

EE450 Discussion #6



- Shannon's Theorem
- Modulations
- Multiplexing



Shannon's Theorem



- $C = B \log_2(1 + \text{SNR})$
- Theoretical Maximum Capacity that can be obtained on a line
- Sets an Upper Bound on the capacity given the conditions
 - Used for Calculating the
 - Signal to Noise Ratio – Given the Bandwidth and capacity of the channel
 - Bandwidth - Given the SNR and Channel Capacity
 - Capacity - Given the SNR and the Bandwidth



Problem #1

- What SNR is needed to put a T-1 carrier on a 50 khz line?
 - What do we know?
 - T-1 Capacity = 1.544 Mbps
 - Bandwidth = 50 KHz
 - Move them around and Solve:
 - $1,544,000 = 50,000 \log_2 (1 + \text{SNR})$
 - $2^{30.88} - 1 = \text{SNR}$



Continued

- So $\text{SNR} = 1976087931$
 - SNR is typically measured in DB
 - Use $\text{SNR dB} = 10 \log_{10} (\text{SNR})$
 - In this case
 - $\text{SNR dB} = 10 \log_{10} (1976087931)$
 - SNR aprox. 92.9 dB
 - However you must NOT plug SNR into Shannon's theorem in dB format

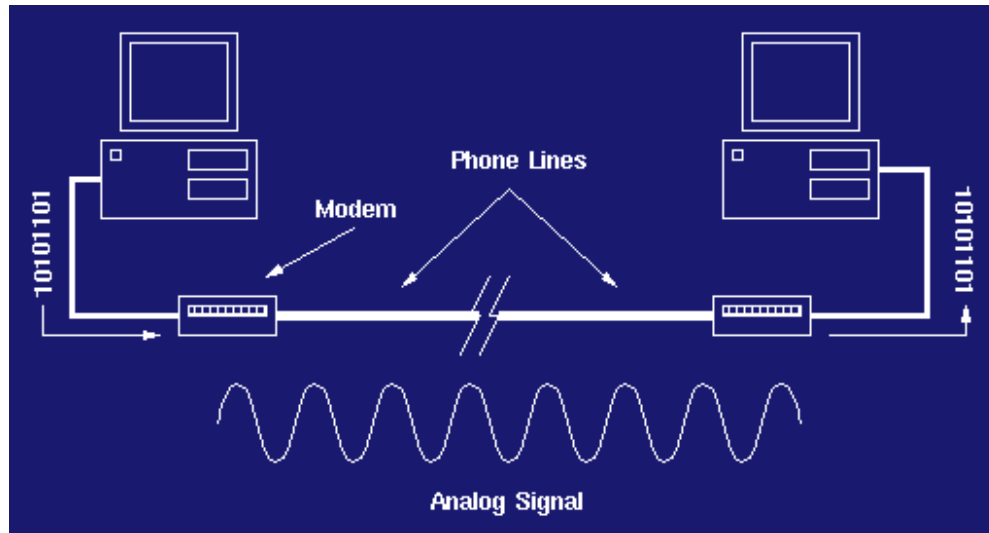


Problem #2

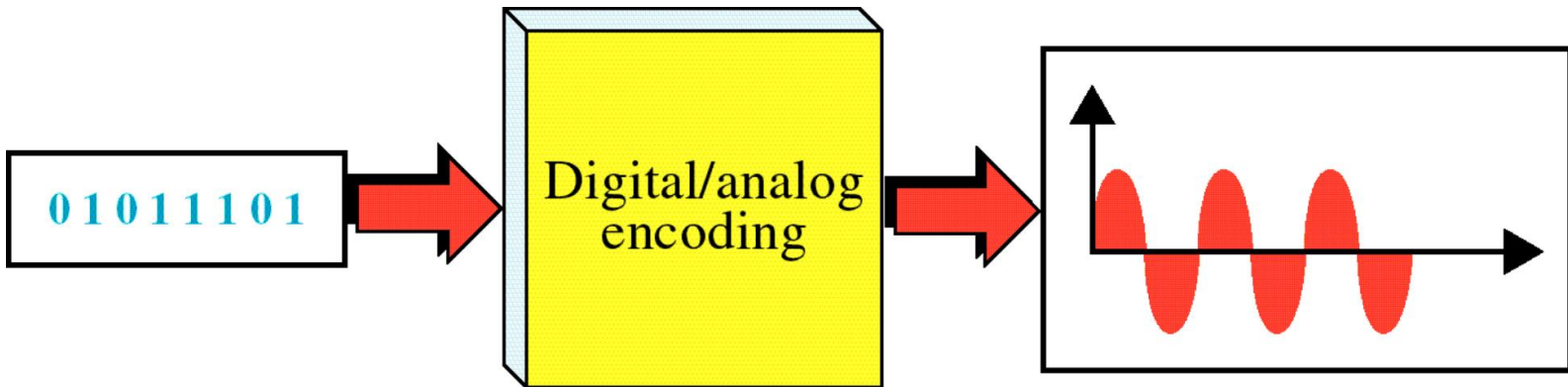
- Calculate the maximum rate supported by a telephone line with BW of 4 KHz. When the signal is 10 volts, the noise is 5 millivolts.
- $\text{SNR} = \text{Signal power} / \text{Noise Power}$
- Power is proportional to square of the voltage
- $S/N = (10^2) / (0.005^2) = 4000000$
- $B = 4000 \text{ Hz}$
- $C = B \log_2 (1 + S/N)$
 - Reminder: $\log_2 x = \ln x / \ln 2$
- $C = 4000 \log_2 (1 + 4000000) = 87726 \text{ bps}$

Review on Modems

- Modem Stands for
 - MOdulator / DEModulator
- Uses Sine wave As the carrier Signal



Digital to Analog Encoding



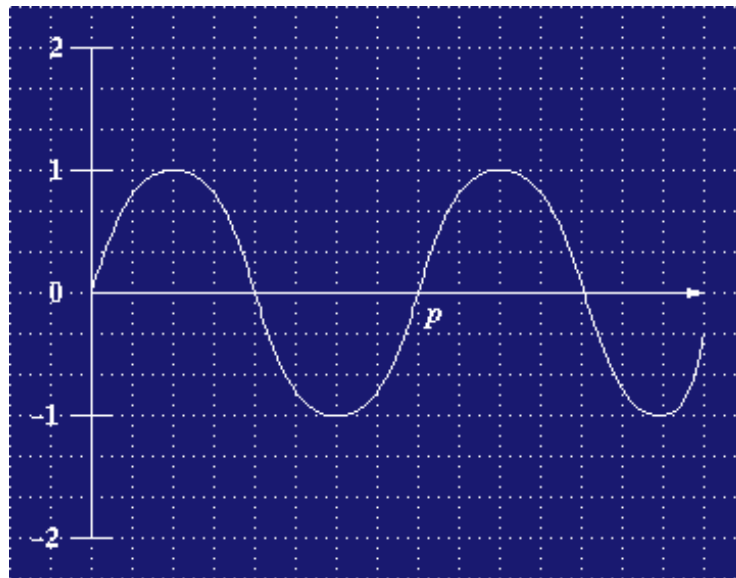


Modulation

- Need to Encode Digital Data in an Analog Signal
- In modem transmission we use different techniques for modulation
 - Amplitude Modulation
 - Frequency Modulation
 - Phase Shift

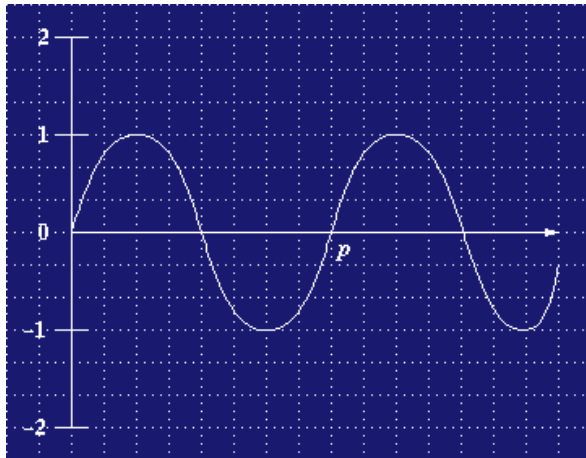
Amplitude Modulation

- Varies the Amplitude of the Signal

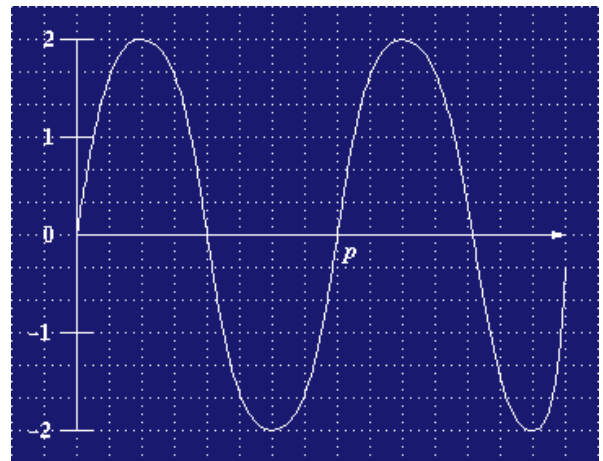


Amplitude Modulation

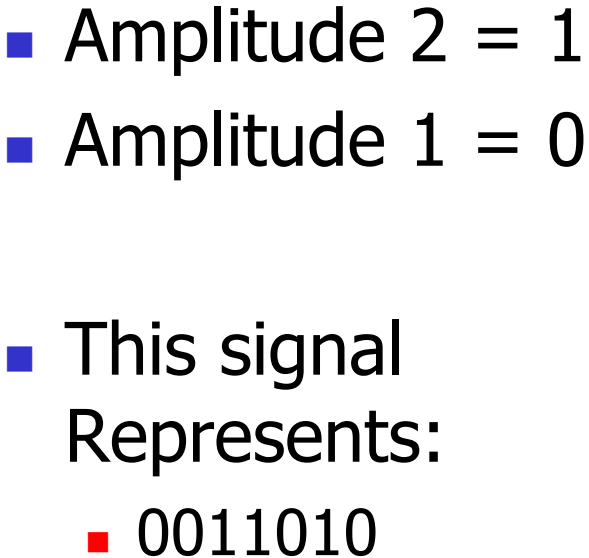
- Same Signal Greater Amplitude



Amplitude = 1

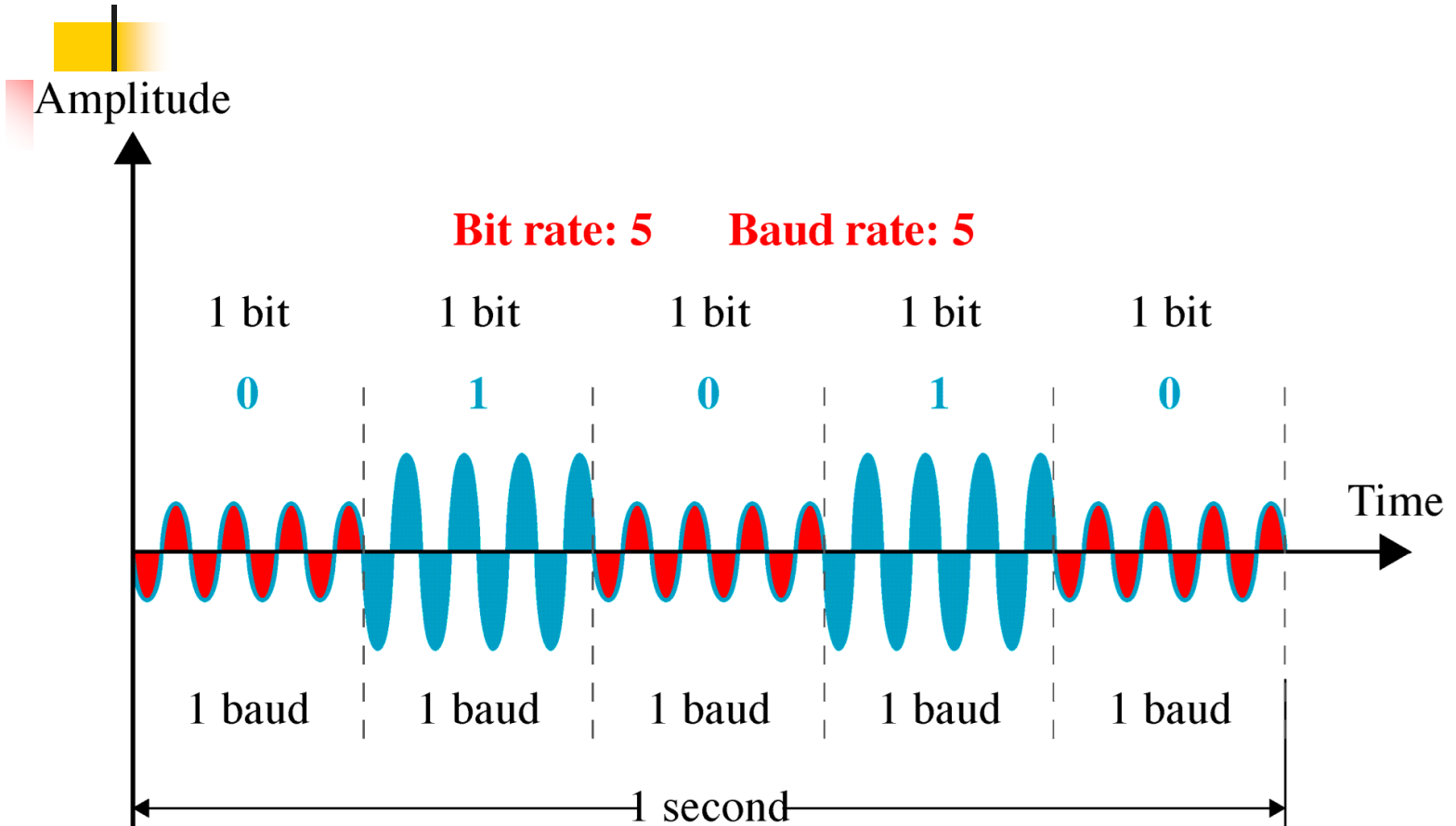


Amplitude = 2

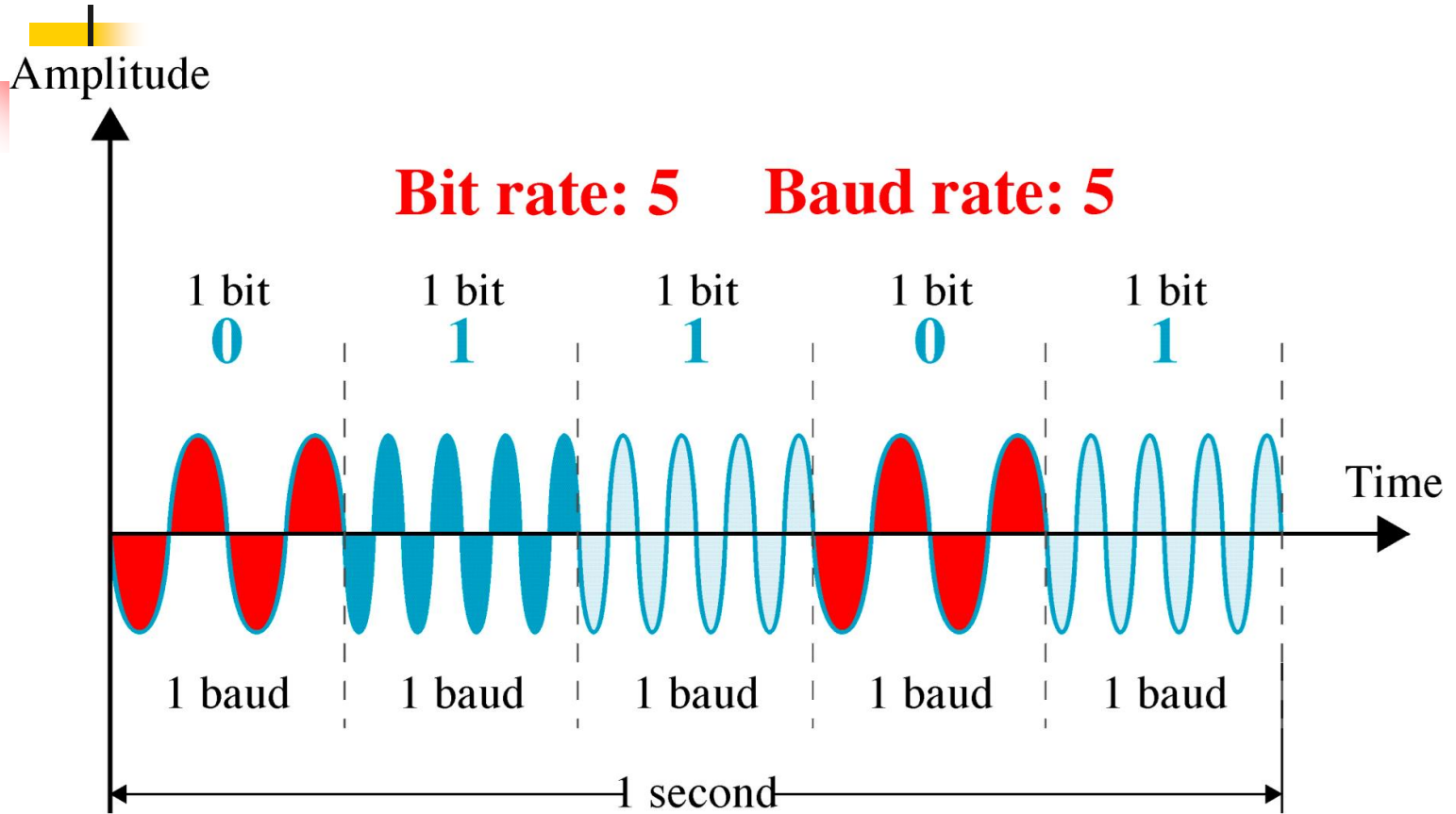


- Amplitude 2 = 1
- Amplitude 1 = 0
- This signal Represents:
 - 0011010

ASK

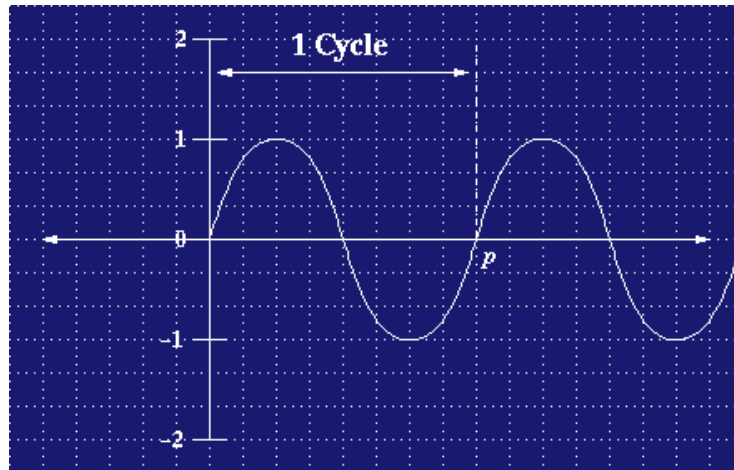


FSK



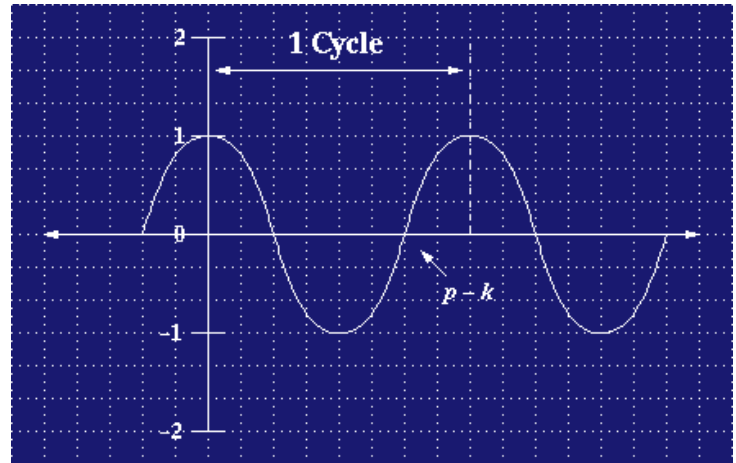
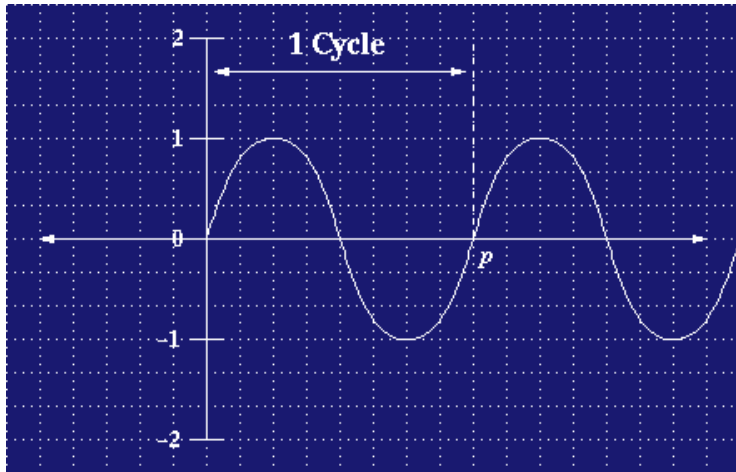
Phase-Shift Modulation

- Start with our normal sine wave
- The sine wave has a period of P
 - P may be denoted as T instead in the equations

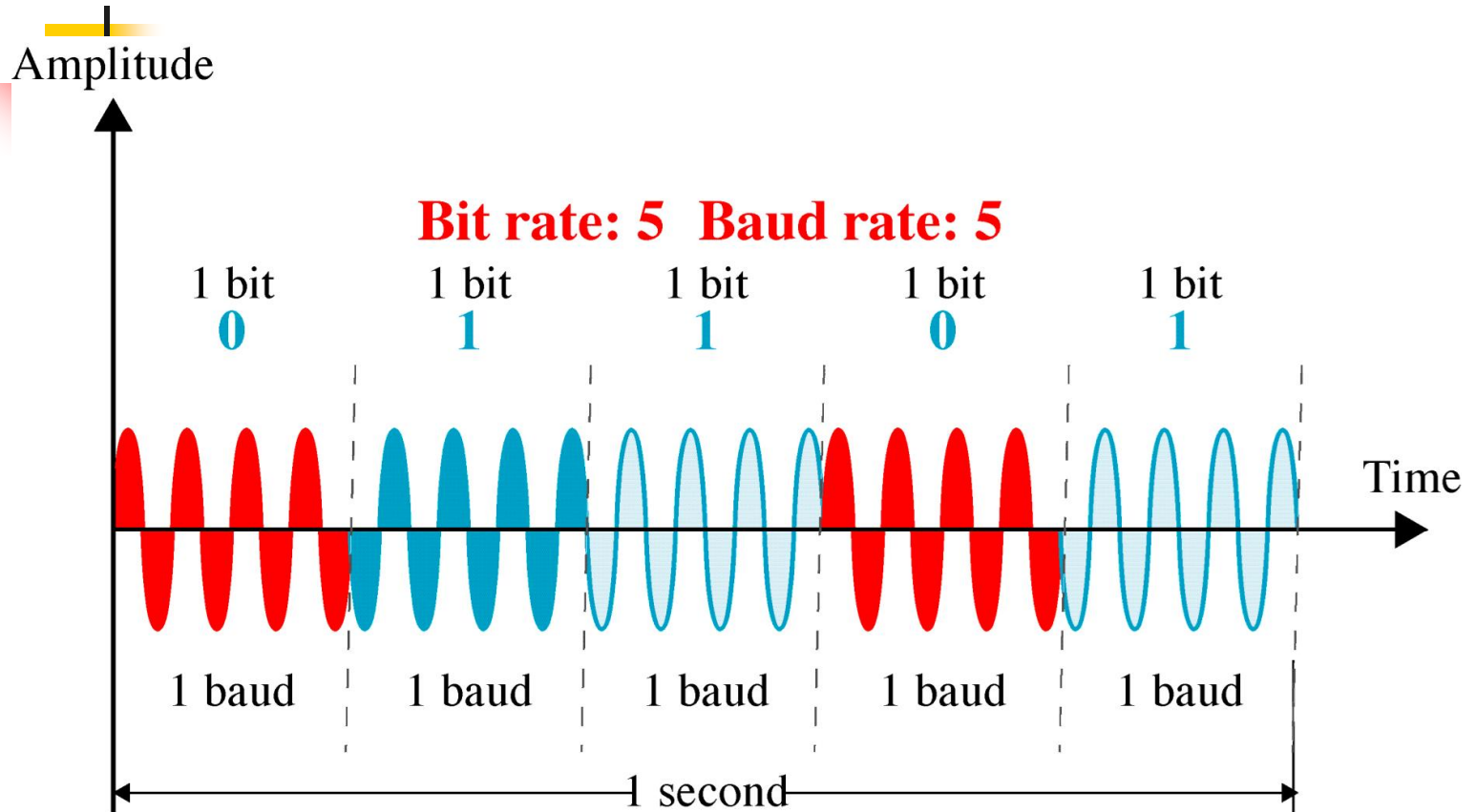


Phase-Shift Modulation

- Shift the Phase of the Sine Wave
- Shifted diagram shows that the cycle starting at 1 vs. starting at 0



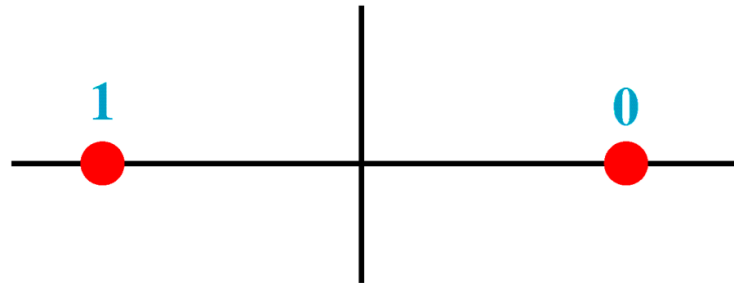
PSK



PSK Constellation

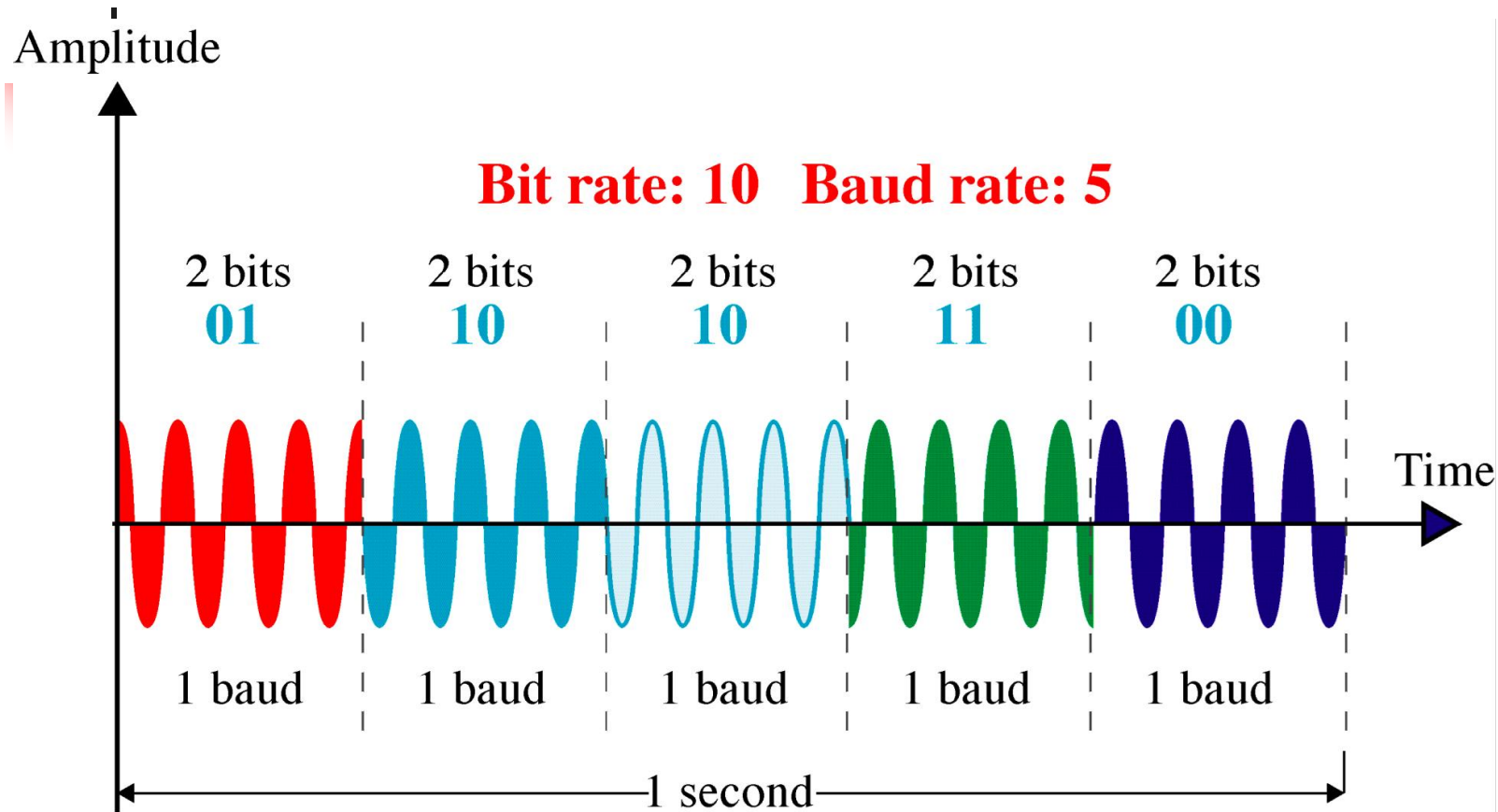
| Bit | Phase |
|-----|-------|
| 0 | 0 |
| 1 | 180 |

Bits



Constellation diagram

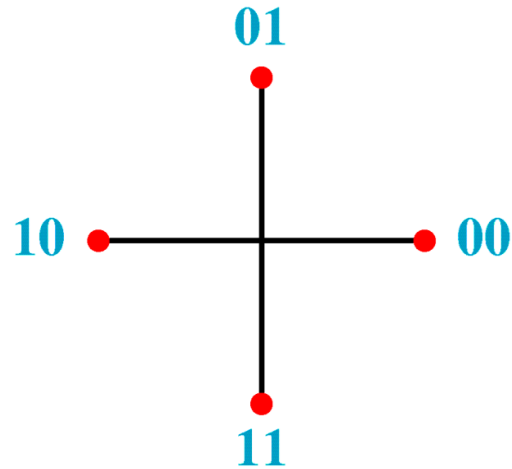
4-PSK



4-PSK Constellation

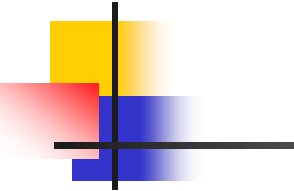
| Dibit | Phase |
|-------|-------|
| 00 | 0 |
| 01 | 90 |
| 10 | 180 |
| 11 | 270 |

Dibit
(2 bits)



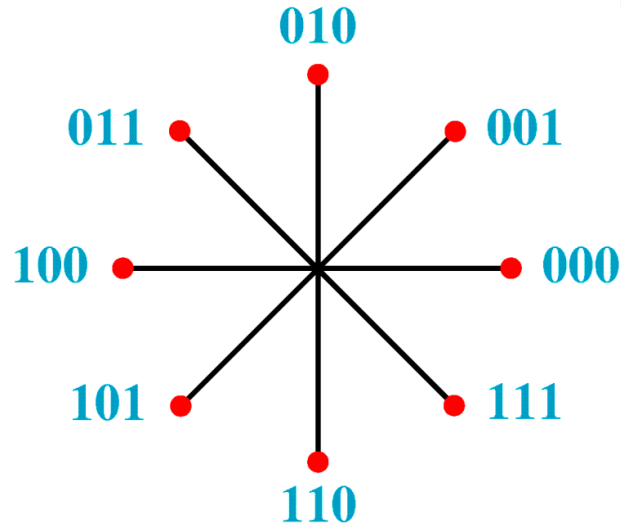
Constellation diagram

8-PSK Constellation



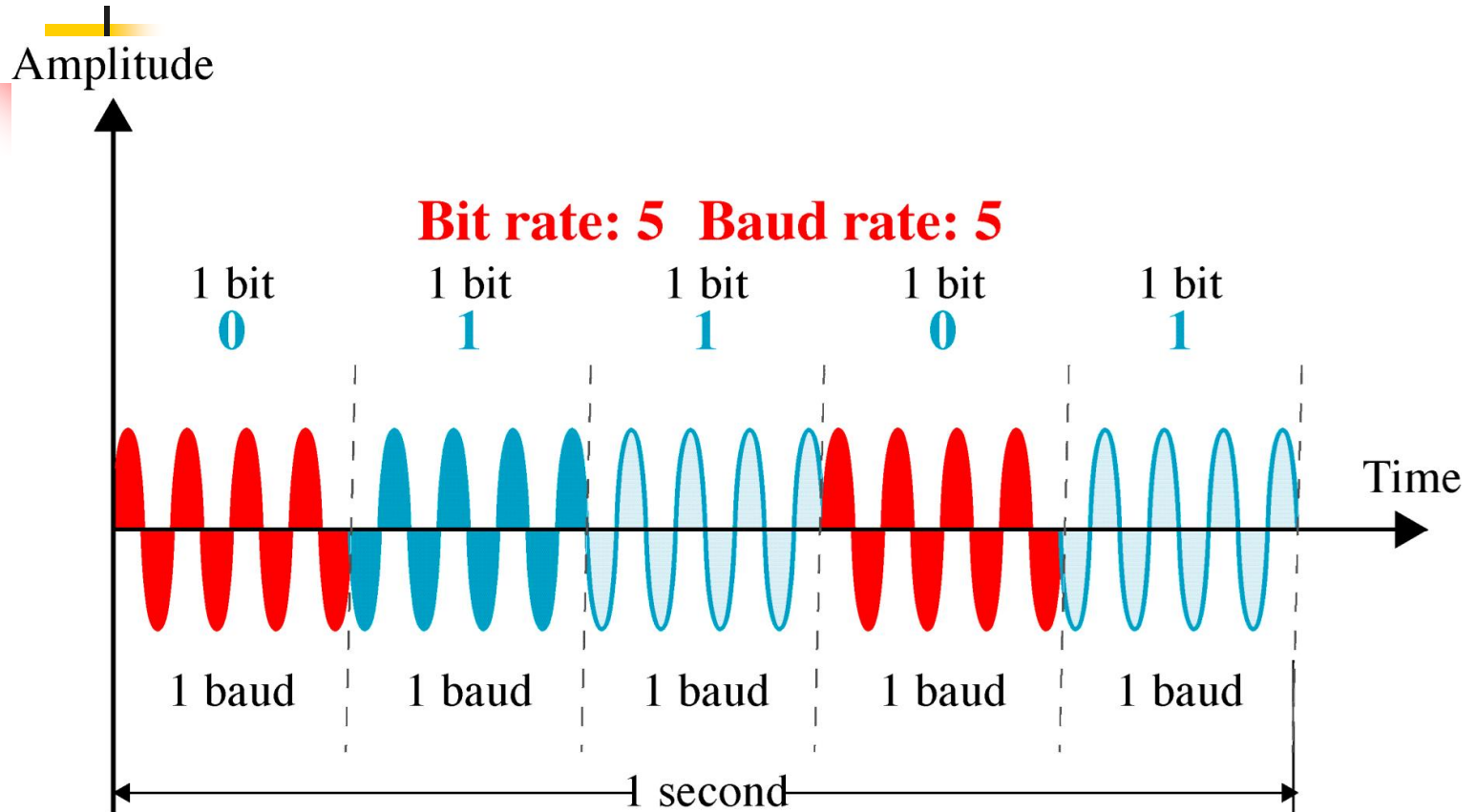
| Tribit | Phase |
|--------|-------|
| 000 | 0 |
| 001 | 45 |
| 010 | 90 |
| 011 | 135 |
| 100 | 180 |
| 101 | 225 |
| 110 | 270 |
| 111 | 315 |

Tribits
(3 bits)



Constellation diagram

PSK

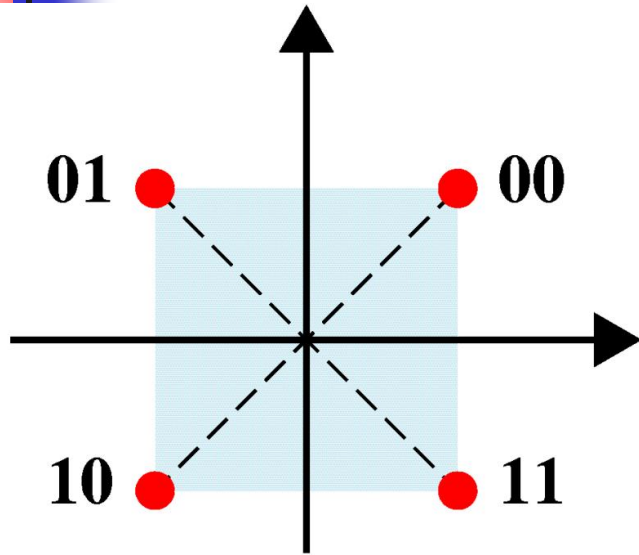




Combining Both

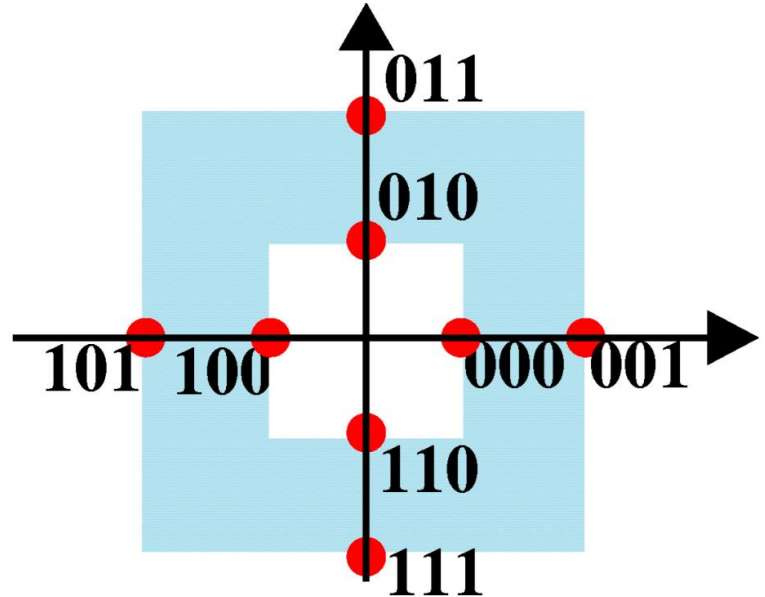
- Modulation used in Modern Modems
 - Uses:
 - Amplitude Modulation
 - Phase Shift Keying
- QAM
 - Quadrature Amplitude Modulation
 - Big Name – Simple Concept

4-QAM and 8-QAM Constellation



4-QAM

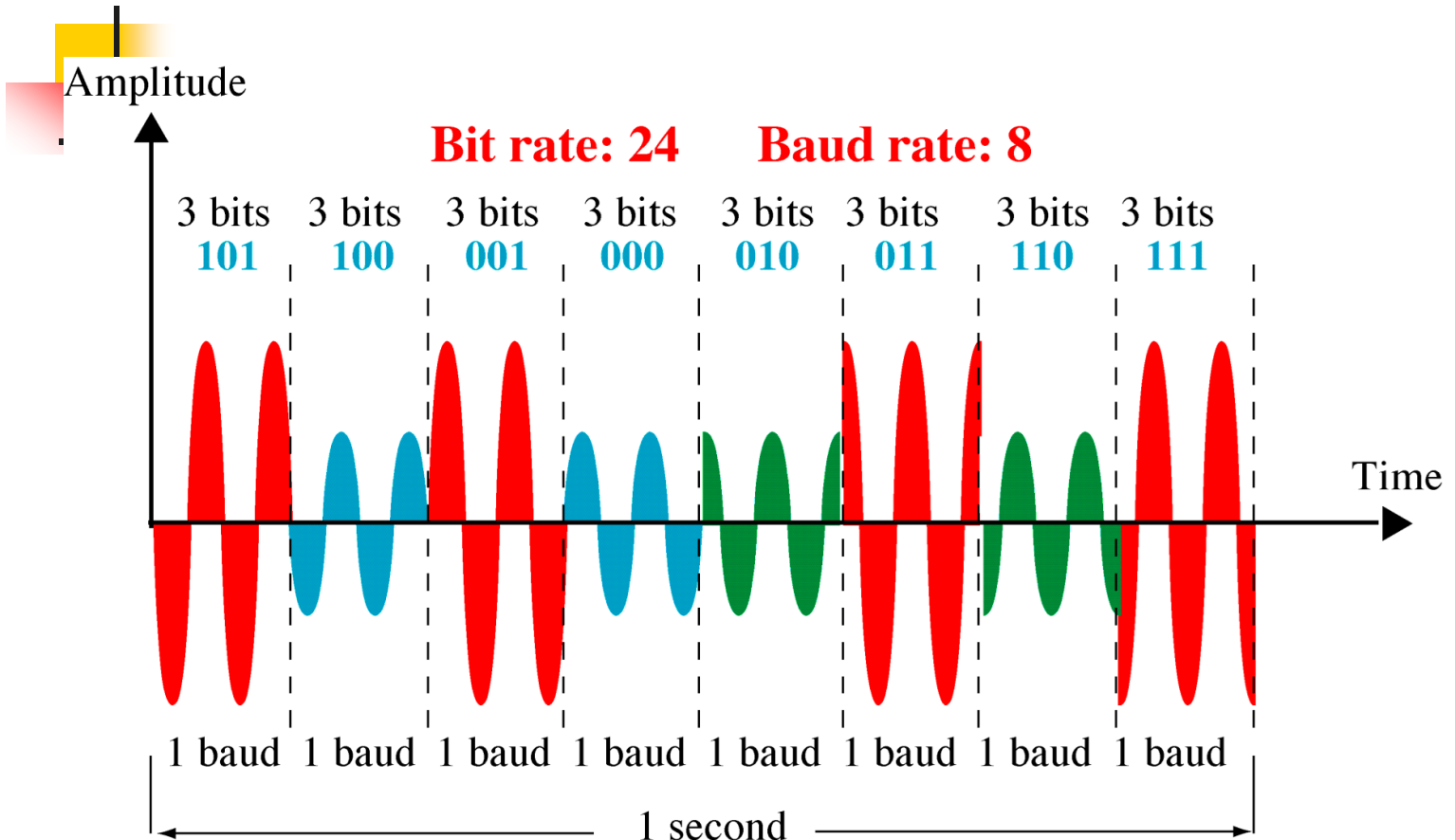
1 amplitude, 4 phases



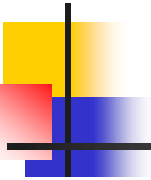
8-QAM

2 amplitudes, 4 phases

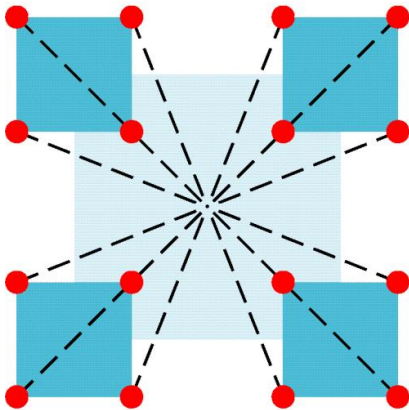
8-QAM Signal



16-QAM Constellation

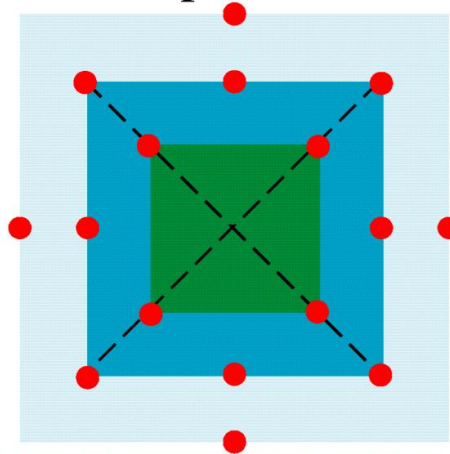


3 amplitudes,
12 phases



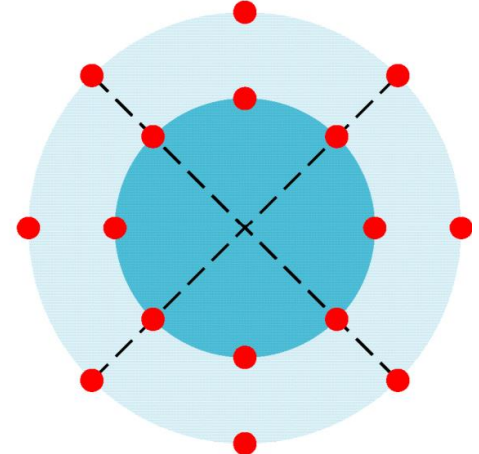
16-QAM

4 amplitudes,
8 phases



16-QAM

2 amplitudes,
8 phases



16-QAM

Bit Rate and Baud Rate

Bit

Baud rate = N

Bit rate = N

| | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 1 | 0 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|

Dibit

Baud rate = N

Bit rate = $2N$

| | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 1 | 0 | 1 | 0 | 1 | 1 | 1 | 0 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|



Problem #3

- A modem uses an 8-PSK modulation scheme supporting data rate of 4800 bps. What is the signaling rate (aka baud rate)?
- 8 PSK – (Phase Shift Keying)
 - 8 different encoding levels
 - Each encoding has $\log_2 8 = 3$ bits
 - $4800 / 3 = 1600$ Baud Rate



Statistical TDM Parameters

- I = Number of Input Sources
- R = Data rate of each source (bps)
- α (Alpha) = mean fraction of time each source is transmitting
- M = Effective capacity of multiplexed line
- $K = M / (I \times R)$ = Ratio of multiplexed line capacity to total input rate
- λ (lambda) = $\alpha \times I \times R$ = Average Arrival rate
- $T_s = 1 / M$ = Service time in seconds



ρ : Line Utilization

- ρ = Fraction of total link capacity being used
- Many different forms to express line utilization
 - $\rho = \lambda T_s$
 - $\rho = (a \times I \times R) / M$
 - $\rho = a / K$
 - $\rho = \lambda / M$



Sample Problem #5

- Ten 9600 bps lines are multiplexed using TDM. Ignoring overhead bits what is the total capacity required for Synchronous TDM?
 - Simple: $10 \times 9600 = 96 \text{ kbps (96,000)}$



Sample Problem #6

- Ten 9600 bps lines are multiplexed using TDM. Assuming that we limit line utilization to 0.8 and each line is busy 50 % of the time. What is the capacity required for Statistical TDM?
 - What do we know?
 - Line utilization – $\rho = .8$
 - Fraction of time transmitting - $\alpha = .5$
 - R Data Rate of each input source = 9600 bps
 - I number of Input Sources = 10



Continued

- The Equation:
- $\rho = a \times I \times R \times /M$
 - Where M is the capacity of the multiplexed line
- Rearrange for M
 - $M = a \times I \times R / \rho$
- Plug in the given parameters
 - $M = 0.5 \times 10 \times 9600 / 0.8$
 - $M = 60 \text{ kbps}$

Four 1 Kbps devices are to be multiplexed using synchronous TDM. The multiplexor will take one bit from each source during each cycle. Find

- a) The duration of the bit before multiplexing
- b) The duration of the bit after multiplexing
- c) The duration of the multiplexed frame
- d) The multiplexer bit rate
- e) The multiplexer frame rate.

Duration of bit before MUX = $1/1K = 1 \text{ msec}$

Multiplexer bit rate is $4 \times 1K = 4K\text{bps}$ and hence the bit duration at output of MUX is $1/4K = 0.25 \text{ msec}$

Duration of multiplexed frame is $4 \times 0.25\text{msec} = 1\text{msec}$

Frame rate is $1/1\text{msec} = 1000 \text{ frames/sec}$
