CSCI 561 - Foundation for Artificial Intelligence

Week 7 Discussion
First-Order Logic

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First-order logic

Syntax of FOL: Basic Elements

Constants	**KingJohn, 2, Crown,...**
Predicates	**Brother, >,...**
Functions	**Sqrt, LeftLeg, +, ...**
Variables	**x, y, a, b,...**
Connectives	$\leftarrow, \Rightarrow, \wedge, \vee, \Leftrightarrow$
Equality	**=**
Quantifiers	$\forall, \exists$

Example Domain:
Arithmetic on Natural Numbers

Objects

- Non-negative numbers (0, 1, …)

Relations

- NatNum, =, <, >, …

Functions

- Successor, +, -, x, integer division, remainder, exponentiation,…

E.g., >(+(5, 20211), -(5111,777)) ∧ =(5,+(3,2))

Convert from English to FOL

Circle True or False. For sentences in English make your judgment of the meaning of the sentence, i.e., you may want to translate it in FOL to conclude.

1. **[True/False] "Bert and Ernie are brothers" is equivalent to "Bert is a brother and Ernie is a brother"** False

2. **[True/False] "p and q are not both true" is equivalent to "p and q are both not true"**
False

3. **[True/False] "Neither p nor q" is equivalent to "both p and q are false"**
True

4. **[True/False] "Not all A's are B's" is equivalent to "$\exists x\ (A(x) \wedge \sim B(x))$"**
True

5. **[True/False] "MS students and PhD students are welcome to apply." is equivalent to "$\forall x\ [(M(x) \wedge P(x)) \Rightarrow Apply(x)]$"** False

Convert from English to FOL

Questions 6 to 9: Attract is a relation from x to y, i.e., A(x,y) says that "x attracts y" or equivalently that "y is attracted by x".

6. [True/False] "Everything attracts something", where "something" means "something or other", is equivalent to "∀x ∃y A(x, y) "

True

7. [True/False] "Something is attracted by everything", where "something" means "something in particular", is equivalent to "∃y ∃x A(x, y) "

False

8. [True/False] "Everything is attracted by something" ", where "something" means "something or other", is equivalent to "∃x ∀y A(x, y) "

False

9. [True/False] "Something attracts everything", where "something" means "something in particular", is equivalent to "∃x ∃y A(x, y) "

False

Exercise 8.24

Represent the following sentences in first-order logic, using a consistent vocabulary (which you must define):

a.Some students took French in spring 2001.

b.Every student who takes French passes it.

c.Only one student took Greek in spring 2001.

d.The best score in Greek is always higher than the best score in French.

e.Every person who buys a policy is smart.

f.No person buys an expensive policy.

g.There is an agent who sells policies only to people who are not insured.

h.There is a barber who shaves all men in town who do not shave themselves

i.A person born in the UK, each of whose parents is a UK citizen or a UK resident, is a UK citizen by birth.

j.A person born outside the UK, one of whose parents is a UK citizen by birth, is a UK citizen by descent.

k.Politicians can fool some of the people all of the time, and they can fool all of the people some of the time, but they can't fool all of the people all of the time.

l.All Greeks speak the same language. (Use Speaks(x,l) to mean that person x speaks language l.)

Exercise 8.24

- **Student(x):** x is a student

- **Takes(x, c, s)**: student x takes course c in semester s;
- **Passes(x, c, s)**: student x passes course c in semester s;
- **Score(x, c, s)**: the score obtained by student x in course c in semester s;

- **Policy(x):** x is a policy
- **Expensive(x):** x is expensive
- **Buys(x, y, z):** x buys y from z (using a binary predicate with unspecified seller is OK but less expressive);

Exercise 8.24

a. Some students took French in spring 2001.

b. Every student who takes French passes it.

c. Only one student took Greek in spring 2001.

f. No person buys an expensive policy.

Exercise 8.24

a. Some students took French in spring 2001.
$\exists x\ Student(x) \wedge Takes(x,F,Spring2001)$.

b. Every student who takes French passes it.
$\forall x,s\ Student(x) \wedge Takes(x,F,s) \Rightarrow Passes(x,F,s)$.

c. Only one student took Greek in spring 2001.
$\exists x\ Student(x) \wedge Takes(x,G,Spring2001) \wedge$
$\forall y\ y \neq x \Rightarrow \neg Takes(y,G,Spring2001)$.

f. No person buys an expensive policy.
$\forall x,y,z\ Person(x) \wedge Policy(y) \wedge Expensive(y) \Rightarrow \neg Buys(x,y,z)$.

What you should know

- What is entailment and inference? How do they differ?
- What are examples of sound or complete inference techniques?
- What does satisfiable or valid mean?
- What is propositional logic? Basic manipulation rules? Inference rules? What are some of its limitations?
- What is first order logic?

Want More?

- **Check out some of these exercises in the book:**

  7.1, 7.4-8, 10

  Chap 8: 8.1-3, 8.6, 8.9-10, 8.14,17, 8.28