Homework #1 (Covers Unit-1, Unit-2 and Unit-3)
CDA Computer Logic Design
Total Points: 100

Notes:

1. **All homework should be done and submitted individually**
2. **Show all steps for each question to get full points (Use extra pages if required)**
3. **Submit electronically in canvas as a single pdf file**
4. **Follow instructions for each question**
5. **A' is the complement of A**

Q1. Write True or False after each question. Subscript defines the base format. (7*2 = 14 points)

(a) Addition in binary: $(100010)_2 + (000111)_2 + (000101)_2 = (100110)_2$
Answer: False

(b) Subtraction in octal: $(7546)_8 - (2154)_8 = (5327)_8$
Answer: False

(c) Division of $(110011)_2$ by $(1100)_2$ produces remainder $(10)_2$
Answer: False

(d) Multiplication in hexadecimal: $(5C3)_{16} \times (32)_{16} = (12116)_{16}$
Answer: False

(e) Conversion: $(1011.101)_2 = (135)_8$
Answer: False

(f) Conversion: $(1011.1101)_2 = (bd)_{16}$
Answer: False

(g) DeMorgan's law is limited to 2 variables.
Answer: False

1

a.
$$\begin{array}{r} 100010 \\ 000111 \\ 000101 \\ \hline 101110 \end{array}$$

b.
$$\begin{array}{r} 7546_8 \\ -\ 2154_8 \\ \hline 5372 \end{array}$$

c. 1001 Reminder 11_2

$$\begin{array}{r|l} 1100 & 110011 \\ & \underline{1100} \\ & \ \ 01 \\ & \ \ \underline{00} \\ & \ \ 011 \\ & \ \ \underline{000} \\ & \ \ 011 \end{array}$$

d.
$$\begin{array}{r} 5C3 \\ \times\ 32 \\ \hline 12016 \end{array}$$

e. 13.5

f. B.D

g. 3 Variables

Q2. Conversion of numbers. (5*2 = 10 points)

(a) Hexadecimal 1FA to decimal
(b) Octal 270 to Binary
(c) Binary 101101 to Hexadecimal
(d) Decimal 627 to Trinary (Base 3)
(e) What is the base x in $(2400)_x = (1010)_7$

Answers:

(a) 1FA

$= (1 \times 16^2) + (15 \times 16^1) + (10 \times 16^0) = \boxed{506}$

(b) 270

2	7	0
010	111	000

$= \boxed{010\ 111\ 000}$

(c) 10 1101

0010	1101
2	D

$= \boxed{2D}$

(d)

$$\begin{array}{r|l l} 3 & 627 & 0 \\ & 209 & 2 \\ & 69 & 0 \\ & 23 & 2 \\ & 7 & 1 \\ & 2 & \end{array}$$

$= \boxed{(212020)_3}$

(e) $(1010)_7$

$(1 \times 7^3) + (0 \times 7^2) + (1 \times 7^1) + (0 \times 7^0)$

$= 350$

$2 \times x^3 + 4 \times x^2 + 0 + 0 = 350$, $\boxed{x = 5}$

$x^3 + 2x^2 = 175$

Q3. (a) A decimal integer is in the range of 102 to -102. How Many bits are required to represent any value in this range in 2's complement representation? (5 points)

(b) Assume the same number of bits as part (a), compute the following using 2's complement method and comment on the correctness of the result. (5 points)

(b1) 75 – 32
(b2) -57 + 99
(b3) -52 – 84
(b4) 37 + 93

Answers:

(a)

102 → 0110 0110

-102 → 1001 1001

$\boxed{\text{8 bits}}$

(b1)

75 → 0100 1011

-32 → 1110 0000

$\boxed{1}$ 0010 1011 = 43

omit 1
Correct

(b2)

-57 → 1100 0111

99 → 0110 0011

$\boxed{1}$ 0010 1010 = 42

omit 1
Correct

(b3)

-52 → 1100 1100

-84 → 1010 1100

$\boxed{1}$ 0111 1000 = 120

omit $\boxed{1}$
overflow
incorrect

(b4)

37 → 0010 0101

93 → 0101 1101

1000 0010 = 130

overflow
incorrect

Q4. Simplify the following Boolean functions (5*2 = 10 points)

(a) F = XY + XY'

(b) F = (X + Y) (X + Y')

(c) F = Y'Z + X'YZ + XYZ

(d) F = (X + Y) (X' + Y + Z) (X' + Y + Z)

(e) F = X + XYZ + X'YZ + X'Y + WX + WX'

Answers:

(a) $XY + XY'$

$= X(Y+Y') = \boxed{X}$

(b) $(x+y)(x+y')$

$= xx + xy' + xy + \cancel{yy'} = x + x(y+y') = x + x = \boxed{x}$

(c) $y'z + x'yz + xyz$

$= y'z + yz(x'+x) = y'z + yz = z(y'+y) = z$

$F = (x+y)(x'+y+z)(x'+y+z)$

(d) $= (x+y)(x'+y+z)$ [as $AA = A$]

$= \cancel{xx'} + xy + xz + x'y + y.y + y.z$

$= xy + xz + x'y + y + yz$ [as $y.y = y$]

$= xy + xz + y + y.z$ [as $y + x'y = y$]

$= y + xy + xz$

$= y + xz$. [as $y + xy = y$]

Ans

(e) $x + xyz + x'yz + x'y + wx + wx'$

$\Rightarrow x + yz(x+x') + x'y + w(x+x')$

$\Rightarrow x + yz + x'y + w$

$\Rightarrow x + x'y + yz + w \Rightarrow x + y + yz + w \Rightarrow \boxed{x + y + w}$

Q5. We can perform logical operations on strings of bits by considering each pair of bits separately (called bitwise operation). (10 points)

Given two strings A and B

A = 10110011

B = 00100111

Perform the bitwise operation using the following functions

(a) NAND

(b) NOR

(c) XOR

Answers:

(a)

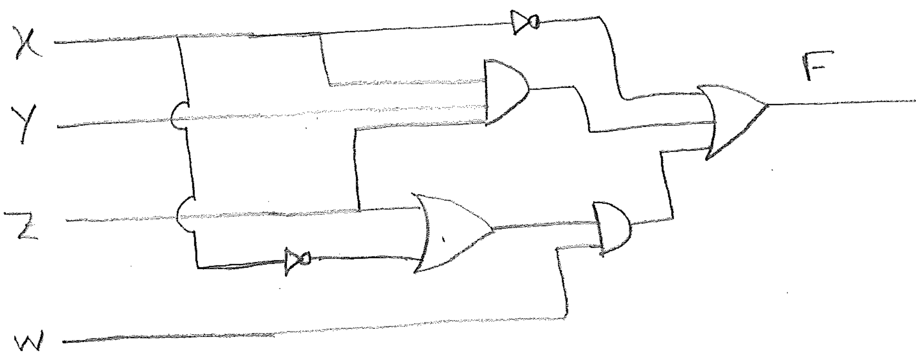
NAND → 1101 1100

(b) NOR → 0100 1000

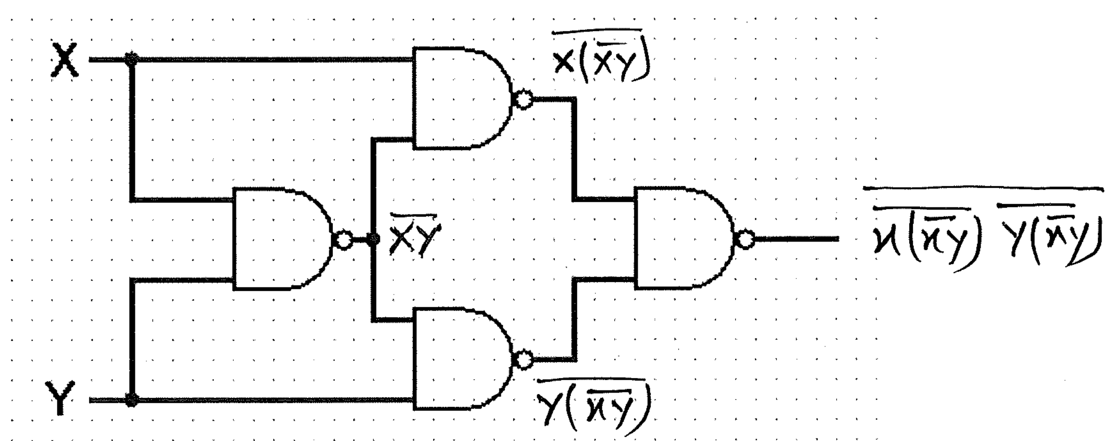
(c) XOR → 1001 0100

Q6. (5*2 = 10 points)

(a) Draw the logic diagram for F = X' + XYZ + W(X'+Z)



(b) Obtain the Reduced Boolean expression for the following circuit diagram (ignore coloring)



$$\overline{\overline{x(\overline{xy})}\ \overline{y(\overline{xy})}}$$

$$\overline{\overline{x(\overline{xy})}} + \overline{\overline{y(\overline{xy})}}$$

$$x(\overline{xy}) + y(\overline{xy})$$

$$x(\bar{x}+\bar{y}) + y(\bar{x}+\bar{y})$$

$$x\bar{x} + x\bar{y} + \bar{x}y + y\bar{y}$$

$$x\cancel{\bar{y}+\bar{y}}\ \ x\bar{y} + \bar{x}y$$

Q7. Use DeMorgan's theorem to simplify the following expressions: (3*2 = 6 points)

(a) ((A' + B)' (C'+D')')'

(b) ((AB'C)' + (AB')')'

(a)

$$\overline{\overline{(\bar{A}+B)}\;\overline{(\bar{C}+\bar{D})}}$$

$$(\overline{(A\bar{B})\,(CD)})$$

$$\bar{A}+B+\bar{C}+\bar{D}$$

(b)

$$\overline{\overline{(A\bar{B}C)}+\overline{(A\bar{B})}}$$

$$\overline{(\bar{A}+B+\bar{C})+(\bar{A}+B)}$$

$$(A\bar{B}C)\,(A\bar{B})$$

$$A\bar{B}C$$

Q8. Kyle, Patrick, Jorge and Steven are hungry college students. They want a quicker way to decide where to go for lunch, the Marshall Center or Juniper. The majority wins, except when Jorge and Steven both agree, then they win. Any other ties end with a trip to Juniper. What would be the design of the logic circuit that automatically selects the restaurant when everyone votes? Show truth table, minimized Boolean expression, and circuit diagram. (10 points)

Kyle A	Patrick B	Steven C	Jorge D	Place to eat
0	0	0	0	0
0	0	0	1	0
0	0	1	0	0
0	0	1	1	1
0	1	0	0	0
0	1	0	1	0
0	1	1	0	0
0	1	1	1	1
1	0	0	0	0
1	0	0	1	0
1	0	1	0	0
1	0	1	1	1
1	1	0	0	0
1	1	0	1	1
1	1	1	0	1
1	1	1	1	1

$$\bar{A}\bar{B}CD + \bar{A}BCD + A\bar{B}CD + AB\bar{C}D + ABC\bar{D} + ABCD$$

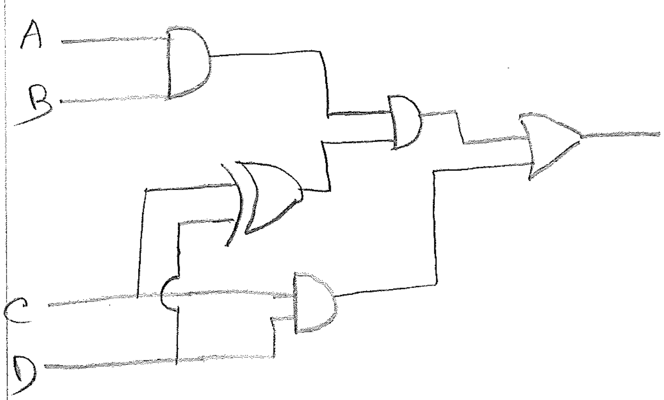
$$CD(\bar{A}\bar{B} + \bar{A}B + A\bar{B} + AB) + AB(\bar{C}D + C\bar{D})$$

$$CD(\bar{A}(\cancel{\bar{B}+B}) + A(\cancel{\bar{B}+B})) + AB(\bar{C}D + C\bar{D})$$

$$CD(\bar{A}+A) + AB(\bar{C}D + C\bar{D})$$

$$CD + AB(\bar{C}D + C\bar{D})$$

$$CD + AB(C \oplus D)$$



9. Draw the schematic for the following functions using NOR gates only:

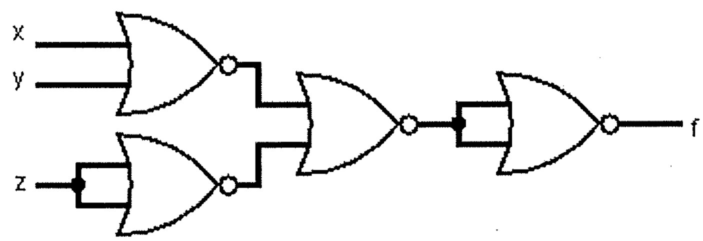
a.

$$\overline{(x+y)}+\overline{z}$$

$$=\overline{x+y}+\overline{z+z}\ (Idempotency\,Theorem)$$

$$=\overline{\overline{\overline{x+y}+\overline{z+z}}}\,(Involution\,Theorem)$$

$$=\overline{\overline{\overline{\overline{x+y}+\overline{z+z}}+\overline{\overline{x+y}+\overline{z+z}}}}\ (Idempotency\,Theorem)$$



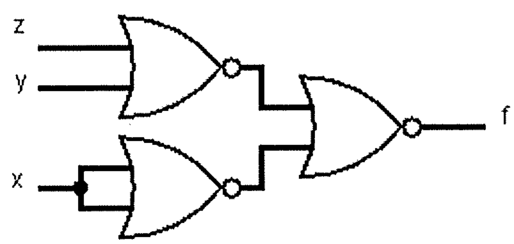
b.

$$xy+xz$$

$$=x(y+z)\ (Distributive\,Law)$$

$$=\overline{\overline{x(y+z)}}\ (Involution\,Theorem)$$

$$=\overline{\overline{x}+\overline{(y+z)}}\ (DeMorgan's\,Theorem)$$



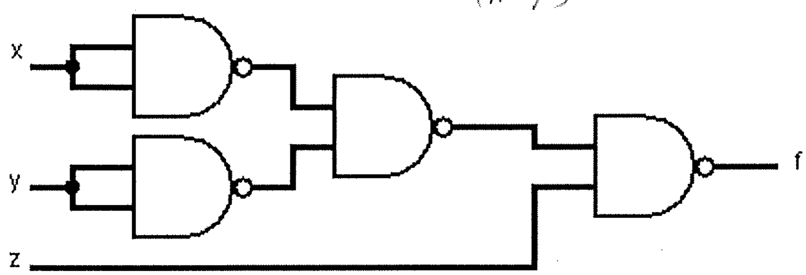
9. Draw the schematic for the following function using NAND gate only:

a.

$$\overline{(x+y)} + \overline{z}$$

$$=\overline{\overline{(\overline{x}\bullet\overline{y})+\overline{z}}} \quad (DeMorgan's\ Theorem\ and\ Involution\ Theorem)$$

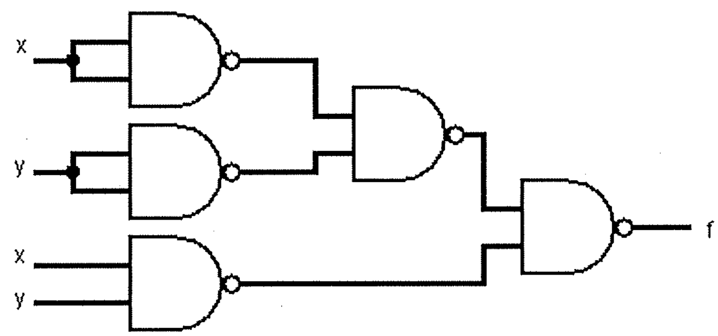
$$=\overline{\overline{(\overline{x}\bullet\overline{y})}\bullet \overline{\overline{z}}} \quad (DeMorgan's\ Theorem) \quad = \overline{(\overline{x}\bullet\overline{y})\bullet z}$$



b. $xy + \overline{x}\,\overline{y}$

$$=\overline{\overline{xy+\overline{x}\overline{y}}} \quad (Involution\ Theorem)$$

$$=\overline{\overline{xy}\bullet\overline{\overline{x}\overline{y}}} \quad (DeMorgan's\ Theorem)$$



10. Determine the minimized realization of the following functions in the sum-of-products form:

a. $f(a,b,c,d) = \sum m(1,7,11,13) + \sum d(2,5,14,15)$

	A'b'	A'b	Ab	ab'
c'd'	0	0	0	0
c'd	1	X	1	0
cd	0	1	X	1
cd'	X	0	X	0

$'a'$ *is the most significant bit and* $'d'$ *is the least significant bit*

$f(a,b,c,d) = \bar{a}\bar{c}d + bd + acd$

b. $f(a,b,c,d) = \prod M(1,2,11,13,14,15) + \sum d(6,7,10)$

	A'b'	A'b	Ab	ab'
c'd'	1	1	1	1
c'd	0	1	0	1
cd	1	X	0	0
cd'	0	X	0	X

$'a'$ *is the most significant bit and* $'d'$ *is the least significant bit*

$f(a,b,c,d) = \bar{c}\bar{d} + \bar{a}b + \bar{a}cd + a\bar{b}\bar{c}$