

An Aside: Matrices and Relations



How do we store a dense matrix?

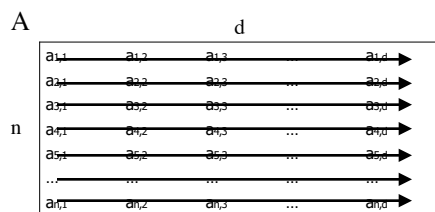
A	d				
	$a_{1,1}$	$a_{1,2}$	$a_{1,3}$...	$a_{1,d}$
	$a_{2,1}$	$a_{2,2}$	$a_{2,3}$...	$a_{2,d}$
	$a_{3,1}$	$a_{3,2}$	$a_{3,3}$...	$a_{3,d}$
n	$a_{4,1}$	$a_{4,2}$	$a_{4,3}$...	$a_{4,d}$
	$a_{5,1}$	$a_{5,2}$	$a_{5,3}$...	$a_{5,d}$

	$a_{n,1}$	$a_{n,2}$	$a_{n,3}$...	$a_{n,d}$

The above is a "logical" model. Storing it (in RAM or disk) is a "physical" model.



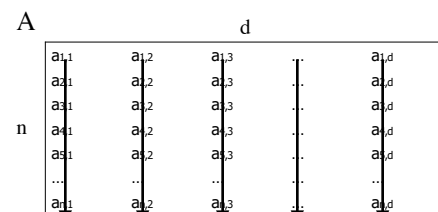
How do we store a dense matrix?



$n \times d$, row-major "linearization" of a matrix



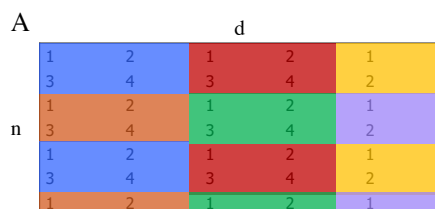
How do we store a dense matrix?



$n \times d$, column-major "linearization" of a matrix



How do we store a dense matrix?



block-oriented linearization of the matrix (e.g. see [ScaLAPACK's block-cyclic linearization](#))



But what if it's sparse?

- I.e. mostly 0
 - Common: one-hot and bag-of-words
- Store a set of triples:

$$\{(\text{row}, \text{col}, \text{val})\}$$
- I.e. a relation!

How do we store a sparse matrix?

A

row	column	val
1	3	a _{1,3}
1	7	a _{1,7}
2	14	a _{2,14}
2	2741	a _{2,2741}
3	14	a _{3,14}
3	116	a _{3,116}
...

store this on disk however you like to store relations... or use special encodings to enhance matrix arithmetic locality (see Wikipedia)

Tradeoffs

A

row	column	val
1	3	a _{1,3}
1	7	a _{1,7}
2	14	a _{2,14}
2	2741	a _{2,2741}
3	14	a _{3,14}
3	116	a _{3,116}
...

- (row,col) not stored
- Many possible layouts
 - For disk storage
 - For parallel partitioning
- Often tuned for matrix operations, e.g. multiplication

- Empty vals not stored
- Many possible layouts
 - For disk storage
 - For parallel partitioning
- Harder to tune for matrix operations
 - Irregular structure
- Rows and columns can be arbitrary values
 - Integers, reals, strings, etc.

Recall this slide

Multidimensional Data: Star Schema

Products: pid, pname, catid, sty

Time: timeid, Date, Day

Sales Fact Table: pid, timeid, locid, sales

Locations: locid, city, state, country

← This looks like a star ...

3D Cube Visualization: Product Id (11, 12, 13), Location Id (1, 2, 5), Time Id (1, 2, 3). Values: 8, 10, 10, 30, 20, 50, 25, 8, 15.

Imagine Computing AB

- Sparse matrix stored as a relation:
 - (row integer, col integer, value float)
- $(AB)_{ij} = \sum_{k=1}^m A_{ik} B_{kj}$

```

SELECT _____, SUM(A.val*B.val)
FROM A, B
WHERE _____
GROUP BY _____;
  
```

Imagine Computing AB

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```


SELECT _____, SUM(A.val*B.val)
FROM A, B
WHERE A.col = B.row
GROUP BY _____;
  
```

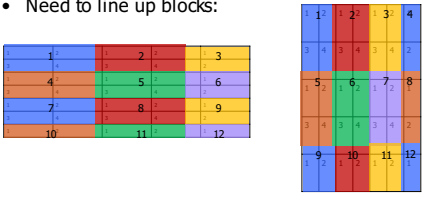
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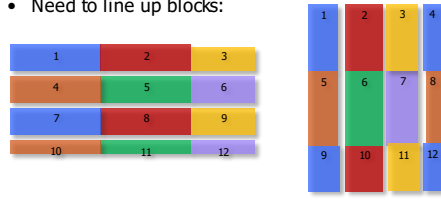
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GROUP BY A.row, B.col;
  
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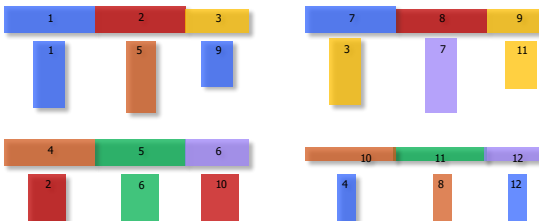
 **Imagine Computing AB**


- Dense matrix might be stored as a relation of blocks:
 - (blockNum integer, subMatrix blob)
- Need to line up blocks:
 

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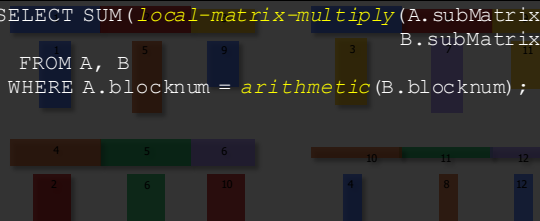
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 **Imagine Computing AB**


- Dense matrix might be stored as a relation of blocks:
 - (blockNum integer, subMatrix blob)
- Need to line up blocks:


```
SELECT SUM(local-matrix-multiply(A.subMatrix,
                                B.subMatrix))
FROM A, B
WHERE A.blocknum = arithmetic(B.blocknum);
```

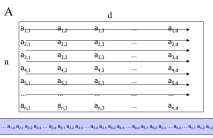


 **Many other variants**

- We looked at:
 - Sparse relations
 - Dense linearizations
 - Dense blocking (relations of dense blocks)
- Also
 - Relations of dense rows (row integer, vals blob)
 - Relations of dense columns (col integer, vals blob)
 - Compression schemes for sparse and dense
 - From the Scientific Computing & DB literature
- And don't forget non-numeric "dimensions"!
- At scale, looks pretty much like DB queries/dataflow!
 - Even for dense matrices, across "blocks"
 - Blocking may be or disk (out-of-core) or for parallelism

 **Quick Summary**

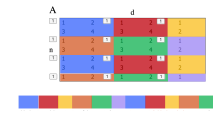
A



A

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A



```
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WHERE A.col = B.row
GROUP BY A.row, B.col;
```

Data Models

R & G, Chaps. 2&3



Steps in Traditional Database Design

- Requirements Analysis
 - user needs; what must database do?
- Conceptual Design
 - high level description
- Logical Design
 - translate into DBMS data model
- Schema Refinement
 - consistency, normalization
- Physical Design
 - indexes, disk layout
- Security Design
 - who accesses what, and how



Describing Data: Data Models

- Data model: collection of concepts for describing data.
- Schema: description of a particular collection of data, using a given data model.



Two Data Models

- Linear Algebra
 - Main concept: matrix
 - Matrix schema:
 - dimensions $n \times d$ with indices from the ordinals
 - domain for all "entries"
- Relational Model
 - Main concept: relation (table), rows and columns
 - Every relation has a schema
 - describes the columns
 - column names and simple domains
 - Values in each column all match the domain



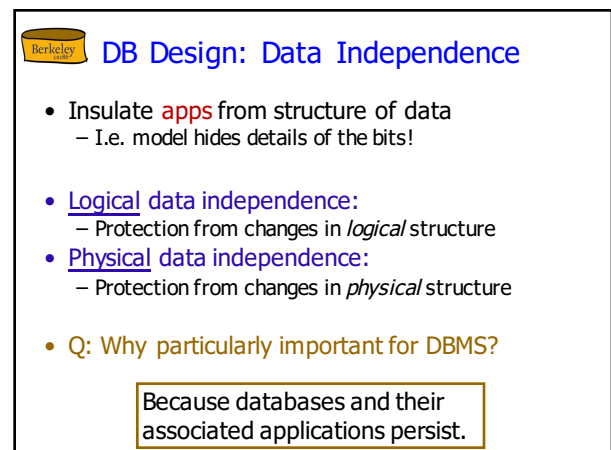
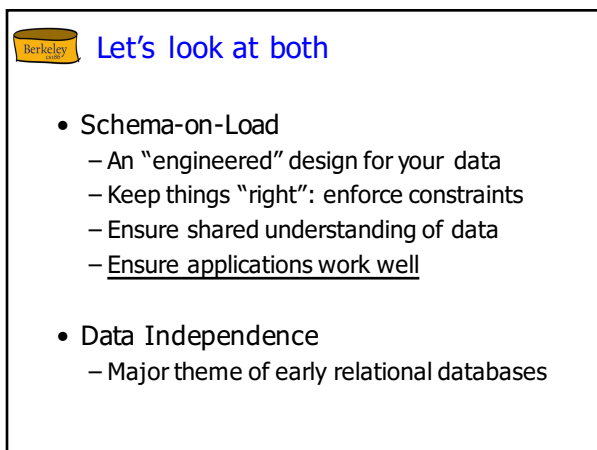
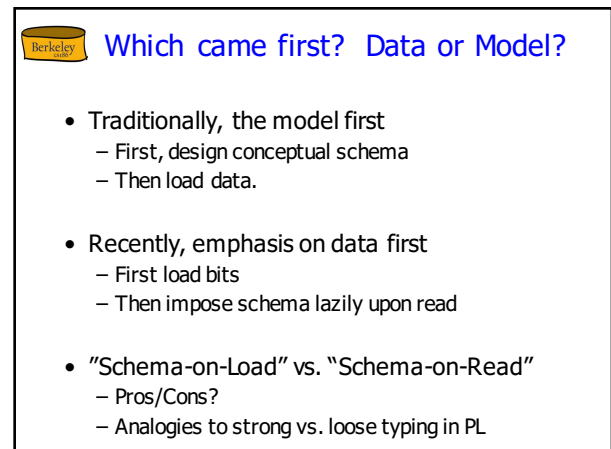
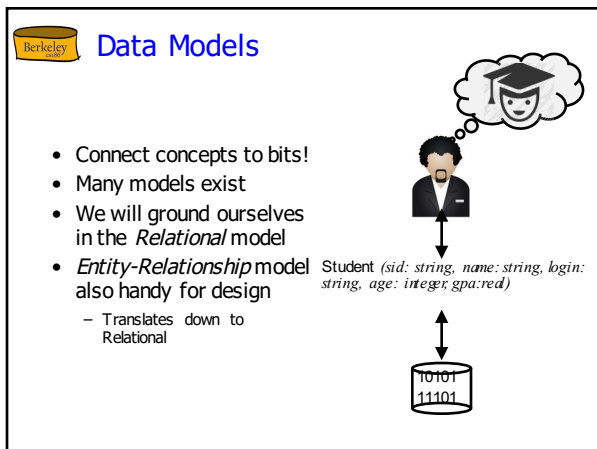
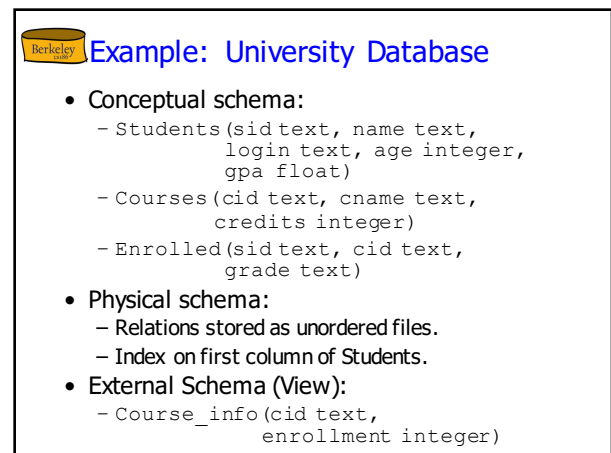
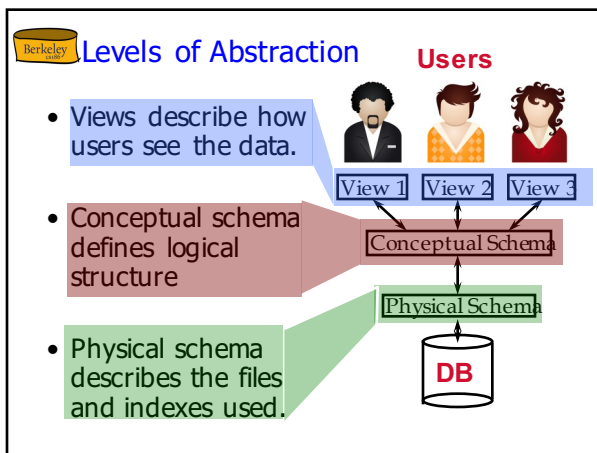
Relational Model

- Very expressive
 - Can represent many data relationships
 - Subsumes matrices, graphs, etc.
- Yet very simple
 - Domains of columns are atomic types
 - No nesting
- Expressive + Simple = Freedom
 - Lots of room for database *design*



Why Focus on Relational Model?

- Most widely used model
 - And many other models are subsets (e.g. key-value stores)
- Other models exist (and co-exist)
 - "Legacy systems" in older models
 - e.g., IBM's IMS
 - Object-Relational mergers
 - Object-Oriented features provided by DBMS
 - Object-Relational Mapping (ORM) outside the DBMS
 - A la Rails (Ruby), Django (Python), Hibernate (Java)
 - Documents: XML, JSON, etc.
 - Nested or "semi-structured" data
 - Languages like Xquery, XSLT, JSONiq
 - Many relational engines now handle these to a degree





Hellerstein's Inequality

Data independence matters when...

$$\frac{dapp}{dt} \ll \frac{denv}{dt}$$

- Not just a database issue!
 - E.g. consider elastic resources in the cloud.
 - E.g. consider Internet-wide performance.
 - ...



Agile Analytics & "Schema on Use"

- What about **agile**, exploratory analytics? $\frac{dapp}{dt} \gg \frac{denv}{dt}$
- First, don't let the lack of schema prevent storing data!
 - Just vomit out binary, text, CSV, JSON, xlsx, etc.
 - Shove into DBMS blobs or a filesystem (e.g. HDFS)
- Wrangle the data into shape as needed
 - Essentially defining physio-logical views over the raw bits
 - "Data Dependence"
 - Each Analyst has their own "opinion" about the data
 - "Opinion" embodied in custom code that is dependent on the bits!
- Fits well with data that is never (re)organized
 - E.g. Big Data, logs, "data exhaust"
 - Less of a fit with app-centric, update-heavy data

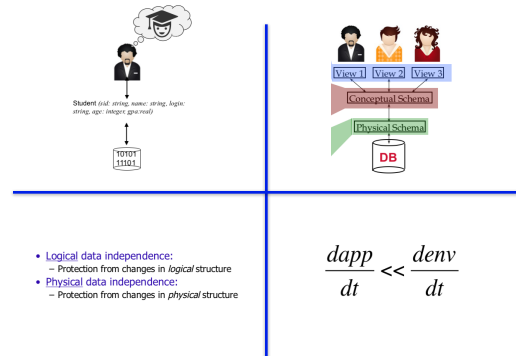


So Which is Better? It depends.

- On the use case *app*
 - Mission-critical? Exploratory?
 - Stable? Fast-changing?
- On the environment *env*
 - Governance requirements?
 - App developers? IT managers? Analysts?



Quick Summary



Entity-Relationship Diagrams

R & G, Chaps. 2&3



Entity-Relationship Model

- Relational model is a great formalism
 - and a clean system framework
- But a bit detailed for design time
 - a bit fussy for brainstorming
 - hard to communicate to customers
- Entity-Relationship model is a popular "shim" over relational model
 - graphical, slightly higher level

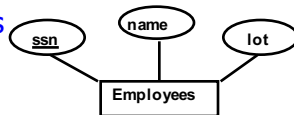
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Conceptual Design

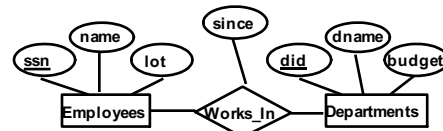
- What are the entities and relationships?
- What info about E's & R's should be in DB?
- What *integrity constraints* (*business rules*) hold?
- ER diagram is the "schema"
- Can map an ER diagram into a relational schema.
- This is where SW/data engineering *begins*
 - Ruby-on-Rails "models"

ER Model Basics



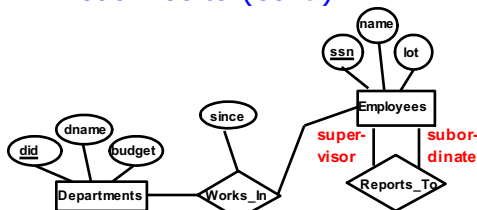
- Entity:
 - A real-world object described by a set of attribute values.
- Entity Set: A collection of similar entities.
 - E.g., all employees.
 - All entities in an entity set have the same attributes.
 - Each entity set has a key (underlined)
 - Each attribute has a domain

ER Model Basics (Contd.)



- Relationship: Association among two or more entities.
 - E.g., Attishoo works in Pharmacy department.
 - relationships can have their own attributes.
- Relationship Set: Collection of similar relationships.
 - An n -ary relationship set R relates n entity sets $E_1 \dots E_n$; each relationship in R involves entities $e_1 \in E_1, \dots, e_n \in E_n$

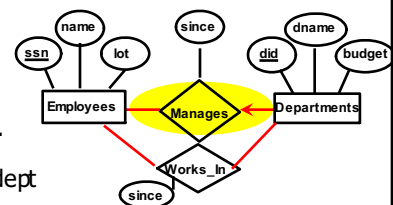
ER Model Basics (Cont.)



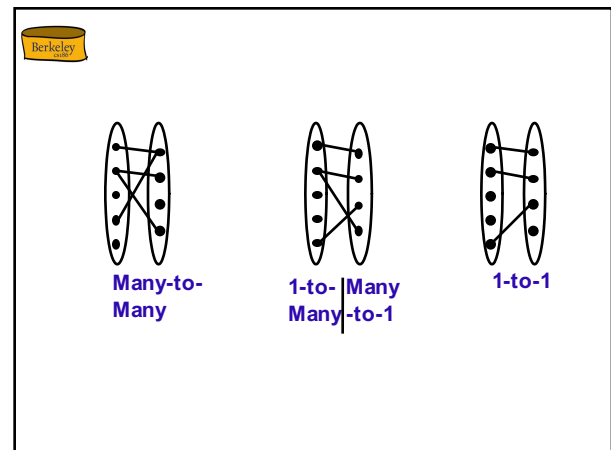
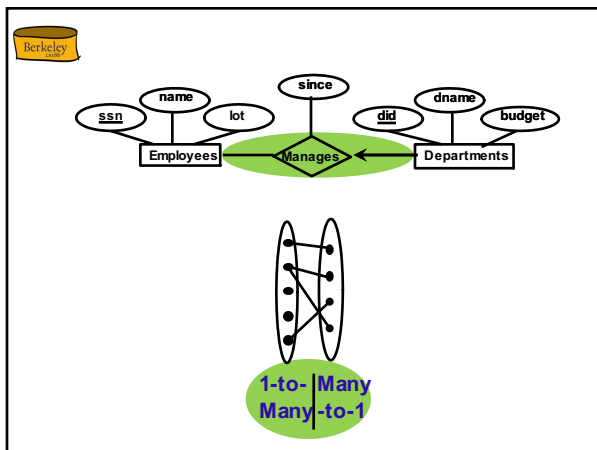
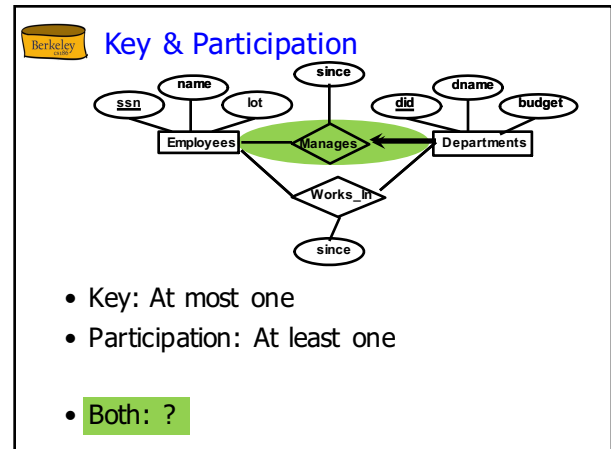
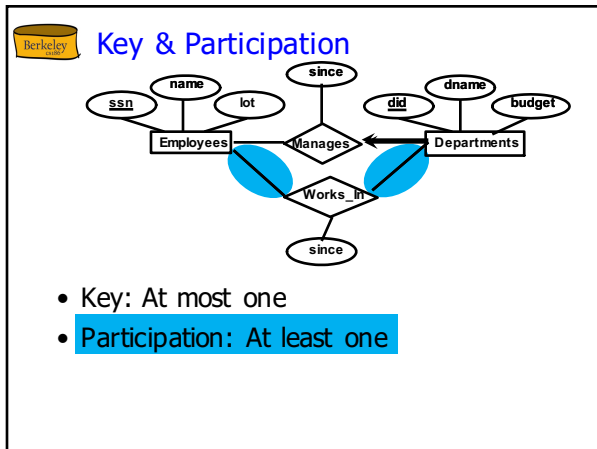
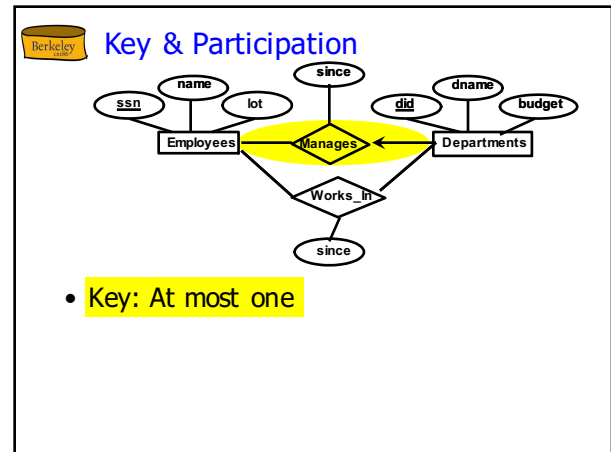
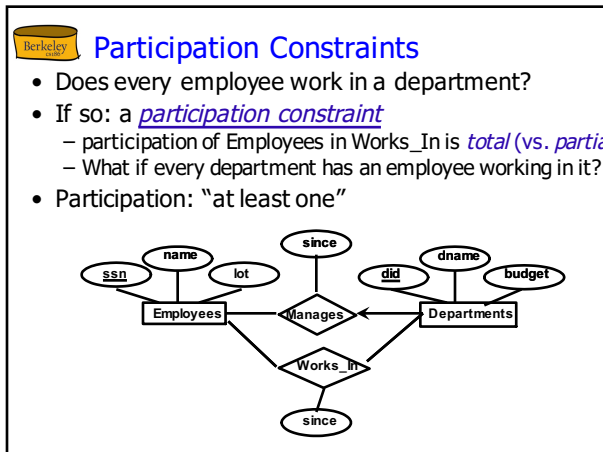
- Same entity set can participate in different relationship sets, or in different "roles" in the same relationship set.

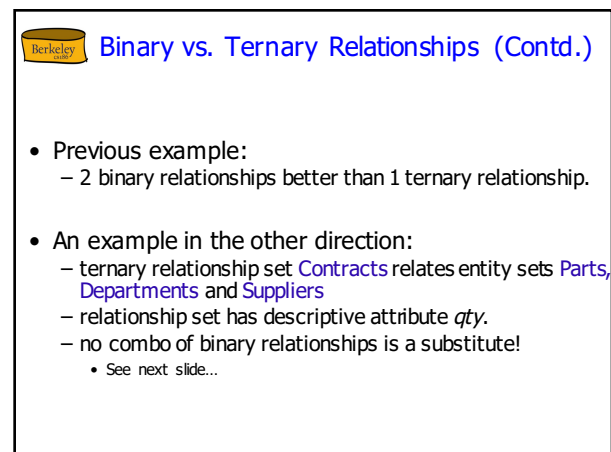
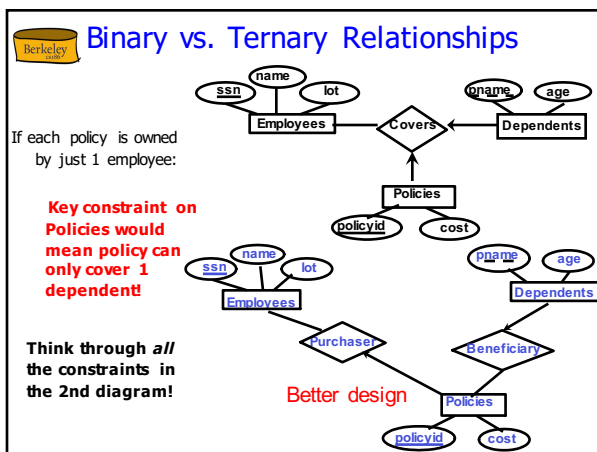
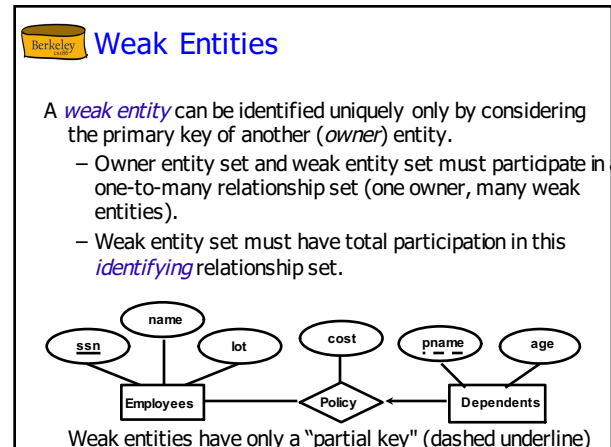
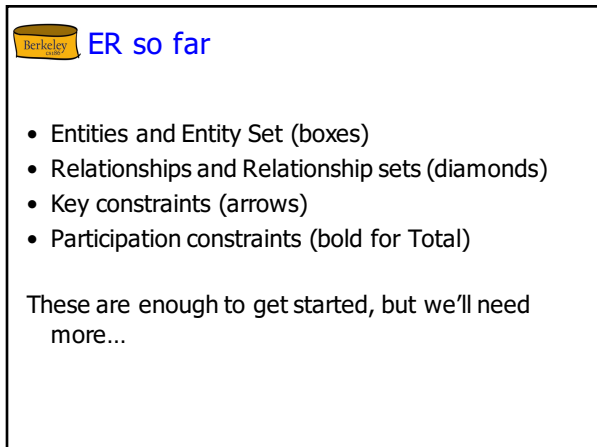
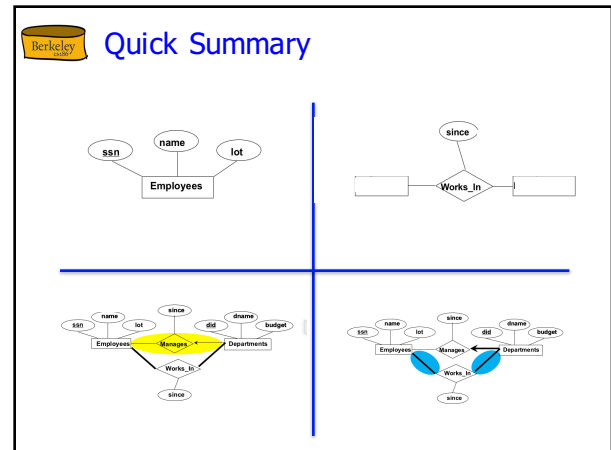
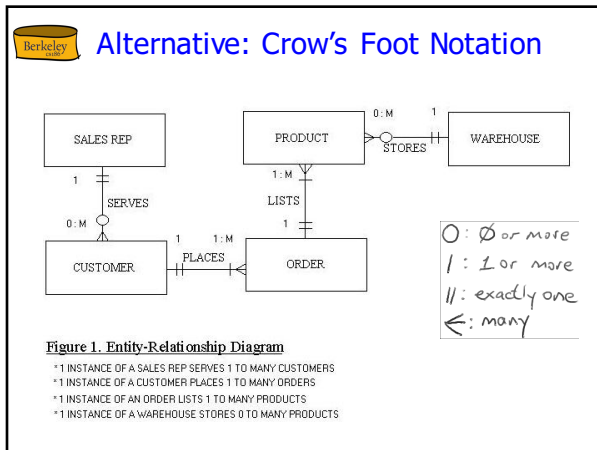
Key Constraints

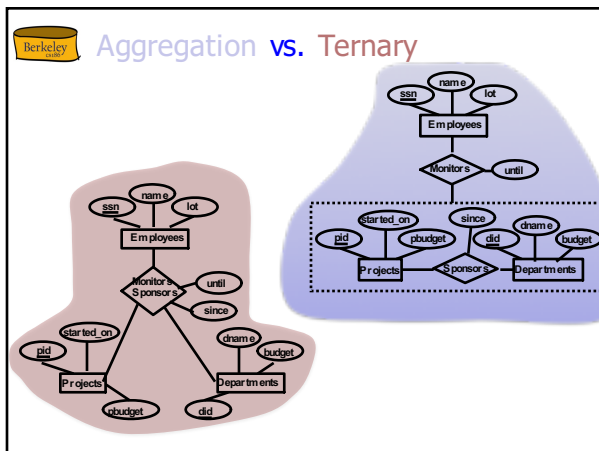
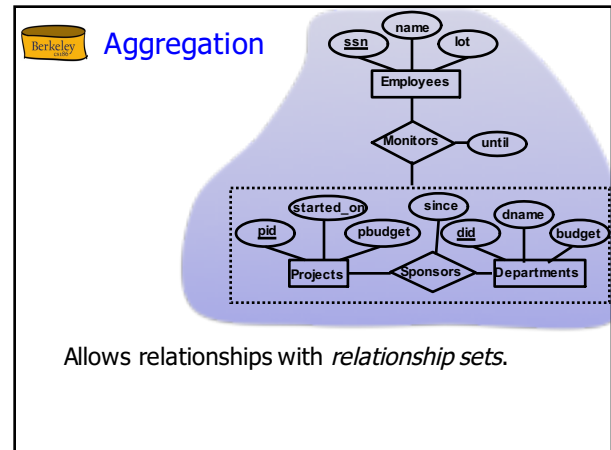
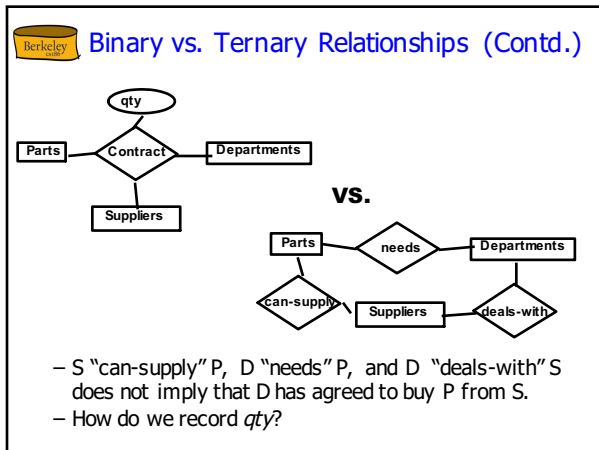
An employee can work in **many** departments; a dept can have **many** employees.



In contrast, each dept has **at most one** manager, according to the key constraint on Manages.

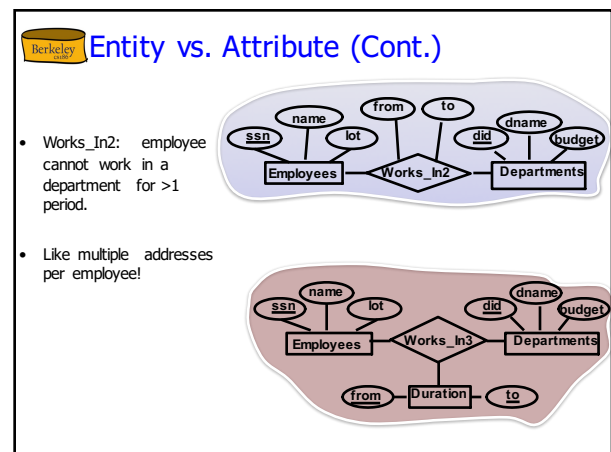
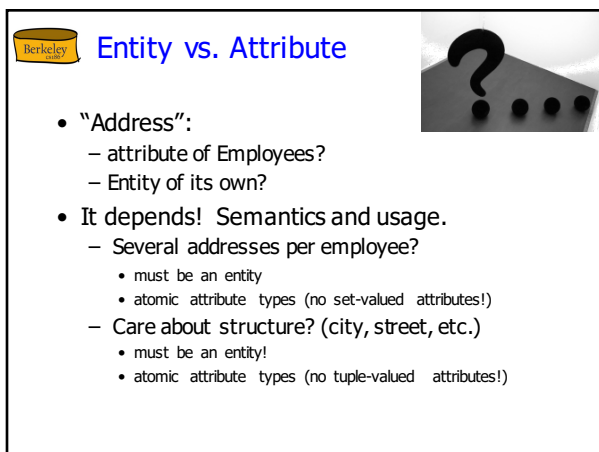






Conceptual Design Using the ER Model

- ER modeling *can* get tricky!
- Design choices:**
 - Entity or attribute?
 - Entity or relationship?
 - Relationships: **Binary or ternary?** **Aggregation?**
- ER Model goals and limitations:
 - Lots of semantics can (and should) be captured.
 - Some constraints *cannot* be captured in ER.
 - We’ll refine things in our logical (relational) design



Entity vs. Relationship

- Separate discretionary budget (dbudget) for each dept.
- What if manager's dbudget covers all managed depts
 - Could repeat value
 - But redundancy = problems
- Better design:**

E-R Diagram as Wallpaper

- Very common for them to be wall-sized

Converting ER to Relational

- Fairly analogous structure
- But many simple concepts in ER are subtle to specify in relations

Logical DB Design: ER to Relational

- Entity sets to tables.

ssn	name	lot
123-22-3666	Attishoo	48
231-31-5368	Smiley	22
131-24-3650	Smethurst	35

```
CREATE TABLE Employees
(ssn CHAR(11),
name CHAR(20),
lot INTEGER,
PRIMARY KEY (ssn))
```

Relationship Sets to Tables

- In translating a **many-to-many** relationship set to a relation, attributes of the relation must include:
 - Keys for each participating entity set (as foreign keys). This set of attributes forms a **superkey** for the relation.
 - All descriptive attributes.

```
CREATE TABLE Works_In(
  ssn CHAR(11),
  did INTEGER,
  since DATE,
  PRIMARY KEY (ssn, did),
  FOREIGN KEY (ssn) REFERENCES Employees,
  FOREIGN KEY (did) REFERENCES Departments)
```

ssn	did	since
123-22-3666	51	1/1/91
123-22-3666	56	3/3/93
231-31-5368	51	2/2/92

Review: Key Constraints

- Each dept has at most one manager, according to the key constraint on Manages.

Translation to relational model?

Translating ER with Key Constraints

- Since each department has a unique manager, we could instead combine Manages and Departments.

<pre>CREATE TABLE Manages(ssn CHAR(11), did INTEGER, since DATE, PRIMARY KEY (did), FOREIGN KEY (ssn) REFERENCES Employees, FOREIGN KEY (did) REFERENCES Departments)</pre>	Vs.	<pre>CREATE TABLE Dept_Mgr(did INTEGER, dname CHAR(20), budget REAL, ssn CHAR(11), since DATE, PRIMARY KEY (did), FOREIGN KEY (ssn) REFERENCES Employees)</pre>
--	-----	--

Review: Participation Constraints

- Does every department have a manager?
 - If so, this is a **participation constraint**: the participation of Departments in Manages is said to be **total (vs. partial)**.
 - Every *did* value in Departments table must appear in a row of the Manages table (with a non-null *ssn* value!)

Participation Constraints in SQL

- We can capture participation constraints involving one entity set in a binary relationship, but little else (without resorting to CHECK constraints).

```
CREATE TABLE Dept_Mgr(
  did INTEGER,
  dname CHAR(20),
  budget REAL,
  ssn CHAR(11) NOT NULL,
  since DATE,
  PRIMARY KEY (did),
  FOREIGN KEY (ssn) REFERENCES
  Employees
  ON DELETE NO ACTION)
```

Review: Weak Entities

- A **weak entity** can be identified uniquely only by considering the primary key of another (*owner*) entity.
 - Owner entity set and weak entity set must participate in a one-to-many relationship set (1 owner, many weak entities).
 - Weak entity set must have total participation in this **identifying** relationship set.

Translating Weak Entity Sets

- Weak entity set and identifying relationship set are translated into a single table.
 - When the owner entity is deleted, all owned weak entities must also be deleted.

```
CREATE TABLE Dep_Policy(
  pname CHAR(20),
  age INTEGER,
  cost REAL,
  ssn CHAR(11) NOT NULL,
  PRIMARY KEY (pname, ssn),
  FOREIGN KEY (ssn) REFERENCES Employees
  ON DELETE CASCADE)
```

Quick Summary

ssn CHAR(11) NOT NULL



Summary of Conceptual Design

- *Conceptual design* follows *requirements analysis*,
 - Yields a high-level description of data to be stored
 - You may want to postpone it for read-only “schema on use”
- ER model popular for conceptual design
 - Constructs are expressive, close to the way people think about their applications.
 - Note: There are many variations on ER model
 - Both graphically and conceptually
- Basic constructs: *entities*, *relationships*, and *attributes* (of entities and relationships).
- Some additional constructs: *weak entities*, *ISA hierarchies* (see text if you're curious), and *aggregation*.



Summary of ER (Cont.)

- Several kinds of integrity constraints:
 - *key constraints*
 - *participation constraints*
- Some *foreign key constraints* are also implicit in the definition of a relationship set.
- Many other constraints (notably, *functional dependencies*) cannot be expressed.
- Constraints play an important role in determining the best database design for an enterprise.



Summary of ER (Cont.)

- ER design is *subjective*. There are often many ways to model a given scenario!
- Analyzing alternatives can be tricky, especially for a large enterprise. Common choices include:
 - Entity vs. attribute, entity vs. relationship, binary or n-ary relationship, whether or not to use ISA hierarchies, aggregation.
- Ensuring good database design: resulting relational schema should be analyzed and refined further.
 - Functional Dependency information and normalization techniques are especially useful.