This code can be used for the arbitrary bunch lengthening study in the triple radio frequency (RF) system with active harmonic cavities (HCs), including analytical calculation, semi-analytical calculation and tracking simulation.

How to use this code?

Part 1: analytical calculation, used to obtain arbitrary symmetrical bunch distribution.

Script main.m

```
%% parameter
cspeed = 299792458;
C = 480;
h = 800;
I0 = 350e-3;
V_rf = 1.2e6;
U0 = 400e3;
n1 = 3;
n2 = 5;
E0 = 2.2e9;
sigma_t = 7e-12;
sigma_E = 7.44e-4;
alpha_c = 9.4e-5;
```

```
%% lower order HC
k_v1 = 0.99;
k_fai1 = 1.01;
x(4) = k_v1*x(4);
x(2) = k_fai1*x(2);
```

1. Setting right parameter and choosing needed voltage amplitude and phase radios of lower order HC is enough. Note that k_v1 + k_fai1 = 2.

Script bestlength.m

```
function F = bestlength(x,V_rf,U0,n1,n2)

    F(1) = V_rf*sin(x(1))+x(4)*V_rf*sin(x(2))+x(5)*V_rf*sin(x(3))-U0;
    F(2) = cos(x(1))+n1*x(4)*cos(x(2))+n2*x(5)*cos(x(3));
    F(3) = sin(x(1))+n1^2*x(4)*sin(x(2))+n2^2*x(5)*sin(x(3));
    F(4) = cos(x(1))+n1^3*x(4)*cos(x(2))+n2^3*x(5)*cos(x(3));
    F(5) = sin(x(1))+n1^4*x(4)*sin(x(2))+n2^4*x(5)*sin(x(3));
end
```

2. Using this equation solver to calculate optimum bunch lengthening parameter.

Part 2: modified semi-analytical calculation, used to evaluate periodic transient beam loading (PTBL) threshold quickly.

Script EquilibriumSolution.m

```
%% main parameters
format long;
cspeed = 299792458;
sigma_t = 10e-12;   % initial set rms bunch length  in ps
sigma_E = 7.44e-4;  % natural energy spread
alpha_c = 9.4e-5;  % momentum compaction factor
C       = 479.86;    % circumference in m
h       = 800;        % harmonic number
I0      = 350e-3;    % beam current in A
U0      = 400e3;    % energy loss per turn in eV
E0      = 2.2e9;      % beam energy in eV
V_mc    = 1.2e6;   % MC voltage in V
n_hc1  = 3;          % harmonic number of HHC
n_hc2  = 5;          % harmonic number of HHC
% Q_hc   = 1e5;     % quality factor of HHC
% R_hc   = Q_hc*10;% shunt impedance of HHC !!!!!
Q_hc2  = 1e5;     % quality factor of HHC
R_hc2  = Q_hc2*5;% shunt impedance of HHC !!!!!
```

```
%% lower HC R, Q, detuning
T0 = C/cspeed;
f_rf = h/T0;
psi_hc = angle(F1)-fais_hc1+pi/2;
R_hc = k1*V_mc/(abs(-2*I0*F1*cos(psi_hc)*exp(-1i*psi_hc)));
Q_hc = R_hc/(10);
fre_shift = -n_hc1*f_rf*tan(psi_hc)/(2*Q_hc);
```

1. Parameter setting. In order to make the beam loading voltage of the lower order HC approach the ideal cavity voltage, its R, Q and detuning are set separately.

```
%% equilibrium distribution using analytical equation or semi-analytical calculation
% analytical equilibrium distribution
Hf = cos(fais_mc)-cos(fais_mc+HALF.fai)+k1/n_hc1*(cos(fais_hc1)-cos(n_hc1*HALF.fai+(fais_hc1)))...
    +k2/n_hc2*(cos(fais_hc2)-cos(n_hc2*HALF.fai+fais_hc2))-U0*HALF.fai/V_mc;
den_dist = exp(HALF.poten_coef*Hf);
ta =HALF.fai/(2*pi*HALF.f_rf);
norm_den_dist_single = den_dist*2/(2*sum(den_dist)-den_dist(1)-den_dist(end))/(ta(2)-ta(1));
norm_den_dist = repmat(norm_den_dist_single',1,HALF.h);
[bunchlength,centroid]=sigma_tau_calc(tau,norm_den_dist_single,0);

% semi-analytical solution for equilibrium distribution
%MainIterationLoopCode; |

figure(4)
plot(HALF.fai/HALF.w_rf*1e12,norm_den_dist(:,1)*delta_tau,'LineWidth',2);hold on;
ylabel('\rho [a.u.]');xlabel('\tau [ps]');
```

2. Obtaining bunch equilibrium distribution. The bunch distribution can be calculated directly using the ideal cavity voltage or by using semi-analytical calculation.

Script MainIterationLoopCode2.m

```
while iteration<500 % the total number of iterations
iteration = iteration + 1;
Fac1 = exp(-1i*tau*HALF.w_r)*norm_den_dist*delta_tau;
Fac2 = exp(-1i*tau*HALF2.w_r)*norm_den_dist*delta_tau;

x = 1:h;
Fac1 = mean(Fac1)*ones(1,HALF.h)-1i*1e-6*sin(x/h*2*pi);
```

3. Adding perturbation. Choosing an appropriate perturbation amplitude and adding to the real part of imaginary part of bunch form factor.

Part 3: STABLE tracking simulation.

Script MPMBTrackBunchLengthingwoHighQ.m

```
%% beam parameters
% HALF 参数
cspeed = 299792458;
sigma_t0 = 10e-12;      % s initial rms bunch length  （用于计算上归一化）
sigma_e0 = 7.44e-4;     %  rms energy spread
alpha_c  = 9.4e-5;      %  momentum compaction factor
tau_s    = 14e-3;       %  radiation damping time
tau_z    = 14e-3;       %  radiation damping time
I0      = 350e-3;       %  beam current
E0      = 2.2e9;        %  beam energy
U0      = 400e3;        %  energy loss per turn
V_mc    = 1.2e6;        %  main cavity voltage
h       = 800;          % harmonic number
n_hc1   = 3;            % harmonic order of HHC1
n_hc2   = 5;            % harmonic order of HHC2
C       = 479.86;       % Circumference of the ring
% Q_hc   = 1e5;           % HHC loaded quality factor
% R_hc   = Q_hc*40;       % HHC loaded shunt impedance
Q_hc2   = 1e5;          % HHC loaded quality factor
R_hc2   = Q_hc2*2;     % HHC loaded shunt impedance
Q_mc    = 1e5;          % shunt impedance of main cavity
R_mc    = Q_mc*45;      % quality factor of main cavity
```

1. Beam parameter setting. Similar with modified semi-analytical calculation, R, Q and detuning of lower order HC are set separately.

```
%% RF cavity parameter settings for different longitudinal bunch equilibrium distribution
% obtained by analytical formula

fais_mc = 2.74028221659595;    %%1.2MV flat potential condition
fais_hc1 = -0.140514110229679;
fais_hc2 = 3.05692778011039;
k1 = 0.464856515347598;
k2 = 0.0923859093693406;

F0 = 0.979626801531162 + 0.00127578071227 4i; %%1.2MV flat potential condition
F1 = 0.826435778739579 + 0.00315886448763 9i;
F2 = 0.567832151901265 + 0.003414962774284i;


V_hc = k1 * V_mc;   %ideal voltage amplitude of lower order HC
V_hc2 = k2 * V_mc;  %ideal voltage amplitude of higher order HC
```

2. RF cavity parameter setting. Data is obtained from analytical calculation in part 1.

```
%% RF cavity detuning setting (Minimum generator power)

T0 = C/cspeed;
f_rf = h/T0;

psi_hc = angle(F1)-fais_hc1+pi/2;                          %detuning angle of lower order HC
R_hc = k1*V_mc/(abs(-2*I0*abs(F1)*cos(psi_hc)*exp(-1i*psi_hc)));
Q_hc = R_hc/15;
fre_shift = -n_hc1*f_rf*tan(psi_hc)/2/Q_hc;

psi_hc2 = atan(abs(F2)*2*I0*R_hc2*cos(fais_hc2)/k2/V_mc);    %detuning angle of higer order HC
fre_shift2 = tan(psi_hc2)*h*cspeed*n_hc2/C/2/Q_hc2;

psi_mc = atan(abs(F0)*2*I0*R_mc*cos(fais_mc)/V_mc);          %detuning angle of MC
fre_shift_mc = tan(psi_mc)*h*cspeed/C/2/Q_mc;
```

3. RF cavity detuning setting. The setting principle is to minimize generator power.

```
%% Working mode setting

% fill pattern
pattern = zeros(1,h);
pattern(1:1:h)=1;    %Uniform filling
fillrate = length(find(pattern==1))/h;

HALF = machine(C,I0,U0,E0,tau_s,tau_z,sigma_t0,sigma_e0,alpha_c,h,...
    V_mc,V_hc,V_hc2,fais_mc,fais_hc1,fais_hc2,n_hc1,n_hc2,R_hc,Q_hc...
    ,R_hc2,Q_hc2,fillrate,fre_shift,fre_shift2,Q_mc,R_mc,fre_shift_mc,F1,F2,F0);
HALF.ShortRange_on = 0; % 0 - neglecting short range effect, 1 - considering.

HALF.MC_mode  = 1; % 1 - ideal RF cavity, 2 - passive RF cavity, 3 - active RF cavity.
HALF.HC_mode  = 3; % 1 - ideal RF cavity, 2 - passive RF cavity, 3 - active RF cavity.
HALF.HC2_mode  = 3; % 1 - ideal RF cavity, 2 - passive RF cavity, 3 - active RF cavity.
```

4. Working mode setting. Including ideal cavity, passive cavity and active cavity.

```
%% bunch generation
Par_num = 1e4; Bun_num = length(find(pattern==1));

% Track number.
Track_num  = 30e4;
```

5. The number of macroparticle per bunch and tracking turn setting.

Script PI_Set.m

6. Setting PI parameters of three RF cavity in this script. The key point lies in the KP and KI Settings of each RF cavity.