Module IT.2407
Digital Transmission Chain

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I. Digital transmission Chain

The purpose of this problem is to follow the evolution of a message in a digital transmission chain

A Emission

The structure of the transmitter is given in the Figure 1:

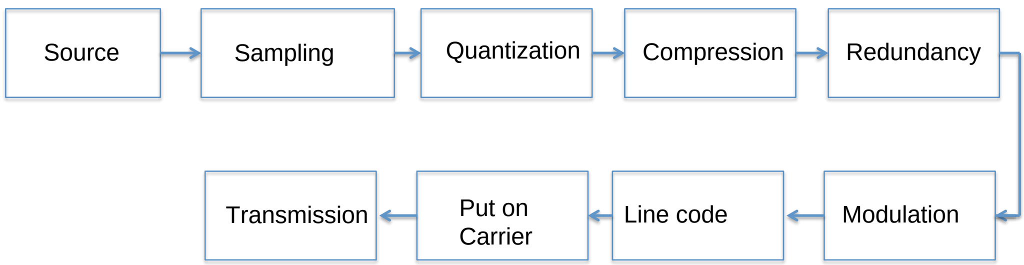


FIGURE 1 – transmission chain

A.1 Sampling and Quantization

We consider a signal x(t) having a limited spectrum between 0 and 4 KHz. It is assumed that the digital processing is performed on sections of the signal x(t) of duration D = 5 ms.

1. Find the minimum sampling period and the number of samples.

We use N = 8 levels of quantization that we designate by N1 ...N8 such that N1 =-N8, N2 =-N7, N3 =-N6 and N4 =-N5. It is assumed that the values of x(t) vary between 0.0032 and -0.0032.

2. Find the quantization step q.
3. Determine the values of the 8 levels according to q.

A.2 Compression des données : codage de source

We assume that for the considered signal, the table of occurrence of the different quantization levels is given in Table 1.

Quantized Values	Occurrence	Uniform Code	Huffman Code
N_1	10		
N_2	5		
N_3	6		
N_4	4		
N_5	2		
N_6	3		
N_7	8		
N_8	2		

TABLE 1 – Occurrence of the quantized values

It is first assumed that a uniform code is used to encode the different quantization levels.

1. Indicate the number of bits per quantization level necessary to perform a uniform coding? Deduce the length of the binary stream obtained in this case.
2. Complete Table 1 with the corresponding uniform coding.

To compress the source (reduce the size of the generated bitstream), we use the Huffman code.

3. Draw the Huffman tree.
4. Complete Table 1 with the corresponding Huffman codes and deduce the average number of bits per quantization level and the Huffman size of the binary stream obtained in this case. Comment on the results obtained.
5. We suppose that the first 4 quantization levels obtained are N_3, N_4, N_2, N_1, N_7, N_8, N_6. Find the corresponding bitstream generated by the Huffman coding.

A.3 Redundancy : error correcting code

We define the following binary code that associates to each combination of k = 4 consecutive information bits b_0, b_1, b_2, b_3 the coded sequence with $n = 8$ bits given by:

$$(b_0, b_1, b_2, b_3) \rightarrow (b_0, b_1, b_2, b_3, b_0 \oplus b_1 \oplus b_2, b_0 \oplus b_1 \oplus b_3, b_1 \oplus b_2 \oplus b_3, b_0 \oplus b_1 \oplus b_2 \oplus b_3)$$

1. Find the coding rate of this code.
2. Complete Table 2 with the binary code associated code, the Hamming weight of each coded sequence.
3. Deduce the minimal Hamming distance, its capacity of correction and detection.

Info Bits	Coded Bits	Weight
0000		
0001		
0010		
0011		
0100		
0101		
0110		
0111		

Info Bits	Coded Bits	Weight
1000		
1001		
1010		
1011		
1100		
1101		
1110		
1111		

TABLE 2 – Redundancy

We add redundancy to the binary message coded with Huffman.

4. Find the length of the message after redundancy.

Hint: If the Huffman length is not divisible by 4, consider adding zero bits at the end of the sequence at the output of the Huffman encoder.

5. Find the binary message corresponding to the sequence N_3, N_4, N_2, N_1, N_7, N_8, N_6 obtained with the Huffman coding (obtained in question A.2.5) followed by the considered channel coding.

A.4 Modulation et mise sous-porteuse

We use the 16QAM modulation (illustrated in Figure 2) associated with a rectangular line code to give a physical representation for the binary stream. The frequency carrier is 900 MHz, and the bandwidth allocated to the transmission of this signal is 200 kHz.

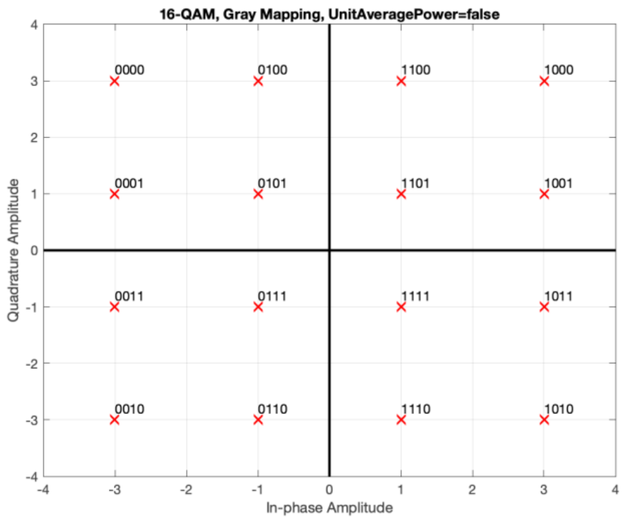


FIGURE 2 – Constellation

1. Check that if the gray mapping is correct on the constellation.
2. Find the average power of the modulated signal using 16QAM.
3. Find the symbol duration, the bit duration and the useful binary data rate.
4. Find the corresponding symbols for the 16 first bits corresponding to N_3, N_4, N_2, N_1, N_7, N_8, N_6. Deduce the corresponding wave.

   Write this wave as $x_e(t) = \left(A_k \cos(2\pi f_0 t) + B_k \sin(2\pi f_0 t)\right)$. Note that A_k is known as the inphase signal and B_k is the quadrature phase one.

B. Transmission and Reception

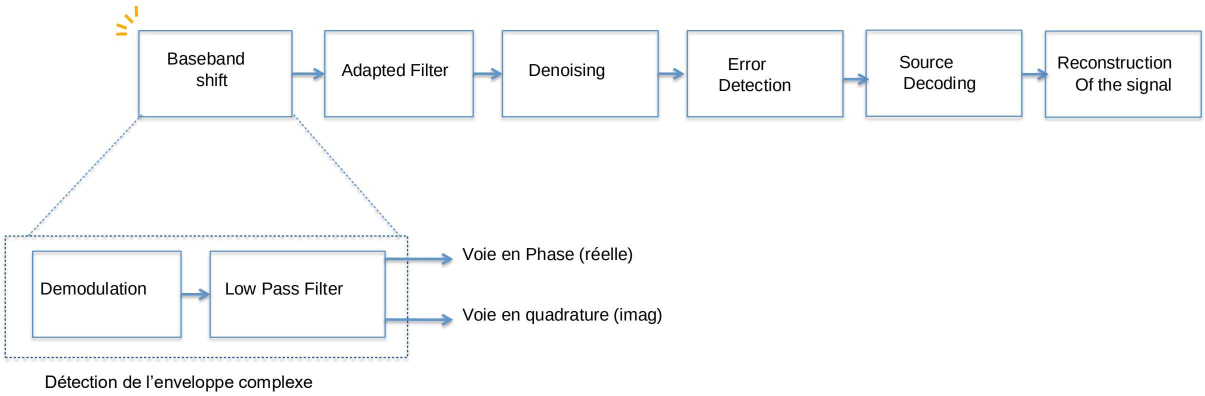
During the wireless transmission, the transmitted signal is attenuated by the path-loss, the fading, the shadowing and is perturbated by an additive noise. The average signal to noise ratio SNR measures the ratio between the power of the attenuated signal and the power of the noise.

After normalizing on the noise power, the received signal can be written as:

$$\mathrm{y}(t) = \sqrt{SNR}\, x_e(t) + \mathrm{n}(\mathrm{t})$$

with n(t) being the normalized additive noise.

The structure of the receiver is given below:



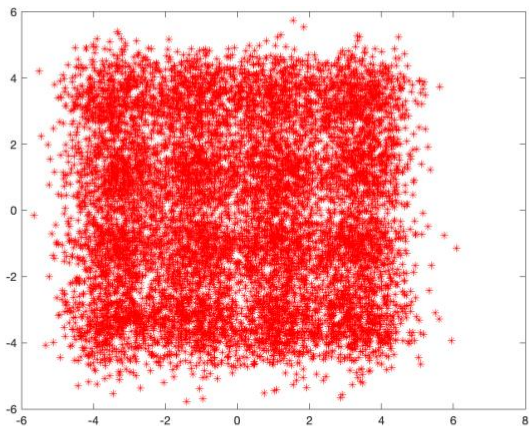
B.1 Demodulation

The demodulation consists in bringing back in base band the signal i.e., to eliminate the oscillations introduced by the carrier f_0. To find the amplitude A_k and B_k, we proceed by two steps: Each symbol time, we multiply the received signal $\mathrm{y}(t)$ by $\cos(2\pi f_0 t)$ and $\sin(2\pi f_0 t)$ and then we filter the resulting signal.

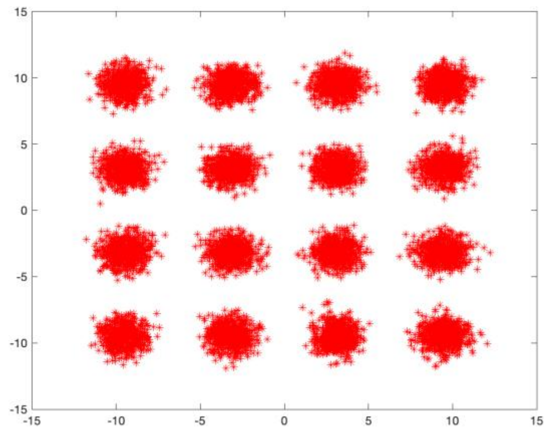
1. Write the expression of the received signal after multiplying by $\cos(2\pi f_0 t)$ and $\sin(2\pi f_0 t)$. Indicate the frequencies around which the spectrum of the resulting signal is found.
2. Choose a suitable cutoff frequency for the low-pass filter.
3. Comment on the way we can find A_k and B_k

B.2 Demapping or denoising

The corresponding amplitude of the received signal for two respective SNR. Two remove the noise, we use a maximum likelihood decoder that approximates the noisy value by the value of the 16QAM constellation.



SNR = 1 dB



SNR = 10 dB

1. Comment on the maximum likelihood decoding in both cases.
2. Find the Shannon capacity and the spectral efficiency in both cases and deduce if the transmission of a 16QAM is compatible.

B.3 Error detection and decompression

1. Take 8 bits from the binary stream obtained in question A.3.5 and inject one random error in a random position. Use the coding to correct this error.
2. Find back the quantized symbol corresponding to the binary Huffman sequence using Huffman tree.

II. OFDM systems

In a WiFi 802.11n system that operates at a frequency carrier $f_0 = 5$ GHz, the bandwidth of 20 MHz is cut into 64 subcarriers: 48 are dedicated for data, 4 for pilots and 12 nulls subcarriers on the border. The spacing between two subcarriers is 312.5 kHz. A 16QAM - 3/4 (meaning that there is only 3 information bits, the 4th is dedicated for redundancy).
The in phase and quadrature phase are denoted by A_k and B_k.

1. Specify the duration T of the OFDM signal and specify the values of the subcarriers:

$$x_e(t) = \sum_{k=1}^{64} (A_k \cos(2\pi f_k t) - B_k \sin(2\pi f_k t))\, rect(t,T)$$

2. Write the Fourier transform of this signal and tell how we can compute A_k and B_k.

Before transmitting another OFDM symbol with 64 subcarriers, the transmitter should blank message to avoid the channel echoes during 0.8µs .

3. Find the data rate of this OFDM signal.

Appendix

Mathematical relationship :

$$\cos^2(x) + \sin^2(x) = 1$$

$$\cos(a+b) = \cos(a)\cos(b) - \sin(a)\sin(b)$$

$$\sin(2x) = 2\sin(x)\cos(x)$$

$$\cos(2x) = 2cos^2(x) - 1$$

$$\cos(2\pi f_0 t) = \frac{1}{2}(e^{j2\pi f_0 t} + e^{-j2\pi f_0 t})$$

$$\sin(2\pi f_0 t) = \frac{1}{2j}(e^{j2\pi f_0 t} - e^{-j2\pi f_0 t})$$

Fourier Transform:

$$\mathrm{TF}(\mathrm{rect}(\mathrm{t}, \mathrm{T})) = \mathrm{Tsinc}(\pi \mathrm{fT})$$

$$TF(cos(2\pi f_O t)) = \frac{1}{2}(\delta(f - f_0) + \delta(f + f_0))$$

$$TF(sin(2\pi f_O t)) = \frac{1}{2j}(\delta(f - f_0) - \delta(f + f_0))$$

Modulation : $TF(x(t)e^{j2\pi f_0 t}) = X(f - f_0)$

$$TF(x(t)\cos(2\pi f_0 t)) = \frac{1}{2}(X(f - f_0) + X(f + f_0))$$

$$TF(x(t)\sin(2\pi f_0 t)) = \frac{1}{2j}(X(f - f_0) - X(f + f_0))$$