Homework 8 PHYS230

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1 Problem 8.40

As an external force is applied to the paper to displace it, friction is exerted on the bowling :

$$\sum F = ma = F_f$$

Using the fact that moment of inertia for a solid sphere is

$$I = \frac{2MR^2}{5}$$

The conservation of torque around the CM of the bowling ball gives:

$$\vec{\tau}_{tot} = I\alpha$$
$$F_f R = \frac{2MR^2}{5}\alpha$$

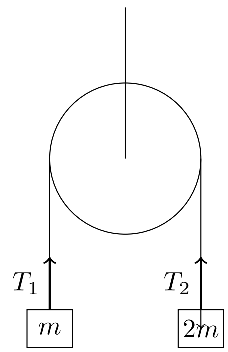
Since $\dfrac{F_f}{m} = a$, we can substitue and solve for a

$$\frac{5F_f}{2m} = R\alpha$$
$$\to \frac{5a}{2} = R\alpha$$

Consider a point p at the contact point between the paper and the ball. Then by definition of no-slipping condition,

$$\vec{a}_p = \vec{a}_{CM} + \vec{a}_{CM,p}$$
$$\iff a_0 = a + R\omega$$
$$\implies a_0 = a + \frac{5a}{2} = \frac{7a}{2}$$
$$\implies \boxed{a = \frac{2a_0}{7}}$$

2 Problem 8.44



Since $m_2 \geq m_1$, and acceleration is same for both objects (conservation of string), we'll assume the acceleration on the right side is positive and that on the left side is negative. Thus, $F = ma$ on both sides yields

$$T_1 - mg = ma$$
$$T_2 - 2mg = 2m(-a) \rightarrow 2mg - T_2 = 2ma$$

Moment of inertia for a pulley of radius R rotating along it's center is defined as $I = \frac{mR^2}{2}$. Therefore using conservation of torque we get

$$-T_1 r + T_2 r - I\alpha = 0$$
$$\rightarrow (T_2 - T_1)r = \frac{mr^2\alpha}{2}$$
$$\rightarrow (T_2 - T_1) = \frac{m(r\alpha)}{2}$$

Since we have a no slipping condition, $v = R\omega \implies a = R\alpha$, so we can substitute this result

$$(T_2 - T_1) = \frac{ma}{2}$$
$$\text{Therefore ,} \quad T_1 = T_2 - \frac{ma}{2}$$

Substituting the previous equation in the initial $F = ma$ equation gives

$$T_1 = ma + mg = T_2 - \frac{ma}{2}$$
$$\rightarrow T_2 = ma + mg + \frac{ma}{2}$$
$$\rightarrow 2mg - 2ma - mg = \frac{3ma}{2}$$
$$\rightarrow mg = \frac{7ma}{2} \qquad \implies \boxed{a = \frac{2g}{7}}$$

3 Problem 11.29

Since Δx and t' are known, we can directly find v. Relativity equations are symmetric between light years and light seconds therefore

$$\Delta x = c \qquad \text{and} \quad t' = 1$$

$$t = \gamma t'$$

$$\rightarrow v = \frac{\Delta x}{\gamma t'} = \frac{\Delta x \sqrt{1 - \frac{v^2}{c^2}}}{t'}$$

$$\begin{aligned} v^2 (t')^2 &= (\Delta x)^2 \left[1 - \frac{v^2}{c^2} \right] \\ &= (\Delta x)^2 - \frac{(\Delta x v)^2}{c^2} \end{aligned}$$

Using $\Delta x = c$ and $t' = 1$,

$$v^2 + \frac{c^2 v^2}{c^2} = c^2 \implies \boxed{v = \frac{c}{\sqrt{2}}}$$

4 Problem 11.32

4.1 a)

The length of the train in the ground's frame is L so the relativistic L_0 is

$$L_0 = \frac{L_0}{\gamma} = L\sqrt{1 - \frac{v^2}{c^2}} = L\sqrt{1 = \frac{\left(3c/5\right)^2}{c^2}}$$

$$\rightarrow \frac{4L}{5} = L_0$$

The train travels a distance of L but also it's whole length $\frac{4L}{5}$, so in total a distance of $\frac{9L}{5}$,thus the time it takes in the ground frame is

$$\frac{\Delta x}{\Delta t} = v \implies \frac{\Delta x}{v} = \Delta t$$

$$\rightarrow \frac{\left(\frac{9L}{5}\right)}{\left(\frac{3c}{5}\right)} = t = \boxed{\frac{3L}{c}}$$

4.2 b)

The person travels a total distance of L at a speed $\frac{3L}{c}$ with respect to the groud frame, thus

$$\frac{\Delta x}{\Delta t} = v \implies \frac{L}{\left(\frac{3L}{c}\right)} = \boxed{\frac{c}{3}}$$

4.3 c)

Since the person is travelling at a speed of $\frac{c}{3}$ with respect to the ground over a distance L

$$\Delta t = \frac{\Delta x}{v} = \frac{L}{\left(\frac{c}{3}\right)} = \frac{3L}{c}$$

Moreover,

$$\begin{aligned} t' = \frac{t}{\gamma_{1/3}} &= \frac{3L}{c}\sqrt{1 - \frac{\left({}^{c}\!/_{3}\right)^2}{c^2}} \\ &= \frac{3L}{c}\frac{\sqrt{8}}{3} = \boxed{\frac{\sqrt{8}L}{c}} \end{aligned}$$