Fundamentals of LTI Systems

Xinyang He

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The Laplace transform of integral control is

$$H_I(s) = \frac{U_I(s)}{E(s)} = \frac{k_I}{s}$$

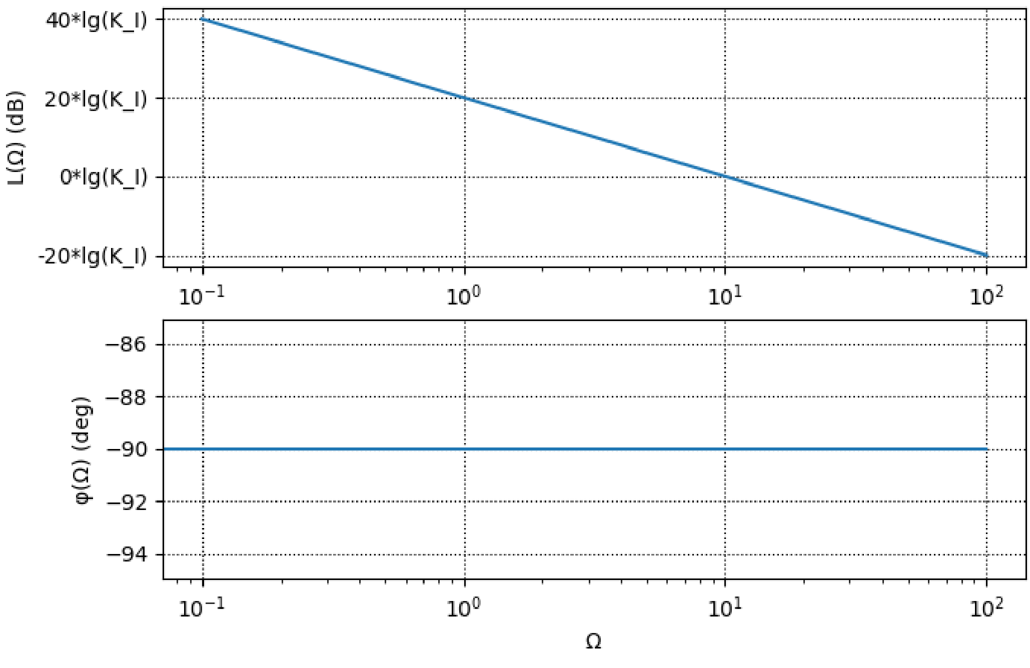
And we have

$$H_I(j\omega) = \frac{k_I}{j\omega} \Rightarrow |H_I(j\omega)| = \frac{k_I}{\omega}$$

So

$$20\log_{10}|H_I(j\omega)| = 20\log_{10}k_I - 20\log_{10}\omega$$

The bode plot is



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$$\begin{cases} sY_D(s) = \omega_d(E(s) - Y_D(s)) \\ u_D(s) = k_D s Y_D(s) \Rightarrow Y_D(s) = \frac{u_D(s)}{k_D s} \end{cases}$$

$$H(s) = \frac{u_D(s)}{E(s)} = \frac{k_D\omega_d s}{s + \omega_d}$$

$$\Rightarrow \text{Closed-loop characteristic equation is } D(s) = (k_D\omega_d + 1)s + \omega_d$$

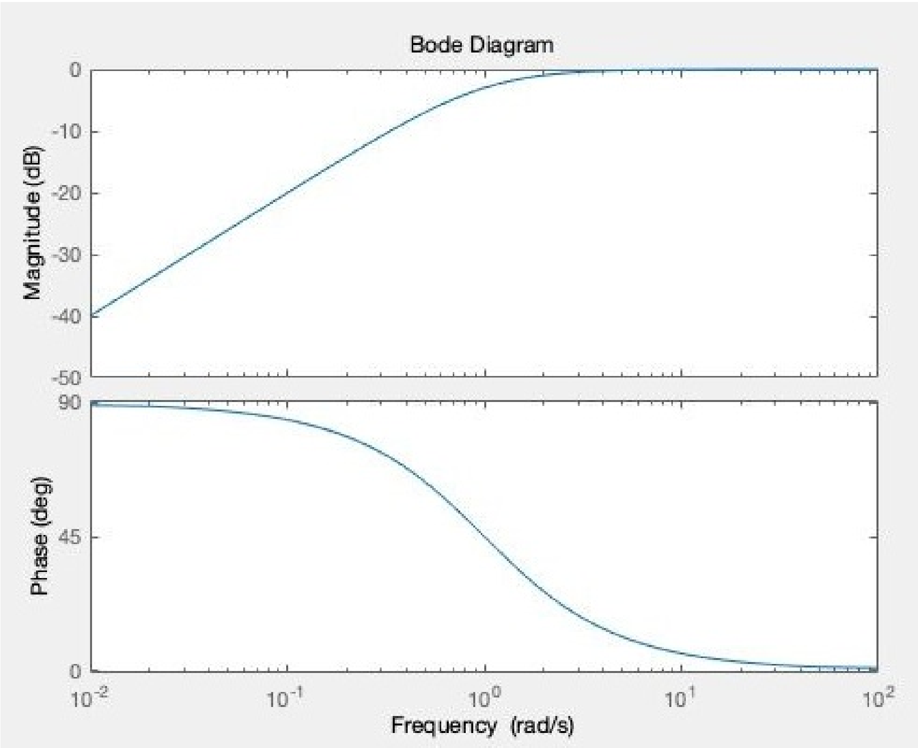
Based on Routh–Hurwitz stability criterion

$$\begin{cases} \omega_d > 0 \\ k_D\omega_d + 1 > 0 \end{cases} \Rightarrow \omega_d > 0$$

Therefore, ω_d should larger than 0, and ω_d don't have the limitation of maximum value.

$$\begin{aligned} H(j\omega) &= 90^\circ - \arctan\frac{1}{\omega_d}\omega \\ &= \begin{cases} \omega \to 0^+, & H(j\omega) = 90^\circ \\ \omega \to +\infty, & H(j\omega) = 0^\circ \end{cases} \end{aligned}$$

In the bode plot, the gradient of low frequency band is $20dB/dec$. And the line crosses the dot $(1, 20\log_{10}(k_D\omega_d))$. When the frequency is larger or equal to $1/\omega_d$, the gradient becomes 0.



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The z-transform of $H_I(z)$ is

$$s = \frac{2}{T_s}\left(\frac{1-z^{-1}}{1+z^{-1}}\right)$$

$$H_I(z) = \frac{u_I(s)}{E(s)}\Big|_{s=\frac{2}{T_s}(\frac{1-z^{-1}}{1+z^{-1}})} = \frac{k_I}{s}\Big|_{s=\frac{2}{T_s}(\frac{1-z^{-1}}{1+z^{-1}})} = \frac{k_I T_s(z+1)}{2(z-1)} = \frac{k_I T_s(1+z^{-1})}{2(1-z^{-1})}$$

$$\begin{aligned} \Rightarrow \quad 2u_I(z) - 2z^{-1}u_I(z) &= k_I T_s E(z) + k_I T_s z^{-1}E(z) \\ \Rightarrow \quad 2u_I(k) - 2u_I(k-1) &= k_I T_s e(k) + k_I T_s z^{-1} e(k-1) \end{aligned}$$

The resulting expression is

$$u_I(k) = \frac{k_I T_s}{2}e(k) + \frac{k_I T_s}{2}e(k-1) + u_I(k-1)$$

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Please check the attached files to see the MATLAB code.

$$
\begin{aligned}
u(k) &= u_p(k) + u_I(k) \\
&= (k_p + \frac{k_I T_s}{2})e(k) + \frac{k_I T_s}{2} e(k-1) + u_I(k-1)
\end{aligned}
$$

The simulation figure is shown as follow: