CORRECTION OF TUTOPRIAL N°4

Infinite Impulse Response (IIR) Filters

EXERCISE N°1

1) $y(n) = K[x(n-1) - y(n-1)] + y(n-1)$

$$H(Z) = \frac{Y(Z)}{X(Z)} = \frac{KZ^{-1}}{1+(K-1)Z^{-1}}$$

2) the output of the filter to the unit level input is:

$$y(n) = K\frac{1-(1-K)^n}{1-(1-K)} = 1-(1-K)^n$$

3) The stability domain is given by :
$|K-1| < 1\ ;\ 0 < K < 2$

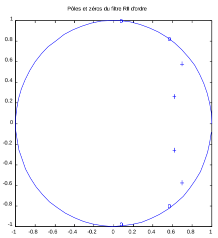
EXERCISE N°2

We have an IIR (Infinite Impulse response) filter because it contains both poles and zeros simultaneously.

The Z transform of the considered filter is :

$$H(Z) = \frac{(Z-Z_1)(Z-Z_1^*)(Z-Z_2)(Z-Z_2^*)}{(Z-P_1)(Z-P_1^*)(Z-P_2)(Z-P_2^*)}.$$

The numerator and the denominator are both of order 4.



The positions of the poles and zeros indicate that we have a low-pass filter.

EXERCISE N°3

1) We have an IIR filter of second order.
2) We have a pure phase shifter, $|H(\omega)| = 1\ \ \forall\omega$.

3) $H(Z) = \dfrac{N(Z)}{D(Z)} = \dfrac{Z^{-2}\ D(Z^{-1})}{D(Z)}$

$$\Rightarrow \varphi(\omega) = 2\,\varphi_D(\omega) - 2\,\omega$$

EXERCISE N°4

$$x(n<0) = 0, x(0) = 1, x(1) = b_1, x(2) = b_2, x(n>2) = 0$$

The input signal $x(n)$ is the Dirac function $\delta(n)$ filtered by the FIR filter with its Z transform

$G(Z) = 1 + b_1 Z^{-1} + b_2 Z^{-2}$. G(z) is the reverse filter of $H(Z)$, i.e. $G(Z).H(Z) = 1$

$$H(Z) = \frac{1}{1-b_1Z^{-1}-b_2Z^{-2}} = \frac{1}{(1-PZ^{-1})(1-P^*Z^{-1})}$$

$$H(Z) = \frac{A}{1-PZ^{-1}} + \frac{B}{1-P^*Z^{-1}}$$

By identification, $A = \dfrac{P}{P-P^*}$ et $B = \dfrac{P^*}{P^*-P}$

By performing the division :

$$H(Z) = \left(\frac{P}{P-P^*}\right)\sum_{n=0}^{\infty} P^n Z^{-n} + \left(\frac{P^*}{P^*-P}\right)\sum_{n=0}^{\infty} P^{*n} Z^{-n}$$

$$H(Z) = \sum_{n=0}^{\infty}\left(\left(\frac{PP^n}{P-P^*}\right) + \left(\frac{P^*P^{*n}}{P^*-P}\right)\right)Z^{-n}$$

Thus, $h_n = \left(\dfrac{PP^n}{P-P^*}\right) + \left(\dfrac{P^*P^{*n}}{P^*-P}\right)$

By using the polar coordinates $P = \rho.e^{j\theta}$, we obtain :

$$h_n = \rho^n\frac{\sin(n+1)\theta}{\sin\theta}$$

If the pole value is outside the unit circle, the function $\sum_{n=-\infty}^{n=+\infty}|h_n|$ diverges.