EXERCISE N°1

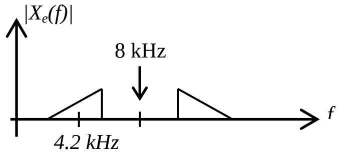
$\frac{F_e}{2} = 8kHz$, consequently $F_e = 16\ kHz$

Thus, $f_1 = f_{1n} \times F_e = 200\ Hz$

$y(nT_e) = x(nT_e)\cos(2\pi f_1 nT_e)$

$y(nT_e) = x(nT_e)\cos\left(\frac{\pi n}{40}\right)$

If we consider a narrow band, and we filter the signal around a frequency of 3.8 kHz, we obtain the following spectrum:



EXERCISE N°2

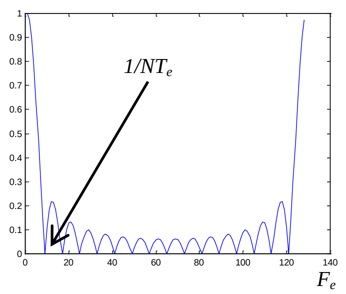
The analog signal has an unbounded spectrum (cardinal sine function), Thus, it is impossible to respect Shannon Theorem.

$$TF(\Pi) = \frac{1}{M}\sum_{i=0}^{N-1} e^{-j2\pi f iT_e}$$

$$= \frac{1}{M}\frac{1-e^{-j2\pi f NT_e}}{1-e^{-j2\pi f T_e}}$$

$$= \frac{1}{M}\frac{e^{-j\pi f NT_e}}{e^{-j\pi f T_e}}\frac{e^{j\pi f NT_e} - e^{-j\pi f NT_e}}{e^{j\pi f T_e} - e^{-j\pi f T_e}}$$

$$TF(\Pi) = \frac{1}{M}e^{-j\pi f(N-1)T_e}\frac{\sin \pi f NT_e}{\sin \pi f T_e}$$



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$$C(f) = \frac{1}{2}(X(f) + X(-f)) = \frac{1}{2}\left(X(f) + X(f)^*\right)$$

$$C(f) = Re\{X(f)\}$$

- In order to have C(f)=X(f), $X(f)$ must be real. Thus, $x(nT_e)$ is symmetric.

It is completely wrong to say that the Fourier transform is real because the signal x(nTe) is symmetric.

We have : $x_0 = x_4 = 0.2$, $x_1 = x_3 = 0.5$, $x_2 = 1$, $N = 5$

$$X\left(\frac{k}{5T_e}\right) = \sum_{n=0}^{4} x_n e^{-j2\pi\frac{nk}{5}T_e} = x_0\left(1 + e^{-j2\pi\frac{4k}{5}}\right)$$

$$+ x_1\left(e^{-j2\pi\frac{k}{5}} + e^{-j2\pi\frac{3k}{5}}\right) + x_2 e^{-j2\pi\frac{2k}{5}}$$

The computed expression is not real.

When we add a zero sample x_0=0, we obtain

$x_0 = 0$, $x_1 = x_5 = 0.2$, $x_2 = x_4 = 0.5$, $x_3 = 1$

$$X\left(\frac{k}{6T_e}\right) = \sum_{n=0}^{5} x_n e^{-j2\pi\frac{nk}{6}T_e} = x_0 + x_3$$

$$+ x_1\left(2\cos\left(\frac{2\pi k}{6}\right)\right) + x_2\left(2\cos\left(\frac{4\pi k}{6}\right)\right)$$

Th result is real.

EXERCISE N°3

The observation duration is = $10\ \mu s$ Thus, the frequency resolution is = $\frac{1}{10\mu s} = 100\ kHz$

An other method to calculate the frequency resolution :

By sampling a signal during $10\ \mu s$ at $10\ MHz$ we obtain 100 samples. Thus, the resolution is :

$\frac{F_e}{N} = \frac{10\ MHz}{100} = 100\ kHz$. Thus, the imprecision is $\frac{1}{2}\frac{F_e}{N}$.

The imprecision on frequency value is expressed as $\Delta f = f_0 \times \frac{\Delta v}{c}$. By using $f_0 = 5\ GHz$, the imprecision on the mobile speed can be calculated as $\Delta v = \pm 3\ km/s = \pm 10.800\ km/h$. This radar gives information about distance and never about the mobile speed.

For the mobile speed, we must observe the received signal much more time in order to decrease the imprecision on the frequency value.