1.15) Air enters a heat exchanger with a velocity of 120 m/s and a temperature and pressure of 225° C and 2.5 MPa. Heat is removed from the air in the heat exchanger and the air leaves with a velocity 30 m/s at a temperature and pressure of 80° C and 2.45 MPa. Find the heat removed per kilogram of air flowing through the heat exchanger and the density of the air at the inlet and the exit to the heat exchanger.

Solution:

Given:

$V_1 = 120\,\text{m/s}$, $T_1 = 225°\,\text{C} = 498\,\text{K}$, $p_1 = 2.5\,\text{MPa}$,
$V_e = 30\,\text{m/s}$, $T_e = 80°\,\text{C} = 353\,\text{K}$, $p_e = 2.45\,\text{MPa}$.

To calculate: $\dot{q}/\dot{m}$ =?, ρ_1 =?, ρ_e =?.

The schematic diagram of the problem description is shown in Fig. 1.

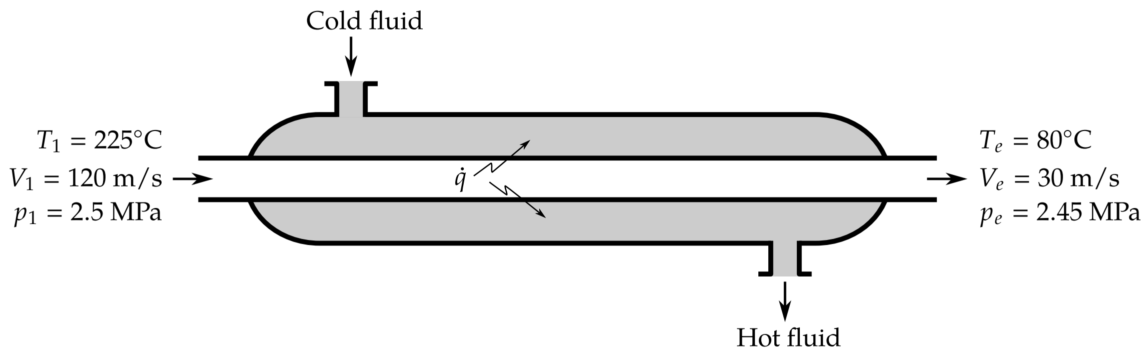

Fig. 1: Schematic diagram for problem description

Applying the conservation of energy,

$$\text{Rate of energy at inlet} + \text{Rate of heat addition} = \text{Rate of energy at exit}$$

$$c_p T_1 + \frac{V_1^2}{2} + \frac{\dot{q}}{\dot{m}} = c_p T_e + \frac{V_e^2}{2}$$

Taking the heat capacity of air at constant pressure,$c_p = 1005\,\text{J/kg-K}$,

$$1005 \times 498 + \frac{120^2}{2} + \frac{\dot{q}}{\dot{m}} = 1005 \times 353 + \frac{30^2}{2}$$

Which can be solved for heat flow rate to be,

$$\frac{\dot{q}}{\dot{m}} = 1005 \times 353 + \frac{30^2}{2} - 1005 \times 498 - \frac{120^2}{2} = -152475$$

The negative sign indicates that the heat is removed. The heat removed per kilogram of air flowing through the heat exchanger,

$$\boxed{\frac{\dot{q}}{\dot{m}} = 152475\,\text{J/kg}}.$$

The density at the inlet and exit can be calculated using the ideal gas equation.

$$\rho_1 = \frac{p_1}{R\,T_1} = \frac{2.5 \times 10^6}{287 \times 498}$$

$$\boxed{\rho_1 = 17.49\,\text{kg/m}^3}.$$

$$\rho_e = \frac{p_e}{R\,T_e} = \frac{2.45 \times 10^6}{287 \times 353}$$

$$\boxed{\rho_e = 24.18\,\text{kg/m}^3}.$$