated or translated to produce a particular movement.

To calculate the forward kinematics of a robotic arm, the position and orientation of each joint must be determined relative to the base of the robot. This is typically done using transformation matrices, which describe how the position and orientation of each joint are affected by the rotation and translation of the previous joint.

Once the transformation matrices are defined, they will be used to calculate the homogeneous transformation matrix. This is the matrix where we can extract the last columns to get the coordinates of the end-effector.

The following is the mathematical representation of the same.

(1.) Forward kinematics :-



$l_1 = 5$ cm
$l_2 = 11.5$ cm
$l_3 = 10$ cm

Link	θ	d	a	α
1	θ_1^*	l_1	0	90
2	θ_2^*	0	l_2	0
3	θ_3^*	0	l_3	0

Replacing l_1, l_2, l_3 values, we get

D-H parameters

Link	θ	d	a	α
1	θ_1^*	5	0	90
2	θ_2^*	0	11.5	0
3	θ_3^*	0	10	0

θ_1: Angle between x_0 and x_1 about z_0 $= \theta_1^*$
θ_2: Angle between x_1 and x_2 about z_1 $= \theta_2^*$
θ_3: Angle between x_2 and x_3 about z_2 $= \theta_3^*$

d_1: Distance between O_0 and $x_1 \cap z_0$ $= l_1$
d_2: Distance between O_1 and $x_2 \cap z_1$ $= 0$
d_3: Distance between O_2 and $x_3 \cap z_2$ $= 0$

a_1: Distance between z_0 and z_1 along x_1 $= 0$
a_2: Distance between z_1 and z_2 along x_2 $= l_2$
a_3: Distance between z_2 and z_3 along x_3 $= l_3$

α_1: Angle from z_0 to z_1 about x_1 $= 90$
α_2: Angle from z_1 to z_2 about x_2 $= 0$
α_3: Angle from z_2 to z_3 about x_3 $= 0$

We know the general A matrix:

$$A_i = \begin{bmatrix} C\theta_i & -S\theta_i C\alpha_i & S\theta_i S\alpha_i & a_i C\theta_i \\ S\theta_i & C\theta_i C\alpha_i & -C\theta_i S\alpha_i & a_i S\theta_i \\ 0 & S\alpha_i & C\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

To find A_1, A_2, A_3 matrices, substituting $i = 1, 2$ and 3 in the above A_i matrix.

$$A_1 = \begin{bmatrix} \cos\theta_1^* & 0 & \sin\theta_1^* & 0 \\ \sin\theta_1^* & 0 & -\cos\theta_1^* & 0 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} \cos\theta_2^* & -\sin\theta_2^* & 0 & 11.5(\cos\theta_2^*) \\ \sin\theta_2^* & \cos\theta_2^* & 0 & 11.5(\sin\theta_2^*) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_3 = \begin{bmatrix} \cos\theta_3^* & -\sin\theta_3^* & 0 & 10\cos\theta_3^* \\ \sin\theta_3^* & \cos\theta_3^* & 0 & 10\sin\theta_3^* \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The next step is to find the homogeneous Matrix,

$$H = T_3^0 = A_1 A_2 A_3$$

By multiplying three matrices in the matlab, we get following equations for dx, dy, dz in the form of $\theta_1^*, \theta_2^*, \theta_3^*$.

$$d_x = (23^* \cos\theta_1^* * \cos\theta_1^* - 10\cos\theta^* \cdot \sin\theta_2^* \cdot \sin\theta_3^* + 10\cos\theta^* \cdot \cos\theta_2^* \cdot \cos\theta_3^*)$$

$$d_y = 23 \cos\hat{\theta}_2 \cdot \sin\hat{\theta}_1/2 + 10 \cos\hat{\theta}_2 \cdot \cos\hat{\theta}_3 \cdot \sin\hat{\theta}_1 - 10 \sin\hat{\theta}_1 \cdot \sin\hat{\theta}_2 \cdot \sin\hat{\theta}_3$$

$$d_z = 23 \sin\hat{\theta}_2/2 + 10 \cos\hat{\theta}_2 \cdot \sin\hat{\theta}_3 + 10 \cos\hat{\theta}_3 \cdot \sin\hat{\theta}_2 + 5$$

- Substituting any values of $\theta_1, \theta_2, \theta_3$ would give corresponding d_x, d_y, d_z end-effector coordinates.

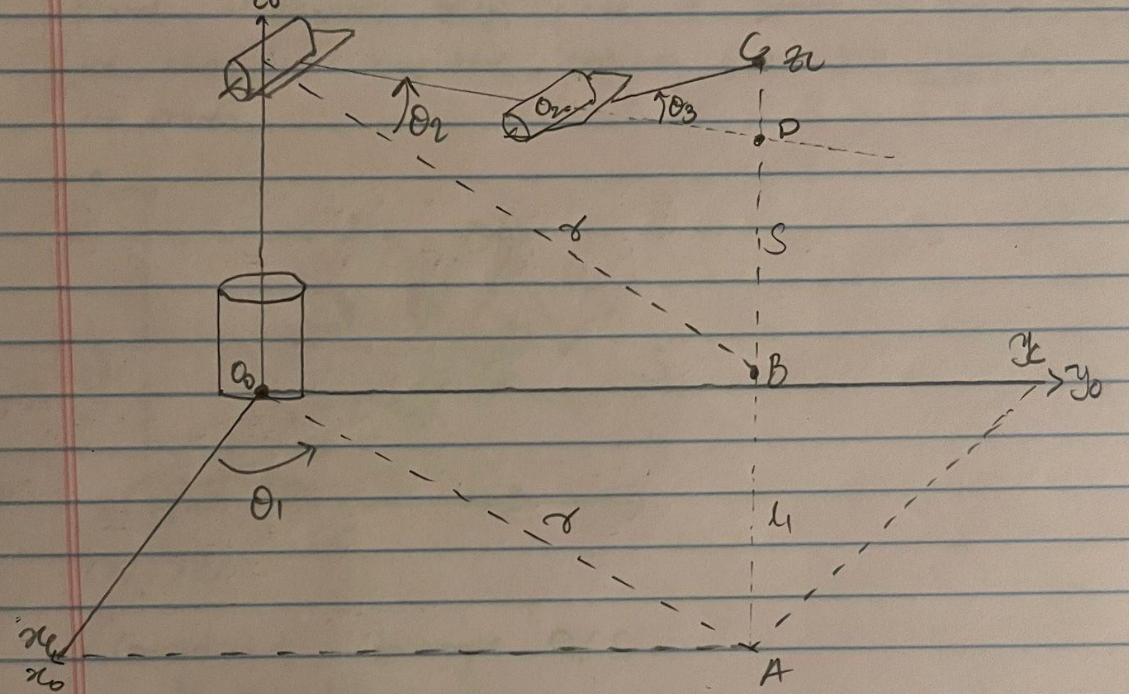
Inverse Kinematics:

Inverse Kinematics is the reverse process of Forward Kinematics.It is a technique used in robotics to determine the joint angles and other parameters needed to achieve a desired position and orientation of the end-effector, such as a robotic arm or gripper. In contrast to forward kinematics, which determines the position and orientation of the end-effector based on the joint angles, inverse kinematics calculates the joint angles needed to achieve a particular position and orientation.

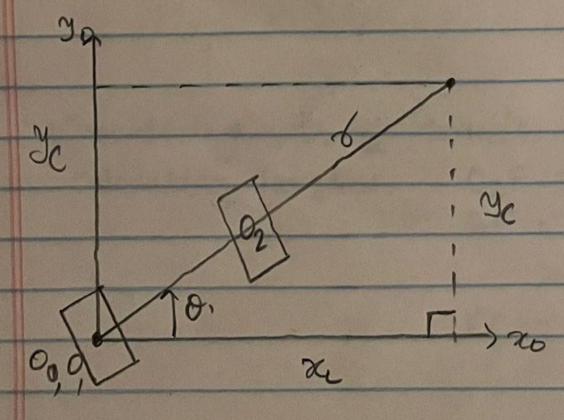
Inverse kinematics is a complex mathematical problem, as there may be multiple solutions or no solution at all depending on the specific robotic system and desired end-effector position. To reach the end effector coordinate, one can reach by many different angles.

The following is the mathematical representation of Inverse Kinematics:

Inverse kinematics :-



i) To find θ_1 , we will project our arm on $x_0 y_0$ plane-

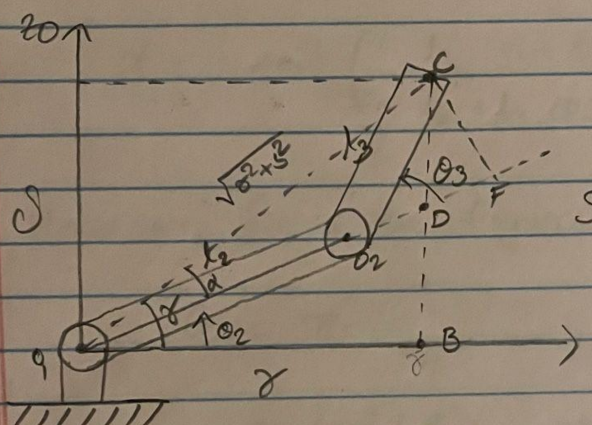


- It is clear that,

$$\tan\theta_1 = y_c / x_c$$

$$\theta_1 = \tan^{-1}\left(\frac{y_c}{x_c}\right)$$

ii) To find θ_2, we need to project the arm on $x_1 y_1$ plane. However, $x_1 y_1$ plane is same as $x_2 y_2$. Or we can also consider, looking from the Top of an arm.



- Considering, triangle O_1CS.

$$\tan \gamma = \frac{s}{r}$$

$$\gamma = \tan^{-1}\left(\frac{s}{r}\right)$$

- Considering O_2FC triangle,

calculating lengths, $O_2F = l_3 \cos\theta_3$ (triangle rule)

$O_1F = (O_1O_2 + O_2F)$

$= (l_2 + l_3 \cos\theta_3)$

$FC = l_3 \sin\theta_3$ (triangle rule)

O_1FC triangle would give

$$\tan\alpha = \frac{l_3 \sin\theta_3}{l_2 + l_3 \cos\theta_3}$$

$$\alpha = \tan^{-1}\left(\frac{l_3 \sin\theta_3}{l_2 + l_3 \cos\theta_3}\right)$$

However, from the figure, it is clear that

$$\theta_2 = \gamma - \alpha$$

$$\theta_2 = \tan^{-1}\left(\frac{s}{r}\right) - \tan^{-1}\left(\frac{l_3 \sin\theta_3}{l_2 + l_3 \cos\theta_3}\right)$$

ii) From the same projection,

$O_1C = r^2 + s^2$ using cosine rule.

$O_1B = x_c^2 + y_c^2 = r^2$

cosine triangle O_1O_2C,

$$\cos(180 - \theta_3) = \frac{-(r^2+s^2) + l_2^2 + l_3^2}{2l_2l_3}$$

$$\cos\theta_3 = \frac{(r^2+s^2) - l^2 - l_3^2}{2l_2 l_3} \approx D$$

(temporary name)

we know,

$$\sin^2\theta + \cos^2\theta = 1$$

$$\therefore \sin^2\theta = 1 - \cos^2\theta$$

$$\hookrightarrow \sin^2\theta_3 = 1 - \cos^2\theta_3$$

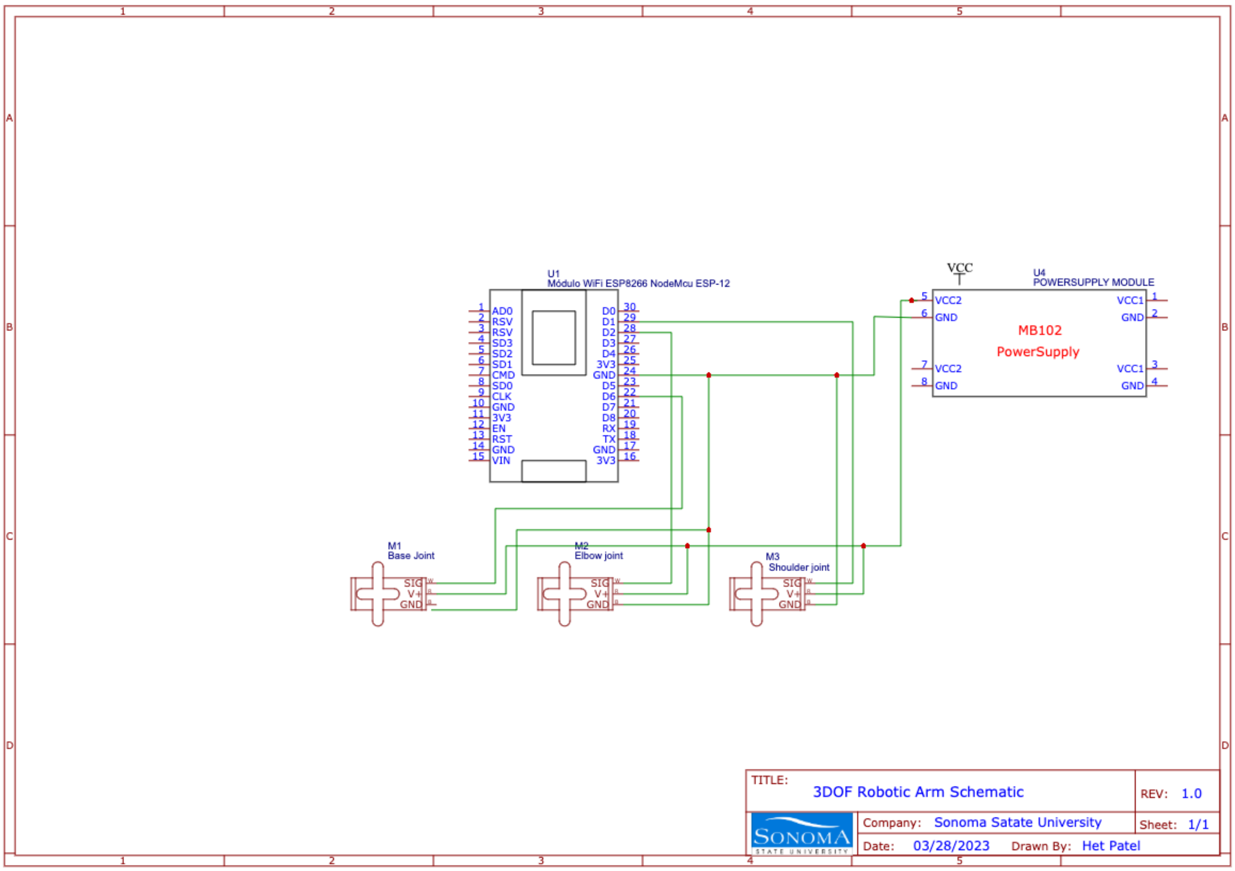
$$\sin\theta_3 = \pm\sqrt{1-D^2}$$

$$\tan\theta_3 = \frac{\sin\theta_3}{\cos\theta_3} = \frac{\pm\sqrt{1-D^2}}{D}$$

$$\theta_3 = \left(\tan^{-1} \pm\frac{\sqrt{1-D^2}}{D}\right)$$

By replacing the value of D, we get equation in the form of known variables. which will be used to compute θ_3.

Schematic Representation:



Conclusion:

Appendix:

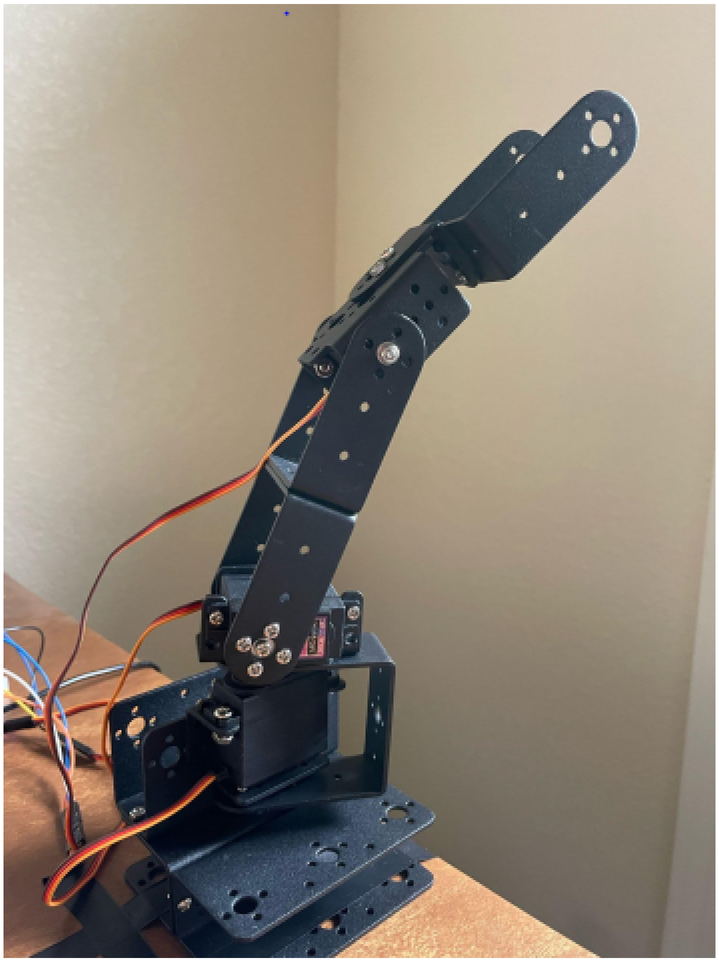


Fig. 3 DOF Robotic Arm