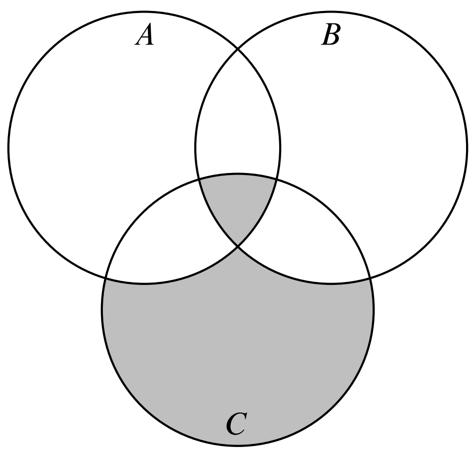
MATH 240 Assignment 1

Mihail Anghelici 260928404

February 8, 2021

Question 1

(a)



(b)

The simplest description of the given Venn diagram is

$$B \oplus (A \cap C).$$

Question 2

(a)

($\subseteq$)

$$\begin{aligned} \text{Let } x \in \overline{A \cup B}. &\Longrightarrow x \notin A \cup B \\ &\Longrightarrow x \notin A \text{ and } x \notin B \\ &\Longrightarrow x \in \overline{A} \text{ and } x \in \overline{B} \\ &\Longrightarrow x \in \overline{A} \cap \overline{B} \end{aligned}$$

($\supseteq$)

$$\begin{aligned} \text{Let } x \in \overline{A} \cap \overline{B}. &\Longrightarrow x \in \overline{A} \text{ and } x \in \overline{B} \\ &\Longrightarrow x \notin A \text{ and } x \notin B \\ &\Longrightarrow x \notin A \cup B \\ &\Longrightarrow x \in \overline{A \cup B} \end{aligned}$$

(b)

$$\begin{aligned} (A \backslash B) \cap (C \backslash B) &= (A \cap \overline{B}) \cap (C \cap \overline{B}) && \text{[Set difference law]} \\ &= A \cap (\overline{B} \cap C) \cap \overline{B} && \text{[Associative law]} \\ &= A \cap (C \cap \overline{B}) \cap \overline{B} && \text{[Commutative law]} \\ &= (A \cap C) \cap (\overline{B} \cap \overline{B}) && \text{[Associative law]} \\ &= (A \cap C) \cap \overline{B} && \text{[Idenpotent law]} \\ &= (A \cap C) \backslash B && \text{[Set difference law]} \end{aligned}$$

Question 3

(a)

(i) The statement is true since $\forall v \in \mathbb{R}$ we can chose $u = 1$ such that the equality $uv = v$ holds.

(ii) $\neg(\exists u \in \mathbb{R}, \forall v \in \mathbb{R}, uv = v) = \forall u \in \mathbb{R}, \exists v \in \mathbb{R}, uv \neq v$. Here we let $u = 1$ then $\nexists v \in \mathbb{R} | uv \neq v$. The statement is therefore false.

(b)

(i) The given statement is false. Indeed, let $x = 1, y = 1$, then $z = 1 - 1 = 0 \notin \mathbb{N}$.

(ii) $\neg(\forall x \in \mathbb{N}, \forall y \in \mathbb{N}, \exists z \in \mathbb{N}, z = x - y) = \exists x \in \mathbb{N}, \exists y \in \mathbb{N}, \forall z \in \mathbb{N}, z \neq x - y$. Here to disprove the statement we have to show $\nexists z \in \mathbb{N}$ such that there is at least one x and y in $\mathbb{N}$ for which $z \neq x - y$. This is impossible since we can just pick $z = x = y$ which satisfies $z \neq x - y$. The statement is therefore true.

Question 4

(a)

p	q	r	$p \implies r$	$q \implies r$	$(p \implies r) \iff (q \implies r)$
T	T	T	T	T	T
T	T	F	F	F	T
T	F	T	T	T	T
T	F	F	F	T	F
F	T	T	T	T	T
F	T	F	T	F	F
F	F	T	T	T	T
F	F	F	T	T	T

Not all values are true in the last column,therefore this is not a tautology.

(b)

p	q	$\overline{p}$	$\overline{q}$	$\overline{p} \vee q$	$p \wedge \overline{q}$	$(\overline{p} \vee q) \vee (p \wedge \overline{q})$
T	T	F	F	T	T	T
T	F	F	T	F	T	T
F	T	T	F	T	F	T
F	F	T	T	T	T	T

All values are true in the last column, therefore this is a tautology.

(c)

p	q	r	$p \oplus q$	$\overbrace{(p \oplus q) \wedge r}^{:=P}$	$p \wedge r$	$q \wedge r$	$\overbrace{p \wedge r \oplus q \wedge r}^{:=Q}$	$P \iff Q$
T	T	T	F	F	T	T	F	T
T	T	F	F	F	F	F	F	T
T	F	T	T	T	T	F	T	T
T	F	F	T	F	F	F	F	T
F	T	T	T	T	F	T	T	T
F	T	F	T	F	F	F	F	T
F	F	T	F	F	F	F	F	T
F	F	F	F	F	F	F	F	T

All values are true in the last column, therefore this is a tautology.

Question 5

(a)

$$
\begin{aligned}
p \implies (q \implies r) &\equiv \overline{p} \vee (q \implies r) && \text{[Definition of } \implies\text{]}\\
&\equiv \overline{p} \vee (\overline{q} \vee r) && \text{[Definition of } \implies\text{]}\\
&\equiv \overline{p} \vee \overline{q} \vee r && \text{[Commutativity of } \vee\text{]}\\
&\equiv \overline{p \wedge q} \vee r && \text{[De Morgan]}\\
&\equiv (p \wedge q) \implies r && \text{[Definition of } \implies\text{]}
\end{aligned}
$$

(b)

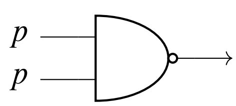
$$
\overline{p \implies q} \equiv \underbrace{\overline{\overline{p} \vee q}}_{\text{[Definition of } \implies\text{]}} \equiv \overbrace{p \wedge \overline{q}}^{\text{[De Morgan]}}
$$

(c)

$$
\begin{aligned}
(\overline{p \vee \overline{q}}) \vee (\overline{p} \wedge \overline{q}) &\equiv (\overline{p} \wedge q) \vee (\overline{p} \wedge \overline{q}) && \text{[De Morgan]}\\
&\equiv \overline{p} \wedge (q \vee \overline{q}) && \text{[Distributive law]}\\
&\equiv \overline{p} \wedge 1 && \text{[Complement rule]}\\
&\equiv \overline{p} && \text{[Identity rule]}
\end{aligned}
$$

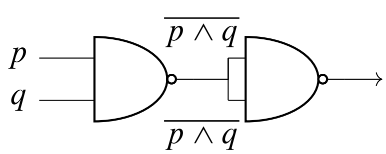
Question 6

(a)



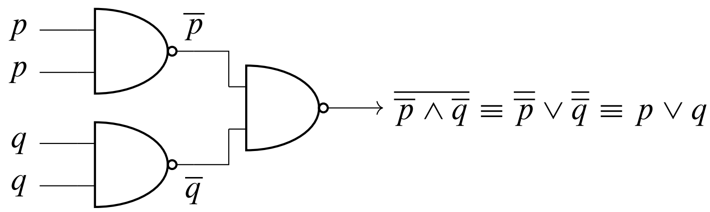
$$\overline{p \wedge p} \equiv \overline{p}$$

(b)



$$\overline{\overline{p \wedge q} \wedge \overline{p \wedge q}} \equiv \overline{\overline{p \wedge q}} \equiv p \wedge q$$

(c)


p
p
$\overline{p}$
q
q
$\overline{q}$
$\overline{\overline{p} \wedge \overline{q}} \equiv \overline{\overline{p}} \vee \overline{\overline{q}} \equiv p \vee q$