Time Delay Estimation in Gravitationally Lensed Photon Stream Pairs

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Outline

What is Gravitational Lensing?

- The bending of light due to gravitational effects
- Objects such as galaxy clusters affect the path of light
- Multiple images of the lensed object can be observed
- Source has a characteristic signal
- Images have the same signal, but with some time delay Δ

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Strong vs. Weak Lensing

Strong Lensing

Time delays can be on the order of hundreds of days

- Daily measurements of photon flux used to observe variation

Weak Lensing

Time delays are on a much shorter timescale

- Variation in the signal observed on the order of hours rather than days
- Track individual photon arrival times (streams of photons)

Strong vs. Weak Lensing

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Aim of the Project

Create a system to estimate the time delay Δ between pairs of photon streams from weakly lensed objects

Motivation

1. Form the base for a system to automatically flag potential lensed objects
   - Lots of data, but analysing it all is difficult
   - Flag interesting-looking objects for further investigation
2. Better estimates of time delay are useful
   - Improved estimates of H_0
   - Dark matter measurements
   - Mass distribution for regions of space

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Three main parts of the system

1. Photon stream simulation
2. Function estimation
3. Time delay estimation

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Photon Simulation

We use a nonhomogeneous poisson process to simulate arrival times.

- Rate parameter λ is the expected number of arrivals per unit time
- Waiting time until the next event has an exponential distribution
- Time to next event in homogeneous process $t = -\frac{1}{\lambda}\ln(U)$, where $U \sim U(0,1)$
- Use thinning on events generated using the above to generate times based on a nonhomogeneous process

Function Generation

To generate events, need some function $\lambda(t)$

- Randomly generate function by using Gaussians
- Centre Gaussians at uniform intervals Δt, with standard deviation $\alpha \cdot \Delta t$
- Sum the Gaussians to give a continuous function

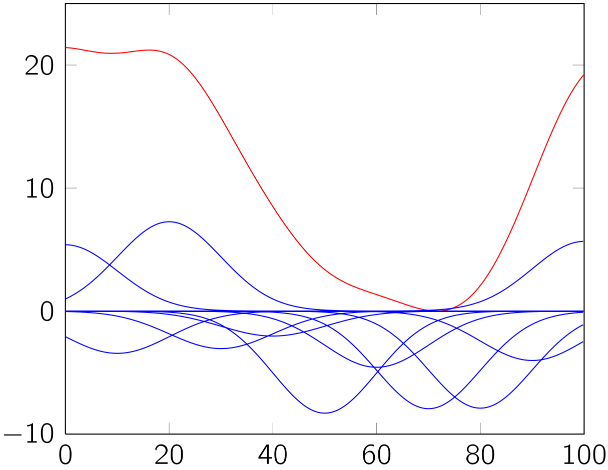

Figure: The red function is generated by summing the blue Gaussians. Gaussian values are multiplied by 3. Function is shifted so all $y \geq 0$

Three main parts of the system

1. Photon stream simulation
2. Function estimation
3. Time delay estimation

Show what residuals are

- Split the interval into bins
- Count the number of events that occur in each bin
- Estimate functions based on these counts

Iterative Weighted Least Squares

Estimate linear functions of the form $y = a + bx$ using Iterative Weighted Least Squares (IWLS)

- Find
$$\min_{\alpha,\beta} \sum_{k=1}^{n} w_k \cdot (Y_k - [\alpha + \beta x])^2$$
- α and β are estimators for a and b, w_k is the weight assigned to each value Y_k, which is the event count for the kth bin. x is the midpoint of the sub-interval.
- Update weights at each iteration by using estimated values of λ in each sub-interval.

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Piecewise

Some parts of functions can be reasonably approximated by straight lines

- Split the interval into several subintervals and estimate each in turn
- Once an estimate is done, extend the line to probe the next interval
- If the extension matches the data, keep it

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Baseline

Characteristic functions of photon streams are continuous - must make the piecewise estimate continuous as well.

- Modify each interval estimate to make a continuous function
- At each breakpoint, find the midpoint between the estimates
- Modify function values to make the end point of one interval estimate meet the start of the next

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Piecewise Estimate Example

Example piecewise estimate

Baseline Estimate vs Piecewise Estimate

Example of piecewise estimate compared to baseline for the same function

Kernel Density

- Centre a Gaussian kernel at each event time
- Sum Gaussians to approximate the function
- Must be normalised depending on standard deviation used
- Use probability density function to automatically calculate normalisation constant

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General Idea

The actual Δ is not known, so we make guesses and check to see how good they are.

- Pick a value of Δ and shift the function estimate
- Compare it to the other estimate and see how good the match is
- Hierarchical - coarse first pass, improve estimate with finer second pass

Area Between Curves

1. Approximate the area between the two function estimates $\hat{\lambda}_1$ and $\hat{\lambda}_2$

$$d(\hat{\lambda}_1, \hat{\lambda}_2) = \int (\hat{\lambda}_1(t) - \hat{\lambda}_2(t))^2 \, dt$$
$$\approx \frac{1}{N} \sum_{i=1}^{N} (\hat{\lambda}_1(t) - \hat{\lambda}_2(t))^2$$

2. Find the value of Δ for which $d(\hat{\lambda}_1, \hat{\lambda}_2)$ is minimised

Time Delay Estimation

Diagram for area between curves

Probability Density

1. Pick a value of Δ
2. Combine function estimates $\hat{\lambda}_2$ and $\hat{\lambda}_2$ into an "average" function $\overline{\lambda}$, where

$$\overline{\lambda}(t) = \frac{\hat{\lambda}_1(t) + \hat{\lambda}_2(t+\Delta)}{2}$$

3. See how well $\overline{\lambda}$ matches the data from the two streams

$$P(S_A, S_B \mid \overline{\lambda}(t)) = \sum_{t=\Delta}^{T-\Delta} \log P(S_A(t) \mid \overline{\lambda}(t)) + \log P(S_B(t+\Delta) \mid \overline{\lambda}(t))$$

Experimental Setup

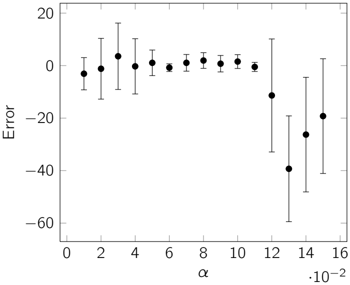
Three sets of experiments

1. Preliminary sine function experiments
   - Vary α in $y = a - b\sin(\alpha t)$
2. Experiments on a smaller range to show degradation
3. Random functions
   - Vary α where standard deviation of Gaussian $\sigma = \alpha \cdot \Delta t$

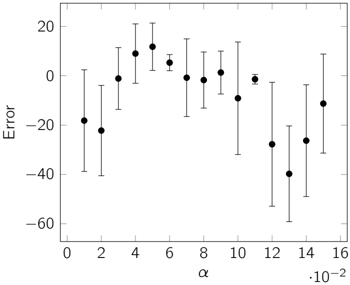
Results

	Gaussian	Baseline
Area	14.30 ± 1.48	14.22 ± 1.46
PDF	19.71 ± 3.09	20.31 ± 3.27

Figure: Experimental results for $\alpha = 0.06$ in the second set of sine function experiments ($\mu \pm \sigma$, $n = 10$). Actual delay is 15.

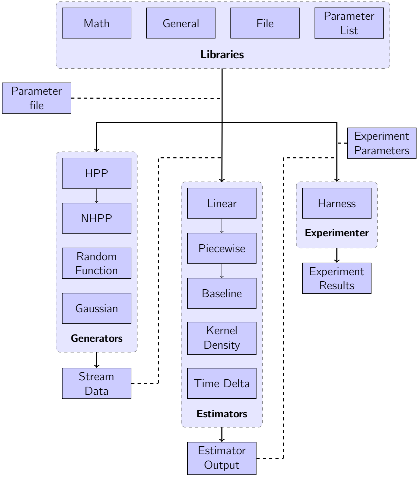


(a) Baseline area



(b) Baseline PDF

Code Structure



Summary

- We want to find the value of Δ, the time delay between photon stream arrival times
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- Estimation of characteristic function of stream using baseline or kernel density estimators
- Estimation of time delay with PDF or area estimators
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