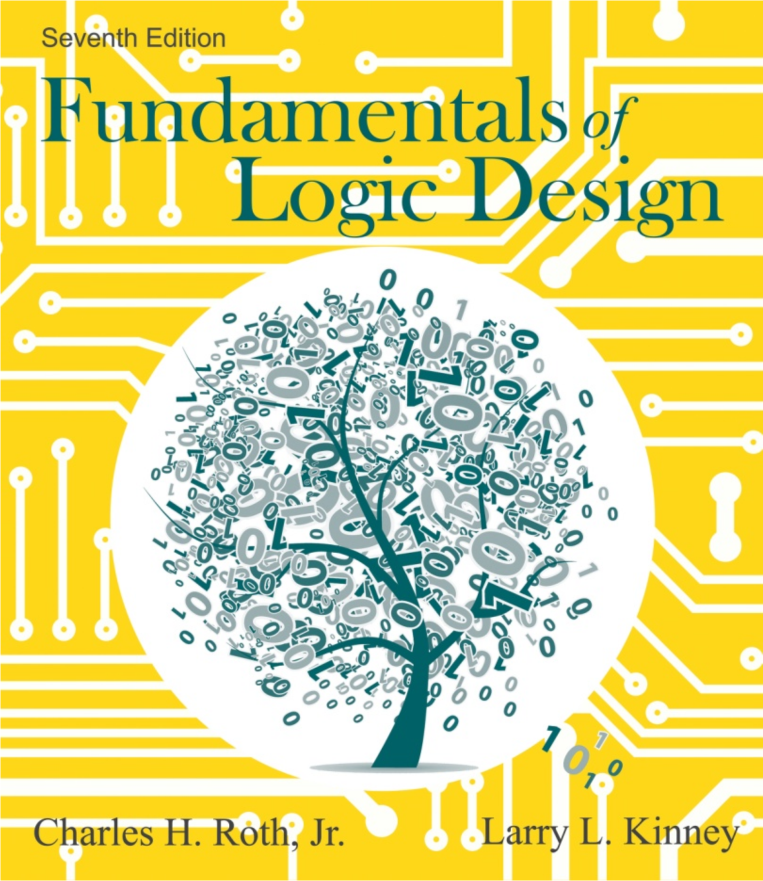

UNIT 6

Quine-McCluskey Method

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This chapter includes:

6.1 Determination of Prime Implicants
6.2 The Prime Implicant Chart
6.3 Petrick’s Method
6.4 Simplification of Incompletely Specified Functions
6.5 Simplification Using Map-Entered Variables
6.6 Conclusion

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Learning Objectives

1. Find the prime implicants of a function by using the Quine-McCluskey method. Explain the reasons for the procedures used.
2. Define *prime implicant and essential prime implicant.*
3. Given the prime implicants, find the essential prime implicants and a minimum sum-of-products expression for a function, using a prime implicant chart and using Petrick's method.
4. Minimize an incompletely specified function, using the Quine-McCluskey method.
5. Find a minimum sum-of-products expression for a function, using the method of map-entered variables.

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Introduction

Chapter Introduction:

- The Quine-McCluskey method presented in this unit provides a systematic simplification procedure which can be readily programmed for a digital computer.
- The Quine-McCluskey method reduces the minterm expansion (standard sum-of-products form) of a function to obtain a minimum sum of products.

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Introduction

Chapter Introduction (continued):

- The Quine-McCluskey procedure consists of two main steps:

  1. Eliminate as many literals as possible from each term by systematically applying

     the theorem $XY + XY = X$. *The resulting terms are called prime implicants.*

  2. Use a prime implicant chart to select a minimum set of prime implicants which,

     when ORed together, are equal to the function being simplified and which contain a minimum number of literals.

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Determination of Prime Implicants

Forming Prime Implicants:

- In the first part of the Quine-McCluskey method, all of the prime implicants of a function are systematically formed by combining minterms.
- The minterms are represented in binary notation and combined using

$$XY + XY' = X$$

where X represents a product of literals and Y is a single variable.

- Two minterms will combine if they differ in exactly one variable.

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Determination of Prime Implicants

Determining Prime Implicants:

To reduce the required number of comparisons, the binary minterms are sorted into groups according to the number of 1’s in each term. Thus,

$$f(a, b, c, d) = \Sigma\, m(0, 1, 2, 5, 6, 7, 8, 9, 10, 14) \tag{6-2}$$

is represented by the following list of minterms:

group 0	0	0000
group 1	1	0001
	2	0010
	8	1000
group 2	5	0101
	6	0110
	9	1001
	10	1010
group 3	7	0111
	14	1110

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Determination of Prime Implicants

Determination of Prime Implicants:

TABLE 6-1
Determination of Prime Implicants

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	Column I		Column II			Column III	
group 0	0	0000 ✓	0, 1	000– ✓		0, 1, 8, 9	–00–
group 1	1	0001 ✓	0, 2	00–0 ✓		0, 2, 8, 10	–0–0
	2	0010 ✓	0, 8	–000 ✓		~~0, 8, 1, 9~~	~~–00–~~
	8	1000 ✓	1, 5	0–01		~~0, 8, 2, 10~~	~~–0–0~~
group 2	5	0101 ✓	1, 9	–001 ✓		2, 6, 10, 14	-- 10
	6	0110 ✓	2, 6	0–10 ✓		~~2, 10, 6, 14~~	~~-- 10~~
	9	1001 ✓	2, 10	–010 ✓			
	10	1010 ✓	8, 9	100– ✓			
group 3	7	0111 ✓	8, 10	10–0 ✓			
	14	1110 ✓	5, 7	01–1			
			6, 7	011–			
			6, 14	–110 ✓			
			10, 14	1–10 ✓			

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Determination of Prime Implicants

Implicant and Prime Implicant:

Given a function F of n variables, a product term P is an *implicant* of F iff for every combination of values of the n variables for which $P = 1$, F is also equal to 1.

A *prime implicant* of a function F is a product term implicant which is no longer an implicant if any literal is deleted from it.

The Quine-McCluskey method finds all of the product term implicants of a function. The implicants which are nonprime are checked off

in the process of combining terms so that the remaining terms are prime implicants.

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The Prime Implicant Chart

Prime Implicant Chart:

- The prime implicant chart can be used to select a minimum set of prime implicants.
- The minterms of the function are listed across the top of the chart, and the prime implicants are listed down the side.
- A prime implicant is equal to a sum of minterms, and the prime implicant is said to cover these minterms.

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The Prime Implicant Chart

Prime Implicant Chart:

- If a prime implicant covers a given minterm, an X is placed at the intersection of the corresponding row and column.
- Table 6-2 shows the prime implicant chart derived from Table 6-1. All of the prime implicants (terms which have not been checked off in Table 6-1) are listed on the left.
- If a given column contains only one X, then the corresponding row is an essential prime implicant.
- See next slide for Table 6-2.

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The Prime Implicant Chart

Prime Implicant Chart:

TABLE 6-2
Prime Implicant Chart
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		0	1	2	5	6	7	8	9	10	14
(0, 1, 8, 9)	$b'c'$	X	X					X	⊗		
(0, 2, 8, 10)	$b'd'$	X		X				X		X	
(2, 6, 10, 14)	cd'			X		X				X	⊗
(1, 5)	$a'c'd$		X		X						
(5, 7)	$a'bd$				X		X				
(6, 7)	$a'bc$					X	X				

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The Prime Implicant Chart

Prime Implicant Chart:

- Each time a prime implicant is selected for inclusion in the minimum sum, the corresponding row should be crossed out. After doing this, the columns which correspond to all minterms covered by that prime implicant should also be crossed out.
- Table 6-3 shows the resulting chart when the essential prime implicants and the corresponding rows and columns of Table 6-2 are crossed out.

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The Prime Implicant Chart

Prime Implicant Chart:

TABLE 6-3

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		0	1	2	5	6	7	8	9	10	14
(0, 1, 8, 9)	$b'c'$	X	X					X	X		
(0, 2, 8, 10)	$b'd'$	X		X				X		X	
(2, 6, 10, 14)	cd'			X		X				X	X
(1, 5)	$a'c'd$		X		X						
(5, 7)	$a'bd$				X		X				
(6, 7)	$a'bc$					X	X				

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The Prime Implicant Chart

Example 1:

A prime implicant chart which has two or more X's in every column is called a *cyclic* prime implicant chart. The following function has such a chart:

$$F = \Sigma\, m(0, 1, 2, 5, 6, 7) \tag{6-6}$$

Derivation of prime implicants:

0	000	✓	0, 1	00–
1	001	✓	0, 2	0–0
2	010	✓	1, 5	–01
5	101	✓	2, 6	–10
6	110	✓	5, 7	1–1
7	111	✓	6, 7	11–

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The Prime Implicant Chart

Example 1 (continued):

Table 6-4 shows the resulting prime implicant chart. All columns have two X's, so we will proceed by trial and error. Both (0, 1) and (0, 2) cover column 0, so we will try (0, 1). After crossing out row (0, 1) and columns 0 and 1, we examine column 2, which is covered by (0, 2) and (2, 6). The best choice is (2, 6) because it covers two of the remaining columns while (0, 2) covers only one of the remaining columns. After crossing out row (2, 6) and columns 2 and 6, we see that (5, 7) covers the remaining columns and completes the solution. Therefore, one solution is $F = a'b' + bc' + ac$.

TABLE 6-4

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				0	1	2	5	6	7
①	→	(0, 1)	$a'b'$	X	X				
		(0, 2)	$a'c'$	X		X			
		(1, 5)	$b'c$		X		X		
②	→	(2, 6)	bc'			X		X	
③	→	(5, 7)	ac				X		X
		(6, 7)	ab					X	X

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The Prime Implicant Chart

Example 1 (continued):

However, we are not guaranteed that this solution is minimum. We must go back and solve the problem over again starting with the other prime implicant that covers column 0. The resulting table (Table 6-5) is

TABLE 6-5

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			0	1	2	5	6	7
P_1	(0, 1)	$a'b'$	×	×				
P_2	(0, 2)	$a'c'$	×		×			
P_3	(1, 5)	$b'c$		×		×		
P_4	(2 6)	bc'			×		×	
P_5	(5, 7)	ac				×		×
P_6	(6, 7)	ab					×	×

Finish the solution and show that $F = a'c' + b'c + ab$. Because this has the same number of terms and same number of literals as the expression for F derived in Table 6-4, there are two minimum sum-of-products solutions to this problem. Compare these two minimum solutions for Equation (6-6) with the solutions obtained in Figure 5-9 using Karnaugh maps. Note that each minterm on the map can be covered by two different loops. Similarly, each column of the prime implicant chart (Table 6-4) has two X's, indicating that each minterm can be covered by two different prime implicants.

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Petrick’s Method

General procedure:

1. Reduce the prime implicant chart by eliminating the essential prime implicant rows and the corresponding columns.
2. Label the rows of the reduced prime implicant chart P_1, P_2, P_3, etc.
3. Form a logic function P which is true when all columns are covered. P consists of a product of sum terms, each sum term having the form $(P_{i0} + P_{i1} + \cdots)$, where $P_{i0}, P_{i1} \ldots$ represent the rows which cover column i.
4. Reduce P to a minimum sum of products by multiplying out and applying $X + XY = X$.

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Petrick's Method

General procedure (continued):

5. Each term in the result represents a solution, that is, a set of rows which covers all of the minterms in the table. To determine the minimum solutions (as defined in Section 5.1), find those terms which contain a minimum number of variables. Each of these terms represents a solution with a minimum number of prime implicants.
6. For each of the terms found in step 5, count the number of literals in each prime implicant and find the total number of literals. Choose the term or terms which correspond to the minimum total number of literals, and write out the corresponding sums of prime implicants.

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Simplification of Incompletely Specified Functions

Handling "Don't Cares":

- In this section, we will show how to modify the Quine-McCluskey method in order to obtain a minimum solution when don't-care terms are present.
- In the process of finding the prime implicants, we will treat the don't-care terms as if they were required minterms.
- When forming the prime implicant chart, the "don't cares" are not listed at the top.

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Simplification of Incompletely Specified Functions

Example 2:

$$F(A, B, C, D) = \Sigma\, m(2, 3, 7, 9, 11, 13) + \Sigma\, d(1, 10, 15)$$

(the terms following d are don't-care terms)

The don't-care terms are treated like required minterms when finding the prime implicants:

1	0001	✓
2	0010	✓
3	0011	✓
9	1001	✓
10	1010	✓
7	0111	✓
11	1011	✓
13	1101	✓
5	1111	✓

(1, 3)	00–1	✓
(1, 9)	–001	✓
(2, 3)	001–	✓
(2, 10)	–010	✓
(3, 7)	0–11	✓
(3, 11)	–011	✓
(9, 11)	10–1	✓
(9, 13)	1–01	✓
(10, 11)	101–	✓
(7, 15)	–111	✓
(11, 15)	1–11	✓
(13, 15)	11–1	✓

(1, 3, 9, 11)	–0–1
(2, 3, 10, 11)	–01–
(3, 7, 11, 15)	- -11
(9, 11, 13, 15)	1- -1

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Simplification of Incompletely Specified Functions

Example 2 (continued):

The don't-care columns are omitted when forming the prime implicant chart:

	2	3	7	9	11	13
(1, 3, 9, 11)		X		X	X	
*(2, 3, 10, 11)	X	X			X	
*(3, 7, 11, 15)		X	X		X	
*(9, 11, 13, 15)				X	X	X

*Indicates an essential prime implicant.

$$F = B'C + CD + AD$$

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Simplification of Incompletely Specified Functions

Example 2 (continued):

Note that although the original function was incompletely specified, the final simplified expression for F is defined for all combinations of values for A, B, C, and D and is therefore completely specified. In the process of simplification, we have automatically assigned values to the don't-cares in the original truth table for F. If we replace each term in the final expression for F by its corresponding sum of minterms, the result is

$$F = (m_2 + m_3 + m_{10} + m_{11}) + (m_3 + m_7 + m_{11} + m_{15}) + (m_9 + m_{11} + m_{13} + m_{15})$$

Because m_{10} and m_{15} appear in this expression and m_1 does not, this implies that the don't-care terms in the original truth table for F have been assigned as follows:

for $ABCD = 0001$, $F = 0$; for 1010, $F = 1$; for 1111, $F = 1$

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Simplification Using Map-Entered Variables

- By using map-entered variables, Karnaugh map techniques can be extended to simplify functions with more than four or five variables.
- Figure 6-1(a) shows a four-variable map with two additional variables entered in the squares in the map.

FIGURE 6-1 Use of Map-Entered Variables

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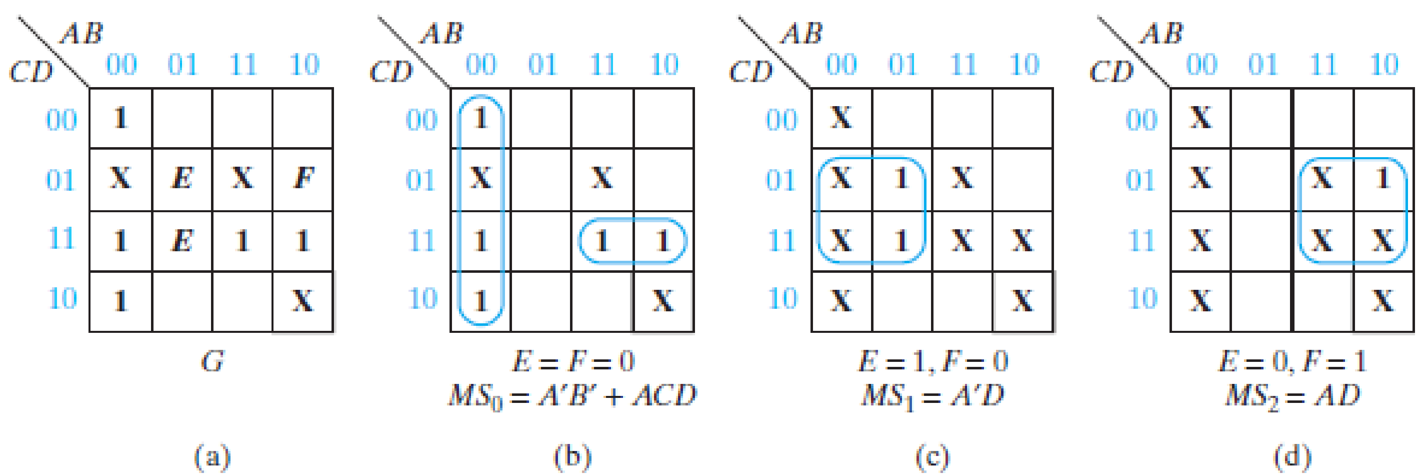

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Simplification Using Map-Entered Variables

- When E appears in a square, this means that if E=1, the corresponding minterm is present in the function G, and if E=0, the minterm is absent. Thus the map represents the six-variable function

$$G(A, B, C, D, E, F) = m_0 + m_2 + m_3 + Em_5 + Em_7 + Fm_9 + m_{11} + m_{15}$$
$$(+\text{don't-care terms})$$

where the minterms are the minterms of variables A,B,C, and D. Note that m_9 is only present in G when F=1.

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Simplification Using Map-Entered Variables

- To simplify the following function,

$$F(A,B,C,D) = A'B'C + A'BC + A'BC'D + ABCD + (AB'C)$$

D is chosen as the map-entered variable. See page 183 for further explanation.

FIGURE 6-2 Simplification Using a Map-Entered Variable

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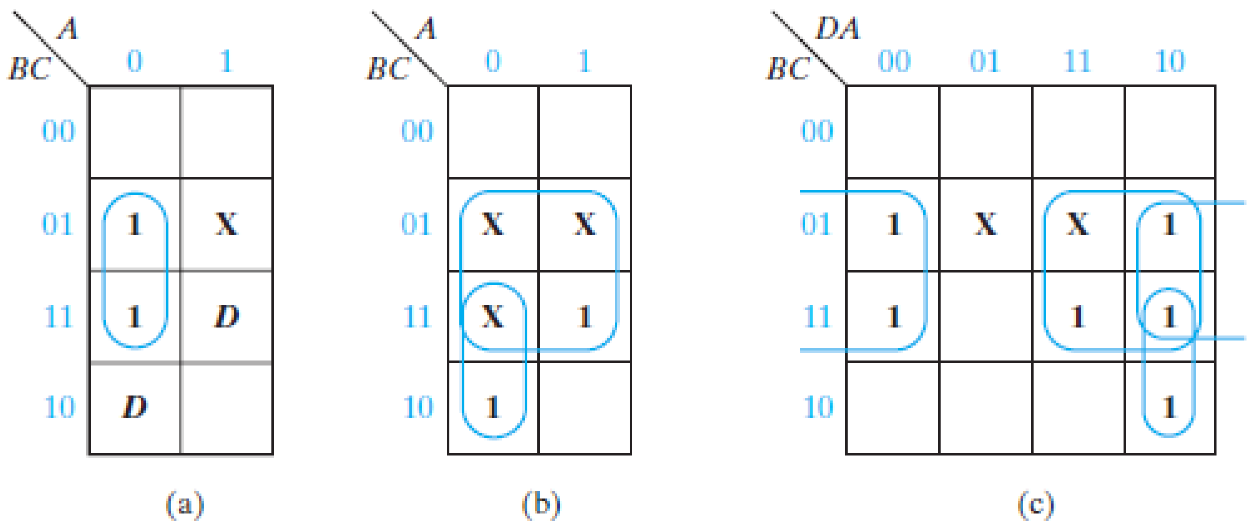

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Simplification Using Map-Entered Variables

General Method of Simplifying Expressions Using Map-Entered Variables:

Find a sum-of-products expression for F of the form

$$F = MS_0 + P_1MS_1 + P_2MS_2 + \cdots$$

where

MS_0 is the minimum sum obtained by setting $P_1 = P_2 = \cdots = 0$.

MS_1 is the minimum sum obtained by setting $P_1 = 1$, $P_j = 0$ $(j \neq 1)$, and replacing all 1's on the map with don't-cares.

MS_2 is the minimum sum obtained by setting $P_2 = 1$, $P_j = 0$ $(j \neq 2)$ and replacing all 1's on the map with don't-cares.

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