es. 1) Sia:

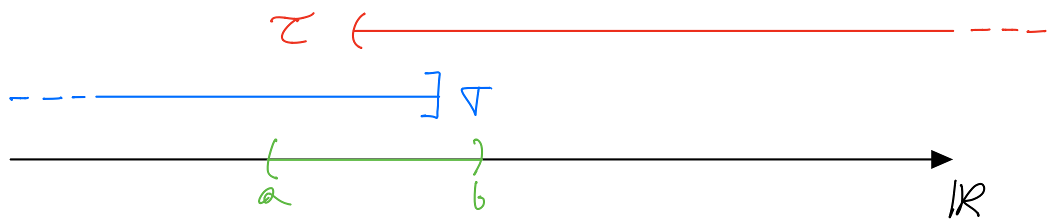
$$X = \mathbb{R}, \quad \tau = \{\emptyset, X = \mathbb{R}\} \cup \{(a, +\infty) \mid a \in \mathbb{R}\}$$

$$\Rightarrow \nabla = \{(-\infty, a] \mid a \in \mathbb{R}\} \cup \{\emptyset, \mathbb{R}\}$$

$\Rightarrow$ Determinare chiusura ed interno dei seguenti insiemi:

1) $Y = (a, b)$
2) $Y = [c, +\infty)$
3) $Y = (-\infty, d]$
4) $Y = \{0\}$

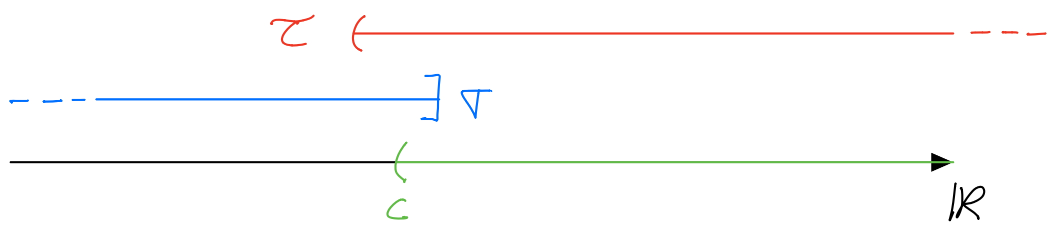
$\Rightarrow$ 1)



$\Rightarrow \mathring{Y} = \emptyset$ poiché è l'unico aperto contenuto in (a, b)

$\Rightarrow \overline{Y} = (-\infty, b]$ è il più piccolo chiuso contenente (a, b)

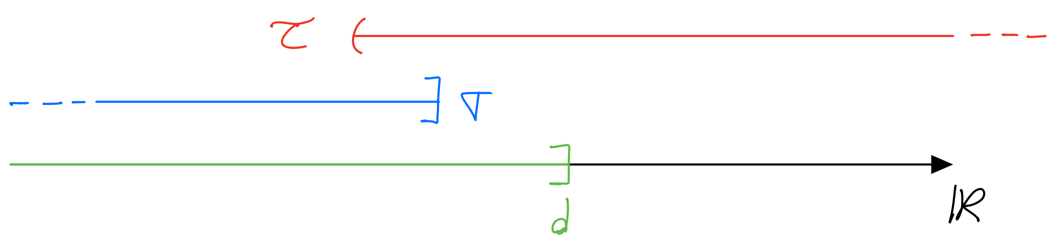
$\Rightarrow$ 2)



$\Rightarrow \mathring{Y} = Y = (c, +\infty) \quad (Y \in \tau \Rightarrow Y = \mathring{Y})$

$\Rightarrow \overline{Y} = \mathbb{R}$ (poiché è l'unico chiuso contenente Y)

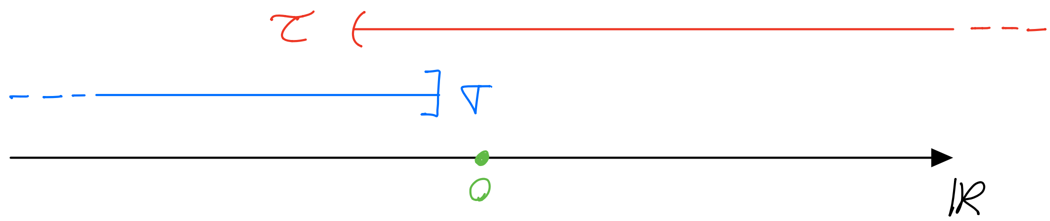
$\Rightarrow$ 3)



$\Rightarrow \mathring{Y} = \emptyset$

$\Rightarrow \overline{Y} = Y = (-\infty, d] \quad (Y \in \mathcal{T} \Rightarrow \overline{Y} = Y)$

$\Rightarrow$ 4)



$\Rightarrow \mathring{Y} = \emptyset \subseteq \{0\}$

$\Rightarrow \overline{Y} = (-\infty, 0]$

es. 2) Sia:

$X = \mathbb{R}, \quad \tau = \{A \mid \forall x \in A \ \exists y > x \text{ t.c. } [x, y) \subseteq A\} \cup \{\emptyset, X\}$

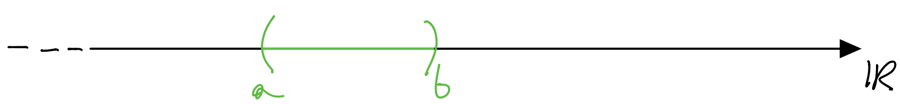
τ contiene gli insiemi ottenuti come unione di intervalli e semirette con estremo destro escluso.

$\mathcal{T}$ contiene gli insiemi ottenuti come unione di intervalli e semirette con estremo sinistro incluso.

Determinare interno e chiusura dei seguenti insiemi:

$$\left.\begin{array}{ll} 1) & (a, b) \\ 2) & [a, b) \\ 3) & (a, b] \\ 4) & [a, b] \end{array}\right\} \text{ con } a < b$$

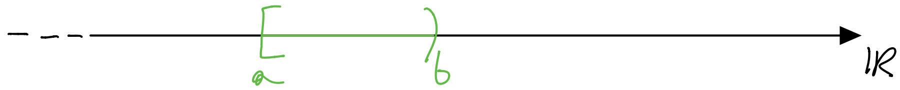
$\Rightarrow$ 1)



$$\Rightarrow \quad \overset{\circ}{(a,b)} = (a,b) \quad ((a,b) \in \tau)$$

$$\Rightarrow \quad \overline{(a,b)} = [a,b)$$

$\Rightarrow$ 2)



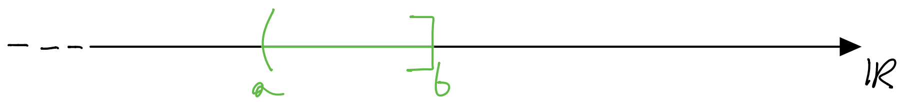
$$\Rightarrow \quad \overset{\circ}{[a,b)} = [a,b) \quad ([a,b) \in \tau)$$

$$\Rightarrow \quad \overline{[a,b)} = [a,b) \quad ([a,b) \in \mathcal{T})$$

<u>Oss.</u>

$[a,b) \in \tau \cap \mathcal{T}$ ($[a,b)$ è sia aperto che chiuso)

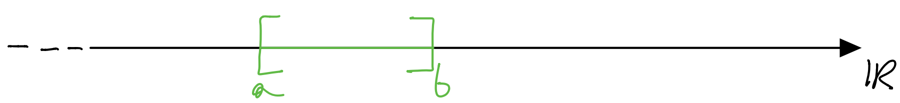
$\Rightarrow$ 3)



$$\Rightarrow \quad \overset{\circ}{(a,b]} = (a,b)$$

$$\Rightarrow \quad \overline{(a,b]} = [a,b]$$

$\Rightarrow$ 4)



$$\Rightarrow \quad \overset{\circ}{[a,b]} = [a,b)$$

$$\Rightarrow \quad \overline{[a,b]} = [a,b] \quad ([a,b] \in \mathcal{T})$$