CORRECTION OF TUTOPRIAL N°5

Correlation of random variables

EXERCISE N°1

$r_{xx}(0) = 4.3\sigma^2$, $r_{xx}(1) = 0.88\sigma^2$

$r_{xx}(2) = -0.35\sigma^2$, $r_{xx}(3) = 0.2\sigma^2$

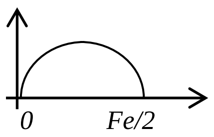
The signal $x(n)$ isn't white because it has been filtered. We note that its autorrelation function is not a dirac function.

The signal is a gaussian because it a linear combination of Gaussian signals.

$$R_3 = \sigma^2 \begin{pmatrix} 4.3 & 0.88 & -0.35 \\ 0.88 & 4.3 & 0.88 \\ -0.35 & 0.88 & 4.3 \end{pmatrix}$$

$$H(Z) = 1 - Z^2 = \left(1 - Z^{-1}\right)\left(1 + Z^{-1}\right)$$

$Z_0 = 1$, $Z_0 = -1$, the cut-off frequencies are 0 and $\frac{F_e}{2}$



The signal $y(n)$ is not white.

$r_{xy}(0) = 2.2\sigma^2$, $r_{xy}(1) = 0.4\sigma^2$, $r_{xy}(2) = -0.2\sigma^2$

EXERCISE N°2

Noticing that in this exercise the noise $b(n)$ is not white because it is not centred and it has a raie on zero frequency which is more important than other ones situated at other frequencies. The temporal autocorrelation function is not a dirac.

$$H(Z) = \left(1 - Z_0 Z^{-1}\right)\left(1 - Z_0^* Z^{-1}\right)$$
$$= 1 - \sqrt{2} Z^{-1} + Z^{-2}$$

$$x(n) = b(n) - \sqrt{2} b(n-1) + b(n-2)$$

$E(b(n)) = \frac{1}{2}$ et $E\left(b(n)^2\right) = \frac{1}{3}$

$$r_{xx}(0) = \frac{4}{3} + \frac{1}{2} - \sqrt{2} = 0.419$$

$$r_{xx}(1) = \frac{3}{2} - \frac{\sqrt{2}}{2} - \frac{2}{3}\sqrt{2} \approx -0.137$$

EXERCISE N°3

Express the Signa-to-Noise-Ratio on each sensor. We note $x_1(t)$ and $x_2(t)$ the corresponding signals for the two sensors.

$x_1(t) = s(t) + b_1(t)$

$x_2(t) = s(t) + b_2(t)$

$\frac{S}{B} = \frac{1}{\sigma^2}$ on each sensor.

$y(t) = 2s(t) + b_1(t) + b_2(t)$

$\frac{S}{B} = \frac{2}{\sigma^2}$ sur le signal $y(t)$

With the attenuation coefficient α :

$\frac{S}{B} = \frac{(1+\alpha)^2}{2\sigma^2}$ for the signal $y(t)$

$\alpha = 0$, waste of 3 dB

$\alpha \approx 0.414$, gain of 0 dB

$\alpha = 1$, gain of 3 dB