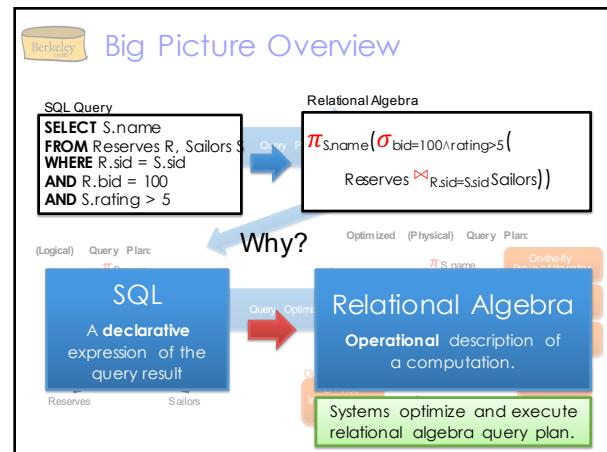
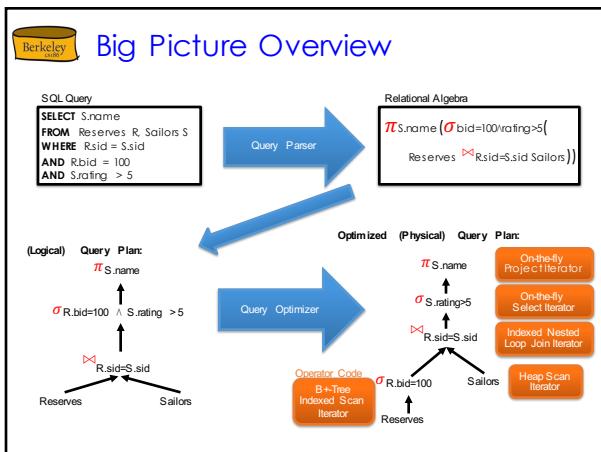
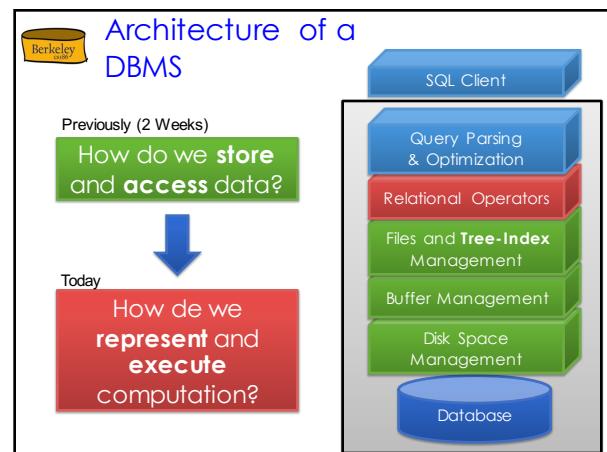
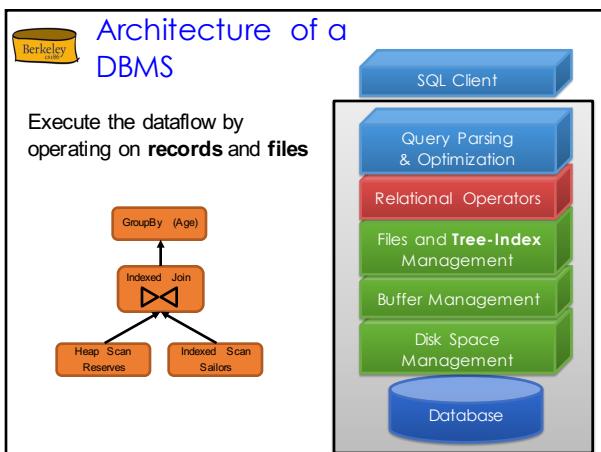
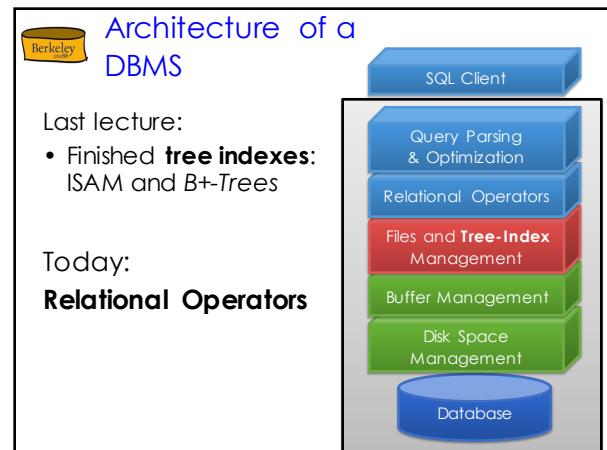


Relational Algebra, Iterators, & Operator Implementations

R & G, Chapters

- Relational Algebra: 4.1 - 4.2
- Relational → Query Plan: 12
- Select, Project, Join operators: 14



 **Big Picture Overview**

SQL Query

```
SELECT S.name
FROM Reserves R, Sailors S
WHERE R.sid = S.sid
AND R.bid = 100
AND S.rating > 5
```

Relational Algebra

$$\pi_{Sname}(\sigma_{bid=100 \wedge rating>5(Reserves \bowtie R.sid=S.sid Sailors)})$$

Why?

(Logical) Query Plan Optimized (Physical) Query Plan

SQL
A **declarative** expression of the query result

Relational Algebra
Operational description of a computation.

Systems optimize and execute relational algebra query plan.

SQL (Structured Query Language)



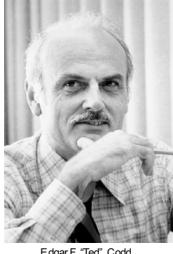
- One of most **widely used** languages
- Introduced in **1970s**
 - IBM System R
- Key **System Features**: *Why do we like SQL*
 - Declarative:
 - Say **what** you want, not **how** to get it
 - Enables system to optimize the **how**
 - Limited DSL → formal reasoning and optimization
- Foundation in formal Query Languages
 - Relational Calculus**

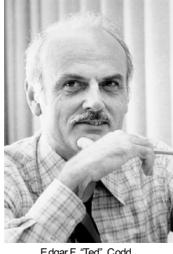
```
SELECT S.name
FROM Reserves R, Sailors S
WHERE R.sid = S.sid
AND R.bid = 100
AND S.rating > 5
```

 **Formal Relational QL's**

- Relational Calculus:** (Basis for SQL)
 - Describe the result of computation
 - Based on first order logic
 - Tuple Relational Calculus (**TRC**)
 - $\{S \mid S \in \text{Sailors } \exists R \in \text{Reserves } (R.sid = S.sid \wedge R.bid = 103)\}$
- Relational Algebra:**
 - Algebra on sets
 - Operational description of transformations

Are these equivalent?
Can we go from one to the other?

 **Codd's Theorem**



Established equivalence in expressivity between :

- Relational Calculus***
- Relational Algebra**

Why an import result?

- Connects **declarative** representation of queries with **operational description**
- Constructive**: we can compile SQL into relational algebra

*Domain Independent Relational Calculus

Image obtained from https://en.wikipedia.org/wiki/Edgar_F._Codd

 **Big Picture Overview**

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AND S.rating > 5
```

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Relational Algebra, Iterators, & Operator Implementations



R & G, Chapters

- Relational Algebra: 4.1 - 4.2
- Relational → Query Plan: 12
- Select, Project, **Join** operators: 14
- Day 2 (2/18/2016)

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Relational Algebra Preliminaries

- Algebra of **operators** on **relational instances**

$$\pi_{S.name}(\sigma_{R.bid=100 \wedge S.rating > 5}(R \bowtie_{R.sid=S.sid} S))$$

- Closed:** result is also a relational instance
 - Enables rich composition!
- Typed:** input schema determines output
- Why is this important?

- Pure relational algebra has **set semantics**
 - No **duplicate** tuples in a relation instance
 - vs. SQL, which has **multiset** (bag) semantics
 - We will relax this in the system discussion

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Relational Algebra Operators

Unary Operators: operate on **single relation instance**

- Projection (π): Retains only desired columns (vertical)
- Selection (σ): Selects a subset of rows (horizontal)
- Renaming (ρ): Rename attributes and relations.

Binary Operators: operate on **pairs of relation instances**

- Union (\cup): Tuples in $r1$ or in $r2$.
- Intersection (\cap): Tuples in $r1$ and in $r2$.
- Set-difference ($-$): Tuples in $r1$, but not in $r2$.
- Cross-product (\times): Allows us to combine two relations.
- Joins (\bowtie , \bowtie_1): Combine relations that satisfy predicates

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Relational Algebra Operators

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Compound Operators: operators built from earlier operators

- Intersection (\cap): Tuples in $r1$ and in $r2$.
- Joins (\bowtie , \bowtie_1): Combine relations that satisfy predicates

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Nautical Example Instances

R1	sid	bid	day
22	101	10/10/96	
58	103	11/12/96	

Boats	sid	sname	rating	age
22	dustin	7	45.0	
31	lubber	8	55.5	
58	rusty	10	35.0	

S1	bid	bname	color
101	Interlake	blue	
102	Interlake	red	
103	Clipper	green	
104	Marine	red	

S2	sid	sname	rating	age
28	yuppy	9	35.0	
31	lubber	8	55.5	
44	guppy	5	35.0	
58	rusty	10	35.0	

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Projection (π)

Selects a subset of columns (vertical)

$$\pi_{sname, rating}(S2)$$

List of Attributes

Relational Instance **S2**

sid	sname	rating	age
28	yuppy	9	35.0
31	lubber	8	55.5
44	guppy	5	35.0
58	rusty	10	35.0

Relational Instance **S2**

sname	age
yuppy	35.0
lubber	55.5
guppy	35.0
rusty	35.0

• Corresponds to the **SELECT** list

• Schema determined by schema of attribute list

- Names and types correspond to input attributes

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Projection (π)

Selects a subset of columns (vertical)

$$\pi_{age}(S2)$$

Relational Instance **S2**

sid	sname	rating	age
28	yuppy	9	35.0
31	lubber	8	55.5
44	guppy	5	35.0
58	rusty	10	35.0

Multiset

age
35.0
55.5
35.0
35.0

Set

age
35.0
55.5

• Set semantics → results in fewer rows

- Real systems don't automatically remove duplicates
- why?

Selection (σ)

Selects a subset of rows (horizontal)

$\sigma_{\text{rating} > 8}(\text{S2})$

Selection Condition (Boolean Expression)

Relational Instance S2

sid	sname	rating	age
28	yuppy	9	35.0
31	lubber	8	55.5
44	guppy	5	35.0
58	rusty	10	35.0

sid	sname	rating	age
28	yuppy	9	35.0
58	rusty	10	35.0

- Corresponds to the WHERE clause
- Output schema same as input
- Duplicate Elimination?

Composing Select and Project

- Names of sailors with rating > 8

$\pi_{\text{name}}(\sigma_{\text{rating} > 8}(\text{S2}))$

Relational Instance S2

sid	sname	rating	age
28	yuppy	9	35.0
31	lubber	8	55.5
44	guppy	5	35.0
58	rusty	10	35.0

Relational Instance $\sigma_{\text{rating} > 8}(\text{S2})$

sid	sname	rating	age
28	yuppy	9	35.0
58	rusty	10	35.0

Relational Instance $\pi_{\text{name}}(\sigma_{\text{rating} > 8}(\text{S2}))$

sname
yuppy
rusty

• What about:

$\sigma_{\text{rating} > 8}(\pi_{\text{name}}(\text{S2}))$

Invalid types. Input to $\sigma_{\text{rating} > 8}$ does not contain rating.

Union (\cup)

$S1 \cup S2$

Two input relations, must be *compatible*:

- Same number of fields.
- “Corresponding” fields have same **type**
- SQL Expression: UNION

Union (\cup)

$S1 \cup S2$

Relational Instance S1

sid	sname	rating	age
22	dustin	7	45.0
31	lubber	8	55.5
58	rusty	10	35.0

Relational Instance S2

sid	sname	rating	age
28	yuppy	9	35.0
31	lubber	8	55.5
44	guppy	5	35.0
58	rusty	10	35.0

Duplicate elimination in practice?

- UNION vs UNION ALL?

Set Difference ($-$)

$S1 - S2$

Same as with union, both input relations must be *compatible*.

SQL Expression: EXCEPT

Set Difference ($-$)

$S1 - S2$

Relational Instance S1

sid	sname	rating	age
22	dustin	7	45.0
31	lubber	8	55.5
58	rusty	10	35.0

Relational Instance S2

sid	sname	rating	age
28	yuppy	9	35.0
31	lubber	8	55.5
44	guppy	5	35.0
58	rusty	10	35.0

Symmetric?

$S2 - S1$

sid	sname	rating	age
22	dustin	7	45.0

Duplicate elimination?

- Not required
- EXCEPT vs EXCEPT ALL



Monotonicity and Set Difference

- A relational op. Q is monotone in R if:

$$R_1 \subseteq R_2 \Rightarrow Q(R_1, S, T, \dots) \subseteq Q(R_2, S, T, \dots)$$

- For example UNION (\cup) is monotone:

$$R_1 \subseteq R_2 \Rightarrow S \cup R_1 \subseteq S \cup R_2$$

- Set difference?



Monotonicity and Set Difference

- A relational op. Q is monotone in R if:

$$R_1 \subseteq R_2 \Rightarrow Q(R_1, S, T, \dots) \subseteq Q(R_2, S, T, \dots)$$

- Set difference ($-$) where $S = \{a,b,c\}$:

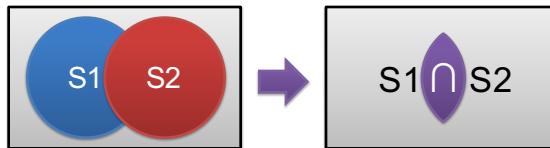
$$R_1 = \{a\} \subseteq R_2 = \{a,b\} \quad ? \quad S - \{a\} \subseteq S - \{a,b\} \\ \{b,c\} \not\subseteq \{c\}$$

- Implication:* set difference is **blocking**

– For $S - R$, need to have full contents of R before emitting **any** results

Intersection (\cap)

$$S1 \cap S2$$



Same as with union, both input relations must be *compatible*.

SQL Expression: INTERSECT



Intersection (\cap)

$$S1 \cap S2$$

Relational Instance S1

sid	sname	rating	age
22	dustin	7	45.0
31	lubber	8	55.5
58	rusty	10	35.0

Relational Instance S2

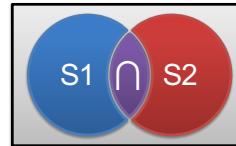
sid	sname	rating	age
28	yuppy	9	35.0
31	lubber	8	55.5
44	guppy	5	35.0
58	rusty	10	35.0



Is intersection essential?

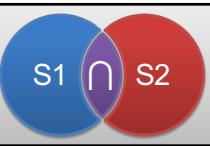
- Implement it with earlier ops. ?

Intersection (\cap)



$$S1 \cap S2 = ?$$

Intersection (\cap)



$$S1 \cap S2 = S1 - ?$$



Intersection (\cap)

$$S1 \cap S2 = S1 - (S1 - S2)$$

Is intersection monotonic?

$$R_1 \subseteq R_2 \Rightarrow S \cap R_1 \subseteq S \cap R_2$$

Yes!

Cross-Product (\times)

R1 × S1: Each row of R1 paired with each row of S1

R1:			S1:			R1 × S1							
sid	bid	day	sid	sname	rating	sid	sname	rating	age				
22	101	10/10/96	22	dustin	7	45.0	22	101	10/10/96	22	dustin	7	45.0
58	103	11/12/96	31	lubber	8	55.5	58	103	11/12/96	31	lubber	8	55.5

\times

sid	sname	rating	age
22	dustin	7	45.0
31	lubber	8	55.5
58	rusty	10	35.0

=

sid	bid	day	sid2	sname	rating	age
22	101	10/10/96	22	dustin	7	45.0
22	101	10/10/96	31	lubber	8	55.5
58	103	11/12/96	22	dustin	7	45.0
58	103	11/12/96	31	lubber	8	55.5
58	103	11/12/96	58	rusty	10	35.0

How many rows in the result? $|R1| * |R2|$

Sometimes also called **Cartesian Product**:

4	4.a	4.b	4.c	4.d
3	3.a	3.b	3.c	3.d
2	2.a	2.b	2.c	2.d
1	1.a	1.b	1.c	1.d
a	b	c	d	

Schema compatibility? No requirements. One field per field in original schemas.

What about duplicate names? Renaming operator

Renaming (ρ = "rho")

Renames relations and their attributes:

$\rho(\text{Temp1}(\underset{\substack{\text{Output Relation} \\ \text{Name}}}{1} \rightarrow \text{sid1}, 4 \rightarrow \text{sid2}), \text{R1} \times \text{S1})$

Renaming List: position \rightarrow New Name

Input Relation: Temp1

sid	bid	day	sid	sname	rating	age
22	101	10/10/96	22	dustin	7	45.0
22	101	10/10/96	31	lubber	8	55.5
22	101	10/10/96	58	rusty	10	35.0
58	103	11/12/96	22	dustin	7	45.0
58	103	11/12/96	31	lubber	8	55.5
58	103	11/12/96	58	rusty	10	35.0

\rightarrow

sid1	bid	day	sid2	sname	rating	age
22	101	10/10/96	22	dustin	7	45.0
22	101	10/10/96	31	lubber	8	55.5
22	101	10/10/96	58	rusty	10	35.0
58	103	11/12/96	22	dustin	7	45.0
58	103	11/12/96	31	lubber	8	55.5
58	103	11/12/96	58	rusty	10	35.0

- Note that relational algebra doesn't require names.
 - We could just use positional arguments. $\pi_{f5}(\sigma_{f6>f8}(S2))$
 - Difficult to read ...

Compound Operator: Join

- Joins are compound operators (like intersection):
 - Built using **cross product** & **selection** and sometimes **projection** (for natural join)
- Hierarchy of common kinds:
 - Theta Join (\bowtie_θ):** join on logical expression θ
 - Equi-Join:** theta join with conjunction equalities
 - Natural Join (\bowtie):** equi-join on all matching column names
- Note: we should use a good join algorithm, not a cross-product if we can avoid it!!

Theta Join (\bowtie_θ)

$R \bowtie_\theta S = \sigma_\theta(R \times S)$

Example: More senior sailors for each sailor.

$S1 \bowtie_\theta age < age S1$

This predicate doesn't make any sense!

Could switch to positional arguments

f1	f2	f3	f4
22	dustin	7	45.0
31	lubber	8	55.5
58	rusty	10	35.0

Theta Join (\bowtie_θ)

$R \bowtie_\theta S = \sigma_\theta(R \times S)$

Example: More senior sailors for each sailor.

$S1 \bowtie_\theta f4 < f8 S1$

S1:

f1	f2	f3	f4	f5	f6	f7	f8
22	dustin	7	45.0	22	dustin	7	45.0
22	dustin	7	45.0	31	lubber	8	55.5
22	dustin	7	45.0	58	rusty	10	35.0
31	lubber	8	55.5	22	dustin	7	45.0
31	lubber	8	55.5	31	lubber	8	55.5
31	lubber	8	55.5	58	rusty	10	35.0
58	rusty	10	35.0	22	dustin	7	45.0
58	rusty	10	35.0	31	lubber	8	55.5
58	rusty	10	35.0	58	rusty	10	35.0

S1 × S1

S1				S1			
f1	f2	f3	f4	f5	f6	f7	f8
22	dustin	7	45.0	22	dustin	7	45.0
22	dustin	7	45.0	31	lubber	8	55.5
22	dustin	7	45.0	58	rusty	10	35.0
31	lubber	8	55.5	22	dustin	7	45.0
31	lubber	8	55.5	31	lubber	8	55.5
31	lubber	8	55.5	58	rusty	10	35.0
58	rusty	10	35.0	22	dustin	7	45.0
58	rusty	10	35.0	31	lubber	8	55.5
58	rusty	10	35.0	58	rusty	10	35.0

Difficult to read
→ use ρ operator

Theta Join (\bowtie_θ)

$$R \bowtie_\theta S = \sigma_\theta(R \times S)$$

Example: More senior sailors for each sailor.

$$S1 \bowtie_{age < age2} \rho((8 \rightarrow age2), S1) \quad S1 \times S1$$

S1:

S1				S1			
sid	sname	rating	age	sid	sname	rating	age2
22	dustin	7	45.0	22	dustin	7	45.0
31	lubber	8	55.5	22	dustin	7	45.0
58	rusty	10	35.0	22	dustin	7	45.0



Theta Join (\bowtie_θ)

$$R \bowtie_\theta S = \sigma_\theta(R \times S)$$

Example: More senior sailors for each sailor.

$$S1 \bowtie_{age < age2} \rho((8 \rightarrow age2), S1)$$

S1:

S1				S1			
sid	sname	rating	age	sid	sname	rating	age2
22	dustin	7	45.0	31	lubber	8	55.5
31	lubber	8	55.5	58	rusty	10	35.0
58	rusty	10	35.0	22	dustin	7	45.0



• Result schema same as that of cross-product.

• Special Case:

• **Equi-Join:** theta join with conjunction equalities

• Special special case **Natural Join ...**

Natural Join (\bowtie)



Special case of **equi-join** in which equalities are specified for all matching fields and duplicate fields are projected away

$$R \bowtie S = \pi_{\text{unique fld.}} \sigma_{\text{eq. matching fld.}}(R \times S)$$

- Compute $R \times S$
- Select rows where fields appearing in both relations have equal values
- Project onto the set of all unique fields.

Natural Join (\bowtie)



$$R \bowtie S = \pi_{\text{unique fld.}} \sigma_{\text{eq. matching fld.}}(R \times S)$$

Example:

Natural Join (\bowtie)



$$R \bowtie S = \pi_{\text{unique fld.}} \sigma_{\text{eq. matching fld.}}(R \times S)$$

Example:

$$R1 \bowtie S1$$

R1:

sid	b_id	day
22	101	10/10/96
58	103	11/12/96

sid	b_id	day	sname	rating	age
22	101	10/10/96	dustin	7	45.0
58	103	11/12/96	rusty	10	35.0

S1:

sid	sname	rating	age
22	dustin	7	45.0
31	lubber	8	55.5
58	rusty	10	35.0

Commonly used for foreign key joins (as above).

Boats(bid,bname,color)
Sailors(sid, sname, rating, age)
Reserves(sid, bid, day)

Exercise:

Find names of sailors who've reserved boat #103

- Solution 1:

$$\pi_{\text{sname}}(\sigma_{\text{bid}=103}(\text{Sailors} \bowtie \text{Reserves}))$$

- Solution 2:

$$\pi_{\text{sname}}(\text{Sailors} \bowtie \sigma_{\text{bid}=103}(\text{Reserves}))$$

Exercise:

$\text{Boats}(\underline{\text{bid}}, \text{bname}, \text{color})$
$\text{Sailors}(\underline{\text{sid}}, \text{sname}, \text{rating}, \text{age})$
$\text{Res}(\underline{\text{sid}}, \underline{\text{bid}}, \underline{\text{day}})$

Find names of sailors who've reserved a red boat

- Solution 1:

$$\pi_{\text{sname}}(\sigma_{\text{color}=\text{'red'}}(\text{Boats}) \bowtie \text{Res} \bowtie \text{Sailors})$$
- More “efficient” Solution 2:

$$\pi_{\text{sname}}(\pi_{\text{sid}}(\pi_{\text{bid}}(\sigma_{\text{color}=\text{'red'}}(\text{Boats})) \bowtie \text{Res} \bowtie \text{Sailors})$$

In general many possible equivalent expressions: **algebra...**

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Relational Algebra Rules

- Operator Precedence:**
 - Unary operators before binary operators
- Selections:**
 - $\sigma_{c_1 \wedge \dots \wedge c_n}(R) = \sigma_{c_1}(\dots(\sigma_{c_n}(R))\dots)$ (cascade)
 - $\sigma_{c_1}(\sigma_{c_2}(R)) = \sigma_{c_2}(\sigma_{c_1}(R))$ (commute)
- Projections:**
 - $\pi_{a_1}(R) = \pi_{a_1}(\dots(\pi_{a_l}, \dots, a_{n-1})(R))\dots)$ (cascade)
- Cartesian Product**
 - $R \times (S \times T) = (R \times S) \times T$ (associative)
 - $R \times S = S \times R$ (commutative)
 - Applies for joins as well but be careful with join predicates ...

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Caution with Natural Join

$\text{Boats}(\underline{\text{bid}}, \text{bname}, \text{color})$
$\text{Sailors}(\underline{\text{sid}}, \text{sname}, \text{rating}, \text{age})$
$\text{Reserves}(\underline{\text{sid}}, \underline{\text{bid}}, \underline{\text{day}})$

- Consider the following:

$$\text{Boats} \bowtie_{\text{bid}} \text{Reserves} \bowtie_{\text{sid}} \text{Sailors}$$
- Commute and Associate:

$$\text{Boats} \bowtie_{\text{bid}} \left(\text{Sailors} \bowtie_{\text{sid}} \text{Reserves} \right)$$
 - Incompatible join predicate:

$$\left(\text{Boats} \bowtie_{\text{bid}} \text{Sailors} \right) \bowtie_{\text{sid}} \text{Reserves}$$

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Caution with Natural Join

$\text{Boats}(\underline{\text{bid}}, \text{bname}, \text{color})$
$\text{Sailors}(\underline{\text{sid}}, \text{sname}, \text{rating}, \text{age})$
$\text{Reserves}(\underline{\text{sid}}, \underline{\text{bid}}, \underline{\text{day}})$

- Consider the following:

$$\text{Boats} \bowtie_{\text{bid}} \text{Reserves} \bowtie_{\text{sid}} \text{Sailors}$$
- Commute and Associate:

$$\text{Boats} \bowtie_{\text{bid}} \left(\text{Sailors} \bowtie_{\text{sid}} \text{Reserves} \right)$$
 - Incompatible join predicate:

$$\left(\text{Boats} \times \text{Sailors} \right) \bowtie_{\text{sid}, \text{bid}} \text{Reserves}$$

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Relational Algebra Rules

Commuting of selection operators

- $\sigma_c(R \times S) = \sigma_c(R) \times S$ (c only has fields in R)
- $\sigma_c(R \bowtie S) = \sigma_c(R) \bowtie S$ (c only has fields in R)

Commuting of projection operators

- $\pi_a(R \times S) = \pi_{a_1}(R) \times \pi_{a_2}(S)$
 - a_1 is subset of a that mentions R and a_2 is subset of a that mentions S
 - Similar result holds for joins

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Division (/) Compound Operator

“Find the names of sailors that reserved all boats.”

- Consider Relations: A(x,y) and B(y)
 - $A / B = \{x \mid \forall y \in B \ ((x,y) \in A) \}$
 - all entries **x** in **A** such that for all **y** in **B** there is an **(x,y)** in **A**
- Pictorially:

A		B		A / B

x	y			

x ₁	y ₁			

x ₁	y ₂			

x ₂	y ₁			

x ₃	y ₂			
- How do we implement division in relational algebra?

Division (/) Compound Operator

- If all the **x** values in **A** are in the result what would **A** look like?

$\pi_x(A) \times B$	
x	y
x₁	y₁
x₂	y₁
x₃	y₂
x₁	y₂
x₂	y₂
x₃	y₁
x₁	y₁
x₂	y₁
x₃	y₂

Division (/) Compound Operator

- If all the **x** values in **A** were in the result what would **A** look like?

$$\pi_x(A) \times B$$

- Which of these tuples are we missing from **A**:

A	B	$(\pi_x(A) \times B) - A$	Missing (x,y) values
x	y	x	x
x₁	y₁	y₁	x₁
x₁	y₂	y₂	x₁
x₂	y₁		x₂
x₃	y₂		x₃
x₁	y₁		
x₂	y₁		
x₃	y₂		

Division (/) Compound Operator

- If all the **x** values in **A** were in the result what would **A** look like?

$$\pi_x(A) \times B$$

- Which of these tuples are we missing from **A** and what are their **x** values:

A	B	$\pi_x((\pi_x(A) \times B) - A)$
x	y	x
x₁	y₁	y₁
x₁	y₂	y₂
x₂	y₁	
x₃	y₂	

$=$ Disqualified x values

Division (/) Compound Operator

- If all the **x** values in **A** were in the result what would **A** look like?

$$\pi_x(A) \times B$$

- Which of these tuples are we missing from **A** and what are their **x** values:

A	B	$\pi_x((\pi_x(A) \times B) - A)$
x	y	x
x₁	y₁	y₁
x₁	y₂	y₂
x₂	y₁	
x₃	y₂	

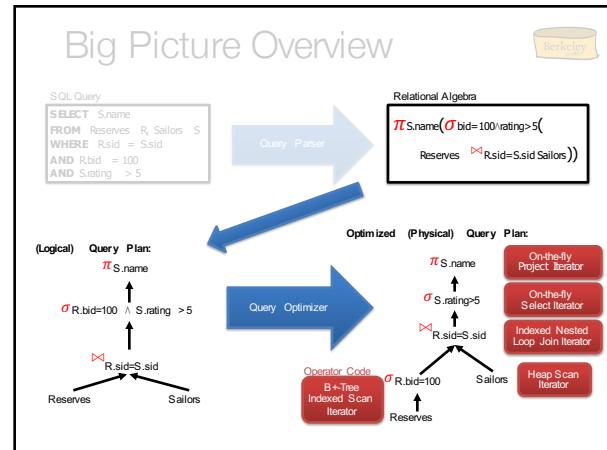
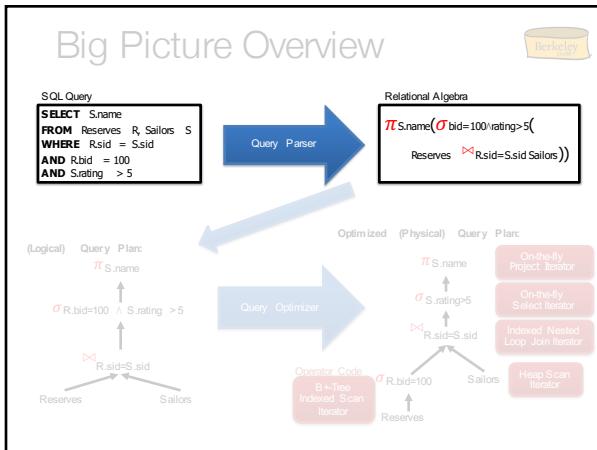
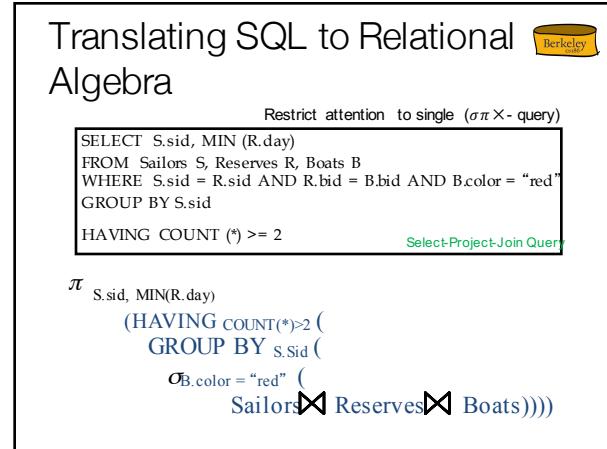
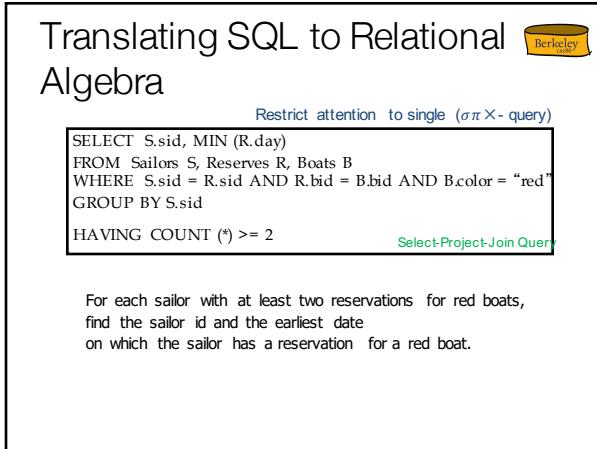
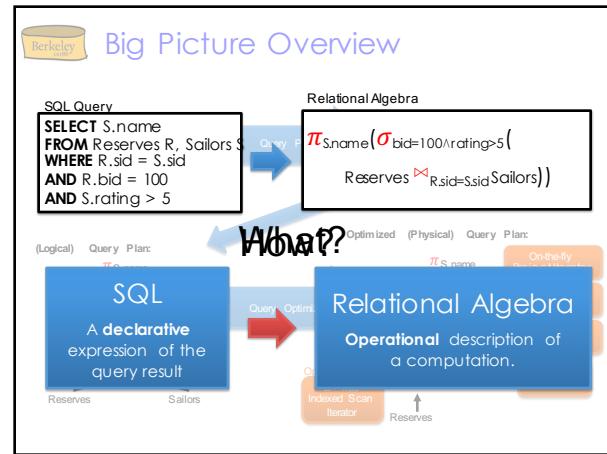
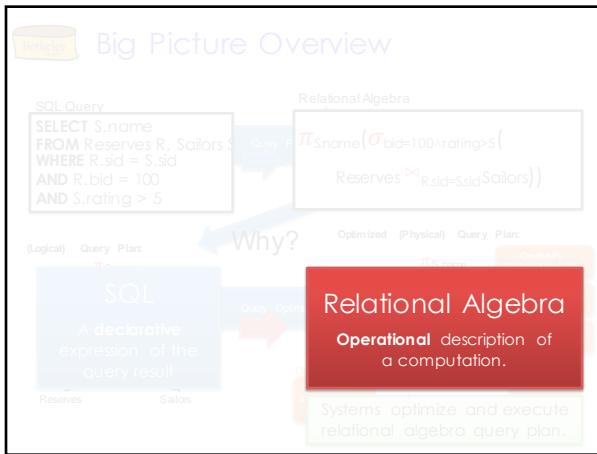
• Remove disqualified **x** values:
 $A / B = \pi_x(A) - \pi_x((\pi_x(A) \times B) - A)$

Operators in Extended Algebra

- Group By / Aggregation Operator (γ):
 $\gamma_{age, AVG(rating)}(Sailors)$
 - With selection (HAVING clause):
 $\gamma_{age, AVG(rating), COUNT(*) > 2}(Sailors)$
- Textbook uses two operators:
 $GROUP BY_{age, AVG(rating)}(Sailors)$
 $HAVING_{COUNT(*) > 2}(GROUP BY_{age, AVG(rating)}(Sailors))$

Summary

- Relational Algebra: a small set of operators mapping relations to relations
 - Operational, in the sense that you specify the explicit order of operations
 - A closed set of operators! Mix and match.
- Basic ops include: $\sigma, \pi, \times, \cup, -$
- Important compound ops: \cap, \bowtie



Berkeley

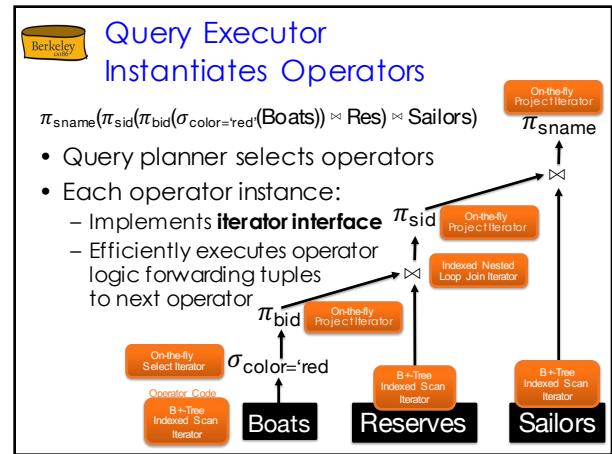
Relational Operators as a Graph

$$\pi_{\text{sname}}(\pi_{\text{sid}}(\pi_{\text{bid}}(\sigma_{\text{color}=\text{red}}(\text{Boats})) \bowtie \text{Res}) \bowtie \text{Sailors})$$

- Query plan
 - Edges encode “flow” of tuples
 - Vertices = Operators
 - Source vertices = table access operators ...
- Also called dataflow graph

```

    graph TD
      Boats[Boats] -- "σcolor='red'" --> Reserves[Reserves]
      Reserves -- "πbid" --> Sailors[Sailors]
      Sailors -- "πsid" --> πsid["πsid"]
      πsid -- "πsname" --> πsname["πsname"]
  
```



Iterators Interface

The relational operators are all subclasses of the class iterator:

```

abstract class iterator {
    // Invoked when “wiring” dataflow
    void setup(List<Iterator> inputs, args);
    void init(args); // Invoked before calling next
    tuple next(); // Invoked repeatedly
    void close(); // Invoked when finished
}
  
```

Note:

- Pull:** based computation model
 - e.g., Console calls `init` and `next` which propagates down graph
- Blocking:** `init` and `next` don't return until done.
 - Asynchronous** models more complex but could be more efficient
- Encapsulation:** any iterator can be input to any other!
- State:** iterators may maintain substantial state
 - e.g., hash tables, running counts, large sorted files ...

Example: Scan Heap File

```

class heap_scan_iterator extends iterator {
    void init() {
        current_page = file_manager.begin(heap_file, 'MRU')
        pageIter = current_page.iterator()
    }
    tuple next() { // Invoked repeatedly
        if (!pageIter.hasNext()) {
            current_page = file_manager.next(current_page);
            pageIter = current_page.iterator();
        }
        if (current_page == null) { return EOF; }
        else { return pageIter.next(); }
    }
    void close() { // Invoked when finished
        file_manager.close(heap_file);
    }
}
  
```

pseudocode ... might have pseudobugs

Example: Select

```

class select_iterator extends iterator {
    void setup(List<Iterator> inputs, predicate) ...
    void init() { input[0].init(); }

    tuple next() { // Invoked repeatedly
        res = input[0].next();
        while( res not EOF && not predicate(res) ) {
            res = input[0].next();
        }
        return res;
    }

    void close() { // Invoked when finished
        input[0].close();
    }
}
  
```

pseudocode ... might have pseudobugs

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Selective Access Paths

- Access Path:** How data is accessed from input relations:
 - File scan
 - Indexed lookup: Attr op Value
- Selectivity of Access Path:** Number of pages retrieved
- Goal:** Use the most selective access path

Example: B+-Tree (by Ref Clustered)

```
class index_select_iterator extends iterator {
    void init(keyRange) { // might be invoked repeatedly
        index = file_manager.getIndex(file)
        (leaf_page, offset) = index.findBegin(keyRange)
    }
    tuple next() { // Invoked repeatedly
        if (offset == END_OF_PAGE) {
            (leaf_page, offset) = index.next_leaf(leaf_page)
        }
        (k, rid) = leaf_page.read(offset)
        if (rid == EOF || k not in keyRange) { return EOF; }
        return file_manager.get_record(rid);
    }
    void close() { // Invoked when finished
        file_manager.close(index);
    }
}
```

pseudocode ... might have pseudobugs

Example: B+-Tree (by Ref Unclustered)

```
class index_select_iterator extends iterator {
    void init(keyRange) { // might be invoked repeatedly
        index = file_manager.getIndex(file)
        tmpFile = index.scanRecordIdsToTmpFile(keyRange)
        iter = new sort_iterator(tmpFile)
        iter.init()
    }
    tuple next() { // Invoked repeatedly
        rid = iter.next()
        if (rid == EOF) { return EOF; }
        return file_manager.get_record(rid);
    }
    void close() { // Invoked when finished
        file_manager.close(index);
        iter.close()
        file_manager.destroy(tmpFile);
    }
}
```

pseudocode ... might have pseudobugs

Example: Sort

- **init():**
 - generate the sorted runs on disk
 - Open each sorted run file
 - load (sorted) heap with first tuple in each run
- **next():**
 - Get top tuple from heap
 - Load next tuple from file which top record came from and place in heap
 - return tuple (or EOF -- "End of Fun" -- if no tuples remain)
- **close():**
 - deallocate the runs files

Get to do this in the homework but with an additional distributed partitioning step!

Example: Sort → Group By

- 
- The Sort iterator ensures all its tuples are output in sequence
 - The Aggregate iterator keeps running info ("transition values") on agg functions in the SELECT list, per group
 - E.g., for COUNT, it keeps count-so-far
 - For SUM, it keeps sum-so-far
 - For AVG it keeps sum-so-far and count-so-far
 - As soon as the Aggregate iterator sees a tuple from a new group:
 1. It produces output for the old group based on agg function
E.g. for AVG it returns (sum-so-far / count-so-far)
 2. It resets its running info.
 3. It updates the running info with the new tuple's info

Example: Hash Group By

You get to do this in your homework!

Iterators in PySpark (Homework)

```
def init(iteratorA, iteratorB):
    a = list(iteratorA)
    b = list(iteratorB)
    return iterator(zip(a, b))

rdd = sc.parallelize(range(10), 2)

print rdd.zipPartitions(rdd, init).collect()

[(0, 0), (1, 1), (2, 2), (3, 3), (4, 4), (5, 5),
 (6, 6), (7, 7), (8, 8), (9, 9)]
```

Berkeley

Uses a lot of memory (avoid "materializing" partition)

Iterators in PySpark (Homework)

```

def __init__(iterA, iterB):
    for a in iterA:
        yield (a, iterB.next())

rdd = sc.parallelize(range(10), 2)

print rdd.zipPartitions(rdd, __init__).collect()
[(0, 0), (1, 1), (2, 2), (3, 3), (4, 4), (5, 5),
 (6, 6), (7, 7), (8, 8), (9, 9)]

```

Join Operators

R&G 14.4

Joins

$$R \bowtie_{\theta} S = \sigma_{\theta}(R \times S)$$

- Joins are very common
- $R \times S$ is large →
 - Minimize IO
 - Exploit θ to reduce computation (selectivity)
- Join techniques we will cover today:
 - Nested-loops join
 - Index-nested loops join
 - Sort-merge join
 - Hash Joins

Schema for Examples

Cost Notation

- $[R]$: the number of **pages** to store R
- p_R : number of **records per page** of R
- $|R|$: the **cardinality** (*number of records*) of R
 - $|R| = p_R * [R]$

Reserves (*sid*: int, *bid*: int, *day*: date, *rname*: string)

- $[R]=1000$, $p_R=100$, $|R|=100,000$

Sailors (*sid*: int, *sname*: string, *rating*: int, *age*: real)

- $[S]=500$, $p_S=80$, $|S|=40,000$

Simple Nested Loops Join

$R \bowtie S$: foreach record r in R do
foreach record s in S do
if $\theta(r, s)$ then add $\langle r, s \rangle$ to result

Cost?

- $(p_R * |R|) * |S| + |R|$
- $100,000 * 500 + 1,000$
- $50,001,000$

Swap R and S?

- $(p_S * |S|) * |R| + |S|$
- $40,000 * 1,000 + 500$
- $40,000,500$
- $@10ms \rightarrow 4.63 Days!$

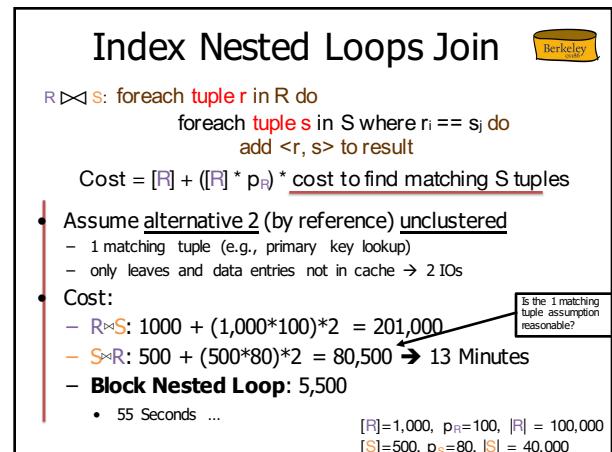
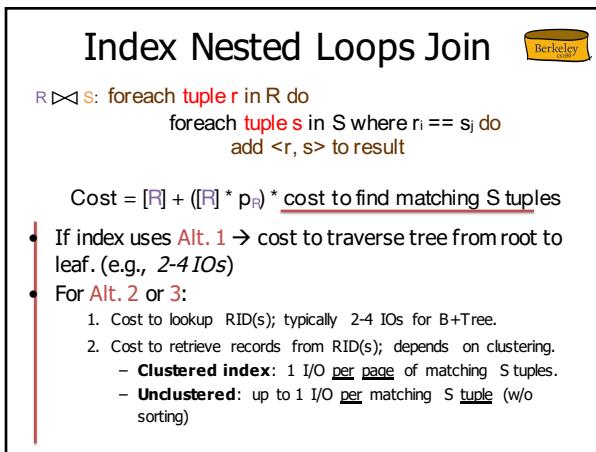
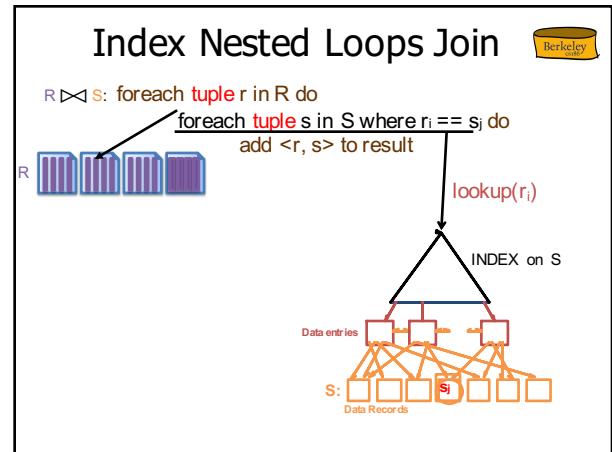
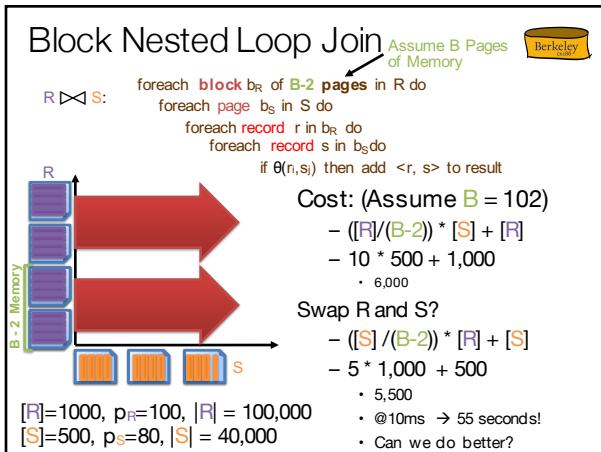
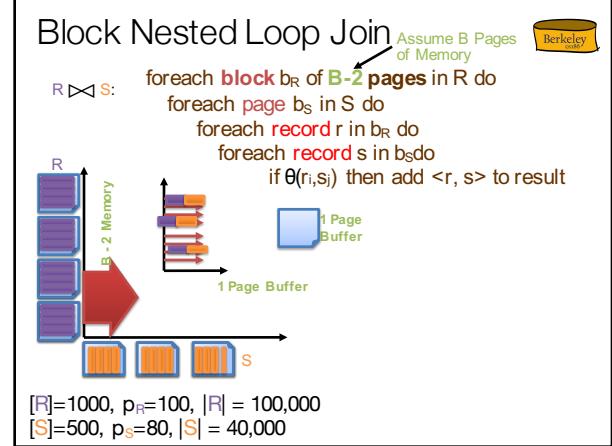
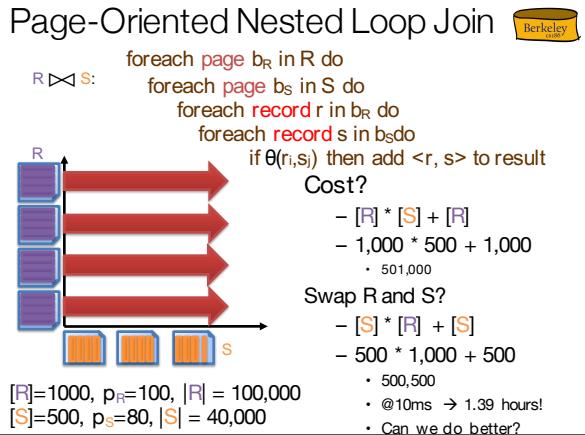
$[R]=1000$, $p_R=100$, $|R|=100,000$
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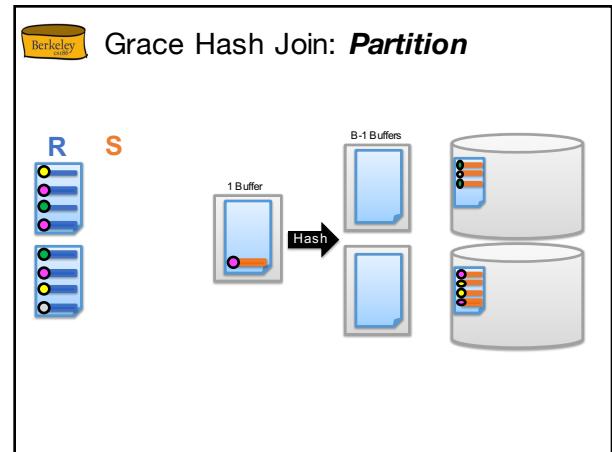
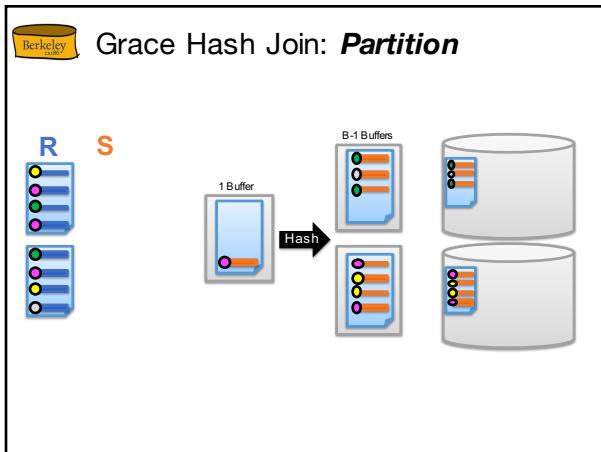
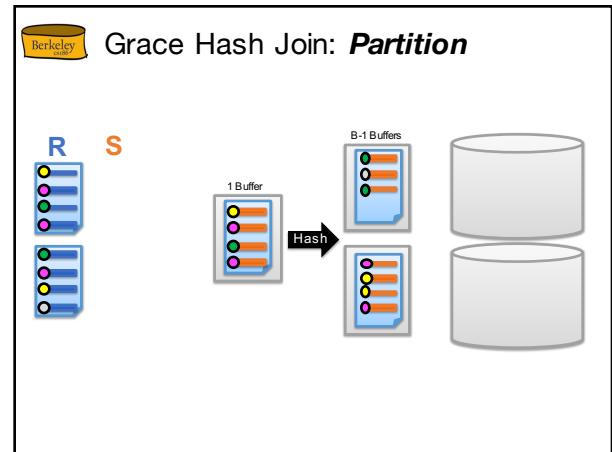
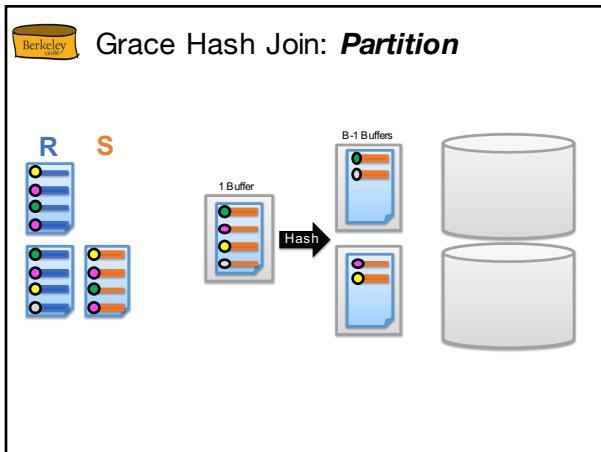
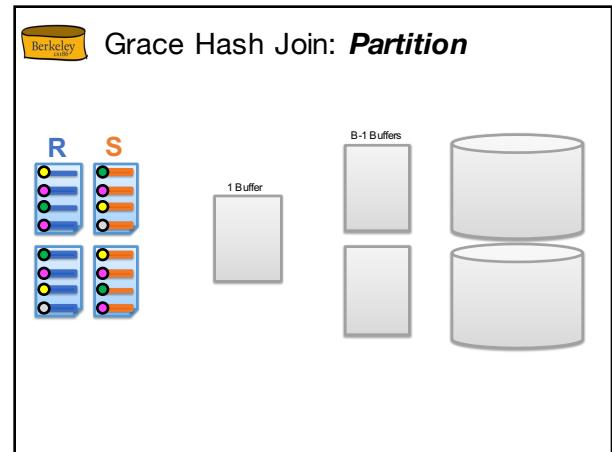
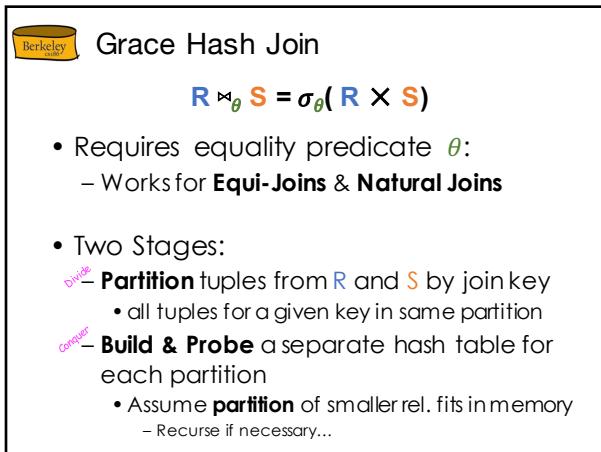
• Can we do better?

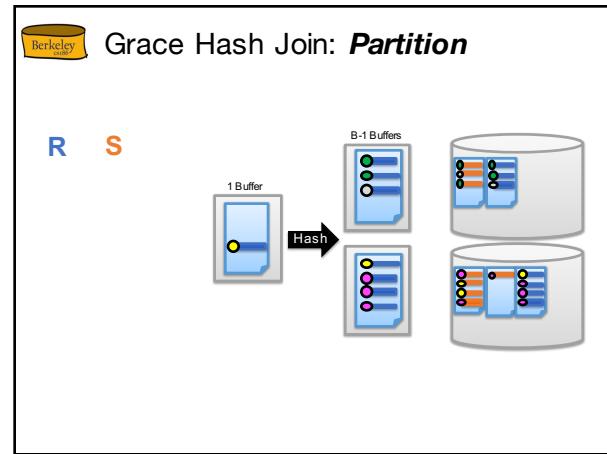
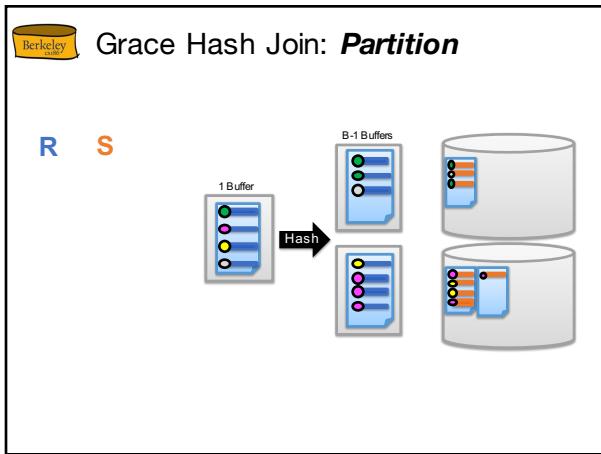
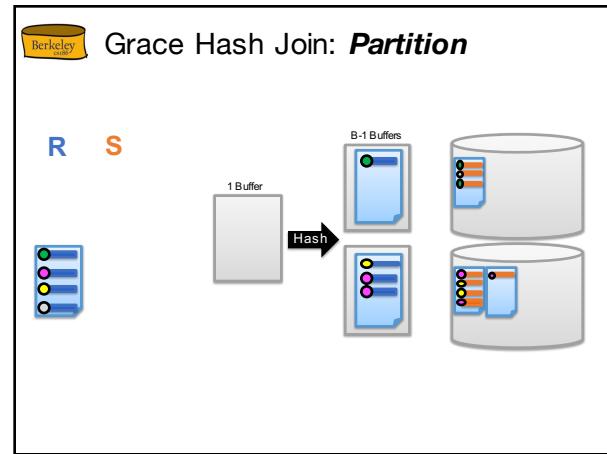
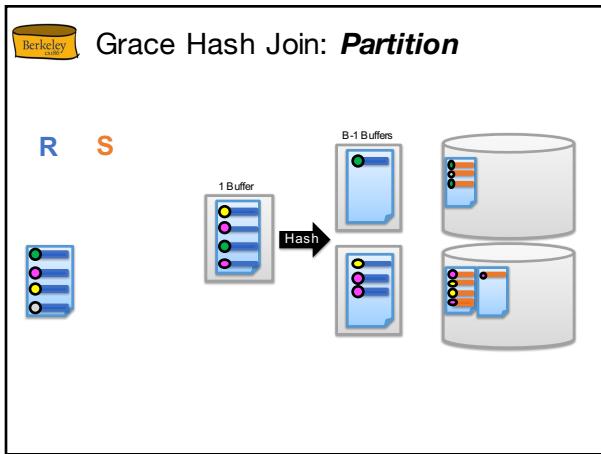
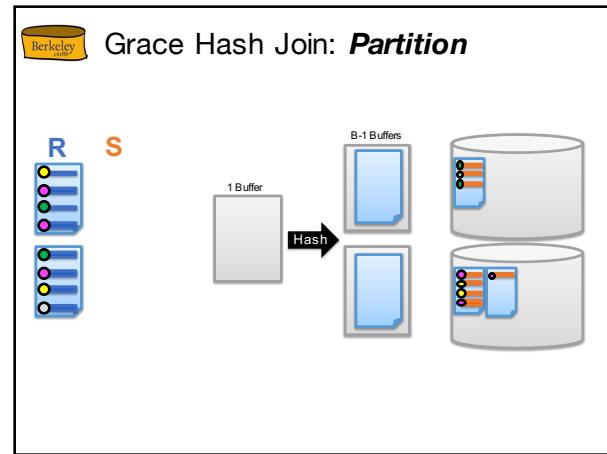
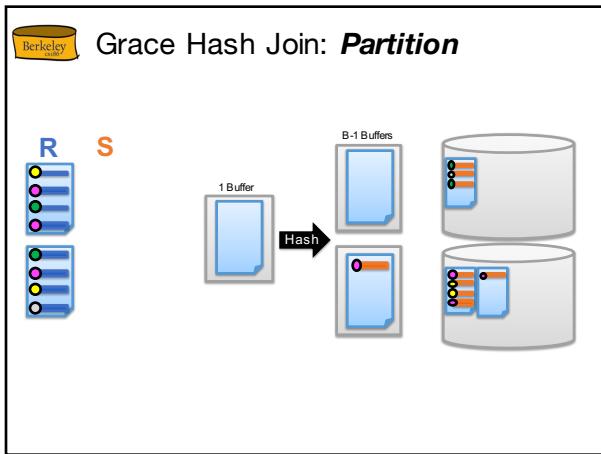
Page-Oriented Nested Loop Join

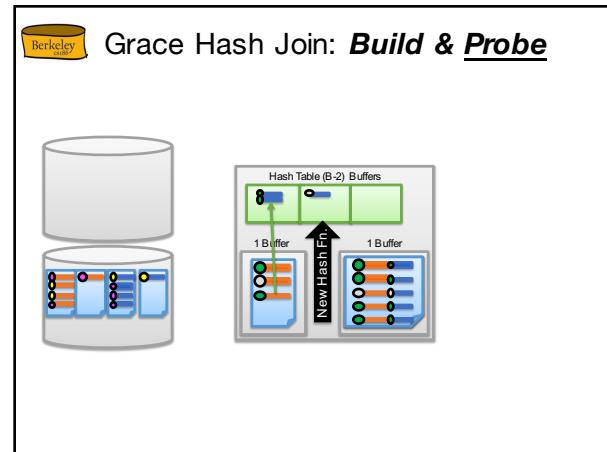
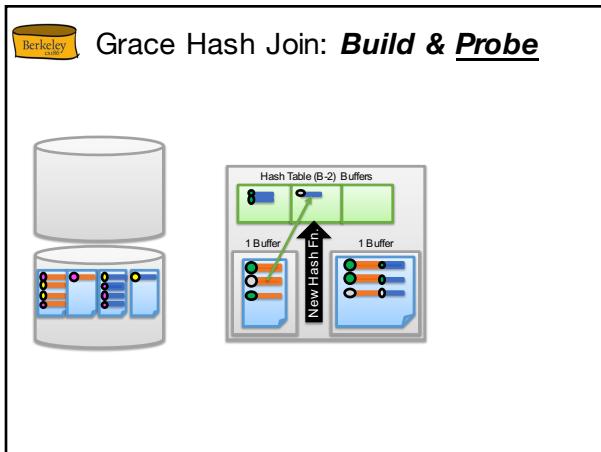
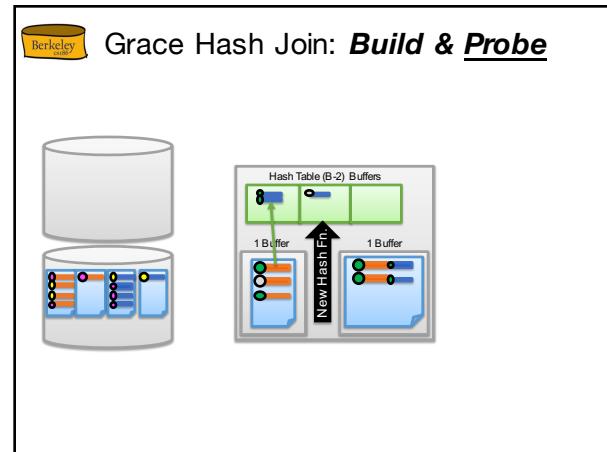
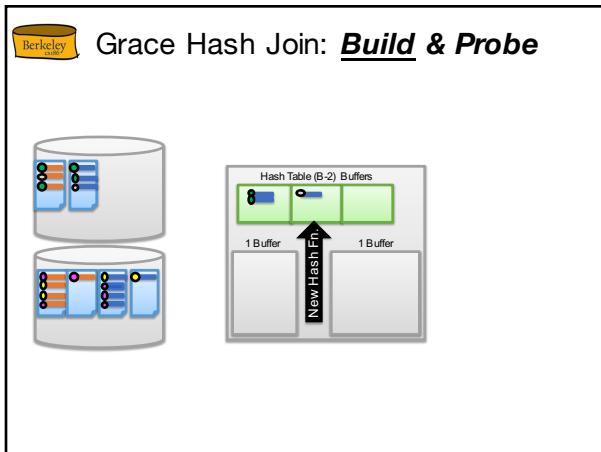
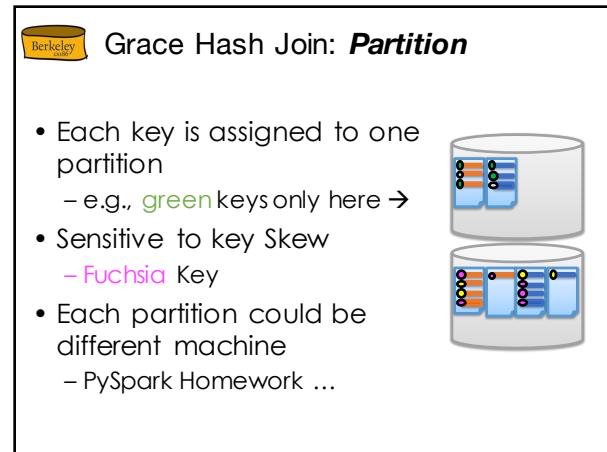
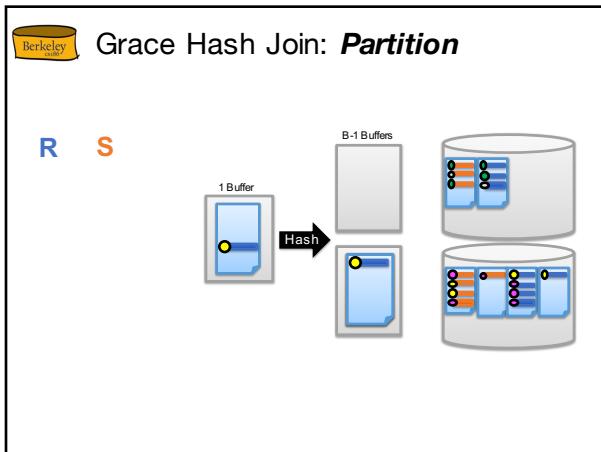
$R \bowtie S$: foreach page b_R in R do
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foreach record r in b_R do
foreach record s in b_S do
if $\theta(r, s)$ then add $\langle r, s \rangle$ to result

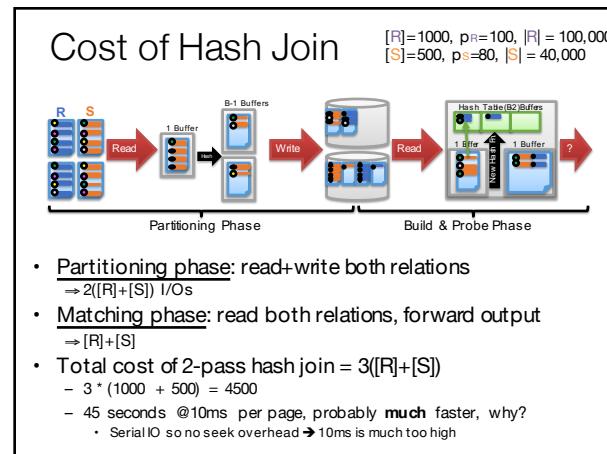
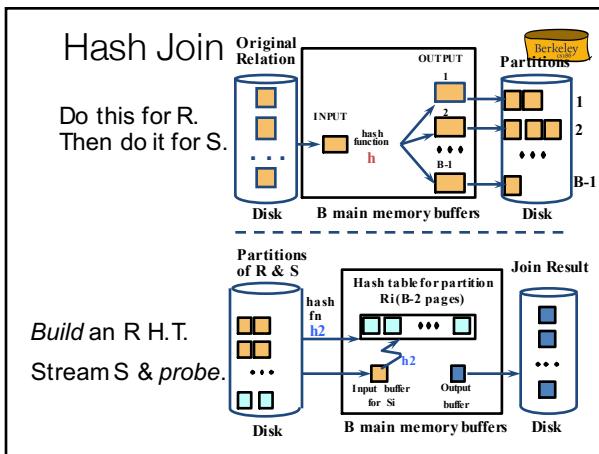
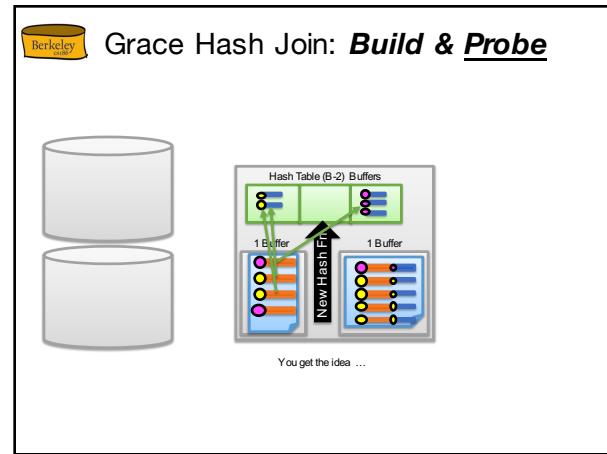
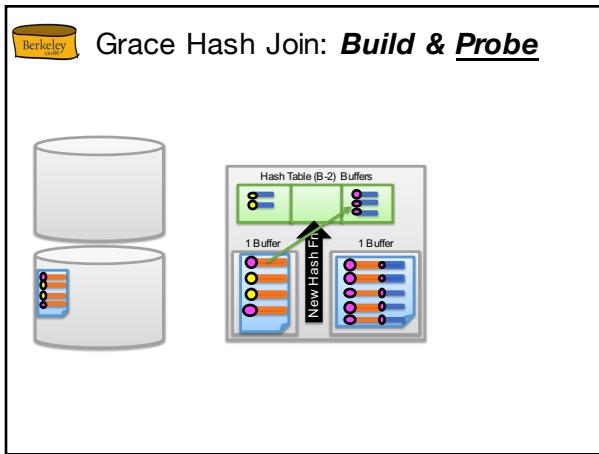
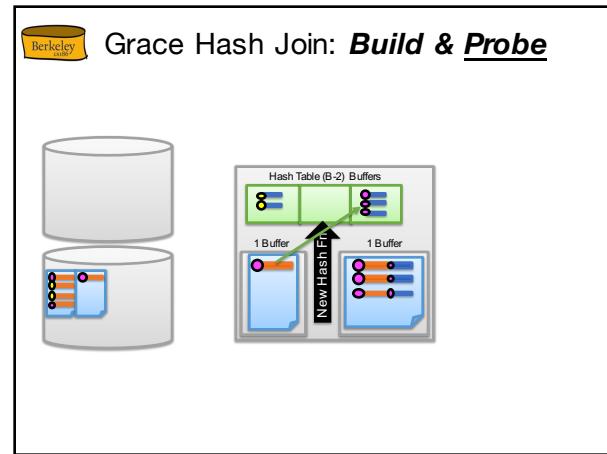
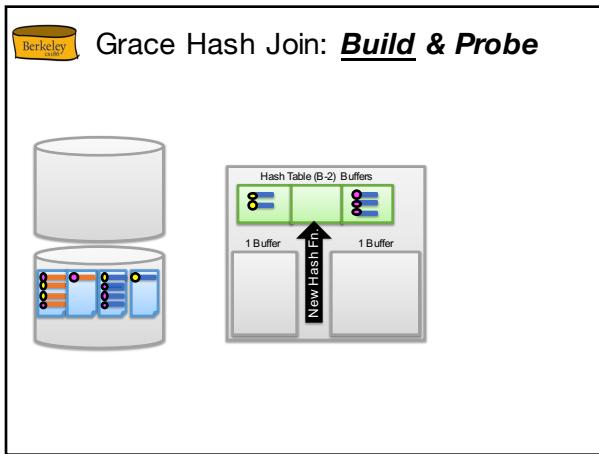
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Mechanisms of Rendezvous

Hashing & Sorting

 Sort-Merge Join

$$R \bowtie_{\theta} S = \sigma_{\theta}(R \times S)$$

- Requires equality predicate θ :
 - Works for **Equi-Joins** & **Natural Joins**
- Two Stages:
 - **Sort** tuples in R and S by join key
 - all tuples with same key in consecutive order
 - input might already sorted ... why?
 - **Merge** scan the sorted partitions and emit tuples that match

 Sort-Merge Join

- We already covered this (Lecture 2)

In case you forgot:

- You get to do this in your homework!

 Sort-Merge Join

```
while not done {
  while (r < s) { advance r }
  while (r > s) { advance s }
  // assert r == s
  mark s // save start of "block"
  while (r == s) {
    // Outer loop over r
    while (r == s) {
      // Inner loop over s
      yield r, s
      advance s
    }
    reset s to mark
    advance r
  }
}
```

sid	sname	sid	bid
22	dustin	28	103
28	yuppy	28	104
31	lubber	31	101
31	lubber2	31	102
44	guppy	42	142
58	rusty	58	107

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Sort-Merge Join



```

while not done {
    while ( $r < s$ ) { advance  $r$  }
    while ( $r > s$ ) { advance  $s$  }
    // assert  $r == s$ 
    mark  $s$  // save start of "block"
    while ( $r == s$ ) {
        // Outer loop over  $r$ 
        while ( $r == s$ ) {
            // Inner loop over  $s$ 
            yield  $\langle r, s \rangle$ 
            advance  $s$ 
        }
        reset  $s$  to mark
        advance  $r$ 
    }
}

```

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Sort-Merge Join



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            advance  $s$ 
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        advance  $r$ 
    }
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```

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            // Inner loop over  $s$ 
            yield  $\langle r, s \rangle$ 
            advance  $s$ 
        }
        reset  $s$  to mark
        advance  $r$ 
    }
}

```

sid	sname	sid	bid
22	dustin	28	103
28	yuppy	28	104
31	lubber	31	101
31	lubber2	31	102
44	guppy	42	142
58	rusty	58	107

sid	sname	bid
28	yuppy	103
28	yuppy	104
31	lubber	101
31	lubber	102

Sort-Merge Join



```

while not done {
    while ( $r < s$ ) { advance  $r$  }
    while ( $r > s$ ) { advance  $s$  }
    // assert  $r == s$ 
    mark  $s$  // save start of "block"
    while ( $r == s$ ) {
        // Outer loop over  $r$ 
        while ( $r == s$ ) {
            // Inner loop over  $s$ 
            yield  $\langle r, s \rangle$ 
            advance  $s$ 
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Sort-Merge Join



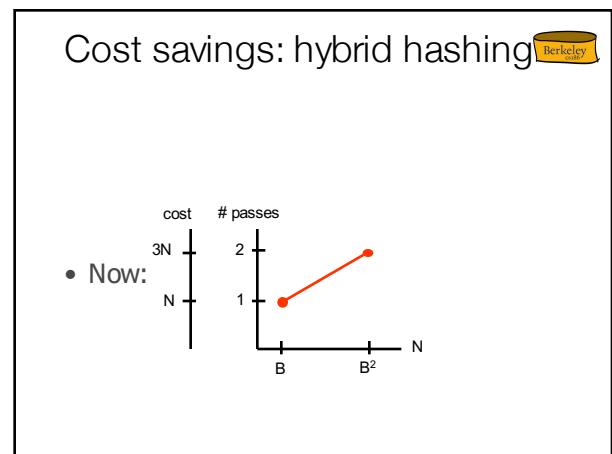
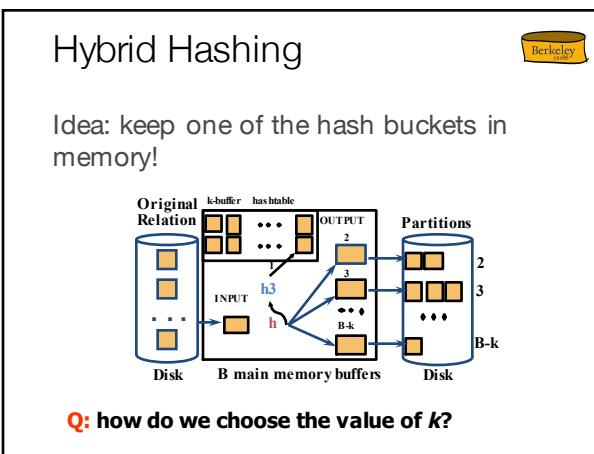
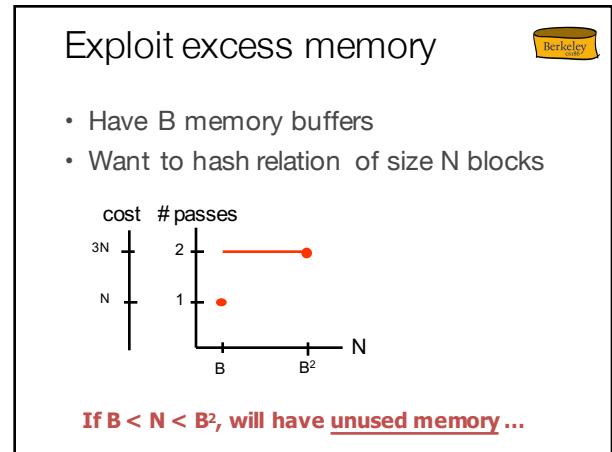
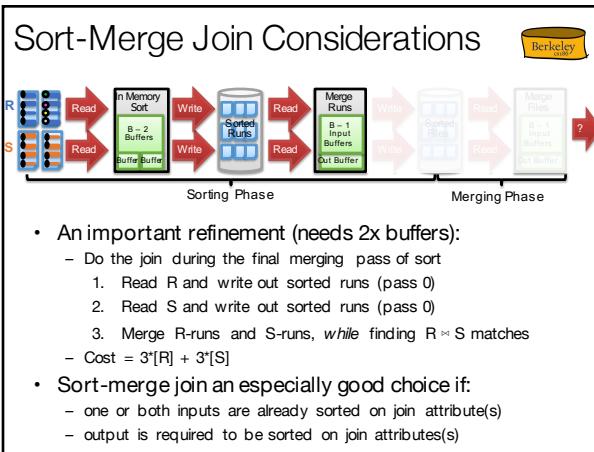
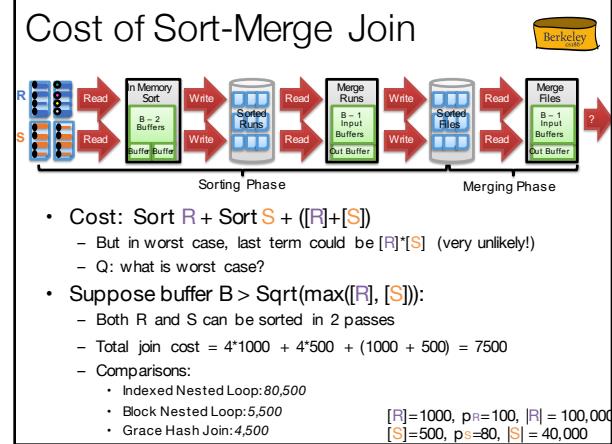
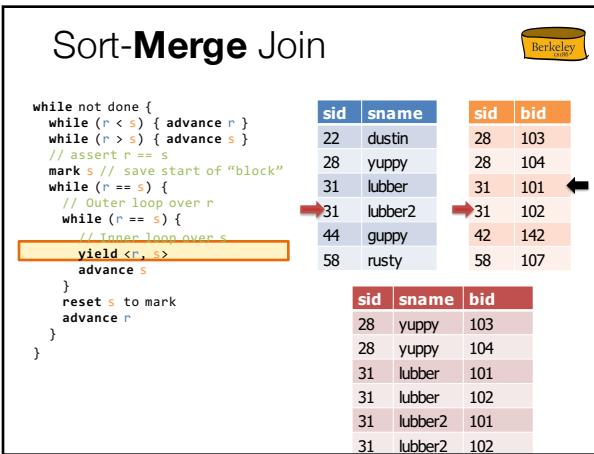
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Hash Join vs. Sort-Merge Join



- Sorting pros:
 - Good if input already sorted, or need output sorted
 - Not sensitive to data skew or bad hash functions

- Hashing pros:
 - Can be cheaper due to hybrid hashing
 - For join: # passes depends on size of smaller relation
 - Good if input already hashed, or need output hashed

Recap



- Nested Loops Join
 - Works for arbitrary Θ
 - Make sure to utilize memory in blocks
- Index Nested Loops
 - For equi-joins
 - When you already have an index on one side
- Sort/Hash
 - For equi-joins
 - No index required
- No clear winners – may want to implement them all
- Be sure you know the cost model for each
 - You will need it for query optimization!