PHYS 230 Homework 9

Mihail Anghelici 260928404

1 Question 11.41

1.1 a)

In the ground frame the length of the train is percieved as being shorter

$$L = \frac{L_0}{\gamma}$$

In the ground frame the photon has relative velocity of $c - v$ since the train moves away from the stationnary observer. Thus,

Since $vt = L \implies t = {}^{L}\!/_{v}$

$$t = \frac{L}{\gamma(c - v)}$$

Since the photon travels at speed c we have that

$$d = ct = \frac{cL}{(c - v)\gamma}$$

If we define $\beta = {}^{v}\!/_{c}$ then,

$$d = \frac{L}{\gamma(1 - \frac{v}{c})} = \frac{L\sqrt{1 - \beta^2}}{1 - \beta}$$

$$d = \frac{L(1 + \beta)(1 - \beta)}{(1 - \beta)} = \frac{L\sqrt{1 + \beta}}{\sqrt{1 - \beta}} = \boxed{L\sqrt{\frac{1 + \beta}{1 - \beta}}}$$

Mihail Anghelici
260 928 404

1.2 b)

In the train's frame the total distance traveled that we found in a) is relativisticaly contracted therefore,

$$
\begin{aligned}
d' &= \frac{d}{\gamma} = \frac{L}{\gamma}\sqrt{\frac{1+\beta}{1-\beta}} \\
&= L\sqrt{1-\beta^2}\sqrt{\frac{1+\beta}{1-\beta}} \\
\implies d'^2 &= L^2(1-\beta)(1+\beta)\frac{1+\beta}{1-\beta} \implies d' = \boxed{L(1+\beta)}
\end{aligned}
$$

So the total distance between the tree and the house is

$$d' = L + \frac{Lv}{c}$$

Since the length of the train is L , in this frame the train already starts at a distance of L therefore the time it takes for the front of the train to reach the house is

$$t = \frac{Lv/c}{v} = \frac{L}{c}.$$

1.3 Question 11.52

In A's frame the distance L for the mark is contracted to

$$
\begin{aligned}
L = L\sqrt{1 - v^2/c^2} &= L_0\sqrt{1 - \frac{(3c/5)^2}{c^2}} \\
&= L\sqrt{25 - \frac{9}{25}} \\
&= \frac{4}{5}L
\end{aligned}
$$

The time A takes to reach the x mark is

$$t = \frac{d}{v} \implies t = \frac{4L/5}{3c/5} = \frac{4L}{3c}$$

Mihail Anghelici
260 928 404

Since time is relativistic, in A's frame B has relative speed :

$$v'_B = \frac{v_B - v}{1 + \dfrac{vu'_x}{c^2}}$$

$$v' = \frac{\dfrac{3c}{5} - \dfrac{-3c}{5}}{1 - \dfrac{-9c^2}{25c^2}} = \frac{30c}{34} = \frac{15c}{17}$$

Therefore in time $\dfrac{4L}{3c}$, the train travels

$$\left(\frac{4L}{3c}\right)\left(\frac{15c}{17}\right) = \boxed{\frac{20L}{17}}$$

1.4 Question 11.53

Since the speed of light is c in all frames, in S' frame the x and y components of the speed of light are $u'_x = c\cos\theta$ and $u'_y = c\sin\theta$ respectively, therefore we have

$$u_x = \frac{ux' + v}{1 + \dfrac{vu_x}{c^2}} = \frac{c\cos\theta + v}{1 + \dfrac{vc\cos\theta}{c^2}} = \frac{c\cos\theta + v}{1 + \dfrac{v\cos\theta}{c}}$$

$$u_y = \frac{uy'}{\gamma\left(1 + \dfrac{vu_x}{c^2}\right)} = \frac{c\sin\theta}{\gamma\left(1 + \dfrac{vc\cos\theta}{c^2}\right)} = \frac{c\sin\theta}{\gamma\left(1 + \dfrac{v\cos\theta}{c}\right)}$$

To show that the speed is c in S frame, we'll show that $u_x^2 + u_y^2 = c^2$ i.e the magnitude is c

$$
\begin{aligned}
u_x^2 + u_y^2 &= \left(\frac{c\cos\theta + v}{1 + \dfrac{v\cos\theta}{c}}\right)^2 + \left(\frac{c\sin\theta}{\gamma\left(1 + \dfrac{v\cos\theta}{c}\right)}\right)^2 \\
&= \frac{(c\cos\theta + v)^2}{\dfrac{(c + v\cos\theta)^2}{c^2}} + \frac{c^2\sin^2(\theta)}{\dfrac{\gamma^2}{c^2}(c + v\cos\theta)^2} \\
&= \frac{c^2}{(c + v\cos\theta)^2}\left((c\cos\theta + v)^2 + \frac{c^2\sin^2(\theta)}{\gamma^2}\right) \\
&= \frac{c^2}{(c + v\cos\theta)^2}\left(c^2\cos^2(\theta) + 2vc\cos\theta + v^2 + \sin^2(\theta)c^2\left(1 - \frac{v^2}{c^2}\right)\right) \\
&= \frac{c^2}{(c + v\cos\theta)^2}\left(c^2\cos^2(\theta) + 2vc\cos\theta + v^2 + \sin^2(\theta)c^2 - \sin^2(\theta)v^2\right) \\
&= \frac{c^2}{(c + v\cos\theta)^2}\left(c^2(\sin^2(\theta) + \cos^2(\theta)) + v^2(1 - \sin^2(\theta)) + 2vc\cos\theta\right)
\end{aligned}
$$

Since $\cos^2(\theta) + \sin^2(\theta) = 1$ and $1 - \sin^2(\theta) = \cos^2(\theta)$,we have

$$
\begin{aligned}
&= \frac{c^2}{(c + v\cos\theta)^2}\left(v^2\cos^2(\theta) + c^2 + 2vc\cos\theta\right) \\
&= \frac{c^2}{(c + v\cos\theta)^2}(c + v\cos\theta)^2 = \boxed{c^2} \\
&\implies \sqrt{c^2} = c \qquad \text{As we wanted to show}
\end{aligned}
$$

1.5 Question 11.60

1.6 a)

The relative velocity in A's frame is :

$$
v_B = \frac{v + v}{1 + \dfrac{v^2}{c^2}} = \frac{2vc^2}{c^2 + v^2}
$$

Mihail Anghelici
260 928 404

The length of B train is then contracted by a factor gamma :

$$\gamma_{B,A} = \frac{1}{\sqrt{1 - \frac{\left(\frac{2vc^2}{c^2+v^2}\right)^2}{c^2}}} = \frac{1}{\sqrt{1 - \frac{\frac{4v^2c^4}{(c^2+v^2)^2}}{c^2}}}$$
$$= \frac{c^2+v^2}{\sqrt{c^4 + 2v^2c^2 + v^4 - 4v^2c^2}}$$
$$= \frac{c^2+v^2}{c^2-v^2}$$

Therefore, the time it takes to pass each other is the length of train A and contracted length of train B

$$t = \frac{L + 2L\left(\frac{c^2-v^2}{c^2+v^2}\right)}{v_B}$$
$$= \frac{L + 2L\left(\frac{c^2-v^2}{c^2+v^2}\right)}{\left(\frac{2vc^2}{c^2+v^2}\right)}$$
$$= \frac{L(c^2+v^2) + 2L(c^2-v^2)}{2vc^2}$$
$$= \boxed{\frac{3Lc^2 - Lv^2}{2vc^2}}$$

Mihail Anghelici
260 928 404

1.7 b)

In B's frame A's length is contracted and the relative velocity is the same therefore the time it takes is

$$\begin{aligned}
t &= \frac{L\left(\dfrac{c^2-v^2}{c^2+v^2}\right)+2L}{v_A} \\
&= \frac{L\left(\dfrac{c^2-v^2}{c^2+v^2}\right)+2L}{\left(\dfrac{2vc^2}{c^2+v^2}\right)} \\
&= \frac{L(c^2-v^2)+2L(c^2+v^2)}{2vc^2} \\
&= \boxed{\frac{3Lc^2+Lv^2}{2vc^2}}
\end{aligned}$$

1.8 c)

In the ground frame, a stationnary observer sees both length of A and B contracted and the relative velocity is $2v$, therefore the time it takes is both lengths contracted

$$\begin{aligned}
t &= \frac{L\left(\dfrac{c^2-v^2}{c^2+v^2}\right)+2L\left(\dfrac{c^2-v^2}{c^2+v^2}\right)}{2v} \\
&= \frac{3L\left(\dfrac{c^2-v^2}{c^2+v^2}\right)}{2v} \\
&= \boxed{\frac{3L\gamma_{B,A}}{2v}}
\end{aligned}$$