CSCI 561 - Foundation for Artificial Intelligence

Week 7 Discussion
Propositional Logic

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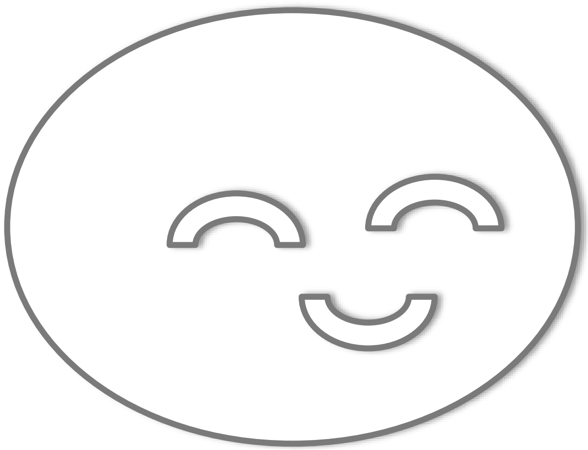
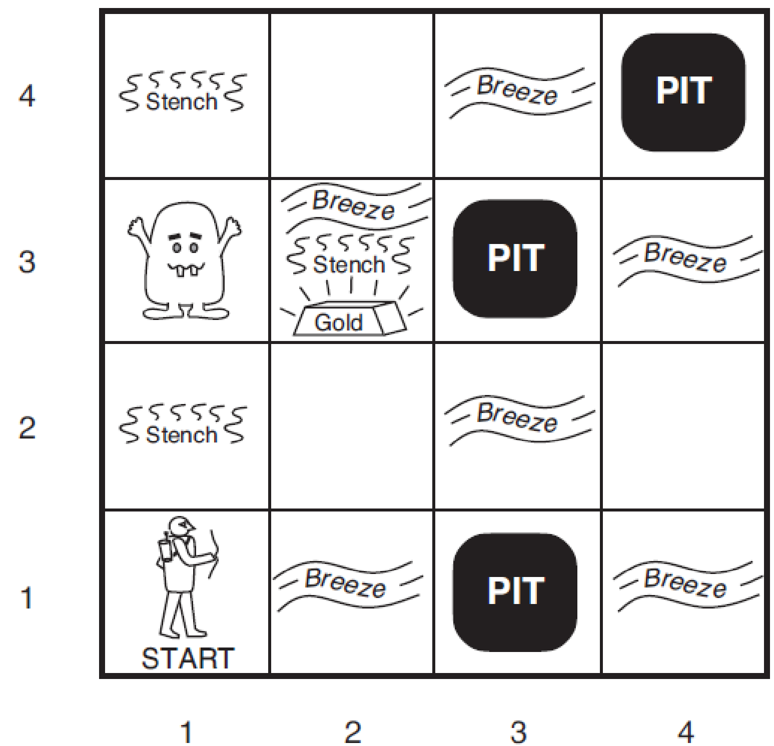
Logic Concepts

- *Entailment* $\models$
- *Inference* $\vdash$



Your head



A possible “real” world

Entailment
(between your head and the universe)

$KB \models \alpha$

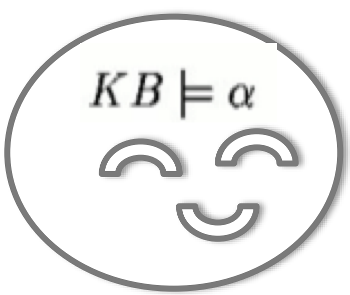
Knowledge base KB <u>entails</u> sentence α (in your head)

if and only if

α is true in all worlds where KB is true (in the universe)

E.g., the KB containing "the Giants won" and "the Reds won" entails "Either the Giants won or the Reds won"

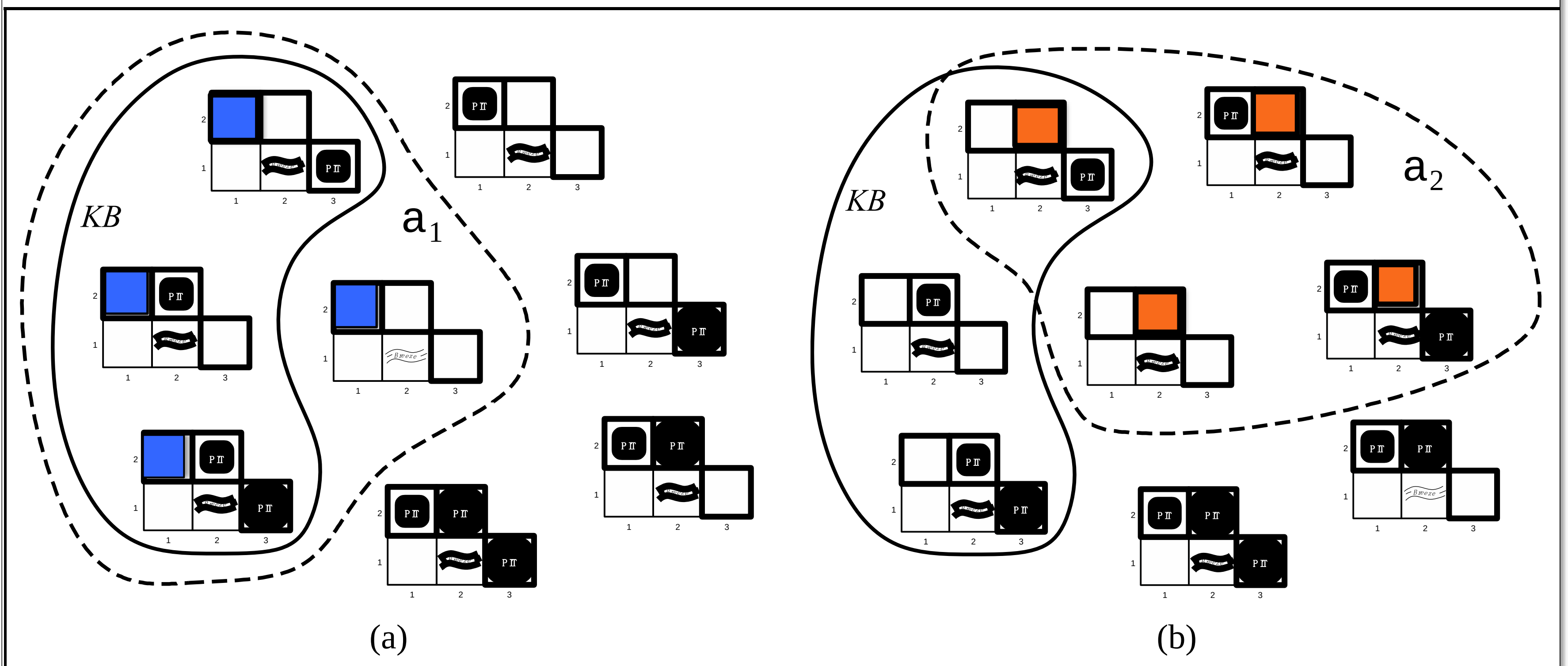
Entailment means it is impossible for this case to occur:
premises are true and the consequence is false.



You must check in the whole univese for this !!!

$$M(KB) \subseteq M(a)$$

Entailment in Wumpus World



$\alpha_1 = \neg P_{1,2}$

$\alpha_2 = \neg P_{2,2}$

$$KB \overset{?}{\models} \alpha_1$$

$$KB \overset{?}{\models} \alpha_2$$

$$M(KB) \subseteq M(a_1)$$

$$M(KB) \subseteq M(a_2)$$

Inference (all in your head)

$KB \vdash_i \alpha$ = sentence α can be derived from KB by procedure i

<u>Soundness</u>: i is sound if
whenever $KB \vdash_i \alpha$, it is also true that $KB \models \alpha$

<u>Completeness</u>: i is complete if
whenever $KB \models \alpha$, it is also true that $KB \vdash_i \alpha$

Exercise 7.4

1. False |= True
2. True |= False
3. $(A \wedge B) \models (A \Leftrightarrow B)$
4. $A \Leftrightarrow B \models A \vee B$
5. $A \Leftrightarrow B \models \neg A \vee B$
6. $(A \wedge B) \Rightarrow C \models (A \Rightarrow C) \vee (B \Rightarrow C)$. $(C \vee (\neg A \wedge \neg B)) \equiv ((A \Rightarrow C) \wedge (B \Rightarrow C))$
7. $(A \vee B) \wedge (\neg C \vee \neg D \vee E) \models (A \vee B)$
8. $(A \vee B) \wedge (\neg C \vee \neg D \vee E) \models (A \vee B) \wedge (\neg D \vee E)$
9. $(A \vee B) \wedge \neg (A \Rightarrow B)$ is satisfiable
10. $(A \Leftrightarrow B) \wedge (\neg A \vee B)$ Is satisfiable
11. $(A \Leftrightarrow B) \Leftrightarrow C$ has the same number of models as $(A \Leftrightarrow B)$ for any fixed set of proposition symbols that includes A, B, C.

Exercise 7.4

False $\models$ True.

Exercise 7.4

False ⊨ True. TRUE

- **False has no models and hence entails every sentence**

- **True is true in all models and hence is entailed by every sentence.**

Exercise 7.4

True $\models$ False.

Exercise 7.4

True $\models$ False. FALSE

- False is not true in any models

Exercise 7.4

$(A \wedge B) \models (A \Leftrightarrow B)$.

Exercise 7.4

$(A \wedge B) \models (A \Leftrightarrow B)$. TRUE

- **The left-hand side $(A \wedge B)$ has exactly one model: A=True and B=True then $(A \wedge B)$=True**
- **That model is one of the two models of the right-hand side $(A \Leftrightarrow B)$. Two models:**
  - A=True and B=True then $(A \Leftrightarrow B)$ =True
  - A=False and B=False then $(A \Leftrightarrow B)$ =True

Exercise 7.4

$A \Leftrightarrow B \models A \vee B$.

Exercise 7.4

$A \Leftrightarrow B \models A \vee B$. FALSE

- **$(A \Leftrightarrow B)$ has two models:**
  - A=True and B=True then $(A \Leftrightarrow B)$ =True
  - A=False and B=False then $(A \Leftrightarrow B)$ =True
- **(A V B) is not True in this model**
  - A=False and B=False then (A V B) = False

Exercise 7.4

$(A \Leftrightarrow B) \wedge (\neg A \vee B)$ is satisfiable.

Validity and satisfiability

A sentence is valid if it is true in all models

e.g., $A \lor \neg A$, $A \Rightarrow A$, $(A \land (A \Rightarrow B)) \Rightarrow B$

Validity is connected to inference via the Deduction Theorem

$KB \models \alpha$ if and only if $(KB \Rightarrow \alpha)$ is valid

A sentence is satisfiable if it is true in some model

e.g., $A \lor B$, C

A sentence is unsatisfiable if it is true in no models

e.g., $A \land \neg A$

Satisfiability is connected to inference via the following:

$KB \models \alpha$ if and only if $(KB \land \neg\alpha)$ is unsatisfiable

i.e., prove α by *reductio ad absurdum*

Exercise 7.4

$(A \Leftrightarrow B) \wedge (\neg A \vee B)$ is satisfiable. **TRUE**

- **This sentence $(A \Leftrightarrow B) \wedge (\neg A \vee B)$ is True, when A=True and B=True**

Exercise 7.4

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6. $(A \wedge B) \Rightarrow C \models (A \Rightarrow C) \vee (B \Rightarrow C)$. $(C \vee (\neg A \wedge \neg B)) \equiv ((A \Rightarrow C) \wedge (B \Rightarrow C))$.
7. $(A \vee B) \wedge (\neg C \vee \neg D \vee E) \models (A \vee B)$.
8. $(A \vee B) \wedge (\neg C \vee \neg D \vee E) \models (A \vee B) \wedge (\neg D \vee E)$.
9. $(A \vee B) \wedge \neg (A \Rightarrow B)$ is satisfiable.
10. $(A \Leftrightarrow B) \wedge (\neg A \vee B)$ Is satisfiable.
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Proof methods

Proof methods divide into (roughly) two kinds:

<u>Model checking</u>

truth table enumeration (sound and complete for propositional)

heuristic search in model space (sound but incomplete)

e.g., the GSAT algorithm (Ex. 6.15)

<u>Application of inference rules</u>

Legitimate (sound) generation of new sentences from old

<u>Proof</u> = a sequence of inference rule applications

Can use inference rules as operators in a standard search alg.

Basic manipulation rules

$\neg(\neg A) = A$	Double negation
$\neg(A \wedge B) = (\neg A) \vee (\neg B)$	Negated “and”
$\neg(A \vee B) = (\neg A) \wedge (\neg B)$	Negated “or”
$A \wedge (B \vee C) = (A \wedge B) \vee (A \wedge C)$	Distributivity of ^ on V
$A \vee (B \wedge C) = (A \vee B) \wedge (A \vee C)$	Distributivity of V on ^
$A \Rightarrow B = (\neg A) \vee B$	by definition
$\neg(A \Rightarrow B) = A \wedge (\neg B)$	using negated or
$A \quad B = (A \Rightarrow B) \wedge (B \Rightarrow A)$	by definition

$\neg(A \quad B) = (A \wedge (\neg B)) \vee (B \wedge (\neg A))$ using negated and & or

…

Inference Rules

- **Modus Ponens** or **Implication-Elimination**: (From an implication and the premise of the implication, you can infer the conclusion.)

$$\frac{\alpha \Rightarrow \beta, \quad \alpha}{\beta}$$

Modus Tollens:z $\dfrac{\alpha \Rightarrow \beta, \quad \neg \beta}{\neg \alpha}$

- **And-Elimination**: (From a conjunction, you can infer any of the conjuncts.)

$$\frac{\alpha_1 \wedge \alpha_2 \wedge \ldots \wedge \alpha_n}{\alpha_i}$$

- **And-Introduction**: (From a list of sentences, you can infer their conjunction.)

$$\frac{\alpha_1, \quad \alpha_2, \quad \ldots, \quad \alpha_n}{\alpha_1 \wedge \alpha_2 \wedge \ldots \wedge \alpha_n}$$

- **Or-Introduction**: (From a sentence, you can infer its disjunction with anything else at all.)

$$\frac{\alpha_i}{\alpha_1 \vee \alpha_2 \vee \ldots \vee \alpha_n}$$

Inference Rules

◇ **Double-Negation Elimination**: (From a doubly negated sentence, you can infer a positive sentence.)

$$\frac{\neg\neg\alpha}{\alpha}$$

◇ **Unit Resolution**: (From a disjunction, if one of the disjuncts is false, then you can infer the other one is true.)

$$\frac{\alpha \vee \beta, \qquad \neg\beta}{\alpha}$$

◇ **Resolution**: (This is the most difficult. Because β cannot be both true and false, one of the other disjuncts must be true in one of the premises. Or equivalently, implication is transitive.)

$$\frac{\alpha \vee \beta, \qquad \neg\beta \vee \gamma}{\alpha \vee \gamma} \quad \text{or equivalently} \quad \frac{\neg\alpha \Rightarrow \beta, \qquad \beta \Rightarrow \gamma}{\neg\alpha \Rightarrow \gamma}$$

What you should know

- What is entailment and inference? How do they differ?
- What are examples of sound or complete inference techniques?
- What does satisfiable or valid mean?
- What is propositional logic? Basic manipulation rules? Inference rules? What are some of its limitations?

Want More?

- **Check out some of these exercises in the book:**

  7.1, 7.4-8, 10

  Chap 8: 8.1-3, 8.6, 8.9-10, 8.14,17, 8.28