Convex Optimization: Homework #3

Due on November 13, 2017 at 11:59pm

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Problem 1

Textbook Exercises 4.1

Solution

the sketch of the feasible set:

(a) $x^* = (2/5, 1/5)$, $p^* = 3/5$
(b) With these constrains, $inf\,\{f_0(x_1, x_2)\} = -\infty$
(c) $X_{opt} = \{(0, x_2)\,|\,x_2 \geq 1\}$, $P^* = 0$
(d) $x^* = (1/3, 1/3)$, $P^* = 1/3$
(e) when the line $x_1 + 3x_2 = 1$ and the ellipsoid $x_2^2 + 9x_2^2 = R^2$ are tangent, the point of tangency (x_1^*, x_2^*) satisfy:

$$
\begin{aligned}
x_1^* + 3x_2^* &= 1 \\
-x_1^*/9x_2^* &= -1/3
\end{aligned}
$$

Thus, $x^* = (1/2, 1/6)$ and $p^* = 1/2$

Problem 2

Textbook exercise 4.8 a-e

Solution

(a) If the problem is infeasible, the optimal value is ∞. If the problem is feasible, the optimal value is $-\infty$.
(b) If the vector c is not parallel to a, then the optimal value is $-\infty$.
If the vector c is equal to λa, the optimal value is $-\infty$ when the $\lambda > 0$ and the optimal value is λb when $\lambda < 0$.
(c)

$$p^* = \sum_{i=1}^{j} c_i l + \sum_{i=j+1}^{n} c_i u$$

where $c_1, \dots, c_j \geq 0$ and $c_{j+1}, \dots, c_n \leq 0$
(d)

$$c^T x = \sum_{i=1}^{n} c_i x_i$$

Let $c_j = c_{min}$. Thus $p^* = c_j$ when $x_j^* = 1$ and $x_i^* = 0$, $i \neq j$
When the constraint is replaced by the inequality and $c_j \leq 0$, the optimal value is achieved with $x_j = 1$ and $x_i = 0$, $i \neq j$. If $c_j \geq 0$ the optimal value is achieved when $x_i = 0$, $i = 1, \dots, n$.
(e)

Problem 3

Textbook exercise 4.33

Solution

(a) It is equivalent to:

$$\begin{aligned} &\text{minimize } t \\ &\text{subject to } \log\left(\sum_{k=1}^{K} exp(a_{ik}^T y + b_{ik})\right)/t \leq 1,\ i = 1, 2 \end{aligned}$$

(b) It is equivalent to:

$$\begin{aligned} &\text{minimize } exp(t_1) + exp(t_2) \\ &\text{subject to } \log\left(\sum_{k=1}^{K} exp(a_{ik}^T y + b_{ik})\right) \leq t_i,\ i = 1, 2 \end{aligned}$$

(c)It is equivalent to:

$$\begin{aligned} &\text{minimize } t \\ &\text{subject to } \frac{\left(\log\left(\sum_{k=1}^{K} exp(a_{1k}^T y + b_{1k})\right)\right)/t + \log\left(\sum_{k=1}^{K} exp(a_{2k}^T y + b_{2k})\right)}{a^T y + b} \leq 1 \end{aligned}$$

Problem 4

Textbook exercise 4.40 a-c

Solution

(a) It is equivalent to:

$$\begin{aligned} &\text{minimize } c^T x + d \\ &\text{subject to } \mathbf{diag}(Gx - h) \preceq 0 \\ &Ax = b \end{aligned}$$

(b) QP. Let $P_i = W_i W_i^T$

$$\begin{aligned} &\text{minimize } t + 2q^T x + r \\ &\text{subject to } \begin{bmatrix} I & W_T x \\ x^T W & tI \end{bmatrix} \succeq 0 \\ &\mathbf{diag}(Gx - h) \preceq 0 \\ &Ax = b \end{aligned}$$

QCQP. Let $P_i = W_i W_i^T$

$$\begin{aligned} &\text{minimize } t_0 + 2q_0^T x + r_0 \\ &\text{subject to } \begin{bmatrix} I & W_i^T x \\ x^T W_i & t_i I \end{bmatrix} \succeq 0,\ i = 0, 1, \ldots, m \\ &t_i + 2q_i^T x + r_i \leq 0,\ i = 1, \ldots, m \\ &Ax = b \end{aligned}$$

SOCP.

$$\begin{aligned} &\text{minimize } c^T x \\ &\text{subject to } \begin{bmatrix} (c_i^T x + d_i)I & A_i x + b_i \\ (Ax_i + b_i)^T & (c_i^T + d_i)I \end{bmatrix} \succeq 0,\ i = 1, \ldots, N \\ &Fx = g \end{aligned}$$

(c)

$$\begin{aligned} &\text{minimize } t \\ &\text{subject to } \begin{bmatrix} F(x) & Ax + b \\ (Ax + b)^T & t \end{bmatrix} \succeq 0 \end{aligned}$$