

1 Inference rules

1.1 Formation of contexts, types, and their elements

MLTT rule:

$$\frac{\Gamma, x : A \vdash B(x) \text{ type}}{\Gamma \vdash A \text{ type}}$$

MLTT rule:

$$\frac{\Gamma \vdash A \equiv B \text{ type}}{\Gamma \vdash A \text{ type}}$$

MLTT rule:

$$\frac{\Gamma \vdash A \equiv B \text{ type}}{\Gamma \vdash B \text{ type}}$$

MLTT rule:

$$\frac{\Gamma \vdash a : A}{\Gamma \vdash A \text{ type}}$$

MLTT rule:

$$\frac{\Gamma \vdash a \equiv b : A}{\Gamma \vdash A \text{ type}}$$

MLTT rule:

$$\frac{\Gamma \vdash a \equiv b : A}{\Gamma \vdash B \text{ type}}$$

1.2 Judgemental equality is an equivalence relation

MLTT rule:

$$\frac{\Gamma \vdash A \text{ type}}{\Gamma \vdash A \equiv A \text{ type}}$$

MLTT rule:

$$\frac{\Gamma \vdash A \equiv B \text{ type}}{\Gamma \vdash B \equiv A \text{ type}}$$

MLTT rule:

$$\frac{\Gamma \vdash A \equiv B \text{ type} \quad \Gamma \vdash B \equiv C \text{ type}}{\Gamma \vdash A \equiv C \text{ type}}$$

MLTT rule:

$$\frac{\Gamma \vdash a : A}{\Gamma \vdash a \equiv a : A}$$

MLTT rule:

$$\frac{\Gamma \vdash a \equiv b : A}{\Gamma \vdash b \equiv a : A}$$

MLTT rule:

$$\frac{\Gamma \vdash a \equiv b : A \quad \Gamma \vdash b \equiv c : A}{\Gamma \vdash a \equiv c : A}$$

1.3 Variable conversion

MLTT rule:

$$\frac{\Gamma \vdash A \equiv A' \text{ type} \quad \Gamma, x : A, \Delta \vdash B(x) \text{ type}}{\Gamma, x : A', \Delta \vdash B(x) \text{ type}}$$

MLTT rule:

$$\frac{\Gamma \vdash A \equiv A' \text{ type} \quad \Gamma, x : A, \Delta \vdash \mathcal{J}}{\Gamma, x : A', \Delta \vdash \mathcal{J}}$$

1.4 Substitution

MLTT rule:

$$\frac{\Gamma \vdash a : A \quad \Gamma, x : A, \Delta \vdash \mathcal{J}}{\Gamma, \Delta[a/x] \vdash \mathcal{J}[a/x]} S$$

MLTT rule:

$$\frac{\Gamma \vdash a \equiv a' : A \quad \Gamma, x : A, \Delta \vdash B \text{ type}}{\Gamma, \Delta[a/x] \vdash B[a/x] \equiv B[a'/x] \text{ type}}$$

MLTT rule:

$$\frac{\Gamma \vdash a \equiv a' : A \quad \Gamma, x : A, \Delta \vdash b : B}{\Gamma, \Delta[a/x] \vdash b[a/x] \equiv b[a'/x] : B[a/x]}$$

1.5 Weakening

MLTT rule:

$$\frac{\Gamma \vdash A \text{ type} \quad \Gamma, \Delta \vdash \mathcal{J}}{\Gamma, x : A, \Delta \vdash \mathcal{J}} W$$

MLTT rule:

$$\frac{\Gamma \vdash A \text{ type} \quad \Gamma \vdash B \text{ type}}{\Gamma, x : A \vdash B \text{ type}} W$$

1.6 The generic elements

MLTT rule:

$$\frac{\Gamma \vdash A \text{ type}}{\Gamma, x : A \vdash x : A} \delta$$

1.7 Manipulating variables

Derived rule:

$$\frac{\Gamma, x : A, \Delta \vdash \mathcal{J}}{\Gamma, x' : A, \Delta[x'/x] \vdash \mathcal{J}[x'/x]} x'/x$$

Derived rule:

$$\frac{\Gamma \vdash B \text{ type} \quad \Gamma, x : A, y : B, \Delta \vdash \mathcal{J}}{\Gamma, y : B, x : A, \Delta \vdash \mathcal{J}}$$

2 Dependent function types

2.1 Π formation

MLTT rule:

$$\frac{\Gamma, x : A \vdash B(x) \text{ type}}{\Gamma \vdash \Pi_{(x:A)} B(x) \text{ type}} \Pi$$

MLTT rule:

$$\frac{\Gamma \vdash A \equiv A' \text{ type} \quad \Gamma, x : A \vdash B(x) \equiv B'(x) \text{ type}}{\Gamma \vdash \Pi_{(x:A)} B(x) \equiv \Pi_{(x:A')} B'(x) \text{ type}} \Pi\text{-eq}$$