PROBABILITY THEORY – EXERCISES SESSION N° 1

About chapters:

- Chapter 1: NOTION OF EVENT AND OF PROBABILITY
- Chapter 2: CONDITIONAL PROBABILITIES & INDEPENDENCE

TD1 – 1 * :

The Items obtained from a manufacturing chain must pass a pass/fail test. The inspector in charge of quality control examines each article, then classifies it. This process continues until either two consecutive defective items are observed or until three non-defective items are observed. Describe the set Ω of possible outcomes associated with this experiment.

TD1 – 2 :

1) Let *A* and *B* be two independent events associated to a random experiment. Demonstrate that the events $\overline{A}$ and *B* are also independent.

2) Application: Every morning of class, Peter, a student at ISEP, can be the victim of two independent events:

• R: "he does not hear his alarm clock ringing";

• S: “his scooter, poorly maintained, breaks down”.

For a given day of class, the probability of R is 0.1 and that of S is 0.05. If at least one of the two events occurs, Peter is late at ISEP, otherwise he is on time.

a) Calculate the probability that: "a given day Peter hears his alarm clock and his scooter breaks down”.
b) Calculate the probability that “Peter is on time at ISEP”.

TD1 – 3 :

Permutations – Combinations

a) In how many ways can 4 interchangeable tires be placed on 4 wheels of a car? What about 5 tires on 4 wheels?
b) What is the number of different “melodies” (series of different musical notes) of 3 notes that can be made from 8 different notes if no note is repeated?

TD1 – 2 : independence of A&B : $P(A\cap B) = P(A)\cdot P(B)$

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1) $\because$ A&B be independent

$\therefore P(B) = P(B\cap A) + P(B\cap \bar{A}) \Rightarrow P(B) - P(B\cap A) = P(B\cap \bar{A})$

$P(B)\cdot P(A) \uparrow$

for $P(B\cap\bar{A}) = P(B) - P(B)\cdot P(A)$

$= P(B)\cdot[1-P(A)] = P(B)\cdot P(\bar{A})$

$\because P(B\cap\bar{A}) = P(B)\cdot P(\bar{A}) \therefore \bar{A}$&B independent

2a) $\because P(R) = 0.1$, $P(S) = 0.05$ (R&S independent)

$P(\bar{R}\cap S) = P(\bar{R})\cdot P(S)$

$= [1-P(R)]\cdot P(S)$

$= 0.9\times 0.05 = 0.045$

2b) $P(\bar{R}\cap\bar{S}) = [1-P(R)]\cdot[1-P(S)]$

$= 0.9\times 0.95$

$= 0.855$

TD 1-3

a) $P_1 = 4 \times 3 \times 2 \times 1 = 24$

$P_2 = 5 \times 4 \times 3 \times 2 \times 1 = 120$

b) $A_8^3 = \dfrac{876\cancel{54321}}{\cancel{54321}} = 56 \times 6 = 336$

c) $C_{32}^5 = \dfrac{\cancel{32}\ 31\ \cancel{30}\ 29\ 28\ \ 8}{8\cancel{4}\cancel{3}\cancel{2}\ 1} = 201376$

d)

$A_3^3 \times C_3^3 \times C_2^2 \times C_1^1 \qquad \dfrac{6!}{3!\,2!} = \dfrac{654\cancel{321}}{\cancel{321}\ 21} = 60$

4 6
5 5
5 6
6 4
6 5
6 6

TD 1-4.

M: equal ≥ 10 A: first $= 5$ B: at least one $= 5$

$P(M) = \dfrac{6}{C_6^1 \cdot C_6^1} = \dfrac{6}{36} = \dfrac{1}{6}$

$P(A|M) = \dfrac{C_1^1 \times C_2^1}{C_6^1} = \dfrac{1}{3}$

$P(B|M) = \dfrac{C_1^1 \times C_2^1 \times C_2^1 \times C_1^1 \times C_1^1 \times C_1^1}{C_6^1} = \dfrac{2}{3}$?

TD1-5.

a) $P(E) = 0.6$ $\quad P(R) = 0.6 \times 0.8 + 0.4 \times 0.3$

$P(\bar{E}) = 0.4$

$$= 0.48 + 0.12$$

$$= 0.6$$

b) $P(R|E) = 0.8$

d) $P(E|R) = \dfrac{P(R|E) \cdot P(E)}{P(R)} = \dfrac{0.8 \times 0.6}{0.6} = 0.8$

TD1-6

a) $P_1 =$

TD1-8

$H: 0.3$ — $0.25N$, $0.75m$

$M: 0.5$ — $0.4N$, $0.6m$

$L: 0.2$ — $0.8N$, $0.2m$

a) $P_N = 0.3 \times 0.25 + 0.5 \times 0.4 + 0.2 \times 0.8$

$= 0.075 + 0.2 + 0.16 = 0.435$

b) $P_m = C_3^1 \times 0.75 + C_3^1 \times 0.6 + C_3^1 \times 0.2$

$= 0.25 + 0.2 + 0.06 \approx 0.52$

c) $P(M|m) = \dfrac{P(M \cap \bar{N})}{P(\bar{N})} = \dfrac{P(\bar{N}|M) \cdot P(M)}{1 - P(N)}$

$= \dfrac{0.6 \times 0.5}{0.565} = 0.53$