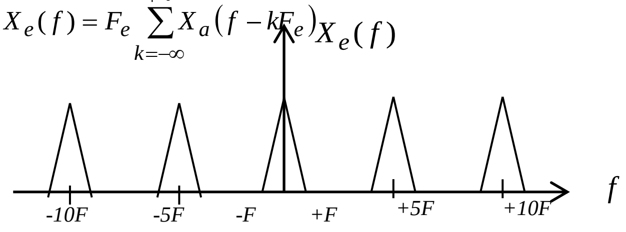
DIGITAL SIGNAL PROCESSING - CORRECTION TD N°1 -

EXERCISE N°1

$$X_e(f) = F_e \sum_{k=-\infty}^{+\infty} X_a\left(f - kF_e\right)$$



The sampled signal is written :

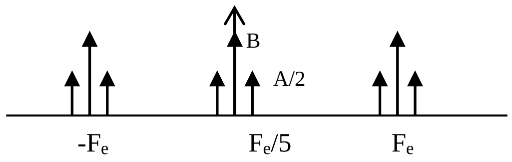
$$x(nT_e) = A\cos\left(2\pi f_0 nT_e\right) + B$$

The Fourier Transform is written :

$$X_a(f) = \frac{A}{2}\left[\delta\left(f - f_0\right) + \delta\left(f + f_0\right)\right] + B\delta\left(f\right)$$

The Discrete Fourier Transform is expressed :

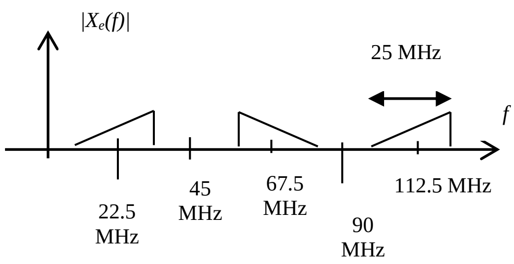
$$X_e(f) = \frac{A}{2}\left[W_{F_e}\left(f - f_0\right) + W_{F_e}\left(f + f_0\right)\right] + BW_{F_e}\left(f\right)$$



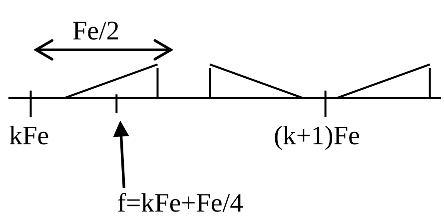
EXERCISE N°2

1. The Shannon theorem is respected because $F_e > 2B$ ($90\ MHz > 50\ MHz$)

2.



3.
The solution is not valid for 100 MHz and Fe=50 MHz.
A sufficient condition by down-sampling at $F_e > 2B$ and $f = kF_e + \frac{F_e}{4}$

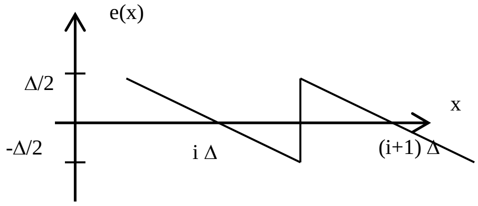


EXERCISE N°3

the maximal frequency value is 22 kHz. Thus, the minimal sampling frequency is 44 kHz. If we consider two microphones at 44 kHz and 16 bits, we obtain 1.408 Mbits/s. Consequently for 70 minutes, we obtain : 739.2 Mbytes.

Exercise n°4

1)



2) The mean :

$$m_e = \int_{-\infty}^{+\infty} p_e(u)udu = \int_{-\frac{\Delta}{2}}^{+\frac{\Delta}{2}} \frac{1}{\Delta}udu = 0$$

The variance

$$\sigma_e^2 = E\left[e^2\right] = \int_{-\frac{\Delta}{2}}^{+\frac{\Delta}{2}} p_e(u)u^2du = \int_{-\frac{\Delta}{2}}^{+\frac{\Delta}{2}} \frac{1}{\Delta}u^2du = \frac{\Delta^2}{12}$$

1) The signal-to-quantization noise ratio:

$$\Gamma = 10\log_{10}\left(\frac{12\sigma_x^2 2^{2b}}{A^2}\right),$$ d'où

$$\Gamma = 6.02b + 20\log_{10}\left(\frac{\sigma_x}{A}\right) + 10.8$$

6.02 dB per additional bit.

5) For a sinusoidal signal, the amplitude (Peak value) la is $\sqrt{2}\sigma_x$ and the condition of non-saturation can be written : $\sqrt{2}\sigma_x < \frac{A}{2}$. Thus, $\Gamma < 97.8\ dB$.

For a Gaussian signal with an estimation of its amplitude of $4\sigma_x$, we obtain: $4\sigma_x < \frac{A}{2}$, and consequently $\Gamma < 88.7\ dB$.