CS570
Analysis of Algorithms
Fall 2015
Exam II

Name: ______________________

Student ID: __________________

Email Address:________________

_____Check if DEN Student

	Maximum	Received
Problem 1	20	
Problem 2	20	
Problem 3	20	
Problem 4	20	
Problem 5	20	
Total	100	

Instructions:
1. This is a 2-hr exam. Closed book and notes
2. If a description to an algorithm or a proof is required please limit your description or proof to within 150 words, preferably not exceeding the space allotted for that question.
3. No space other than the pages in the exam booklet will be scanned for grading.
4. If you require an additional page for a question, you can use the extra page provided within this booklet. However please indicate clearly that you are continuing the solution on the additional page.

1) 20 pts
Mark the following statements as **TRUE** or **FALSE**. No need to provide any justification.

F **[TRUE/FALSE]** ??????
The Ford-Fulkerson Algorithm finds a maximum flow of a unit-capacity flow network with n vertices and m edges in time $O(mn)$.

O(m)

F **[TRUE/FALSE]**
In a flow network, if maximum flow is unique then min cut must also be unique.

T **[TRUE/FALSE]** HOW??????
In a flow network, if min cut is unique then maximum flow must also be unique.

F **[TRUE/FALSE]**
In dynamic programming you must calculate the optimal value of a sub-problem twice, once during the bottom up pass and once during the top down pass.

T **[TRUE/FALSE]** ???????
Bellman-Ford algorithm solves the shortest path problem in graphs with negative cost edges in polynomial time.

T **[TRUE/FALSE]**
The problem of deciding whether a given flow f of a given flow network G is a maximum flow can be solved in linear time. create a residual graph G' and find if there is any path between s and t

F **[TRUE/FALSE]** ????????
An optimal solution to a 0/1 knapsack problem will always contain the object i with the greatest value-to-cost ratio V_i/C_i

T **[TRUE/FALSE]**
The Ford-Fulkerson algorithm is based on greedy.

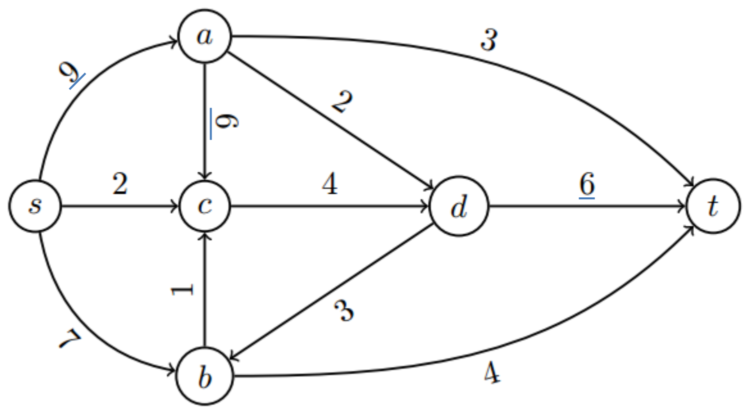
F **[TRUE/FALSE]** ???????
A flow network with unique edge capacities may have several min cuts.

F **[TRUE/FALSE]** ???????
Complexity of a dynamic programming algorithm is equal to the number of unique sub-problems in the solution space.

2) 20 pts

Consider the flow network G below with source s and sink t. The edge capacities are the numbers given near each edge.

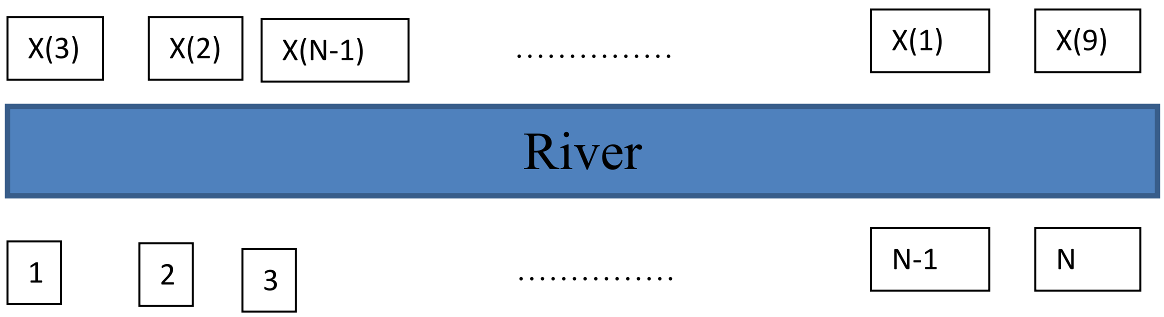
(a) Find a maximum flow in this network using the Ford Fulkerson algorithm. Show all steps of augmentation. Once you have found a maximum flow, draw a copy of the original network G and clearly indicate the flow on each edge of G in your maximum flow. (12 pts)



(b) Find a minimum s-t cut in the network, i.e. name the two (nonempty) sets of vertices that define a minimum cut. (8 pts)

3) 20 pts ?????????

There is a river going from east to west, on the south bank of the river there are a set of cities 1…N as shown in the figure below. On the northern bank there are also N cities. Each city on the south bank has a unique sister city on the north bank of the river. We use X(i) to denote the sister city corresponding to city i. Suppose you are assigned a task to build bridges to connect the southern cities with their sister cities such that as many bridges as possible are built without any bridges crossing each other. Solve this problem using dynamic programming.



a) Recurrence formula (7 pts)

b) Provide the algorithm to find the value of the optimal solution using the above recurrence formula (6 pts)

c) Provide the algorithm to list the actual bridges corresponding to the value found in part b (7 pts) survey problem?????

4) 20 pts

There are n reservoirs and m cities. Due to the drought you have to bring water from the reservoirs to the cities through a network of pipes. Each city *i* has a request for *Ci* gallons of water every day. To make matters worse, engineers have detected a leaking pipe in the water network. If the leaking pipe is used, they will lose L gallons of water per day through that pipe regardless of how much water flows through it (i.e., as long as the pipe is used, it leaks L gallon, we cannot control the leak by trying to push less water through that pipe). The other option is to shut down the leaking pipe at both ends, but that might reduce the capacity of their network. The water network is represented by a graph. Each edge represents the capacity of the pipe in gallons per day. Provide algorithms to determine if it is possible to:

a) Meet all demands for water at all cities without using the leaking pipe (7 pts)

lower bound is Ci

b) Meet all demands for water at all cities by using the leaking pipe (6 pts)

lower bound is L

c) Meet all demands for water at all cities after fixing the leaking pipe (6 pts)

5) 20 pts ?????????

Assume you want to ski down the mountain. You want the total length of your run to be as long as possible, but you can only go down, i.e. you can only ski from a higher position to a lower position. The height of the mountain is represented by an n X n matrix A. A[i][j] is the height of the mountain at position (i,j). At position (i,j), you can potentially ski to four adjacent positions (i-1,j) (i,j-1), (i,j+1), and (i+1,j) (only if the adjacent position is lower than current position). Movements in any of the four directions will add 1 unit to the length of your run. Provide a dynamic programming solution to find the longest possible downhill ski path starting at any location within the given n X n grid.

Additional Space

Additional Space