MATH475 Weekly Work 8

Mihail Anghelici 260928404

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Question 1

$$\frac{\partial}{\partial x}u_0(x-ut) = u_0'\left(\frac{\partial}{\partial x}(x-ut)\right) = u_0'\left(1-\frac{\partial}{\partial x}(ut)\right) = u_0'(1-u_x t)$$

Rearranging the expression and setting the argument of u_0' to x_0 we get

$$u_x(x,t) = \frac{u_0'(x_0)}{1+u_0'(x_0)t}$$

Question 2

$$\begin{aligned} &\frac{\mathrm{d}x}{\mathrm{d}\tau} = q'(z); \qquad &&\frac{\mathrm{d}t}{\mathrm{d}\tau} = 1; \qquad &&\frac{\mathrm{d}z}{\mathrm{d}t} = 0 \\ &x(s,0) = s; &&t(s,0) = 0; &&z(s,0) = u_0(s) \end{aligned}$$

Thus, we obtain

$$z = u_0(s); \qquad t = \tau; \qquad x = q'(u)t + s.$$

Therefore we convert to $u(x,t)$

$$s = x - q'(u)t \implies u(x,t) = u_0(x - q'(u)t).$$

As a double check,

$$\frac{\partial}{\partial x}u_0(x-q'(u)t) \implies u_x(x,t) = \frac{u_0'(x_0)}{1+q''(u)u_0'(x_0)t}$$

Question 3

A sufficient condition for $u(x,t)$ to remain smooth for all t is to require $q''(u)u_0'(x) \geq 0 \; \forall \, x \in \mathbb{R}$.

If this condition is violated, i.e., $\exists \, x_0$ such that $q''(u)u_0'(x_0) < 0$ then since

$$u_x \to -\infty \quad \text{as } t \to \frac{-1}{q''(u)u_0'(x_0)},$$

then

$$t^* = \min_{x\in\mathbb{R}} \left\{ \frac{-1}{q''(u)u_0'(x_0)} \,\middle|\, q''(u)u_0'(x) < 0 \right\} = \frac{-1}{q''(u)u_0(x_0)}.$$