PHYS230 Homework 10

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1 Question 12.20

1.1 Part a)

$$\begin{aligned} E_{tot} &= E_1 + E_2 \\ &= M_0 c^2 \gamma_V + M_0 c^2 \gamma_V \\ &= \boxed{2\gamma_V M_0 c^2} \end{aligned}$$

1.2 Part b)

Let V be the corresponding velocities of each particles in the S' frame. We'll convert these velocities in the S frame and compute the total energy of the system relative to this frame. We'll also drop the c^2 terms in the relative velocity and gamma equations as this is algebraicly allowed and will make

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the algebra easier.

$$
\begin{aligned}
v_1 &= \frac{u+V}{1+uV} \\
\gamma_{v_1} &= \frac{1}{\sqrt{1-v_1^2}} \\
\gamma_{v_1} &= \frac{1}{\sqrt{1-\dfrac{(u+V)^2}{(1+uV)^2}}} \\
&= \frac{1+uV}{\sqrt{(1+uV)^2-(u+V)^2}} \\
&= \frac{1+uV}{\sqrt{1+2uV+u^2V^2-u^2-2uV-V^2}} \\
&= \frac{1+uV}{\sqrt{(1-u^2)}\sqrt{(1-V^2)}} = \boxed{\gamma_u\gamma_V(1+uV)}
\end{aligned}
$$

Simillarly for v_2 ,

$$
\begin{aligned}
v_2 &= \frac{u-V}{1-uV} \\
\gamma_{v_2} &= \frac{1}{\sqrt{1-v_2^2}} \\
\gamma_{v_2} &= \frac{1}{\sqrt{1-\dfrac{(u-V)^2}{(1-uV)^2}}} \\
&= \frac{1-uV}{\sqrt{(1-uV)^2-(u-V)^2}} \\
&= \frac{1-uV}{\sqrt{1-2uV+u^2V^2-u^2+2uV-V^2}} \\
&= \frac{1-uV}{\sqrt{(1-u^2)}\sqrt{(1-V^2)}} = \boxed{\gamma_u\gamma_V(1-uV)}
\end{aligned}
$$

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Finally, we compute the total energy of the system in the new frame :

$$\begin{aligned} E_{tot} = E_1 + E_2 &= (\gamma_u \gamma_V (1 + uV))M_0 c^2 + (\gamma_u \gamma_V (1 - uV))M_0 c^2 \\ &= \boxed{2M_0 c^2 \gamma_u \gamma_V} \\ \implies \quad & \text{In this new frame the energy is larger by a factor } \gamma_u \end{aligned}$$

2 Question 12.22

2.1 a)

Let E_1, E_2, p_1, p_2 be the energy and the momentum of each particle respectively.

$$\gamma_{3/5} = \frac{1}{\sqrt{1 - \frac{\left(3c/5\right)^2}{c^2}}} = \frac{1}{\sqrt{\frac{25-9}{25}}} = \frac{5}{4}$$

$$\begin{aligned} E_1 &= \gamma_{3/5} mc^2 = \left(\frac{5}{4}\right) mc^2 & \qquad p_1 &= \gamma_{3/5} m \frac{3c}{5} = \frac{3}{4} mc \\ E_2 &= mc^2 & p_2 &= 0 \end{aligned}$$

2.2 b)

Let CM move with speed v relative to the lab frame. In the CM frame both particles move with the same velocities v, including the CM itself.

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Let $u_1 = \dfrac{3c}{5}$ the velocity of the first particle with respect to the lab frame

Let $u_2' = v$ the velocity of the first particle with respect to the CM frame ,then

$$u_1 = \frac{u_1' + v}{1 + \dfrac{u_1' v}{c^2}} \implies \frac{3c}{5} = \frac{2v}{1 + \dfrac{v^2}{c^2}}$$

$$\implies \frac{3c}{5} = \frac{2vc^2}{c^2 + v^2} \implies \frac{3}{5} = \frac{2vc}{c^2 + v^2}$$

$$\rightarrow 3c^2 + 3v^2 = 10vc \implies 3c^2 - 10vc + 3v^2 = 0$$

using the quadratic formula to solve for v yields solutions :

$$v_+ = \frac{18c}{6} \quad v_- = \frac{c}{3}.$$

the first solution is rejected as it is larger than the speed of light which is impossible, therefore

$$\boxed{v = \frac{c}{3}}$$

3 Question 12.25

When the particle decays, assume one photon is sent to the right and the new particle is sent to the left. Conservation of energy and momentum yields:

Using the fact that for a photon $E = pc$,

$$E_i = M_0 c^2 \qquad\qquad E_f = pc + \gamma m_0 c^2$$

$$p_1 = 0 \qquad\qquad p_f = \frac{E}{c} - \gamma m_0 v$$

$$\Delta E \implies M_0 c^2 = pc + \gamma m_0 c^2 \tag{1}$$

$$\Delta p \implies 0 = \frac{E}{c} - \gamma m_0 v \tag{2}$$

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Using (1) $- \; c \times$(2) yields ,

$$M_0c^2 = pc + \gamma m_0 c^2 - E + \gamma m_0 vc \implies M_0 c^2 = m_0(\gamma c^2 + \gamma vc)$$

$$\implies m_0 = \frac{M_0 c}{\gamma(1+v)} = \frac{M_0 c\sqrt{1 - v^2/c^2}}{(c+v)}$$

$$\implies m_0 = \frac{M_0 c\sqrt{c^2 - v^2}}{c(c+v)} = \frac{M_0\sqrt{c-v}\sqrt{c+v}}{(c+v)}$$

$$\implies \boxed{m_0 = M_0\sqrt{\frac{c-v}{c+v}}}$$

For the energy of the photon we'll insert this result in equation 2 .

$$E = m_0\gamma vc = M_0\sqrt{\frac{c-v}{c+v}}\gamma vc$$

$$E = \frac{M_0\sqrt{c-v}vc}{\sqrt{c+v}\sqrt{1-(v^2/c^2)}}$$

$$\implies E = \frac{M_0\sqrt{c-v}vc^2}{\sqrt{c+v}\sqrt{c-v}\sqrt{c+v}} \implies \boxed{E = \frac{M_0 vc^2}{(c+v)}}$$

4 Question 12.29

Let E_1 and E_2 be the energies of the top and bottom photons respectively. Then conservation of energy gives

$$\gamma mc^2 = E_1 + E_2$$

Since photons are maseless, $E^2 = (pc)^2 \implies p = \frac{E}{c}$. Conservation of momentum is decoupled along the x and y axis and the corresponding conservation equations are

$$p_x : \gamma mv = \frac{E_1}{c}\cos\theta$$

$$P_y : 0 = \frac{E_1}{c}\sin\theta - \frac{E_2}{c} \implies E_2 = E_1\sin\theta$$

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Substituting the previous two equations into the initial energy conservation equation gives

$$\gamma mc^2 = E_1 + E_1 \sin\theta = E_1(1 + \sin\theta)$$

Replacing E_1 with the result found from p_x conservation yields

$$\begin{aligned} \gamma mc^2 &= \frac{c\gamma mv}{\cos\theta}(1 + \sin\theta) \\ \rightarrow \frac{c}{v} &= \left(\frac{1}{\cos\theta} + \tan\theta\right) \end{aligned}$$

Since $\tan\theta = {}^1\!/\!_2 \implies \cos\theta = {}^2\!/\!_{\sqrt{5}}$

$$\begin{aligned} \frac{c}{v} &= \frac{\sqrt{5}+1}{2} \\ \implies \frac{v}{c} &= \frac{2}{\sqrt{5}+1}, \frac{2}{\sqrt{5}+1}\frac{(\sqrt{5}-1)}{(\sqrt{5}-1)} = \frac{2(\sqrt{5}-1)}{4} \\ \implies & \boxed{\frac{v}{c} = \frac{\sqrt{5}-1}{2}} \end{aligned}$$