CSCI 561
Foundation for Artificial Intelligence

15. Logic Reasoning Systems

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Logical Reasoning Systems

- Logic programming languages and Theorem provers
- Production systems
- Frame systems and semantic networks
- Description logic systems

Logical Reasoning Systems

- Theorem provers and logic programming languages – Provers: use resolution to prove sentences in full FOL. Languages: use backward chaining on restricted set of FOL constructs.
- Production systems – based on implications, with consequents interpreted as action (e.g., insertion & deletion in KB). Based on forward chaining + conflict resolution if several possible actions.
- Frame systems and semantic networks – objects as nodes in a graph, nodes organized as taxonomy, links represent binary relations.
- Description logic systems – evolved from semantic nets. Reason with object classes & relations among them.

Basic Tasks

- Add a new fact to KB – TELL
- Given KB and new facts, derive facts implied by conjunction of KB and the new facts. In forward chaining: part of TELL
- Decide if query entailed by KB – ASK
- Decide if query explicitly stored in KB – restricted ASK
- Remove sentence from KB: distinguish between correcting false sentence, forgetting useless sentence, or updating KB re. change in the world.

Indexing, retrieval & unification

- Implementing sentences & terms: define syntax and map sentences onto machine representation.

  Compound: has operator & arguments.

  e.g., $c = P(x) \wedge Q(x)$ $\quad\quad$ $Op[c] = \wedge$; $Args[c] = [P(x), Q(x)]$

- FETCH: find sentences in KB that have same structure as query.

  ASK makes multiple calls to FETCH.

- STORE: add each conjunct of sentence to KB. Used by TELL.

  e.g., implement KB as list of conjuncts

  TELL(KB, $A \wedge \neg B$) $\quad\quad$ TELL(KB, $\neg C \wedge D$)

  then KB contains: [A, $\neg B$, $\neg C$, D]

Complexity

- With the previous approaches:

    FETCH takes O(n) time on n-element KB

    STORE takes O(n) time on n-element KB (if check for duplicates)

- Faster Solutions = ?

Table-Based Indexing

- **What are you indexing on? Predicates (relations/functions). Example:**

Key	Positive	Negative	Conclusion	Premise
Mother	Mother(ann,sam) Mother(grace,joe)	-Mother(ann,al)	xxxx	xxxx
dog	dog(rover) dog(fido)	-dog(alice)	xxxx	xxxx

Table-Based Indexing

- Use hash table to avoid looping over entire KB for each TELL or FETCH

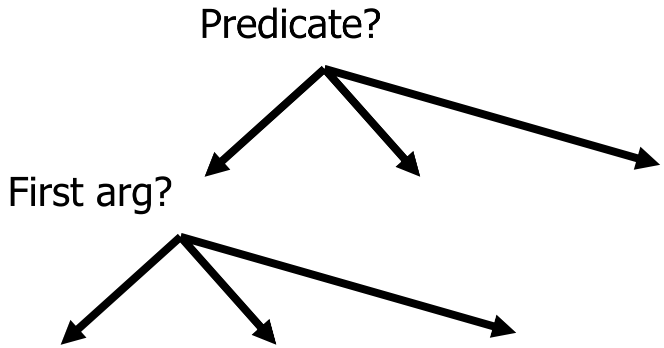
  e.g., if only allowed literals are single letters, use a 26-element array to store their values.

- More generally:
  - convert to Horn form
  - index table by predicate symbol
  - for each symbol, store:
    - list of positive literals
    - list of negative literals
    - list of sentences in which predicate is in conclusion
    - list of sentences in which predicate is in premise

Tree-Based Indexing

- Hash table impractical if many clauses for a given predicate symbol

- Tree-based indexing (or more generally combined indexing):
  compute indexing key from predicate and argument symbols



Tree-Based Indexing

Example:

Person(age,height,weight,income)

Person(30,72,210,45000)

Fetch(Person(age,72,210,income))

Fetch(Person(age,height>72,weight<210,income))

Unification Algorithm: Example

Understands(mary,x) implies Loves(mary,x)

Understands(mary,pete) allows the system to substitute pete for x, and make the implication that

IF Understands(mary,pete) THEN Loves(mary,pete)

Unification Algorithm

- Using clever indexing, can reduce number of calls to unification

- Still, unification is called very often (at basis of modus ponens) => need efficient implementation.

- See AIMA p. 303 for example of algorithm with $O(n^2)$ complexity (n being size of expressions being unified).

LOGIC PROGRAMMING

Logic Programming

Remember: knowledge engineering vs. programming…

Sound bite: computation as inference on logical KBs

	Logic programming	Ordinary programming
1.	Identify problem	Identify problem
2.	Assemble information	Assemble information
3.	Tea break	Figure out solution
4.	Encode information in KB	Program solution
5.	Encode problem instance as facts	Encode problem instance as data
6.	Ask queries	Apply program to data
7.	Find false facts	Debug procedural errors

Should be easier to debug $Capital(NewYork, US)$ than $x := x + 2$!

Logic Programming Systems

e.g., **Prolog:**

- Program = sequence of sentences (implicitly conjoined)
- All variables implicitly universally quantified
- Variables in different sentences considered distinct
- Horn clause sentences only (= atomic sentences or sentences with no negated antecedent and atomic consequent)
- Terms = constant symbols, variables or functional terms
- Queries = conjunctions, disjunctions, variables, functional terms
- Instead of negated antecedents, use negation as failure operator: goal NOT P considered proved if system fails to prove P
- Syntactically distinct objects refer to distinct objects
- Many built-in predicates (arithmetic, I/O, etc)

Prolog Systems

Basis: backward chaining with Horn clauses + bells & whistles
Widely used in Europe, Japan (basis of 5th Generation project)
Compilation techniques ⇒ 10 million LIPS

Program = set of clauses = `head :- literal`$_1$`, ... literal`$_n$`.`
Efficient unification by open coding
Efficient retrieval of matching clauses by direct linking
Depth-first, left-to-right backward chaining
Built-in predicates for arithmetic etc., e.g., `X is Y*Z+3`
Closed-world assumption ("negation as failure")
 e.g., `not PhD(X)` succeeds if `PhD(X)` fails

Basic syntax of facts, rules and queries

```
<fact>  ::= <term> .
<rule>  ::= <term> :- <term> .
<query> ::= <term> .
<term>  ::= <number> | <atom> | <variable> | <atom>
(<terms>)
<terms> ::= <term> | <term>, <terms>
```

Nice very concise intro to prolog:
https://www.cis.upenn.edu/~matuszek/Concise%20Guides/Concise%20Prolog.html

Basic syntax of facts, rules and queries

```
/* program P                       clause #      */

p(a).                              /* #1 */
p(X) :- q(X), r(X).                /* #2 */
p(X) :- u(X).                      /* #3 */

q(X) :- s(X).                      /* #4 */


r(a).                              /* #5 */
r(b).                              /* #6 */


s(a).                              /* #7 */
s(b).                              /* #8 */
s(c).                              /* #9 */

u(d).                              /* #10 */
```

q(X) ^ r(X) -> p(X)

https://www.csupomona.edu/~jrfisher/www/prolog_tutorial/contents.html

Basic syntax of facts, rules and queries

```
/* program P                     clause #      */

p(a).                            /* #1 */
p(X) :- q(X), r(X).              /* #2 */
p(X) :- u(X).                    /* #3 */

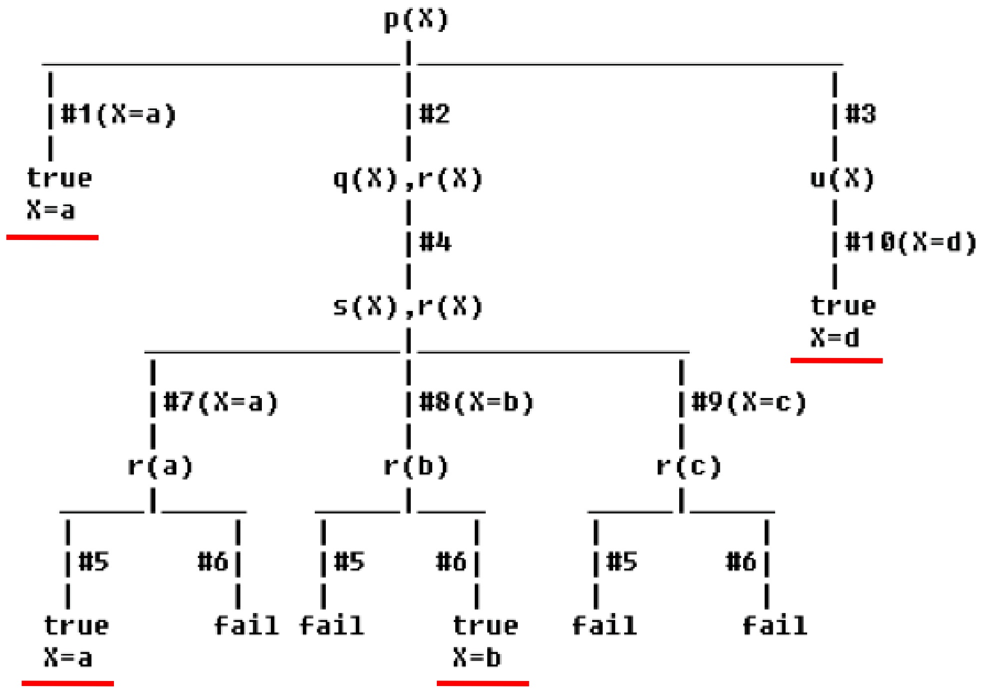
q(X) :- s(X).                    /* #4 */


r(a).                            /* #5 */
r(b).                            /* #6 */


s(a).                            /* #7 */
s(b).                            /* #8 */
s(c).                            /* #9 */

u(d).                            /* #10 */
```

Query for p(X) (i.e., for which values of X is p(X) true):



A PROLOG Program

- A PROLOG program is a set of ***facts*** and ***rules***.
- A simple program with just facts :

```
parent(alice, jim).
parent(jim,   tim).
parent(jim,   dave).
parent(jim,   sharon).
parent(tim,   james).
parent(tim,   thomas).
```

A PROLOG Program

- c.f. a table in a relational database.
- Each line is a ***fact*** (a.k.a. a tuple or a row).
- Each line states that some person `X` is a parent of some (other) person `Y`.
- In GNU PROLOG the program is kept in an ASCII file.

A PROLOG Query

- Now we can ask PROLOG questions :

```
| ?- parent(alice, jim).
yes
| ?- parent(jim, herbert).
no
| ?-
```

A PROLOG Query

- Not very exciting. But what about this :

```
| ?- parent(alice, Who).
Who = jim
yes
| ?-
```

- `Who` is called a ***logical variable***.
  - PROLOG will set a logical variable to any value which makes the query succeed.

A PROLOG Query II

- Sometimes there is more than one correct answer to a query.
- PROLOG gives the answers one at a time. To get the next answer type `;`.

```
| ?- parent(jim, Who).
Who = tim ? ;
Who = dave ? ;
Who = sharon ? ;
yes
| ?-
```

NB : The `;` do not actually appear on the screen.

- After finding that `jim` was a parent of `sharon` GNU PROLOG detects that there are no more alternatives for `parent` and ends the search.

conjunction

Prolog Example

Depth-first search from a start state X:

```
dfs(X) :- goal(X).
dfs(X) :- successor(X,S),dfs(S).
```

No need to loop over S: `successor` succeeds for each

Appending two lists to produce a third:

```
append([],Y,Y).
append([X|L],Y,[X|Z]) :- append(L,Y,Z).
```

“cons”

```
query:   append(A,B,[1,2]) ?
answers: A=[]     B=[1,2]
         A=[1]    B=[2]
         A=[1,2]  B=[]
```

Append

- **append([], L, L)**
- **append([H| L1], L2, [H| L3]) :- append(L1, L2, L3)**

- Example join [a, b, c] with [d, e].
  - [a, b, c] has the recursive structure [a| [b, c]].
  - Then the rule says:
  - IF [b, c] appends with [d, e] to form [b, c, d, e],
    THEN [a|[b, c]] appends with [d, e] to form [a|[b, c, d, e]]
  - i.e. [a, b, c] [a, b, c, d, e]

https://www.csupomona.edu/~jrfisher/www/prolog_tutorial/contents.html
http://www.cs.toronto.edu/~sheila/384/w11/simple-prolog-examples.html

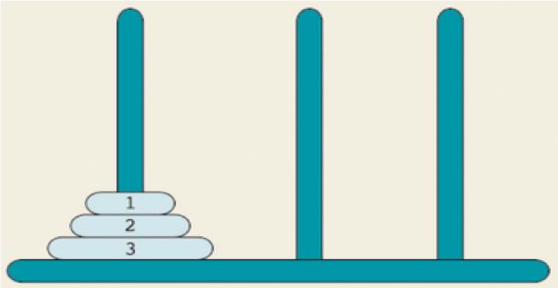
Tower of Hanoi in Prolog

```
% move(N,X,Y,Z) - move N disks from peg X to peg Y, with peg Z being the
%                 auxilliary peg
%
% Strategy:
% Base Case: One disc - To transfer a stack consisting of 1 disc from
%    peg X to peg Y, simply move that disc from X to Y
% Recursive Case: To transfer n discs from X to Y, do the following:
         Transfer the first n-1 discs to some other peg X
         Move the last disc on X to Y
         Transfer the n-1 discs from X to peg Y
```

Anonymous variable _

```
    move(1,X,Y,_) :-
        write('Move top disk from '),
        write(X),
        write(' to '),
        write(Y),
        nl.
    move(N,X,Y,Z) :-
        N>1,
        M is N-1,
        move(M,X,Z,Y),
        move(1,X,Y,_),
        move(M,Z,Y,X).
```

Write a newline



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        M is N-1,
        move(M,X,Z,Y),
        move(1,X,Y,_),
        move(M,Z,Y,X).
```

Here is what happens when Prolog solves the case N=3.

```
?-  move(3,left,right,center).
Move top disk from left to right
Move top disk from left to center
Move top disk from right to center
Move top disk from left to right
Move top disk from center to left
Move top disk from center to right
Move top disk from left to right

yes
```

http://web.cse.ohio-state.edu/~stiff.4/cse3521/prolog-examples.html

Sudoku in Prolog

The key to solving Sudoku puzzles with Prolog is to use the `clpfd` (constraint logic programming over finite domains) library to restrict the search space to numbers 1-9. Then, it's just a matter of describing what a solution looks like.

Availability: `:- use_module(library(apply)).` *(can be autoloaded)*

maplist(*:Goal, ?List*)

True if *Goal* can successfully be applied on all elements of *List*. Arguments are reordered to gain performance as well as to make the predicate deterministic under normal circumstances.

Availability: `:- use_module(library(clpfd)).`

+Vars **ins** *+Domain*

The variables in the list *Vars* are elements of *Domain*. See in/2 for the syntax of *Domain*.

https://www.swi-prolog.org/pldoc/doc/_CWD_/index.html

http://web.cse.ohio-state.edu/~stiff.4/cse3521/prolog-examples.html

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```
%% need the module "clpfd" (constraint logic programming over finite domains)
%% so that we can specify the range of numbers to search
?- use_module(library(clpfd)).

sudoku(Rows) :-
        length(Rows, 9),  % ensure there are 9 rows
        maplist(length_(9), Rows),  % ensure each row has 9 elements (see below for length_)
        append(Rows, Vs),  % combined all rows into the variable Vs
        Vs ins 1..9,  % ensure that the elements of Vs should be numbers 1-9
        maplist(all_distinct, Rows),  % ensure each row is distinct
        transpose(Rows, Columns),  % flip the matrix
        maplist(all_distinct, Columns),  % ensure each column is distinct
        Rows = [A,B,C,D,E,F,G,H,I],  % create variables A-I for each row
        blocks(A, B, C),  % make sure all values in these three rows (three 3x3 blocks) are distinct
        blocks(D, E, F),  % ... and these rows/blocks
        blocks(G, H, I).  % ... and these rows/blocks
```

http://web.cse.ohio-state.edu/~stiff.4/cse3521/prolog-examples.html

Sudoku in Prolog

```
length_(L, Ls) :- length(Ls, L).  % version of length that's easierto use with maplist (above)

% this predicate ensures a "block" (3x3 grid) contains only distinct values
blocks([], [], []).
blocks([A,B,C|Bs1], [D,E,F|Bs2], [G,H,I|Bs3]) :-
        all_distinct([A,B,C,D,E,F,G,H,I]),
        blocks(Bs1, Bs2, Bs3).

problem([[_,_,_,_,_,_,_,_,_],
          [_,_,_,_,_,3,_,8,5],
          [_,_,1,_,2,_,_,_,_],
          [_,_,_,5,_,7,_,_,_],
          [_,_,4,_,_,_,1,_,_],
          [_,9,_,_,_,_,_,_,_],
          [5,_,_,_,_,_,_,7,3],
          [_,_,2,_,1,_,_,_,_],
          [_,_,_,_,4,_,_,_,9]]).
```

Sudoku in Prolog

```
solve_problems :-
        problem(Rows),
        statistics(runtime, _),  % builtin function, establishes "runtime" variable
        sudoku(Rows),  % solve the puzzle
        maplist(writeln, Rows),  % show the solution (writeln, i.e., println each row)
        statistics(runtime, [_,T]),  % use "runtime" variable to compute total time
        write('CPU time = '), write(T), write(' msec'), nl, nl, false. % write total time
```

http://web.cse.ohio-state.edu/~stiff.4/cse3521/prolog-examples.html

Sudoku in Prolog

```
itti@iLab0:~$ prolog sudoku.pl
Welcome to SWI-Prolog (threaded, 64 bits, version 7.6.4)
SWI-Prolog comes with ABSOLUTELY NO WARRANTY. This is free software.
Please run ?- license. for legal details.

For online help and background, visit http://www.swi-prolog.org
For built-in help, use ?- help(Topic). or ?- apropos(Word).

?- solve_problems.
[9,8,7,6,5,4,3,2,1]
[2,4,6,1,7,3,9,8,5]
[3,5,1,9,2,8,7,4,6]
[1,2,8,5,3,7,6,9,4]
[6,3,4,8,9,2,1,5,7]
[7,9,5,4,6,1,8,3,2]
[5,1,9,2,8,6,4,7,3]
[4,7,2,3,1,9,5,6,8]
[8,6,3,7,4,5,2,1,9]
CPU time = 44 msec

false.

?-
```

Note: returned false here only so that the program would then try to solve any other declared Instance of problem( ).

Expanding Prolog

- Parallelization:
  OR-parallelism: goal may unify with many different literals and implications in KB
  AND-parallelism: solve each conjunct in body of an implication in parallel

- Compilation: generate built-in theorem prover for different predicates in KB

- Optimization: for example through re-ordering
  e.g., "what is the income of the spouse of the president?"
  $Income(s, i) \land Married(s, p) \land Occupation(p, President)$
  faster if re-ordered as:
  $Occupation(p, President) \land Married(s, p) \land Income(s, i)$

THEOREM PROVERS

Theorem Provers

- Differ from logic programming languages in that:
  - accept full FOL
  - results independent of form in which KB entered

OTTER

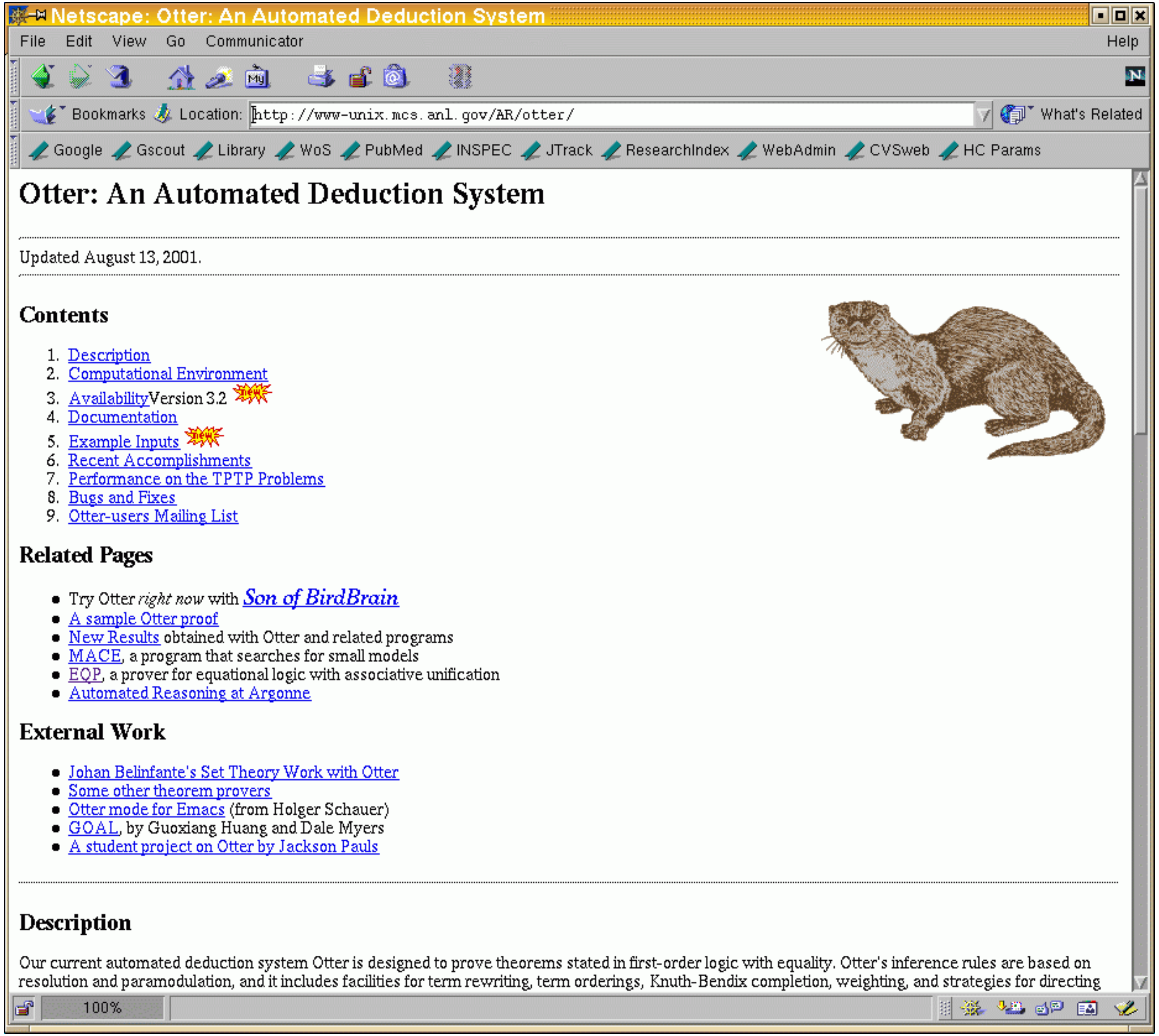
- Organized Techniques for Theorem Proving and Effective Research (McCune, 1992)

- Set of Support (SOS): set of clauses defining facts about a given problem
- Each resolution step: resolves member of sos against other axiom
- Usable axioms (outside sos): provide background knowledge about domain
- Rewrites (or demodulators): define canonical forms into which terms can be simplified. E.g., x+0=x
- Control strategy: defined by set of parameters and clauses. E.g., heuristic function to control search, filtering function to eliminate uninteresting subgoals.

OTTER

- Operation: resolve elements of SOS against usable axioms
- Use best-first search: heuristic function measures “weight” of each clause (lighter weight preferred; thus in general weight correlated with size/difficulty)
- At each step: move lightest clause in SOS to usable list, and add to usable list consequences of resolving that clause against usable list
- Halt: when refutation found or SOS is empty

OTTER


Netscape: Otter: An Automated Deduction System
File Edit View Go Communicator Help
Bookmarks Location: http://www-unix.mcs.anl.gov/AR/otter/ What's Related
Google Gscout Library WoS PubMed INSPEC JTrack ResearchIndex WebAdmin CVSweb HC Params
Otter: An Automated Deduction System
Updated August 13, 2001.
Contents
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Related Pages
• Try Otter *right now* with Son of BirdBrain
• A sample Otter proof
• New Results obtained with Otter and related programs
• MACE, a program that searches for small models
• EQP, a prover for equational logic with associative unification
• Automated Reasoning at Argonne
External Work
• Johan Belinfante's Set Theory Work with Otter
• Some other theorem provers
• Otter mode for Emacs (from Holger Schauer)
• GOAL, by Guoxiang Huang and Dale Myers
• A student project on Otter by Jackson Pauls
Description
Our current automated deduction system Otter is designed to prove theorems stated in first-order logic with equality. Otter's inference rules are based on resolution and paramodulation, and it includes facilities for term rewriting, term orderings, Knuth-Bendix completion, weighting, and strategies for directing
100%

Example: Robbins Algebras Are Boolean

- The Robbins problem---are all Robbins algebras Boolean?---has been solved: Every Robbins algebra is Boolean. This theorem was proved automatically by EQP, a theorem proving program developed at Argonne National Laboratory

https://www.mcs.anl.gov/research/projects/AR/eqp/

Example: Robbins Algebras Are Boolean

```
-----------------------------------------------------
                         0 --x-- -----y-----|
      |-----y----- --x--                           // ¬ (x+y)
                          --x—
                |-------y-------                   // x + ¬ (y)
                |-(y-x)-|
                          -(y-x)-|                 // ¬ (x + ¬ (y))
```

Historical Background

- In 1933, E. V. Huntington presented the following basis for Boolean algebra:

x + y = y + x.	**[commutativity]**	**// + is ∨**
(x + y) + z = x + (y + z).	**[associativity]**	
n(n(x) + y) + n(n(x) + n(y)) = x.	**[Huntington equation]**	**// n is ¬**

- Shortly thereafter, Herbert Robbins conjectured that the Huntington equation can be replaced with a simpler one:

n(n(x + y) + n(x + n(y))) = x.	**[Robbins equation]**

- Robbins and Huntington could not find a proof, and the problem was later studied by Tarski and his students

~α

KB: Given to the system

```
Searching ...

Success, in 1.28 seconds!

---------------- PROOF ----------------

1         n(n(A)+B)+n(n(A)+n(B))!=A.                 []
2         x=x.                                       []
3         x+y=y+x.                                   []
5,4        (x+y)+z=x+ (y+z).                         []
6         n(n(x+y)+n(x+n(y)))=x.                     []
8         x+x=x.                                     []

10        n(n(A)+n(B))+n(n(A)+B)!=A.                 [para_from,3,1]
13        x+ (x+y)=x+y.                              [para_into,4,8,flip.1]
15        x+ (y+z)=y+ (x+z).                         [para_into,4,3,demod,5]
23,22     x+ (y+x)=x+y.                              [para_into,13,3]
26        n(n(x)+n(x+n(x)))=x.                       [para_into,6,8]
36        n(n(n(x)+x)+n(n(x)))=n(x).                 [para_into,6,8]
42        n(n(x+n(y))+n(x+y))=x.                     [para_into,6,3]
52        x+ (y+z)=x+ (z+y).                         [para_into,15,3,demod,5]
81,80     n(n(x+n(x))+n(x))=x.                       [para_into,26,3]
82        n(n(n(x)+x)+x)=n(x).                       [para_from,26,6,demod,23]
125       n(n(n(x+n(x))+ (n(x)+x))+x)=n(x+n(x))+n(x).  [para_into,80,80,demod,5,81]
139       n(n(n(x+n(x))+x)+x)=n(x+n(x)).             [para_from,80,6]
166,165   n(n(x+n(x))+x)=n(x).                       [para_into,82,3]
180,179   n(n(x)+x)=n(x+n(x)).                       [back_demod,139,demod,166]
195       n(n(x+n(x))+n(n(x)))=n(x).                 [back_demod,36,demod,180]
197       n(n(x+ (n(x)+n(x+n(x))))+ (n(x+n(x))+x))=n(x).  [para_into,165,165,demod,5,180,5,166]
206,205   n(n(x+ (n(x)+n(x+n(x))))+n(x))=n(x+n(x))+x.  [para_from,165,80,demod,166,5,180,5]
223,222   n(n(x+y)+ (y+x))=n(x+ (y+n(x+y))).         [para_into,179,52,demod,5]
231,230   n(n(x+ (n(x)+n(x+n(x))))+x)=n(x+n(x))+n(x).  [back_demod,125,demod,223]
564,563   n(x+n(x))+x=x.                             [para_into,195,80,demod,5,223,81,206,81]
582,581   n(x+n(x))+n(x)=n(x).                       [back_demod,197,demod,564,231]
586,585   n(n(x))=x.                                 [back_demod,80,demod,582]
606,605   n(x+n(y))+n(x+y)=n(x).                     [para_into,585,42,flip.1]
621       A!=A.                                      [back_demod,10,demod,606,586]
622       $F.                                        [binary,621,2]

------------ end of proof -------------
```

Logical Reasoning Systems

- Logic programming languages and Theorem provers
- <u>Production systems</u>
- Frame systems and semantic networks
- Description logic systems

PRODUCTION SYSTEMS

Forward-chaining Production Systems

- Prolog & other programming languages: rely on backward-chaining (I.e., given a query, find substitutions that satisfy it)

- Forward-chaining systems: infer everything that can be inferred from KB each time new sentence is TELL'ed

- Appropriate for agent design: as new percepts come in, forward-chaining returns best action

Implementation (Production System)

- One possible approach: use a theorem prover, using resolution to forward-chain over KB
- More restricted systems can be more efficient.
- Typical components:
  - KB called “working memory” (positive literals, no variables)
  - rule memory (set of inference rules in form

    $p1 \land p2 \land \ldots \Rightarrow act1 \land act2 \land \ldots$
  - at each cycle: find rules whose premises satisfied by working memory (match phase)
  - decide which should be executed (conflict resolution phase)
  - execute actions of chosen rule (act phase)

Match phase

- Unification can do it, but inefficient

- Rete algorithm (used in OPS-5 system): example

rule memory:

$A(x) \wedge B(x) \wedge C(y) \Rightarrow$ add $D(x)$

$A(x) \wedge B(y) \wedge D(x) \Rightarrow$ add $E(x)$

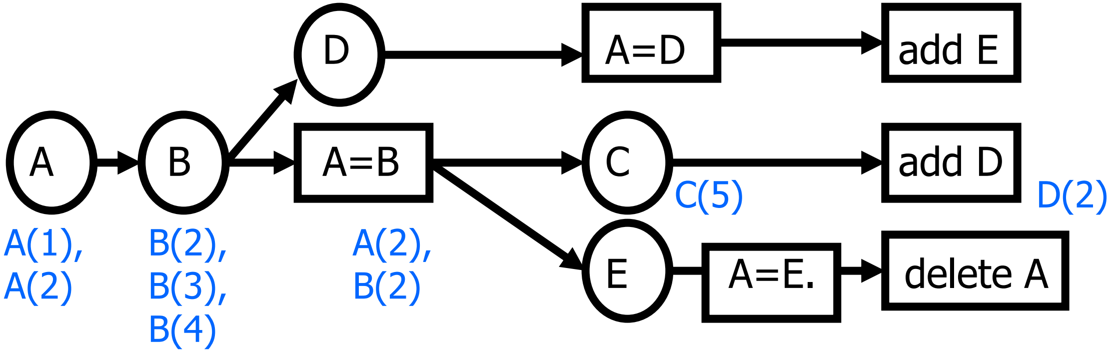
$A(x) \wedge B(x) \wedge E(x) \Rightarrow$ delete $A(x)$

working memory:

$\{A(1), A(2), B(2), B(3), B(4), C(5)\}$

- Build Rete network from rule memory, then pass working memory through it

Rete network



Circular nodes: fetches to WM; rectangular nodes: unifications

$A(x) \wedge B(x) \wedge C(y) \Rightarrow$ add $D(x)$

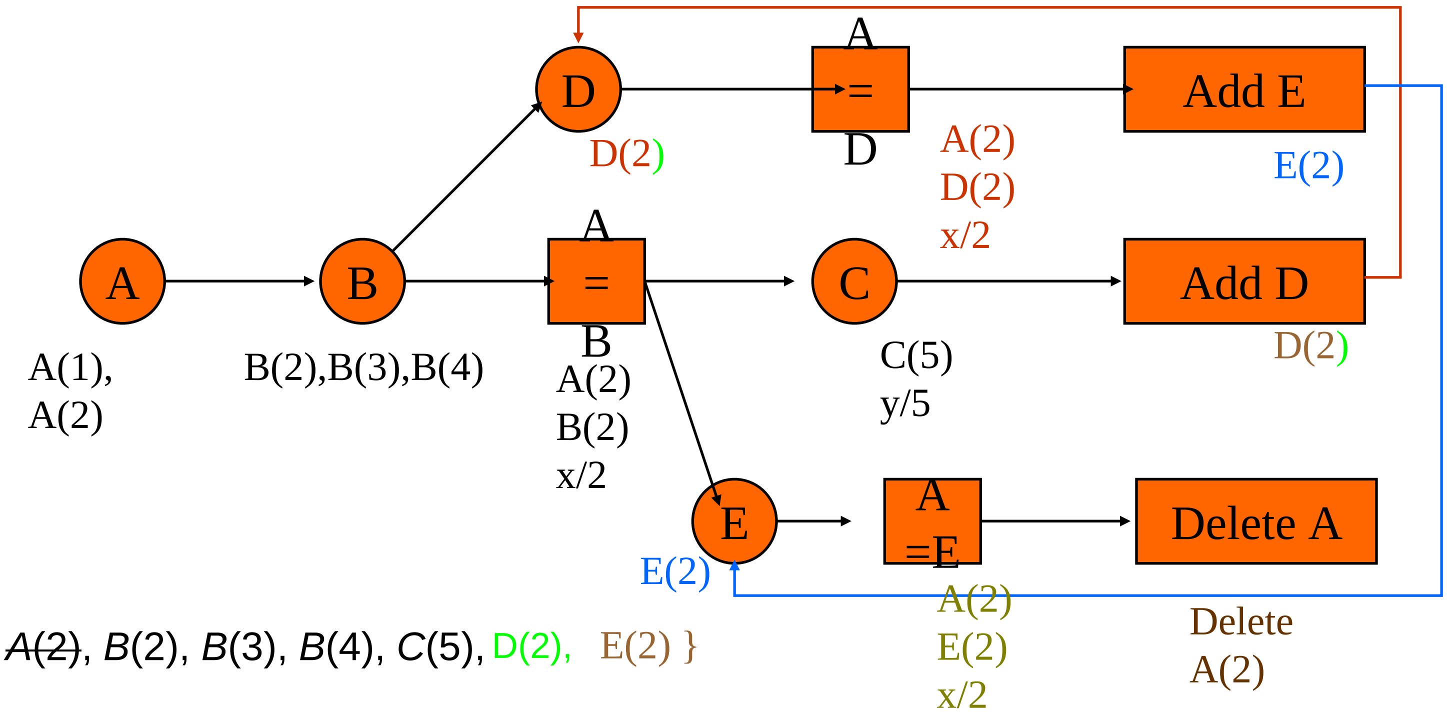
$A(x) \wedge B(y) \wedge D(x) \Rightarrow$ add $E(x)$

$A(x) \wedge B(x) \wedge E(x) \Rightarrow$ delete $A(x)$

{A(1), A(2), B(2), B(3), B(4), C(5)}

Rete match

$A(x) \wedge B(x) \wedge C(y) \Rightarrow \text{add } D(x)$
$A(x) \wedge B(y) \wedge D(x) \Rightarrow \text{add } E(x)$
$A(x) \wedge B(x) \wedge E(x) \Rightarrow \text{delete } A(x)$



{ A(1), ~~A(2)~~, B(2), B(3), B(4), C(5), D(2), E(2) }

Advantages of Rete networks

- Share common parts of rules
- Eliminate duplication over time (since for most production systems only a few rules change at each time step)

Conflict resolution phase

- one strategy: execute all actions for all satisfied rules
- or, treat them as suggestions and use conflict resolution to pick one action.
- Strategies:
  - no duplication (do not execute twice same rule on same args)
  - regency (prefer rules involving recently created WM elements)
  - specificity (prefer more specific rules)
  - operation priority (rank actions by priority and pick highest)

Logical Reasoning Systems

- Theorem provers and logic programming languages
- Production systems
- <u>Frame systems and semantic networks ☐</u>
- Description logic systems

FRAME SYSTEMS AND SEMANTIC NETWORKS

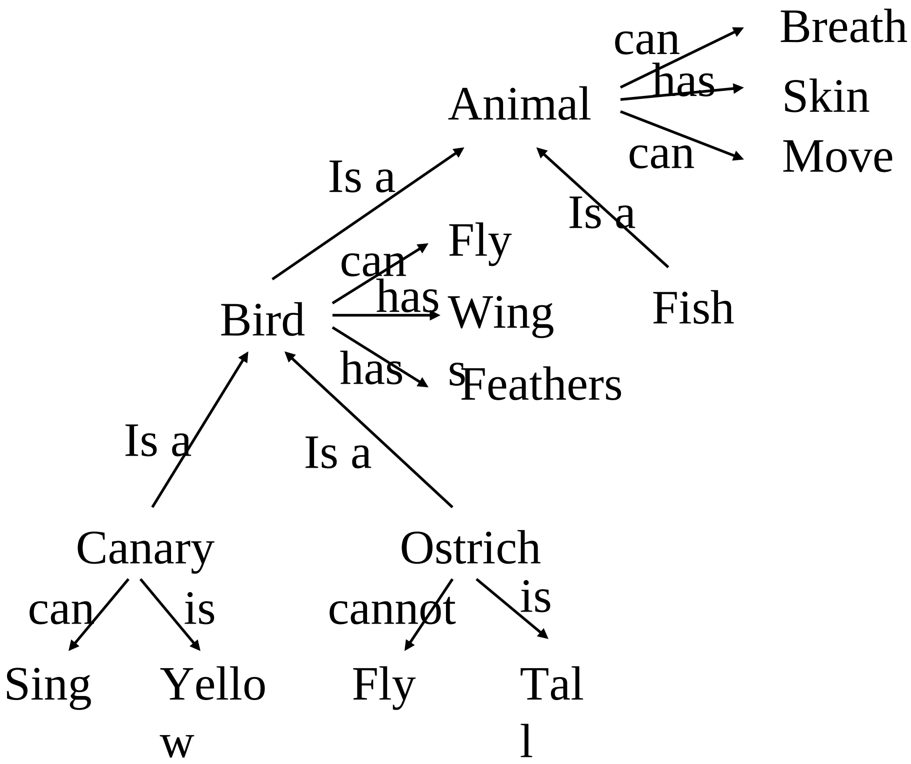
Frame systems & semantic networks

- Other notation for logic; equivalent to sentence notation
- Focus on categories and relations between them (remember ontologies)
- e.g., Cats $\xrightarrow{\textit{Subset}}$ Mammals

Syntax and Semantics

Link	Semantics
Type Subset	$A \subset B$
$A \xrightarrow{\text{Member}} B$	$A \in B$
$A \xrightarrow{R} B$	$R(A,B)$
$A \xrightarrow{\boxed{R}} B$	$\forall x\ [\ x \in A \Rightarrow R(x,y)\ ]$
$A \xrightarrow{\boxed{\boxed{R}}} B$	$\forall x\ \exists y\ [\ x \in A \Rightarrow y \in B \wedge R(x,y)$
$A \rightarrow B$	]

Semantic Network Representation



Semantic network link types

Link type	Semantics	Example
$A \xrightarrow{\mathit{Subset}} B$	$A \subset B$	$\text{Cats} \xrightarrow{\mathit{Subset}} \text{Mammals}$
$A \xrightarrow{\mathit{Member}} B$	$A \in B$	$\text{Bill} \xrightarrow{\mathit{Member}} \text{Cats}$
$A \xrightarrow{R} B$	$R(A, B)$	$\text{Bill} \xrightarrow{\mathit{Age}} 12$
$A \xrightarrow{\boxed{R}} B$	$\forall x \quad x \in A \Rightarrow R(x, B)$	$\text{Birds} \xrightarrow{\boxed{\mathit{Legs}}} 2$
$A \xrightarrow{\boxed{\boxed{R}}} B$	$\forall x\ \exists y\ x \in A \Rightarrow y \in B \wedge R(x, y)$	$\text{Birds} \xrightarrow{\boxed{\boxed{\mathit{Parent}}}} \text{Birds}$

Logical Reasoning Systems

- Theorem provers and logic programming languages

- Production systems

- Frame systems and semantic networks

- Description logic systems ☐

DESCRIPTION LOGIC SYSTEMS

http://www.cs.ox.ac.uk/ian.horrocks/Seminars/download/Horrocks_Ian_pt1.pdf

Description logic

Goal: Make it easier to describe categories (as opposed to objects)
Popularized by projects such as the Semantic Web

DL Syntax

- Signature
  - **Concept** (aka class) names, e.g., Cat, Animal, Doctor
    - Equivalent to FOL unary predicates
  - **Role** (aka property) names, e.g., sits-on, hasParent, loves
    - Equivalent to FOL binary predicates
  - **Individual** names, e.g., Felix, John, Mary, Boston, Italy
    - Equivalent to FOL constants
- Operators
  - Many kinds available, e.g.,
    - Standard FOL Boolean operators ($\sqcap$, $\sqcup$, $\neg$)
    - Restricted form of quantifiers ($\exists$, $\forall$)
    - Counting ($\geq$, $\leq$, $=$)
    - …

Description logic

- Concept expressions, e.g.,
  - $\text{Doctor} \sqcup \text{Lawyer}$
  - $\text{Rich} \sqcap \text{Happy}$
  - $\text{Cat} \sqcap \exists \text{sits-on}.\text{Mat}$
- Equivalent to FOL formulae with one free variable
  - $\text{Doctor}(x) \vee \text{Lawyer}(x)$
  - $\text{Rich}(x) \wedge \text{Happy}(x)$
  - $\exists y.(\text{Cat}(x) \wedge \text{sits-on}(x, y))$
- Special concepts
  - $\top$ (aka top, Thing, most general concept)
  - $\bot$ (aka bottom, Nothing, inconsistent concept)

  used as abbreviations for
  - $(A \sqcup \neg A)$ for any concept A
  - $(A \sqcap \neg A)$ for any concept A

Description logic

- Role expressions, e.g.,
  - loves^-
  - $\text{hasParent} \circ \text{hasBrother}$
- Equivalent to FOL formulae with two free variables
  - $\text{loves}(y, x)$
  - $\exists z.(\text{hasParent}(x, z) \land \text{hasBrother}(z, y))$
- "Schema" Axioms, e.g.,
  - $\text{Rich} \sqsubseteq \neg\text{Poor}$ (concept inclusion)
  - $\text{Cat} \sqcap \exists\text{sits-on.Mat} \sqsubseteq \text{Happy}$ (concept inclusion)
  - $\text{BlackCat} \equiv \text{Cat} \sqcap \exists\text{hasColour.Black}$ (concept equivalence)
  - $\text{sits-on} \sqsubseteq \text{touches}$ (role inclusion)
  - Trans(part-of) (transitivity)
- Equivalent to (particular form of) FOL sentence, e.g.,
  - $\forall x.(\text{Rich}(x) \rightarrow \neg\text{Poor}(x))$
  - $\forall x.(\text{Cat}(x) \land \exists y.(\text{sits-on}(x,y) \land \text{Mat}(y)) \rightarrow \text{Happy}(x))$
  - $\forall x.(\text{BlackCat}(x) \leftrightarrow (\text{Cat}(x) \land \exists y.(\text{hasColour}(x,y) \land \text{Black}(y)))$
  - $\forall x,y.(\text{sits-on}(x,y) \rightarrow \text{touches}(x,y))$
  - $\forall x,y,z.((\text{sits-on}(x,y) \land \text{sits-on}(y,z)) \rightarrow \text{sits-on}(x,z))$

Description logic

- "Data" Axioms (aka Assertions or Facts), e.g.,
  - BlackCat(Felix) (concept assertion)
  - Mat(Mat1) (concept assertion)
  - Sits-on(Felix,Mat1) (role assertion)
- Directly equivalent to FOL "ground facts"
  - Formulae with no variables

Knowledge base = set of axioms (TBox) + set of facts (ABox)

http://www.cs.ox.ac.uk/ian.horrocks/Seminars/download/Horrocks_Ian_pt1.pdf

The DL family

- Many different DLs, often with "strange" names
  - E.g., $\mathcal{EL}$, $\mathcal{ALC}$, $\mathcal{SHIQ}$
- Particular DL defined by:
  - Concept operators ($\sqcap$, $\sqcup$, $\neg$, $\exists$, $\forall$, etc.)
  - Role operators ($^-$, $\circ$, etc.)
  - Concept axioms ($\sqsubseteq$, $\equiv$, etc.)
  - Role axioms ($\sqsubseteq$, Trans, etc.)

The DL family

- E.g., $\mathcal{EL}$ is a well known "sub-Boolean" DL
  - Concept operators: $\sqcap$, $\neg$, $\exists$
  - No role operators (only atomic roles)
  - Concept axioms: $\sqsubseteq$, $\equiv$
  - No role axioms
- E.g.:

$$\text{Parent} \equiv \text{Person} \sqcap \exists \text{hasChild.Person}$$

- $\mathcal{ALC}$ is the smallest propositionally closed DL
  - Concept operators: $\sqcap$, $\sqcup$, $\neg$, $\exists$, $\forall$
  - No role operators (only atomic roles)
  - Concept axioms: $\sqsubseteq$, $\equiv$
  - No role axioms
- E.g.:

$$\text{ProudParent} \equiv \text{Person} \sqcap \forall \text{hasChild.}(\text{Doctor} \sqcup \exists \text{hasChild.Doctor})$$

The DL family

- $\mathcal{S}$ used for $\mathcal{ALC}$ extended with (role) transitivity axioms
- Additional letters indicate various extensions, e.g.:
  - $\mathcal{H}$ for role hierarchy (e.g., hasDaughter $\sqsubseteq$ hasChild)
  - $\mathcal{R}$ for role box (e.g., hasParent $\circ$ hasBrother $\sqsubseteq$ hasUncle)
  - $\mathcal{O}$ for nominals/singleton classes (e.g., {Italy})
  - $\mathcal{I}$ for inverse roles (e.g., isChildOf $\equiv$ hasChild^{-})
  - $\mathcal{N}$ for number restrictions (e.g., $\geqslant$2hasChild, $\leqslant$3hasChild)
  - $\mathcal{Q}$ for qualified number restrictions (e.g., $\geqslant$2hasChild.Doctor)
  - $\mathcal{F}$ for functional number restrictions (e.g., $\leqslant$1hasMother)
- E.g., $\mathcal{SHIQ}$ = $\mathcal{S}$ + role hierarchy + inverse roles + QNRs

Example

- W3C's OWL 2 (like OWL, DAML+OIL & OIL) based on DL
  - OWL 2 based on $\mathcal{SROIQ}$, i.e., $\mathcal{ALC}$ extended with transitive roles, a role box nominals, inverse roles and qualified number restrictions
    - OWL 2 EL based on $\mathcal{EL}$
    - OWL 2 QL based on DL-Lite
    - OWL 2 EL based on $\mathcal{DLP}$
  - OWL was based on $\mathcal{SHOIN}$
    - only simple role hierarchy, and unqualified NRs

http://www.cs.ox.ac.uk/ian.horrocks/Seminars/download/Horrocks_Ian_pt1.pdf

Example: OWL2

OWL Constructor	DL Syntax	Example
intersectionOf	$C_1 \sqcap \ldots \sqcap C_n$	Human $\sqcap$ Male
unionOf	$C_1 \sqcup \ldots \sqcup C_n$	Doctor $\sqcup$ Lawyer
complementOf	$\neg C$	$\neg$Male
oneOf	$\{x_1\} \sqcup \ldots \sqcup \{x_n\}$	{john} $\sqcup$ {mary}
allValuesFrom	$\forall P.C$	$\forall$hasChild.Doctor
someValuesFrom	$\exists P.C$	$\exists$hasChild.Lawyer
maxCardinality	$\leqslant n P$	$\leqslant$1hasChild
minCardinality	$\geqslant n P$	$\geqslant$2hasChild

OWL Syntax	DL Syntax	Example
subClassOf	$C_1 \sqsubseteq C_2$	Human $\sqsubseteq$ Animal $\sqcap$ Biped
equivalentClass	$C_1 \equiv C_2$	Man $\equiv$ Human $\sqcap$ Male
subPropertyOf	$P_1 \sqsubseteq P_2$	hasDaughter $\sqsubseteq$ hasChild
equivalentProperty	$P_1 \equiv P_2$	cost $\equiv$ price
transitiveProperty	$P^+ \sqsubseteq P$	ancestor$^+$ $\sqsubseteq$ ancestor

OWL Syntax	DL Syntax	Example
type	$a : C$	John : Happy-Father
property	$\langle a, b \rangle : R$	$\langle$John, Mary$\rangle$: has-child

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Example: OWL2

OWL RDF/XML Exchange Syntax

E.g., Person $\sqcap$ $\forall$hasChild.(Doctor $\sqcup$ $\exists$hasChild.Doctor):

```
<owl:Class>
  <owl:intersectionOf rdf:parseType=" collection">
    <owl:Class rdf:about="#Person"/>
    <owl:Restriction>
      <owl:onProperty rdf:resource="#hasChild"/>
      <owl:allValuesFrom>
        <owl:unionOf rdf:parseType=" collection">
          <owl:Class rdf:about="#Doctor"/>
          <owl:Restriction>
            <owl:onProperty rdf:resource="#hasChild"/>
            <owl:someValuesFrom rdf:resource="#Doctor"/>
          </owl:Restriction>
        </owl:unionOf>
      </owl:allValuesFrom>
    </owl:Restriction>
  </owl:intersectionOf>
</owl:Class>
```

Why description logics?

- OWL exploits results of 20+ years of DL research
  - Well defined (model theoretic) **semantics**
  - **Formal properties** well understood (complexity, decidability)

I can't find an efficient algorithm, but neither can all these famous people.

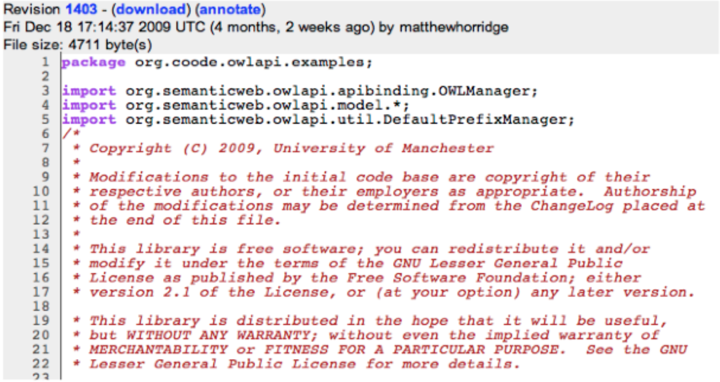
[Garey & Johnson. Computers and Intractability: A Guide to the Theory of NP-Completeness. Freeman, 1979.]

  - Known **reasoning algorithms**
  - **Scalability** demonstrated by **implemented systems**

Why description logics?

Major benefit of OWL has been huge increase in range and sophistication of tools and infrastructure:

- Editors/development environments
- Reasoners
- Explanation, justification and pinpointing
- Integration and modularisation
- APIs, in particular the OWL API



Revision 1403 - (download) (annotate)
Fri Dec 18 17:14:37 2009 UTC (4 months, 2 weeks ago) by matthewhorridge
File size: 4711 byte(s)

```
package org.coode.owlapi.examples;

import org.semanticweb.owlapi.apibinding.OWLManager;
import org.semanticweb.owlapi.model.*;
import org.semanticweb.owlapi.util.DefaultPrefixManager;
/*
 * Copyright (C) 2009, University of Manchester
 *
 * Modifications to the initial code base are copyright of their
 * respective authors, or their employers as appropriate.  Authorship
 * of the modifications may be determined from the ChangeLog placed at
 * the end of this file.
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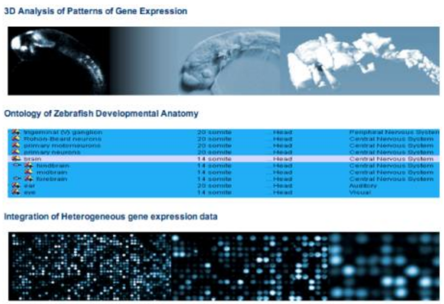
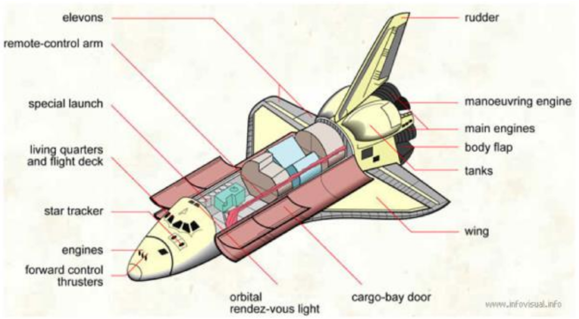
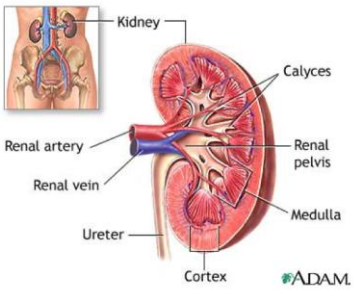
```

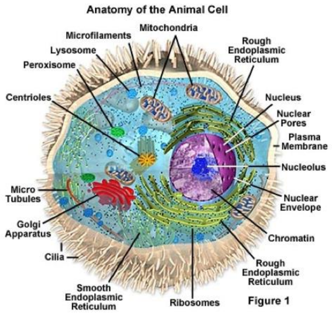
Many Applications of Logic Systems

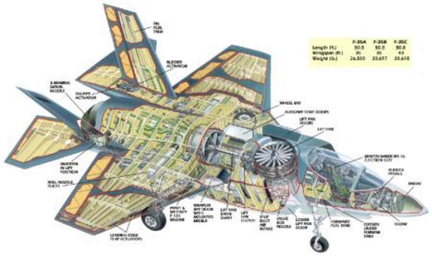




Science (e.g., genomics), geography, engineering







Medicine, biology, defense