Chapter 4

Finite Volume Method

The classical method of solving any problem is breaking it down into smaller manageable chunks. It is no different when solving problems on a computer. When solving a partial differential equation (PDE) using a computer, we first divide the domain (space) into smaller pieces. Then solve the PDE on each of these pieces and stitch the pieces together to get a global (complete domain) solution. Numerous methodologies have been developed to do this, each having its own novelty, advantages and limitations. The finite difference method (FDM) is probably the oldest of such methods, where orthogonal points are laid out in the domain and simple approximations are applied to each of the partial derivatives using the local point values. Another method in this family is the finite element method (FEM). This is very well worked out and widely used for solving structural problems. In this method, the domain is divided into smaller parts, called elements, and approximate distribution of variables inside the element is assumed. This assumed distribution is then substituted into the PDE and the residue (integral of PDE) is equated to zero. This results in the so called "local stiffness matrix" for each of the element. These smaller matrices are assembled together to solve the global problem. A relatively new advancement of the FEM is the discontinuous-Galerkin method, which does not assume a continuous solution at the faces of elements. Hence, it can tackle problems with discontinuous solutions in a natural way. The finite volume method (FVM), which is the topic of this chapter, is the most widely used method for solving convection dominated flows. In this method the domain is divided into smaller pieces, called finite volumes or cells, and the PDE is integrated over each cell. Using the divergence theorem the volume integrals are converted to surface integrals. Thus, the complete problem boils down to calculation of flux at the cell faces. This method automatically ensures conservation if the cells cover the domain completely without overlapping.

Many other methods exist for solving PDEs such as the meshless methods which do not store connection information between the points in the domain. Hence making them very useful in handling domains with changing shapes. Another family of methods use the Lagrangian frame of reference, where each particle is tracked. These methods are very accurate in calculation of convection of particles, especially if high order integration is used. In this book and code the finite volume method is used therefore we will look at this method in more detail.

4.1 Formulation of finite volume method