1 Inference rules

1.1 Formation of contexts, types, and their elements

MLTT rule:
$$\frac{\Gamma,\, x\!:\!A \;\vdash\; B(x) \text{ type}}{\Gamma \;\vdash\; A \text{ type}}$$

MLTT rule:
$$\frac{\Gamma \;\vdash\; A \equiv B \text{ type}}{\Gamma \;\vdash\; A \text{ type}}$$

MLTT rule:
$$\frac{\Gamma \;\vdash\; A \equiv B \text{ type}}{\Gamma \;\vdash\; B \text{ type}}$$

MLTT rule:
$$\frac{\Gamma \;\vdash\; a\!:\!A}{\Gamma \;\vdash\; A \text{ type}}$$

MLTT rule:
$$\frac{\Gamma \;\vdash\; a \equiv b\!:\!A}{\Gamma \;\vdash\; a\!:\!A}$$

MLTT rule:
$$\frac{\Gamma \;\vdash\; a \equiv b\!:\!A}{\Gamma \;\vdash\; b\!:\!A}$$

1.2 Judgemental equality is an equivalence relation

MLTT rule:
$$\frac{\Gamma \;\vdash\; A \text{ type}}{\Gamma \;\vdash\; A \equiv A \text{ type}}$$

MLTT rule:
$$\frac{\Gamma \;\vdash\; A \equiv B \text{ type}}{\Gamma \;\vdash\; B \equiv A \text{ type}}$$

MLTT rule:
$$\frac{\Gamma \;\vdash\; A \equiv B \text{ type} \qquad \Gamma \;\vdash\; B \equiv C \text{ type}}{\Gamma \;\vdash\; A \equiv C \text{ type}}$$

MLTT rule:

$$\frac{\Gamma \;\vdash\; a : A}{\Gamma \;\vdash\; a \equiv a : A}$$

MLTT rule:

$$\frac{\Gamma \;\vdash\; a \equiv b : A}{\Gamma \;\vdash\; b \equiv a : A}$$

MLTT rule:

$$\frac{\Gamma \;\vdash\; a \equiv b : A \qquad \Gamma \;\vdash\; b \equiv c : A}{\Gamma \;\vdash\; a \equiv c : A}$$

1.3 Variable conversion

MLTT rule:

$$\frac{\Gamma \;\vdash\; A \equiv A' \text{ type} \qquad \Gamma,\, x : A,\, \Delta \;\vdash\; B(x) \text{ type}}{\Gamma,\, x : A',\, \Delta \;\vdash\; B(x) \text{ type}}$$

MLTT rule:

$$\frac{\Gamma \;\vdash\; A \equiv A' \text{ type} \qquad \Gamma,\, x : A,\, \Delta \vdash \mathcal{J}}{\Gamma,\, x : A',\, \Delta \vdash \mathcal{J}}$$

Derived rule:

$$\frac{\Gamma \;\vdash\; A \equiv A' \text{ type} \qquad \Gamma \;\vdash\; a : A}{\Gamma \;\vdash\; a : A'}$$

1.4 Substitution

Metasyntactic notation (used in many places):

$$t[a/x] := t \text{ with all occurences of } x \text{ replaced by } a$$

MLTT rule:

$$\frac{\Gamma \;\vdash\; a : A \qquad \Gamma,\, x : A,\, \Delta \vdash \mathcal{J}}{\Gamma,\, \Delta[a/x] \;\vdash\; \mathcal{J}[a/x]} \; S$$

MLTT rule:

$$\frac{\Gamma \;\vdash\; a \equiv a' : A \qquad \Gamma,\, x : A,\, \Delta \;\vdash\; B \text{ type}}{\Gamma,\, \Delta[a/x] \;\vdash\; B[a/x] \equiv B[a'/x] \text{ type}}$$

MLTT rule:

$$\frac{\Gamma \;\vdash\; a \equiv a' : A \qquad \Gamma,\, x : A,\, \Delta \;\vdash\; b : B}{\Gamma,\, \Delta[a/x] \;\vdash\; b[a/x] \equiv b[a'/x] : B[a/x]}$$

1.5   Weakening

MLTT rule:
$$\frac{\Gamma \vdash A \text{ type} \qquad \Gamma, \Delta \vdash \mathcal{J}}{\Gamma, x{:}A, \Delta \vdash \mathcal{J}} \, W$$

MLTT rule:
$$\frac{\Gamma \vdash A \text{ type} \qquad \Gamma \vdash B \text{ type}}{\Gamma, x{:}A \vdash B \text{ type}} \, W$$

1.6   Generic elements

MLTT rule:
$$\frac{\Gamma \vdash A \text{ type}}{\Gamma, x{:}A \vdash x{:}A} \, \delta$$

1.7   Manipulating variables

Derived rule:
$$\frac{\Gamma, x{:}A, \Delta \vdash \mathcal{J}}{\Gamma, x'{:}A, \Delta[x'/x] \vdash \mathcal{J}[x'/x]} \, x'/x$$

Derived rule:
$$\frac{\Gamma \vdash B \text{ type} \qquad \Gamma, x{:}A, y{:}B, \Delta \vdash \mathcal{J}}{\Gamma, y{:}B, x{:}A, \Delta \vdash \mathcal{J}}$$

2   Π types

2.1   Dependent functions

2.1.1   Π formation

MLTT rule:
$$\frac{\Gamma, x{:}A \vdash B(x) \text{ type}}{\Gamma \vdash \Pi_{(x:A)}B(x) \text{ type}} \, \Pi$$

MLTT rule:
$$\frac{\Gamma \vdash A \equiv A' \text{ type} \qquad \Gamma, x{:}A \vdash B(x) \equiv B'(x) \text{ type}}{\Gamma \vdash \Pi_{(x:A)}B(x) \equiv \Pi_{(x:A')}B'(x) \text{ type}} \, \Pi\text{-eq}$$

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2.1.2 Π introduction

MLTT rule:

$$\frac{\Gamma,\, x\!:\!A\ \vdash\ b(x) : B(x)}{\Gamma\ \vdash\ \lambda x.b(x) : \Pi_{(x:A)}B(x)}\ \lambda$$

MLTT rule:

$$\frac{\Gamma,\, x\!:\!A\ \vdash\ b(x) \equiv b'(x) : B(x)}{\Gamma\ \vdash\ \lambda x.b(x)\ \equiv\ \lambda x.b'(x) : \Pi_{(x:A)}B(x)}\ \lambda\text{-eq}$$

2.1.3 Π elimination

MLTT rule:

$$\frac{\Gamma\ \vdash\ f : \Pi_{(x:A)}B(x)}{\Gamma,\, x\!:\!A\ \vdash\ f(x) : B(x)}\ \text{ev}$$

MLTT rule:

$$\frac{\Gamma\ \vdash\ f \equiv f' : \Pi_{(x:A)}B(x)}{\Gamma,\, x\!:\!A\ \vdash\ f(x) \equiv f'(x) : B(x)}\ \text{ev-eq}$$

2.1.4 Π computation

MLTT rule:

$$\frac{\Gamma,\, x\!:\!A\ \vdash\ b(x) : B(x)}{\Gamma,\, x\!:\!A\ \vdash\ (\lambda y.b(y))(x) \equiv b(x) : B(x)}\ \beta$$

MLTT rule:

$$\frac{\Gamma\ \vdash\ f : \Pi_{(x:A)}B(x)}{\Gamma\ \vdash\ \lambda x.f(x)\ \equiv\ f : \Pi_{(x:A)}B(x)}\ \eta$$

2.2 Ordinary functions

2.2.1 Basic

MLTT rule:

$$\frac{\Gamma\ \vdash\ A\ \text{type} \qquad \Gamma\ \vdash\ B\ \text{type}}{\Gamma\ \vdash\ A \to B\ \text{type}}\ \to$$

MLTT rule:

$$\frac{\Gamma\ \vdash\ A\ \text{type} \qquad \Gamma\ \vdash\ B\ \text{type}}{\Gamma\ \vdash\ A \to B \equiv \Pi_{(x:A)}B\ \text{type}}$$

Derived rule:

$$\frac{\Gamma \;\vdash\; B \text{ type} \qquad \Gamma, \, x\!:\!A \;\vdash\; b(x) : B}{\Gamma \;\vdash\; \lambda x.b(x) : A \to B} \; \lambda$$

Derived rule:

$$\frac{\Gamma \;\vdash\; f : A \to B}{\Gamma, \, x\!:\!A \;\vdash\; f(x) : B} \; \text{ev}$$

Derived rule:

$$\frac{\Gamma \;\vdash\; B \text{ type} \qquad \Gamma, \, x\!:\!A \;\vdash\; b(x) : B}{\Gamma, \, x\!:\!A \;\vdash\; (\lambda y.b(y))(x) \equiv b(x) : B} \; \beta$$

Derived rule:

$$\frac{\Gamma \;\vdash\; f : A \to B}{\Gamma \;\vdash\; \lambda x.f(x) \equiv f : A \to B} \; \eta$$

The symbol $\to$ is right-associative:

$$X \to Y \to Z \quad := \quad X \to (Y \to Z)$$

2.2.2 Identity function

MLTT rule:

$$\frac{\Gamma \;\vdash\; A \text{ type}}{\Gamma \;\vdash\; \text{id}_A : A \to A}$$

MLTT rule:

$$\frac{\Gamma \;\vdash\; A \text{ type} \qquad \Gamma \;\vdash\; B \text{ type}}{\Gamma \;\vdash\; \text{id}_A \equiv \lambda x.x : A \to A}$$

2.2.3 Function composition

MLTT rule:

$$\frac{\Gamma \;\vdash\; A \text{ type} \qquad \Gamma \;\vdash\; B \text{ type} \qquad \Gamma \;\vdash\; C \text{ type}}{\Gamma \;\vdash\; \text{comp} : (B \to C) \to (A \to B) \to (A \to C)}$$

MLTT rule:

$$\frac{\Gamma \;\vdash\; A \text{ type} \qquad \Gamma \;\vdash\; B \text{ type} \qquad \Gamma \;\vdash\; C \text{ type}}{\Gamma \;\vdash\; \text{comp} \equiv \lambda g.\lambda f.\lambda x.g(f(x)) : (B \to C) \to (A \to B) \to (A \to C)}$$

Convention:
$$g \circ f \quad := \quad \mathrm{comp}(g, f) \equiv \lambda x.g(f(x))$$

Derived rule:
$$\frac{\Gamma \vdash f : A \to B \qquad \Gamma \vdash g : B \to C \qquad \Gamma \vdash h : C \to D}{\Gamma \vdash (h \circ g) \circ f \equiv h \circ (g \circ f) : A \to D}$$

3 The natural numbers

3.1 Basic

MLTT rule:
$$\frac{}{\vdash \mathbb{N} \text{ type}} \ \mathbb{N}\text{-form}$$

MLTT rule:
$$\frac{}{\vdash 0_{\mathbb{N}} : \mathbb{N}}$$

MLTT rule:
$$\frac{}{\vdash \mathrm{succ}_{\mathbb{N}} : \mathbb{N} \to \mathbb{N}}$$

3.2 Induction principle

Convention (used in many places):
$$f(x, y) \quad := \quad f(x)(y) \equiv (f(x))(y)$$

Metasyntactic shortcut (only for this document):
$$\text{ⓟ} \quad := \quad \Gamma, n : \mathbb{N} \vdash P(n) \text{ type}$$

MLTT rule:
$$\frac{\text{ⓟ} \qquad \Gamma \vdash p_0 : P(0_{\mathbb{N}}) \qquad \Gamma \vdash p_S : \Pi_{(n:\mathbb{N})} \left(P(n) \to P(\mathrm{succ}_{\mathbb{N}}(n)) \right)}{\Gamma \vdash \mathrm{ind}_{\mathbb{N}}(p_0, p_S) : \Pi_{(n:\mathbb{N})} P(n)} \ \mathbb{N}\text{-ind}$$

MLTT rule:
$$\frac{\text{ⓟ} \qquad \Gamma \vdash p_0 : P(0_{\mathbb{N}}) \qquad \Gamma \vdash p_S : \Pi_{(n:\mathbb{N})} \left(P(n) \to P(\mathrm{succ}_{\mathbb{N}}(n)) \right)}{\Gamma \vdash \mathrm{ind}_{\mathbb{N}}(p_0, p_S, 0_{\mathbb{N}}) \equiv p_0 : P(0_{\mathbb{N}})}$$

MLTT rule:

$$\dfrac{\text{(P)} \quad \Gamma \;\vdash\; p_0 : P(0_\mathbb{N}) \quad\quad \Gamma \;\vdash\; p_S : \Pi_{(n:\mathbb{N})}\,(P(n)\rightarrow P(\text{succ}_\mathbb{N}(n)))}{\Gamma,\, n:\mathbb{N} \;\vdash\; \text{ind}_\mathbb{N}(p_0, p_S, \text{succ}_\mathbb{N}(n)) \equiv p_S(n, \text{ind}_\mathbb{N}(p_0, p_S, n)) : P(\text{succ}_\mathbb{N}(n))}$$

3.3   Addition

MLTT rule:

$$\overline{m : \mathbb{N} \;\vdash\; \text{add-zero}_\mathbb{N}(m) : \mathbb{N}}$$

MLTT rule:

$$\overline{m : \mathbb{N} \;\vdash\; \text{add-zero}_\mathbb{N}(m) \equiv m : \mathbb{N}}$$

MLTT rule:

$$\overline{m : \mathbb{N} \;\vdash\; \text{add-succ}_\mathbb{N}(m) : \mathbb{N}\rightarrow\mathbb{N}\rightarrow\mathbb{N}}$$

MLTT rule:

$$\overline{m : \mathbb{N} \;\vdash\; \text{add-succ}_\mathbb{N}(m) \equiv \lambda n.\,\text{succ}_\mathbb{N} : \mathbb{N}\rightarrow\mathbb{N}\rightarrow\mathbb{N}}$$

MLTT rule:

$$\overline{m : \mathbb{N} \;\vdash\; \text{add}_\mathbb{N}(m) : \mathbb{N}\rightarrow\mathbb{N}}$$

MLTT rule:

$$\overline{m : \mathbb{N} \;\vdash\; \text{add}_\mathbb{N}(m) \equiv \text{ind}_\mathbb{N}(\text{add-zero}_\mathbb{N}(m), \text{add-succ}_\mathbb{N}(m)) : \mathbb{N}\rightarrow\mathbb{N}}$$

Remark:
$$m + 0_\mathbb{N} \equiv \text{add}_\mathbb{N}(m, 0_\mathbb{N}) \equiv m$$

Remark:

$$m + \text{succ}_\mathbb{N}(n) \equiv \text{add}_\mathbb{N}(m, \text{succ}_\mathbb{N}(n)) \equiv \text{succ}_\mathbb{N}(m + n)$$

4 More inductive types

4.1 The unit type

MLTT rule:

$$\overline{\vdash \mathbf{1}\ \text{type}}$$

MLTT rule:

$$\overline{\vdash \bigstar : \mathbf{1}}$$

MLTT rule:

$$\frac{\Gamma,\ x\!:\!\mathbf{1}\ \vdash\ P(x)\ \text{type}}{\Gamma\ \vdash\ \text{ind}_{\mathbf{1}} : P(\bigstar) \to \Pi_{(x:\mathbf{1})}P(x)}$$

MLTT rule:

$$\frac{\Gamma,\ x\!:\!\mathbf{1}\ \vdash\ P(x)\ \text{type} \qquad \Gamma\ \vdash\ p : P(\bigstar)}{\Gamma\ \vdash\ \text{ind}_{\mathbf{1}}(p, \bigstar) \equiv p : P(\bigstar)}$$

Derived rule:

$$\frac{\Gamma\ \vdash\ A\ \text{type}}{\Gamma\ \vdash\ \text{ind}_{\mathbf{1}} : A \to (\mathbf{1} \to A)}$$

Derived rule:

$$\frac{\Gamma\ \vdash\ A\ \text{type}}{\Gamma\ \vdash\ \text{ind}_{\mathbf{1}} \equiv \lambda a.\lambda x.a : A \to (\mathbf{1} \to A)}$$

4.2 The empty type

MLTT rule:

$$\overline{\vdash \emptyset\ \text{type}}$$

MLTT rule:

$$\frac{\Gamma,\ x\!:\!\emptyset\ \vdash\ P(x)\ \text{type}}{\Gamma\ \vdash\ \text{ind}_{\emptyset} : \Pi_{(x:\emptyset)}P(x)}$$

MLTT rule:

$$\frac{\Gamma\ \vdash\ A\ \text{type}}{\Gamma\ \vdash\ \text{ex-falso} : \emptyset \to A}$$

MLTT rule:

$$\frac{\Gamma\ \vdash\ A\ \text{type}}{\Gamma\ \vdash\ \text{ex-falso} \equiv \text{ind}_{\emptyset} : \emptyset \to A}$$

4.3   Negation

MLTT rule:
$$\frac{\Gamma \;\vdash\; A \text{ type}}{\Gamma \;\vdash\; \neg A \text{ type}}$$

MLTT rule:
$$\frac{\Gamma \;\vdash\; A \text{ type}}{\Gamma \;\vdash\; \neg A \equiv (A \to \emptyset) \text{ type}}$$

4.4   Coproducts

MLTT rule:
$$\frac{\Gamma \;\vdash\; A \text{ type} \qquad \Gamma \;\vdash\; B \text{ type}}{\Gamma \;\vdash\; A{+}B \text{ type}}$$

MLTT rule:
$$\frac{\Gamma \;\vdash\; A \text{ type} \qquad \Gamma \;\vdash\; B \text{ type}}{\Gamma \;\vdash\; \text{inl} : A \to A{+}B}$$

MLTT rule:
$$\frac{\Gamma \;\vdash\; A \text{ type} \qquad \Gamma \;\vdash\; B \text{ type}}{\Gamma \;\vdash\; \text{inr} : B \to A{+}B}$$

MLTT rule:
$$\frac{\Gamma \;\vdash\; A \text{ type} \qquad \Gamma \;\vdash\; B \text{ type} \qquad \Gamma, z : A{+}B \;\vdash\; P(z) \text{ type}}{\Gamma \;\vdash\; \text{ind}_{+} : \big(\Pi_{(x:A)}P(\text{inl}(x))\big) \to \big(\Pi_{(y:B)}P(\text{inr}(y))\big) \to \big(\Pi_{(z:A{+}B)}P(z)\big)}$$

MLTT rule:
$$\frac{\Gamma \;\vdash\; A \text{ type} \qquad \Gamma \;\vdash\; B \text{ type} \qquad \Gamma, z : A{+}B \;\vdash\; P(z) \text{ type}}{\Gamma,\, f : \Pi_{(x:A)}P(\text{inl}(x)),\, g : \Pi_{(y:B)}P(\text{inr}(y)),\, a : A \;\vdash\; \text{ind}_{+}(f, g, \text{inl}(a)) \equiv f(a) : P(\text{inl}(a))}$$

MLTT rule:
$$\frac{\Gamma \;\vdash\; A \text{ type} \qquad \Gamma \;\vdash\; B \text{ type} \qquad \Gamma, z : A{+}B \;\vdash\; P(z) \text{ type}}{\Gamma,\, f : \Pi_{(x:A)}P(\text{inl}(x)),\, g : \Pi_{(y:B)}P(\text{inr}(y)),\, b : B \;\vdash\; \text{ind}_{+}(f, g, \text{inr}(b)) \equiv g(b) : P(\text{inr}(b))}$$

Derived rule:
$$\frac{\Gamma \;\vdash\; A \text{ type} \qquad \Gamma \;\vdash\; B \text{ type} \qquad \Gamma \;\vdash\; X \text{ type}}{\Gamma \;\vdash\; \text{ind}_{+} : (A \to X) \to (B \to Y) \to (A{+}B \to X)}$$

Derived rule:

$$\frac{\Gamma \;\vdash\; A \text{ type} \qquad \Gamma \;\vdash\; A' \text{ type} \qquad \Gamma \;\vdash\; B \text{ type} \qquad \Gamma \;\vdash\; B' \text{ type}}{\Gamma,\, f : A \to A',\, g : B \to B' \;\vdash\; f + g \,:\, A + B \to A' + B'}$$

Derived rule:

$$\frac{\Gamma \;\vdash\; A \text{ type} \qquad \Gamma \;\vdash\; A' \text{ type} \qquad \Gamma \;\vdash\; B \text{ type} \qquad \Gamma \;\vdash\; B' \text{ type}}{\Gamma,\, f : A \to A',\, g : B \to B',\, x : A \;\vdash\; (f+g)(\mathrm{inl}(x)) \equiv \mathrm{inl}(f(x)) : A' + B'}$$

Derived rule:

$$\frac{\Gamma \;\vdash\; A \text{ type} \qquad \Gamma \;\vdash\; A' \text{ type} \qquad \Gamma \;\vdash\; B \text{ type} \qquad \Gamma \;\vdash\; B' \text{ type}}{\Gamma,\, f : A \to A',\, g : B \to B',\, y : B \;\vdash\; (f+g)(\mathrm{inr}(y)) \equiv \mathrm{inr}(g(y)) : A' + B'}$$

4.5 Σ types

4.5.1 Dependent pairs

MLTT rule:

$$\frac{\Gamma,\, x : A \;\vdash\; B(x) \text{ type}}{\Gamma \;\vdash\; \Sigma_{(x:A)} B(x) \text{ type}}$$

MLTT rule:

$$\frac{\Gamma,\, x : A \;\vdash\; B(x) \text{ type}}{\Gamma \;\vdash\; \mathrm{pr}_1 : \big(\Sigma_{(x:A)} B(x)\big) \to A}$$

MLTT rule:

$$\frac{\Gamma,\, x : A \;\vdash\; B(x) \text{ type}}{\Gamma,\, x : A,\, y : B(x) \;\vdash\; \mathrm{pr}_1((x,y)) \equiv x : A}$$

MLTT rule:

$$\frac{\Gamma,\, x : A \;\vdash\; B(x) \text{ type}}{\Gamma \;\vdash\; \mathrm{pr}_2 : \Pi_{\big(p : \Sigma_{(x:A)} B(x)\big)} B(\mathrm{pr}_1(p))}$$

MLTT rule:

$$\frac{\Gamma,\, x : A \;\vdash\; B(x) \text{ type}}{\Gamma,\, x : A,\, y : B(x) \;\vdash\; \mathrm{pr}_2((x,y)) \equiv y : B(x)}$$

Disclaimer: Many rules about dependent pairs are missing (hopefully, none of them is needed for the homework.

4.5.2 Ordinary pairs

MLTT rule:
$$\frac{\Gamma \;\vdash\; A \text{ type} \qquad \Gamma \;\vdash\; B \text{ type}}{\Gamma \;\vdash\; A \times B \text{ type}}$$

MLTT rule:
$$\frac{\Gamma \;\vdash\; A \text{ type} \qquad \Gamma \;\vdash\; B \text{ type}}{\Gamma \;\vdash\; A \times B \equiv \Sigma_{(x:A)} B \text{ type}}$$

MLTT rule:
$$\frac{\Gamma \;\vdash\; A \text{ type} \qquad \Gamma \;\vdash\; B \text{ type} \qquad \Gamma, x\!:\!A, y\!:\!B \;\vdash\; P(x,y) \text{ type}}{\Gamma \;\vdash\; \mathrm{ind}_\times : \big(\Pi_{(x:A)} \Pi_{(y:B)} P(x,y)\big) \to \big(\Pi_{(z:A\times B)} P(z)\big)}$$

MLTT rule:
$$\frac{\Gamma \;\vdash\; A \text{ type} \qquad \Gamma \;\vdash\; B \text{ type} \qquad \Gamma, x\!:\!A, y\!:\!B \;\vdash\; P(x,y) \text{ type}}{\Gamma, g\!:\!\Pi_{(x:A)} \Pi_{(y:B)} P(x,y), x\!:\!A, y\!:\!B \;\vdash\; \mathrm{ind}_\times(g,(x,y)) \equiv g(x,y)\!:\!P(x,y)}$$

Derived rule:
$$\frac{\Gamma \;\vdash\; A \text{ type} \qquad \Gamma \;\vdash\; B \text{ type}}{\Gamma \;\vdash\; \mathrm{pr}_1 : A \times B \to A}$$

Derived rule:
$$\frac{\Gamma \;\vdash\; A \text{ type} \qquad \Gamma \;\vdash\; B \text{ type}}{\Gamma, x\!:\!A, y\!:\!B \;\vdash\; \mathrm{pr}_1((x,y)) \equiv x : A}$$

Derived rule:
$$\frac{\Gamma \;\vdash\; A \text{ type} \qquad \Gamma \;\vdash\; B \text{ type}}{\Gamma \;\vdash\; \mathrm{pr}_2 : A \times B \to B}$$

Derived rule:
$$\frac{\Gamma \;\vdash\; A \text{ type} \qquad \Gamma \;\vdash\; B \text{ type}}{\Gamma, x\!:\!A, y\!:\!B \;\vdash\; \mathrm{pr}_2((x,y)) \equiv y : B}$$

5   Propositional equality

MLTT rule:
$$\frac{\Gamma \vdash a : A \qquad \Gamma \vdash b : A}{\Gamma \vdash a =_A b \;\; \text{type}}$$

MLTT rule:
$$\frac{\Gamma \vdash a : A}{\Gamma \vdash \text{refl}_a : \; a =_A a}$$

MLTT rule:

$$\frac{\Gamma, \, x : A, \, y : A, \, p : x =_A y \; \vdash \; P(x, y, p) \;\text{type}}{\Gamma \vdash J : \big(\Pi_{(x:A)} P(x, x, \text{refl}_x)\big) \to \big(\Pi_{(a:A)}\Pi_{(b:A)}\Pi_{(q:a=_A b)} P(a, b, q)\big)}$$

MLTT rule:

$$\frac{\Gamma, x : A, y : A, p : x =_A y \vdash P(x, y, p) \;\text{type} \qquad \Gamma \vdash d : \Pi_{(x:A)} P(x, x, \text{refl}_x) \qquad \Gamma \vdash a : A}{\Gamma \vdash J(d, a, a, \text{refl}_a) \equiv d(a) : P(a, a, \text{refl}_a)}$$