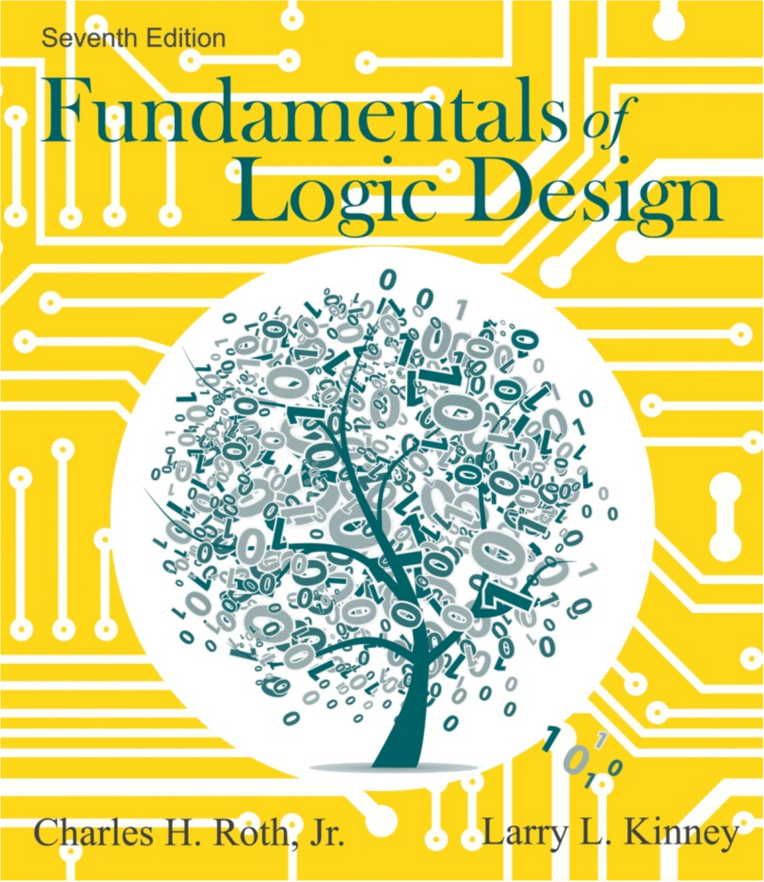

UNIT 5

Karnaugh Maps

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This chapter includes:

5.1 Minimum Form of Switching Functions

5.2 Two- and Three- Variable Karnaugh Maps

5.3 Four-Variable Karnaugh Maps

5.4 Determination of Minimum Expressions Using Essential Prime Implicants

5.5 Five-Variable Karnaugh Maps

5.6 Other Uses of Karnaugh Maps

5.7 Other Forms of Karnaugh Maps

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Learning Objectives

1. Given a function (completely or incompletely specified) of three to five variables, plot it on a Karnaugh map. The function may be given in minterm, maxterm, or algebraic form.
2. Determine the essential prime implicants of a function from a map.
3. Obtain the minimum sum-of-products or minimum product-of-sums form of a function from the map.
4. Determine all of the prime implicants of a function from a map.
5. Understand the relation between operations performed using the map and the corresponding algebraic operations.

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Minimum Form of Switching Functions

Minimum Sum-of-Products:

- A **minimum sum-of-products** expression for a function is defined as a sum of product terms which (a) has a minimum number of terms and (b) of all those expressions which have the same minimum number of terms, has a minimum number of literals.
- It corresponds directly to a minimum two-level gate circuit which has a minimum number of gates and gate inputs.

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Minimum Form of Switching Functions

How to Find a Minimum Sum-of-Products Given a Minterm Expansion:

1. Combine terms by using the uniting theorem *XY + XY' = X. Do this repeatedly* to eliminate as many literals as possible. A given term may be used more than once because *X + X = X.*

2. Eliminate redundant terms by using the consensus theorem or other theorems.

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Minimum Form of Switching Functions

Example 1:

Find a minimum sum-of-products expression for

$$F(a, b, c) = \Sigma\, m\,(0, 1, 2, 5, 6, 7)$$

$$\begin{aligned} F &= a'b'c' + a'b'c + a'bc' + ab'c + abc' + abc \\ &= a'b' + b'c + bc' + ab \end{aligned} \tag{5-1}$$

None of the terms in the above expression can be eliminated by consensus. However, combining terms in a different way leads directly to a minimum sum of products:

$$\begin{aligned} F &= a'b'c' + a'b'c + a'bc' + ab'c + abc' + abc \\ &= a'b' + bc' + ac \end{aligned} \tag{5-2}$$

If the uniting theorem is applied to all possible pairs of minterms, six two-literal products are obtained: $a'b'$, $a'c'$, $b'c$, bc', ac, ab. Then, the consensus theorem can be applied to obtain a second minimal solution:

$$a'c' + b'c + ab \tag{5-3}$$

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Minimum Form of Switching Functions

Minimum Product-of-Sums:

- A **minimum product-of-sums** expression for a function is defined as a product of sum terms which (a) has a minimum number of terms and (b) of all those expressions which have the same minimum number of terms, has a minimum number of literals.
- Given a maxterm expansion, the minimum product of sums can often be obtained by a procedure similar to that used in the minimum sum-of-products case, except that the uniting theorem $(X + Y)(X + Y') = X$ is used to combine terms.

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Minimum Form of Switching Functions

Example 2:

Example

$$(A + B' + C + D')(A + B' + C' + D')(A + B' + C' + D)(A' + B' + C' + D)(A + B + C' + D)(A' + B + C' + D)$$

$$= (A + B' + D') \quad (A + B' + C') \quad (B' + C' + D) \quad (B + C' + D)$$

$$= (A + B' + D') \quad \underbrace{(A + B' + C')}_{\text{eliminate by consensus}} \quad (C' + D)$$

$$= (A + B' + D')(C' + D) \qquad (5\text{-}4)$$

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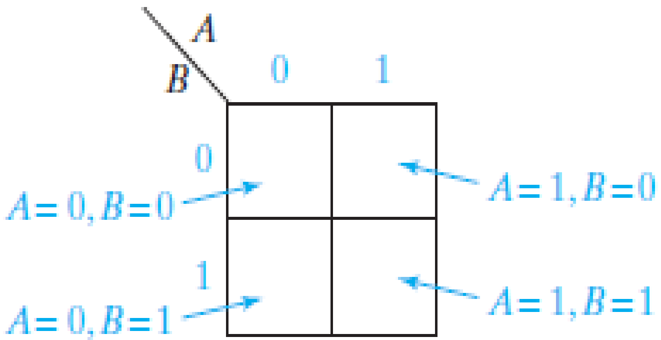
Two- and Three-Variable Karnaugh Maps

Karnaugh Maps:

- A **Karnaugh map** is a systematic way of simplifying switching functions and lead directly to minimum cost two-level circuits composed of AND and OR gates.
- It specifies the value of the function for every combination of values of the independent variables.

Two variable K-map →



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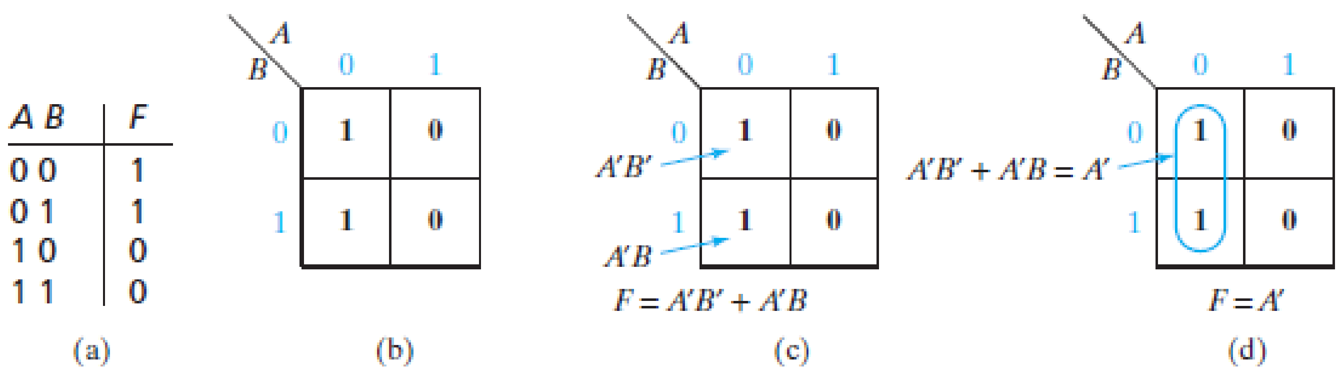
Two- and Three-Variable Karnaugh Maps

Two Variable Karnaugh Maps:

- Note that the value of F for A = B = 0 is plotted in the upper left square, and the other map entries are plotted in a similar way in the figure below (Figure 5-1 in book).
- Each 1 on the map corresponds to a minterm of F. For example, a 1 in square 01 indicates that AB is a minterm.
- Minterms in adjacent squares of the map can be combined since they differ in only one variable.



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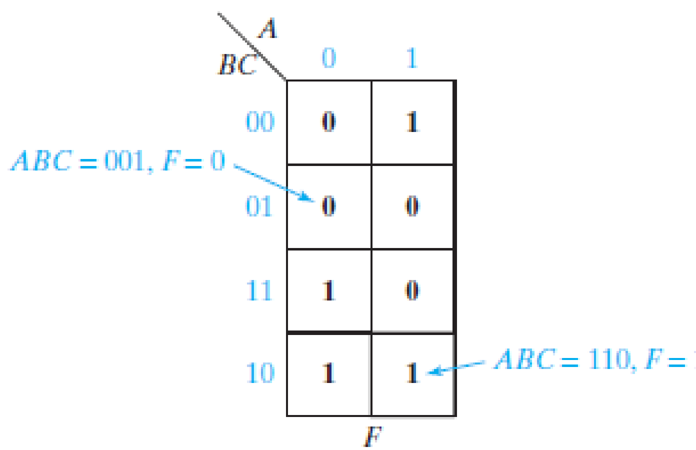
Two- and Three-Variable Karnaugh Maps

Three-Variable Karnaugh Maps:

- A three-variable Karnaugh map can be plotted in a similar way to the two-variable map.
- The value of one variable, A, is listed on the top and the vales of the other two, B and C, are listed on the side

A B C	F
0 0 0	0
0 0 1	0
0 1 0	1
0 1 1	1
1 0 0	1
1 0 1	0
1 1 0	1
1 1 1	0

(a)



(b)

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Two- and Three-Variable Karnaugh Maps

Locations of Minterms on a Karnaugh Map:

- Minterms in adjacent squares of the map differ in only one variable and therefore can be combined using the uniting theorem $XY + XY' = X$.

FIGURE 5-3 Location of Minterms on a Three-Variable Karnaugh Map

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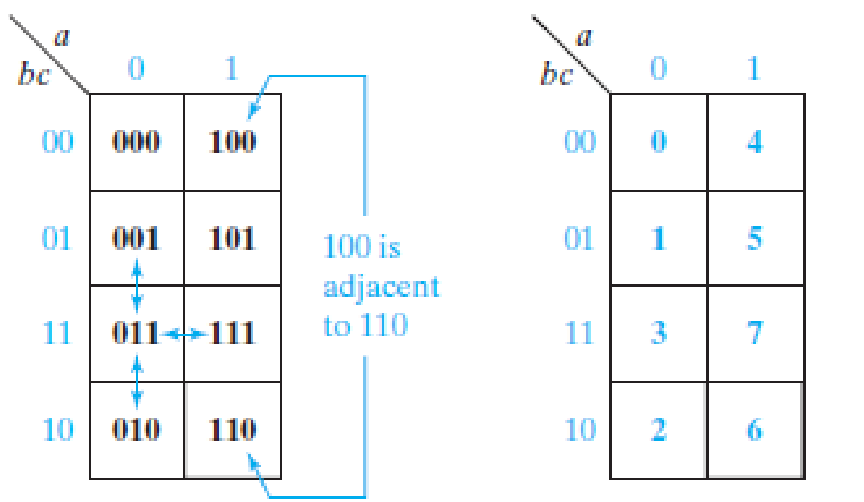

(a) Binary notation (b) Decimal notation

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Two- and Three-Variable Karnaugh Maps

Mapping Minterm and Maxterm Expressions on Karnaugh Maps:

- Given the minterm or maxterm expansion of a function, it can be mapped on a Karnaugh map as follows:

FIGURE 5-4
Karnaugh Map of $F(a, b, c) = \Sigma\, m(1, 3, 5) = \Pi\, M(0, 2, 4, 6, 7)$

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bc \ a	0	1
00	0 [0]	0 [4]
01	1 [1]	1 [5]
11	1 [3]	0 [7]
10	0 [2]	0 [6]

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Two- and Three-Variable Karnaugh Maps

Plotting Product Terms:

To plot the term b, 1's are entered in the four squares of the map where b = 1 as shown below:

FIGURE 5-5
Karnaugh Maps for Product Terms
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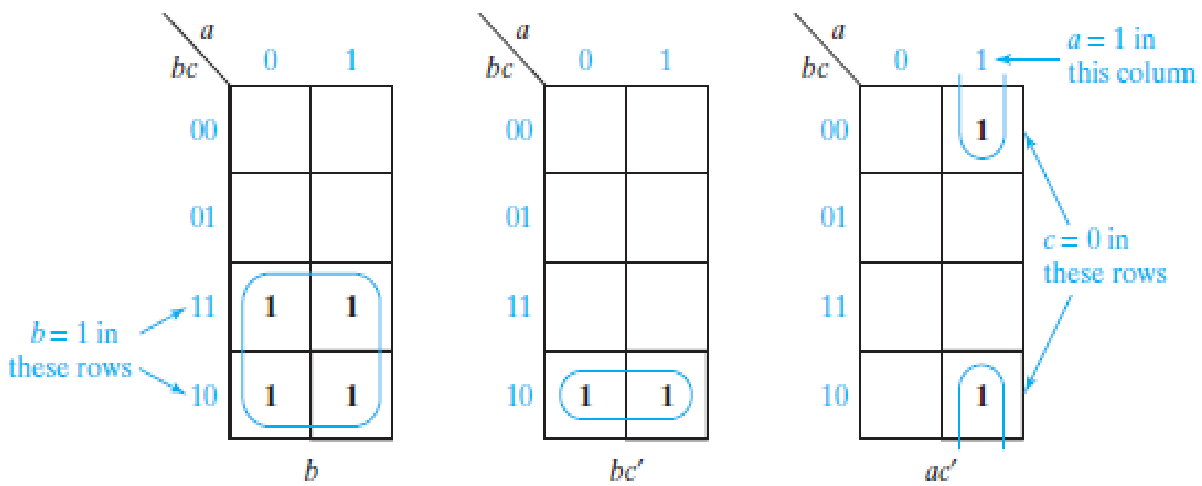

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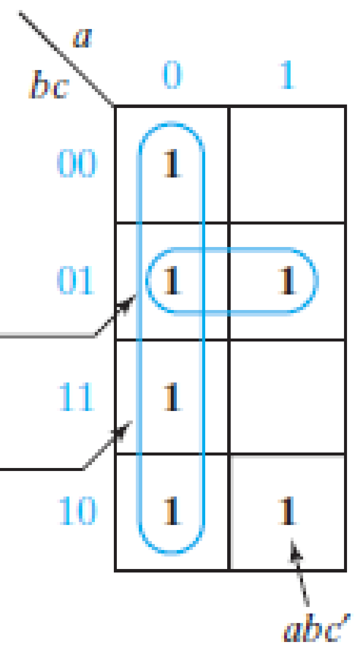
Two- and Three-Variable Karnaugh Maps

Plotting A Karnaugh Map using an Expression in Algebraic form:

- Given f(a,b,c)=abc’+b’c+a’, we would plot the map:

1. The term abc' is 1 when $a = 1$ and $bc = 10$, so we place a 1 in the square which corresponds to the $a = 1$ column and the $bc = 10$ row of the map.
2. The term $b'c$ is 1 when $bc = 01$, so we place 1’s in both squares of the $bc = 01$ row of the map.
3. The term a' is 1 when $a = 0$, so we place 1’s in all the squares of the $a = 0$ column of the map. (Note: Since there already is a 1 in the $abc =$ 001 square, we do not have to place a second 1 there because $x + x = x$.)



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Two- and Three-Variable Karnaugh Maps

Simplifying Expressions:

FIGURE 5-6
Simplification of a Three-Variable Function

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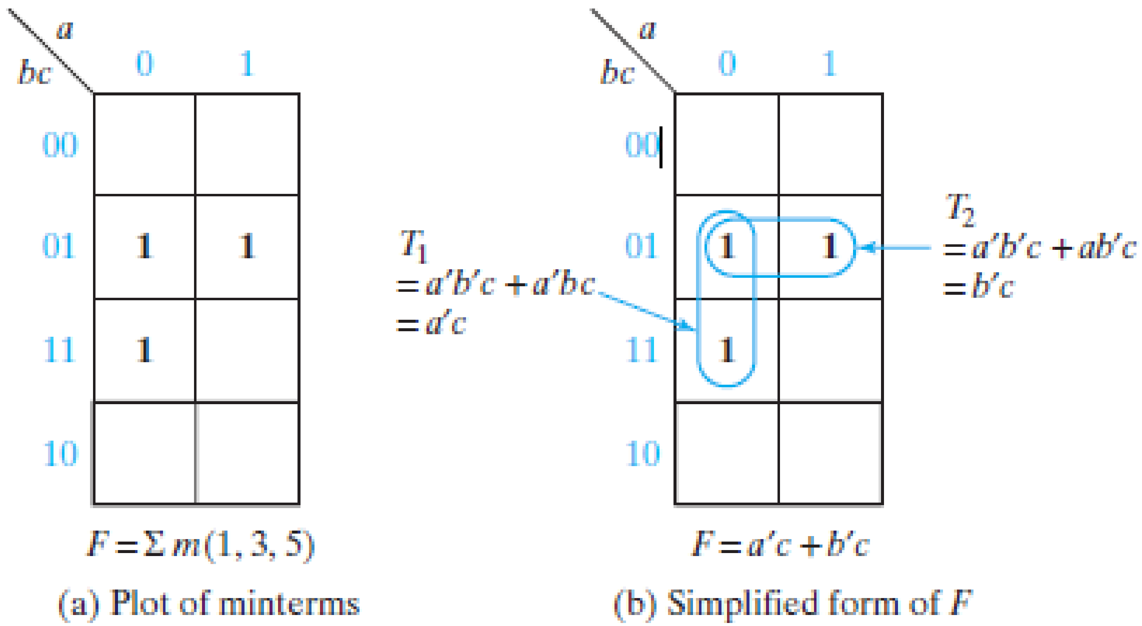

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Two- and Three-Variable Karnaugh Maps

Consensus Theorem in Karnaugh Maps:

FIGURE 5-8
Karnaugh Maps that Illustrate the Consensus Theorem

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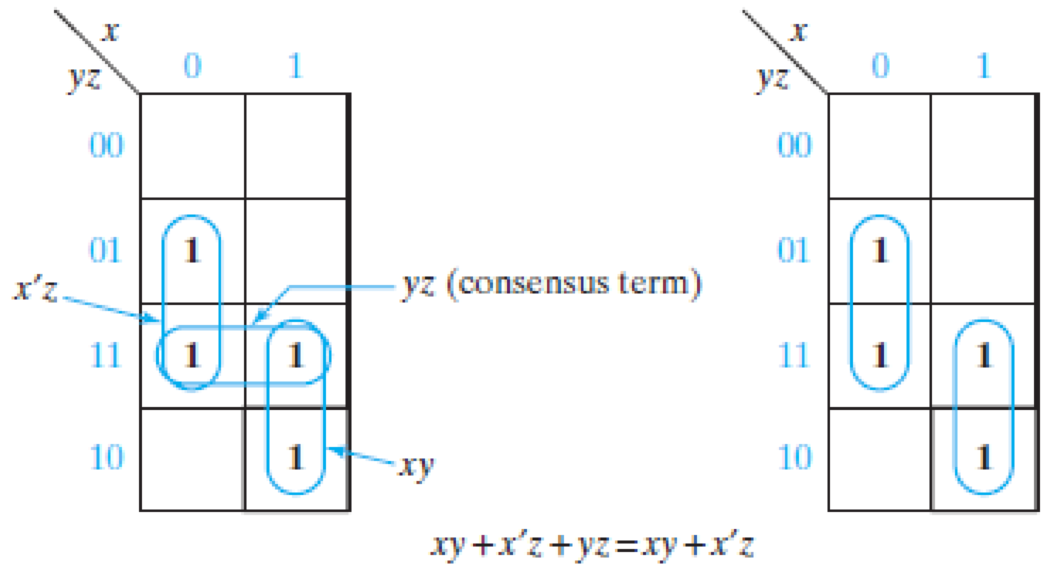

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Two- and Three-Variable Karnaugh Maps

If a function has two or more minimum sum-of-products forms, all of these forms can be determined from a map. Figure 5-9 shows the two minimum solutions for $F = \Sigma\, m(0, 1, 2, 5, 6, 7)$.

FIGURE 5-9
Function with Two Minimum Forms

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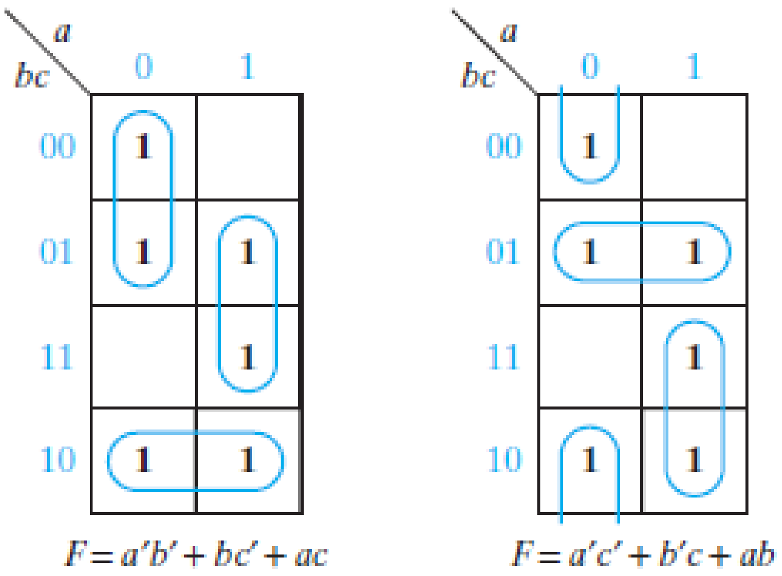

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Four-Variable Karnaugh Maps

Location of terms on a Four-Variable K-map:

FIGURE 5-10 Location of Minterms on Four-Variable Karnaugh Map

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CD \ AB	00	01	11	10
00	0	4	12	8
01	1	5	13	9
11	3	7	15	11
10	2	6	14	10

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Four-Variable Karnaugh Maps

Plotting functions on a Four-Variable Karnaugh Map:

- This is accomplished in the same way as for two- or three-variable Karnaugh maps.
- "1"s are plotted for whichever values of the variables would result in the expression yielding "1".

FIGURE 5-11 Plot of $acd + a'b + d'$

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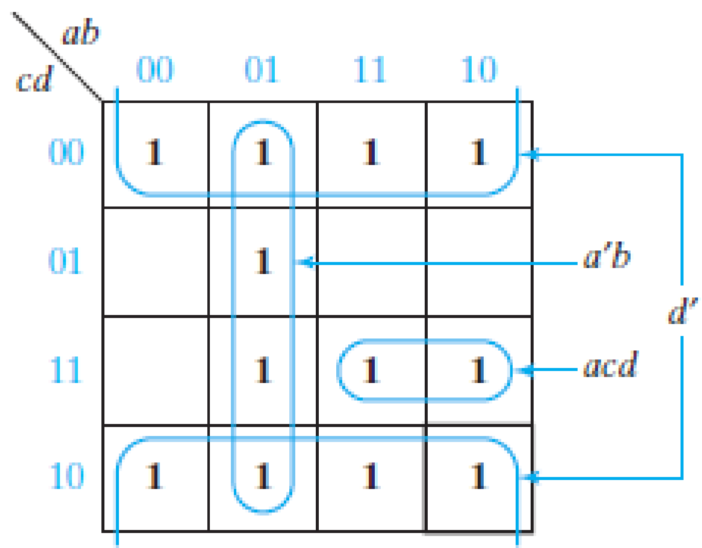

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Four-Variable Karnaugh Maps

Simplifying Expressions in Four-Variable Karnaugh Maps:

FIGURE 5-12
Simplification of Four-Variable Functions

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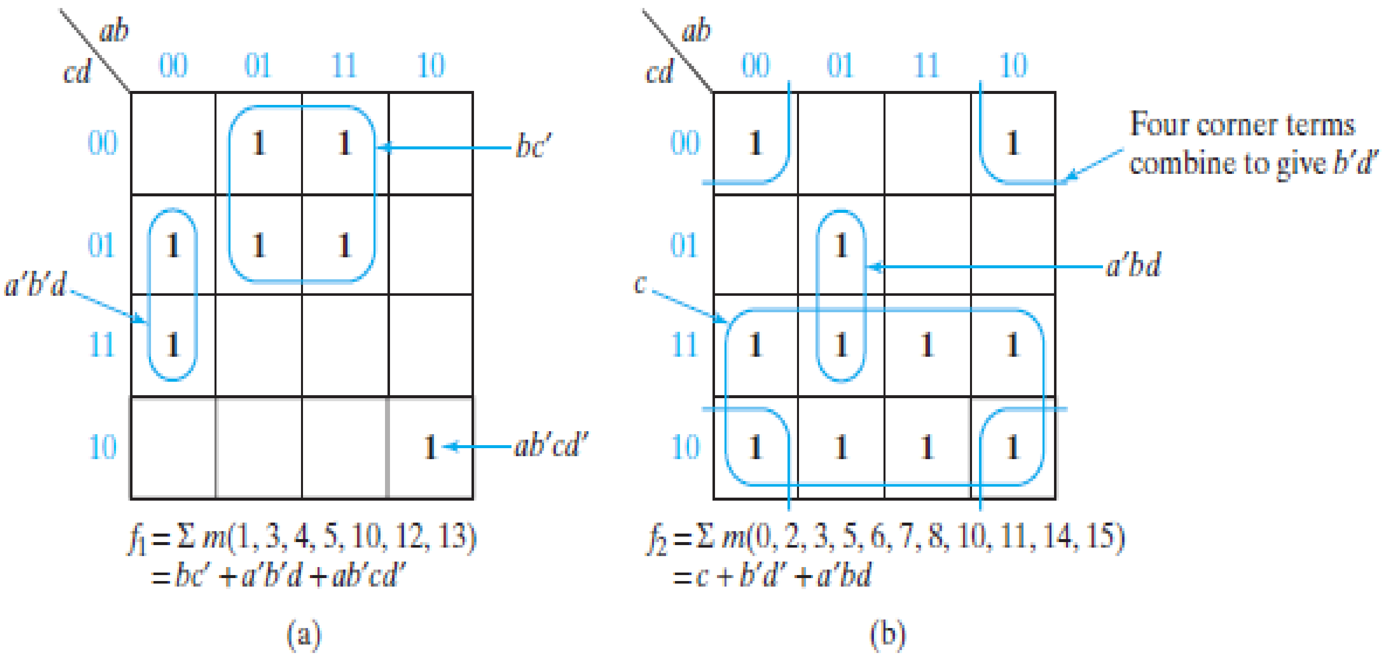

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Four-Variable Karnaugh Maps

Expressions with “don’t cares”:

- “Don’t cares” are noted as X’s in Karnaugh maps. See below:

FIGURE 5-13 Simplification of an Incompletely Specified Function

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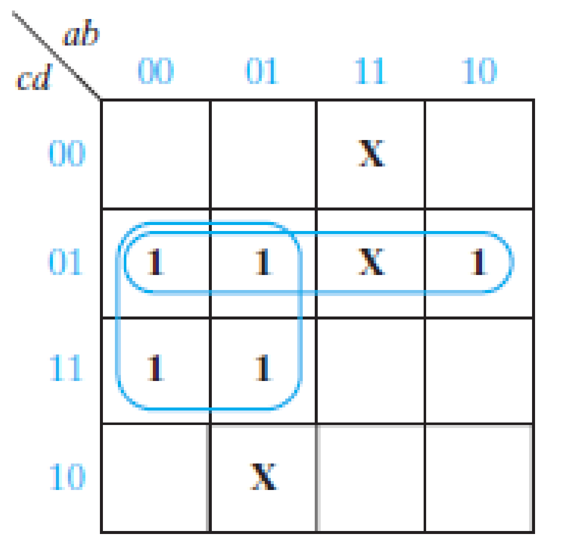

$$f = \Sigma\, m(1, 3, 5, 7, 9) + \Sigma\, d(6, 12, 13)$$
$$= a'd + c'd$$

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Four-Variable Karnaugh Maps

From SOP to POS form using Karnaugh Maps:'

- For the function specified below as f, the process of finding the product-of-sums from the sum-of-products is shown.

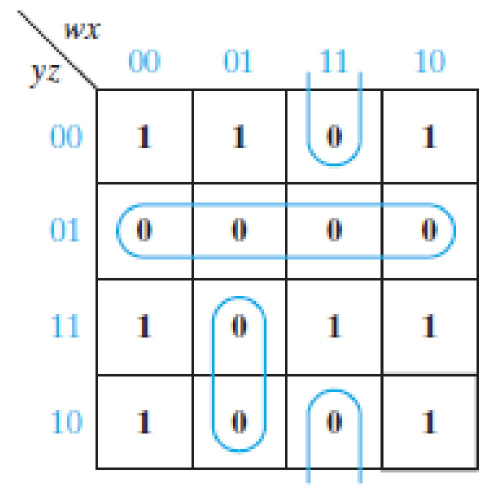
$$f = x'z' + wyz + w'y'z' + x'y$$

First, the 1's of f are plotted in Figure 5-14. Then, from the 0's,

$$f' = y'z + wxz' + w'xy$$

and the minimum product of sums for f is

$$f = (y + z')(w' + x' + z)(w + x' + y')$$



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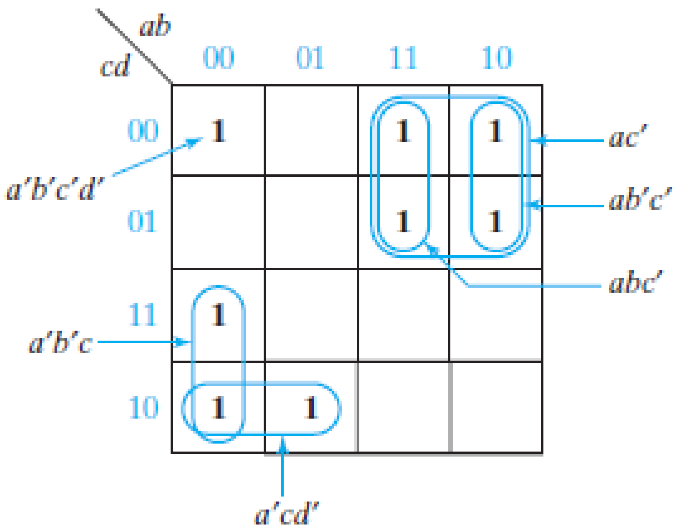
Determination of Minimum Expressions Using Essential Prime Implicants

Prime Implicants:

- A product term implicant is called a **prime implicant** if it cannot be combined with another term to eliminate a variable.



- a’b’c, a’cd’, and ac’ are prime implicants
- a’b’c’d’, abc’, and ab’c’ are not prime implicants

A sum-of-products expression containing a term which is not a prime implicant cannot be minimum.

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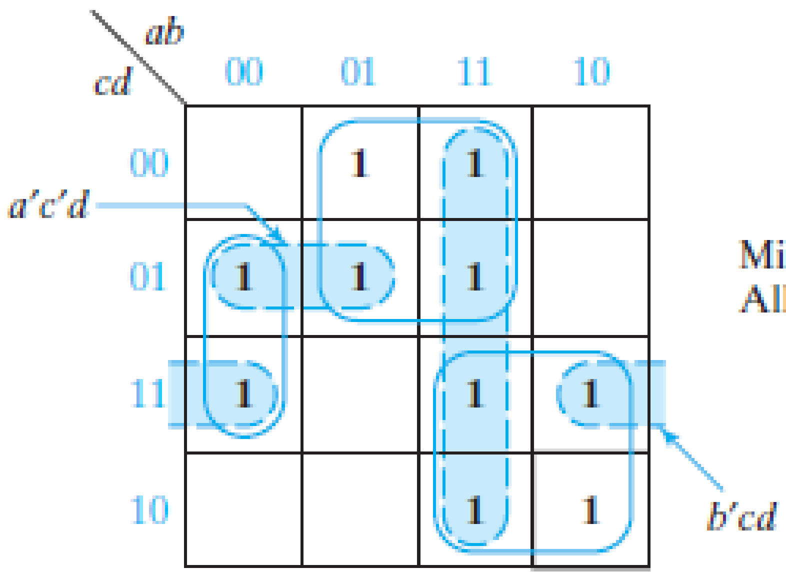
Determination of Minimum Expressions Using Essential Prime Implicants

Determination of All Prime Implicants:

The minimum solution may not include all prime implicants, as shown below:



Minimum solution: $F = a'b'd + bc' + ac$
All prime implicants: $a'b'd, bc', ac, a'c'd, ab, b'cd$

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Determination of Minimum Expressions Using Essential Prime Implicants

Essential Prime Implicants:

- If a minterm is covered by only one prime implicant, that prime implicant is said to be **essential**, and it must be included in the minimum sum of products.
- In order to find a minimum sum of products from a map, we should first loop all of the essential prime implicants.

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Determination of Minimum Expressions Using Essential Prime Implicants

Finding Essential Prime Implicants:

- Sometimes essential prime implicants can be found by inspection.
- Other times, we must look at all squares adjacent to that minterm. If the given minterm and all of the 1's adjacent to it are covered by a single term, then that term is an *essential prime implicant.*
- If all of the 1's adjacent to a given minterm are *not covered by a single term, then* we cannot say whether these prime implicants are essential or not without checking the other minterms.
- See figure on next page.

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Determination of Minimum Expressions Using Essential Prime Implicants

FIGURE 5-18

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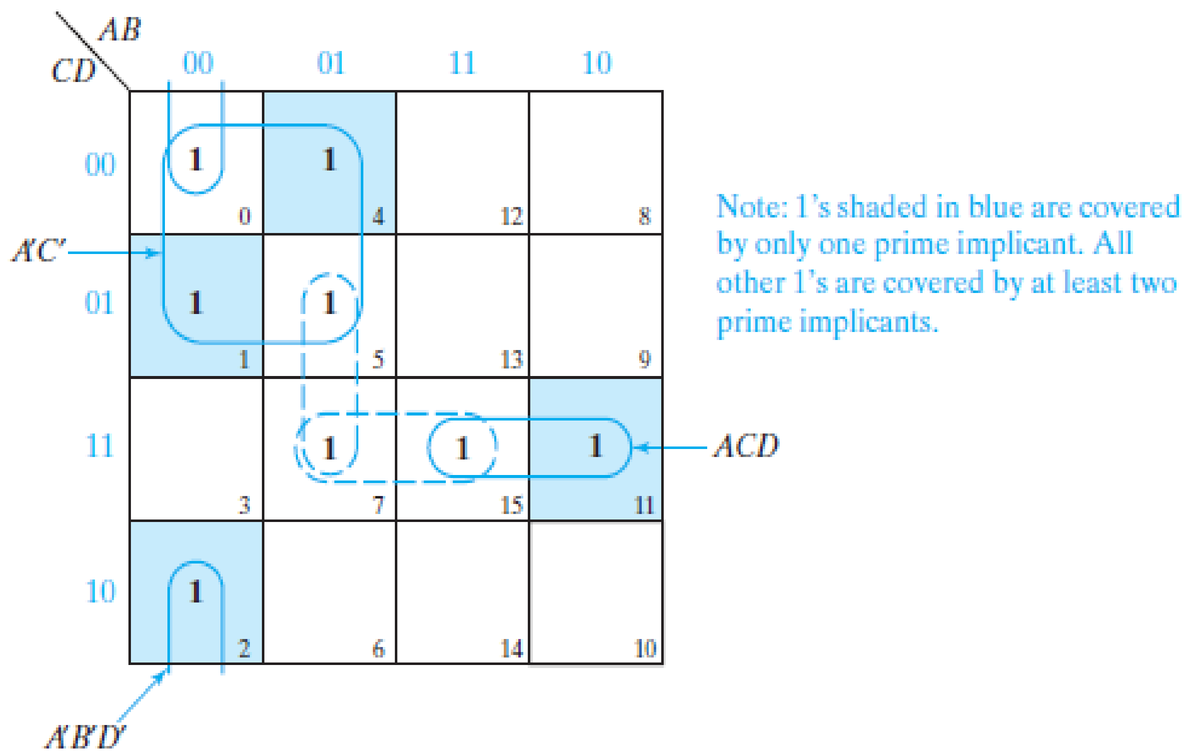

[1]This statement is proved in Appendix D.

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Determination of Minimum Expressions Using Essential Prime Implicants

Procedure to Obtain a Minimum Sum of Products from a Karnaugh Map:

1. Choose a minterm (a 1) which has not yet been covered.

2. Find all 1's and X's adjacent to that minterm. (Check the *n adjacent squares on* an *n-variable map.)*

3. If a single term covers the minterm and all of the adjacent 1's and X's, then that term is an essential prime implicant, so select that term. (Note that "don't-care" terms are treated like 1's in steps 2 and 3 but not in step 1.)

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Determination of Minimum Expressions Using Essential Prime Implicants

Procedure (continued):

4. Repeat steps 1, 2, and 3 until all essential prime implicants have been chosen.

5. Find a minimum set of prime implicants which cover the remaining 1's on the map. If there is more than one such set, choose a set with a minimum number of literals.

See figure 5-19 in book for a flowchart of this procedure (pg 148).

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Five-Variable Karnaugh Maps

Five-Variable Karnaugh Maps:

- A five-variable map can be constructed in three dimensions by placing one four-variable map on top of a second one.
- Terms in the bottom layer are numbered 0 through 15 and corresponding terms in the top layer are numbered 16 through 31, so that terms in the bottom layer contain *A' and those in the top layer contain A.*
- *To represent* the map in two dimensions, we will divide each square in a four-variable map by a diagonal line and place terms in the bottom layer below the line and terms in the top layer above the line (Figure 5-21).

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Five-Variable Karnaugh Maps

FIGURE 5-21
A Five-Variable Karnaugh Map

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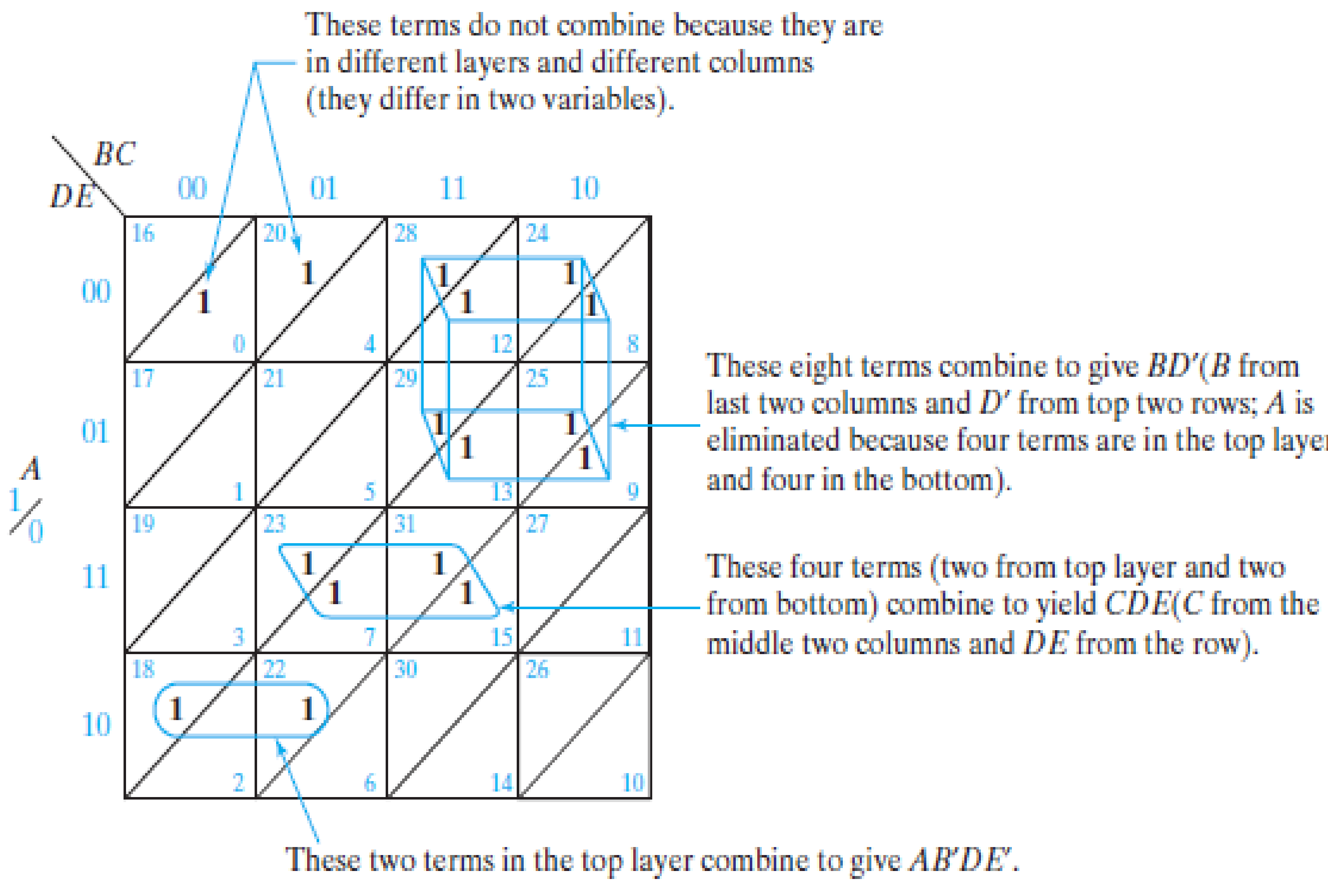

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Five-Variable Karnaugh Maps

Example of Five-Variable Karnaugh Map:

Figure 5-23 is a map of

$$F(A, B, C, D, E) = \Sigma\, m(0, 1, 4, 5, 13, 15, 20, 21, 22, 23, 24, 26, 28, 30, 31)$$

FIGURE 5-23

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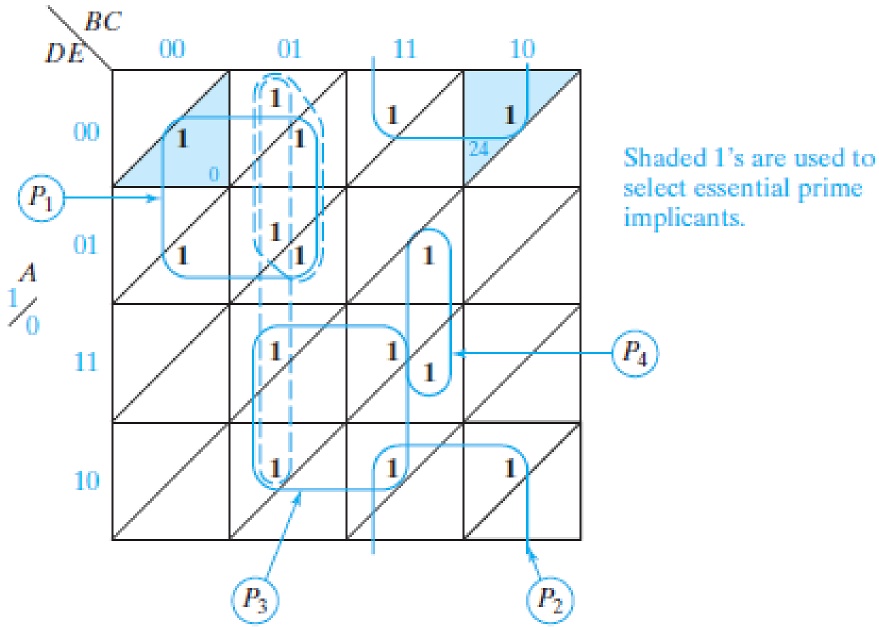

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Other Uses of Karnaugh Maps

Other Uses:

- We can prove that two functions are equal by plotting them on maps and showing that they have the same Karnaugh map.
- We can perform the AND operation (or the OR operation) on two functions by ANDing (or ORing) the 1's and 0's which appear in corresponding positions on their maps.
- A Karnaugh map can facilitate factoring an expression.
- When simplifying a function algebraically, the Karnaugh map can be used as a guide in determining what steps to take (see pages 152-153).

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Other Forms of Karnaugh Maps

Veitch Diagrams:

Instead of labeling the sides of a Karnaugh map with 0's and 1's, some people prefer to use the labeling shown in Figure 5-27.

For the half of the map labeled A, A=1; and for the other half, A=0.

FIGURE 5-27
Veitch Diagrams
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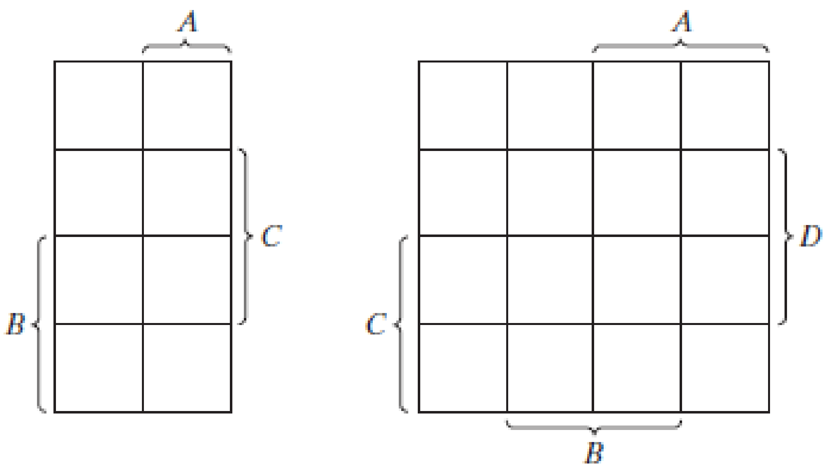

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Other Forms of Karnaugh Maps

Other forms of Five-Variable Karnaugh Maps:

- One form simply consists of two four-variable maps side-by-side as in Figure 5-28(a).
- Figure 5-28(b) shows *mirror image map, in which the first and eighth columns* are "adjacent" as are second and seventh columns, third and sixth columns, and fourth and fifth columns.

FIGURE 5-28 Other Forms of Five-Variable Karnaugh Maps

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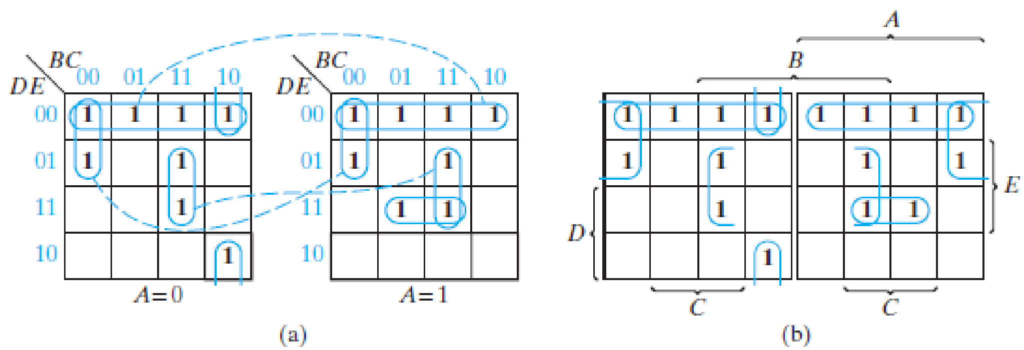

$$F = D'E' + B'C'D' + BCE + A'BC'E' + ACDE$$

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