The American Mathematical Monthly, Vol. 60, No. 7 (Aug. - Sep., 1953), p. 474

Elementary proof that e is irrational

L. L. Pennisi, University of Illinois

The following variation on the usual proof of the irrationality of e is perhaps slightly simpler. Suppose that e is rational, say $e = a/b$. Then

$$\frac{b}{a} = \frac{1}{e} = \sum_{n=0}^{\infty} \frac{(-1)^n}{n!}$$

and multiplication by $(-1)^{a+1}a!$ and transposition of terms gives

$$(-1)^{a+1}\left\{b(a-1)! - \sum_{n=0}^{a}(-1)^n \frac{a!}{n!}\right\} = \frac{1}{(a+1)} - \frac{1}{(a+1)(a+2)} + \frac{1}{(a+1)(a+2)(a+3)} - \cdots$$

The right side has a value between 0 and 1 since the alternating series clearly converges to a value between its first term and the sum of its first two terms. But the left side is an integer, so we have a contradiction.