PHYS 241 Assignment 1

Mihail Anghelici 260928404

1 Question 1

First, since the stove operates at 450°C let us transform the temeprature coefficient of resistance.

$$\begin{aligned}\Delta\rho &= \alpha\Delta T\rho_0 \\ &= (3.5\times10^{-3\circ}\mathrm{C}^{-1})(450^\circ\mathrm{C}-20^\circ\mathrm{C})(3.2\times10^{-8}\Omega\,\mathrm{m}) \\ &= 4.8\times10^{-8}\Omega\,\mathrm{m}\end{aligned}$$

$$\implies \rho = 8.0\times10^{-8}\ \Omega\,\mathrm{m}\quad \text{at } 450^\circ\mathrm{C}$$

Power dessipation in a resistor is given by the following relationship

$$P = i^2R = \frac{V^2}{R}$$

Therefore we can compute the resistor's resistance :

$$R = \frac{V^2}{P} = \frac{(220)^2}{1000}\left[\frac{\mathrm{V}^2}{\mathrm{W}}\right] = 48.4\ \Omega$$

from the relationship between a wire's shape and it's resistance, we can find the legnth of the wire required.

$$R = \rho\frac{L}{A} \implies L = \frac{RA}{\rho} = \frac{\pi d^2\ 48.4\ \Omega}{4(8.0\times10^{-8})}\left[\frac{\mathrm{m}^2\,\mathrm{V}^2}{\Omega\,\mathrm{m}}\right] = \boxed{= 29.6\ \mathrm{m}}$$

2 Question 2

2.1 a)

The resistors are presented in series, therefore

$$V_s = I_s(R_1+R_2) \implies I_s = \frac{V_s}{R_1+R_2} = \frac{5\ \mathrm{V}}{300\ \Omega} = 16.7\ \mathrm{mA}$$

2.2 b

For resistors in series, the voltage dropped across them is directly proportional to their size ,i.e, their resistance. Thus,

$$\Delta V_2 = I_s R_2 = \frac{1}{60}\ \mathrm{A}\ (200)\ \Omega = \frac{10}{3}\ \mathrm{V} = 3.33\ \mathrm{V}$$

2.3 c)

The power dissipated in R_2 is given by the following equation :

$$P_2 = i^2 R_2 = \left(\frac{1}{60}\right)^2 \mathrm{A}^2\ 200\ \Omega = \frac{1}{18}\ \mathrm{W} = \frac{1}{18} \times 10^{-3}\ \mathrm{W} = 55.6\ \mathrm{mW}$$

3 Question 3

The upward portion of the given circuit acts as if it had a very small resistance, let us denote it R'. Since R' and R_1 are parallel,

$$R_T = \left(\frac{1}{R'} + \frac{1}{R_1}\right)^{-1}$$
$$R_1 >> R' \implies \frac{1}{R'} \to \infty \implies R_T = \frac{1}{\infty}$$

Effectively, R_1 acts as if it were absent due to the given configuration.

3.1 a)

Consequently we can compute the current in the circuit :

$$i = \frac{V_0}{R_2} = \frac{5\ \mathrm{V}}{200\ \Omega} = 25\ \mathrm{mA}$$

3.2 b)

$$\begin{aligned} I = \frac{V}{R_2} \implies V &= R_2 I \\ &= 200\ \Omega\ 0.025\ \mathrm{A} \\ &= 5\ \mathrm{V} \end{aligned}$$

3.3 c)

The power dissipated in R_2 is given by the following equation :

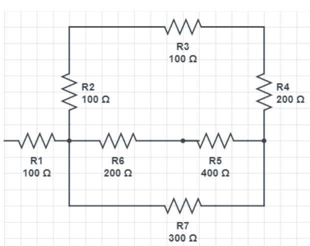
$$\begin{aligned} P_2 &= i^2 R_2 \\ &= (0.025\ \text{A})^2\ 200\ \Omega \\ &= 0.125\ \text{W} = 125\ \text{mW} \end{aligned}$$

4 Question 4

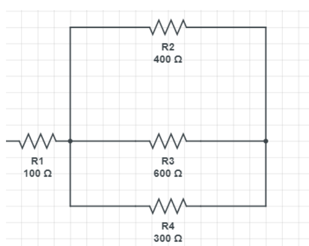
The current I pasing through the right-ward portion of the parallel circuit is given by the following equation :

$$I_2 = \frac{V}{R_2} = \frac{5\ \text{V}}{300\ \Omega} = \frac{1}{60}\ \text{A} = 16.7\ \text{mA}$$

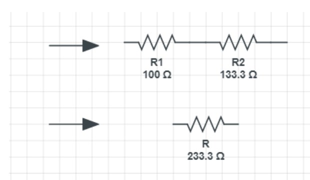
5 Question 5



(a)



(b)



(c)

Figure a) : Original circuit schematic.
Figure b) : R_2 , R_3 and R_4 are resistors in series, therefore they recombine to form a single resistor. Define $R_2 = 400\ \Omega$ the sum of the series resistors.
Figure c) : R_2, R_3 and R_4 from figure b) are resistors in parallel therefore they recombine as $R_2 = {}^{400}/_{3}\ \Omega$. Finally, R_1 and the recombined R_2 are in series thus the final equivalent resistor is $R = {}^{700}/_{3}\ \Omega$.

$$R = \left(\frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4}\right)^{-1} = \frac{700}{3}\ \Omega = 233.3\ \Omega$$

6 Question 6.

Since there's a circuit ground inserted at point B in the middle.

6.1 a)

If $R_1 = R_2$, equal voltage will be dissipated in both resistors due to symmetry. Moreover because the source is set-up that way, $\Delta V_A = -3$ V , $\Delta V_C = 0$ and $\Delta V_C = 3$ V.

6.2 b)

If $R_1 = 2R_2$, $\Delta V_A = -4$ V , $\Delta V_B = 0$ and $\Delta V_C = 2$ V.

7 Question 7.

7.1 a)

Let i be the current flowing through the circuit, V_0 the $\mathcal{E}_{\text{fem}}$, R the resistance of the right-ward resitor , v the voltage drop across R and finally r the resistance of the resistor upwards next to the photovoltaic cell.
Then ,since the two resistors are in series, the the current flow through the circuit is given by :

$$i = \frac{V_0}{R+r}$$

Moreover, the voltage drop across R is :

$$i = \frac{v}{R}$$

. Let us rearrange these equations to find the desired expression

$$i(R+r) = V_0 \implies i = \frac{V_0 - v}{r} \tag{1}$$

Equation 1. can be rearranged , in the form of $y = mx + b$:

$$i = \frac{V_0}{r} - \frac{v}{r}$$

hence a plot of I against V will have the form of a decreasing linear function, as initially given.

7.2 b)

When $i = 0$, $v = 1$ and when $v = 0$, $i = 3.5$ A. Plugging those in Equation 1 yields

$$\begin{aligned} 3.5 &= \frac{V_0}{r} \\ 0 &= \frac{V_0 - 1}{r} \implies V_0 = 1 \text{ V} \end{aligned}$$

Solving for r gives $r = 1/3.5 = 0.286 \text{ V A}^{-1}$

7.3 c)

Using Ohm's law,

$$I = \left(\frac{V_0}{r+R}\right)$$

. Since, $P_R = I^2 R$ we can substitute for P_R yielding

$$P = \left(\frac{V_0}{r+R}\right)^2 R \implies P = \frac{V^2}{\frac{r^2}{R} + 2r + R}$$

. The maximum occurs when the derivative of the denominator is set to zero.

$$\begin{aligned} \frac{d}{dR}\left(\frac{r^2}{R} + 2r + R\right) &= 0 \\ \frac{-r^2}{R^2} + 0 + 1 &= 0 \\ \frac{R^2 - r^2}{R^2} &= 0 \\ \implies R &= \pm\, r \end{aligned}$$

Since resistance is positive by definition, the maximum power in R occurs at $R = r$. Moreover,using the values found in b), the maximal power is given by

$$\begin{aligned} P &= \frac{V_0^2}{1 + 3R} \\ &= \frac{1 \text{ V}^2}{1 + 3(0.286\ \Omega)} \\ &= 539 \text{ mW} \\ \implies R = 0.286\ \Omega \end{aligned}$$

8 Question 8

By the hydrolic law and since R_1 is in series with R and R_2 in series with another R,

- $Q_1 = \frac{\Delta P}{R_H} = \frac{\Delta P}{R_1+R}$
- $Q_2 = \frac{\Delta P}{R_H} = \frac{\Delta P}{R_2+R}$

Since the final pressure is 0 $\implies$ $\Delta P = P_0$. For a pipe of radius r , the hydraulic resistance is given by

$$R_H = \frac{8\eta L}{\pi r^4}$$

So we have the following equalities

$$\begin{aligned} R_1 + R &= \frac{8\eta(l_1 + l_0)}{(d/2)^4} \implies Q_1 = \frac{8\pi(l_1 + l_0)2^4}{\pi d^4 P_0} \\ R_2 + R &= \frac{8\eta(l_2 + l_0)}{(d/2)^4} \implies Q_2 = \frac{8\pi(l_2 + l_0)2^4}{\pi d^4 P_0} \end{aligned}$$

Using the given relationship $Q_1 = Q_1/Q_1 + Q_2$, we have

$$\begin{aligned} \frac{\left(\frac{8\pi(l_1+l_0)2^4}{\pi d^4 P_0}\right)}{\left(\frac{8\pi(l_1+l_0)2^4}{\pi d^4 P_0}\right) + \left(\frac{8\pi(l_2+l_0)2^4}{\pi d^4 P_0}\right)} &= 0.25 \\ \frac{\left(\frac{8\eta 2^4}{\pi d^4 P_0}\right)}{\left(\frac{8\eta 2^4}{\pi d^4 P_0}\right)} \frac{(l_1 + l_0)}{l_1 + l_0 + l_2 + l_0} &= 0.25 \\ \implies l_1 &= \frac{l_0 - 0.25(2l_0 + l_2)}{-0.75} \\ \implies l_1 &= \frac{10^{-6}(50 - 0.25(100 + 500))}{-0.75} \\ \implies l_1 &= 1.33 \times 10^{-6} \text{ m} = 133 \text{ µm} \end{aligned}$$