Homework 7 PHYS230

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1 Problem 8.26

Key : ω is the same for every infinitesimal part along the stick.

For the stick around the pivot .Let

$$T_i = 0 \qquad V_i = Mg\frac{L}{2}$$
$$T_f = \frac{I_{pivot}\omega^2}{2} \qquad V_f = 0$$

Conservation of energy gives

$$K_i + V_i = K_f + V_f$$

Using $I_{pivot} = \dfrac{ML^2}{3}$,

$$\frac{mgL}{2} = \frac{I_{pivot}\omega^2}{2}$$
$$\frac{mgL}{2} = \frac{ML^2\omega^2}{6}$$
$$\rightarrow \omega = \sqrt{\frac{3g}{L}}$$

Having found ω, conservation of energy along the whole stick gives

$$\frac{MgL}{2} = \frac{Mv^2}{2} + \frac{I_z^{CM}\omega^2}{2}$$
$$\frac{MgL}{2} = \frac{Mv^2}{2} + \left(\frac{1}{2}\right)\left(\frac{1}{12}\right)ML^2\left(\frac{3g}{L}\right)$$
$$\rightarrow \frac{MgL}{2} = \frac{Mv^2}{2} + \left(\frac{1}{8}\right)MLg$$
$$\rightarrow v = \frac{\sqrt{3gL}}{2}$$

After $\dfrac{3\pi}{4}$ of a turn, since rotational velocity is perpendicular to the axis of rotation, if the pivot disappears CM has velocity only along the y axis. Thus using kinematics,

$$\begin{aligned} v_y = v_{0,y} - gt &\rightarrow 0 = v_{0,y} - gt \\ &\rightarrow v_{0,y} = gt \\ &\rightarrow t = \frac{v_{0,y}}{g} \end{aligned}$$

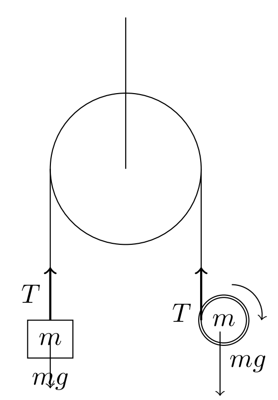
Substituting the velocity found previously,

$$t = \frac{\sqrt{3gL}}{2} \times \frac{1}{g}$$

We can now find the maximum heigth

$$\begin{aligned} &y = y_0 + v_y t - \frac{gt^2}{2} \\ &y = \frac{\sqrt{3gL}}{2}\frac{\sqrt{3gL}}{2} \times \frac{1}{g} - \frac{1}{2}g\left(\frac{\sqrt{3gL}}{2}\frac{1}{g}\right)^2 \\ &y = \frac{3gL}{4g} - \frac{3g^2L}{8g^2} \\ &\boxed{y = \frac{3L}{8}} \end{aligned}$$

2 Problem 8.27



$$\begin{aligned} &\sum F_{m_1} = -mg + T = ma \rightarrow a = \frac{T - mg}{m} \\ &\sum F_{m_2} = -mg + T = ma \rightarrow a = \frac{T - mg}{m} \\ &\rightarrow a_1 = a_2 = a \end{aligned}$$

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As the system is released from it's stationary position, the cylinder will unroll due to gravity. But since the other mass is going down as well and since tension is constant along the string, the cylinder will unroll more. So if there's Δy on the left and Δy on the right, the net length of the string scales by 2v. Meaning that for the cylinder $\omega r = 2v$.
We can then use conservation of energy to find an expression for v.
Let h be the distance from the ground, then

$$\begin{aligned} K_{1,i} &= 0 & K_{1,f} &= \frac{mv^2}{2} \\ K_{2,i} &= 0 & K_{2,f} &= \frac{mv^2}{2} + \frac{I_z^{CM}\omega^2}{2} \\ V_{1,i} &= mgh & V_{1,f} &= 0 \\ V_{2,i} &= mgh & V_{2,f} &= 0 \end{aligned}$$

Then conservation of energy gives

$$\begin{aligned} &K_{1,i} + K_{2,i} + V_{1,i} + V_{2,i} = K_{1,f} + K_{2,f} + V_{1,f} + V_{2,f} \\ &mgh + mgh = \frac{mv^2}{2} + \left(\frac{mv^2}{2} + \frac{I\omega^2}{2}\right) \\ &2mgh = \frac{mv^2}{2} + \frac{1}{2}\left(\frac{mr^2}{2}\right)\frac{4v^2}{r^2} \\ &2mgh = \frac{mv^2}{2} + \left(\frac{mv^2}{2} + mv^2\right) \\ &\rightarrow v = \sqrt{gh} \rightarrow \boxed{a = \frac{g}{2}} \end{aligned}$$

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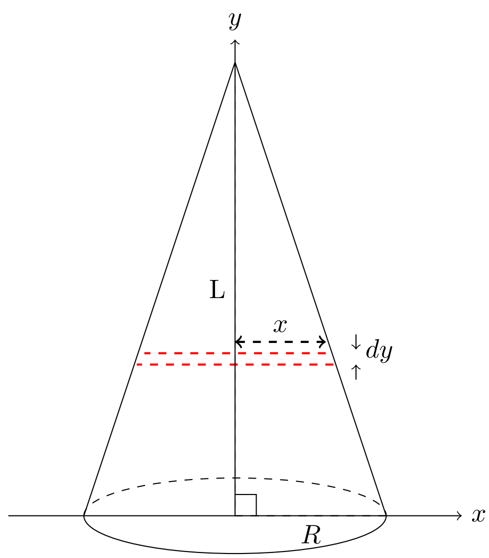
3 Problem 8.32



The volume of the infinitesimal disk Highlighted in the figure above is

$$dV = \pi x^2 dy$$

It's mass is then

$$\begin{aligned} dm &= \rho dV \\ &= \rho\pi x^2 dy \end{aligned}$$

The moment of inertia of this thin disk is also

$$\begin{aligned} dI &= \frac{(dm)x^2}{2} \\ &= \frac{\rho\pi x^2 dy x^2}{2} \end{aligned}$$

We can then integrate over the whole length of the cone.

$$I = \int_0^L dI = \int_0^L \frac{\rho\pi x^4}{2}\ dy$$

To complete this integral we must find a relationship between x^4 and dy.
From the figure we see that $y = mx$, set $m = L/R$ since both triangles are similar with common ration between the opposite side and adjacent side. This

then gives $y = \dfrac{Lx}{R}$ and so $x = \dfrac{Ry}{L}$
We can now substitute this relationship in the integral and solve.

$$\begin{aligned} I =& \frac{1}{2}\rho\pi \int_0^L \frac{R^4 y^4}{L^4}\ dy \\ &= \frac{L^5}{5}\frac{1}{2}\rho\pi\frac{1}{L^4}R^4 \\ &= \frac{L\rho\pi R^4}{10} \end{aligned}$$

Since we're integrating over a cone,

$$\rho = \frac{M}{V} = \frac{3M}{\pi R^2 L} \rightarrow \boxed{I = \frac{3MR^2}{10}}$$

4 Problem 8.36

$$\text{Let } \vec{L} = \vec{r} \times \vec{p}$$

If someone were in space, rotating his arms in one direction would result him spinning in the opposite direction due to conservation of angular momentum. Same thing applies in this problem. If the person falling spins his arms in the falling direction he would have a negative $\vec{L}$ component with respect to the body's $\vec{L}$. Therefore there is a chance is a chance the person prevents himself from falling if he rotates his arms fast enough and $\vec{L}$ with respect to the body is not sufficiently high.