# **CQF** Final Project

# Pairs Trading Strategy Design & Backtest (TS)



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#### I. Introduction

In the dynamic world of financial markets, traders and investors are always seeking innovative strategies to maximize returns while minimizing risks. One such strategy that has gained attention is the concept of **pairs trading**. This strategy is grounded in the idea that certain assets tend to move in tandem over time, and deviations from this historical relationship can present trading opportunities.

Pairs trading hinges on the principle of **mean reversion**. Let's break that down in simpler terms. Imagine two friends on a seesaw. If one friend goes much higher than the other, eventually, they'll come back to an equilibrium point – that's mean reversion. In finance, it means that if two related assets drift apart from their historical average relationship, they tend to revert, or come back, to that average over time. Pairs trading aims to capitalize on this tendency.

# A. The Objective of This Project

The main goal of this project is to design and rigorously test a pairs trading strategy. We'll focus on Exchange-Traded Funds (ETFs), which are collections of assets that represent a particular market index or sector. By carefully selecting pairs of ETFs that historically move together, we'll create a systematic trading approach to exploit opportunities when the pair deviates from its historical norm.

## B. Key Questions and Goals

As we delve into this project, we'll seek answers to crucial questions such as:

- How do we identify suitable pairs of ETFs for trading?
- What is the best way to determine if a pair of ETFs has deviated from its historical relationship?
- How can we quantify the potential profits and risks of our pairs trading strategy?
- Can this strategy outperform a passive investment in a broad market index like the S&P 500?

# C. Significance and Implications

Understanding and implementing pairs trading not only provides insights into market dynamics but also opens the door to a systematic approach that could potentially yield consistent profits. By testing this strategy rigorously, we aim to contribute to the growing body of knowledge in quantitative finance and provide a practical tool for investors and traders looking to enhance their portfolio returns.

As we progress through the project, we will explore the data, methodology, implementation, results, and ultimately draw meaningful conclusions about the feasibility and effectiveness of our pairs trading strategy.

In the following sections, we will discuss the data collection process, the methods used to identify potential pairs, the detailed steps of our mean reversion trading strategy, and the analytical tools employed to evaluate its performance.

### II. METHODOLOGY

## A. Pairs Trading

Pairs trading is a trading strategy where investors aim to profit from the relative price movements between two related assets, like stocks or commodities. The idea is to identify pairs of assets that historically tend to move together in price. When the prices of these assets deviate from their historical relationship – for example, if one asset becomes more expensive than the other – the investor takes advantage of this divergence by simultaneously buying the cheaper asset and selling the more expensive one.

The goal is to benefit from the eventual reversion of their prices back to their typical relationship. Pairs trading is a market-neutral strategy, meaning it aims to generate returns regardless of whether the overall market is going up or down, as long as the relative prices of the chosen assets return to their usual pattern.

#### B. Mean Reversion

Mean reversion is a financial concept that suggests that over time, the prices or values of assets tend to move back towards their historical average or "mean." In other words, when an asset's price or value deviates significantly from its average, there is a tendency for it to eventually revert, or return, to that average. This phenomenon is based on the idea that extreme price movements are temporary and that markets have a tendency to self-correct.

Investors who use mean reversion strategies often take advantage of these temporary deviations by buying assets that have recently experienced price drops (assuming they will rebound) or selling assets that have seen rapid price increases (expecting them to decline). This strategy assumes that the market's overreaction to news or events will eventually subside, causing the asset's price to move back closer to its historical average.

#### C. ETFs

Exchange-Traded Funds (ETFs) are a type of investment that bundles together a collection of stocks, bonds, or other assets, allowing investors to buy a single unit representing a diversified portfolio. Think of ETFs as a "basket" of various investments. They are traded on stock exchanges, similar to individual stocks, and their prices fluctuate throughout the trading day. ETFs provide a convenient way for investors to gain exposure to a specific sector, market index, or asset class without needing to buy each individual security separately. They offer flexibility, transparency, and often have lower fees compared to traditional mutual funds. ETFs have gained popularity due to their ease of trading, cost-effectiveness, and ability to track various market indices or investment strategies.

# D. Portfolio Optimization

Portfolio optimization is a method used in finance to create an investment portfolio that aims to achieve the best possible balance between expected returns and risk. It involves carefully selecting a combination of different assets, such as stocks, bonds, and other investments, in order to maximize potential profits while minimizing the overall level of risk. By analyzing historical data and using mathematical models, portfolio optimization helps investors find the optimal allocation of assets that can provide the highest expected return for a given level of risk tolerance.

The goal is to construct a diversified portfolio that takes advantage of the potential for higher returns from riskier assets while also providing some level of protection through the inclusion of less risky assets. It's important to note that portfolio optimization doesn't eliminate risk entirely, but rather aims to find a well-balanced mix of investments.

#### III. IMPLEMENTATION (DESIGN AND BACKTEST)

In this section, we delve into the practical execution of our pairs trading strategy. We will explore the step-by-step process of designing and backtesting the strategy, ensuring a comprehensive understanding of its implementation. The foundation of our strategy lies in the selection of appropriate pairs of ETFs. We identify potential pairs by conducting a thorough analysis of historical price data. Pairs are chosen based on their demonstrated cointegration, indicating a strong historical relationship between the ETFs. This cointegration is crucial for the mean reversion principle to hold.

# A. Selection of ETFs for Analysis

In this study, we have carefully curated a list of Exchange-Traded Funds (ETFs) representing a diverse range of global markets and sectors. Our selection is based on crucial factors such as Asset Under Management (AUM) and liquidity. These ETFs have been chosen to provide a comprehensive overview of different geographical regions and industries. The selected ETFs for analysis are as follows:

- iShares MSCI Japan ETF (EWJ): This ETF tracks the performance of the Japanese equity market, offering insights into one of the world's leading economies.
- iShares MSCI China ETF (MCHI): Focusing on China's stock market, this ETF provides a window into the dynamic and rapidly growing Chinese economy.
- iShares MSCI India ETF (INDA): By tracking Indian equities, this ETF allows us to explore the trends within the vibrant and diverse Indian market.
- iShares MSCI Brazil ETF (EWZ): With a focus on Brazil's stock market, this ETF helps us understand the economic dynamics of South America's largest economy.
- iShares MSCI South Korea ETF (EWY): This ETF provides insights into South Korea's equity market, known for its technological innovation and global brands.
- iShares MSCI Taiwan ETF (EWT): By tracking Taiwanese equities, this ETF offers a glimpse into Taiwan's essential role in the global tech supply chain.
- iShares MSCI Canada ETF (EWC): Focusing on Canadian stocks, this ETF enables us to analyze one of the world's resource-rich economies.

- iShares MSCI United Kingdom ETF (EWU): With a focus on the UK's stock market, this ETF provides insights into Europe's financial hub and a major global economy.
- iShares MSCI Australia ETF (EWA): By tracking Australian equities, this ETF helps us explore trends within a commodity-driven economy and a key Asia-Pacific region player.

The chosen ETFs hold significance due to their representation of major global economies and industries. AUM and liquidity were key factors guiding our selection, ensuring that the ETFs chosen are actively traded and reflective of market trends. By analyzing these ETFs, we aim to gain valuable insights into how different markets and sectors interact and perform over time. This approach allows us to explore potential relationships and dependencies that can offer valuable strategic insights for investors and decision-makers.

## B. Visualizing Close Prices and Daily Returns

To conduct a thorough analysis, we retrieved historical data for each of the selected ETFs. This data includes their daily closing prices over a specific time period. We utilized the Yahoo Finance API to ensure accurate and up-to-date information. The collected data has been organized and stored for further examination.

To understand how the ETFs have performed over time, we create an Plotly figure that displays the close prices of each ETF. Close prices are the prices at which each ETF ended each trading day. This visualization helps us see the overall trends and fluctuations in the ETF prices.



Fig. 1. Close Prices of ETFs

Next, we calculate the daily returns for each ETF. Daily returns represent the percentage change in the ETF's price from one day to the next. These returns are important for assessing the performance and risk of each ETF. We create another Plotly figure that displays the daily returns over time for each ETF.

Daily Returns of ETFs

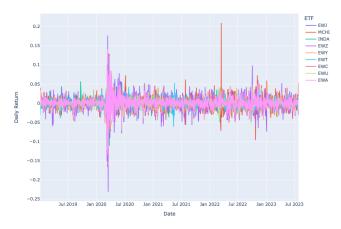


Fig. 2. Daily Returns of ETFs

## C. Ordinary Least Squares (OLS) Regression

Now we move into the realm of quantitative analysis. We implement the Ordinary Least Squares (OLS) regression, a statistical technique used to understand the relationship between two variables. In this case, we use it to explore how one ETF's returns are influenced by another ETF's returns.

OLS regression aims to find the best-fitting line (linear equation) that minimizes the sum of the squared differences between the actual and predicted values. In this context, we're trying to find the relationship between the returns of two ETFs:  $\mathbf{y}$  (dependent variable) and  $\mathbf{x}$  (independent variable). The linear equation for OLS is:

$$y = \beta_0 + \beta_1 \cdot x + \varepsilon \tag{1}$$

where,

- y is the dependent variable (returns of one ETF).
- x is the independent variable (returns of another ETF).
- $\beta_0$  is the intercept (y-intercept).
- β<sub>1</sub> is the slope (how much y changes for a unit change in x).
- $\varepsilon$  represents the error term (unexplained variability).

The goal is to find the values of  $\beta_0$  and  $\beta_1$  that minimize the sum of squared errors  $(\varepsilon)$ . These values indicate the strength and direction of the relationship between the two ETFs returns.

#### D. Augmented Dickey-Fuller (ADF) Test for Stationarity

Stationarity is an important concept in time series analysis. A stationary time series has statistical properties that remain constant over time. The Augmented Dickey-Fuller (ADF) test helps determine whether a time series is stationary. A price is stationary if it doesn't deviate much but stays around the mean. The ADF test involves estimating the following regression equation:

$$\Delta Y(t) = \alpha + \beta \cdot t + \gamma \cdot Y(t-1) + \delta_1 \cdot \Delta Y(t-1) + \dots + \delta_k \cdot \Delta Y(t-k) + \varepsilon$$
(2)

where,

- Y(t) is the time series at time t.
- \( \Delta\) represents the first difference (change from one time period to the next).
- $\alpha$ ,  $\beta$ ,  $\gamma$  and  $\delta$  are coefficients to be estimated.
- $\varepsilon$  is the error term.

The simplified equation,

$$\Delta P_t = \lambda P_{t-1} + \varepsilon \tag{3}$$

The test checks whether the coefficient  $\gamma$  is significantly different from zero. If it is, the series is non-stationary; if not, the series is stationary.

#### E. Engle-Granger Cointegration Test

Cointegration is a long-term equilibrium relationship between two time series. Engle-Granger two-step cointegration test helps determine whether two non-stationary time series move together in the long run. A portfolio of two or more instruments such that portfolio is stationary. Then, the instruments in the portfolio are said to be cointegrated.

**Step 1 - OLS Regression:** First, we perform OLS regression between the two non-stationary time series. The residuals  $(\varepsilon)$  obtained from this regression represent the short-term deviations from the long-term equilibrium relationship.

**Step 2 - ADF Test:** We then test the residuals for stationarity using the ADF test. If the residuals are stationary, it suggests cointegration between the two series, indicating a long-term relationship.

# F. Finding Cointegrated ETF Pairs

We generate a list of all possible pairs of ETFs and apply the Engle-Granger cointegration test to each pair. This helps identify pairs of ETFs that are likely to be cointegrated, meaning they have a long-term equilibrium relationship.

To summarize the cointegration test results for all ETF pairs, we create a heatmap using Plotly. The heatmap visually represents the p-values obtained from the cointegration tests between different pairs of ETFs. A lower p-value indicates a stronger likelihood of cointegration.

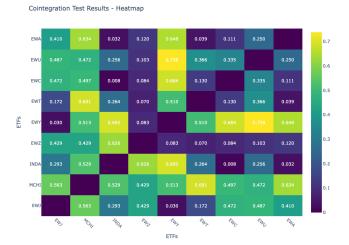


Fig. 3. Cointegration Test Results - Heatmap

Below are the list of pairs that are likely to be cointegrated which have p-value < 0.05.

| Cointegrated Pairs (Sorted by p-value, Unique):          |
|----------------------------------------------------------|
| Pair: EWC - INDA<br>Cointegration test p-value: 0.008284 |
| Pair: EWJ - EWY Cointegration test p-value: 0.029559     |
| Pair: EWA - INDA<br>Cointegration test p-value: 0.031841 |
| Pair: EWA - EWT<br>Cointegration test p-value: 0.0392    |

Fig. 4. Cointegrated Pairs

The p-value is a number that helps researchers decide if their ideas are supported by evidence. It shows how likely their results would happen by chance. If the p-value is very small, it means their results are probably not random and their idea might be true. But if the p-value is big, their results could easily happen by chance, so their idea might not be as likely.

## G. Calculating Spread between Pairs of ETFs

The spread between pairs of Exchange-Traded Funds (ETFs) is a fundamental concept in pairs trading. It helps us identify deviations from the historical relationship between the prices of two ETFs, which can potentially indicate trading opportunities. The spread is calculated using a statistical method called Ordinary Least Squares (OLS) regression.

Let's consider two ETFs, ETF1 and ETF2, and denote their daily closing prices as P1 and P2 respectively. The spread between these two ETFs, denoted as S, can be calculated as follows:

$$S = P1 - \beta_0 - \beta_1 \cdot P2 \tag{4}$$

where.

- $\beta_0$  is the intercept of the OLS regression line.
- $\beta_1$  is the slope of the OLS regression line.

The spread calculation is based on the concept of fitting a linear equation to the historical prices of the two ETFs. The OLS regression aims to find the best-fitting line that minimizes the sum of squared differences between the actual prices and the predicted values. This line represents the spread between the two ETFs. The spread represents the deviation from the expected value of P1 based on the OLS regression line. If the spread is positive, it indicates that ETF1 is relatively more expensive than expected compared to ETF2. Conversely, if the spread is negative, it suggests that ETF1 is relatively cheaper than expected compared to ETF2. Below is the spread plot between EWC-INDA:



Fig. 5. Spread between EWC and INDA

In the context of our project, we gather historical data for two ETFs, ETF1 and ETF2, and perform OLS regression to obtain the coefficients  $\beta_0$  and  $\beta_1$ . Subsequently, we use these coefficients to calculate the spread S for each data point over the specified time period. This spread data helps us visualize and analyze the deviations from the historical relationship between the two ETFs. By examining the calculated spread, we can identify potential trading opportunities where the spread significantly deviates from its historical norm. These deviations are essential signals for executing pairs trading strategies, as they suggest that the two ETFs may revert to their typical relationship, presenting opportunities for profit. Below is the one more spread plot between EWJ-EWY:

Spread between EWJ and EWY



Fig. 6. Spread between EWJ and EWY

## H. Mean Reversion Trading Strategy

A key aspect of our pairs trading project involves implementing a mean reversion trading strategy using a powerful tool called Bollinger Bands. Bollinger Bands are a statistical indicator that helps us identify potential buy and sell signals based on the historical spread between two ETFs. Bollinger Bands consist of three lines: the middle line, which is the moving average of the spread, and two outer bands that represent a certain number of standard deviations away from the moving average. These bands provide a dynamic framework for identifying periods of potential overvaluation or undervaluation of the spread.

Let's delve into the mean reversion strategy entry and exit conditions:

- Moving Average (MA): We calculate the moving average
  of the spread over a specified lookback period. This
  moving average represents the typical value of the spread
  during that period.
- Moving Standard Deviation (MSD): The moving standard deviation of the spread is computed over the same lookback period. It indicates the variability or volatility of the spread.
- Upper Band (UB) and Lower Band (LB): The upper band is calculated by adding a certain number of standard deviations (given by  $std_dev$ ) to the moving average. The lower band is calculated by subtracting the same number of standard deviations from the moving average.
- Long Entry and Exit Signals: A long entry signal is generated when the spread falls below the lower band. A long exit signal occurs when the spread rises above the moving average.
- Short Entry and Exit Signals: Conversely, a short entry signal is triggered when the spread exceeds the upper band. A short exit signal occurs when the spread drops below the moving average.
- **Position Indicators:** We use position indicators to represent the trading stance. For long positions, the indicator is set to 1, and for short positions, it is set to -1. A value of 0 indicates no position.

# **Strategy Flow:**

- Calculate the moving average (MA) and moving standard deviation (MSD) of the spread.
- 2) Compute the upper band (UB) and lower band (LB) using the MA and MSD.
- 3) Determine long and short entry and exit signals based on the spread's relation to the bands.
- 4) Assign position indicators for long and short positions, and ensure continuity of positions using forward filling.

Bollinger Bands serve as dynamic thresholds that help us identify potential trading signals. When the spread deviates significantly from its moving average and crosses the bands, it suggests a potential reversion to the mean. This is a core principle of mean reversion trading, where we anticipate that extreme price movements will eventually subside and the spread will return to its typical value. By implementing this mean reversion strategy with Bollinger Bands, we aim to systematically identify and capitalize on trading opportunities arising from deviations in the spread between ETF pairs.

The Bollinger Bands formula involves calculating the moving average (MA) and moving standard deviation (MSD), and then computing the upper band (UB) and lower band (LB) as follows:

$$MA = \frac{\sum_{i=1}^{N} Spread_i}{N}$$
 (5)

$$MSD = \sqrt{\frac{\sum_{i=1}^{N} (Spread_i - MA)^2}{N}}$$
 (6)

$$UB = MA + std\_dev \times MSD \tag{7}$$

$$LB = MA - std\_dev \times MSD \tag{8}$$

where,

- N is the lookback period.
- $Spread_i$  is the spread at time i.
- std\_dev is the number of standard deviations for the bands.

# I. Parameter Optimization

Parameter optimization is a critical aspect of quantitative trading strategy development. It involves systematically searching for the best combination of parameter values within a given range to enhance the performance of a trading strategy.

In the context of our mean reversion trading strategy, parameter optimization plays a vital role in determining the optimal values for the lookback period and standard deviation used in the Bollinger Bands.

In the realm of algorithmic trading, selecting appropriate parameter values is essential to ensure that a trading strategy performs optimally under various market conditions. However, there is no one-size-fits-all approach, as the optimal parameter values may vary based on the specific dataset, assets being traded, and market dynamics. Parameter optimization helps us find the values that maximize returns, minimize risk, and lead to better trading decisions.

The performance of a trading strategy is heavily influenced by the parameter values used. Suboptimal parameter values can lead to poor trading results, including lower returns and higher risk. Therefore, it is crucial to systematically explore different parameter combinations to identify those that offer the best potential outcomes. By conducting parameter optimization, we aim to enhance the effectiveness and robustness of our mean reversion trading strategy.

Parameter optimization is a fundamental step in the development of any quantitative trading strategy. It allows us to finetune the strategy's parameters to achieve optimal performance in real-world trading scenarios. By systematically searching for the best parameter values, we can enhance the effectiveness of our mean reversion trading strategy and increase its potential for generating consistent profits while managing risks. We will see more on this in results and analysis section.

# J. Portfolio Optimization

In the realm of finance, portfolio optimization plays a pivotal role in constructing investment portfolios that aim to achieve optimal returns while managing risk. The process involves carefully selecting a combination of assets to achieve the best possible trade-off between expected returns and risk. The goal is to diversify investments to reduce risk while maximizing potential gains.

Investors face a fundamental challenge on how to allocate their capital among various assets to achieve their financial goals. This challenge is compounded by the fact that different assets have different levels of risk and return. Portfolio optimization addresses this challenge by mathematically determining the optimal allocation of funds among assets in a way that seeks to maximize returns for a given level of risk.

The Expected return of a portfolio is the weighted sum of the expected returns of its individual assets. Mathematically, it is given by:

$$E(R_p) = \sum_{i=1}^{n} w_i \cdot E(R_i)$$
(9)

- $E(R_p)$  is the expected return of the portfolio.
- $w_i$  is the weight of asset i in the portfolio.
- $E(R_i)$  is the expected return of asset i.

The Variance of a portfolio measures the dispersion of its returns from the mean. It considers both the individual asset returns and their pairwise covariances. Mathematically, it is given by:

$$\sigma_p^2 = \sum_{i=1}^n \sum_{j=1}^n w_i \cdot w_j \cdot \sigma_{ij}$$
 (10)

- $\sigma_p^2$  is the portfolio variance.  $w_i$  and  $w_j$  are the weights of assets i and j in the
- $\sigma_{ij}$  is the covariance between the returns of assets i and

Portfolio simulation involves generating a large number of portfolios with different combinations of asset weights. For each portfolio, its expected return and risk (usually measured by standard deviation or volatility) are calculated. The goal is to identify the set of portfolios that lie on the efficient frontier, a curve representing the highest possible return for a given level of risk.

The **Sharpe ratio** is a widely used metric in portfolio management that assesses the risk-adjusted return of a portfolio. It measures the excess return of the portfolio over the risk-free rate per unit of portfolio risk. Mathematically, it is given by:

Sharpe Ratio = 
$$\frac{E(R_p) - R_f}{\sigma_p}$$
 (11)

where,

- $E(R_p)$  is the expected return of the portfolio.
- $R_f$  is the risk-free rate.
- $\sigma_p$  is the portfolio standard deviation.

Let's take a look at some figures that illustrate the key aspects of our portfolio analysis. We present the annualized return and volatility for our strategies. This provides a quick snapshot of how the strategies has performed over time in terms of both potential returns and risk.

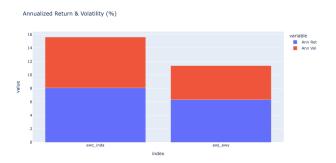


Fig. 7. Portfolio Returns and Risk

Below figure showcases the contribution of each individual strategy to the overall annualized return of the portfolio. This visualization helps us understand which strategy play a significant role in driving the portfolio's returns.

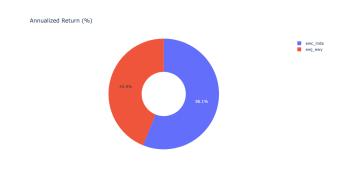


Fig. 8. Annualized Return Contribution

These figures offer valuable insights into the performance and composition of our portfolio, enabling us to make informed decisions based on the balance between potential returns and associated risks. We will see the optimal portfolio allocation in the later section.

# IV. RESULTS AND ANALYSIS

In this section, we delve into the outcomes and implications of our pairs trading strategy. We present and analyze the performance of the strategy, examining key metrics, charts, and insights that provide a comprehensive understanding of its effectiveness.

## A. Strategy Performance Metrics

We begin by showcasing most important performance metrics that help evaluate the success of our pairs trading strategy. These metrics include:

- Cumulative returns: Visual representation of the strategy's overall performance over time.
- Sharpe ratio: Risk-adjusted return, indicating how well the strategy compensates for risk.
- Maximum drawdown: Measure of the largest drop in portfolio value from a peak to a trough.

The strategy involving the EWC and INDA pair, with a **lookback of 30** for the moving average and a **standard deviation of 2** for Bollinger Bands before parameter optimization, yielded the following results:

Fig. 9. EWC\_INDA Performance before Optimization

Below is the Cumulative Returns Plot before Optimization:



Fig. 10. EWC\_INDA Cumulative Returns Plot before Optimization

Below is the Drawdown Plot before Optimization:



Fig. 11. EWC\_INDA Drawdown Plot before Optimization

The strategy involving the EWC and INDA pair, with a **lookback of 10** for the moving average and a **standard deviation of 1** for Bollinger Bands after parameter optimization, yielded the following results:

Fig. 12. EWC\_INDA Performance after Optimization

Below is the Cumulative Returns Plot after Optimization:



Fig. 13. EWC\_INDA Cumulative Returns Plot after Optimization

Below is the Drawdown Plot after Optimization:

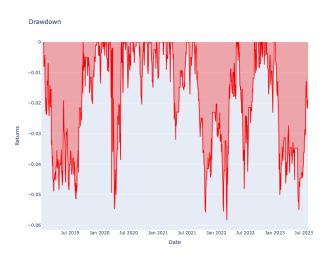


Fig. 14. EWC\_INDA Drawdown Plot after Optimization

The strategy involving the EWJ and EWY pair, with a **lookback of 30** for the moving average and a **standard deviation of 2** for Bollinger Bands before parameter optimization, yielded the following results:

Fig. 15. EWJ\_EWY Performance before Optimization

Below is the Cumulative Returns Plot before Optimization:



Fig. 16. EWJ\_EWY Cumulative Returns Plot before Optimization

Below is the Drawdown Plot before Optimization:



Fig. 17. EWJ\_EWY Drawdown Plot before Optimization

The strategy involving the EWJ and EWY pair, with a **lookback of 25** for the moving average and a **standard deviation of 1.5** for Bollinger Bands after parameter optimization, yielded the following results:

Fig. 18. EWJ\_EWY Performance after Optimization

Below is the Cumulative Returns Plot after Optimization:



Fig. 19. EWJ\_EWY Cumulative Returns Plot after Optimization

Below is the Drawdown Plot after Optimization:

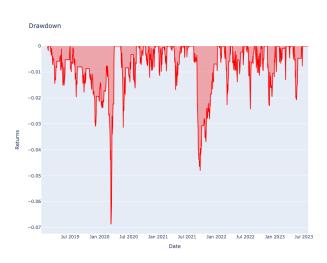


Fig. 20. EWJ\_EWY Drawdown Plot after Optimization

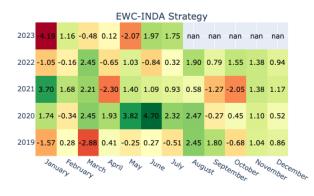
Let's see plot of both the strategies cumulative returns:



Fig. 21. EWC\_INDA & EWJ\_EWY Cumulative Returns Plot

## Monthly Returns Heatmap:

Monthly Returns Heatmap of Mean Reversion Strategies



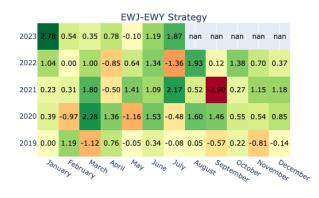


Fig. 22. Monthly Returns Heatmap of EWC\_INDA & EWJ\_EWY

The monthly returns heatmap provides a visual representa-

tion of the performance of our trading strategy across different months. It's like a dashboard that highlights the highs and lows of our strategy's monthly returns.

#### **Trade Metrics:**

#### EWC\_INDA EWJ\_EWY Metrics total\_wins 439.000 377.000 total losses 402.000 336,000 total\_trades 841.000 713.000 win ratio 0.522 0.529 0.478 0.471 loss\_ratio max\_profit 4.018 1.748 max loss -4.684-2.524

Fig. 23. Trade Metrics of EWC\_INDA & EWJ\_EWY strategies

To evaluate the effectiveness of our trading strategies, we examine a set of key performance metrics for two pairs: EWC\_INDA and EWJ\_EWY. These metrics shed light on various aspects of our strategies performance. For the EWC\_INDA pair, we observe 439 winning trades and 402 losing trades, while the EWJ\_EWY pair has 377 winning trades and 336 losing trades. The total number of trades executed for each pair. We conducted a total of 841 trades for EWC\_INDA and 713 trades for EWJ\_EWY.

The win ratio represents the proportion of winning trades out of the total trades. In the case of EWC\_INDA, the win ratio is approximately 0.522 (52.2%), indicating that slightly more than half of the trades were profitable. Similarly, for EWJ\_EWY, the win ratio is around 0.529 (52.9%). The loss ratio complements the win ratio by showing the proportion of losing trades. For the EWC\_INDA pair, the maximum profit achieved was 4.018%, while the maximum loss was -4.684%. In the case of EWJ\_EWY, the maximum profit and loss were 1.748% and -2.524%, respectively.

## B. Comparison with Benchmarks

In this section, we will compare the strategies with respective benchmarks and S&P500 index. Let's see the EWC\_INDA strategy performance with buy and hold returns of respective ETF:

The above metrics offer valuable insights into the performance and risk characteristics of different investment strategies. Starting with the returns, we observe that the





Fig. 24. EWC\_INDA, EWC, and INDA Cumulative Returns Plot

|         | EWC_INDA | EWC   | INDA  |
|---------|----------|-------|-------|
| Metrics |          |       |       |
| returns | 42.26    | 62.81 | 46.08 |
| risk    | 7.53     | 22.91 | 25.65 |

Fig. 25. EWC\_INDA, EWC, and INDA Returns-Risk Table

"EWC\_INDA" strategy has yielded a return of 42.26% over the analyzed period. This indicates a successful outcome where the strategy has generated a positive percentage gain. However, it's noteworthy that the buy-and-hold investments in "EWC" and "INDA" have achieved even higher returns of 62.81% and 46.08%, respectively. These higher returns from the buy-and-hold approach signify the potential benefits of long-term investment in these assets.

Delving deeper into the risk aspect, we examine the standard deviation of returns, which serves as a measure of volatility and variability. Remarkably, the "EWC\_INDA" strategy demonstrates a notably lower risk level with a standard deviation of 7.53%. This implies that the strategy's returns have exhibited less fluctuation and volatility compared to the buyand-hold investments. In contrast, the buy-and-hold strategy for "EWC" comes with a higher risk, as indicated by its standard deviation of 22.91%. Similarly, the buy-and-hold approach for "INDA" also presents a similar level of higher risk with a standard deviation of 25.65%.

Considering these findings, it becomes evident that the "EWC\_INDA" strategy prioritizes risk management by aiming for lower volatility, even if it entails a trade-off in terms of slightly lower returns. On the other hand, the buy-and-hold investments in "EWC" and "INDA" have generated higher returns, but this comes at the cost of greater exposure to market fluctuations and heightened risk.

Now, let's see the EWJ\_EWY strategy performance with

buy and hold returns of respective ETF:

Cumulative Returns of EWJ, EWY, and EWJ\_EWY Strategy



Fig. 26. EWJ\_EWY, EWJ, and EWY Cumulative Returns Plot

|         | EWJ_EWY | EWJ   | EWY   |
|---------|---------|-------|-------|
| Metrics |         |       |       |
| returns | 31.83   | 34.11 | 23.70 |
| risk    | 5.04    | 18.32 | 27.81 |

Fig. 27. EWJ\_EWY, EWJ, and EWY Returns-Risk Table

The presented metrics shed light on the performance and risk characteristics of investment strategies involving the ETF pairs "EWJ\_EWY," as well as their individual constituents "EWJ" and "EWY." First, let's examine the returns. The "EWJ\_EWY" strategy has delivered a return of 31.83% during the analyzed period. This signifies a positive outcome where the strategy has generated a respectable percentage gain. Notably, the individual ETFs "EWJ" and "EWY" have also achieved returns of 34.11% and 23.70%, respectively, by pursuing a buy-and-hold approach. These higher returns from the individual ETFs highlight the potential advantages of long-term investment in these assets.

Turning our attention to risk assessment, we evaluate the standard deviation of returns. Impressively, the "EWJ\_EWY" strategy demonstrates a lower risk profile, boasting a standard deviation of 5.04%. This suggests that the strategy's returns have exhibited relatively stable and less volatile behavior compared to the individual ETFs. Conversely, the buy-and-hold strategy for "EWJ" carries a higher risk, evident from its standard deviation of 18.32%. Similarly, the buy-and-hold approach for "EWY" presents an even higher risk level, as indicated by its standard deviation of 27.81%.

In light of these findings, it becomes apparent that the "EWJ\_EWY" strategy prioritizes risk mitigation by aiming

for lower volatility, albeit with a modest trade-off in terms of slightly lower returns. On the other hand, the buy-and-hold investments in "EWJ" and "EWY" have yielded higher returns, accompanied by a higher level of exposure to market fluctuations and elevated risk.

Next, let's dive into analyzing EWC\_INDA and EWJ\_EWY both the strategies performance with S&P500 index:



Fig. 28. S&P500, EWC\_INDA, and EWJ\_EWY Cumulative Returns Plot

|         | S&P500 | EWC_INDA | EWJ_EWY |
|---------|--------|----------|---------|
| Metrics |        |          |         |
| returns | 82.56  | 42.26    | 31.83   |
| risk    | 21.98  | 7.53     | 5.04    |

Fig. 29. S&P500, EWC\_INDA, and EWJ\_EWY Returns-Risk Table

The above metrics offer valuable insights into the performance and risk characteristics of distinct investment approaches, featuring the S&P 500 index as well as the strategies "EWC\_INDA" and "EWJ\_EWY." Beginning with returns, we observe that the S&P 500 index has delivered an impressive return of 82.56% over the analyzed period. This notable return underscores the potential gains achievable through a passive investment in a diversified index like the S&P 500. In comparison, the "EWC\_INDA" strategy has generated a return of 42.26%, showcasing a positive outcome and indicating the strategy's ability to capture a substantial portion of market upswings. Similarly, the "EWJ\_EWY" strategy has achieved a return of 31.83%, signifying favorable returns through its investment approach.

Shifting focus to risk evaluation, the standard deviation of returns is utilized as a measure of volatility. Notably, the S&P 500 index demonstrates a higher risk profile with a standard deviation of 21.98%. This indicates that the index's returns

have exhibited relatively higher volatility and fluctuations. In contrast, the "EWC\_INDA" strategy showcases a lower risk profile with a standard deviation of 7.53%. This suggests that the strategy's returns have demonstrated relatively stable behavior, highlighting risk mitigation compared to the broader market index. Impressively, the "EWJ\_EWY" strategy boasts the lowest risk, with a remarkably low standard deviation of 5.04%. This underscores the strategy's ability to achieve stable returns and mitigate volatility effectively.

In summary, these metrics provide a comprehensive view of investment outcomes. While the S&P 500 index has delivered substantial returns, it is accompanied by higher volatility, potentially requiring investors to navigate increased market fluctuations. The "EWC\_INDA" strategy offers attractive returns with a notable reduction in risk, making it an appealing choice for risk-conscious investors seeking a balance between returns and stability. Finally, the "EWJ\_EWY" strategy excels in both returns and risk mitigation, positioning it as a compelling option for those aiming to achieve stable performance while minimizing exposure to market volatility.

#### C. Portfolio Allocation

The portfolio simulation process involves the generation and evaluation of a large number of portfolios to gain insights into their potential performance. In this analysis, 5,000 portfolios are simulated, each consisting of a combination of two strategies. The goal is to identify the portfolio with the most favorable risk-return profile, aiding in the decision-making process for portfolio allocation.

For each portfolio, two essential metrics are calculated: annualized return and risk, represented as annualized volatility. The annualized return indicates the expected average annual gain expressed as a percentage, providing an estimate of potential profits. On the other hand, annualized volatility quantifies the variability of returns over time, serving as a measure of risk. These metrics play a pivotal role in assessing the attractiveness of different portfolios.

The simulation also yields valuable insights into the portfolio composition. The "Maximum Sharpe Ratio Portfolio" is a notable outcome of this analysis. It signifies the portfolio that achieves the optimal balance between risk and return, making it an attractive choice for investors seeking efficient allocation strategies. In this context, the term "Sharpe ratio" is of significance. It represents the risk-adjusted return, indicating how well a portfolio generates excess returns per unit of risk taken.

The "Maximum Sharpe Ratio Portfolio" stands out due to its compelling performance metrics. It demonstrates an annualized return of 7.02%, suggesting the potential for favorable gains. Simultaneously, the annualized volatility is calculated at 4.31%, reflecting effective risk management. The weight allocation for this portfolio indicates the distribution of investments across different assets, illustrating the diversified nature of the strategy.

| Maximum  | Sharpe | Ratio | Poi | rtfolio: |
|----------|--------|-------|-----|----------|
|          |        |       |     |          |
| port_ret | s      |       |     | 7.02     |
| port_vol | s      |       |     | 4.31     |
| weights  |        | [39.9 | 94, | 60.06]   |
| sharpe r | ratio  |       |     | 1.63     |

Fig. 30. EWC\_INDA and EWJ\_EWY Portfolio Allocation Performance

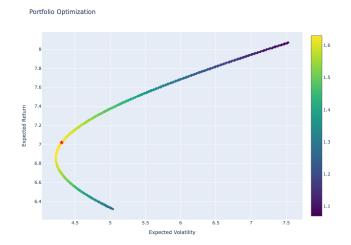


Fig. 31. EWC\_INDA and EWJ\_EWY Portfolio Allocation Chart

Incorporating these findings into the portfolio allocation section equips investors with actionable insights. The identification of the "Maximum Sharpe Ratio Portfolio" showcases a well-structured allocation strategy that seeks to optimize returns while effectively managing risk. By leveraging these results, investors can make informed decisions aligned with their investment goals and risk tolerance, fostering a more strategic and effective approach to portfolio construction.

## V. CONCLUSION

Upon analyzing the metrics for different strategies and benchmark indices, we gain valuable insights into their riskreturn profiles, enabling informed decision-making for future investment strategies.

The EWC\_INDA strategy demonstrates promising returns of 42.26%, outperforming both individual EWC and INDA investments. Additionally, the relatively lower risk of 7.53% further enhances its attractiveness. This strategy showcases an appealing balance between returns and risk, making it a compelling option for investors seeking steady growth while managing potential volatility.

Conversely, the EWJ\_EWY strategy yields a return of 31.83%, performing favorably compared to its individual components, EWJ and EWY. Notably, the risk associated with this strategy is remarkably low at 5.04%, indicating effective risk mitigation. Investors looking for a strategy with consistent

returns and minimized risk might find EWJ\_EWY to be an appealing choice.

Comparing these strategies to the benchmark S&P 500 index, we observe that while S&P 500 boasts an impressive return of 82.56%, its risk, represented by a high value of 21.98%, underscores the potential for substantial fluctuations. In contrast, both EWC\_INDA and EWJ\_EWY strategies achieve competitive returns while maintaining comparatively lower risks, implying more stable investment avenues.

The calculated Sharpe ratios further reinforce these observations. The EWC\_INDA strategy achieves a Sharpe ratio of 5.62, indicating a solid risk-adjusted performance. Similarly, EWJ\_EWY boasts a Sharpe ratio of 6.95, showcasing its effective risk management.

In the context of portfolio optimization, the "Maximum Sharpe Ratio Portfolio" stands out, delivering an annualized return of 7.02% with a relatively low risk of 4.31%. The weight allocation of 39.94% and 60.06% between the two assets further highlights the diversification strategy. This portfolio's Sharpe ratio of 1.63 solidifies its potential for favorable risk-adjusted returns.

In conclusion, the analysis of these strategies and the portfolio optimization emphasizes the importance of balancing risk and return. While S&P 500 exhibits impressive returns, the examined strategies provide viable alternatives with commendable performance and lower risk. Going forward, investors may consider further exploration of the EWC\_INDA, EWJ\_EWY, and the "Maximum Sharpe Ratio Portfolio" strategies. The insights gained from this analysis serve as a foundation for strategic investment decisions, enabling investors to navigate the complex landscape of financial markets with enhanced confidence and informed choices.

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