1.8) The engine of a small jet aircraft develops a thrust of  $18 \, kN$  when the aircraft is flying at a speed of  $900 \, km/h$  at an altitude where the ambient pressure is  $50 \, kPa$ . The air flow rate through the engine is  $75 \, kg/s$  and the engine uses fuel at a rate of  $3 \, kg/s$ . The pressure on the engine discharge plane is  $55 \, kPa$  and the area of the engine exit is  $0.2 \, m^2$ . Find the jet efflux velocity.

## **Solution:**

Given:

Thrust=18 kN, fuel rate=3 kg/s Inlet  $\rightarrow V_1 = 900 \,\text{km/h} = 900/3.6 = 250 \,\text{m/s}, p_1 = 50 \,\text{kPa}, \dot{m}_1 = 75 \,\text{kg/s}.$  Outlet  $\rightarrow p_2 = 55 \,\text{kPa}, A_{\text{exit}} = 0.2 \,\text{m}^2, V_2 =? \,\text{m/s}.$ 

The schematic diagram of the problem description is shown in Fig. 1.

 $m_1 V_1$   $p_1$   $m_2 V_2$   $p_2$ 

Fig. 1: Schematic diagram for problem description

Using the mass conservation equation,

$$\dot{m}_2 = \dot{m}_1 + \dot{m}_{\text{fuel}}$$
 $\dot{m}_2 = 75 + 3 = 78 \,\text{kg/s}$ 

Using the conservation of momentum along horizontal direction,

Thrust = rate of momentum exiting - rate of momentum entering + pressure force at exit - pressure force at inlet   
Thrust = 
$$\dot{m}_2 V_2 - \dot{m}_1 V_1 + (p_2 - p_1) A_{\rm exit}$$
  
 $18 \times 10^3 = 78 \times V_2 - 75 \times 250 + \left(55 \times 10^3 - 50 \times 10^3\right) \times 0.2$   
 $V_2 = \frac{18 \times 10^3 + 75 \times 250 - \left(55 \times 10^3 - 50 \times 10^3\right) \times 0.2}{78}$ 

$$\boxed{V_2 = 458.3333 \, \text{m/s}}.$$