PHYS 241 Lab 2.

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Section 22524 Thursday

Experiment performed with Guillaume Payeur

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Question 1.

a)

The voltage across a capacitor is defined as

$$V_C = V_0 \left(1 - e^{-t/\tau}\right)$$

Let t_0 be the time at which the capacitor is charged by $0.1V_0$ V and t_f the time at which the capacitor is charged to $0.9V_0$ V. Then we have

$$0.1V_0 = V_0 \left(1 - e^{-t_0/\tau}\right)$$
$$0.9V_0 = V_0 \left(1 - e^{-t_f/\tau}\right)$$

$$\implies t_f - t_0 = -\tau \ln |-0.9 + 1| + \tau \ln |-0.1 + 1| = 2.3\tau - 0.1\tau = 2.2\tau.$$

The theoretical circuit rise time (using $C = 0.68$ µF and $R = 1$ kΩ) is then 1.5 ms.

In our experiment ,using a multimeter, the capacitance along with the resistance were found to be respectively 0.701 µF and 0.9726 kΩ. Multiplying these yields a value for the time constant , $\tau = 0.000681$ s.

$$\therefore t_r = 2.2\tau = 1.520 \text{ ms}.$$

This value is in good agreement with the theoretical rise time which we initially expected.

b)

It was found experimentally that the frequency at which V_C waveform's amplitude starts to decrease is at approximately 200 Hz. This is a sensible answer because at frequencies close to the time response the decrease gets progressively noticeable. Indeed, t_r is the time required to charge the capacitor up from 10% to 90% of its maximal capacity. 200 Hz corresponds to 0.005 s which is higher than t_r , such that the capacitor doesn't have time to fully charge since the voltage threshold is not attained.

c)

The input voltage v_i is 1.0 V and the time divisions correspond to 1 µs, therefore we can immediately compute V_C

$$V_C = \frac{1}{RC}\int v_i\, dt = \frac{1}{0.000681 \text{ s}}\int (1)\, dt = \frac{t}{0.000681 \text{ s}} = \frac{10 \times 10^{-6} \text{ V s}^{-1}}{0.000681 \text{ s}} = 14,6 \text{ mV}.$$

Experimentally it was found graphically that the peak to peak $V_C = 15$ mV , which is in good agreement with the theoretical expectation.

Question 2.

The rise time for this circuit was found to be 0.0015 s. Since the input square wave is at 100 Hz, that converts to 0.01 s which is higher than t_r, thus allowing the capacitor to always fully charge. In that case, adding a DC voltage doesn't affect its DC level of the capacitor since that is already at its maximal value.

Question 3.

a)

The rate of change was evaluated graphically by looking at the input waveform's voltage along with the time division 10 ms. The rate of change was found to be

$$v_i = \left(\frac{2 \text{ V}}{5 \times 10 \text{ ms}}\right) = 40 \text{ V s}^{-1}.$$

Multiplying by 2 to account for a full wave "peak-to-peak" yields

$$v_i = 80 \text{ V s}^{-1}$$

Then we compute V_C :

$$V_C = RC\frac{dv_i}{dt} = 0.000681 \text{ s } \frac{d}{dt}\left(80t \text{ V s}^{-1}\right) = 0.05448 \text{ V}.$$

Experimentally we found 0.0572 V which is in agreement with the theoretical value.

b)

The first choice (i) ,i.e., $R = 1$ kΩ, $C = 10$ µF is more appropriate for two reasons. First, since it was experimentally verified that for the initial given set up, 20 Hz is the frequency at which the output waveform starts to deform, choosing a larger capacitance will allow the capacitor to charge further given a large frequency. Moreover, the RC constant is larger such that 100 Hz won't be the high frequency limit, this way the output voltage won't be distorted .

Question 4.

Let V_T be the voltage at the point in between the resistor and the inductor in the studied circuit.Using complex impedences ,we find the value of the inductance as follows,

$$\frac{\widetilde{V_0}}{\widetilde{V_T}} = \frac{j\omega L}{R + j\omega L} = \left|\frac{\widetilde{V_0}}{\widetilde{V_T}}\right|^2 = \left|\frac{(\omega L)^2}{(R+\omega L)^2}\right|.$$

Let $V_0/V_T = 1/2$, then we have

$$\left|\frac{1}{2}\right|^2 = \frac{(\omega L)^2}{R^2 + (\omega L)^2}$$

$$\implies 4(\omega L)^2 = R^2 + (\omega L)^2 \implies R = \omega L\sqrt{3}$$

$$\therefore L = \frac{RT}{2\pi\sqrt{3}}.$$

Using $R = 0.9728$ kΩ and a wave generated of 150 Hz, we get

$$L = \frac{0.9728 \times 10^3 \ \Omega}{2\pi\sqrt{3} \ 150000 \text{ Hz}} = 595.9 \text{ µH}.$$