TD2-1.

a) ~~$X\in[0,n]$~~ $X\in\{0,1,2,\dots n\}$, X/n : frequency $A \to$ relative frequency of event A

b) $P(X=$

X: discrete r.v.

X: follows a binomial distribution with parameters n & p.

$X\sim B(n,p)$

where n is the numbers of trials and p is the probability of su

For a binomial r.v. the, probability distribution is.

$\forall k\in\{0,1,\dots,n\}$ $\qquad C_n^k=\dfrac{n!}{k!\,(n-k)!}$

$P(X=k)=C_n^k p^k(1-p)^{n-k}$

$n=1 \quad X\in\{0,1\} \quad P(X=1)=p \quad P(X=0)=1-p$

$n=2 \quad X\in\{0,1,2\} \quad P(X=0)=P(TT)=p^0(1-p)^2=(1-p)^2$

$P(X=1)=P(TH)+P(HT)=2p(1-p)$

$P(X=2)=P(HH)=p^2$

$\vdots$

$n=k \qquad P(X=k)=C_n^k p^k(1-p)^{n-k}$

c) 1° $0\le P(X=k)\le 1 \quad k\in\{0,1,\dots,n\}$

2° $\sum_{k=0}^{n}P(X=k)=1$

$\sum_{k=0}^{n}C_n^k p^k(1-k)^{n-k}$: brimial expension of the binomial $(p+(1-p))$

$\vdots$

$(a+b)^n=\sum_{k=0}^{n}C_n^k a^k b^{n-k}$ binomial expension of $(a+b)$

$\sum_{k=0}^{n}C_n^k p^k(1-p)^{n-k}=(p+(1-p))^n=1^n=1$

TD2-2.

a) X : a random variable

b)

$$P(x=1\text{-}5) = \frac{11+13+15+17+19}{100} = \frac{75}{100}$$

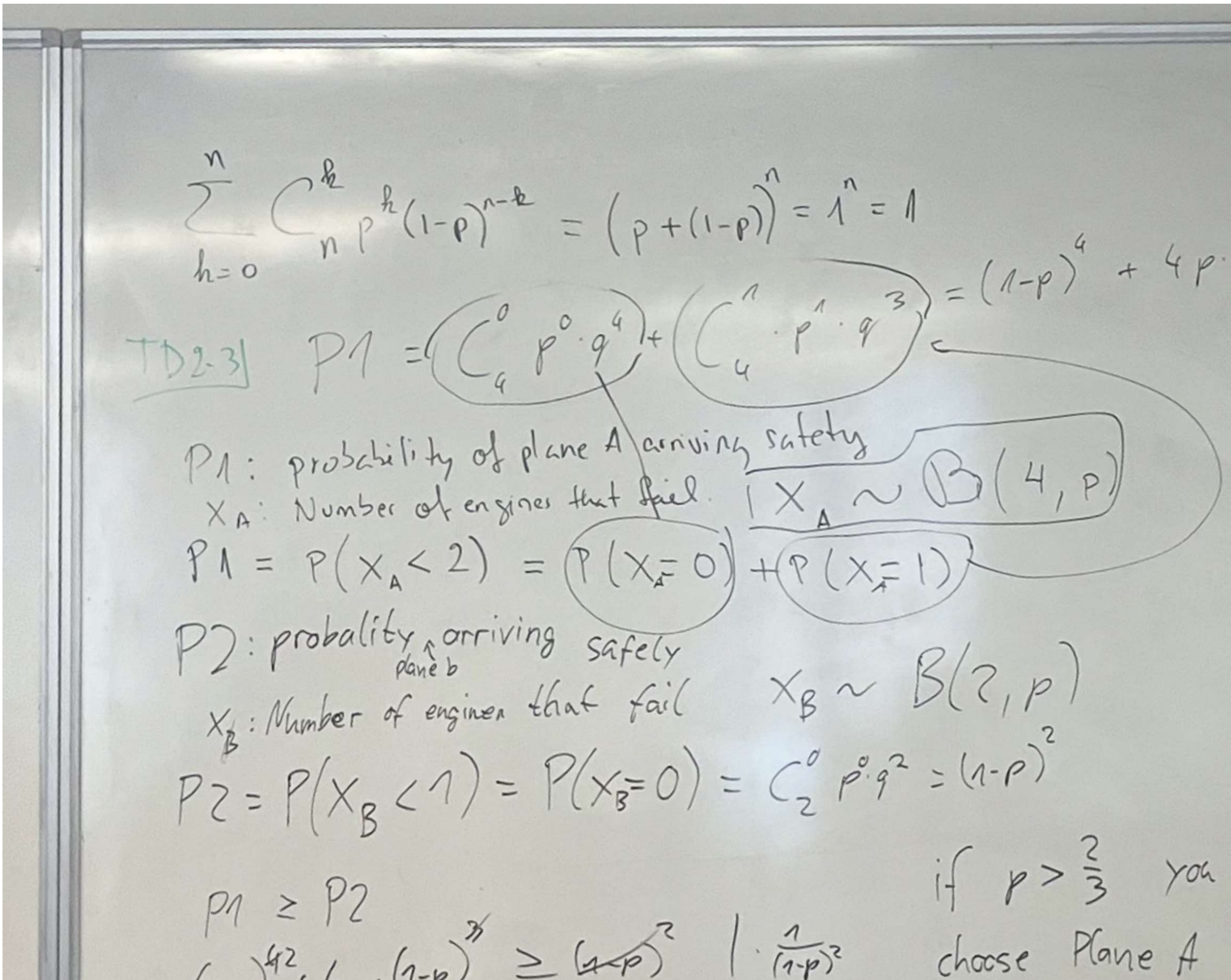
$$A = (-2)\times\frac{75}{100} + \frac{1}{100}\times 10 + \frac{3}{100}\times 9 + \frac{5}{100}\times 8 + \frac{7}{100}\times 7 + \frac{9}{100}\times 6$$

$$= -1.5 + 0.01 + 0.27 + 0.4 + 0.49 + 0.54$$

$$= 0.21 > 0$$

favorable to the player.

T2-3



T2-4

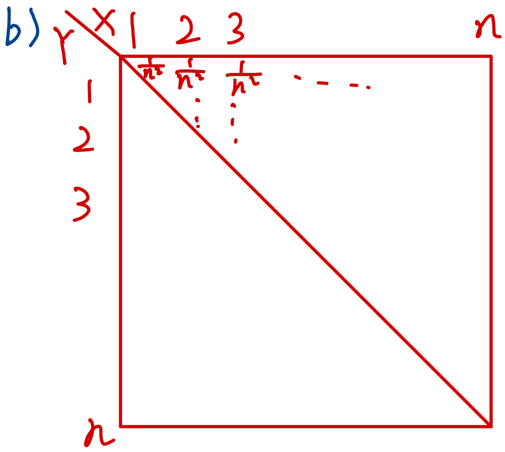
$P(X=k)=\frac{1}{n}, \forall k=\{1,\dots,n\}$

$P(Y=k)=\frac{1}{n}, \forall k=\{1,\dots,n\}$

a) $P(X=Y)=C_k^{0}C_k^{1}=\frac{1}{n}$ $\quad P(X=Y)=\sum_{k=1}^{n}P(X=k\cap Y=k) \quad \forall k\in\{1,\dots,n\}$

$$=\sum_{k=1}^{n}P(X=k)\,P(X=k)=\sum_{k=1}^{n}\cdot\frac{1}{n}\cdot\frac{1}{n}=\frac{1}{n}$$

b)



$$P(X\geq Y)=\frac{n(n+1)}{2}\times\frac{1}{n^2}=\frac{n+1}{2n}$$

By symetry:

$P(X>Y)=P(Y>X)$

$P(X>Y)+P(Y>X)+P(Y=X)=1$

$2P(X>Y)=1-P(X=Y)\quad=1-\frac{1}{n}$

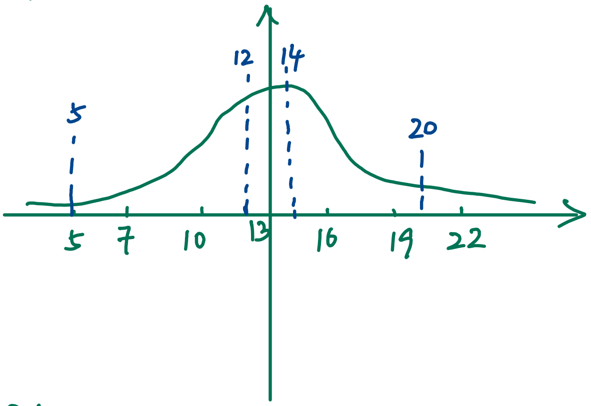
$\therefore P(X>Y)=\frac{1}{2}\left(1-\frac{1}{n}\right)$

$\therefore P(X\geq Y)=\left(\frac{1}{2}-\frac{1}{2n}\right)+\frac{1}{n}=\frac{1}{2}+\frac{1}{2n}$

TD2-5

X r.v. follows the Gaussian distribution with parametres m and σ^2

$$X \sim N(m, \sigma^2) \rightarrow X \sim N(13, 3^2)$$



$$P(X > 20) = P\left(\frac{X-m}{\sigma} > \frac{20-13}{3}\right)$$

$$Z = \frac{X-m}{\sigma} \sim N(0,1)$$

$$= P\left(Z > \frac{7}{3}\right) = P(Z > 2.33)$$

from table: CDF $P(Z \leq 2.33) = 0.9901$

$$\therefore P(Z > 2.33) = 1 - 0.9901 = 0.0099$$

$$P(X < 5) = P\left(\frac{X-m}{\sigma} < \frac{5-13}{3}\right)$$

$$Z = \frac{X-m}{\sigma} \sim N(0,1)$$

$$= P\left(Z < -\frac{8}{3}\right) = P(Z < -2.67)$$

CDF: $P(Z < -2.67) \Leftrightarrow P(Z > 2.67) = 1 - P(Z \leq 2.67)$

by symetry

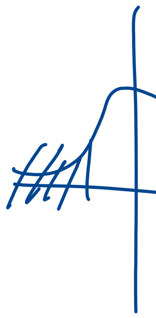
$$= 1 - 0.9962$$

$$= 0.0038$$

$$P(12 < X < 14) = P\left(\frac{12-13}{3} < \frac{X-m}{\sigma} < \frac{14-13}{3}\right)$$

$$P(X>12) = P\left(\frac{X-m}{\sigma} > \frac{12-13}{3}\right)$$

$$Z = P(Z > -0.33)$$



$$= P(Z < 0.33)$$

$$= 0.6293$$

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for 10% : $Z = 0.1 \Rightarrow$ make the

$$P(X>Y) = P\left(\frac{X-m}{\sigma} > \frac{Y-13}{3}\right)$$

$$Z = P\left(Z > \frac{Y-13}{3}\right) = 0.1$$

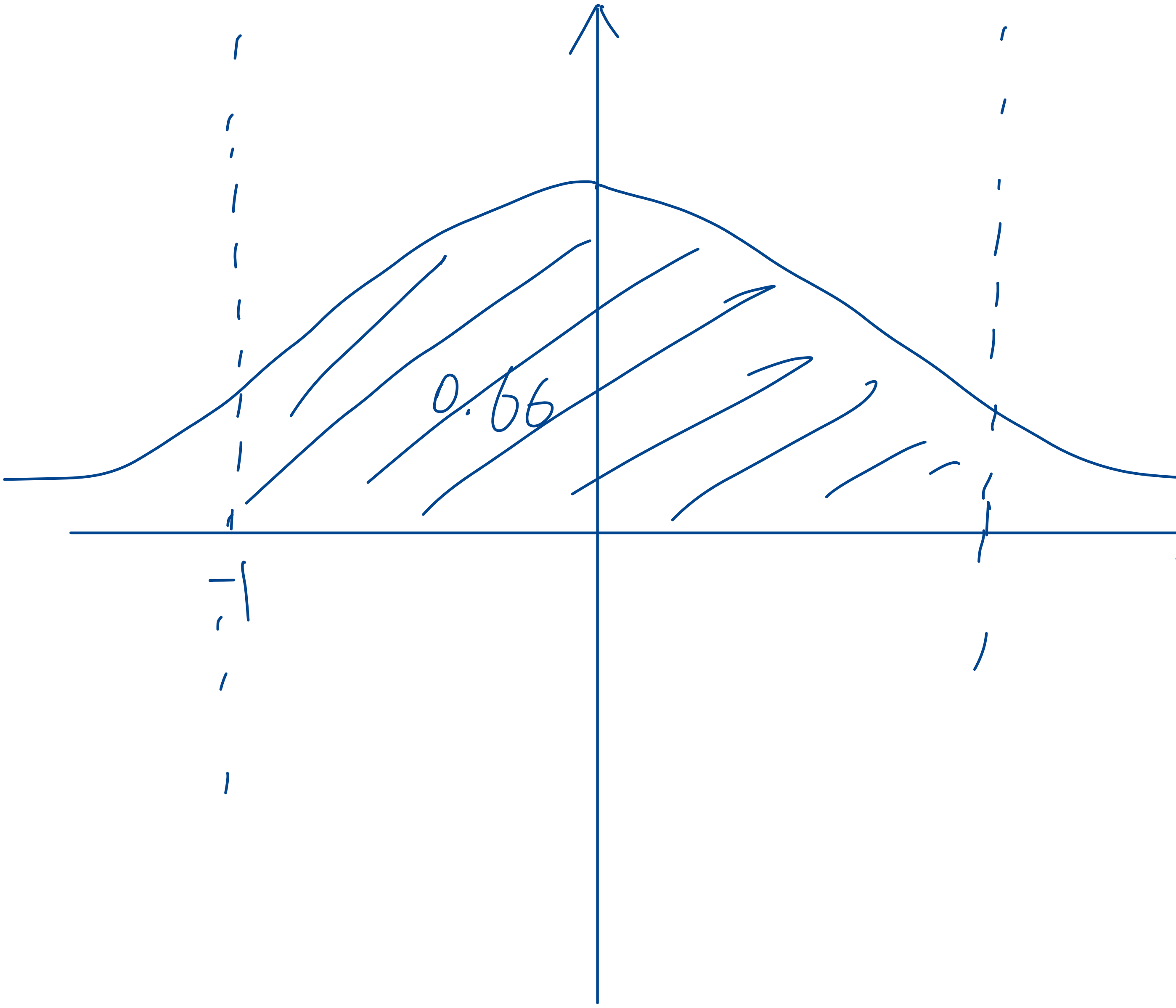
$$= 1 - P\left(Z < \frac{Y-13}{3}\right) = 0.1$$

$$\therefore P\left(Z < \frac{Y-13}{3}\right) = 0.9$$

from formula

TD2-6:

a)



b) ① $P(0 \le X \le 1) = \frac{1}{2} P(-1 \le X \le$

② $P(X \le 1) = P(-1 \le X \le 1) + P$

$= P(-1 \le X \le 1) + \frac{1}{2}[$

$= 0.66 + \frac{1}{2} \times 0.3$

⑤ $P(X \geq 1) = P(X > 1) = 0$

⑥ $P(|X| \leq 1) = P(-1 \leq X$

$= 0.66$

⑦ $P(X^2 \leq 1) = P(-1 \leq$

$= 0.66$

⑧ $P(X^3 \leq 1) = P(-1 \leq$

$+ P(-1 <$

$= P(X \leq$