

# Supplementary Appendix: R&D Networks under Heterogeneous Firm Productivities

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## A Derivations

Throughout the provided derivations, we solve for a more general version of the model by introducing a new parameter that governs the efficacy of collaboration. That is, the marginal cost reduction equation becomes:

$$c_i(\mathbf{e}, \mathbf{G}) = \bar{c} - \theta_i e_i - \delta \sum_{j \in \mathcal{N}_i(\mathbf{G})} \theta_j e_j. \quad (1)$$

### A.1 Quantity Competition with Heterogeneous Firms

We solve for the final stage of the game, in which the marginal cost of production is heterogeneous among the set of  $\mathcal{N} = \{1, 2, \dots, n\}$  competing firms. Each firm  $i$  decides how much to produce ( $q_i(\mathbf{c})$ ), given the production cost of all firms  $\mathbf{c} = \{c_j\}_{j \in \mathcal{N}}$ . That is to solve:

$$\max_{q_i} \{q_i \cdot [p(\mathbf{q}) - c_i]\} \quad \text{s.t.} \quad p(\mathbf{q}) = \alpha - Q,$$

where  $Q = \sum_{j \in \mathcal{N}} q_j$  is the total amount of supply <sup>1</sup>, and  $\alpha$  is a constant parameter characterising the size of the market. Each firm  $i$  maximizes its profit by selecting the best production amount  $q_i^*$  given the amount of production by others, i.e.  $Q^{(-i)} = \sum_{j \neq i} q_j$ , and its own production cost. Thus, firm  $i$ 's profit is given by:

$$\begin{aligned} \pi_i^{(m)} &= \max_{q_i} \{q_i \cdot p(\mathbf{q}) - q_i \cdot c_i\} \\ &= \max_{q_i} \left\{ \alpha q_i - Q^{(-i)} q_i - q_i^2 - c_i q_i \right\}. \end{aligned}$$

Solving the first-order condition yields:

$$\left[ \frac{\partial \pi_i^{(m)}}{\partial q_i} \right] : \quad \alpha - Q^{(-i)} - 2q_i^* - c_i = 0,$$

which results in a system of  $n$  equations and  $n$  unknowns, where each equation looks like:

$$2q_i^* + \sum_{j \neq i} q_j^* = \alpha - c_i.$$

Rearranging the equations into matrix form, we have:

$$\mathbf{B}_n \mathbf{q}^* = \mathbf{h},$$

where  $\mathbf{q}^* = (q_1^*, \dots, q_n^*)^\top$  is the vector of production quantities,  $\mathbf{h} = (\alpha - c_1, \dots, \alpha - c_n)^\top$  is a vector related to the firms' production costs, and  $\mathbf{B}_n$  is a constant  $n \times n$  matrix which its diagonal elements are equal to 2 and all other elements are equal to 1, i.e.

$$\mathbf{B}_n = \mathbf{I}_n + \mathbf{1}_n \mathbf{1}_n^\top = \begin{pmatrix} 2 & 1 & \dots & 1 \\ 1 & 2 & & \vdots \\ \vdots & & \ddots & \\ 1 & 1 & \dots & 2 \end{pmatrix},$$

where  $\mathbf{I}_n$  is an  $n \times n$  identity matrix and  $\mathbf{1}_n$  is an  $n \times 1$  vector of ones.

Therefore, the equilibrium production amount could be determined using the following equation:

$$\mathbf{q}^* = \mathbf{B}_n^{-1} \mathbf{h}$$

Using the [Sherman and Morrison \(1950\)](#) formula, we know that the determinant of  $\mathbf{B}_n$  is

$$|\mathbf{B}_n| = 1 + \mathbf{1}_n^\top \mathbf{I}_n^{-1} \mathbf{1}_n = n + 1,$$

indicating that  $\mathbf{B}_n$  is invertible, and the inverse is:

$$\mathbf{B}_n^{-1} = \mathbf{I}_n^{-1} - \frac{\mathbf{I}_n^{-1} \mathbf{1}_n \mathbf{1}_n^\top \mathbf{I}_n^{-1}}{1 + \mathbf{1}_n^\top \mathbf{I}_n^{-1} \mathbf{1}_n} = \mathbf{I}_n - \frac{1}{n+1} \mathbf{1}_n \mathbf{1}_n^\top = \frac{1}{n+1} \begin{pmatrix} n & -1 & \dots & -1 \\ -1 & n & & \vdots \\ \vdots & & \ddots & \\ -1 & -1 & \dots & n \end{pmatrix},$$

---

<sup>1</sup>Here we assume that the amount of production ( $q_i$ ) is continuous.

which is an  $n \times n$  matrix with diagonal elements equal to  $\frac{n}{n+1}$  and non-diagonal elements equal to  $\frac{-1}{n+1}$ . Thus, the equilibrium production amount for each firm  $i$  is

$$\begin{aligned} q_i^* &= \frac{1}{n+1} \left( n(\alpha - c_i) - \sum_{j \neq i} (\alpha - c_j) \right) \\ &= \frac{1}{n+1} \left( \alpha - n c_i + \sum_{j \neq i} c_j \right) \\ &= \frac{1}{n+1} \left( \alpha - c_i + \sum_{j=1}^n (c_j - c_i) \right). \end{aligned} \quad (2)$$

Moreover, the total amount of production in the equilibrium is

$$Q^* = \sum_{i=1}^n q_i^* = \frac{1}{n+1} \sum_{i=1}^n (\alpha - c_i),$$

and the equilibrium price is<sup>2</sup>

$$p^*(\mathbf{c}) = \alpha - Q^* = \frac{\alpha}{n+1} + \frac{\sum_{i=1}^n c_i}{n+1} = \frac{1}{n+1} \alpha + \frac{n}{n+1} \left( \frac{1}{n} \sum_{i=1}^n c_i \right). \quad (3)$$

Using equation (1), we can rewrite equation (2) as:

$$\begin{aligned} q_i^* &= \frac{1}{n+1} \left[ \alpha - n \left( \bar{c} - \theta_i e_i - \delta \sum_{j=1}^n \theta_j e_j \mathbf{G}_{ij} \right) + \sum_{j \neq i} \left( \bar{c} - \theta_j e_j - \delta \sum_{k=1}^n \theta_k e_k \mathbf{G}_{kj} \right) \right] \\ &= \frac{1}{n+1} \left[ (\alpha - \bar{c}) + (n+1) \theta_i e_i + (n+1) \delta \sum_{j \in \mathcal{N}_i(\mathbf{G})} \theta_j e_j - \sum_{k=1}^n \theta_k e_k - \delta \sum_{k=1}^n \theta_k e_k \sum_{j=1}^n \mathbf{G}_{kj} \right] \\ &= \frac{1}{n+1} \left[ (\alpha - \bar{c}) + (n+1) \theta_i e_i + (n+1) \delta \sum_{j \in \mathcal{N}_i(\mathbf{G})} \theta_j e_j - \sum_{k=1}^n \theta_k e_k - \delta \sum_{k=1}^n \theta_k e_k d_k(\mathbf{G}) \right] \end{aligned}$$

Therefore:

$$\begin{aligned} q_i^* &= \frac{1}{n+1} \left( (\alpha - \bar{c}) + (n+1) \theta_i e_i + (n+1) \delta \sum_{j \in \mathcal{N}_i(\mathbf{G})} \theta_j e_j - \sum_{k=1}^n \theta_k e_k (1 + \delta d_k(\mathbf{G})) \right) \\ &= \underbrace{\frac{\alpha - \bar{c}}{n+1}}_{\text{baseline output}} + \underbrace{\frac{n - \delta d_i(\mathbf{G})}{n+1} \theta_i e_i}_{\text{own effort effect}} + \underbrace{\delta \sum_{j \in \mathcal{N}_i(\mathbf{G})} \theta_j e_j}_{\text{neighbor benefit}} - \underbrace{\frac{1}{n+1} \sum_{k \neq i} (1 + \delta d_k(\mathbf{G})) \theta_k e_k}_{\text{general competition effect}} \end{aligned} \quad (4)$$

$$\begin{aligned} &= \underbrace{\frac{\alpha - \bar{c}}{n+1}}_{\text{baseline}} + \underbrace{\frac{n - \delta d_i(\mathbf{G})}{n+1} \theta_i e_i}_{\text{own effort effect}} + \underbrace{\sum_{j \in \mathcal{N}_i(\mathbf{G})} \frac{\delta (n+1 - d_j(\mathbf{G})) - 1}{n+1} \theta_j e_j}_{\text{neighbors' effort effect}} - \underbrace{\sum_{k \in \mathcal{N}_{-i}(\mathbf{G})} \frac{1 + \delta d_k(\mathbf{G})}{n+1} \theta_k e_k}_{\text{non-neighbors' effort effect}}, \end{aligned} \quad (5)$$

where  $\mathcal{N}_{-i}(\mathbf{G}) = \{j \in \mathcal{N} \mid \mathbf{G}_{ij} = 0, j \neq i\}$  is the set of firm  $i$ 's non-neighbors in network  $\mathbf{G}$ .

Finally, firm  $i$ 's profit is:

$$\begin{aligned} \pi_i^{(m)} &= q_i^* (p^* - c_i) = \frac{1}{n+1} \left( \alpha - c_i + \sum_{j=1}^n (c_j - c_i) \right) \left( \frac{\alpha + \sum_{j=1}^n c_j}{n+1} - c_i \right) \\ &= \frac{1}{(n+1)^2} \left( \alpha - c_i + \sum_{j=1}^n (c_j - c_i) \right)^2 = q_i^{*2}. \end{aligned} \quad (6)$$

<sup>2</sup>There is a possibility that a firm will have higher marginal production costs than the equilibrium prices, which consequently results in a corner solution to the firm's maximization problem. We assume this case does not happen in this study.

## A.2 Optimal Effort

Given the solution of the final stage, we solve for the optimal R&D effort ( $e_i$ ) in firms' profit maximization problem. That is:

$$\pi_i = \max_{e_i} \left\{ \pi_i^{(m)} - \phi e_i^2 \right\} \quad \text{s.t.} \quad \text{Equation (6)}.$$

Using equations (2) and (6) we get the following first-order condition:

$$\left[ \frac{\partial \pi_i}{\partial e_i} \right] : \quad \left. \frac{\partial \pi_i^{(m)}}{\partial e_i} \right|_{e_i^*} - 2 \phi e_i^* = 0,$$

where  $e_i^* = \arg \max_{e_i} \{ \pi_i^{(m)} - \phi e_i^2 \}$ , and the partial derivative of market profit with respect to the R&D effort is given by:

$$\frac{\partial \pi_i^{(m)}}{\partial e_i} = 2 q_i^* \frac{\partial}{\partial e_i} \left( \frac{1}{n+1} \left[ \alpha - n c_i + \sum_{j \neq i} c_j \right] \right) = \frac{2 q_i^*}{n+1} \left( -n \frac{\partial c_i}{\partial e_i} + \sum_{j \neq i} \frac{\partial c_j}{\partial e_i} \right).$$

Using the equation for marginal production cost((??), we have the following:

$$\left. \begin{array}{l} \partial c_i / \partial e_i = -\theta_i \\ \partial c_j / \partial e_i = -\delta \theta_i \mathbf{G}_{ij} \end{array} \right\} \Rightarrow \frac{\partial \pi_i}{\partial e_i} = \frac{2 q_i^*}{n+1} \theta_i \left( n - \delta \sum_{j \neq i} \mathbf{G}_{ij} \right) - 2 \phi e_i.$$

Therefore, the first-order condition can be reformulated as:

$$\left[ \frac{\partial \pi_i}{\partial e_i} \right] : \quad \frac{2 q_i^*}{n+1} \theta_i (n - \delta d_i(\mathbf{G})) - 2 \phi e_i^* = 0, \quad (7)$$

where  $q_i$  denotes the equilibrium production amount in equation (2), and  $d_i(\mathbf{G}) = |\mathcal{N}_i(\mathbf{G})| = \sum_{j \in \mathcal{N}} \mathbf{G}_{ij}$  is firm  $i$ 's degree in the network  $\mathbf{G}$ .

Replacing equation (4) into (7), we have:

$$\phi e_i^* = \frac{n - \delta d_i(\mathbf{G})}{n+1} \theta_i \left( \frac{\alpha - \bar{c}}{n+1} + \frac{n - \delta d_i(\mathbf{G})}{n+1} \theta_i e_i^* + \delta \sum_{j \in \mathcal{N}_i(\mathbf{G})} \theta_j e_j - \frac{1}{n+1} \sum_{k \neq i} (1 + \delta d_k(\mathbf{G})) \theta_k e_k \right).$$

By defining  $\eta_i(\mathbf{G}) := \frac{n - \delta d_i(\mathbf{G})}{n+1}$ , the first-order condition becomes:

$$e_i^* = \underbrace{\frac{1}{\frac{\phi}{\theta_i \eta_i(\mathbf{G})} - \theta_i \eta_i(\mathbf{G})}}_{\text{connectivity}} \left( \underbrace{\frac{\alpha - \bar{c}}{n+1}}_{\text{baseline}} + \underbrace{\delta \sum_{j \in \mathcal{N}_i(\mathbf{G})} \theta_j e_j}_{\text{collaboration benefit}} - \underbrace{\frac{1}{n+1} \sum_{k \neq i} (1 + \delta d_k(\mathbf{G})) \theta_k e_k}_{\text{competition effect}} \right) \quad (8)$$

$$= \underbrace{\frac{\theta_i \eta_i(\mathbf{G})}{\phi - \theta_i^2 \eta_i(\mathbf{G})}}_{\text{connectivity}} \left( \underbrace{\frac{\alpha - \bar{c}}{n+1}}_{\text{baseline}} + \underbrace{\sum_{j \in \mathcal{N}_i(\mathbf{G})} \frac{\delta (n - d_j(\mathbf{G}) + 1) - 1}{n+1} \theta_j e_j}_{\text{neighbor effect}} - \underbrace{\sum_{k \in \mathcal{N}_{-i}(\mathbf{G})} \frac{1 + \delta d_k(\mathbf{G})}{n+1} \theta_k e_k}_{\text{non-neighbor effect}} \right). \quad (9)$$

Equation (9) denotes the best response function of each firm  $i$  ( $e_i^*$ ) given the action of other firms ( $e_j$ ) given the structure of the collaboration network, which results in a system of  $n$  equations and  $n$  unknowns. That is:

$$\left[ \frac{1}{\theta_i (n - \delta d_i(\mathbf{G}))} - (n+1) \theta_i \right] e_i^* = (\alpha - \bar{c}) + \sum_{j=1}^n \theta_j e_j^* [(n+1) \delta \mathbf{G}_{ij} - (1 + \delta d_j(\mathbf{G}))]. \quad (10)$$

## A.3 Deviation from a Complete network

Without the loss of generality, we consider a network  $\mathbf{G} := \mathbf{G}_{-1,2}^C$ , in which all firms are connected to each other except for firms 1 and 2.

Rewriting the initial system of equations, we have:

$$\left[ \frac{(n+1)^2\phi}{2\theta_1} - 2\theta_1 \right] e_1 = (\alpha - \bar{c}) + \sum_{j=3}^n \theta_j e_j - (n-1)\theta_2 e_2 \quad (11)$$

$$\left[ \frac{(n+1)^2\phi}{2\theta_2} - 2\theta_2 \right] e_2 = (\alpha - \bar{c}) + \sum_{j=3}^n \theta_j e_j - (n-1)\theta_1 e_1 \quad (12)$$

For  $i \geq 3$ , the equation is:

$$\frac{(n+1)^2\phi}{\theta_i} e_i = (\alpha - \bar{c}) + \sum_{j=3}^n \theta_j e_j + 2\theta_1 e_1 + 2\theta_2 e_2 \quad (13)$$

We define the coefficients  $b_i$  as follows:

$$\begin{aligned} b_1 &= \frac{(n+1)^2\phi}{2\theta_1} - 2\theta_1 \\ b_2 &= \frac{(n+1)^2\phi}{2\theta_2} - 2\theta_2 \\ b_i &= \frac{(n+1)^2\phi}{\theta_i} \quad \text{for } i \geq 3 \end{aligned}$$

Substituting these definitions into the system:

$$\begin{cases} b_1 e_1 + (n-1)\theta_2 e_2 = (\alpha - \bar{c}) + \sum_{j=3}^n \theta_j e_j \\ b_2 e_2 + (n-1)\theta_1 e_1 = (\alpha - \bar{c}) + \sum_{j=3}^n \theta_j e_j \end{cases} \quad (14)$$

Since the right-hand sides are identical, we equate the left-hand sides:

$$\begin{aligned} b_1 e_1 + (n-1)\theta_2 e_2 &= b_2 e_2 + (n-1)\theta_1 e_1 \\ [b_1 - (n-1)\theta_1] e_1 &= [b_2 - (n-1)\theta_2] e_2 \\ e_1 &= \frac{b_2 - (n-1)\theta_2}{b_1 - (n-1)\theta_1} e_2 \end{aligned}$$

Substituting the expressions for  $b_1$  and  $b_2$  back into the ratio:

$$e_1 = \frac{\theta_1 (n+1)\phi - 2\theta_2^2}{\theta_2 (n+1)\phi - 2\theta_1^2} e_2 \quad (15)$$

We define  $\lambda$  to represent this ratio:

$$\lambda = \frac{\theta_1 (n+1)\phi - 2\theta_2^2}{\theta_2 (n+1)\phi - 2\theta_1^2} \implies e_1 = \lambda e_2 \quad (16)$$

Define  $T$  as the sum over the remaining terms:

$$T = \sum_{j=3}^n \theta_j e_j \quad (17)$$

From the first equation of the system:

$$\begin{aligned} b_1 e_1 + (n-1)\theta_2 e_2 &= (\alpha - \bar{c}) + T \\ b_1 (\lambda e_2) + (n-1)\theta_2 e_2 &= (\alpha - \bar{c}) + T \\ e_2 [b_1 \lambda + (n-1)\theta_2] &= (\alpha - \bar{c}) + T \end{aligned} \quad (18)$$

Define  $S$  to include the first two terms:

$$S = T + 2\theta_1 e_1 + 2\theta_2 e_2 \quad (19)$$

For  $i \geq 3$ , the equation becomes:

$$b_i e_i = (\alpha - \bar{c}) + S \implies e_i = \frac{(\alpha - \bar{c}) + S}{b_i} \quad (20)$$

now, substitute  $e_i$  back into the definition of  $T$ :

$$T = \sum_{j=3}^n \theta_j e_j = \sum_{j=3}^n \frac{\theta_j}{b_j} [(\alpha - \bar{c}) + S]$$

$$T = \left( \sum_{j=3}^n \frac{\theta_j}{b_j} \right) ((\alpha - \bar{c}) + S)$$

Define

$$Q := \sum_{j=3}^n \frac{\theta_j}{b_j} = \sum_{j=3}^n \frac{\theta_j^2}{(n+1)^2 \phi} = \frac{\sum_{j=3}^n \theta_j^2}{(n+1)^2 \phi}.$$

Then:

$$T = Q(\alpha - \bar{c}) + QS \quad (21)$$

Substitute  $T$  back into the expression for  $S$ :

$$S = [Q(\alpha - \bar{c}) + QS] + 2\theta_1 e_1 + 2\theta_2 e_2$$

$$S(1 - Q) = Q(\alpha - \bar{c}) + 2\theta_1 e_1 + 2\theta_2 e_2$$

$$S = \frac{Q}{1 - Q}(\alpha - \bar{c}) + \frac{2\theta_1 e_1 + 2\theta_2 e_2}{1 - Q}$$

Recall  $e_1 = \lambda e_2$ . Substituting this into  $S$ :

$$S = \frac{Q}{1 - Q}(\alpha - \bar{c}) + \frac{2(\lambda\theta_1 + \theta_2)e_2}{1 - Q} \quad (22)$$

From equation (18), we use the consistent sign convention:

$$e_2 = \frac{(\alpha - \bar{c}) + T}{b_1 \lambda + (n-1)\theta_2} = \frac{(\alpha - \bar{c})(1 + Q) + QS}{b_1 \lambda + (n-1)\theta_2} \quad (23)$$

Substituting  $e_2$  into the  $S$  equation:

$$S = \frac{Q}{1 - Q}(\alpha - \bar{c}) + \frac{2(\lambda\theta_1 + \theta_2)}{1 - Q} \left[ \frac{(\alpha - \bar{c})(1 + Q) + QS}{b_1 \lambda + (n-1)\theta_2} \right] \quad (24)$$

Define  $\Delta$ :

$$\Delta = \frac{2(\lambda\theta_1 + \theta_2)}{b_1 \lambda + (n-1)\theta_2} \quad (25)$$

The equation for  $S$  becomes:

$$S = \frac{Q}{1 - Q}(\alpha - \bar{c}) + \frac{\Delta}{1 - Q} [(\alpha - \bar{c})(1 + Q) + QS]$$

$$S \left( 1 - \frac{\Delta Q}{1 - Q} \right) = (\alpha - \bar{c}) \left[ \frac{Q}{1 - Q} + \frac{\Delta(1 + Q)}{1 - Q} \right]$$

Simplifying the terms:

$$1 - \frac{\Delta Q}{1 - Q} = \frac{1 - Q - \Delta Q}{1 - Q} = \frac{1 - (1 + \Delta)Q}{1 - Q}$$

$$\frac{Q + \Delta(1 + Q)}{1 - Q} = \frac{Q + \Delta + \Delta Q}{1 - Q} = \frac{\Delta + (1 + \Delta)Q}{1 - Q}$$

Thus:

$$S \left( \frac{1 - (1 + \Delta)Q}{1 - Q} \right) = (\alpha - \bar{c}) \left( \frac{\Delta + (1 + \Delta)Q}{1 - Q} \right) \quad (26)$$

Solving for  $S$ :

$$S = (\alpha - \bar{c}) \frac{\Delta + (1 + \Delta)Q}{1 - (1 + \Delta)Q} \quad (27)$$

Substitute the derived  $S$  back into the expression for  $T$ :

$$\begin{aligned}
T &= Q(\alpha - \bar{c}) + QS \\
&= Q(\alpha - \bar{c}) \left[ 1 + \frac{\Delta + (1 + \Delta)Q}{1 - (1 + \Delta)Q} \right] \\
&= Q(\alpha - \bar{c}) \left[ \frac{1 - (1 + \Delta)Q + \Delta + (1 + \Delta)Q}{1 - (1 + \Delta)Q} \right] \\
&= Q(\alpha - \bar{c}) \left[ \frac{1 + \Delta}{1 - (1 + \Delta)Q} \right] \\
\boxed{T &= (\alpha - \bar{c}) \frac{Q(1 + \Delta)}{1 - (1 + \Delta)Q}} \tag{28}
\end{aligned}$$

### Solution for $e_2$

We substitute the value of  $T$  into the expression for  $e_2$ :

$$\begin{aligned}
e_2 &= \frac{(\alpha - \bar{c}) + T}{b_1\lambda + (n - 1)\theta_2} \\
&= \frac{1}{b_1\lambda + (n - 1)\theta_2} \left[ (\alpha - \bar{c}) + (\alpha - \bar{c}) \frac{Q(1 + \Delta)}{1 - (1 + \Delta)Q} \right] \\
&= \frac{\alpha - \bar{c}}{b_1\lambda + (n - 1)\theta_2} \left[ 1 + \frac{Q(1 + \Delta)}{1 - (1 + \Delta)Q} \right] \\
&= \frac{\alpha - \bar{c}}{b_1\lambda + (n - 1)\theta_2} \left[ \frac{1 - (1 + \Delta)Q + Q(1 + \Delta)}{1 - (1 + \Delta)Q} \right] \\
&= \frac{\alpha - \bar{c}}{b_1\lambda + (n - 1)\theta_2} \left[ \frac{1}{1 - (1 + \Delta)Q} \right]
\end{aligned}$$

Thus, the final expression is:

$$\boxed{e_2 = \frac{\alpha - \bar{c}}{b_1\lambda + (n - 1)\theta_2} \left( \frac{1}{1 - (1 + \Delta)Q} \right)} \tag{29}$$

### Solution for $e_1$

Substituting the closed-form equation for  $e_2$  into  $e_1 = \lambda e_2$  we get:

$$\boxed{e_1 = \frac{\alpha - \bar{c}}{b_1\lambda + (n - 1)\theta_2} \left( \frac{\lambda}{1 - (1 + \Delta)Q} \right)} \tag{30}$$

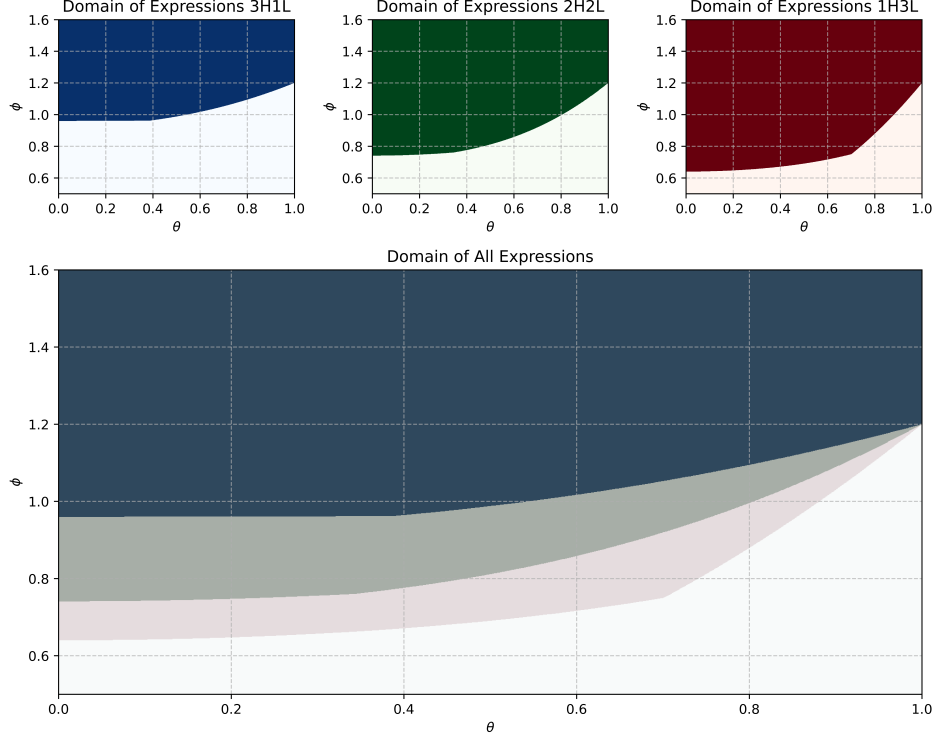
### Solution for $e_i$ ( $i \geq 3$ )

$$\begin{aligned}
e_i &= \frac{1}{b_i} [(\alpha - \bar{c}) + S] \\
&= \frac{\alpha - \bar{c}}{b_i} \left[ 1 + \frac{\Delta + (1 + \Delta)Q}{1 - (1 + \Delta)Q} \right] \\
&= \frac{\alpha - \bar{c}}{b_i} \left[ \frac{1 - (1 + \Delta)Q + \Delta + (1 + \Delta)Q}{1 - (1 + \Delta)Q} \right] \\
\boxed{e_i &= \frac{\alpha - \bar{c}}{b_i} \left( \frac{1 + \Delta}{1 - (1 + \Delta)Q} \right)} \tag{31}
\end{aligned}$$

## B Stability Domain

For the system of equations in all of the cases studied in the following sections, the value of the cost controlling parameter  $\phi$  should be high enough to guarantee that the result of the effort optimizations are positive. In order to achieve this, the numerator of all the optimal efforts in Section D should be positive. Figure 1 depicts the domain in which the said conditions hold for different configurations of the market. This figure suggests the value of the cost parameter  $\phi$  should be larger than 1.2 in order to guarantee the non-corner solution of the system in all possible configurations.

Figure 1: Non-corner Solution Domain



In order to prove the stability of each network configuration, we should check all possible deviations from the current structure, which is the number of possible edges in that network. If a link exists, we should check the structure in which it is not and vice versa. However, due to symmetry, the possible number of deviations is less than all possible deviations. The following figures (2, 3, 4), plots the transition graph of all unique structures for  $n = 4$ . That is:

- [1H3L]: 1 High- and 3 Low-productive firms ( $\rho = 0.25$ )
- [1H3L]: 2 High- and 2 Low-productive firms ( $\rho = 0.50$ )
- [1H3L]: 3 High- and 1 Low-productive firms ( $\rho = 0.75$ )



Transition Graph for 4 Nodes with 1 H-Type Nodes (with Images)

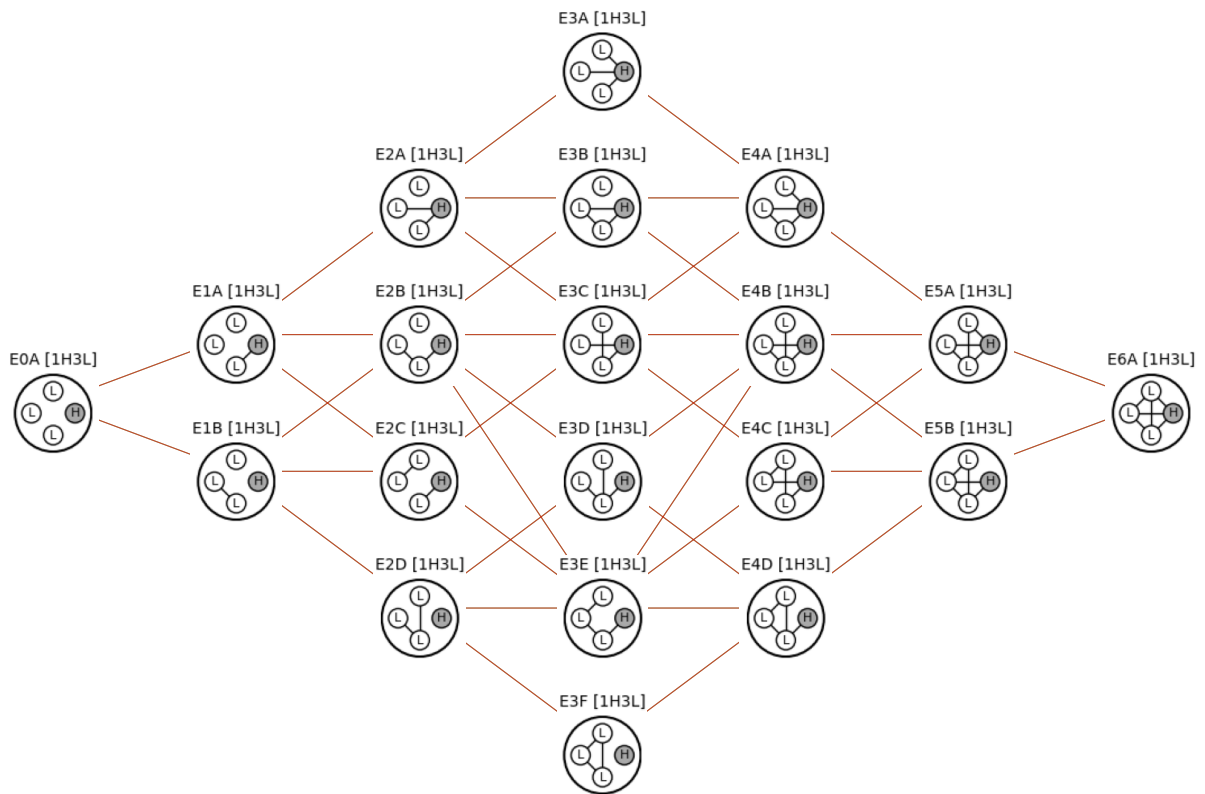


Figure 2: Transition graph for the 1H3L case.

Transition Graph for 4 Nodes with 2 H-Type Nodes (with Images)

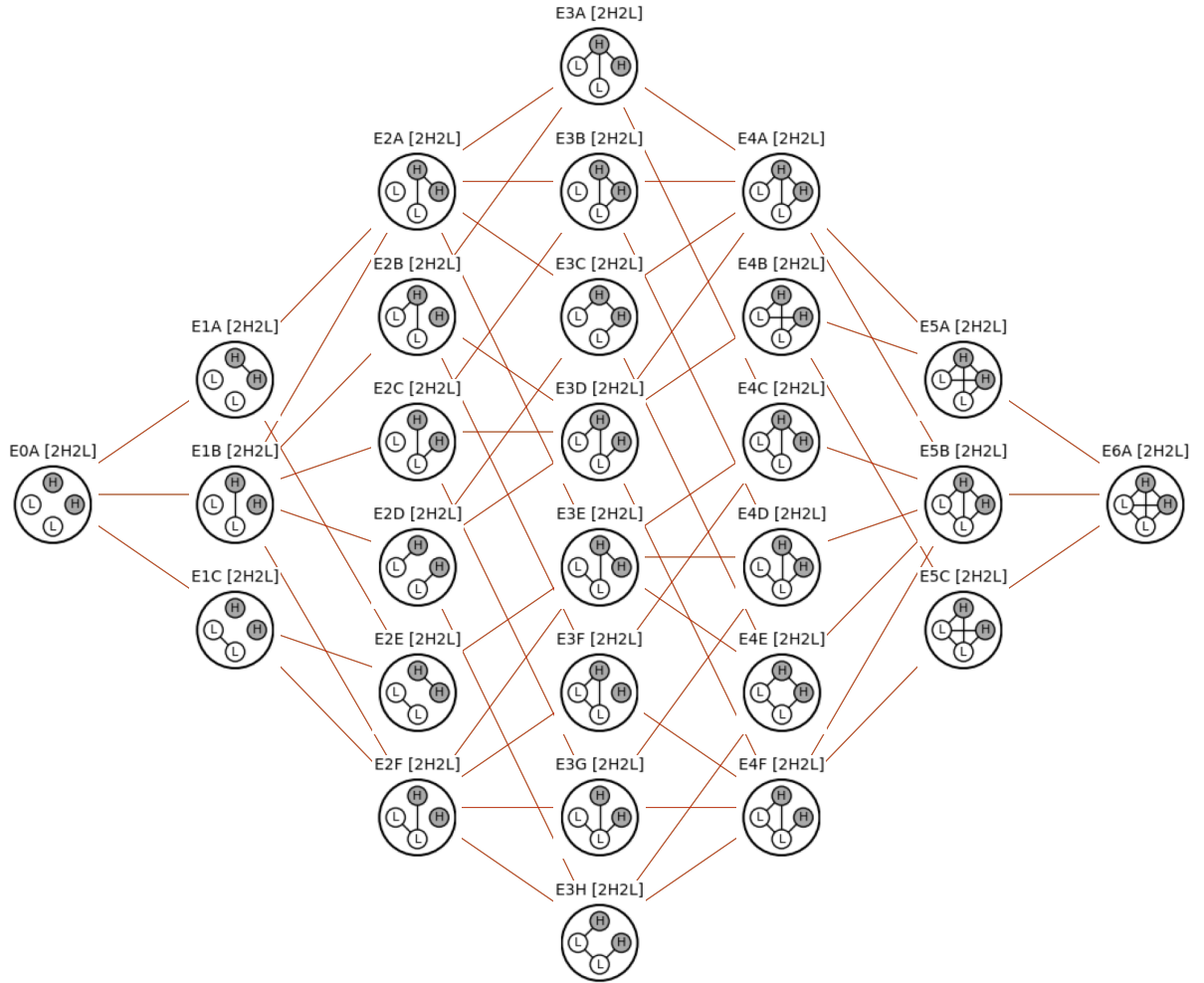


Figure 3: Transition graph for the 2H2L case.

Transition Graph for 4 Nodes with 3 H-Type Nodes (with Images)

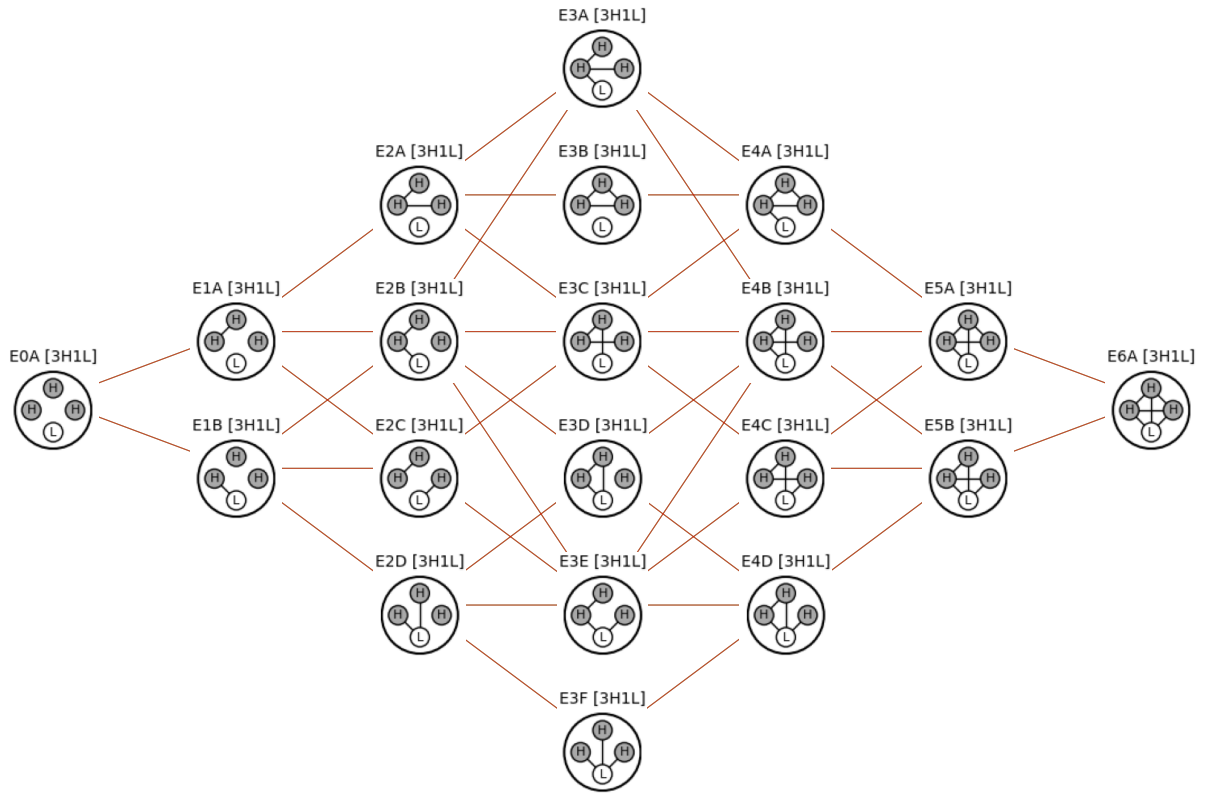


Figure 4: Transition graph for the 3H1L case.

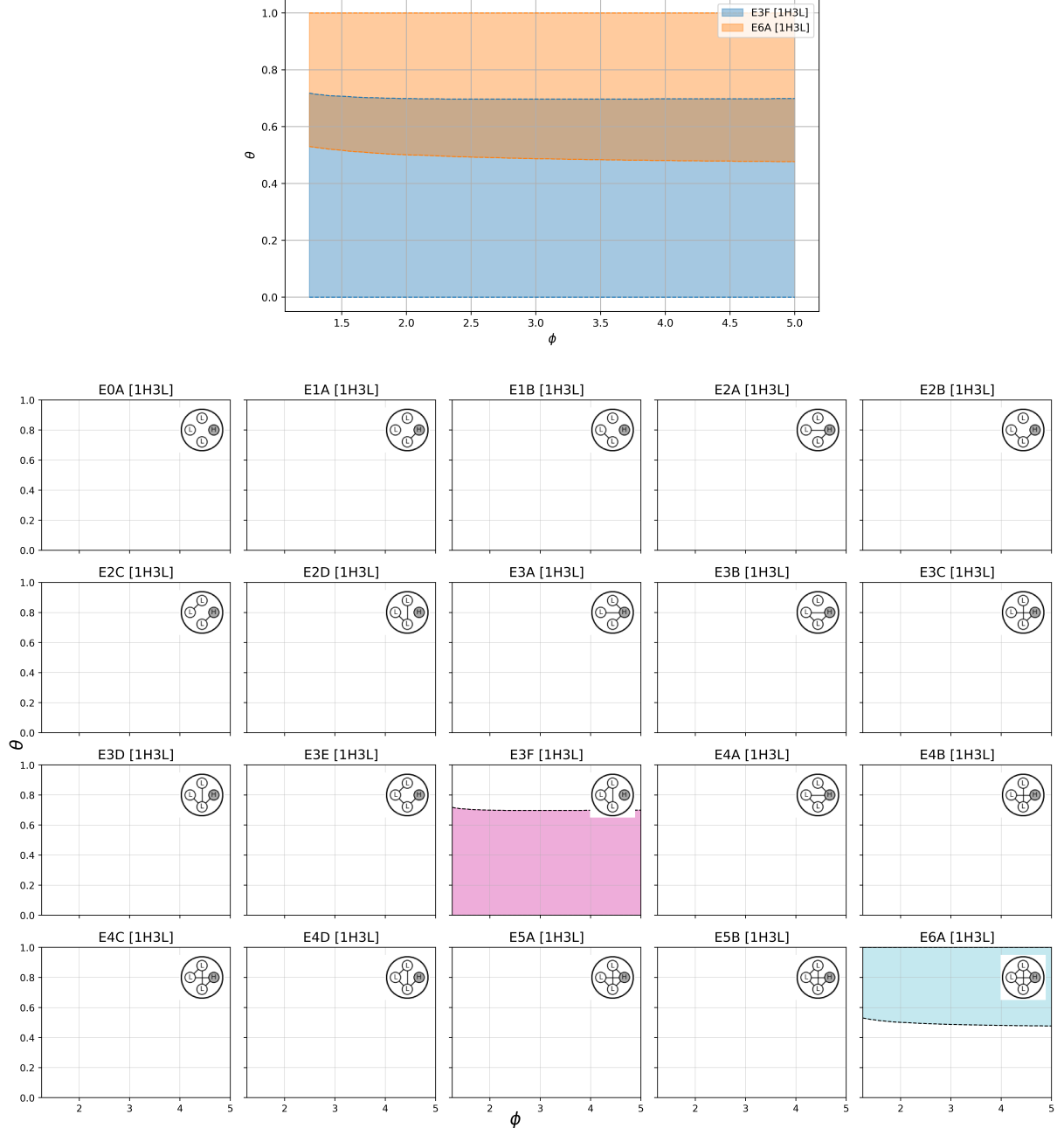


Figure 5: Stability domain (top) and full cluster visualization (bottom) for the 1H3L case.

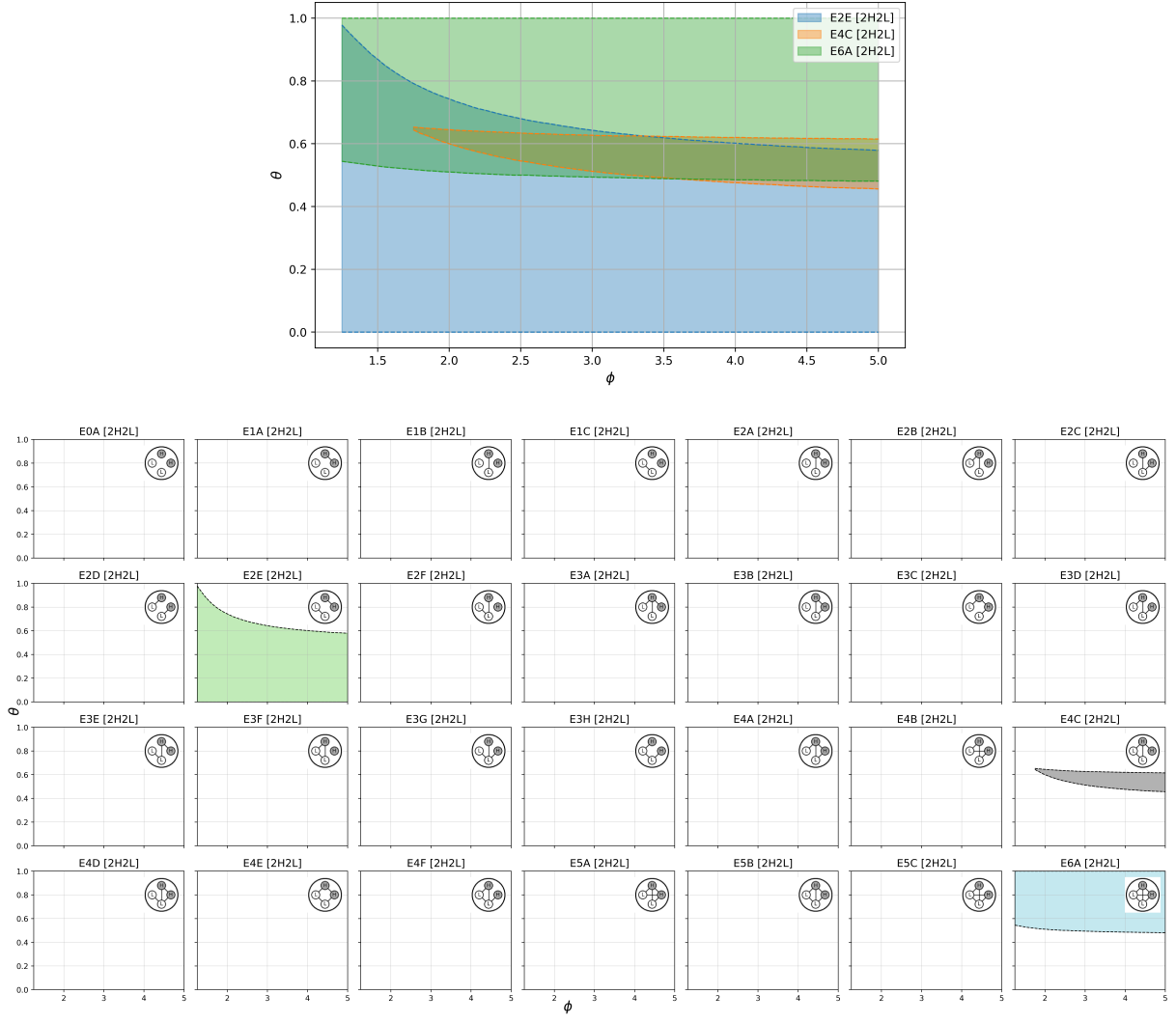


Figure 6: Stability domain (top) and full cluster visualization (bottom) for the 2H2L case.

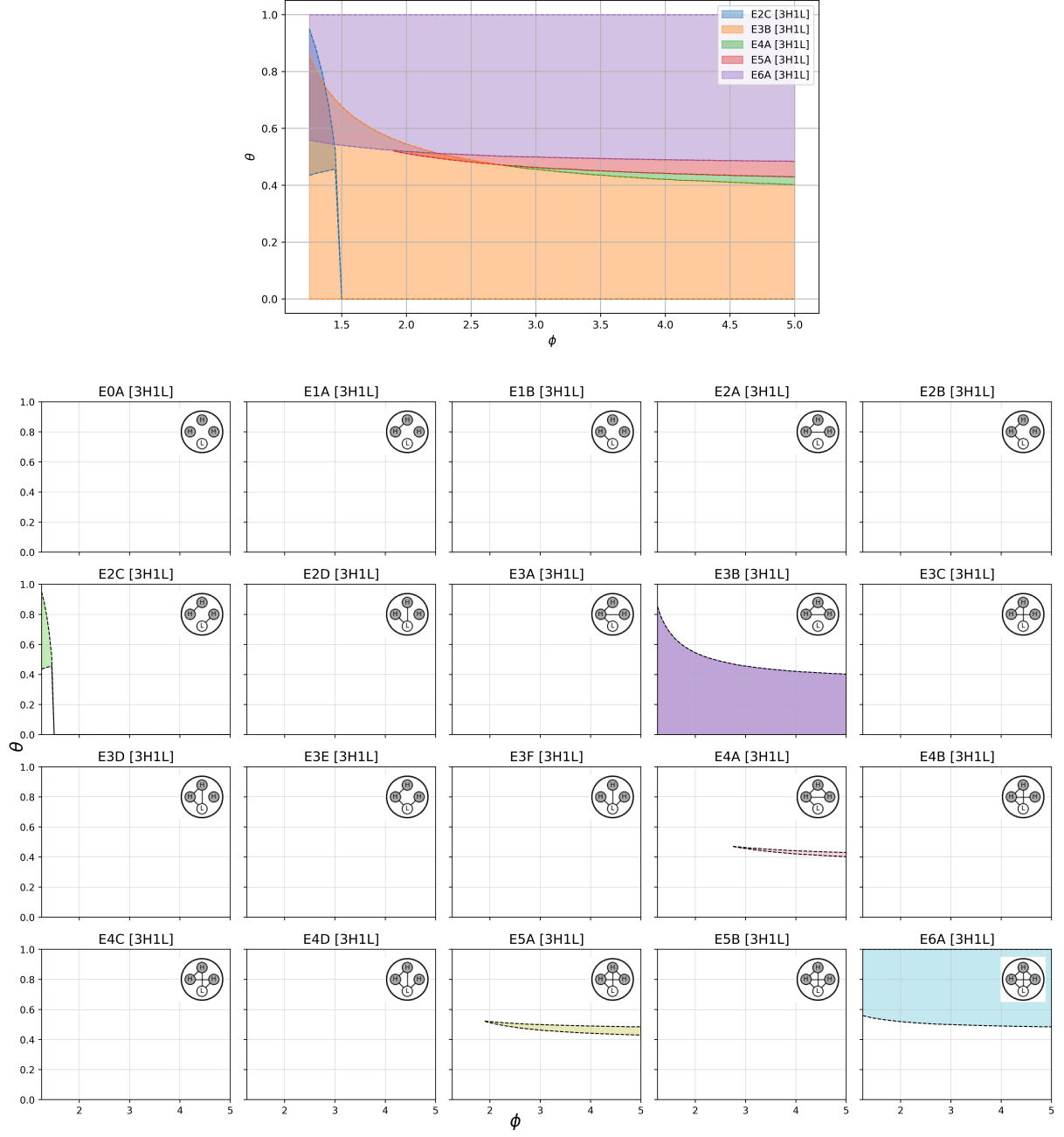


Figure 7: Stability domain (top) and full cluster visualization (bottom) for the 3H1L case.

## C Stability conditions

### C.1 One *high*- and three *low*-productive firms (1H3L)

#### C.1.1 Stability conditions for: E0A [1H3L]

- Adding an edge between nodes  $H_1, L_1$ : E0A [1H3L]  $\rightarrow$  E1A [1H3L]

– For  $H_1$  not to be profitable:

$$\frac{\phi(\alpha - \bar{c})^2 (5\phi - 4\theta^2)^2 \cdot (25\phi - 16)}{(125\phi^2 - 40\phi\theta^2 - 80\phi + 16\theta^2)^2} > \frac{\phi(\alpha - \bar{c})^2 (5\phi - 4\theta^2)^2 \cdot (25\phi - 9)}{(125\phi^2 - 105\phi\theta^2 - 45\phi + 12\theta^4 + 12\theta^2)^2}$$

– For  $L_1$  not to be profitable:

$$\frac{\phi(\alpha - \bar{c})^2 (5\phi - 4)^2 \cdot (25\phi - 16\theta^2)}{(125\phi^2 - 40\phi\theta^2 - 80\phi + 16\theta^2)^2} > \frac{\phi(\alpha - \bar{c})^2 (5\phi - 4\theta^2)^2 \cdot (25\phi - 9\theta^2)}{(125\phi^2 - 105\phi\theta^2 - 45\phi + 12\theta^4 + 12\theta^2)^2}$$

- Adding an edge between nodes  $L_1, L_2$ : E0A [1H3L]  $\rightarrow$  E1B [1H3L]

– For  $L_1$  not to be profitable:

$$\frac{\phi(\alpha - \bar{c})^2 (5\phi - 4)^2 \cdot (25\phi - 16\theta^2)}{(125\phi^2 - 40\phi\theta^2 - 80\phi + 16\theta^2)^2} > \frac{\phi(\alpha - \bar{c})^2 (5\phi - 4)^2 (5\phi - 4\theta^2)^2 \cdot (25\phi - 9\theta^2)}{(625\phi^3 - 850\phi^2\theta^2 - 400\phi^2 + 240\phi\theta^4 + 480\phi\theta^2 - 96\theta^4)^2}$$

– For  $L_2$  not to be profitable:

$$\frac{\phi(\alpha - \bar{c})^2 (5\phi - 4)^2 \cdot (25\phi - 16\theta^2)}{(125\phi^2 - 40\phi\theta^2 - 80\phi + 16\theta^2)^2} > \frac{\phi(\alpha - \bar{c})^2 (5\phi - 4)^2 (5\phi - 4\theta^2)^2 \cdot (25\phi - 9\theta^2)}{(625\phi^3 - 850\phi^2\theta^2 - 400\phi^2 + 240\phi\theta^4 + 480\phi\theta^2 - 96\theta^4)^2}$$

#### C.1.2 Stability conditions for: E1A [1H3L]

- Adding an edge between nodes  $H_1, L_1$ : E1A [1H3L]  $\rightarrow$  E2A [1H3L]

– For  $H_1$  not to be profitable:

$$\frac{\phi(\alpha - \bar{c})^2 (5\phi - 4\theta^2)^2 \cdot (25\phi - 9)}{(125\phi^2 - 105\phi\theta^2 - 45\phi + 12\theta^4 + 12\theta^2)^2} > \frac{\phi(\alpha - \bar{c})^2 (5\phi - 4\theta^2)^2 (5\phi + 3\theta^2)^2 \cdot (25\phi - 4)}{(625\phi^3 - 475\phi^2\theta^2 - 100\phi^2 - 20\phi\theta^2 + 24\theta^4)^2}$$

– For  $L_1$  not to be profitable:

$$\frac{\phi(\alpha - \bar{c})^2 \cdot (25\phi - 16\theta^2) (5\phi - 3\theta^2 - 3)^2}{(125\phi^2 - 105\phi\theta^2 - 45\phi + 12\theta^4 + 12\theta^2)^2} > \frac{25\phi^3 (\alpha - \bar{c})^2 (5\phi - 4\theta^2)^2 \cdot (25\phi - 9\theta^2)}{(625\phi^3 - 475\phi^2\theta^2 - 100\phi^2 - 20\phi\theta^2 + 24\theta^4)^2}$$

- Adding an edge between nodes  $L_1, L_3$ : E1A [1H3L]  $\rightarrow$  E2B [1H3L]

– For  $L_1$  not to be profitable:

$$\frac{\phi(\alpha - \bar{c})^2 \cdot (25\phi - 16\theta^2) (5\phi - 3\theta^2 - 3)^2}{(125\phi^2 - 105\phi\theta^2 - 45\phi + 12\theta^4 + 12\theta^2)^2} > \frac{25\phi^3 (\alpha - \bar{c})^2 (5\phi - 3)^2 (5\phi - 4\theta^2)^2 \cdot (25\phi - 9\theta^2)}{(3125\phi^4 - 3625\phi^3\theta^2 - 1125\phi^3 + 800\phi^2\theta^4 + 825\phi^2\theta^2 + 180\phi\theta^4 - 72\theta^6)^2}$$

– For  $L_3$  not to be profitable:

$$\frac{\phi(\alpha - \bar{c})^2 (5\phi - 4\theta^2)^2 \cdot (25\phi - 9\theta^2)}{(125\phi^2 - 105\phi\theta^2 - 45\phi + 12\theta^4 + 12\theta^2)^2} > \frac{\phi(\alpha - \bar{c})^2 (5\phi - 3\theta^2)^2 (5\phi + 3\theta^2)^2 (5\phi - 4\theta^2)^2 \cdot (25\phi - 4\theta^2)}{(3125\phi^4 - 3625\phi^3\theta^2 - 1125\phi^3 + 800\phi^2\theta^4 + 825\phi^2\theta^2 + 180\phi\theta^4 - 72\theta^6)^2}$$

- Adding an edge between nodes  $L_1, L_2$ : E1A [1H3L]  $\rightarrow$  E2C [1H3L]

– For  $L_1$  not to be profitable:

$$\frac{\phi(\alpha - \bar{c})^2 \cdot (25\phi - 16\theta^2) (5\phi - 3\theta^2 - 3)^2}{(125\phi^2 - 105\phi\theta^2 - 45\phi + 12\theta^4 + 12\theta^2)^2} > \frac{\phi(\alpha - \bar{c})^2 \cdot (25\phi - 9\theta^2) (5\phi - 3\theta^2 - 3)^2}{(125\phi^2 - 135\phi\theta^2 - 45\phi + 18\theta^4 + 18\theta^2)^2}$$

– For  $L_2$  not to be profitable:

$$\frac{\phi(\alpha - \bar{c})^2 \cdot (25\phi - 16\theta^2) (5\phi - 3\theta^2 - 3)^2}{(125\phi^2 - 105\phi\theta^2 - 45\phi + 12\theta^4 + 12\theta^2)^2} > \frac{\phi(\alpha - \bar{c})^2 \cdot (25\phi - 9\theta^2) (5\phi - 3\theta^2 - 3)^2}{(125\phi^2 - 135\phi\theta^2 - 45\phi + 18\theta^4 + 18\theta^2)^2}$$

- Removing an edge between nodes  $H_1, L_3$ : E1A [1H3L]  $\rightarrow$  E0A [1H3L]

- For  $H_1$  not to be profitable:

$$\frac{\phi(\alpha - \bar{c})^2 (5\phi - 4\theta^2)^2 \cdot (25\phi - 9)}{(125\phi^2 - 105\phi\theta^2 - 45\phi + 12\theta^4 + 12\theta^2)^2} \geq \frac{\phi(\alpha - \bar{c})^2 (5\phi - 4\theta^2)^2 \cdot (25\phi - 16)}{(125\phi^2 - 40\phi\theta^2 - 80\phi + 16\theta^2)^2}$$

- For  $L_3$  not to be profitable:

$$\frac{\phi(\alpha - \bar{c})^2 (5\phi - 4\theta^2)^2 \cdot (25\phi - 9\theta^2)}{(125\phi^2 - 105\phi\theta^2 - 45\phi + 12\theta^4 + 12\theta^2)^2} \geq \frac{\phi(\alpha - \bar{c})^2 (5\phi - 4)^2 \cdot (25\phi - 16\theta^2)}{(125\phi^2 - 40\phi\theta^2 - 80\phi + 16\theta^2)^2}$$

### C.1.3 Stability conditions for: E1B [1H3L]

- Adding an edge between nodes  $H_1, L_2$ : E1B [1H3L]  $\rightarrow$  E2B [1H3L]

- For  $H_1$  not to be profitable:

$$\frac{\phi(\alpha - \bar{c})^2 (5\phi - 6\theta^2)^2 (5\phi - 4\theta^2)^2 \cdot (25\phi - 16)}{(625\phi^3 - 850\phi^2\theta^2 - 400\phi^2 + 240\phi\theta^4 + 480\phi\theta^2 - 96\theta^4)^2} > \frac{25\phi^3 (\alpha - \bar{c})^2 (5\phi - 4\theta^2)^2 (5\phi - 3\theta^2)^2 \cdot (25\phi - 9)}{(3125\phi^4 - 3625\phi^3\theta^2 - 1125\phi^3 + 800\phi^2\theta^4 + 825\phi^2\theta^2 + 180\phi\theta^4 - 72\theta^6)^2}$$

- For  $L_2$  not to be profitable:

$$\frac{\phi(\alpha - \bar{c})^2 (5\phi - 4)^2 (5\phi - 4\theta^2)^2 \cdot (25\phi - 9\theta^2)}{(625\phi^3 - 850\phi^2\theta^2 - 400\phi^2 + 240\phi\theta^4 + 480\phi\theta^2 - 96\theta^4)^2} > \frac{\phi(\alpha - \bar{c})^2 (5\phi - 3\theta)^2 (5\phi + 3\theta)^2 (5\phi - 4\theta^2)^2 \cdot (25\phi - 4\theta^2)}{(3125\phi^4 - 3625\phi^3\theta^2 - 1125\phi^3 + 800\phi^2\theta^4 + 825\phi^2\theta^2 + 180\phi\theta^4 - 72\theta^6)^2}$$

- Adding an edge between nodes  $H_1, L_1$ : E1B [1H3L]  $\rightarrow$  E2C [1H3L]

- For  $H_1$  not to be profitable:

$$\frac{\phi(\alpha - \bar{c})^2 (5\phi - 6\theta^2)^2 (5\phi - 4\theta^2)^2 \cdot (25\phi - 16)}{(625\phi^3 - 850\phi^2\theta^2 - 400\phi^2 + 240\phi\theta^4 + 480\phi\theta^2 - 96\theta^4)^2} > \frac{\phi(\alpha - \bar{c})^2 (5\phi - 6\theta^2)^2 \cdot (25\phi - 9)}{(125\phi^2 - 135\phi\theta^2 - 45\phi + 18\theta^4 + 18\theta^2)^2}$$

- For  $L_1$  not to be profitable:

$$\frac{\phi(\alpha - \bar{c})^2 (5\phi - 4)^2 (5\phi - 6\theta^2)^2 \cdot (25\phi - 16\theta^2)}{(625\phi^3 - 850\phi^2\theta^2 - 400\phi^2 + 240\phi\theta^4 + 480\phi\theta^2 - 96\theta^4)^2} > \frac{\phi(\alpha - \bar{c})^2 (5\phi - 6\theta^2)^2 \cdot (25\phi - 9\theta^2)}{(125\phi^2 - 135\phi\theta^2 - 45\phi + 18\theta^4 + 18\theta^2)^2}$$

- Adding an edge between nodes  $L_1, L_2$ : E1B [1H3L]  $\rightarrow$  E2D [1H3L]

- For  $L_1$  not to be profitable:

$$\frac{\phi(\alpha - \bar{c})^2 (5\phi - 4)^2 (5\phi - 6\theta^2)^2 \cdot (25\phi - 16\theta^2)}{(625\phi^3 - 850\phi^2\theta^2 - 400\phi^2 + 240\phi\theta^4 + 480\phi\theta^2 - 96\theta^4)^2} > \frac{25\phi^3 (\alpha - \bar{c})^2 (5\phi - 4)^2 \cdot (25\phi - 9\theta^2)}{(5\phi + \theta^2)^2 (125\phi^2 - 60\phi\theta^2 - 80\phi + 24\theta^2)^2}$$

- For  $L_2$  not to be profitable:

$$\frac{\phi(\alpha - \bar{c})^2 (5\phi - 4)^2 (5\phi - 4\theta^2)^2 \cdot (25\phi - 9\theta^2)}{(625\phi^3 - 850\phi^2\theta^2 - 400\phi^2 + 240\phi\theta^4 + 480\phi\theta^2 - 96\theta^4)^2} > \frac{\phi(\alpha - \bar{c})^2 (5\phi - 4)^2 (5\phi + 3\theta^2)^2 \cdot (25\phi - 4\theta^2)}{(5\phi + \theta^2)^2 (125\phi^2 - 60\phi\theta^2 - 80\phi + 24\theta^2)^2}$$

- Removing an edge between nodes  $L_2, L_3$ : E1B [1H3L]  $\rightarrow$  E0A [1H3L]

- For  $L_2$  not to be profitable:

$$\frac{\phi(\alpha - \bar{c})^2 (5\phi - 4)^2 (5\phi - 4\theta^2)^2 \cdot (25\phi - 9\theta^2)}{(625\phi^3 - 850\phi^2\theta^2 - 400\phi^2 + 240\phi\theta^4 + 480\phi\theta^2 - 96\theta^4)^2} \geq \frac{\phi(\alpha - \bar{c})^2 (5\phi - 4)^2 \cdot (25\phi - 16\theta^2)}{(125\phi^2 - 40\phi\theta^2 - 80\phi + 16\theta^2)^2}$$

- For  $L_3$  not to be profitable:

$$\frac{\phi(\alpha - \bar{c})^2 (5\phi - 4)^2 (5\phi - 4\theta^2)^2 \cdot (25\phi - 9\theta^2)}{(625\phi^3 - 850\phi^2\theta^2 - 400\phi^2 + 240\phi\theta^4 + 480\phi\theta^2 - 96\theta^4)^2} \geq \frac{\phi(\alpha - \bar{c})^2 (5\phi - 4)^2 \cdot (25\phi - 16\theta^2)}{(125\phi^2 - 40\phi\theta^2 - 80\phi + 16\theta^2)^2}$$

### C.1.4 Stability conditions for: E2A [1H3L]

- Adding an edge between nodes  $H_1, L_1$ : E2A [1H3L]  $\rightarrow$  E3A [1H3L]

- For  $H_1$  not to be profitable:

$$\frac{\phi(\alpha - \bar{c})^2 (5\phi - 4\theta^2)^2 (5\phi + 3\theta^2)^2 \cdot (25\phi - 4)}{(625\phi^3 - 475\phi^2\theta^2 - 100\phi^2 - 20\phi\theta^2 + 24\theta^4)^2} > \frac{\phi(\alpha - \bar{c})^2 (5\phi + 6\theta^2)^2 \cdot (25\phi - 1)}{(125\phi^2 + 15\phi\theta^2 - 5\phi - 6\theta^2)^2}$$

- For  $L_1$  not to be profitable:



$$\frac{\phi(\alpha - \bar{c})^2 \cdot (25\phi - 16\theta^2) (25\phi^2 - 15\phi\theta^2 - 10\phi - 6\theta^2)^2}{(625\phi^3 - 475\phi^2\theta^2 - 100\phi^2 - 20\phi\theta^2 + 24\theta^4)^2} > \frac{25\phi^3(\alpha - \bar{c})^2 \cdot (25\phi - 9\theta^2)}{(125\phi^2 + 15\phi\theta^2 - 5\phi - 6\theta^2)^2}$$

- Adding an edge between nodes  $L_2, L_3$ : **E2A [1H3L]**  $\rightarrow$  **E3B [1H3L]**

– For  $L_2$  not to be profitable:

$$\frac{25\phi^3(\alpha - \bar{c})^2 (5\phi - 4\theta^2)^2 \cdot (25\phi - 9\theta^2)}{(625\phi^3 - 475\phi^2\theta^2 - 100\phi^2 - 20\phi\theta^2 + 24\theta^4)^2} > \frac{\phi(\alpha - \bar{c})^2 (5\phi - 4\theta^2)^2 \cdot (25\phi - 4\theta^2)}{(125\phi^2 - 120\phi\theta^2 - 20\phi + 16\theta^4 + 8\theta^2)^2}$$

– For  $L_3$  not to be profitable:

$$\frac{25\phi^3(\alpha - \bar{c})^2 (5\phi - 4\theta^2)^2 \cdot (25\phi - 9\theta^2)}{(625\phi^3 - 475\phi^2\theta^2 - 100\phi^2 - 20\phi\theta^2 + 24\theta^4)^2} > \frac{\phi(\alpha - \bar{c})^2 (5\phi - 4\theta^2)^2 \cdot (25\phi - 4\theta^2)}{(125\phi^2 - 120\phi\theta^2 - 20\phi + 16\theta^4 + 8\theta^2)^2}$$

- Adding an edge between nodes  $L_1, L_2$ : **E2A [1H3L]**  $\rightarrow$  **E3C [1H3L]**

– For  $L_1$  not to be profitable:

$$\frac{\phi(\alpha - \bar{c})^2 \cdot (25\phi - 16\theta^2) (25\phi^2 - 15\phi\theta^2 - 10\phi - 6\theta^2)^2}{(625\phi^3 - 475\phi^2\theta^2 - 100\phi^2 - 20\phi\theta^2 + 24\theta^4)^2} > \frac{\phi(\alpha - \bar{c})^2 \cdot (25\phi - 9\theta^2) (125\phi^3 - 75\phi^2\theta^2 - 50\phi^2 + 12\theta^4)^2}{(3125\phi^4 - 2750\phi^3\theta^2 - 500\phi^3 + 225\phi^2\theta^4 + 240\phi\theta^4 - 36\theta^6)^2}$$

– For  $L_2$  not to be profitable:

$$\frac{25\phi^3(\alpha - \bar{c})^2 (5\phi - 4\theta^2)^2 \cdot (25\phi - 9\theta^2)}{(625\phi^3 - 475\phi^2\theta^2 - 100\phi^2 - 20\phi\theta^2 + 24\theta^4)^2} > \frac{25\phi^3(\alpha - \bar{c})^2 \cdot (25\phi - 4\theta^2) (25\phi^2 - 15\phi\theta^2 - 6\theta^2)^2}{(3125\phi^4 - 2750\phi^3\theta^2 - 500\phi^3 + 225\phi^2\theta^4 + 240\phi\theta^4 - 36\theta^6)^2}$$

- Removing an edge between nodes  $H_1, L_2$ : **E2A [1H3L]**  $\rightarrow$  **E1A [1H3L]**

– For  $H_1$  not to be profitable:

$$\frac{\phi(\alpha - \bar{c})^2 (5\phi - 4\theta^2)^2 (5\phi + 3\theta^2)^2 \cdot (25\phi - 4)}{(625\phi^3 - 475\phi^2\theta^2 - 100\phi^2 - 20\phi\theta^2 + 24\theta^4)^2} \geq \frac{\phi(\alpha - \bar{c})^2 (5\phi - 4\theta^2)^2 \cdot (25\phi - 9)}{(125\phi^2 - 105\phi\theta^2 - 45\phi + 12\theta^4 + 12\theta^2)^2}$$

– For  $L_2$  not to be profitable:

$$\frac{25\phi^3(\alpha - \bar{c})^2 (5\phi - 4\theta^2)^2 \cdot (25\phi - 9\theta^2)}{(625\phi^3 - 475\phi^2\theta^2 - 100\phi^2 - 20\phi\theta^2 + 24\theta^4)^2} \geq \frac{\phi(\alpha - \bar{c})^2 \cdot (25\phi - 16\theta^2) (5\phi - 3\theta^2 - 3)^2}{(125\phi^2 - 105\phi\theta^2 - 45\phi + 12\theta^4 + 12\theta^2)^2}$$

### C.1.5 Stability conditions for: **E2B [1H3L]**

- Adding an edge between nodes  $H_1, L_2$ : **E2B [1H3L]**  $\rightarrow$  **E3B [1H3L]**

– For  $H_1$  not to be profitable:

$$\frac{25\phi^3(\alpha - \bar{c})^2 (5\phi - 4\theta^2)^2 (5\phi - 3\theta^2)^2 \cdot (25\phi - 9)}{(3125\phi^4 - 3625\phi^3\theta^2 - 1125\phi^3 + 800\phi^2\theta^4 + 825\phi^2\theta^2 + 180\phi\theta^4 - 72\theta^6)^2} > \frac{\phi(\alpha - \bar{c})^2 (5\phi - 4\theta^2)^2 \cdot (25\phi - 4)}{(125\phi^2 - 120\phi\theta^2 - 20\phi + 16\theta^4 + 8\theta^2)^2}$$

– For  $L_2$  not to be profitable:

$$\frac{25\phi^3(\alpha - \bar{c})^2 (5\phi - 3)^2 (5\phi - 4\theta^2)^2 \cdot (25\phi - 9\theta^2)}{(3125\phi^4 - 3625\phi^3\theta^2 - 1125\phi^3 + 800\phi^2\theta^4 + 825\phi^2\theta^2 + 180\phi\theta^4 - 72\theta^6)^2} > \frac{\phi(\alpha - \bar{c})^2 (5\phi - 4\theta^2)^2 \cdot (25\phi - 4\theta^2)}{(125\phi^2 - 120\phi\theta^2 - 20\phi + 16\theta^4 + 8\theta^2)^2}$$

- Adding an edge between nodes  $H_1, L_1$ : **E2B [1H3L]**  $\rightarrow$  **E3C [1H3L]**

– For  $H_1$  not to be profitable:

$$\frac{25\phi^3(\alpha - \bar{c})^2 (5\phi - 4\theta^2)^2 (5\phi - 3\theta^2)^2 \cdot (25\phi - 9)}{(3125\phi^4 - 3625\phi^3\theta^2 - 1125\phi^3 + 800\phi^2\theta^4 + 825\phi^2\theta^2 + 180\phi\theta^4 - 72\theta^6)^2} > \frac{25\phi^3(\alpha - \bar{c})^2 \cdot (25\phi - 4) (25\phi^2 - 15\phi\theta^2 - 6\theta^4)^2}{(3125\phi^4 - 2750\phi^3\theta^2 - 500\phi^3 + 225\phi^2\theta^4 + 240\phi\theta^4 - 36\theta^6)^2}$$

– For  $L_1$  not to be profitable:

$$\frac{\phi(\alpha - \bar{c})^2 \cdot (25\phi - 16\theta^2) (125\phi^3 - 125\phi^2\theta^2 - 75\phi^2 + 45\phi\theta^2 + 18\theta^4)^2}{(3125\phi^4 - 3625\phi^3\theta^2 - 1125\phi^3 + 800\phi^2\theta^4 + 825\phi^2\theta^2 + 180\phi\theta^4 - 72\theta^6)^2} > \frac{\phi(\alpha - \bar{c})^2 \cdot (25\phi - 9\theta^2) (125\phi^3 - 125\phi^2\theta^2 + 12\theta^4)^2}{(3125\phi^4 - 2750\phi^3\theta^2 - 500\phi^3 + 225\phi^2\theta^4 + 240\phi\theta^4 - 36\theta^6)^2}$$

- Adding an edge between nodes  $L_1, L_3$ : **E2B [1H3L]**  $\rightarrow$  **E3D [1H3L]**

- For  $L_1$  not to be profitable:

$$\frac{\phi(\alpha - \bar{c})^2 \cdot (25\phi - 16\theta^2) (125\phi^3 - 125\phi^2\theta^2 - 75\phi^2 + 45\phi\theta^2 + 18\theta^4)^2}{(3125\phi^4 - 3625\phi^3\theta^2 - 1125\phi^3 + 800\phi^2\theta^4 + 825\phi^2\theta^2 + 180\phi\theta^4 - 72\theta^6)^2} > \frac{25\phi^3(\alpha - \bar{c})^2(5\phi - 3)^2 \cdot (25\phi - 9\theta^2)}{(625\phi^3 - 100\phi^2\theta^2 - 225\phi^2 - 15\phi\theta^4 - 45\phi\theta^2 + 18\theta^4)^2}$$

- For  $L_3$  not to be profitable:

$$\frac{\phi(\alpha - \bar{c})^2(5\phi - 3\theta^2)(5\phi + 3\theta^2)(5\phi - 4\theta^2)^2 \cdot (25\phi - 4\theta^2)}{(3125\phi^4 - 3625\phi^3\theta^2 - 1125\phi^3 + 800\phi^2\theta^4 + 825\phi^2\theta^2 + 180\phi\theta^4 - 72\theta^6)^2} > \frac{\phi(\alpha - \bar{c})^2 \cdot (25\phi - \theta^2)(25\phi^2 + 15\phi\theta^2 - 18\theta^2)^2}{(625\phi^3 - 100\phi^2\theta^2 - 225\phi^2 - 15\phi\theta^4 - 45\phi\theta^2 + 18\theta^4)^2}$$

- Adding an edge between nodes  $L_1, L_2$ : **E2B [1H3L]**  $\rightarrow$  **E3E [1H3L]**

- For  $L_1$  not to be profitable:

$$\frac{\phi(\alpha - \bar{c})^2 \cdot (25\phi - 16\theta^2) (125\phi^3 - 125\phi^2\theta^2 - 75\phi^2 + 45\phi\theta^2 + 18\theta^4)^2}{(3125\phi^4 - 3625\phi^3\theta^2 - 1125\phi^3 + 800\phi^2\theta^4 + 825\phi^2\theta^2 + 180\phi\theta^4 - 72\theta^6)^2} > \frac{\phi(\alpha - \bar{c})^2(5\phi - 2\theta^2)^2 \cdot (25\phi - 9\theta^2)(25\phi^2 - 15\phi - 6\theta^2)^2}{(3125\phi^4 - 2125\phi^3\theta^2 - 1125\phi^3 + 225\phi^2\theta^2 + 120\phi\theta^6 + 120\phi\theta^4 - 36\theta^6)^2}$$

- For  $L_2$  not to be profitable:

$$\frac{25\phi^3(\alpha - \bar{c})^2(5\phi - 3)^2(5\phi - 4\theta^2)^2 \cdot (25\phi - 9\theta^2)}{(3125\phi^4 - 3625\phi^3\theta^2 - 1125\phi^3 + 800\phi^2\theta^4 + 825\phi^2\theta^2 + 180\phi\theta^4 - 72\theta^6)^2} > \frac{25\phi^3(\alpha - \bar{c})^2 \cdot (25\phi - 4\theta^2)(25\phi^2 - 15\phi - 6\theta^4)^2}{(3125\phi^4 - 2125\phi^3\theta^2 - 1125\phi^3 + 225\phi^2\theta^2 + 120\phi\theta^6 + 120\phi\theta^4 - 36\theta^6)^2}$$

- Removing an edge between nodes  $L_2, L_3$ : **E2B [1H3L]**  $\rightarrow$  **E1A [1H3L]**

- For  $L_2$  not to be profitable:

$$\frac{25\phi^3(\alpha - \bar{c})^2(5\phi - 3)^2(5\phi - 4\theta^2)^2 \cdot (25\phi - 9\theta^2)}{(3125\phi^4 - 3625\phi^3\theta^2 - 1125\phi^3 + 800\phi^2\theta^4 + 825\phi^2\theta^2 + 180\phi\theta^4 - 72\theta^6)^2} \geq \frac{\phi(\alpha - \bar{c})^2 \cdot (25\phi - 16\theta^2)(5\phi - 3\theta^2 - 3)^2}{(125\phi^2 - 105\phi\theta^2 - 45\phi + 12\theta^4 + 12\theta^2)^2}$$

- For  $L_3$  not to be profitable:

$$\frac{\phi(\alpha - \bar{c})^2(5\phi - 3\theta^2)(5\phi + 3\theta^2)(5\phi - 4\theta^2)^2 \cdot (25\phi - 4\theta^2)}{(3125\phi^4 - 3625\phi^3\theta^2 - 1125\phi^3 + 800\phi^2\theta^4 + 825\phi^2\theta^2 + 180\phi\theta^4 - 72\theta^6)^2} \geq \frac{\phi(\alpha - \bar{c})^2(5\phi - 4\theta^2)^2 \cdot (25\phi - 9\theta^2)}{(125\phi^2 - 105\phi\theta^2 - 45\phi + 12\theta^4 + 12\theta^2)^2}$$

- Removing an edge between nodes  $H_1, L_3$ : **E2B [1H3L]**  $\rightarrow$  **E1B [1H3L]**

- For  $H_1$  not to be profitable:

$$\frac{25\phi^3(\alpha - \bar{c})^2(5\phi - 4\theta^2)^2(5\phi - 3\theta^2)^2 \cdot (25\phi - 9)}{(3125\phi^4 - 3625\phi^3\theta^2 - 1125\phi^3 + 800\phi^2\theta^4 + 825\phi^2\theta^2 + 180\phi\theta^4 - 72\theta^6)^2} \geq \frac{\phi(\alpha - \bar{c})^2(5\phi - 6\theta^2)^2(5\phi - 4\theta^2)^2 \cdot (25\phi - 16)}{(625\phi^3 - 850\phi^2\theta^2 - 400\phi^2 + 240\phi\theta^4 + 480\phi\theta^2 - 96\theta^4)^2}$$

- For  $L_3$  not to be profitable:

$$\frac{\phi(\alpha - \bar{c})^2(5\phi - 3\theta^2)(5\phi + 3\theta^2)(5\phi - 4\theta^2)^2 \cdot (25\phi - 4\theta^2)}{(3125\phi^4 - 3625\phi^3\theta^2 - 1125\phi^3 + 800\phi^2\theta^4 + 825\phi^2\theta^2 + 180\phi\theta^4 - 72\theta^6)^2} \geq \frac{\phi(\alpha - \bar{c})^2(5\phi - 4)^2(5\phi - 4\theta^2)^2 \cdot (25\phi - 9\theta^2)}{(625\phi^3 - 850\phi^2\theta^2 - 400\phi^2 + 240\phi\theta^4 + 480\phi\theta^2 - 96\theta^4)^2}$$

### C.1.6 Stability conditions for: **E2C [1H3L]**

- Adding an edge between nodes  $H_1, L_1$ : **E2C [1H3L]**  $\rightarrow$  **E3C [1H3L]**

- For  $H_1$  not to be profitable:

$$\frac{\phi(\alpha - \bar{c})^2(5\phi - 6\theta^2)^2 \cdot (25\phi - 9)}{(125\phi^2 - 135\phi\theta^2 - 45\phi + 18\theta^4 + 18\theta^2)^2} > \frac{25\phi^3(\alpha - \bar{c})^2 \cdot (25\phi - 4)(25\phi^2 - 15\phi\theta^2 - 6\theta^4)^2}{(3125\phi^4 - 2750\phi^3\theta^2 - 500\phi^3 + 225\phi^2\theta^4 + 240\phi\theta^4 - 36\theta^6)^2}$$

- For  $L_1$  not to be profitable:

$$\frac{\phi(\alpha - \bar{c})^2 \cdot (25\phi - 9\theta^2)(5\phi - 3\theta^2 - 3)^2}{(125\phi^2 - 135\phi\theta^2 - 45\phi + 18\theta^4 + 18\theta^2)^2} > \frac{25\phi^3(\alpha - \bar{c})^2 \cdot (25\phi - 4\theta^2)(25\phi^2 - 15\phi\theta^2 - 6\theta^2)^2}{(3125\phi^4 - 2750\phi^3\theta^2 - 500\phi^3 + 225\phi^2\theta^4 + 240\phi\theta^4 - 36\theta^6)^2}$$

- Adding an edge between nodes  $L_1, L_3$ : **E2C [1H3L]**  $\rightarrow$  **E3E [1H3L]**

- For  $L_1$  not to be profitable:

$$\frac{\phi(\alpha - \bar{c})^2 \cdot (25\phi - 9\theta^2)(5\phi - 3\theta^2 - 3)^2}{(125\phi^2 - 135\phi\theta^2 - 45\phi + 18\theta^4 + 18\theta^2)^2} > \frac{25\phi^3(\alpha - \bar{c})^2 \cdot (25\phi - 4\theta^2)(25\phi^2 - 15\phi - 6\theta^4)^2}{(3125\phi^4 - 2125\phi^3\theta^2 - 1125\phi^3 + 225\phi^2\theta^2 + 120\phi\theta^6 + 120\phi\theta^4 - 36\theta^6)^2}$$

- For  $L_3$  not to be profitable:

$$\frac{\phi(\alpha - \bar{c})^2 (5\phi - 6\theta^2)^2 \cdot (25\phi - 9\theta^2)}{(125\phi^2 - 135\phi\theta^2 - 45\phi + 18\theta^4 + 18\theta^2)^2} > \frac{25\phi^3 (\alpha - \bar{c})^2 \cdot (25\phi - 4\theta^2) (25\phi^2 - 15\phi\theta^2 - 6\theta^2)^2}{(3125\phi^4 - 2125\phi^3\theta^2 - 1125\phi^3 + 225\phi^2\theta^2 + 120\phi\theta^6 + 120\phi\theta^4 - 36\theta^6)^2}$$

- Removing an edge between nodes  $L_1, L_2$ : **E2C [1H3L] → E1A [1H3L]**

– For  $L_1$  not to be profitable:

$$\frac{\phi(\alpha - \bar{c})^2 \cdot (25\phi - 9\theta^2) (5\phi - 3\theta^2 - 3)^2}{(125\phi^2 - 135\phi\theta^2 - 45\phi + 18\theta^4 + 18\theta^2)^2} \geq \frac{\phi(\alpha - \bar{c})^2 \cdot (25\phi - 16\theta^2) (5\phi - 3\theta^2 - 3)^2}{(125\phi^2 - 105\phi\theta^2 - 45\phi + 12\theta^4 + 12\theta^2)^2}$$

– For  $L_2$  not to be profitable:

$$\frac{\phi(\alpha - \bar{c})^2 \cdot (25\phi - 9\theta^2) (5\phi - 3\theta^2 - 3)^2}{(125\phi^2 - 135\phi\theta^2 - 45\phi + 18\theta^4 + 18\theta^2)^2} \geq \frac{\phi(\alpha - \bar{c})^2 \cdot (25\phi - 16\theta^2) (5\phi - 3\theta^2 - 3)^2}{(125\phi^2 - 105\phi\theta^2 - 45\phi + 12\theta^4 + 12\theta^2)^2}$$

- Removing an edge between nodes  $H_1, L_3$ : **E2C [1H3L] → E1B [1H3L]**

– For  $H_1$  not to be profitable:

$$\frac{\phi(\alpha - \bar{c})^2 (5\phi - 6\theta^2)^2 \cdot (25\phi - 9)}{(125\phi^2 - 135\phi\theta^2 - 45\phi + 18\theta^4 + 18\theta^2)^2} \geq \frac{\phi(\alpha - \bar{c})^2 (5\phi - 6\theta^2)^2 (5\phi - 4\theta^2)^2 \cdot (25\phi - 16)}{(625\phi^3 - 850\phi^2\theta^2 - 400\phi^2 + 240\phi\theta^4 + 480\phi\theta^2 - 96\theta^4)^2}$$

– For  $L_3$  not to be profitable:

$$\frac{\phi(\alpha - \bar{c})^2 (5\phi - 6\theta^2)^2 \cdot (25\phi - 9\theta^2)}{(125\phi^2 - 135\phi\theta^2 - 45\phi + 18\theta^4 + 18\theta^2)^2} \geq \frac{\phi(\alpha - \bar{c})^2 (5\phi - 4)^2 (5\phi - 6\theta^2)^2 \cdot (25\phi - 16\theta^2)}{(625\phi^3 - 850\phi^2\theta^2 - 400\phi^2 + 240\phi\theta^4 + 480\phi\theta^2 - 96\theta^4)^2}$$

### C.1.7 Stability conditions for: **E2D [1H3L]**

- Adding an edge between nodes  $H_1, L_3$ : **E2D [1H3L] → E3D [1H3L]**

– For  $H_1$  not to be profitable:

$$\frac{\phi(\alpha - \bar{c})^2 (5\phi - 6\theta^2)^2 \cdot (25\phi - 16)}{(125\phi^2 - 60\phi\theta^2 - 80\phi + 24\theta^2)^2} > \frac{25\phi^3 (\alpha - \bar{c})^2 (5\phi - 3\theta^2)^2 \cdot (25\phi - 9)}{(625\phi^3 - 100\phi^2\theta^2 - 225\phi^2 - 15\phi\theta^4 - 45\phi\theta^2 + 18\theta^4)^2}$$

– For  $L_3$  not to be profitable:

$$\frac{\phi(\alpha - \bar{c})^2 (5\phi - 4)^2 (5\phi + 3\theta^2)^2 \cdot (25\phi - 4\theta^2)}{(5\phi + \theta^2)^2 (125\phi^2 - 60\phi\theta^2 - 80\phi + 24\theta^2)^2} > \frac{\phi(\alpha - \bar{c})^2 \cdot (25\phi - \theta^2) (25\phi^2 + 15\phi\theta^2 - 18\theta^2)^2}{(625\phi^3 - 100\phi^2\theta^2 - 225\phi^2 - 15\phi\theta^4 - 45\phi\theta^2 + 18\theta^4)^2}$$

- Adding an edge between nodes  $H_1, L_1$ : **E2D [1H3L] → E3E [1H3L]**

– For  $H_1$  not to be profitable:

$$\frac{\phi(\alpha - \bar{c})^2 (5\phi - 6\theta^2)^2 \cdot (25\phi - 16)}{(125\phi^2 - 60\phi\theta^2 - 80\phi + 24\theta^2)^2} > \frac{\phi(\alpha - \bar{c})^2 (5\phi - 2\theta^2)^2 \cdot (25\phi - 9) (25\phi^2 - 15\phi\theta^2 - 6\theta^4)^2}{(3125\phi^4 - 2125\phi^3\theta^2 - 1125\phi^3 + 225\phi^2\theta^2 + 120\phi\theta^6 + 120\phi\theta^4 - 36\theta^6)^2}$$

– For  $L_1$  not to be profitable:

$$\frac{25\phi^3 (\alpha - \bar{c})^2 (5\phi - 4)^2 \cdot (25\phi - 9\theta^2)}{(5\phi + \theta^2)^2 (125\phi^2 - 60\phi\theta^2 - 80\phi + 24\theta^2)^2} > \frac{25\phi^3 (\alpha - \bar{c})^2 \cdot (25\phi - 4\theta^2) (25\phi^2 - 15\phi\theta^2 - 6\theta^2)^2}{(3125\phi^4 - 2125\phi^3\theta^2 - 1125\phi^3 + 225\phi^2\theta^2 + 120\phi\theta^6 + 120\phi\theta^4 - 36\theta^6)^2}$$

- Adding an edge between nodes  $L_1, L_2$ : **E2D [1H3L] → E3F [1H3L]**

– For  $L_1$  not to be profitable:

$$\frac{25\phi^3 (\alpha - \bar{c})^2 (5\phi - 4)^2 \cdot (25\phi - 9\theta^2)}{(5\phi + \theta^2)^2 (125\phi^2 - 60\phi\theta^2 - 80\phi + 24\theta^2)^2} > \frac{\phi(\alpha - \bar{c})^2 (5\phi - 4)^2 \cdot (25\phi - 4\theta^2)}{(125\phi^2 - 60\phi\theta^2 - 80\phi + 24\theta^2)^2}$$

– For  $L_2$  not to be profitable:

$$\frac{25\phi^3 (\alpha - \bar{c})^2 (5\phi - 4)^2 \cdot (25\phi - 9\theta^2)}{(5\phi + \theta^2)^2 (125\phi^2 - 60\phi\theta^2 - 80\phi + 24\theta^2)^2} > \frac{\phi(\alpha - \bar{c})^2 (5\phi - 4)^2 \cdot (25\phi - 4\theta^2)}{(125\phi^2 - 60\phi\theta^2 - 80\phi + 24\theta^2)^2}$$

- Removing an edge between nodes  $L_1, L_3$ : **E2D [1H3L] → E1B [1H3L]**

– For  $L_1$  not to be profitable:

$$\frac{25\phi^3 (\alpha - \bar{c})^2 (5\phi - 4)^2 \cdot (25\phi - 9\theta^2)}{(5\phi + \theta^2)^2 (125\phi^2 - 60\phi\theta^2 - 80\phi + 24\theta^2)^2} \geq \frac{\phi (\alpha - \bar{c})^2 (5\phi - 4)^2 (5\phi - 6\theta^2)^2 \cdot (25\phi - 16\theta^2)}{(625\phi^3 - 850\phi^2\theta^2 - 400\phi^2 + 240\phi\theta^4 + 480\phi\theta^2 - 96\theta^4)^2}$$

– For  $L_3$  not to be profitable:

$$\frac{\phi (\alpha - \bar{c})^2 (5\phi - 4)^2 (5\phi + 3\theta^2)^2 \cdot (25\phi - 4\theta^2)}{(5\phi + \theta^2)^2 (125\phi^2 - 60\phi\theta^2 - 80\phi + 24\theta^2)^2} \geq \frac{\phi (\alpha - \bar{c})^2 (5\phi - 4)^2 (5\phi - 4\theta^2)^2 \cdot (25\phi - 9\theta^2)}{(625\phi^3 - 850\phi^2\theta^2 - 400\phi^2 + 240\phi\theta^4 + 480\phi\theta^2 - 96\theta^4)^2}$$

### C.1.8 Stability conditions for: E3A [1H3L]

- Adding an edge between nodes  $L_1, L_2$ : E3A [1H3L]  $\rightarrow$  E4A [1H3L]

– For  $L_1$  not to be profitable:

$$\frac{25\phi^3 (\alpha - \bar{c})^2 \cdot (25\phi - 9\theta^2)}{(125\phi^2 + 15\phi\theta^2 - 5\phi - 6\theta^2)^2} > \frac{25\phi^3 (\alpha - \bar{c})^2 (5\phi - 3\theta^2)^2 \cdot (25\phi - 4\theta^2)}{(625\phi^3 - 425\phi^2\theta^2 - 25\phi^2 + 12\theta^4)^2}$$

– For  $L_2$  not to be profitable:

$$\frac{25\phi^3 (\alpha - \bar{c})^2 \cdot (25\phi - 9\theta^2)}{(125\phi^2 + 15\phi\theta^2 - 5\phi - 6\theta^2)^2} > \frac{25\phi^3 (\alpha - \bar{c})^2 (5\phi - 3\theta^2)^2 \cdot (25\phi - 4\theta^2)}{(625\phi^3 - 425\phi^2\theta^2 - 25\phi^2 + 12\theta^4)^2}$$

- Removing an edge between nodes  $H_1, L_1$ : E3A [1H3L]  $\rightarrow$  E2A [1H3L]

– For  $H_1$  not to be profitable:

$$\frac{\phi (\alpha - \bar{c})^2 (5\phi + 6\theta^2)^2 \cdot (25\phi - 1)}{(125\phi^2 + 15\phi\theta^2 - 5\phi - 6\theta^2)^2} \geq \frac{\phi (\alpha - \bar{c})^2 (5\phi - 4\theta^2)^2 (5\phi + 3\theta^2)^2 \cdot (25\phi - 4)}{(625\phi^3 - 475\phi^2\theta^2 - 100\phi^2 - 20\phi\theta^2 + 24\theta^4)^2}$$

– For  $L_1$  not to be profitable:

$$\frac{25\phi^3 (\alpha - \bar{c})^2 \cdot (25\phi - 9\theta^2)}{(125\phi^2 + 15\phi\theta^2 - 5\phi - 6\theta^2)^2} \geq \frac{\phi (\alpha - \bar{c})^2 \cdot (25\phi - 16\theta^2) (25\phi^2 - 15\phi\theta^2 - 10\phi - 6\theta^2)^2}{(625\phi^3 - 475\phi^2\theta^2 - 100\phi^2 - 20\phi\theta^2 + 24\theta^4)^2}$$

### C.1.9 Stability conditions for: E3B [1H3L]

- Adding an edge between nodes  $H_1, L_1$ : E3B [1H3L]  $\rightarrow$  E4A [1H3L]

– For  $H_1$  not to be profitable:

$$\frac{\phi (\alpha - \bar{c})^2 (5\phi - 4\theta^2)^2 \cdot (25\phi - 4)}{(125\phi^2 - 120\phi\theta^2 - 20\phi + 16\theta^4 + 8\theta^2)^2} > \frac{\phi (\alpha - \bar{c})^2 \cdot (25\phi - 1) (25\phi^2 - 12\theta^4)^2}{(625\phi^3 - 425\phi^2\theta^2 - 25\phi^2 + 12\theta^4)^2}$$

– For  $L_1$  not to be profitable:

$$\frac{\phi (\alpha - \bar{c})^2 \cdot (25\phi - 16\theta^2) (5\phi - 4\theta^2 - 2)^2}{(125\phi^2 - 120\phi\theta^2 - 20\phi + 16\theta^4 + 8\theta^2)^2} > \frac{25\phi^3 (\alpha - \bar{c})^2 (5\phi - 4\theta^2)^2 \cdot (25\phi - 9\theta^2)}{(625\phi^3 - 425\phi^2\theta^2 - 25\phi^2 + 12\theta^4)^2}$$

- Adding an edge between nodes  $L_1, L_2$ : E3B [1H3L]  $\rightarrow$  E4B [1H3L]

– For  $L_1$  not to be profitable:

$$\frac{\phi (\alpha - \bar{c})^2 \cdot (25\phi - 16\theta^2) (5\phi - 4\theta^2 - 2)^2}{(125\phi^2 - 120\phi\theta^2 - 20\phi + 16\theta^4 + 8\theta^2)^2} > \frac{25\phi^3 (\alpha - \bar{c})^2 \cdot (25\phi - 9\theta^2) (5\phi - 2\theta^2 - 2)^2}{(625\phi^3 - 350\phi^2\theta^2 - 100\phi^2 + 6\theta^6 + 6\theta^4)^2}$$

– For  $L_2$  not to be profitable:

$$\frac{\phi (\alpha - \bar{c})^2 (5\phi - 4\theta^2)^2 \cdot (25\phi - 4\theta^2)}{(125\phi^2 - 120\phi\theta^2 - 20\phi + 16\theta^4 + 8\theta^2)^2} > \frac{\phi (\alpha - \bar{c})^2 \cdot (25\phi - \theta^2) (25\phi^2 - 6\theta^4 - 6\theta^2)^2}{(625\phi^3 - 350\phi^2\theta^2 - 100\phi^2 + 6\theta^6 + 6\theta^4)^2}$$

- Removing an edge between nodes  $L_2, L_3$ : E3B [1H3L]  $\rightarrow$  E2A [1H3L]

– For  $L_2$  not to be profitable:

$$\frac{\phi (\alpha - \bar{c})^2 (5\phi - 4\theta^2)^2 \cdot (25\phi - 4\theta^2)}{(125\phi^2 - 120\phi\theta^2 - 20\phi + 16\theta^4 + 8\theta^2)^2} \geq \frac{25\phi^3 (\alpha - \bar{c})^2 (5\phi - 4\theta^2)^2 \cdot (25\phi - 9\theta^2)}{(625\phi^3 - 475\phi^2\theta^2 - 100\phi^2 - 20\phi\theta^2 + 24\theta^4)^2}$$

– For  $L_3$  not to be profitable:

$$\frac{\phi(\alpha - \bar{c})^2 (5\phi - 4\theta^2)^2 \cdot (25\phi - 4\theta^2)}{(125\phi^2 - 120\phi\theta^2 - 20\phi + 16\theta^4 + 8\theta^2)^2} \geq \frac{25\phi^3 (\alpha - \bar{c})^2 (5\phi - 4\theta^2)^2 \cdot (25\phi - 9\theta^2)}{(625\phi^3 - 475\phi^2\theta^2 - 100\phi^2 - 20\phi\theta^2 + 24\theta^4)^2}$$

- Removing an edge between nodes  $H_1, L_2$ : **E3B** [1H3L]  $\rightarrow$  **E2B** [1H3L]

– For  $H_1$  not to be profitable:

$$\frac{\phi(\alpha - \bar{c})^2 (5\phi - 4\theta^2)^2 \cdot (25\phi - 4)}{(125\phi^2 - 120\phi\theta^2 - 20\phi + 16\theta^4 + 8\theta^2)^2} \geq \frac{25\phi^3 (\alpha - \bar{c})^2 (5\phi - 4\theta^2)^2 (5\phi - 3\theta^2)^2 \cdot (25\phi - 9)}{(3125\phi^4 - 3625\phi^3\theta^2 - 1125\phi^3 + 800\phi^2\theta^4 + 825\phi^2\theta^2 + 180\phi\theta^4 - 72\theta^6)^2}$$

– For  $L_2$  not to be profitable:

$$\frac{\phi(\alpha - \bar{c})^2 (5\phi - 4\theta^2)^2 \cdot (25\phi - 4\theta^2)}{(125\phi^2 - 120\phi\theta^2 - 20\phi + 16\theta^4 + 8\theta^2)^2} \geq \frac{25\phi^3 (\alpha - \bar{c})^2 (5\phi - 3)^2 (5\phi - 4\theta^2)^2 \cdot (25\phi - 9\theta^2)}{(3125\phi^4 - 3625\phi^3\theta^2 - 1125\phi^3 + 800\phi^2\theta^4 + 825\phi^2\theta^2 + 180\phi\theta^4 - 72\theta^6)^2}$$

#### C.1.10 Stability conditions for: **E3C** [1H3L]

- Adding an edge between nodes  $H_1, L_1$ : **E3C** [1H3L]  $\rightarrow$  **E4A** [1H3L]

– For  $H_1$  not to be profitable:

$$\frac{25\phi^3 (\alpha - \bar{c})^2 \cdot (25\phi - 4) (25\phi^2 - 15\phi\theta^2 - 6\theta^4)^2}{(3125\phi^4 - 2750\phi^3\theta^2 - 500\phi^3 + 225\phi^2\theta^4 + 240\phi\theta^4 - 36\theta^6)^2} > \frac{\phi(\alpha - \bar{c})^2 \cdot (25\phi - 1) (25\phi^2 - 12\theta^4)^2}{(625\phi^3 - 425\phi^2\theta^2 - 25\phi^2 + 12\theta^4)^2}$$

– For  $L_1$  not to be profitable:

$$\frac{\phi(\alpha - \bar{c})^2 \cdot (25\phi - 9\theta^2) (125\phi^3 - 75\phi^2\theta^2 - 50\phi^2 + 12\theta^4)^2}{(3125\phi^4 - 2750\phi^3\theta^2 - 500\phi^3 + 225\phi^2\theta^4 + 240\phi\theta^4 - 36\theta^6)^2} > \frac{25\phi^3 (\alpha - \bar{c})^2 (5\phi - 3\theta^2)^2 \cdot (25\phi - 4\theta^2)}{(625\phi^3 - 425\phi^2\theta^2 - 25\phi^2 + 12\theta^4)^2}$$

- Adding an edge between nodes  $L_2, L_3$ : **E3C** [1H3L]  $\rightarrow$  **E4B** [1H3L]

– For  $L_2$  not to be profitable:

$$\frac{\phi(\alpha - \bar{c})^2 \cdot (25\phi - 9\theta^2) (125\phi^3 - 125\phi^2\theta^2 + 12\theta^4)^2}{(3125\phi^4 - 2750\phi^3\theta^2 - 500\phi^3 + 225\phi^2\theta^4 + 240\phi\theta^4 - 36\theta^6)^2} > \frac{25\phi^3 (\alpha - \bar{c})^2 (5\phi - 3\theta^2)^2 \cdot (25\phi - 4\theta^2)}{(625\phi^3 - 350\phi^2\theta^2 - 100\phi^2 + 6\theta^6 + 6\theta^4)^2}$$

– For  $L_3$  not to be profitable:

$$\frac{25\phi^3 (\alpha - \bar{c})^2 \cdot (25\phi - 4\theta^2) (25\phi^2 - 15\phi\theta^2 - 6\theta^2)^2}{(3125\phi^4 - 2750\phi^3\theta^2 - 500\phi^3 + 225\phi^2\theta^4 + 240\phi\theta^4 - 36\theta^6)^2} > \frac{\phi(\alpha - \bar{c})^2 \cdot (25\phi - \theta^2) (25\phi^2 - 6\theta^4 - 6\theta^2)^2}{(625\phi^3 - 350\phi^2\theta^2 - 100\phi^2 + 6\theta^6 + 6\theta^4)^2}$$

- Adding an edge between nodes  $L_1, L_2$ : **E3C** [1H3L]  $\rightarrow$  **E4C** [1H3L]

– For  $L_1$  not to be profitable:

$$\frac{\phi(\alpha - \bar{c})^2 \cdot (25\phi - 9\theta^2) (125\phi^3 - 75\phi^2\theta^2 - 50\phi^2 + 12\theta^4)^2}{(3125\phi^4 - 2750\phi^3\theta^2 - 500\phi^3 + 225\phi^2\theta^4 + 240\phi\theta^4 - 36\theta^6)^2} > \frac{\phi(\alpha - \bar{c})^2 (5\phi - 2)^2 (5\phi + 2\theta^2)^2 \cdot (25\phi - 4\theta^2)}{(625\phi^3 - 50\phi^2\theta^2 - 100\phi^2 - 40\phi\theta^4 - 60\phi\theta^2 + 24\theta^4)^2}$$

– For  $L_2$  not to be profitable:

$$\frac{\phi(\alpha - \bar{c})^2 \cdot (25\phi - 9\theta^2) (125\phi^3 - 125\phi^2\theta^2 + 12\theta^4)^2}{(3125\phi^4 - 2750\phi^3\theta^2 - 500\phi^3 + 225\phi^2\theta^4 + 240\phi\theta^4 - 36\theta^6)^2} > \frac{\phi(\alpha - \bar{c})^2 (5\phi - 2\theta)^2 (5\phi + 2\theta)^2 \cdot (25\phi - 4\theta^2)}{(625\phi^3 - 50\phi^2\theta^2 - 100\phi^2 - 40\phi\theta^4 - 60\phi\theta^2 + 24\theta^4)^2}$$

- Removing an edge between nodes  $L_1, L_3$ : **E3C** [1H3L]  $\rightarrow$  **E2A** [1H3L]

– For  $L_1$  not to be profitable:

$$\frac{\phi(\alpha - \bar{c})^2 \cdot (25\phi - 9\theta^2) (125\phi^3 - 75\phi^2\theta^2 - 50\phi^2 + 12\theta^4)^2}{(3125\phi^4 - 2750\phi^3\theta^2 - 500\phi^3 + 225\phi^2\theta^4 + 240\phi\theta^4 - 36\theta^6)^2} \geq \frac{\phi(\alpha - \bar{c})^2 \cdot (25\phi - 16\theta^2) (25\phi^2 - 15\phi\theta^2 - 10\phi - 6\theta^2)^2}{(625\phi^3 - 475\phi^2\theta^2 - 100\phi^2 - 20\phi\theta^2 + 24\theta^4)^2}$$

– For  $L_3$  not to be profitable:

$$\frac{25\phi^3 (\alpha - \bar{c})^2 \cdot (25\phi - 4\theta^2) (25\phi^2 - 15\phi\theta^2 - 6\theta^2)^2}{(3125\phi^4 - 2750\phi^3\theta^2 - 500\phi^3 + 225\phi^2\theta^4 + 240\phi\theta^4 - 36\theta^6)^2} \geq \frac{25\phi^3 (\alpha - \bar{c})^2 (5\phi - 4\theta^2)^2 \cdot (25\phi - 9\theta^2)}{(625\phi^3 - 475\phi^2\theta^2 - 100\phi^2 - 20\phi\theta^2 + 24\theta^4)^2}$$

- Removing an edge between nodes  $H_1, L_2$ : **E3C** [1H3L]  $\rightarrow$  **E2B** [1H3L]

- For  $H_1$  not to be profitable:

$$\frac{25\phi^3 (\alpha - \bar{c})^2 \cdot (25\phi - 4) (25\phi^2 - 15\phi\theta^2 - 6\theta^4)^2}{(3125\phi^4 - 2750\phi^3\theta^2 - 500\phi^3 + 225\phi^2\theta^4 + 240\phi\theta^4 - 36\theta^6)^2} \geq \frac{25\phi^3 (\alpha - \bar{c})^2 (5\phi - 4\theta^2)^2 (5\phi - 3\theta^2)^2 \cdot (25\phi - 9)}{(3125\phi^4 - 3625\phi^3\theta^2 - 1125\phi^3 + 800\phi^2\theta^4 + 825\phi^2\theta^2 + 180\phi\theta^4 - 72\theta^6)^2}$$

- For  $L_2$  not to be profitable:

$$\frac{\phi (\alpha - \bar{c})^2 \cdot (25\phi - 9\theta^2) (125\phi^3 - 125\phi^2\theta^2 + 12\theta^4)^2}{(3125\phi^4 - 2750\phi^3\theta^2 - 500\phi^3 + 225\phi^2\theta^4 + 240\phi\theta^4 - 36\theta^6)^2} \geq \frac{\phi (\alpha - \bar{c})^2 \cdot (25\phi - 16\theta^2) (125\phi^3 - 125\phi^2\theta^2 - 75\phi^2 + 45\phi\theta^2 + 18\theta^4)^2}{(3125\phi^4 - 3625\phi^3\theta^2 - 1125\phi^3 + 800\phi^2\theta^4 + 825\phi^2\theta^2 + 180\phi\theta^4 - 72\theta^6)^2}$$

- Removing an edge between nodes  $H_1, L_3$ : **E3C** [1H3L]  $\rightarrow$  **E2C** [1H3L]

- For  $H_1$  not to be profitable:

$$\frac{25\phi^3 (\alpha - \bar{c})^2 \cdot (25\phi - 4) (25\phi^2 - 15\phi\theta^2 - 6\theta^4)^2}{(3125\phi^4 - 2750\phi^3\theta^2 - 500\phi^3 + 225\phi^2\theta^4 + 240\phi\theta^4 - 36\theta^6)^2} \geq \frac{\phi (\alpha - \bar{c})^2 (5\phi - 6\theta^2)^2 \cdot (25\phi - 9)}{(125\phi^2 - 135\phi\theta^2 - 45\phi + 18\theta^4 + 18\theta^2)^2}$$

- For  $L_3$  not to be profitable:

$$\frac{25\phi^3 (\alpha - \bar{c})^2 \cdot (25\phi - 4\theta^2) (25\phi^2 - 15\phi\theta^2 - 6\theta^2)^2}{(3125\phi^4 - 2750\phi^3\theta^2 - 500\phi^3 + 225\phi^2\theta^4 + 240\phi\theta^4 - 36\theta^6)^2} \geq \frac{\phi (\alpha - \bar{c})^2 \cdot (25\phi - 9\theta^2) (5\phi - 3\theta^2 - 3)^2}{(125\phi^2 - 135\phi\theta^2 - 45\phi + 18\theta^4 + 18\theta^2)^2}$$

### C.1.11 Stability conditions for: **E3D** [1H3L]

- Adding an edge between nodes  $H_1, L_1$ : **E3D** [1H3L]  $\rightarrow$  **E4B** [1H3L]

- For  $H_1$  not to be profitable:

$$\frac{25\phi^3 (\alpha - \bar{c})^2 (5\phi - 3\theta^2)^2 \cdot (25\phi - 9)}{(625\phi^3 - 100\phi^2\theta^2 - 225\phi^2 - 15\phi\theta^4 - 45\phi\theta^2 + 18\theta^4)^2} > \frac{25\phi^3 (\alpha - \bar{c})^2 (5\phi - 3\theta^2)^2 \cdot (25\phi - 4)}{(625\phi^3 - 350\phi^2\theta^2 - 100\phi^2 + 6\theta^6 + 6\theta^4)^2}$$

- For  $L_1$  not to be profitable:

$$\frac{25\phi^3 (\alpha - \bar{c})^2 (5\phi - 3)^2 \cdot (25\phi - 9\theta^2)}{(625\phi^3 - 100\phi^2\theta^2 - 225\phi^2 - 15\phi\theta^4 - 45\phi\theta^2 + 18\theta^4)^2} > \frac{25\phi^3 (\alpha - \bar{c})^2 (5\phi - 3\theta^2)^2 \cdot (25\phi - 4\theta^2)}{(625\phi^3 - 350\phi^2\theta^2 - 100\phi^2 + 6\theta^6 + 6\theta^4)^2}$$

- Adding an edge between nodes  $L_1, L_2$ : **E3D** [1H3L]  $\rightarrow$  **E4D** [1H3L]

- For  $L_1$  not to be profitable:

$$\frac{25\phi^3 (\alpha - \bar{c})^2 (5\phi - 3)^2 \cdot (25\phi - 9\theta^2)}{(625\phi^3 - 100\phi^2\theta^2 - 225\phi^2 - 15\phi\theta^4 - 45\phi\theta^2 + 18\theta^4)^2} > \frac{25\phi^3 (\alpha - \bar{c})^2 (5\phi - 3)^2 \cdot (25\phi - 4\theta^2)}{(625\phi^3 - 225\phi^2\theta^2 - 225\phi^2 + 12\theta^4)^2}$$

- For  $L_2$  not to be profitable:

$$\frac{25\phi^3 (\alpha - \bar{c})^2 (5\phi - 3)^2 \cdot (25\phi - 9\theta^2)}{(625\phi^3 - 100\phi^2\theta^2 - 225\phi^2 - 15\phi\theta^4 - 45\phi\theta^2 + 18\theta^4)^2} > \frac{25\phi^3 (\alpha - \bar{c})^2 (5\phi - 3)^2 \cdot (25\phi - 4\theta^2)}{(625\phi^3 - 225\phi^2\theta^2 - 225\phi^2 + 12\theta^4)^2}$$

- Removing an edge between nodes  $L_1, L_3$ : **E3D** [1H3L]  $\rightarrow$  **E2B** [1H3L]

- For  $L_1$  not to be profitable:

$$\frac{25\phi^3 (\alpha - \bar{c})^2 (5\phi - 3)^2 \cdot (25\phi - 9\theta^2)}{(625\phi^3 - 100\phi^2\theta^2 - 225\phi^2 - 15\phi\theta^4 - 45\phi\theta^2 + 18\theta^4)^2} \geq \frac{\phi (\alpha - \bar{c})^2 \cdot (25\phi - 16\theta^2) (125\phi^3 - 125\phi^2\theta^2 - 75\phi^2 + 45\phi\theta^2 + 18\theta^4)^2}{(3125\phi^4 - 3625\phi^3\theta^2 - 1125\phi^3 + 800\phi^2\theta^4 + 825\phi^2\theta^2 + 180\phi\theta^4 - 72\theta^6)^2}$$

- For  $L_3$  not to be profitable:

$$\frac{\phi (\alpha - \bar{c})^2 \cdot (25\phi - \theta^2) (25\phi^2 + 15\phi\theta^2 - 18\theta^2)^2}{(625\phi^3 - 100\phi^2\theta^2 - 225\phi^2 - 15\phi\theta^4 - 45\phi\theta^2 + 18\theta^4)^2} \geq \frac{\phi (\alpha - \bar{c})^2 (5\phi - 3\theta^2)^2 (5\phi + 3\theta^2)^2 (5\phi - 4\theta^2)^2 \cdot (25\phi - 4\theta^2)}{(3125\phi^4 - 3625\phi^3\theta^2 - 1125\phi^3 + 800\phi^2\theta^4 + 825\phi^2\theta^2 + 180\phi\theta^4 - 72\theta^6)^2}$$

- Removing an edge between nodes  $H_1, L_3$ : **E3D** [1H3L]  $\rightarrow$  **E2D** [1H3L]

- For  $H_1$  not to be profitable:

$$\frac{25\phi^3 (\alpha - \bar{c})^2 (5\phi - 3\theta^2)^2 \cdot (25\phi - 9)}{(625\phi^3 - 100\phi^2\theta^2 - 225\phi^2 - 15\phi\theta^4 - 45\phi\theta^2 + 18\theta^4)^2} \geq \frac{\phi (\alpha - \bar{c})^2 (5\phi - 6\theta^2)^2 \cdot (25\phi - 16)}{(125\phi^2 - 60\phi\theta^2 - 80\phi + 24\theta^2)^2}$$

- For  $L_3$  not to be profitable:

$$\frac{\phi(\alpha - \bar{c})^2 \cdot (25\phi - \theta^2) (25\phi^2 + 15\phi\theta^2 - 18\theta^2)^2}{(625\phi^3 - 100\phi^2\theta^2 - 225\phi^2 - 15\phi\theta^4 - 45\phi\theta^2 + 18\theta^4)^2} \geq \frac{\phi(\alpha - \bar{c})^2 (5\phi - 4)^2 (5\phi + 3\theta^2)^2 \cdot (25\phi - 4\theta^2)}{(5\phi + \theta^2)^2 (125\phi^2 - 60\phi\theta^2 - 80\phi + 24\theta^2)^2}$$

### C.1.12 Stability conditions for: E3E [1H3L]

- Adding an edge between nodes  $H_1, L_2$ : E3E [1H3L]  $\rightarrow$  E4B [1H3L]

– For  $H_1$  not to be profitable:

$$\frac{\phi(\alpha - \bar{c})^2 (5\phi - 2\theta^2)^2 \cdot (25\phi - 9) (25\phi^2 - 15\phi\theta^2 - 6\theta^4)^2}{(3125\phi^4 - 2125\phi^3\theta^2 - 1125\phi^3 + 225\phi^2\theta^2 + 120\phi\theta^6 + 120\phi\theta^4 - 36\theta^6)^2} > \frac{25\phi^3 (\alpha - \bar{c})^2 (5\phi - 3\theta^2)^2 \cdot (25\phi - 4)}{(625\phi^3 - 350\phi^2\theta^2 - 100\phi^2 + 6\theta^6 + 6\theta^4)^2}$$

– For  $L_2$  not to be profitable:

$$\frac{25\phi^3 (\alpha - \bar{c})^2 \cdot (25\phi - 4\theta^2) (25\phi^2 - 15\phi - 6\theta^4)^2}{(3125\phi^4 - 2125\phi^3\theta^2 - 1125\phi^3 + 225\phi^2\theta^2 + 120\phi\theta^6 + 120\phi\theta^4 - 36\theta^6)^2} > \frac{\phi(\alpha - \bar{c})^2 \cdot (25\phi - \theta^2) (25\phi^2 - 6\theta^4 - 6\theta^2)^2}{(625\phi^3 - 350\phi^2\theta^2 - 100\phi^2 + 6\theta^6 + 6\theta^4)^2}$$

- Adding an edge between nodes  $H_1, L_1$ : E3E [1H3L]  $\rightarrow$  E4C [1H3L]

– For  $H_1$  not to be profitable:

$$\frac{\phi(\alpha - \bar{c})^2 (5\phi - 2\theta^2)^2 \cdot (25\phi - 9) (25\phi^2 - 15\phi\theta^2 - 6\theta^4)^2}{(3125\phi^4 - 2125\phi^3\theta^2 - 1125\phi^3 + 225\phi^2\theta^2 + 120\phi\theta^6 + 120\phi\theta^4 - 36\theta^6)^2} > \frac{\phi(\alpha - \bar{c})^2 (5\phi - 2\theta^2)^2 (5\phi + 2\theta^2)^2 \cdot (25\phi - 4)}{(625\phi^3 - 50\phi^2\theta^2 - 100\phi^2 - 40\phi\theta^4 - 60\phi\theta^2 + 24\theta^4)^2}$$

– For  $L_1$  not to be profitable:

$$\frac{\phi(\alpha - \bar{c})^2 (5\phi - 2\theta^2)^2 \cdot (25\phi - 9\theta^2) (25\phi^2 - 15\phi - 6\theta^2)^2}{(3125\phi^4 - 2125\phi^3\theta^2 - 1125\phi^3 + 225\phi^2\theta^2 + 120\phi\theta^6 + 120\phi\theta^4 - 36\theta^6)^2} > \frac{\phi(\alpha - \bar{c})^2 (5\phi - 2\theta^2)^2 (5\phi + 2\theta^2)^2 \cdot (25\phi - 4\theta^2)}{(625\phi^3 - 50\phi^2\theta^2 - 100\phi^2 - 40\phi\theta^4 - 60\phi\theta^2 + 24\theta^4)^2}$$

- Adding an edge between nodes  $L_1, L_3$ : E3E [1H3L]  $\rightarrow$  E4D [1H3L]

– For  $L_1$  not to be profitable:

$$\frac{\phi(\alpha - \bar{c})^2 (5\phi - 2\theta^2)^2 \cdot (25\phi - 9\theta^2) (25\phi^2 - 15\phi - 6\theta^2)^2}{(3125\phi^4 - 2125\phi^3\theta^2 - 1125\phi^3 + 225\phi^2\theta^2 + 120\phi\theta^6 + 120\phi\theta^4 - 36\theta^6)^2} > \frac{25\phi^3 (\alpha - \bar{c})^2 (5\phi - 3)^2 \cdot (25\phi - 4\theta^2)}{(625\phi^3 - 225\phi^2\theta^2 - 225\phi^2 + 12\theta^4)^2}$$

– For  $L_3$  not to be profitable:

$$\frac{25\phi^3 (\alpha - \bar{c})^2 \cdot (25\phi - 4\theta^2) (25\phi^2 - 15\phi\theta^2 - 6\theta^2)^2}{(3125\phi^4 - 2125\phi^3\theta^2 - 1125\phi^3 + 225\phi^2\theta^2 + 120\phi\theta^6 + 120\phi\theta^4 - 36\theta^6)^2} > \frac{\phi(\alpha - \bar{c})^2 \cdot (25\phi - \theta^2) (25\phi^2 - 12\theta^2)^2}{(625\phi^3 - 225\phi^2\theta^2 - 225\phi^2 + 12\theta^4)^2}$$

- Removing an edge between nodes  $L_1, L_2$ : E3E [1H3L]  $\rightarrow$  E2B [1H3L]

– For  $L_1$  not to be profitable:

$$\frac{\phi(\alpha - \bar{c})^2 (5\phi - 2\theta^2)^2 \cdot (25\phi - 9\theta^2) (25\phi^2 - 15\phi - 6\theta^2)^2}{(3125\phi^4 - 2125\phi^3\theta^2 - 1125\phi^3 + 225\phi^2\theta^2 + 120\phi\theta^6 + 120\phi\theta^4 - 36\theta^6)^2} \geq \frac{\phi(\alpha - \bar{c})^2 \cdot (25\phi - 16\theta^2) (125\phi^3 - 125\phi^2\theta^2 - 75\phi^2 + 45\phi\theta^2 + 18\theta^4)^2}{(3125\phi^4 - 3625\phi^3\theta^2 - 1125\phi^3 + 800\phi^2\theta^4 + 825\phi^2\theta^2 + 180\phi\theta^4 - 72\theta^6)^2}$$

– For  $L_2$  not to be profitable:

$$\frac{25\phi^3 (\alpha - \bar{c})^2 \cdot (25\phi - 4\theta^2) (25\phi^2 - 15\phi - 6\theta^4)^2}{(3125\phi^4 - 2125\phi^3\theta^2 - 1125\phi^3 + 225\phi^2\theta^2 + 120\phi\theta^6 + 120\phi\theta^4 - 36\theta^6)^2} \geq \frac{25\phi^3 (\alpha - \bar{c})^2 (5\phi - 3)^2 (5\phi - 4\theta^2)^2 \cdot (25\phi - 9\theta^2)}{(3125\phi^4 - 3625\phi^3\theta^2 - 1125\phi^3 + 800\phi^2\theta^4 + 825\phi^2\theta^2 + 180\phi\theta^4 - 72\theta^6)^2}$$

- Removing an edge between nodes  $L_2, L_3$ : E3E [1H3L]  $\rightarrow$  E2C [1H3L]

– For  $L_2$  not to be profitable:

$$\frac{25\phi^3 (\alpha - \bar{c})^2 \cdot (25\phi - 4\theta^2) (25\phi^2 - 15\phi - 6\theta^4)^2}{(3125\phi^4 - 2125\phi^3\theta^2 - 1125\phi^3 + 225\phi^2\theta^2 + 120\phi\theta^6 + 120\phi\theta^4 - 36\theta^6)^2} \geq \frac{\phi(\alpha - \bar{c})^2 \cdot (25\phi - 9\theta^2) (5\phi - 3\theta^2 - 3)^2}{(125\phi^2 - 135\phi\theta^2 - 45\phi + 18\theta^4 + 18\theta^2)^2}$$

– For  $L_3$  not to be profitable:

$$\frac{25\phi^3(\alpha - \bar{c})^2 \cdot (25\phi - 4\theta^2)(25\phi^2 - 15\phi\theta^2 - 6\theta^2)^2}{(3125\phi^4 - 2125\phi^3\theta^2 - 1125\phi^3 + 225\phi^2\theta^2 + 120\phi\theta^6 + 120\phi\theta^4 - 36\theta^6)^2} \geq \frac{\phi(\alpha - \bar{c})^2(5\phi - 6\theta^2)^2 \cdot (25\phi - 9\theta^2)}{(125\phi^2 - 135\phi\theta^2 - 45\phi + 18\theta^4 + 18\theta^2)^2}$$

- Removing an edge between nodes  $H_1, L_3$ : **E3E** [1H3L]  $\rightarrow$  **E2D** [1H3L]

– For  $H_1$  not to be profitable:

$$\frac{\phi(\alpha - \bar{c})^2(5\phi - 2\theta^2)^2 \cdot (25\phi - 9)(25\phi^2 - 15\phi\theta^2 - 6\theta^4)^2}{(3125\phi^4 - 2125\phi^3\theta^2 - 1125\phi^3 + 225\phi^2\theta^2 + 120\phi\theta^6 + 120\phi\theta^4 - 36\theta^6)^2} \geq \frac{\phi(\alpha - \bar{c})^2(5\phi - 6\theta^2)^2 \cdot (25\phi - 16)}{(125\phi^2 - 60\phi\theta^2 - 80\phi + 24\theta^2)^2}$$

– For  $L_3$  not to be profitable:

$$\frac{25\phi^3(\alpha - \bar{c})^2 \cdot (25\phi - 4\theta^2)(25\phi^2 - 15\phi\theta^2 - 6\theta^2)^2}{(3125\phi^4 - 2125\phi^3\theta^2 - 1125\phi^3 + 225\phi^2\theta^2 + 120\phi\theta^6 + 120\phi\theta^4 - 36\theta^6)^2} \geq \frac{25\phi^3(\alpha - \bar{c})^2(5\phi - 4)^2 \cdot (25\phi - 9\theta^2)}{(5\phi + \theta^2)^2(125\phi^2 - 60\phi\theta^2 - 80\phi + 24\theta^2)^2}$$

### C.1.13 Stability conditions for: **E3F** [1H3L]

- Adding an edge between nodes  $H_1, L_1$ : **E3F** [1H3L]  $\rightarrow$  **E4D** [1H3L]

– For  $H_1$  not to be profitable:

$$\frac{\phi(\alpha - \bar{c})^2(5\phi - 6\theta^2)^2 \cdot (25\phi - 16)}{(125\phi^2 - 60\phi\theta^2 - 80\phi + 24\theta^2)^2} > \frac{25\phi^3(\alpha - \bar{c})^2(5\phi - 4\theta^2)^2 \cdot (25\phi - 9)}{(625\phi^3 - 225\phi^2\theta^2 - 225\phi^2 + 12\theta^4)^2}$$

– For  $L_1$  not to be profitable:

$$\frac{\phi(\alpha - \bar{c})^2(5\phi - 4)^2 \cdot (25\phi - 4\theta^2)}{(125\phi^2 - 60\phi\theta^2 - 80\phi + 24\theta^2)^2} > \frac{\phi(\alpha - \bar{c})^2 \cdot (25\phi - \theta^2)(25\phi^2 - 12\theta^2)^2}{(625\phi^3 - 225\phi^2\theta^2 - 225\phi^2 + 12\theta^4)^2}$$

- Removing an edge between nodes  $L_1, L_2$ : **E3F** [1H3L]  $\rightarrow$  **E2D** [1H3L]

– For  $L_1$  not to be profitable:

$$\frac{\phi(\alpha - \bar{c})^2(5\phi - 4)^2 \cdot (25\phi - 4\theta^2)}{(125\phi^2 - 60\phi\theta^2 - 80\phi + 24\theta^2)^2} \geq \frac{25\phi^3(\alpha - \bar{c})^2(5\phi - 4)^2 \cdot (25\phi - 9\theta^2)}{(5\phi + \theta^2)^2(125\phi^2 - 60\phi\theta^2 - 80\phi + 24\theta^2)^2}$$

– For  $L_2$  not to be profitable:

$$\frac{\phi(\alpha - \bar{c})^2(5\phi - 4)^2 \cdot (25\phi - 4\theta^2)}{(125\phi^2 - 60\phi\theta^2 - 80\phi + 24\theta^2)^2} \geq \frac{25\phi^3(\alpha - \bar{c})^2(5\phi - 4)^2 \cdot (25\phi - 9\theta^2)}{(5\phi + \theta^2)^2(125\phi^2 - 60\phi\theta^2 - 80\phi + 24\theta^2)^2}$$

### C.1.14 Stability conditions for: **E4A** [1H3L]

- Adding an edge between nodes  $L_1, L_2$ : **E4A** [1H3L]  $\rightarrow$  **E5A** [1H3L]

– For  $L_1$  not to be profitable:

$$\frac{25\phi^3(\alpha - \bar{c})^2(5\phi - 4\theta^2)^2 \cdot (25\phi - 9\theta^2)}{(625\phi^3 - 425\phi^2\theta^2 - 25\phi^2 + 12\theta^4)^2} > \frac{25\phi^3(\alpha - \bar{c})^2 \cdot (25\phi - 4\theta^2)}{(125\phi^2 + 5\phi\theta^2 - 5\phi - 2\theta^4 - 2\theta^2)^2}$$

– For  $L_2$  not to be profitable:

$$\frac{25\phi^3(\alpha - \bar{c})^2(5\phi - 3\theta^2)^2 \cdot (25\phi - 4\theta^2)}{(625\phi^3 - 425\phi^2\theta^2 - 25\phi^2 + 12\theta^4)^2} > \frac{\phi(\alpha - \bar{c})^2(5\phi + 2\theta^2)^2 \cdot (25\phi - \theta^2)}{(125\phi^2 + 5\phi\theta^2 - 5\phi - 2\theta^4 - 2\theta^2)^2}$$

- Removing an edge between nodes  $L_2, L_3$ : **E4A** [1H3L]  $\rightarrow$  **E3A** [1H3L]

– For  $L_2$  not to be profitable:

$$\frac{25\phi^3(\alpha - \bar{c})^2(5\phi - 3\theta^2)^2 \cdot (25\phi - 4\theta^2)}{(625\phi^3 - 425\phi^2\theta^2 - 25\phi^2 + 12\theta^4)^2} \geq \frac{25\phi^3(\alpha - \bar{c})^2 \cdot (25\phi - 9\theta^2)}{(125\phi^2 + 15\phi\theta^2 - 5\phi - 6\theta^2)^2}$$

– For  $L_3$  not to be profitable:

$$\frac{25\phi^3(\alpha - \bar{c})^2(5\phi - 3\theta^2)^2 \cdot (25\phi - 4\theta^2)}{(625\phi^3 - 425\phi^2\theta^2 - 25\phi^2 + 12\theta^4)^2} \geq \frac{25\phi^3(\alpha - \bar{c})^2 \cdot (25\phi - 9\theta^2)}{(125\phi^2 + 15\phi\theta^2 - 5\phi - 6\theta^2)^2}$$



- Removing an edge between nodes  $H_1, L_1$ : **E4A [1H3L]**  $\rightarrow$  **E3B [1H3L]**

– For  $H_1$  not to be profitable:

$$\frac{\phi(\alpha - \bar{c})^2 \cdot (25\phi - 1) (25\phi^2 - 12\theta^4)^2}{(625\phi^3 - 425\phi^2\theta^2 - 25\phi^2 + 12\theta^4)^2} \geq \frac{\phi(\alpha - \bar{c})^2 (5\phi - 4\theta^2)^2 \cdot (25\phi - 4)}{(125\phi^2 - 120\phi\theta^2 - 20\phi + 16\theta^4 + 8\theta^2)^2}$$

– For  $L_1$  not to be profitable:

$$\frac{25\phi^3 (\alpha - \bar{c})^2 (5\phi - 4\theta^2)^2 \cdot (25\phi - 9\theta^2)}{(625\phi^3 - 425\phi^2\theta^2 - 25\phi^2 + 12\theta^4)^2} \geq \frac{\phi(\alpha - \bar{c})^2 \cdot (25\phi - 16\theta^2) (5\phi - 4\theta^2 - 2)^2}{(125\phi^2 - 120\phi\theta^2 - 20\phi + 16\theta^4 + 8\theta^2)^2}$$

- Removing an edge between nodes  $H_1, L_2$ : **E4A [1H3L]**  $\rightarrow$  **E3C [1H3L]**

– For  $H_1$  not to be profitable:

$$\frac{\phi(\alpha - \bar{c})^2 \cdot (25\phi - 1) (25\phi^2 - 12\theta^4)^2}{(625\phi^3 - 425\phi^2\theta^2 - 25\phi^2 + 12\theta^4)^2} \geq \frac{25\phi^3 (\alpha - \bar{c})^2 \cdot (25\phi - 4) (25\phi^2 - 15\phi\theta^2 - 6\theta^4)^2}{(3125\phi^4 - 2750\phi^3\theta^2 - 500\phi^3 + 225\phi^2\theta^4 + 240\phi\theta^4 - 36\theta^6)^2}$$

– For  $L_2$  not to be profitable:

$$\frac{25\phi^3 (\alpha - \bar{c})^2 (5\phi - 3\theta^2)^2 \cdot (25\phi - 4\theta^2)}{(625\phi^3 - 425\phi^2\theta^2 - 25\phi^2 + 12\theta^4)^2} \geq \frac{\phi(\alpha - \bar{c})^2 \cdot (25\phi - 9\theta^2) (125\phi^3 - 75\phi^2\theta^2 - 50\phi^2 + 12\theta^4)^2}{(3125\phi^4 - 2750\phi^3\theta^2 - 500\phi^3 + 225\phi^2\theta^4 + 240\phi\theta^4 - 36\theta^6)^2}$$

### C.1.15 Stability conditions for: **E4B [1H3L]**

- Adding an edge between nodes  $H_1, L_1$ : **E4B [1H3L]**  $\rightarrow$  **E5A [1H3L]**

– For  $H_1$  not to be profitable:

$$\frac{25\phi^3 (\alpha - \bar{c})^2 (5\phi - 3\theta^2)^2 \cdot (25\phi - 4)}{(625\phi^3 - 350\phi^2\theta^2 - 100\phi^2 + 6\theta^6 + 6\theta^4)^2} > \frac{\phi(\alpha - \bar{c})^2 (5\phi + 2\theta^2)^2 \cdot (25\phi - 1)}{(125\phi^2 + 5\phi\theta^2 - 5\phi - 2\theta^4 - 2\theta^2)^2}$$

– For  $L_1$  not to be profitable:

$$\frac{25\phi^3 (\alpha - \bar{c})^2 \cdot (25\phi - 9\theta^2) (5\phi - 2\theta^2 - 2)^2}{(625\phi^3 - 350\phi^2\theta^2 - 100\phi^2 + 6\theta^6 + 6\theta^4)^2} > \frac{25\phi^3 (\alpha - \bar{c})^2 \cdot (25\phi - 4\theta^2)}{(125\phi^2 + 5\phi\theta^2 - 5\phi - 2\theta^4 - 2\theta^2)^2}$$

- Adding an edge between nodes  $L_1, L_2$ : **E4B [1H3L]**  $\rightarrow$  **E5B [1H3L]**

– For  $L_1$  not to be profitable:

$$\frac{25\phi^3 (\alpha - \bar{c})^2 \cdot (25\phi - 9\theta^2) (5\phi - 2\theta^2 - 2)^2}{(625\phi^3 - 350\phi^2\theta^2 - 100\phi^2 + 6\theta^6 + 6\theta^4)^2} > \frac{25\phi^3 (\alpha - \bar{c})^2 (5\phi - 2)^2 \cdot (25\phi - 4\theta^2)}{(625\phi^3 - 150\phi^2\theta^2 - 100\phi^2 - 20\phi\theta^2 + 8\theta^4)^2}$$

– For  $L_2$  not to be profitable:

$$\frac{25\phi^3 (\alpha - \bar{c})^2 (5\phi - 3\theta^2)^2 \cdot (25\phi - 4\theta^2)}{(625\phi^3 - 350\phi^2\theta^2 - 100\phi^2 + 6\theta^6 + 6\theta^4)^2} > \frac{\phi(\alpha - \bar{c})^2 (5\phi - 2\theta^2)^2 (5\phi + 2\theta^2)^2 \cdot (25\phi - \theta^2)}{(625\phi^3 - 150\phi^2\theta^2 - 100\phi^2 - 20\phi\theta^2 + 8\theta^4)^2}$$

- Removing an edge between nodes  $L_1, L_3$ : **E4B [1H3L]**  $\rightarrow$  **E3B [1H3L]**

– For  $L_1$  not to be profitable:

$$\frac{25\phi^3 (\alpha - \bar{c})^2 \cdot (25\phi - 9\theta^2) (5\phi - 2\theta^2 - 2)^2}{(625\phi^3 - 350\phi^2\theta^2 - 100\phi^2 + 6\theta^6 + 6\theta^4)^2} \geq \frac{\phi(\alpha - \bar{c})^2 \cdot (25\phi - 16\theta^2) (5\phi - 4\theta^2 - 2)^2}{(125\phi^2 - 120\phi\theta^2 - 20\phi + 16\theta^4 + 8\theta^2)^2}$$

– For  $L_3$  not to be profitable:

$$\frac{\phi(\alpha - \bar{c})^2 \cdot (25\phi - \theta^2) (25\phi^2 - 6\theta^4 - 6\theta^2)^2}{(625\phi^3 - 350\phi^2\theta^2 - 100\phi^2 + 6\theta^6 + 6\theta^4)^2} \geq \frac{\phi(\alpha - \bar{c})^2 (5\phi - 4\theta^2)^2 \cdot (25\phi - 4\theta^2)}{(125\phi^2 - 120\phi\theta^2 - 20\phi + 16\theta^4 + 8\theta^2)^2}$$

- Removing an edge between nodes  $L_2, L_3$ : **E4B [1H3L]**  $\rightarrow$  **E3C [1H3L]**

– For  $L_2$  not to be profitable:

$$\frac{25\phi^3 (\alpha - \bar{c})^2 (5\phi - 3\theta^2)^2 \cdot (25\phi - 4\theta^2)}{(625\phi^3 - 350\phi^2\theta^2 - 100\phi^2 + 6\theta^6 + 6\theta^4)^2} \geq \frac{\phi(\alpha - \bar{c})^2 \cdot (25\phi - 9\theta^2) (125\phi^3 - 125\phi^2\theta^2 + 12\theta^4)^2}{(3125\phi^4 - 2750\phi^3\theta^2 - 500\phi^3 + 225\phi^2\theta^4 + 240\phi\theta^4 - 36\theta^6)^2}$$

– For  $L_3$  not to be profitable:

$$\frac{\phi(\alpha - \bar{c})^2 \cdot (25\phi - \theta^2) (25\phi^2 - 6\theta^4 - 6\theta^2)^2}{(625\phi^3 - 350\phi^2\theta^2 - 100\phi^2 + 6\theta^6 + 6\theta^4)^2} \geq \frac{25\phi^3(\alpha - \bar{c})^2 \cdot (25\phi - 4\theta^2) (25\phi^2 - 15\phi\theta^2 - 6\theta^2)^2}{(3125\phi^4 - 2750\phi^3\theta^2 - 500\phi^3 + 225\phi^2\theta^4 + 240\phi\theta^4 - 36\theta^6)^2}$$

- Removing an edge between nodes  $H_1, L_2$ : **E4B** [1H3L]  $\rightarrow$  **E3D** [1H3L]

– For  $H_1$  not to be profitable:

$$\frac{25\phi^3(\alpha - \bar{c})^2 (5\phi - 3\theta^2)^2 \cdot (25\phi - 4)}{(625\phi^3 - 350\phi^2\theta^2 - 100\phi^2 + 6\theta^6 + 6\theta^4)^2} \geq \frac{25\phi^3(\alpha - \bar{c})^2 (5\phi - 3\theta^2)^2 \cdot (25\phi - 9)}{(625\phi^3 - 100\phi^2\theta^2 - 225\phi^2 - 15\phi\theta^4 - 45\phi\theta^2 + 18\theta^4)^2}$$

– For  $L_2$  not to be profitable:

$$\frac{25\phi^3(\alpha - \bar{c})^2 (5\phi - 3\theta^2)^2 \cdot (25\phi - 4\theta^2)}{(625\phi^3 - 350\phi^2\theta^2 - 100\phi^2 + 6\theta^6 + 6\theta^4)^2} \geq \frac{25\phi^3(\alpha - \bar{c})^2 (5\phi - 3)^2 \cdot (25\phi - 9\theta^2)}{(625\phi^3 - 100\phi^2\theta^2 - 225\phi^2 - 15\phi\theta^4 - 45\phi\theta^2 + 18\theta^4)^2}$$

- Removing an edge between nodes  $H_1, L_3$ : **E4B** [1H3L]  $\rightarrow$  **E3E** [1H3L]

– For  $H_1$  not to be profitable:

$$\frac{25\phi^3(\alpha - \bar{c})^2 (5\phi - 3\theta^2)^2 \cdot (25\phi - 4)}{(625\phi^3 - 350\phi^2\theta^2 - 100\phi^2 + 6\theta^6 + 6\theta^4)^2} \geq \frac{\phi(\alpha - \bar{c})^2 (5\phi - 2\theta^2)^2 \cdot (25\phi - 9) (25\phi^2 - 15\phi\theta^2 - 6\theta^4)^2}{(3125\phi^4 - 2125\phi^3\theta^2 - 1125\phi^3 + 225\phi^2\theta^2 + 120\phi\theta^6 + 120\phi\theta^4 - 36\theta^6)^2}$$

– For  $L_3$  not to be profitable:

$$\frac{\phi(\alpha - \bar{c})^2 \cdot (25\phi - \theta^2) (25\phi^2 - 6\theta^4 - 6\theta^2)^2}{(625\phi^3 - 350\phi^2\theta^2 - 100\phi^2 + 6\theta^6 + 6\theta^4)^2} \geq \frac{25\phi^3(\alpha - \bar{c})^2 \cdot (25\phi - 4\theta^2) (25\phi^2 - 15\phi - 6\theta^4)^2}{(3125\phi^4 - 2125\phi^3\theta^2 - 1125\phi^3 + 225\phi^2\theta^2 + 120\phi\theta^6 + 120\phi\theta^4 - 36\theta^6)^2}$$

#### C.1.16 Stability conditions for: **E4C** [1H3L]

- Adding an edge between nodes  $H_1, L_1$ : **E4C** [1H3L]  $\rightarrow$  **E5A** [1H3L]

– For  $H_1$  not to be profitable:

$$\frac{\phi(\alpha - \bar{c})^2 (5\phi - 2\theta^2)^2 (5\phi + 2\theta^2)^2 \cdot (25\phi - 4)}{(625\phi^3 - 50\phi^2\theta^2 - 100\phi^2 - 40\phi\theta^4 - 60\phi\theta^2 + 24\theta^4)^2} > \frac{\phi(\alpha - \bar{c})^2 (5\phi + 2\theta^2)^2 \cdot (25\phi - 1)}{(125\phi^2 + 5\phi\theta^2 - 5\phi - 2\theta^4 - 2\theta^2)^2}$$

– For  $L_1$  not to be profitable:

$$\frac{\phi(\alpha - \bar{c})^2 (5\phi - 2)^2 (5\phi + 2\theta^2)^2 \cdot (25\phi - 4\theta^2)}{(625\phi^3 - 50\phi^2\theta^2 - 100\phi^2 - 40\phi\theta^4 - 60\phi\theta^2 + 24\theta^4)^2} > \frac{\phi(\alpha - \bar{c})^2 (5\phi + 2\theta^2)^2 \cdot (25\phi - \theta^2)}{(125\phi^2 + 5\phi\theta^2 - 5\phi - 2\theta^4 - 2\theta^2)^2}$$

- Adding an edge between nodes  $L_2, L_3$ : **E4C** [1H3L]  $\rightarrow$  **E5B** [1H3L]

– For  $L_2$  not to be profitable:

$$\frac{\phi(\alpha - \bar{c})^2 (5\phi - 2\theta)^2 (5\phi + 2\theta)^2 \cdot (25\phi - 4\theta^2)}{(625\phi^3 - 50\phi^2\theta^2 - 100\phi^2 - 40\phi\theta^4 - 60\phi\theta^2 + 24\theta^4)^2} > \frac{\phi(\alpha - \bar{c})^2 (5\phi - 2\theta)^2 (5\phi + 2\theta)^2 \cdot (25\phi - \theta^2)}{(625\phi^3 - 150\phi^2\theta^2 - 100\phi^2 - 20\phi\theta^2 + 8\theta^4)^2}$$

– For  $L_3$  not to be profitable:

$$\frac{\phi(\alpha - \bar{c})^2 (5\phi - 2\theta)^2 (5\phi + 2\theta)^2 \cdot (25\phi - 4\theta^2)}{(625\phi^3 - 50\phi^2\theta^2 - 100\phi^2 - 40\phi\theta^4 - 60\phi\theta^2 + 24\theta^4)^2} > \frac{\phi(\alpha - \bar{c})^2 (5\phi - 2\theta)^2 (5\phi + 2\theta)^2 \cdot (25\phi - \theta^2)}{(625\phi^3 - 150\phi^2\theta^2 - 100\phi^2 - 20\phi\theta^2 + 8\theta^4)^2}$$

- Removing an edge between nodes  $L_1, L_2$ : **E4C** [1H3L]  $\rightarrow$  **E3C** [1H3L]

– For  $L_1$  not to be profitable:

$$\frac{\phi(\alpha - \bar{c})^2 (5\phi - 2)^2 (5\phi + 2\theta^2)^2 \cdot (25\phi - 4\theta^2)}{(625\phi^3 - 50\phi^2\theta^2 - 100\phi^2 - 40\phi\theta^4 - 60\phi\theta^2 + 24\theta^4)^2} \geq \frac{\phi(\alpha - \bar{c})^2 \cdot (25\phi - 9\theta^2) (125\phi^3 - 75\phi^2\theta^2 - 50\phi^2 + 12\theta^4)^2}{(3125\phi^4 - 2750\phi^3\theta^2 - 500\phi^3 + 225\phi^2\theta^4 + 240\phi\theta^4 - 36\theta^6)^2}$$

– For  $L_2$  not to be profitable:

$$\frac{\phi(\alpha - \bar{c})^2 (5\phi - 2\theta)^2 (5\phi + 2\theta)^2 \cdot (25\phi - 4\theta^2)}{(625\phi^3 - 50\phi^2\theta^2 - 100\phi^2 - 40\phi\theta^4 - 60\phi\theta^2 + 24\theta^4)^2} \geq \frac{\phi(\alpha - \bar{c})^2 \cdot (25\phi - 9\theta^2) (125\phi^3 - 125\phi^2\theta^2 + 12\theta^4)^2}{(3125\phi^4 - 2750\phi^3\theta^2 - 500\phi^3 + 225\phi^2\theta^4 + 240\phi\theta^4 - 36\theta^6)^2}$$

- Removing an edge between nodes  $H_1, L_2$ : **E4C** [1H3L]  $\rightarrow$  **E3E** [1H3L]

– For  $H_1$  not to be profitable:

$$\frac{\phi(\alpha - \bar{c})^2 (5\phi - 2\theta^2)^2 (5\phi + 2\theta^2)^2 \cdot (25\phi - 4)}{(625\phi^3 - 50\phi^2\theta^2 - 100\phi^2 - 40\phi\theta^4 - 60\phi\theta^2 + 24\theta^4)^2} \geq \frac{\phi(\alpha - \bar{c})^2 (5\phi - 2\theta^2)^2 \cdot (25\phi - 9) (25\phi^2 - 15\phi\theta^2 - 6\theta^4)^2}{(3125\phi^4 - 2125\phi^3\theta^2 - 1125\phi^3 + 225\phi^2\theta^2 + 120\phi\theta^6 + 120\phi\theta^4 - 36\theta^6)^2}$$

– For  $L_2$  not to be profitable:

$$\frac{\phi(\alpha - \bar{c})^2 (5\phi - 2\theta^2)^2 (5\phi + 2\theta^2)^2 \cdot (25\phi - 4\theta^2)}{(625\phi^3 - 50\phi^2\theta^2 - 100\phi^2 - 40\phi\theta^4 - 60\phi\theta^2 + 24\theta^4)^2} \geq \frac{\phi(\alpha - \bar{c})^2 (5\phi - 2\theta^2)^2 \cdot (25\phi - 9\theta^2) (25\phi^2 - 15\phi - 6\theta^2)^2}{(3125\phi^4 - 2125\phi^3\theta^2 - 1125\phi^3 + 225\phi^2\theta^2 + 120\phi\theta^6 + 120\phi\theta^4 - 36\theta^6)^2}$$

### C.1.17 Stability conditions for: E4D [1H3L]

- Adding an edge between nodes  $H_1, L_1$ : E4D [1H3L]  $\rightarrow$  E5B [1H3L]

– For  $H_1$  not to be profitable:

$$\frac{25\phi^3 (\alpha - \bar{c})^2 (5\phi - 4\theta^2)^2 \cdot (25\phi - 9)}{(625\phi^3 - 225\phi^2\theta^2 - 225\phi^2 + 12\theta^4)^2} > \frac{25\phi^3 (\alpha - \bar{c})^2 (5\phi - 2\theta^2)^2 \cdot (25\phi - 4)}{(625\phi^3 - 150\phi^2\theta^2 - 100\phi^2 - 20\phi\theta^2 + 8\theta^4)^2}$$

– For  $L_1$  not to be profitable:

$$\frac{25\phi^3 (\alpha - \bar{c})^2 (5\phi - 3)^2 \cdot (25\phi - 4\theta^2)}{(625\phi^3 - 225\phi^2\theta^2 - 225\phi^2 + 12\theta^4)^2} > \frac{\phi(\alpha - \bar{c})^2 (5\phi - 2\theta^2)^2 (5\phi + 2\theta^2)^2 \cdot (25\phi - \theta^2)}{(625\phi^3 - 150\phi^2\theta^2 - 100\phi^2 - 20\phi\theta^2 + 8\theta^4)^2}$$

- Removing an edge between nodes  $L_1, L_2$ : E4D [1H3L]  $\rightarrow$  E3D [1H3L]

– For  $L_1$  not to be profitable:

$$\frac{25\phi^3 (\alpha - \bar{c})^2 (5\phi - 3)^2 \cdot (25\phi - 4\theta^2)}{(625\phi^3 - 225\phi^2\theta^2 - 225\phi^2 + 12\theta^4)^2} \geq \frac{25\phi^3 (\alpha - \bar{c})^2 (5\phi - 3)^2 \cdot (25\phi - 9\theta^2)}{(625\phi^3 - 100\phi^2\theta^2 - 225\phi^2 - 15\phi\theta^4 - 45\phi\theta^2 + 18\theta^4)^2}$$

– For  $L_2$  not to be profitable:

$$\frac{25\phi^3 (\alpha - \bar{c})^2 (5\phi - 3)^2 \cdot (25\phi - 4\theta^2)}{(625\phi^3 - 225\phi^2\theta^2 - 225\phi^2 + 12\theta^4)^2} \geq \frac{25\phi^3 (\alpha - \bar{c})^2 (5\phi - 3)^2 \cdot (25\phi - 9\theta^2)}{(625\phi^3 - 100\phi^2\theta^2 - 225\phi^2 - 15\phi\theta^4 - 45\phi\theta^2 + 18\theta^4)^2}$$

- Removing an edge between nodes  $L_1, L_3$ : E4D [1H3L]  $\rightarrow$  E3E [1H3L]

– For  $L_1$  not to be profitable:

$$\frac{25\phi^3 (\alpha - \bar{c})^2 (5\phi - 3)^2 \cdot (25\phi - 4\theta^2)}{(625\phi^3 - 225\phi^2\theta^2 - 225\phi^2 + 12\theta^4)^2} \geq \frac{\phi(\alpha - \bar{c})^2 (5\phi - 2\theta^2)^2 \cdot (25\phi - 9\theta^2) (25\phi^2 - 15\phi - 6\theta^2)^2}{(3125\phi^4 - 2125\phi^3\theta^2 - 1125\phi^3 + 225\phi^2\theta^2 + 120\phi\theta^6 + 120\phi\theta^4 - 36\theta^6)^2}$$

– For  $L_3$  not to be profitable:

$$\frac{\phi(\alpha - \bar{c})^2 \cdot (25\phi - \theta^2) (25\phi^2 - 12\theta^2)^2}{(625\phi^3 - 225\phi^2\theta^2 - 225\phi^2 + 12\theta^4)^2} \geq \frac{25\phi^3 (\alpha - \bar{c})^2 \cdot (25\phi - 4\theta^2) (25\phi^2 - 15\phi\theta^2 - 6\theta^2)^2}{(3125\phi^4 - 2125\phi^3\theta^2 - 1125\phi^3 + 225\phi^2\theta^2 + 120\phi\theta^6 + 120\phi\theta^4 - 36\theta^6)^2}$$

- Removing an edge between nodes  $H_1, L_3$ : E4D [1H3L]  $\rightarrow$  E3F [1H3L]

– For  $H_1$  not to be profitable:

$$\frac{25\phi^3 (\alpha - \bar{c})^2 (5\phi - 4\theta^2)^2 \cdot (25\phi - 9)}{(625\phi^3 - 225\phi^2\theta^2 - 225\phi^2 + 12\theta^4)^2} \geq \frac{\phi(\alpha - \bar{c})^2 (5\phi - 6\theta^2)^2 \cdot (25\phi - 16)}{(125\phi^2 - 60\phi\theta^2 - 80\phi + 24\theta^2)^2}$$

– For  $L_3$  not to be profitable:

$$\frac{\phi(\alpha - \bar{c})^2 \cdot (25\phi - \theta^2) (25\phi^2 - 12\theta^2)^2}{(625\phi^3 - 225\phi^2\theta^2 - 225\phi^2 + 12\theta^4)^2} \geq \frac{\phi(\alpha - \bar{c})^2 (5\phi - 4)^2 \cdot (25\phi - 4\theta^2)}{(125\phi^2 - 60\phi\theta^2 - 80\phi + 24\theta^2)^2}$$

### C.1.18 Stability conditions for: E5A [1H3L]

- Adding an edge between nodes  $L_1, L_2$ : E5A [1H3L]  $\rightarrow$  E6A [1H3L]

– For  $L_1$  not to be profitable:

$$\frac{25\phi^3 (\alpha - \bar{c})^2 \cdot (25\phi - 4\theta^2)}{(125\phi^2 + 5\phi\theta^2 - 5\phi - 2\theta^4 - 2\theta^2)^2} > \frac{\phi(\alpha - \bar{c})^2 \cdot (25\phi - \theta^2)}{(25\phi - 3\theta^2 - 1)^2}$$

- For  $L_2$  not to be profitable:

$$\frac{25\phi^3 (\alpha - \bar{c})^2 \cdot (25\phi - 4\theta^2)}{(125\phi^2 + 5\phi\theta^2 - 5\phi - 2\theta^4 - 2\theta^2)^2} > \frac{\phi (\alpha - \bar{c})^2 \cdot (25\phi - \theta^2)}{(25\phi - 3\theta^2 - 1)^2}$$

- Removing an edge between nodes  $L_1, L_3$ : **E5A [1H3L]** → **E4A [1H3L]**

- For  $L_1$  not to be profitable:

$$\frac{25\phi^3 (\alpha - \bar{c})^2 \cdot (25\phi - 4\theta^2)}{(125\phi^2 + 5\phi\theta^2 - 5\phi - 2\theta^4 - 2\theta^2)^2} \geq \frac{25\phi^3 (\alpha - \bar{c})^2 (5\phi - 4\theta^2)^2 \cdot (25\phi - 9\theta^2)}{(625\phi^3 - 425\phi^2\theta^2 - 25\phi^2 + 12\theta^4)^2}$$

- For  $L_3$  not to be profitable:

$$\frac{\phi (\alpha - \bar{c})^2 (5\phi + 2\theta^2)^2 \cdot (25\phi - \theta^2)}{(125\phi^2 + 5\phi\theta^2 - 5\phi - 2\theta^4 - 2\theta^2)^2} \geq \frac{25\phi^3 (\alpha - \bar{c})^2 (5\phi - 3\theta^2)^2 \cdot (25\phi - 4\theta^2)}{(625\phi^3 - 425\phi^2\theta^2 - 25\phi^2 + 12\theta^4)^2}$$

- Removing an edge between nodes  $H_1, L_1$ : **E5A [1H3L]** → **E4B [1H3L]**

- For  $H_1$  not to be profitable:

$$\frac{\phi (\alpha - \bar{c})^2 (5\phi + 2\theta^2)^2 \cdot (25\phi - 1)}{(125\phi^2 + 5\phi\theta^2 - 5\phi - 2\theta^4 - 2\theta^2)^2} \geq \frac{25\phi^3 (\alpha - \bar{c})^2 (5\phi - 3\theta^2)^2 \cdot (25\phi - 4)}{(625\phi^3 - 350\phi^2\theta^2 - 100\phi^2 + 6\theta^6 + 6\theta^4)^2}$$

- For  $L_1$  not to be profitable:

$$\frac{25\phi^3 (\alpha - \bar{c})^2 \cdot (25\phi - 4\theta^2)}{(125\phi^2 + 5\phi\theta^2 - 5\phi - 2\theta^4 - 2\theta^2)^2} \geq \frac{25\phi^3 (\alpha - \bar{c})^2 \cdot (25\phi - 9\theta^2) (5\phi - 2\theta^2 - 2)^2}{(625\phi^3 - 350\phi^2\theta^2 - 100\phi^2 + 6\theta^6 + 6\theta^4)^2}$$

- Removing an edge between nodes  $H_1, L_3$ : **E5A [1H3L]** → **E4C [1H3L]**

- For  $H_1$  not to be profitable:

$$\frac{\phi (\alpha - \bar{c})^2 (5\phi + 2\theta^2)^2 \cdot (25\phi - 1)}{(125\phi^2 + 5\phi\theta^2 - 5\phi - 2\theta^4 - 2\theta^2)^2} \geq \frac{\phi (\alpha - \bar{c})^2 (5\phi - 2\theta^2)^2 (5\phi + 2\theta^2)^2 \cdot (25\phi - 4)}{(625\phi^3 - 50\phi^2\theta^2 - 100\phi^2 - 40\phi\theta^4 - 60\phi\theta^2 + 24\theta^4)^2}$$

- For  $L_3$  not to be profitable:

$$\frac{\phi (\alpha - \bar{c})^2 (5\phi + 2\theta^2)^2 \cdot (25\phi - \theta^2)}{(125\phi^2 + 5\phi\theta^2 - 5\phi - 2\theta^4 - 2\theta^2)^2} \geq \frac{\phi (\alpha - \bar{c})^2 (5\phi - 2)^2 (5\phi + 2\theta^2)^2 \cdot (25\phi - 4\theta^2)}{(625\phi^3 - 50\phi^2\theta^2 - 100\phi^2 - 40\phi\theta^4 - 60\phi\theta^2 + 24\theta^4)^2}$$

### C.1.19 Stability conditions for: **E5B [1H3L]**

- Adding an edge between nodes  $H_1, L_1$ : **E5B [1H3L]** → **E6A [1H3L]**

- For  $H_1$  not to be profitable:

$$\frac{25\phi^3 (\alpha - \bar{c})^2 (5\phi - 2\theta^2)^2 \cdot (25\phi - 4)}{(625\phi^3 - 150\phi^2\theta^2 - 100\phi^2 - 20\phi\theta^2 + 8\theta^4)^2} > \frac{\phi (\alpha - \bar{c})^2 \cdot (25\phi - 1)}{(25\phi - 3\theta^2 - 1)^2}$$

- For  $L_1$  not to be profitable:

$$\frac{25\phi^3 (\alpha - \bar{c})^2 (5\phi - 2)^2 \cdot (25\phi - 4\theta^2)}{(625\phi^3 - 150\phi^2\theta^2 - 100\phi^2 - 20\phi\theta^2 + 8\theta^4)^2} > \frac{\phi (\alpha - \bar{c})^2 \cdot (25\phi - \theta^2)}{(25\phi - 3\theta^2 - 1)^2}$$

- Removing an edge between nodes  $L_1, L_2$ : **E5B [1H3L]** → **E4B [1H3L]**

- For  $L_1$  not to be profitable:

$$\frac{25\phi^3 (\alpha - \bar{c})^2 (5\phi - 2)^2 \cdot (25\phi - 4\theta^2)}{(625\phi^3 - 150\phi^2\theta^2 - 100\phi^2 - 20\phi\theta^2 + 8\theta^4)^2} \geq \frac{25\phi^3 (\alpha - \bar{c})^2 \cdot (25\phi - 9\theta^2) (5\phi - 2\theta^2 - 2)^2}{(625\phi^3 - 350\phi^2\theta^2 - 100\phi^2 + 6\theta^6 + 6\theta^4)^2}$$

- For  $L_2$  not to be profitable:

$$\frac{\phi (\alpha - \bar{c})^2 (5\phi - 2\theta)^2 (5\phi + 2\theta)^2 \cdot (25\phi - \theta^2)}{(625\phi^3 - 150\phi^2\theta^2 - 100\phi^2 - 20\phi\theta^2 + 8\theta^4)^2} \geq \frac{25\phi^3 (\alpha - \bar{c})^2 (5\phi - 3\theta^2)^2 \cdot (25\phi - 4\theta^2)}{(625\phi^3 - 350\phi^2\theta^2 - 100\phi^2 + 6\theta^6 + 6\theta^4)^2}$$

- Removing an edge between nodes  $L_2, L_3$ : **E5B [1H3L]** → **E4C [1H3L]**

- For  $L_2$  not to be profitable:

$$\frac{\phi(\alpha - \bar{c})^2 (5\phi - 2\theta)^2 (5\phi + 2\theta)^2 \cdot (25\phi - \theta^2)}{(625\phi^3 - 150\phi^2\theta^2 - 100\phi^2 - 20\phi\theta^2 + 8\theta^4)^2} \geq \frac{\phi(\alpha - \bar{c})^2 (5\phi - 2\theta)^2 (5\phi + 2\theta)^2 \cdot (25\phi - 4\theta^2)}{(625\phi^3 - 50\phi^2\theta^2 - 100\phi^2 - 40\phi\theta^4 - 60\phi\theta^2 + 24\theta^4)^2}$$

– For  $L_3$  not to be profitable:

$$\frac{\phi(\alpha - \bar{c})^2 (5\phi - 2\theta)^2 (5\phi + 2\theta)^2 \cdot (25\phi - \theta^2)}{(625\phi^3 - 150\phi^2\theta^2 - 100\phi^2 - 20\phi\theta^2 + 8\theta^4)^2} \geq \frac{\phi(\alpha - \bar{c})^2 (5\phi - 2\theta)^2 (5\phi + 2\theta)^2 \cdot (25\phi - 4\theta^2)}{(625\phi^3 - 50\phi^2\theta^2 - 100\phi^2 - 40\phi\theta^4 - 60\phi\theta^2 + 24\theta^4)^2}$$

- Removing an edge between nodes  $H_1, L_2$ : [E5B \[1H3L\]](#) → [E4D \[1H3L\]](#)

– For  $H_1$  not to be profitable:

$$\frac{25\phi^3 (\alpha - \bar{c})^2 (5\phi - 2\theta^2)^2 \cdot (25\phi - 4)}{(625\phi^3 - 150\phi^2\theta^2 - 100\phi^2 - 20\phi\theta^2 + 8\theta^4)^2} \geq \frac{25\phi^3 (\alpha - \bar{c})^2 (5\phi - 4\theta^2)^2 \cdot (25\phi - 9)}{(625\phi^3 - 225\phi^2\theta^2 - 225\phi^2 + 12\theta^4)^2}$$

– For  $L_2$  not to be profitable:

$$\frac{\phi(\alpha - \bar{c})^2 (5\phi - 2\theta)^2 (5\phi + 2\theta)^2 \cdot (25\phi - \theta^2)}{(625\phi^3 - 150\phi^2\theta^2 - 100\phi^2 - 20\phi\theta^2 + 8\theta^4)^2} \geq \frac{25\phi^3 (\alpha - \bar{c})^2 (5\phi - 3)^2 \cdot (25\phi - 4\theta^2)}{(625\phi^3 - 225\phi^2\theta^2 - 225\phi^2 + 12\theta^4)^2}$$

### C.1.20 Stability conditions for: [E6A \[1H3L\]](#)

- Removing an edge between nodes  $L_1, L_2$ : [E6A \[1H3L\]](#) → [E5A \[1H3L\]](#)

– For  $L_1$  not to be profitable:

$$\frac{\phi(\alpha - \bar{c})^2 \cdot (25\phi - \theta^2)}{(25\phi - 3\theta^2 - 1)^2} \geq \frac{25\phi^3 (\alpha - \bar{c})^2 \cdot (25\phi - 4\theta^2)}{(125\phi^2 + 5\phi\theta^2 - 5\phi - 2\theta^4 - 2\theta^2)^2}$$

– For  $L_2$  not to be profitable:

$$\frac{\phi(\alpha - \bar{c})^2 \cdot (25\phi - \theta^2)}{(25\phi - 3\theta^2 - 1)^2} \geq \frac{25\phi^3 (\alpha - \bar{c})^2 \cdot (25\phi - 4\theta^2)}{(125\phi^2 + 5\phi\theta^2 - 5\phi - 2\theta^4 - 2\theta^2)^2}$$

- Removing an edge between nodes  $H_1, L_1$ : [E6A \[1H3L\]](#) → [E5B \[1H3L\]](#)

– For  $H_1$  not to be profitable:

$$\frac{\phi(\alpha - \bar{c})^2 \cdot (25\phi - 1)}{(25\phi - 3\theta^2 - 1)^2} \geq \frac{25\phi^3 (\alpha - \bar{c})^2 (5\phi - 2\theta^2)^2 \cdot (25\phi - 4)}{(625\phi^3 - 150\phi^2\theta^2 - 100\phi^2 - 20\phi\theta^2 + 8\theta^4)^2}$$

– For  $L_1$  not to be profitable:

$$\frac{\phi(\alpha - \bar{c})^2 \cdot (25\phi - \theta^2)}{(25\phi - 3\theta^2 - 1)^2} \geq \frac{25\phi^3 (\alpha - \bar{c})^2 (5\phi - 2)^2 \cdot (25\phi - 4\theta^2)}{(625\phi^3 - 150\phi^2\theta^2 - 100\phi^2 - 20\phi\theta^2 + 8\theta^4)^2}$$

## C.2 Two *high*- and two *low*-productive firms (2H2L)

### C.2.1 Stability conditions for: E0A [2H2L]

- Adding an edge between nodes  $H_1, H_2$ : E0A [2H2L]  $\rightarrow$  E1A [2H2L]

– For  $H_1$  not to be profitable:

$$\frac{\phi(\alpha - \bar{c})^2 (5\phi - 4\theta^2)^2 \cdot (25\phi - 16)}{(125\phi^2 - 60\phi\theta^2 - 60\phi + 16\theta^2)^2} > \frac{\phi(\alpha - \bar{c})^2 (5\phi - 4\theta^2)^2 \cdot (25\phi - 9)}{(125\phi^2 - 60\phi\theta^2 - 90\phi + 24\theta^2)^2}$$

– For  $H_2$  not to be profitable:

$$\frac{\phi(\alpha - \bar{c})^2 (5\phi - 4\theta^2)^2 \cdot (25\phi - 16)}{(125\phi^2 - 60\phi\theta^2 - 60\phi + 16\theta^2)^2} > \frac{\phi(\alpha - \bar{c})^2 (5\phi - 4\theta^2)^2 \cdot (25\phi - 9)}{(125\phi^2 - 60\phi\theta^2 - 90\phi + 24\theta^2)^2}$$

- Adding an edge between nodes  $H_1, L_1$ : E0A [2H2L]  $\rightarrow$  E1B [2H2L]

– For  $H_1$  not to be profitable:

$$\frac{\phi(\alpha - \bar{c})^2 (5\phi - 4\theta^2)^2 \cdot (25\phi - 16)}{(125\phi^2 - 60\phi\theta^2 - 60\phi + 16\theta^2)^2} > \frac{\phi(\alpha - \bar{c})^2 (5\phi - 4)^2 (5\phi - 4\theta^2)^2 \cdot (25\phi - 9)}{(625\phi^3 - 625\phi^2\theta^2 - 625\phi^2 + 120\phi\theta^4 + 480\phi\theta^2 + 120\phi - 48\theta^4 - 48\theta^2)^2}$$

– For  $L_1$  not to be profitable:

$$\frac{\phi(\alpha - \bar{c})^2 (5\phi - 4)^2 \cdot (25\phi - 16\theta^2)}{(125\phi^2 - 60\phi\theta^2 - 60\phi + 16\theta^2)^2} > \frac{\phi(\alpha - \bar{c})^2 (5\phi - 4)^2 (5\phi - 4\theta^2)^2 \cdot (25\phi - 9\theta^2)}{(625\phi^3 - 625\phi^2\theta^2 - 625\phi^2 + 120\phi\theta^4 + 480\phi\theta^2 + 120\phi - 48\theta^4 - 48\theta^2)^2}$$

- Adding an edge between nodes  $L_1, L_2$ : E0A [2H2L]  $\rightarrow$  E1C [2H2L]

– For  $L_1$  not to be profitable:

$$\frac{\phi(\alpha - \bar{c})^2 (5\phi - 4)^2 \cdot (25\phi - 16\theta^2)}{(125\phi^2 - 60\phi\theta^2 - 60\phi + 16\theta^2)^2} > \frac{\phi(\alpha - \bar{c})^2 (5\phi - 4)^2 \cdot (25\phi - 9\theta^2)}{(125\phi^2 - 90\phi\theta^2 - 60\phi + 24\theta^2)^2}$$

– For  $L_2$  not to be profitable:

$$\frac{\phi(\alpha - \bar{c})^2 (5\phi - 4)^2 \cdot (25\phi - 16\theta^2)}{(125\phi^2 - 60\phi\theta^2 - 60\phi + 16\theta^2)^2} > \frac{\phi(\alpha - \bar{c})^2 (5\phi - 4)^2 \cdot (25\phi - 9\theta^2)}{(125\phi^2 - 90\phi\theta^2 - 60\phi + 24\theta^2)^2}$$

### C.2.2 Stability conditions for: E1A [2H2L]

- Adding an edge between nodes  $H_1, L_1$ : E1A [2H2L]  $\rightarrow$  E2A [2H2L]

– For  $H_1$  not to be profitable:

$$\frac{\phi(\alpha - \bar{c})^2 (5\phi - 4\theta^2)^2 \cdot (25\phi - 9)}{(125\phi^2 - 60\phi\theta^2 - 90\phi + 24\theta^2)^2} > \frac{\phi(\alpha - \bar{c})^2 (5\phi - 3\theta)^2 (5\phi + 3\theta)^2 (5\phi - 4\theta^2)^2 \cdot (25\phi - 4)}{(3125\phi^4 - 3125\phi^3\theta^2 - 1625\phi^3 + 600\phi^2\theta^4 + 1025\phi^2\theta^2 + 180\phi\theta^2 - 72\theta^4)^2}$$

– For  $L_1$  not to be profitable:

$$\frac{\phi(\alpha - \bar{c})^2 (5\phi - 6)^2 \cdot (25\phi - 16\theta^2)}{(125\phi^2 - 60\phi\theta^2 - 90\phi + 24\theta^2)^2} > \frac{25\phi^3 (\alpha - \bar{c})^2 (5\phi - 3)^2 (5\phi - 4\theta^2)^2 \cdot (25\phi - 9\theta^2)}{(3125\phi^4 - 3125\phi^3\theta^2 - 1625\phi^3 + 600\phi^2\theta^4 + 1025\phi^2\theta^2 + 180\phi\theta^2 - 72\theta^4)^2}$$

- Adding an edge between nodes  $L_1, L_2$ : E1A [2H2L]  $\rightarrow$  E2E [2H2L]

– For  $L_1$  not to be profitable:

$$\frac{\phi(\alpha - \bar{c})^2 (5\phi - 6)^2 \cdot (25\phi - 16\theta^2)}{(125\phi^2 - 60\phi\theta^2 - 90\phi + 24\theta^2)^2} > \frac{\phi(\alpha - \bar{c})^2 (5\phi - 6)^2 \cdot (25\phi - 9\theta^2)}{(125\phi^2 - 90\phi\theta^2 - 90\phi + 36\theta^2)^2}$$

– For  $L_2$  not to be profitable:

$$\frac{\phi(\alpha - \bar{c})^2 (5\phi - 6)^2 \cdot (25\phi - 16\theta^2)}{(125\phi^2 - 60\phi\theta^2 - 90\phi + 24\theta^2)^2} > \frac{\phi(\alpha - \bar{c})^2 (5\phi - 6)^2 \cdot (25\phi - 9\theta^2)}{(125\phi^2 - 90\phi\theta^2 - 90\phi + 36\theta^2)^2}$$

- Removing an edge between nodes  $H_1, H_2$ : E1A [2H2L]  $\rightarrow$  E0A [2H2L]

- For  $H_1$  not to be profitable:

$$\frac{\phi(\alpha - \bar{c})^2 (5\phi - 4\theta^2)^2 \cdot (25\phi - 9)}{(125\phi^2 - 60\phi\theta^2 - 90\phi + 24\theta^2)^2} \geq \frac{\phi(\alpha - \bar{c})^2 (5\phi - 4\theta^2)^2 \cdot (25\phi - 16)}{(125\phi^2 - 60\phi\theta^2 - 60\phi + 16\theta^2)^2}$$

- For  $H_2$  not to be profitable:

$$\frac{\phi(\alpha - \bar{c})^2 (5\phi - 4\theta^2)^2 \cdot (25\phi - 9)}{(125\phi^2 - 60\phi\theta^2 - 90\phi + 24\theta^2)^2} \geq \frac{\phi(\alpha - \bar{c})^2 (5\phi - 4\theta^2)^2 \cdot (25\phi - 16)}{(125\phi^2 - 60\phi\theta^2 - 60\phi + 16\theta^2)^2}$$

### C.2.3 Stability conditions for: E1B [2H2L]

- Adding an edge between nodes  $H_1, H_2$ : E1B [2H2L]  $\rightarrow$  E2A [2H2L]

- For  $H_1$  not to be profitable:

$$\frac{\phi(\alpha - \bar{c})^2 (5\phi - 4\theta^2)^2 \cdot (25\phi - 16) (5\phi - 3\theta^2 - 3)^2}{(625\phi^3 - 625\phi^2\theta^2 - 625\phi^2 + 120\phi\theta^4 + 480\phi\theta^2 + 120\phi - 48\theta^4 - 48\theta^2)^2} > \frac{25\phi^3 (\alpha - \bar{c})^2 (5\phi - 4\theta^2)^2 (5\phi - 3\theta^2)^2 \cdot (25\phi - 9)}{(3125\phi^4 - 3125\phi^3\theta^2 - 1625\phi^3 + 600\phi^2\theta^4 + 1025\phi^2\theta^2 + 180\phi\theta^2 - 72\theta^4)^2}$$

- For  $H_2$  not to be profitable:

$$\frac{\phi(\alpha - \bar{c})^2 (5\phi - 4)^2 (5\phi - 4\theta^2)^2 \cdot (25\phi - 9)}{(625\phi^3 - 625\phi^2\theta^2 - 625\phi^2 + 120\phi\theta^4 + 480\phi\theta^2 + 120\phi - 48\theta^4 - 48\theta^2)^2} > \frac{\phi(\alpha - \bar{c})^2 (5\phi - 3\theta)^2 (5\phi + 3\theta)^2 (5\phi - 4\theta^2)^2 \cdot (25\phi - 4)}{(3125\phi^4 - 3125\phi^3\theta^2 - 1625\phi^3 + 600\phi^2\theta^4 + 1025\phi^2\theta^2 + 180\phi\theta^2 - 72\theta^4)^2}$$

- Adding an edge between nodes  $H_2, L_1$ : E1B [2H2L]  $\rightarrow$  E2B [2H2L]

- For  $H_2$  not to be profitable:

$$\frac{\phi(\alpha - \bar{c})^2 (5\phi - 4)^2 (5\phi - 4\theta^2)^2 \cdot (25\phi - 9)}{(625\phi^3 - 625\phi^2\theta^2 - 625\phi^2 + 120\phi\theta^4 + 480\phi\theta^2 + 120\phi - 48\theta^4 - 48\theta^2)^2} > \frac{\phi(\alpha - \bar{c})^2 (5\phi - 4)^2 (5\phi + 3\theta^2)^2 \cdot (25\phi - 4)}{(625\phi^3 - 75\phi^2\theta^2 - 500\phi^2 - 60\phi\theta^2 + 40\phi + 24\theta^2)^2}$$

- For  $L_1$  not to be profitable:

$$\frac{\phi(\alpha - \bar{c})^2 (5\phi - 4)^2 \cdot (25\phi - 16\theta^2) (5\phi - 3\theta^2 - 3)^2}{(625\phi^3 - 625\phi^2\theta^2 - 625\phi^2 + 120\phi\theta^4 + 480\phi\theta^2 + 120\phi - 48\theta^4 - 48\theta^2)^2} > \frac{25\phi^3 (\alpha - \bar{c})^2 (5\phi - 4)^2 \cdot (25\phi - 9\theta^2)}{(625\phi^3 - 75\phi^2\theta^2 - 500\phi^2 - 60\phi\theta^2 + 40\phi + 24\theta^2)^2}$$

- Adding an edge between nodes  $H_1, L_2$ : E1B [2H2L]  $\rightarrow$  E2C [2H2L]

- For  $H_1$  not to be profitable:

$$\frac{\phi(\alpha - \bar{c})^2 (5\phi - 4\theta^2)^2 \cdot (25\phi - 16) (5\phi - 3\theta^2 - 3)^2}{(625\phi^3 - 625\phi^2\theta^2 - 625\phi^2 + 120\phi\theta^4 + 480\phi\theta^2 + 120\phi - 48\theta^4 - 48\theta^2)^2} > \frac{25\phi^3 (\alpha - \bar{c})^2 (5\phi - 4\theta^2)^2 \cdot (25\phi - 9)}{(625\phi^3 - 500\phi^2\theta^2 - 75\phi^2 + 40\phi\theta^4 - 60\phi\theta^2 + 24\theta^4)^2}$$

- For  $L_2$  not to be profitable:

$$\frac{\phi(\alpha - \bar{c})^2 (5\phi - 4)^2 (5\phi - 4\theta^2)^2 \cdot (25\phi - 9\theta^2)}{(625\phi^3 - 625\phi^2\theta^2 - 625\phi^2 + 120\phi\theta^4 + 480\phi\theta^2 + 120\phi - 48\theta^4 - 48\theta^2)^2} > \frac{\phi(\alpha - \bar{c})^2 (5\phi + 3)^2 (5\phi - 4\theta^2)^2 \cdot (25\phi - 4\theta^2)}{(625\phi^3 - 500\phi^2\theta^2 - 75\phi^2 + 40\phi\theta^4 - 60\phi\theta^2 + 24\theta^4)^2}$$

- Adding an edge between nodes  $H_1, L_1$ : E1B [2H2L]  $\rightarrow$  E2D [2H2L]

- For  $H_1$  not to be profitable:

$$\frac{\phi(\alpha - \bar{c})^2 (5\phi - 4\theta^2)^2 \cdot (25\phi - 16) (5\phi - 3\theta^2 - 3)^2}{(625\phi^3 - 625\phi^2\theta^2 - 625\phi^2 + 120\phi\theta^4 + 480\phi\theta^2 + 120\phi - 48\theta^4 - 48\theta^2)^2} > \frac{\phi(\alpha - \bar{c})^2 \cdot (25\phi - 9)}{(25\phi - 3\theta^2 - 3)^2}$$

- For  $L_1$  not to be profitable:

$$\frac{\phi(\alpha - \bar{c})^2 (5\phi - 4)^2 \cdot (25\phi - 16\theta^2) (5\phi - 3\theta^2 - 3)^2}{(625\phi^3 - 625\phi^2\theta^2 - 625\phi^2 + 120\phi\theta^4 + 480\phi\theta^2 + 120\phi - 48\theta^4 - 48\theta^2)^2} > \frac{\phi(\alpha - \bar{c})^2 \cdot (25\phi - 9\theta^2)}{(25\phi - 3\theta^2 - 3)^2}$$

- Adding an edge between nodes  $L_1, L_2$ : E1B [2H2L]  $\rightarrow$  E2F [2H2L]

- For  $L_1$  not to be profitable:

$$\frac{\phi(\alpha - \bar{c})^2 (5\phi - 4)^2 \cdot (25\phi - 16\theta^2) (5\phi - 3\theta^2 - 3)^2}{(625\phi^3 - 625\phi^2\theta^2 - 625\phi^2 + 120\phi\theta^4 + 480\phi\theta^2 + 120\phi - 48\theta^4 - 48\theta^2)^2} > \frac{25\phi^3 (\alpha - \bar{c})^2 (5\phi - 4)^2 (5\phi - 3)^2 \cdot (25\phi - 9\theta^2)}{(3125\phi^4 - 1625\phi^3\theta^2 - 3125\phi^3 + 1025\phi^2\theta^2 + 600\phi^2 + 180\phi\theta^4 - 72\theta^4)^2}$$

- For  $L_2$  not to be profitable:

$$\frac{\phi(\alpha - \bar{c})^2 (5\phi - 4)^2 (5\phi - 4\theta^2)^2 \cdot (25\phi - 9\theta^2)}{(625\phi^3 - 625\phi^2\theta^2 - 625\phi^2 + 120\phi\theta^4 + 480\phi\theta^2 + 120\phi - 48\theta^4 - 48\theta^2)^2} > \frac{\phi(\alpha - \bar{c})^2 (5\phi - 4)^2 (5\phi - 3\theta)^2 (5\phi + 3\theta)^2 \cdot (25\phi - 4\theta^2)}{(3125\phi^4 - 1625\phi^3\theta^2 - 3125\phi^3 + 1025\phi^2\theta^2 + 600\phi^2 + 180\phi\theta^4 - 72\theta^4)^2}$$

- Removing an edge between nodes  $H_2, L_2$ : **E1B** [2H2L]  $\rightarrow$  **E0A** [2H2L]

– For  $H_2$  not to be profitable:

$$\frac{\phi(\alpha - \bar{c})^2 (5\phi - 4)^2 (5\phi - 4\theta^2)^2 \cdot (25\phi - 9)}{(625\phi^3 - 625\phi^2\theta^2 - 625\phi^2 + 120\phi\theta^4 + 480\phi\theta^2 + 120\phi - 48\theta^4 - 48\theta^2)^2} \geq \frac{\phi(\alpha - \bar{c})^2 (5\phi - 4\theta^2)^2 \cdot (25\phi - 16)}{(125\phi^2 - 60\phi\theta^2 - 60\phi + 16\theta^2)^2}$$

– For  $L_2$  not to be profitable:

$$\frac{\phi(\alpha - \bar{c})^2 (5\phi - 4)^2 (5\phi - 4\theta^2)^2 \cdot (25\phi - 9\theta^2)}{(625\phi^3 - 625\phi^2\theta^2 - 625\phi^2 + 120\phi\theta^4 + 480\phi\theta^2 + 120\phi - 48\theta^4 - 48\theta^2)^2} \geq \frac{\phi(\alpha - \bar{c})^2 (5\phi - 4)^2 \cdot (25\phi - 16\theta^2)}{(125\phi^2 - 60\phi\theta^2 - 60\phi + 16\theta^2)^2}$$

#### C.2.4 Stability conditions for: **E1C** [2H2L]

- Adding an edge between nodes  $H_1, H_2$ : **E1C** [2H2L]  $\rightarrow$  **E2E** [2H2L]

– For  $H_1$  not to be profitable:

$$\frac{\phi(\alpha - \bar{c})^2 (5\phi - 6\theta^2)^2 \cdot (25\phi - 16)}{(125\phi^2 - 90\phi\theta^2 - 60\phi + 24\theta^2)^2} > \frac{\phi(\alpha - \bar{c})^2 (5\phi - 6\theta^2)^2 \cdot (25\phi - 9)}{(125\phi^2 - 90\phi\theta^2 - 90\phi + 36\theta^2)^2}$$

– For  $H_2$  not to be profitable:

$$\frac{\phi(\alpha - \bar{c})^2 (5\phi - 6\theta^2)^2 \cdot (25\phi - 16)}{(125\phi^2 - 90\phi\theta^2 - 60\phi + 24\theta^2)^2} > \frac{\phi(\alpha - \bar{c})^2 (5\phi - 6\theta^2)^2 \cdot (25\phi - 9)}{(125\phi^2 - 90\phi\theta^2 - 90\phi + 36\theta^2)^2}$$

- Adding an edge between nodes  $H_1, L_1$ : **E1C** [2H2L]  $\rightarrow$  **E2F** [2H2L]

– For  $H_1$  not to be profitable:

$$\frac{\phi(\alpha - \bar{c})^2 (5\phi - 6\theta^2)^2 \cdot (25\phi - 16)}{(125\phi^2 - 90\phi\theta^2 - 60\phi + 24\theta^2)^2} > \frac{25\phi^3 (\alpha - \bar{c})^2 (5\phi - 4)^2 (5\phi - 3\theta^2)^2 \cdot (25\phi - 9)}{(3125\phi^4 - 1625\phi^3\theta^2 - 3125\phi^3 + 1025\phi^2\theta^2 + 600\phi^2 + 180\phi\theta^4 - 72\theta^4)^2}$$

– For  $L_1$  not to be profitable:

$$\frac{\phi(\alpha - \bar{c})^2 (5\phi - 4)^2 \cdot (25\phi - 9\theta^2)}{(125\phi^2 - 90\phi\theta^2 - 60\phi + 24\theta^2)^2} > \frac{\phi(\alpha - \bar{c})^2 (5\phi - 4)^2 (5\phi - 3\theta)^2 (5\phi + 3\theta)^2 \cdot (25\phi - 4\theta^2)}{(3125\phi^4 - 1625\phi^3\theta^2 - 3125\phi^3 + 1025\phi^2\theta^2 + 600\phi^2 + 180\phi\theta^4 - 72\theta^4)^2}$$

- Removing an edge between nodes  $L_1, L_2$ : **E1C** [2H2L]  $\rightarrow$  **E0A** [2H2L]

– For  $L_1$  not to be profitable:

$$\frac{\phi(\alpha - \bar{c})^2 (5\phi - 4)^2 \cdot (25\phi - 9\theta^2)}{(125\phi^2 - 90\phi\theta^2 - 60\phi + 24\theta^2)^2} \geq \frac{\phi(\alpha - \bar{c})^2 (5\phi - 4)^2 \cdot (25\phi - 16\theta^2)}{(125\phi^2 - 60\phi\theta^2 - 60\phi + 16\theta^2)^2}$$

– For  $L_2$  not to be profitable:

$$\frac{\phi(\alpha - \bar{c})^2 (5\phi - 4)^2 \cdot (25\phi - 9\theta^2)}{(125\phi^2 - 90\phi\theta^2 - 60\phi + 24\theta^2)^2} \geq \frac{\phi(\alpha - \bar{c})^2 (5\phi - 4)^2 \cdot (25\phi - 16\theta^2)}{(125\phi^2 - 60\phi\theta^2 - 60\phi + 16\theta^2)^2}$$

#### C.2.5 Stability conditions for: **E2A** [2H2L]

- Adding an edge between nodes  $H_2, L_1$ : **E2A** [2H2L]  $\rightarrow$  **E3A** [2H2L]

– For  $H_2$  not to be profitable:

$$\frac{\phi(\alpha - \bar{c})^2 (5\phi - 3\theta)^2 (5\phi + 3\theta)^2 (5\phi - 4\theta^2)^2 \cdot (25\phi - 4)}{(3125\phi^4 - 3125\phi^3\theta^2 - 1625\phi^3 + 600\phi^2\theta^4 + 1025\phi^2\theta^2 + 180\phi\theta^2 - 72\theta^4)^2} > \frac{\phi(\alpha - \bar{c})^2 \cdot (25\phi - 1) (25\phi^2 + 15\phi\theta^2 - 18\theta^2)^2}{(625\phi^3 - 75\phi^2\theta^2 - 250\phi^2 - 60\phi\theta^2 + 18\theta^2)^2}$$

– For  $L_1$  not to be profitable:



$$\frac{\phi(\alpha - \bar{c})^2 \cdot (25\phi - 16\theta^2) (125\phi^3 - 75\phi^2\theta^2 - 125\phi^2 + 45\phi\theta^2 + 18\theta^2)^2}{(3125\phi^4 - 3125\phi^3\theta^2 - 1625\phi^3 + 600\phi^2\theta^4 + 1025\phi^2\theta^2 + 180\phi\theta^2 - 72\theta^4)^2} > \frac{25\phi^3(\alpha - \bar{c})^2(5\phi - 3)^2 \cdot (25\phi - 9\theta^2)}{(625\phi^3 - 75\phi^2\theta^2 - 250\phi^2 - 60\phi\theta^2 + 18\theta^2)^2}$$

- Adding an edge between nodes  $H_1$ ,  $L_2$ : [E2A \[2H2L\]](#)  $\rightarrow$  [E3B \[2H2L\]](#)

– For  $H_1$  not to be profitable:

$$\frac{25\phi^3(\alpha - \bar{c})^2(5\phi - 4\theta^2)^2(5\phi - 3\theta^2)^2 \cdot (25\phi - 9)}{(3125\phi^4 - 3125\phi^3\theta^2 - 1625\phi^3 + 600\phi^2\theta^4 + 1025\phi^2\theta^2 + 180\phi\theta^2 - 72\theta^4)^2} > \frac{\phi(\alpha - \bar{c})^2(5\phi - 4\theta^2)^2 \cdot (25\phi - 4)}{(125\phi^2 - 100\phi\theta^2 - 40\phi + 8\theta^4 + 16\theta^2)^2}$$

– For  $L_2$  not to be profitable:

$$\frac{25\phi^3(\alpha - \bar{c})^2(5\phi - 3)^2(5\phi - 4\theta^2)^2 \cdot (25\phi - 9\theta^2)}{(3125\phi^4 - 3125\phi^3\theta^2 - 1625\phi^3 + 600\phi^2\theta^4 + 1025\phi^2\theta^2 + 180\phi\theta^2 - 72\theta^4)^2} > \frac{\phi(\alpha - \bar{c})^2(5\phi - 4\theta^2)^2 \cdot (25\phi - 4\theta^2)}{(125\phi^2 - 100\phi\theta^2 - 40\phi + 8\theta^4 + 16\theta^2)^2}$$

- Adding an edge between nodes  $H_1$ ,  $L_1$ : [E2A \[2H2L\]](#)  $\rightarrow$  [E3C \[2H2L\]](#)

– For  $H_1$  not to be profitable:

$$\frac{25\phi^3(\alpha - \bar{c})^2(5\phi - 4\theta^2)^2(5\phi - 3\theta^2)^2 \cdot (25\phi - 9)}{(3125\phi^4 - 3125\phi^3\theta^2 - 1625\phi^3 + 600\phi^2\theta^4 + 1025\phi^2\theta^2 + 180\phi\theta^2 - 72\theta^4)^2} > \frac{25\phi^3(\alpha - \bar{c})^2 \cdot (25\phi - 4)}{(125\phi^2 - 15\phi\theta^2 - 40\phi + 6\theta^2)^2}$$

– For  $L_1$  not to be profitable:

$$\frac{\phi(\alpha - \bar{c})^2 \cdot (25\phi - 16\theta^2) (125\phi^3 - 75\phi^2\theta^2 - 125\phi^2 + 45\phi\theta^2 + 18\theta^2)^2}{(3125\phi^4 - 3125\phi^3\theta^2 - 1625\phi^3 + 600\phi^2\theta^4 + 1025\phi^2\theta^2 + 180\phi\theta^2 - 72\theta^4)^2} > \frac{\phi(\alpha - \bar{c})^2(5\phi - 2)^2 \cdot (25\phi - 9\theta^2)}{(125\phi^2 - 15\phi\theta^2 - 40\phi + 6\theta^2)^2}$$

- Adding an edge between nodes  $L_1$ ,  $L_2$ : [E2A \[2H2L\]](#)  $\rightarrow$  [E3E \[2H2L\]](#)

– For  $L_1$  not to be profitable:

$$\frac{\phi(\alpha - \bar{c})^2 \cdot (25\phi - 16\theta^2) (125\phi^3 - 75\phi^2\theta^2 - 125\phi^2 + 45\phi\theta^2 + 18\theta^2)^2}{(3125\phi^4 - 3125\phi^3\theta^2 - 1625\phi^3 + 600\phi^2\theta^4 + 1025\phi^2\theta^2 + 180\phi\theta^2 - 72\theta^4)^2} > \frac{\phi(\alpha - \bar{c})^2 \cdot (25\phi - 9\theta^2) (125\phi^3 - 125\phi^2 + 12\theta^2)^2}{(3125\phi^4 - 1625\phi^3\theta^2 - 1625\phi^3 + 225\phi^2\theta^2 + 120\phi\theta^4 + 120\phi\theta^2 - 36\theta^4)^2}$$

– For  $L_2$  not to be profitable:

$$\frac{25\phi^3(\alpha - \bar{c})^2(5\phi - 3)^2(5\phi - 4\theta^2)^2 \cdot (25\phi - 9\theta^2)}{(3125\phi^4 - 3125\phi^3\theta^2 - 1625\phi^3 + 600\phi^2\theta^4 + 1025\phi^2\theta^2 + 180\phi\theta^2 - 72\theta^4)^2} > \frac{25\phi^3(\alpha - \bar{c})^2 \cdot (25\phi - 4\theta^2) (25\phi^2 - 15\phi - 6\theta^2)^2}{(3125\phi^4 - 1625\phi^3\theta^2 - 1625\phi^3 + 225\phi^2\theta^2 + 120\phi\theta^4 + 120\phi\theta^2 - 36\theta^4)^2}$$

- Removing an edge between nodes  $H_2$ ,  $L_2$ : [E2A \[2H2L\]](#)  $\rightarrow$  [E1A \[2H2L\]](#)

– For  $H_2$  not to be profitable:

$$\frac{\phi(\alpha - \bar{c})^2(5\phi - 3\theta^2)^2(5\phi + 3\theta^2)^2(5\phi - 4\theta^2)^2 \cdot (25\phi - 4)}{(3125\phi^4 - 3125\phi^3\theta^2 - 1625\phi^3 + 600\phi^2\theta^4 + 1025\phi^2\theta^2 + 180\phi\theta^2 - 72\theta^4)^2} \geq \frac{\phi(\alpha - \bar{c})^2(5\phi - 4\theta^2)^2 \cdot (25\phi - 9)}{(125\phi^2 - 60\phi\theta^2 - 90\phi + 24\theta^2)^2}$$

– For  $L_2$  not to be profitable:

$$\frac{25\phi^3(\alpha - \bar{c})^2(5\phi - 3)^2(5\phi - 4\theta^2)^2 \cdot (25\phi - 9\theta^2)}{(3125\phi^4 - 3125\phi^3\theta^2 - 1625\phi^3 + 600\phi^2\theta^4 + 1025\phi^2\theta^2 + 180\phi\theta^2 - 72\theta^4)^2} \geq \frac{\phi(\alpha - \bar{c})^2(5\phi - 6)^2 \cdot (25\phi - 16\theta^2)}{(125\phi^2 - 60\phi\theta^2 - 90\phi + 24\theta^2)^2}$$

- Removing an edge between nodes  $H_1$ ,  $H_2$ : [E2A \[2H2L\]](#)  $\rightarrow$  [E1B \[2H2L\]](#)

– For  $H_1$  not to be profitable:

$$\frac{25\phi^3(\alpha - \bar{c})^2(5\phi - 4\theta^2)^2(5\phi - 3\theta^2)^2 \cdot (25\phi - 9)}{(3125\phi^4 - 3125\phi^3\theta^2 - 1625\phi^3 + 600\phi^2\theta^4 + 1025\phi^2\theta^2 + 180\phi\theta^2 - 72\theta^4)^2} \geq \frac{\phi(\alpha - \bar{c})^2(5\phi - 4\theta^2)^2 \cdot (25\phi - 16)(5\phi - 3\theta^2 - 3)^2}{(625\phi^3 - 625\phi^2\theta^2 - 625\phi^2 + 120\phi\theta^4 + 480\phi\theta^2 + 120\phi - 48\theta^4 - 48\theta^2)^2}$$

– For  $H_2$  not to be profitable:

$$\frac{\phi(\alpha - \bar{c})^2(5\phi - 3\theta^2)^2(5\phi + 3\theta^2)^2(5\phi - 4\theta^2)^2 \cdot (25\phi - 4)}{(3125\phi^4 - 3125\phi^3\theta^2 - 1625\phi^3 + 600\phi^2\theta^4 + 1025\phi^2\theta^2 + 180\phi\theta^2 - 72\theta^4)^2} \geq \frac{\phi(\alpha - \bar{c})^2(5\phi - 4)^2(5\phi - 4\theta^2)^2 \cdot (25\phi - 9)}{(625\phi^3 - 625\phi^2\theta^2 - 625\phi^2 + 120\phi\theta^4 + 480\phi\theta^2 + 120\phi - 48\theta^4 - 48\theta^2)^2}$$

### C.2.6 Stability conditions for: E2B [2H2L]

- Adding an edge between nodes  $H_1, H_2$ : E2B [2H2L]  $\rightarrow$  E3A [2H2L]

– For  $H_1$  not to be profitable:

$$\frac{\phi(\alpha - \bar{c})^2 \cdot (25\phi - 16) (25\phi^2 - 15\phi\theta^2 - 10\phi - 6\theta^2)^2}{(625\phi^3 - 75\phi^2\theta^2 - 500\phi^2 - 60\phi\theta^2 + 40\phi + 24\theta^2)^2} > \frac{25\phi^3 (\alpha - \bar{c})^2 (5\phi - 3\theta^2)^2 \cdot (25\phi - 9)}{(625\phi^3 - 75\phi^2\theta^2 - 250\phi^2 - 60\phi\theta^2 + 18\theta^2)^2}$$

– For  $H_2$  not to be profitable:

$$\frac{\phi(\alpha - \bar{c})^2 (5\phi - 4)^2 (5\phi + 3\theta^2)^2 \cdot (25\phi - 4)}{(625\phi^3 - 75\phi^2\theta^2 - 500\phi^2 - 60\phi\theta^2 + 40\phi + 24\theta^2)^2} > \frac{\phi(\alpha - \bar{c})^2 \cdot (25\phi - 1) (25\phi^2 + 15\phi\theta^2 - 18\theta^2)^2}{(625\phi^3 - 75\phi^2\theta^2 - 250\phi^2 - 60\phi\theta^2 + 18\theta^2)^2}$$

- Adding an edge between nodes  $H_1, L_1$ : E2B [2H2L]  $\rightarrow$  E3D [2H2L]

– For  $H_1$  not to be profitable:

$$\frac{\phi(\alpha - \bar{c})^2 \cdot (25\phi - 16) (25\phi^2 - 15\phi\theta^2 - 10\phi - 6\theta^2)^2}{(625\phi^3 - 75\phi^2\theta^2 - 500\phi^2 - 60\phi\theta^2 + 40\phi + 24\theta^2)^2} > \frac{\phi(\alpha - \bar{c})^2 \cdot (25\phi - 9) (125\phi^3 - 75\phi^2\theta^2 - 50\phi^2 + 12\theta^4)^2}{(3125\phi^4 - 1625\phi^3\theta^2 - 1625\phi^3 + 225\phi^2\theta^2 + 120\phi\theta^4 + 120\phi\theta^2 - 36\theta^4)^2}$$

– For  $L_1$  not to be profitable:

$$\frac{25\phi^3 (\alpha - \bar{c})^2 (5\phi - 4)^2 \cdot (25\phi - 9\theta^2)}{(625\phi^3 - 75\phi^2\theta^2 - 500\phi^2 - 60\phi\theta^2 + 40\phi + 24\theta^2)^2} > \frac{25\phi^3 (\alpha - \bar{c})^2 \cdot (25\phi - 4\theta^2) (25\phi^2 - 15\phi\theta^2 - 6)^2}{(3125\phi^4 - 1625\phi^3\theta^2 - 1625\phi^3 + 225\phi^2\theta^2 + 120\phi\theta^4 + 120\phi\theta^2 - 36\theta^4)^2}$$

- Adding an edge between nodes  $L_1, L_2$ : E2B [2H2L]  $\rightarrow$  E3F [2H2L]

– For  $L_1$  not to be profitable:

$$\frac{25\phi^3 (\alpha - \bar{c})^2 (5\phi - 4)^2 \cdot (25\phi - 9\theta^2)}{(625\phi^3 - 75\phi^2\theta^2 - 500\phi^2 - 60\phi\theta^2 + 40\phi + 24\theta^2)^2} > \frac{\phi(\alpha - \bar{c})^2 (5\phi - 4)^2 \cdot (25\phi - 4\theta^2)}{(125\phi^2 - 40\phi\theta^2 - 100\phi + 16\theta^2 + 8)^2}$$

– For  $L_2$  not to be profitable:

$$\frac{25\phi^3 (\alpha - \bar{c})^2 (5\phi - 4)^2 \cdot (25\phi - 9\theta^2)}{(625\phi^3 - 75\phi^2\theta^2 - 500\phi^2 - 60\phi\theta^2 + 40\phi + 24\theta^2)^2} > \frac{\phi(\alpha - \bar{c})^2 (5\phi - 4)^2 \cdot (25\phi - 4\theta^2)}{(125\phi^2 - 40\phi\theta^2 - 100\phi + 16\theta^2 + 8)^2}$$

- Removing an edge between nodes  $H_2, L_1$ : E2B [2H2L]  $\rightarrow$  E1B [2H2L]

– For  $H_2$  not to be profitable:

$$\frac{\phi(\alpha - \bar{c})^2 (5\phi - 4)^2 (5\phi + 3\theta^2)^2 \cdot (25\phi - 4)}{(625\phi^3 - 75\phi^2\theta^2 - 500\phi^2 - 60\phi\theta^2 + 40\phi + 24\theta^2)^2} \geq \frac{\phi(\alpha - \bar{c})^2 (5\phi - 4)^2 (5\phi - 4\theta^2)^2 \cdot (25\phi - 9)}{(625\phi^3 - 625\phi^2\theta^2 - 625\phi^2 + 120\phi\theta^4 + 480\phi\theta^2 + 120\phi - 48\theta^4 - 48\theta^2)^2}$$

– For  $L_1$  not to be profitable:

$$\frac{25\phi^3 (\alpha - \bar{c})^2 (5\phi - 4)^2 \cdot (25\phi - 9\theta^2)}{(625\phi^3 - 75\phi^2\theta^2 - 500\phi^2 - 60\phi\theta^2 + 40\phi + 24\theta^2)^2} \geq \frac{\phi(\alpha - \bar{c})^2 (5\phi - 4)^2 \cdot (25\phi - 16\theta^2) (5\phi - 3\theta^2 - 3)^2}{(625\phi^3 - 625\phi^2\theta^2 - 625\phi^2 + 120\phi\theta^4 + 480\phi\theta^2 + 120\phi - 48\theta^4 - 48\theta^2)^2}$$

### C.2.7 Stability conditions for: E2C [2H2L]

- Adding an edge between nodes  $H_1, H_2$ : E2C [2H2L]  $\rightarrow$  E3B [2H2L]

– For  $H_1$  not to be profitable:

$$\frac{25\phi^3 (\alpha - \bar{c})^2 (5\phi - 4\theta^2)^2 \cdot (25\phi - 9)}{(625\phi^3 - 500\phi^2\theta^2 - 75\phi^2 + 40\phi\theta^4 - 60\phi\theta^2 + 24\theta^4)^2} > \frac{\phi(\alpha - \bar{c})^2 (5\phi - 4\theta^2)^2 \cdot (25\phi - 4)}{(125\phi^2 - 100\phi\theta^2 - 40\phi + 8\theta^4 + 16\theta^2)^2}$$

– For  $H_2$  not to be profitable:

$$\frac{25\phi^3 (\alpha - \bar{c})^2 (5\phi - 4\theta^2)^2 \cdot (25\phi - 9)}{(625\phi^3 - 500\phi^2\theta^2 - 75\phi^2 + 40\phi\theta^4 - 60\phi\theta^2 + 24\theta^4)^2} > \frac{\phi(\alpha - \bar{c})^2 (5\phi - 4\theta^2)^2 \cdot (25\phi - 4)}{(125\phi^2 - 100\phi\theta^2 - 40\phi + 8\theta^4 + 16\theta^2)^2}$$

- Adding an edge between nodes  $H_1, L_1$ : E2C [2H2L]  $\rightarrow$  E3D [2H2L]

– For  $H_1$  not to be profitable:

$$\frac{25\phi^3 (\alpha - \bar{c})^2 (5\phi - 4\theta^2)^2 \cdot (25\phi - 9)}{(625\phi^3 - 500\phi^2\theta^2 - 75\phi^2 + 40\phi\theta^4 - 60\phi\theta^2 + 24\theta^4)^2} > \frac{25\phi^3 (\alpha - \bar{c})^2 \cdot (25\phi - 4) (25\phi^2 - 15\phi - 6\theta^4)^2}{(3125\phi^4 - 1625\phi^3\theta^2 - 1625\phi^3 + 225\phi^2\theta^2 + 120\phi\theta^4 + 120\phi\theta^2 - 36\theta^4)^2}$$

– For  $L_1$  not to be profitable:

$$\frac{\phi (\alpha - \bar{c})^2 \cdot (25\phi - 16\theta^2) (25\phi^2 - 10\phi\theta^2 - 15\phi - 6\theta^2)^2}{(625\phi^3 - 500\phi^2\theta^2 - 75\phi^2 + 40\phi\theta^4 - 60\phi\theta^2 + 24\theta^4)^2} > \frac{\phi (\alpha - \bar{c})^2 \cdot (25\phi - 9\theta^2) (125\phi^3 - 50\phi^2\theta^2 - 75\phi^2 + 12\theta^2)^2}{(3125\phi^4 - 1625\phi^3\theta^2 - 1625\phi^3 + 225\phi^2\theta^2 + 120\phi\theta^4 + 120\phi\theta^2 - 36\theta^4)^2}$$

- Adding an edge between nodes  $L_1, L_2$ : **E2C** [2H2L]  $\rightarrow$  **E3G** [2H2L]

– For  $L_1$  not to be profitable:

$$\frac{\phi (\alpha - \bar{c})^2 \cdot (25\phi - 16\theta^2) (25\phi^2 - 10\phi\theta^2 - 15\phi - 6\theta^2)^2}{(625\phi^3 - 500\phi^2\theta^2 - 75\phi^2 + 40\phi\theta^4 - 60\phi\theta^2 + 24\theta^4)^2} > \frac{25\phi^3 (\alpha - \bar{c})^2 (5\phi - 3)^2 \cdot (25\phi - 9\theta^2)}{(625\phi^3 - 250\phi^2\theta^2 - 75\phi^2 - 60\phi\theta^2 + 18\theta^4)^2}$$

– For  $L_2$  not to be profitable:

$$\frac{\phi (\alpha - \bar{c})^2 (5\phi + 3)^2 (5\phi - 4\theta^2)^2 \cdot (25\phi - 4\theta^2)}{(625\phi^3 - 500\phi^2\theta^2 - 75\phi^2 + 40\phi\theta^4 - 60\phi\theta^2 + 24\theta^4)^2} > \frac{\phi (\alpha - \bar{c})^2 \cdot (25\phi - \theta^2) (25\phi^2 + 15\phi - 18\theta^2)^2}{(625\phi^3 - 250\phi^2\theta^2 - 75\phi^2 - 60\phi\theta^2 + 18\theta^4)^2}$$

- Removing an edge between nodes  $H_1, L_2$ : **E2C** [2H2L]  $\rightarrow$  **E1B** [2H2L]

– For  $H_1$  not to be profitable:

$$\frac{25\phi^3 (\alpha - \bar{c})^2 (5\phi - 4\theta^2)^2 \cdot (25\phi - 9)}{(625\phi^3 - 500\phi^2\theta^2 - 75\phi^2 + 40\phi\theta^4 - 60\phi\theta^2 + 24\theta^4)^2} \geq \frac{\phi (\alpha - \bar{c})^2 (5\phi - 4\theta^2)^2 \cdot (25\phi - 16) (5\phi - 3\theta^2 - 3)^2}{(625\phi^3 - 625\phi^2\theta^2 - 625\phi^2 + 120\phi\theta^4 + 480\phi\theta^2 + 120\phi - 48\theta^4 - 48\theta^2)^2}$$

– For  $L_2$  not to be profitable:

$$\frac{\phi (\alpha - \bar{c})^2 (5\phi + 3)^2 (5\phi - 4\theta^2)^2 \cdot (25\phi - 4\theta^2)}{(625\phi^3 - 500\phi^2\theta^2 - 75\phi^2 + 40\phi\theta^4 - 60\phi\theta^2 + 24\theta^4)^2} \geq \frac{\phi (\alpha - \bar{c})^2 (5\phi - 4)^2 (5\phi - 4\theta^2)^2 \cdot (25\phi - 9\theta^2)}{(625\phi^3 - 625\phi^2\theta^2 - 625\phi^2 + 120\phi\theta^4 + 480\phi\theta^2 + 120\phi - 48\theta^4 - 48\theta^2)^2}$$

### C.2.8 Stability conditions for: **E2D** [2H2L]

- Adding an edge between nodes  $H_1, H_2$ : **E2D** [2H2L]  $\rightarrow$  **E3C** [2H2L]

– For  $H_1$  not to be profitable:

$$\frac{\phi (\alpha - \bar{c})^2 \cdot (25\phi - 9)}{(25\phi - 3\theta^2 - 3)^2} > \frac{25\phi^3 (\alpha - \bar{c})^2 \cdot (25\phi - 4)}{(125\phi^2 - 15\phi\theta^2 - 40\phi + 6\theta^2)^2}$$

– For  $H_2$  not to be profitable:

$$\frac{\phi (\alpha - \bar{c})^2 \cdot (25\phi - 9)}{(25\phi - 3\theta^2 - 3)^2} > \frac{25\phi^3 (\alpha - \bar{c})^2 \cdot (25\phi - 4)}{(125\phi^2 - 15\phi\theta^2 - 40\phi + 6\theta^2)^2}$$

- Adding an edge between nodes  $H_1, L_1$ : **E2D** [2H2L]  $\rightarrow$  **E3D** [2H2L]

– For  $H_1$  not to be profitable:

$$\frac{\phi (\alpha - \bar{c})^2 \cdot (25\phi - 9)}{(25\phi - 3\theta^2 - 3)^2} > \frac{25\phi^3 (\alpha - \bar{c})^2 \cdot (25\phi - 4) (25\phi^2 - 15\phi - 6\theta^4)^2}{(3125\phi^4 - 1625\phi^3\theta^2 - 1625\phi^3 + 225\phi^2\theta^2 + 120\phi\theta^4 + 120\phi\theta^2 - 36\theta^4)^2}$$

– For  $L_1$  not to be profitable:

$$\frac{\phi (\alpha - \bar{c})^2 \cdot (25\phi - 9\theta^2)}{(25\phi - 3\theta^2 - 3)^2} > \frac{25\phi^3 (\alpha - \bar{c})^2 \cdot (25\phi - 4\theta^2) (25\phi^2 - 15\phi\theta^2 - 6)^2}{(3125\phi^4 - 1625\phi^3\theta^2 - 1625\phi^3 + 225\phi^2\theta^2 + 120\phi\theta^4 + 120\phi\theta^2 - 36\theta^4)^2}$$

- Adding an edge between nodes  $L_1, L_2$ : **E2D** [2H2L]  $\rightarrow$  **E3H** [2H2L]

– For  $L_1$  not to be profitable:

$$\frac{\phi (\alpha - \bar{c})^2 \cdot (25\phi - 9\theta^2)}{(25\phi - 3\theta^2 - 3)^2} > \frac{25\phi^3 (\alpha - \bar{c})^2 \cdot (25\phi - 4\theta^2)}{(125\phi^2 - 40\phi\theta^2 - 15\phi + 6\theta^2)^2}$$

– For  $L_2$  not to be profitable:

$$\frac{\phi (\alpha - \bar{c})^2 \cdot (25\phi - 9\theta^2)}{(25\phi - 3\theta^2 - 3)^2} > \frac{25\phi^3 (\alpha - \bar{c})^2 \cdot (25\phi - 4\theta^2)}{(125\phi^2 - 40\phi\theta^2 - 15\phi + 6\theta^2)^2}$$

- Removing an edge between nodes  $H_1, L_2$ : **E2D [2H2L]**  $\rightarrow$  **E1B [2H2L]**

– For  $H_1$  not to be profitable:

$$\frac{\phi(\alpha - \bar{c})^2 \cdot (25\phi - 9)}{(25\phi - 3\theta^2 - 3)^2} \geq \frac{\phi(\alpha - \bar{c})^2 (5\phi - 4\theta^2)^2 \cdot (25\phi - 16) (5\phi - 3\theta^2 - 3)^2}{(625\phi^3 - 625\phi^2\theta^2 - 625\phi^2 + 120\phi\theta^4 + 480\phi\theta^2 + 120\phi - 48\theta^4 - 48\theta^2)^2}$$

– For  $L_2$  not to be profitable:

$$\frac{\phi(\alpha - \bar{c})^2 \cdot (25\phi - 9\theta^2)}{(25\phi - 3\theta^2 - 3)^2} \geq \frac{\phi(\alpha - \bar{c})^2 (5\phi - 4)^2 \cdot (25\phi - 16\theta^2) (5\phi - 3\theta^2 - 3)^2}{(625\phi^3 - 625\phi^2\theta^2 - 625\phi^2 + 120\phi\theta^4 + 480\phi\theta^2 + 120\phi - 48\theta^4 - 48\theta^2)^2}$$

### C.2.9 Stability conditions for: **E2E [2H2L]**

- Adding an edge between nodes  $H_1, L_1$ : **E2E [2H2L]**  $\rightarrow$  **E3E [2H2L]**

– For  $H_1$  not to be profitable:

$$\frac{\phi(\alpha - \bar{c})^2 (5\phi - 6\theta^2)^2 \cdot (25\phi - 9)}{(125\phi^2 - 90\phi\theta^2 - 90\phi + 36\theta^2)^2} > \frac{25\phi^3 (\alpha - \bar{c})^2 \cdot (25\phi - 4) (25\phi^2 - 15\phi\theta^2 - 6\theta^2)^2}{(3125\phi^4 - 1625\phi^3\theta^2 - 1625\phi^3 + 225\phi^2\theta^2 + 120\phi\theta^4 + 120\phi\theta^2 - 36\theta^4)^2}$$

– For  $L_1$  not to be profitable:

$$\frac{\phi(\alpha - \bar{c})^2 (5\phi - 6)^2 \cdot (25\phi - 9\theta^2)}{(125\phi^2 - 90\phi\theta^2 - 90\phi + 36\theta^2)^2} > \frac{25\phi^3 (\alpha - \bar{c})^2 \cdot (25\phi - 4\theta^2) (25\phi^2 - 15\phi - 6\theta^2)^2}{(3125\phi^4 - 1625\phi^3\theta^2 - 1625\phi^3 + 225\phi^2\theta^2 + 120\phi\theta^4 + 120\phi\theta^2 - 36\theta^4)^2}$$

- Removing an edge between nodes  $L_1, L_2$ : **E2E [2H2L]**  $\rightarrow$  **E1A [2H2L]**

– For  $L_1$  not to be profitable:

$$\frac{\phi(\alpha - \bar{c})^2 (5\phi - 6)^2 \cdot (25\phi - 9\theta^2)}{(125\phi^2 - 90\phi\theta^2 - 90\phi + 36\theta^2)^2} \geq \frac{\phi(\alpha - \bar{c})^2 (5\phi - 6)^2 \cdot (25\phi - 16\theta^2)}{(125\phi^2 - 60\phi\theta^2 - 90\phi + 24\theta^2)^2}$$

– For  $L_2$  not to be profitable:

$$\frac{\phi(\alpha - \bar{c})^2 (5\phi - 6)^2 \cdot (25\phi - 9\theta^2)}{(125\phi^2 - 90\phi\theta^2 - 90\phi + 36\theta^2)^2} \geq \frac{\phi(\alpha - \bar{c})^2 (5\phi - 6)^2 \cdot (25\phi - 16\theta^2)}{(125\phi^2 - 60\phi\theta^2 - 90\phi + 24\theta^2)^2}$$

- Removing an edge between nodes  $H_1, H_2$ : **E2E [2H2L]**  $\rightarrow$  **E1C [2H2L]**

– For  $H_1$  not to be profitable:

$$\frac{\phi(\alpha - \bar{c})^2 (5\phi - 6\theta^2)^2 \cdot (25\phi - 9)}{(125\phi^2 - 90\phi\theta^2 - 90\phi + 36\theta^2)^2} \geq \frac{\phi(\alpha - \bar{c})^2 (5\phi - 6\theta^2)^2 \cdot (25\phi - 16)}{(125\phi^2 - 90\phi\theta^2 - 60\phi + 24\theta^2)^2}$$

– For  $H_2$  not to be profitable:

$$\frac{\phi(\alpha - \bar{c})^2 (5\phi - 6\theta^2)^2 \cdot (25\phi - 9)}{(125\phi^2 - 90\phi\theta^2 - 90\phi + 36\theta^2)^2} \geq \frac{\phi(\alpha - \bar{c})^2 (5\phi - 6\theta^2)^2 \cdot (25\phi - 16)}{(125\phi^2 - 90\phi\theta^2 - 60\phi + 24\theta^2)^2}$$

### C.2.10 Stability conditions for: **E2F [2H2L]**

- Adding an edge between nodes  $H_1, H_2$ : **E2F [2H2L]**  $\rightarrow$  **E3E [2H2L]**

– For  $H_1$  not to be profitable:

$$\frac{\phi(\alpha - \bar{c})^2 \cdot (25\phi - 16) (125\phi^3 - 125\phi^2\theta^2 - 75\phi^2 + 45\phi\theta^2 + 18\theta^4)^2}{(3125\phi^4 - 1625\phi^3\theta^2 - 3125\phi^3 + 1025\phi^2\theta^2 + 600\phi^2 + 180\phi\theta^4 - 72\theta^4)^2} > \frac{\phi(\alpha - \bar{c})^2 \cdot (25\phi - 9) (125\phi^3 - 125\phi^2\theta^2 + 12\theta^4)^2}{(3125\phi^4 - 1625\phi^3\theta^2 - 1625\phi^3 + 225\phi^2\theta^2 + 120\phi\theta^4 + 120\phi\theta^2 - 36\theta^4)^2}$$

– For  $H_2$  not to be profitable:

$$\frac{25\phi^3 (\alpha - \bar{c})^2 (5\phi - 4)^2 (5\phi - 3\theta^2)^2 \cdot (25\phi - 9)}{(3125\phi^4 - 1625\phi^3\theta^2 - 3125\phi^3 + 1025\phi^2\theta^2 + 600\phi^2 + 180\phi\theta^4 - 72\theta^4)^2} > \frac{25\phi^3 (\alpha - \bar{c})^2 \cdot (25\phi - 4) (25\phi^2 - 15\phi\theta^2 - 6\theta^2)^2}{(3125\phi^4 - 1625\phi^3\theta^2 - 1625\phi^3 + 225\phi^2\theta^2 + 120\phi\theta^4 + 120\phi\theta^2 - 36\theta^4)^2}$$

- Adding an edge between nodes  $H_2, L_1$ : **E2F [2H2L]**  $\rightarrow$  **E3F [2H2L]**

– For  $H_2$  not to be profitable:

$$\frac{25\phi^3 (\alpha - \bar{c})^2 (5\phi - 4)^2 (5\phi - 3\theta^2)^2 \cdot (25\phi - 9)}{(3125\phi^4 - 1625\phi^3\theta^2 - 3125\phi^3 + 1025\phi^2\theta^2 + 600\phi^2 + 180\phi\theta^4 - 72\theta^4)^2} > \frac{\phi (\alpha - \bar{c})^2 (5\phi - 4)^2 \cdot (25\phi - 4)}{(125\phi^2 - 40\phi\theta^2 - 100\phi + 16\theta^2 + 8)^2}$$

– For  $L_1$  not to be profitable:

$$\frac{25\phi^3 (\alpha - \bar{c})^2 (5\phi - 4)^2 (5\phi - 3)^2 \cdot (25\phi - 9\theta^2)}{(3125\phi^4 - 1625\phi^3\theta^2 - 3125\phi^3 + 1025\phi^2\theta^2 + 600\phi^2 + 180\phi\theta^4 - 72\theta^4)^2} > \frac{\phi (\alpha - \bar{c})^2 (5\phi - 4)^2 \cdot (25\phi - 4\theta^2)}{(125\phi^2 - 40\phi\theta^2 - 100\phi + 16\theta^2 + 8)^2}$$

- Adding an edge between nodes  $H_1$ ,  $L_2$ : **E2F [2H2L]** → **E3G [2H2L]**

– For  $H_1$  not to be profitable:

$$\frac{\phi (\alpha - \bar{c})^2 \cdot (25\phi - 16) (125\phi^3 - 125\phi^2\theta^2 - 75\phi^2 + 45\phi\theta^2 + 18\theta^4)^2}{(3125\phi^4 - 1625\phi^3\theta^2 - 3125\phi^3 + 1025\phi^2\theta^2 + 600\phi^2 + 180\phi\theta^4 - 72\theta^4)^2} > \frac{25\phi^3 (\alpha - \bar{c})^2 (5\phi - 3\theta^2)^2 \cdot (25\phi - 9)}{(625\phi^3 - 250\phi^2\theta^2 - 75\phi^2 - 60\phi\theta^2 + 18\theta^4)^2}$$

– For  $L_2$  not to be profitable:

$$\frac{\phi (\alpha - \bar{c})^2 (5\phi - 4)^2 (5\phi - 3\theta^2)^2 (5\phi + 3\theta^2)^2 \cdot (25\phi - 4\theta^2)}{(3125\phi^4 - 1625\phi^3\theta^2 - 3125\phi^3 + 1025\phi^2\theta^2 + 600\phi^2 + 180\phi\theta^4 - 72\theta^4)^2} > \frac{\phi (\alpha - \bar{c})^2 \cdot (25\phi - \theta^2) (25\phi^2 + 15\phi - 18\theta^2)^2}{(625\phi^3 - 250\phi^2\theta^2 - 75\phi^2 - 60\phi\theta^2 + 18\theta^4)^2}$$

- Adding an edge between nodes  $H_1$ ,  $L_1$ : **E2F [2H2L]** → **E3H [2H2L]**

– For  $H_1$  not to be profitable:

$$\frac{\phi (\alpha - \bar{c})^2 \cdot (25\phi - 16) (125\phi^3 - 125\phi^2\theta^2 - 75\phi^2 + 45\phi\theta^2 + 18\theta^4)^2}{(3125\phi^4 - 1625\phi^3\theta^2 - 3125\phi^3 + 1025\phi^2\theta^2 + 600\phi^2 + 180\phi\theta^4 - 72\theta^4)^2} > \frac{\phi (\alpha - \bar{c})^2 (5\phi - 2\theta^2)^2 \cdot (25\phi - 9)}{(125\phi^2 - 40\phi\theta^2 - 15\phi + 6\theta^2)^2}$$

– For  $L_1$  not to be profitable:

$$\frac{25\phi^3 (\alpha - \bar{c})^2 (5\phi - 4)^2 (5\phi - 3)^2 \cdot (25\phi - 9\theta^2)}{(3125\phi^4 - 1625\phi^3\theta^2 - 3125\phi^3 + 1025\phi^2\theta^2 + 600\phi^2 + 180\phi\theta^4 - 72\theta^4)^2} > \frac{25\phi^3 (\alpha - \bar{c})^2 \cdot (25\phi - 4\theta^2)}{(125\phi^2 - 40\phi\theta^2 - 15\phi + 6\theta^2)^2}$$

- Removing an edge between nodes  $L_1$ ,  $L_2$ : **E2F [2H2L]** → **E1B [2H2L]**

– For  $L_1$  not to be profitable:

$$\frac{25\phi^3 (\alpha - \bar{c})^2 (5\phi - 4)^2 (5\phi - 3)^2 \cdot (25\phi - 9\theta^2)}{(3125\phi^4 - 1625\phi^3\theta^2 - 3125\phi^3 + 1025\phi^2\theta^2 + 600\phi^2 + 180\phi\theta^4 - 72\theta^4)^2} \geq \frac{\phi (\alpha - \bar{c})^2 (5\phi - 4)^2 \cdot (25\phi - 16\theta^2) (5\phi - 3\theta^2 - 3)^2}{(625\phi^3 - 625\phi^2\theta^2 - 625\phi^2 + 120\phi\theta^4 + 480\phi\theta^2 + 120\phi - 48\theta^4 - 48\theta^2)^2}$$

– For  $L_2$  not to be profitable:

$$\frac{\phi (\alpha - \bar{c})^2 (5\phi - 4)^2 (5\phi - 3\theta^2)^2 (5\phi + 3\theta^2)^2 \cdot (25\phi - 4\theta^2)}{(3125\phi^4 - 1625\phi^3\theta^2 - 3125\phi^3 + 1025\phi^2\theta^2 + 600\phi^2 + 180\phi\theta^4 - 72\theta^4)^2} \geq \frac{\phi (\alpha - \bar{c})^2 (5\phi - 4)^2 (5\phi - 4\theta^2)^2 \cdot (25\phi - 9\theta^2)}{(625\phi^3 - 625\phi^2\theta^2 - 625\phi^2 + 120\phi\theta^4 + 480\phi\theta^2 + 120\phi - 48\theta^4 - 48\theta^2)^2}$$

- Removing an edge between nodes  $H_2$ ,  $L_2$ : **E2F [2H2L]** → **E1C [2H2L]**

– For  $H_2$  not to be profitable:

$$\frac{25\phi^3 (\alpha - \bar{c})^2 (5\phi - 4)^2 (5\phi - 3\theta^2)^2 \cdot (25\phi - 9)}{(3125\phi^4 - 1625\phi^3\theta^2 - 3125\phi^3 + 1025\phi^2\theta^2 + 600\phi^2 + 180\phi\theta^4 - 72\theta^4)^2} \geq \frac{\phi (\alpha - \bar{c})^2 (5\phi - 6\theta^2)^2 \cdot (25\phi - 16)}{(125\phi^2 - 90\phi\theta^2 - 60\phi + 24\theta^2)^2}$$

– For  $L_2$  not to be profitable:

$$\frac{\phi (\alpha - \bar{c})^2 (5\phi - 4)^2 (5\phi - 3\theta^2)^2 (5\phi + 3\theta^2)^2 \cdot (25\phi - 4\theta^2)}{(3125\phi^4 - 1625\phi^3\theta^2 - 3125\phi^3 + 1025\phi^2\theta^2 + 600\phi^2 + 180\phi\theta^4 - 72\theta^4)^2} \geq \frac{\phi (\alpha - \bar{c})^2 (5\phi - 4)^2 \cdot (25\phi - 9\theta^2)}{(125\phi^2 - 90\phi\theta^2 - 60\phi + 24\theta^2)^2}$$

### C.2.11 Stability conditions for: **E3A [2H2L]**

- Adding an edge between nodes  $H_1$ ,  $L_1$ : **E3A [2H2L]** → **E4A [2H2L]**

– For  $H_1$  not to be profitable:

$$\frac{25\phi^3 (\alpha - \bar{c})^2 (5\phi - 3\theta^2)^2 \cdot (25\phi - 9)}{(625\phi^3 - 75\phi^2\theta^2 - 250\phi^2 - 60\phi\theta^2 + 18\theta^2)^2} > \frac{25\phi^3 (\alpha - \bar{c})^2 (5\phi - 3\theta^2)^2 \cdot (25\phi - 4)}{(625\phi^3 - 325\phi^2\theta^2 - 125\phi^2 + 6\theta^4 + 6\theta^2)^2}$$

– For  $L_1$  not to be profitable:

$$\frac{25\phi^3 (\alpha - \bar{c})^2 (5\phi - 3)^2 \cdot (25\phi - 9\theta^2)}{(625\phi^3 - 75\phi^2\theta^2 - 250\phi^2 - 60\phi\theta^2 + 18\theta^2)^2} > \frac{25\phi^3 (\alpha - \bar{c})^2 (5\phi - 3\theta^2)^2 \cdot (25\phi - 4\theta^2)}{(625\phi^3 - 325\phi^2\theta^2 - 125\phi^2 + 6\theta^4 + 6\theta^2)^2}$$

- Adding an edge between nodes  $L_1, L_2$ : **E3A** [2H2L]  $\rightarrow$  **E4C** [2H2L]

– For  $L_1$  not to be profitable:

$$\frac{25\phi^3 (\alpha - \bar{c})^2 (5\phi - 3)^2 \cdot (25\phi - 9\theta^2)}{(625\phi^3 - 75\phi^2\theta^2 - 250\phi^2 - 60\phi\theta^2 + 18\theta^2)^2} > \frac{25\phi^3 (\alpha - \bar{c})^2 (5\phi - 3)^2 \cdot (25\phi - 4\theta^2)}{(625\phi^3 - 200\phi^2\theta^2 - 250\phi^2 + 12\theta^2)^2}$$

– For  $L_2$  not to be profitable:

$$\frac{25\phi^3 (\alpha - \bar{c})^2 (5\phi - 3)^2 \cdot (25\phi - 9\theta^2)}{(625\phi^3 - 75\phi^2\theta^2 - 250\phi^2 - 60\phi\theta^2 + 18\theta^2)^2} > \frac{25\phi^3 (\alpha - \bar{c})^2 (5\phi - 3)^2 \cdot (25\phi - 4\theta^2)}{(625\phi^3 - 200\phi^2\theta^2 - 250\phi^2 + 12\theta^2)^2}$$

- Removing an edge between nodes  $H_2, L_1$ : **E3A** [2H2L]  $\rightarrow$  **E2A** [2H2L]

– For  $H_2$  not to be profitable:

$$\frac{\phi (\alpha - \bar{c})^2 \cdot (25\phi - 1) (25\phi^2 + 15\phi\theta^2 - 18\theta^2)^2}{(625\phi^3 - 75\phi^2\theta^2 - 250\phi^2 - 60\phi\theta^2 + 18\theta^2)^2} \geq \frac{\phi (\alpha - \bar{c})^2 (5\phi - 3\theta^2)^2 (5\phi + 3\theta^2)^2 (5\phi - 4\theta^2)^2 \cdot (25\phi - 4)}{(3125\phi^4 - 3125\phi^3\theta^2 - 1625\phi^3 + 600\phi^2\theta^4 + 1025\phi^2\theta^2 + 180\phi\theta^2 - 72\theta^4)^2}$$

– For  $L_1$  not to be profitable:

$$\frac{25\phi^3 (\alpha - \bar{c})^2 (5\phi - 3)^2 \cdot (25\phi - 9\theta^2)}{(625\phi^3 - 75\phi^2\theta^2 - 250\phi^2 - 60\phi\theta^2 + 18\theta^2)^2} \geq \frac{\phi (\alpha - \bar{c})^2 \cdot (25\phi - 16\theta^2) (125\phi^3 - 75\phi^2\theta^2 - 125\phi^2 + 45\phi\theta^2 + 18\theta^2)^2}{(3125\phi^4 - 3125\phi^3\theta^2 - 1625\phi^3 + 600\phi^2\theta^4 + 1025\phi^2\theta^2 + 180\phi\theta^2 - 72\theta^4)^2}$$

- Removing an edge between nodes  $H_1, H_2$ : **E3A** [2H2L]  $\rightarrow$  **E2B** [2H2L]

– For  $H_1$  not to be profitable:

$$\frac{25\phi^3 (\alpha - \bar{c})^2 (5\phi - 3\theta^2)^2 \cdot (25\phi - 9)}{(625\phi^3 - 75\phi^2\theta^2 - 250\phi^2 - 60\phi\theta^2 + 18\theta^2)^2} \geq \frac{\phi (\alpha - \bar{c})^2 \cdot (25\phi - 16) (25\phi^2 - 15\phi\theta^2 - 10\phi - 6\theta^2)^2}{(625\phi^3 - 75\phi^2\theta^2 - 500\phi^2 - 60\phi\theta^2 + 40\phi + 24\theta^2)^2}$$

– For  $H_2$  not to be profitable:

$$\frac{\phi (\alpha - \bar{c})^2 \cdot (25\phi - 1) (25\phi^2 + 15\phi\theta^2 - 18\theta^2)^2}{(625\phi^3 - 75\phi^2\theta^2 - 250\phi^2 - 60\phi\theta^2 + 18\theta^2)^2} \geq \frac{\phi (\alpha - \bar{c})^2 (5\phi - 4)^2 (5\phi + 3\theta^2)^2 \cdot (25\phi - 4)}{(625\phi^3 - 75\phi^2\theta^2 - 500\phi^2 - 60\phi\theta^2 + 40\phi + 24\theta^2)^2}$$

### C.2.12 Stability conditions for: **E3B** [2H2L]

- Adding an edge between nodes  $H_1, L_1$ : **E3B** [2H2L]  $\rightarrow$  **E4A** [2H2L]

– For  $H_1$  not to be profitable:

$$\frac{\phi (\alpha - \bar{c})^2 (5\phi - 4\theta^2)^2 \cdot (25\phi - 4)}{(125\phi^2 - 100\phi\theta^2 - 40\phi + 8\theta^4 + 16\theta^2)^2} > \frac{\phi (\alpha - \bar{c})^2 \cdot (25\phi - 1) (25\phi^2 - 6\theta^4 - 6\theta^2)^2}{(625\phi^3 - 325\phi^2\theta^2 - 125\phi^2 + 6\theta^4 + 6\theta^2)^2}$$

– For  $L_1$  not to be profitable:

$$\frac{\phi (\alpha - \bar{c})^2 \cdot (25\phi - 16\theta^2) (5\phi - 2\theta^2 - 4)^2}{(125\phi^2 - 100\phi\theta^2 - 40\phi + 8\theta^4 + 16\theta^2)^2} > \frac{25\phi^3 (\alpha - \bar{c})^2 \cdot (25\phi - 9\theta^2) (5\phi - 2\theta^2 - 2)^2}{(625\phi^3 - 325\phi^2\theta^2 - 125\phi^2 + 6\theta^4 + 6\theta^2)^2}$$

- Adding an edge between nodes  $L_1, L_2$ : **E3B** [2H2L]  $\rightarrow$  **E4D** [2H2L]

– For  $L_1$  not to be profitable:

$$\frac{\phi (\alpha - \bar{c})^2 \cdot (25\phi - 16\theta^2) (5\phi - 2\theta^2 - 4)^2}{(125\phi^2 - 100\phi\theta^2 - 40\phi + 8\theta^4 + 16\theta^2)^2} > \frac{25\phi^3 (\alpha - \bar{c})^2 (5\phi - 4)^2 \cdot (25\phi - 9\theta^2)}{(625\phi^3 - 250\phi^2\theta^2 - 200\phi^2 + 12\theta^4)^2}$$

– For  $L_2$  not to be profitable:

$$\frac{\phi (\alpha - \bar{c})^2 (5\phi - 4\theta^2)^2 \cdot (25\phi - 4\theta^2)}{(125\phi^2 - 100\phi\theta^2 - 40\phi + 8\theta^4 + 16\theta^2)^2} > \frac{\phi (\alpha - \bar{c})^2 \cdot (25\phi - \theta^2) (25\phi^2 - 12\theta^2)^2}{(625\phi^3 - 250\phi^2\theta^2 - 200\phi^2 + 12\theta^4)^2}$$

- Removing an edge between nodes  $H_1, L_2$ : **E3B** [2H2L]  $\rightarrow$  **E2A** [2H2L]

– For  $H_1$  not to be profitable:

$$\frac{\phi(\alpha - \bar{c})^2 (5\phi - 4\theta^2)^2 \cdot (25\phi - 4)}{(125\phi^2 - 100\phi\theta^2 - 40\phi + 8\theta^4 + 16\theta^2)^2} \geq \frac{25\phi^3 (\alpha - \bar{c})^2 (5\phi - 4\theta^2)^2 (5\phi - 3\theta^2)^2 \cdot (25\phi - 9)}{(3125\phi^4 - 3125\phi^3\theta^2 - 1625\phi^3 + 600\phi^2\theta^4 + 1025\phi^2\theta^2 + 180\phi\theta^2 - 72\theta^4)^2}$$

– For  $L_2$  not to be profitable:

$$\frac{\phi(\alpha - \bar{c})^2 (5\phi - 4\theta^2)^2 \cdot (25\phi - 4\theta^2)}{(125\phi^2 - 100\phi\theta^2 - 40\phi + 8\theta^4 + 16\theta^2)^2} \geq \frac{25\phi^3 (\alpha - \bar{c})^2 (5\phi - 3)^2 (5\phi - 4\theta^2)^2 \cdot (25\phi - 9\theta^2)}{(3125\phi^4 - 3125\phi^3\theta^2 - 1625\phi^3 + 600\phi^2\theta^4 + 1025\phi^2\theta^2 + 180\phi\theta^2 - 72\theta^4)^2}$$

- Removing an edge between nodes  $H_1, H_2$ : [E3B \[2H2L\]](#) → [E2C \[2H2L\]](#)

– For  $H_1$  not to be profitable:

$$\frac{\phi(\alpha - \bar{c})^2 (5\phi - 4\theta^2)^2 \cdot (25\phi - 4)}{(125\phi^2 - 100\phi\theta^2 - 40\phi + 8\theta^4 + 16\theta^2)^2} \geq \frac{25\phi^3 (\alpha - \bar{c})^2 (5\phi - 4\theta^2)^2 \cdot (25\phi - 9)}{(625\phi^3 - 500\phi^2\theta^2 - 75\phi^2 + 40\phi\theta^4 - 60\phi\theta^2 + 24\theta^4)^2}$$

– For  $H_2$  not to be profitable:

$$\frac{\phi(\alpha - \bar{c})^2 (5\phi - 4\theta^2)^2 \cdot (25\phi - 4)}{(125\phi^2 - 100\phi\theta^2 - 40\phi + 8\theta^4 + 16\theta^2)^2} \geq \frac{25\phi^3 (\alpha - \bar{c})^2 (5\phi - 4\theta^2)^2 \cdot (25\phi - 9)}{(625\phi^3 - 500\phi^2\theta^2 - 75\phi^2 + 40\phi\theta^4 - 60\phi\theta^2 + 24\theta^4)^2}$$

### C.2.13 Stability conditions for: [E3C \[2H2L\]](#)

- Adding an edge between nodes  $H_1, L_1$ : [E3C \[2H2L\]](#) → [E4A \[2H2L\]](#)

– For  $H_1$  not to be profitable:

$$\frac{25\phi^3 (\alpha - \bar{c})^2 \cdot (25\phi - 4)}{(125\phi^2 - 15\phi\theta^2 - 40\phi + 6\theta^2)^2} > \frac{\phi(\alpha - \bar{c})^2 \cdot (25\phi - 1) (25\phi^2 - 6\theta^4 - 6\theta^2)^2}{(625\phi^3 - 325\phi^2\theta^2 - 125\phi^2 + 6\theta^4 + 6\theta^2)^2}$$

– For  $L_1$  not to be profitable:

$$\frac{\phi(\alpha - \bar{c})^2 (5\phi - 2)^2 \cdot (25\phi - 9\theta^2)}{(125\phi^2 - 15\phi\theta^2 - 40\phi + 6\theta^2)^2} > \frac{25\phi^3 (\alpha - \bar{c})^2 (5\phi - 3\theta^2)^2 \cdot (25\phi - 4\theta^2)}{(625\phi^3 - 325\phi^2\theta^2 - 125\phi^2 + 6\theta^4 + 6\theta^2)^2}$$

- Adding an edge between nodes  $L_1, L_2$ : [E3C \[2H2L\]](#) → [E4E \[2H2L\]](#)

– For  $L_1$  not to be profitable:

$$\frac{\phi(\alpha - \bar{c})^2 (5\phi - 2)^2 \cdot (25\phi - 9\theta^2)}{(125\phi^2 - 15\phi\theta^2 - 40\phi + 6\theta^2)^2} > \frac{\phi(\alpha - \bar{c})^2 (5\phi - 2)^2 \cdot (25\phi - 4\theta^2)}{(125\phi^2 - 40\phi\theta^2 - 40\phi + 12\theta^2)^2}$$

– For  $L_2$  not to be profitable:

$$\frac{\phi(\alpha - \bar{c})^2 (5\phi - 2)^2 \cdot (25\phi - 9\theta^2)}{(125\phi^2 - 15\phi\theta^2 - 40\phi + 6\theta^2)^2} > \frac{\phi(\alpha - \bar{c})^2 (5\phi - 2)^2 \cdot (25\phi - 4\theta^2)}{(125\phi^2 - 40\phi\theta^2 - 40\phi + 12\theta^2)^2}$$

- Removing an edge between nodes  $H_1, L_2$ : [E3C \[2H2L\]](#) → [E2A \[2H2L\]](#)

– For  $H_1$  not to be profitable:

$$\frac{25\phi^3 (\alpha - \bar{c})^2 \cdot (25\phi - 4)}{(125\phi^2 - 15\phi\theta^2 - 40\phi + 6\theta^2)^2} \geq \frac{25\phi^3 (\alpha - \bar{c})^2 (5\phi - 4\theta^2)^2 (5\phi - 3\theta^2)^2 \cdot (25\phi - 9)}{(3125\phi^4 - 3125\phi^3\theta^2 - 1625\phi^3 + 600\phi^2\theta^4 + 1025\phi^2\theta^2 + 180\phi\theta^2 - 72\theta^4)^2}$$

– For  $L_2$  not to be profitable:

$$\frac{\phi(\alpha - \bar{c})^2 (5\phi - 2)^2 \cdot (25\phi - 9\theta^2)}{(125\phi^2 - 15\phi\theta^2 - 40\phi + 6\theta^2)^2} \geq \frac{\phi(\alpha - \bar{c})^2 \cdot (25\phi - 16\theta^2) (125\phi^3 - 75\phi^2\theta^2 - 125\phi^2 + 45\phi\theta^2 + 18\theta^2)^2}{(3125\phi^4 - 3125\phi^3\theta^2 - 1625\phi^3 + 600\phi^2\theta^4 + 1025\phi^2\theta^2 + 180\phi\theta^2 - 72\theta^4)^2}$$

- Removing an edge between nodes  $H_1, H_2$ : [E3C \[2H2L\]](#) → [E2D \[2H2L\]](#)

– For  $H_1$  not to be profitable:

$$\frac{25\phi^3 (\alpha - \bar{c})^2 \cdot (25\phi - 4)}{(125\phi^2 - 15\phi\theta^2 - 40\phi + 6\theta^2)^2} \geq \frac{\phi(\alpha - \bar{c})^2 \cdot (25\phi - 9)}{(25\phi - 3\theta^2 - 3)^2}$$

– For  $H_2$  not to be profitable:

$$\frac{25\phi^3 (\alpha - \bar{c})^2 \cdot (25\phi - 4)}{(125\phi^2 - 15\phi\theta^2 - 40\phi + 6\theta^2)^2} \geq \frac{\phi (\alpha - \bar{c})^2 \cdot (25\phi - 9)}{(25\phi - 3\theta^2 - 3)^2}$$

#### C.2.14 Stability conditions for: E3D [2H2L]

- Adding an edge between nodes  $H_1, H_2$ : E3D [2H2L]  $\rightarrow$  E4A [2H2L]

– For  $H_1$  not to be profitable:

$$\frac{\phi (\alpha - \bar{c})^2 \cdot (25\phi - 9) (125\phi^3 - 75\phi^2\theta^2 - 50\phi^2 + 12\theta^4)^2}{(3125\phi^4 - 1625\phi^3\theta^2 - 1625\phi^3 + 225\phi^2\theta^2 + 120\phi\theta^4 + 120\phi\theta^2 - 36\theta^4)^2} > \frac{25\phi^3 (\alpha - \bar{c})^2 (5\phi - 3\theta^2)^2 \cdot (25\phi - 4)}{(625\phi^3 - 325\phi^2\theta^2 - 125\phi^2 + 6\theta^4 + 6\theta^2)^2}$$

– For  $H_2$  not to be profitable:

$$\frac{25\phi^3 (\alpha - \bar{c})^2 \cdot (25\phi - 4) (25\phi^2 - 15\phi - 6\theta^4)^2}{(3125\phi^4 - 1625\phi^3\theta^2 - 1625\phi^3 + 225\phi^2\theta^2 + 120\phi\theta^4 + 120\phi\theta^2 - 36\theta^4)^2} > \frac{\phi (\alpha - \bar{c})^2 \cdot (25\phi - 1) (25\phi^2 - 6\theta^4 - 6\theta^2)^2}{(625\phi^3 - 325\phi^2\theta^2 - 125\phi^2 + 6\theta^4 + 6\theta^2)^2}$$

- Adding an edge between nodes  $H_1, L_1$ : E3D [2H2L]  $\rightarrow$  E4B [2H2L]

– For  $H_1$  not to be profitable:

$$\frac{\phi (\alpha - \bar{c})^2 \cdot (25\phi - 9) (125\phi^3 - 75\phi^2\theta^2 - 50\phi^2 + 12\theta^4)^2}{(3125\phi^4 - 1625\phi^3\theta^2 - 1625\phi^3 + 225\phi^2\theta^2 + 120\phi\theta^4 + 120\phi\theta^2 - 36\theta^4)^2} > \frac{\phi (\alpha - \bar{c})^2 (5\phi + 2\theta^2)^2 \cdot (25\phi - 4)}{(125\phi^2 + 10\phi\theta^2 + 10\phi - 12\theta^2)^2}$$

– For  $L_1$  not to be profitable:

$$\frac{\phi (\alpha - \bar{c})^2 \cdot (25\phi - 9\theta^2) (125\phi^3 - 50\phi^2\theta^2 - 75\phi^2 + 12\theta^2)^2}{(3125\phi^4 - 1625\phi^3\theta^2 - 1625\phi^3 + 225\phi^2\theta^2 + 120\phi\theta^4 + 120\phi\theta^2 - 36\theta^4)^2} > \frac{\phi (\alpha - \bar{c})^2 (5\phi + 2)^2 \cdot (25\phi - 4\theta^2)}{(125\phi^2 + 10\phi\theta^2 + 10\phi - 12\theta^2)^2}$$

- Adding an edge between nodes  $L_1, L_2$ : E3D [2H2L]  $\rightarrow$  E4F [2H2L]

– For  $L_1$  not to be profitable:

$$\frac{\phi (\alpha - \bar{c})^2 \cdot (25\phi - 9\theta^2) (125\phi^3 - 50\phi^2\theta^2 - 75\phi^2 + 12\theta^2)^2}{(3125\phi^4 - 1625\phi^3\theta^2 - 1625\phi^3 + 225\phi^2\theta^2 + 120\phi\theta^4 + 120\phi\theta^2 - 36\theta^4)^2} > \frac{25\phi^3 (\alpha - \bar{c})^2 (5\phi - 3)^2 \cdot (25\phi - 4\theta^2)}{(625\phi^3 - 125\phi^2\theta^2 - 325\phi^2 + 6\theta^4 + 6\theta^2)^2}$$

– For  $L_2$  not to be profitable:

$$\frac{25\phi^3 (\alpha - \bar{c})^2 \cdot (25\phi - 4\theta^2) (25\phi^2 - 15\phi\theta^2 - 6)^2}{(3125\phi^4 - 1625\phi^3\theta^2 - 1625\phi^3 + 225\phi^2\theta^2 + 120\phi\theta^4 + 120\phi\theta^2 - 36\theta^4)^2} > \frac{\phi (\alpha - \bar{c})^2 \cdot (25\phi - \theta^2) (25\phi^2 - 6\theta^2 - 6)^2}{(625\phi^3 - 125\phi^2\theta^2 - 325\phi^2 + 6\theta^4 + 6\theta^2)^2}$$

- Removing an edge between nodes  $H_1, L_2$ : E3D [2H2L]  $\rightarrow$  E2B [2H2L]

– For  $H_1$  not to be profitable:

$$\frac{\phi (\alpha - \bar{c})^2 \cdot (25\phi - 9) (125\phi^3 - 75\phi^2\theta^2 - 50\phi^2 + 12\theta^4)^2}{(3125\phi^4 - 1625\phi^3\theta^2 - 1625\phi^3 + 225\phi^2\theta^2 + 120\phi\theta^4 + 120\phi\theta^2 - 36\theta^4)^2} \geq \frac{\phi (\alpha - \bar{c})^2 \cdot (25\phi - 16) (25\phi^2 - 15\phi\theta^2 - 10\phi - 6\theta^2)^2}{(625\phi^3 - 75\phi^2\theta^2 - 500\phi^2 - 60\phi\theta^2 + 40\phi + 24\theta^2)^2}$$

– For  $L_2$  not to be profitable:

$$\frac{25\phi^3 (\alpha - \bar{c})^2 \cdot (25\phi - 4\theta^2) (25\phi^2 - 15\phi\theta^2 - 6)^2}{(3125\phi^4 - 1625\phi^3\theta^2 - 1625\phi^3 + 225\phi^2\theta^2 + 120\phi\theta^4 + 120\phi\theta^2 - 36\theta^4)^2} \geq \frac{25\phi^3 (\alpha - \bar{c})^2 (5\phi - 4)^2 \cdot (25\phi - 9\theta^2)}{(625\phi^3 - 75\phi^2\theta^2 - 500\phi^2 - 60\phi\theta^2 + 40\phi + 24\theta^2)^2}$$

- Removing an edge between nodes  $H_2, L_1$ : E3D [2H2L]  $\rightarrow$  E2C [2H2L]

– For  $H_2$  not to be profitable:

$$\frac{25\phi^3 (\alpha - \bar{c})^2 \cdot (25\phi - 4) (25\phi^2 - 15\phi - 6\theta^4)^2}{(3125\phi^4 - 1625\phi^3\theta^2 - 1625\phi^3 + 225\phi^2\theta^2 + 120\phi\theta^4 + 120\phi\theta^2 - 36\theta^4)^2} \geq \frac{25\phi^3 (\alpha - \bar{c})^2 (5\phi - 4\theta^2)^2 \cdot (25\phi - 9)}{(625\phi^3 - 500\phi^2\theta^2 - 75\phi^2 + 40\phi\theta^4 - 60\phi\theta^2 + 24\theta^4)^2}$$

– For  $L_1$  not to be profitable:

$$\frac{\phi (\alpha - \bar{c})^2 \cdot (25\phi - 9\theta^2) (125\phi^3 - 50\phi^2\theta^2 - 75\phi^2 + 12\theta^2)^2}{(3125\phi^4 - 1625\phi^3\theta^2 - 1625\phi^3 + 225\phi^2\theta^2 + 120\phi\theta^4 + 120\phi\theta^2 - 36\theta^4)^2} \geq \frac{\phi (\alpha - \bar{c})^2 \cdot (25\phi - 16\theta^2) (25\phi^2 - 10\phi\theta^2 - 15\phi - 6\theta^2)^2}{(625\phi^3 - 500\phi^2\theta^2 - 75\phi^2 + 40\phi\theta^4 - 60\phi\theta^2 + 24\theta^4)^2}$$



- Removing an edge between nodes  $H_2, L_2$ : **E3D [2H2L]**  $\rightarrow$  **E2D [2H2L]**

– For  $H_2$  not to be profitable:

$$\frac{25\phi^3 (\alpha - \bar{c})^2 \cdot (25\phi - 4) (25\phi^2 - 15\phi - 6\theta^4)^2}{(3125\phi^4 - 1625\phi^3\theta^2 - 1625\phi^3 + 225\phi^2\theta^2 + 120\phi\theta^4 + 120\phi\theta^2 - 36\theta^4)^2} \geq \frac{\phi (\alpha - \bar{c})^2 \cdot (25\phi - 9)}{(25\phi - 3\theta^2 - 3)^2}$$

– For  $L_2$  not to be profitable:

$$\frac{25\phi^3 (\alpha - \bar{c})^2 \cdot (25\phi - 4\theta^2) (25\phi^2 - 15\phi\theta^2 - 6)^2}{(3125\phi^4 - 1625\phi^3\theta^2 - 1625\phi^3 + 225\phi^2\theta^2 + 120\phi\theta^4 + 120\phi\theta^2 - 36\theta^4)^2} \geq \frac{\phi (\alpha - \bar{c})^2 \cdot (25\phi - 9\theta^2)}{(25\phi - 3\theta^2 - 3)^2}$$

### C.2.15 Stability conditions for: **E3E [2H2L]**

- Adding an edge between nodes  $H_2, L_1$ : **E3E [2H2L]**  $\rightarrow$  **E4C [2H2L]**

– For  $H_2$  not to be profitable:

$$\frac{25\phi^3 (\alpha - \bar{c})^2 \cdot (25\phi - 4) (25\phi^2 - 15\phi\theta^2 - 6\theta^2)^2}{(3125\phi^4 - 1625\phi^3\theta^2 - 1625\phi^3 + 225\phi^2\theta^2 + 120\phi\theta^4 + 120\phi\theta^2 - 36\theta^4)^2} > \frac{\phi (\alpha - \bar{c})^2 \cdot (25\phi - 1) (25\phi^2 - 12\theta^2)^2}{(625\phi^3 - 200\phi^2\theta^2 - 250\phi^2 + 12\theta^2)^2}$$

– For  $L_1$  not to be profitable:

$$\frac{\phi (\alpha - \bar{c})^2 \cdot (25\phi - 9\theta^2) (125\phi^3 - 125\phi^2 + 12\theta^2)^2}{(3125\phi^4 - 1625\phi^3\theta^2 - 1625\phi^3 + 225\phi^2\theta^2 + 120\phi\theta^4 + 120\phi\theta^2 - 36\theta^4)^2} > \frac{25\phi^3 (\alpha - \bar{c})^2 (5\phi - 3)^2 \cdot (25\phi - 4\theta^2)}{(625\phi^3 - 200\phi^2\theta^2 - 250\phi^2 + 12\theta^2)^2}$$

- Adding an edge between nodes  $H_1, L_2$ : **E3E [2H2L]**  $\rightarrow$  **E4D [2H2L]**

– For  $H_1$  not to be profitable:

$$\frac{\phi (\alpha - \bar{c})^2 \cdot (25\phi - 9) (125\phi^3 - 125\phi^2\theta^2 + 12\theta^4)^2}{(3125\phi^4 - 1625\phi^3\theta^2 - 1625\phi^3 + 225\phi^2\theta^2 + 120\phi\theta^4 + 120\phi\theta^2 - 36\theta^4)^2} > \frac{25\phi^3 (\alpha - \bar{c})^2 (5\phi - 3\theta^2)^2 \cdot (25\phi - 4)}{(625\phi^3 - 250\phi^2\theta^2 - 200\phi^2 + 12\theta^4)^2}$$

– For  $L_2$  not to be profitable:

$$\frac{25\phi^3 (\alpha - \bar{c})^2 \cdot (25\phi - 4\theta^2) (25\phi^2 - 15\phi - 6\theta^2)^2}{(3125\phi^4 - 1625\phi^3\theta^2 - 1625\phi^3 + 225\phi^2\theta^2 + 120\phi\theta^4 + 120\phi\theta^2 - 36\theta^4)^2} > \frac{\phi (\alpha - \bar{c})^2 \cdot (25\phi - \theta^2) (25\phi^2 - 12\theta^2)^2}{(625\phi^3 - 250\phi^2\theta^2 - 200\phi^2 + 12\theta^4)^2}$$

- Adding an edge between nodes  $H_1, L_1$ : **E3E [2H2L]**  $\rightarrow$  **E4E [2H2L]**

– For  $H_1$  not to be profitable:

$$\frac{\phi (\alpha - \bar{c})^2 \cdot (25\phi - 9) (125\phi^3 - 125\phi^2\theta^2 + 12\theta^4)^2}{(3125\phi^4 - 1625\phi^3\theta^2 - 1625\phi^3 + 225\phi^2\theta^2 + 120\phi\theta^4 + 120\phi\theta^2 - 36\theta^4)^2} > \frac{\phi (\alpha - \bar{c})^2 (5\phi - 2\theta^2)^2 \cdot (25\phi - 4)}{(125\phi^2 - 40\phi\theta^2 - 40\phi + 12\theta^2)^2}$$

– For  $L_1$  not to be profitable:

$$\frac{\phi (\alpha - \bar{c})^2 \cdot (25\phi - 9\theta^2) (125\phi^3 - 125\phi^2 + 12\theta^2)^2}{(3125\phi^4 - 1625\phi^3\theta^2 - 1625\phi^3 + 225\phi^2\theta^2 + 120\phi\theta^4 + 120\phi\theta^2 - 36\theta^4)^2} > \frac{\phi (\alpha - \bar{c})^2 (5\phi - 2)^2 \cdot (25\phi - 4\theta^2)}{(125\phi^2 - 40\phi\theta^2 - 40\phi + 12\theta^2)^2}$$

- Removing an edge between nodes  $L_1, L_2$ : **E3E [2H2L]**  $\rightarrow$  **E2A [2H2L]**

– For  $L_1$  not to be profitable:

$$\frac{\phi (\alpha - \bar{c})^2 \cdot (25\phi - 9\theta^2) (125\phi^3 - 125\phi^2 + 12\theta^2)^2}{(3125\phi^4 - 1625\phi^3\theta^2 - 1625\phi^3 + 225\phi^2\theta^2 + 120\phi\theta^4 + 120\phi\theta^2 - 36\theta^4)^2} \geq \frac{\phi (\alpha - \bar{c})^2 \cdot (25\phi - 16\theta^2) (125\phi^3 - 75\phi^2\theta^2 - 125\phi^2 + 45\phi\theta^2 + 18\theta^2)^2}{(3125\phi^4 - 3125\phi^3\theta^2 - 1625\phi^3 + 600\phi^2\theta^4 + 1025\phi^2\theta^2 + 180\phi\theta^2 - 72\theta^4)^2}$$

– For  $L_2$  not to be profitable:

$$\frac{25\phi^3 (\alpha - \bar{c})^2 \cdot (25\phi - 4\theta^2) (25\phi^2 - 15\phi - 6\theta^2)^2}{(3125\phi^4 - 1625\phi^3\theta^2 - 1625\phi^3 + 225\phi^2\theta^2 + 120\phi\theta^4 + 120\phi\theta^2 - 36\theta^4)^2} \geq \frac{25\phi^3 (\alpha - \bar{c})^2 (5\phi - 3)^2 (5\phi - 4\theta^2)^2 \cdot (25\phi - 9\theta^2)}{(3125\phi^4 - 3125\phi^3\theta^2 - 1625\phi^3 + 600\phi^2\theta^4 + 1025\phi^2\theta^2 + 180\phi\theta^2 - 72\theta^4)^2}$$

- Removing an edge between nodes  $H_2, L_2$ : **E3E [2H2L]**  $\rightarrow$  **E2E [2H2L]**

- For  $H_2$  not to be profitable:

$$\frac{25\phi^3 (\alpha - \bar{c})^2 \cdot (25\phi - 4) (25\phi^2 - 15\phi\theta^2 - 6\theta^2)^2}{(3125\phi^4 - 1625\phi^3\theta^2 - 1625\phi^3 + 225\phi^2\theta^2 + 120\phi\theta^4 + 120\phi\theta^2 - 36\theta^4)^2} \geq \frac{\phi (\alpha - \bar{c})^2 (5\phi - 6\theta^2)^2 \cdot (25\phi - 9)}{(125\phi^2 - 90\phi\theta^2 - 90\phi + 36\theta^2)^2}$$

- For  $L_2$  not to be profitable:

$$\frac{25\phi^3 (\alpha - \bar{c})^2 \cdot (25\phi - 4\theta^2) (25\phi^2 - 15\phi - 6\theta^2)^2}{(3125\phi^4 - 1625\phi^3\theta^2 - 1625\phi^3 + 225\phi^2\theta^2 + 120\phi\theta^4 + 120\phi\theta^2 - 36\theta^4)^2} \geq \frac{\phi (\alpha - \bar{c})^2 (5\phi - 6)^2 \cdot (25\phi - 9\theta^2)}{(125\phi^2 - 90\phi\theta^2 - 90\phi + 36\theta^2)^2}$$

- Removing an edge between nodes  $H_1, H_2$ : **E3E** [2H2L]  $\rightarrow$  **E2F** [2H2L]

- For  $H_1$  not to be profitable:

$$\frac{\phi (\alpha - \bar{c})^2 \cdot (25\phi - 9) (125\phi^3 - 125\phi^2\theta^2 + 12\theta^4)^2}{(3125\phi^4 - 1625\phi^3\theta^2 - 1625\phi^3 + 225\phi^2\theta^2 + 120\phi\theta^4 + 120\phi\theta^2 - 36\theta^4)^2} \geq \frac{\phi (\alpha - \bar{c})^2 \cdot (25\phi - 16) (125\phi^3 - 125\phi^2\theta^2 - 75\phi^2 + 45\phi\theta^2 + 18\theta^4)^2}{(3125\phi^4 - 1625\phi^3\theta^2 - 3125\phi^3 + 1025\phi^2\theta^2 + 600\phi^2 + 180\phi\theta^4 - 72\theta^4)^2}$$

- For  $H_2$  not to be profitable:

$$\frac{25\phi^3 (\alpha - \bar{c})^2 \cdot (25\phi - 4) (25\phi^2 - 15\phi\theta^2 - 6\theta^2)^2}{(3125\phi^4 - 1625\phi^3\theta^2 - 1625\phi^3 + 225\phi^2\theta^2 + 120\phi\theta^4 + 120\phi\theta^2 - 36\theta^4)^2} \geq \frac{25\phi^3 (\alpha - \bar{c})^2 (5\phi - 4)^2 (5\phi - 3\theta^2)^2 \cdot (25\phi - 9)}{(3125\phi^4 - 1625\phi^3\theta^2 - 3125\phi^3 + 1025\phi^2\theta^2 + 600\phi^2 + 180\phi\theta^4 - 72\theta^4)^2}$$

### C.2.16 Stability conditions for: **E3F** [2H2L]

- Adding an edge between nodes  $H_1, H_2$ : **E3F** [2H2L]  $\rightarrow$  **E4C** [2H2L]

- For  $H_1$  not to be profitable:

$$\frac{\phi (\alpha - \bar{c})^2 \cdot (25\phi - 16) (5\phi - 4\theta^2 - 2)^2}{(125\phi^2 - 40\phi\theta^2 - 100\phi + 16\theta^2 + 8)^2} > \frac{25\phi^3 (\alpha - \bar{c})^2 (5\phi - 4\theta^2)^2 \cdot (25\phi - 9)}{(625\phi^3 - 200\phi^2\theta^2 - 250\phi^2 + 12\theta^2)^2}$$

- For  $H_2$  not to be profitable:

$$\frac{\phi (\alpha - \bar{c})^2 (5\phi - 4)^2 \cdot (25\phi - 4)}{(125\phi^2 - 40\phi\theta^2 - 100\phi + 16\theta^2 + 8)^2} > \frac{\phi (\alpha - \bar{c})^2 \cdot (25\phi - 1) (25\phi^2 - 12\theta^2)^2}{(625\phi^3 - 200\phi^2\theta^2 - 250\phi^2 + 12\theta^2)^2}$$

- Adding an edge between nodes  $H_1, L_1$ : **E3F** [2H2L]  $\rightarrow$  **E4F** [2H2L]

- For  $H_1$  not to be profitable:

$$\frac{\phi (\alpha - \bar{c})^2 \cdot (25\phi - 16) (5\phi - 4\theta^2 - 2)^2}{(125\phi^2 - 40\phi\theta^2 - 100\phi + 16\theta^2 + 8)^2} > \frac{25\phi^3 (\alpha - \bar{c})^2 \cdot (25\phi - 9) (5\phi - 2\theta^2 - 2)^2}{(625\phi^3 - 125\phi^2\theta^2 - 325\phi^2 + 6\theta^4 + 6\theta^2)^2}$$

- For  $L_1$  not to be profitable:

$$\frac{\phi (\alpha - \bar{c})^2 (5\phi - 4)^2 \cdot (25\phi - 4\theta^2)}{(125\phi^2 - 40\phi\theta^2 - 100\phi + 16\theta^2 + 8)^2} > \frac{\phi (\alpha - \bar{c})^2 \cdot (25\phi - \theta^2) (25\phi^2 - 6\theta^2 - 6)^2}{(625\phi^3 - 125\phi^2\theta^2 - 325\phi^2 + 6\theta^4 + 6\theta^2)^2}$$

- Removing an edge between nodes  $L_1, L_2$ : **E3F** [2H2L]  $\rightarrow$  **E2B** [2H2L]

- For  $L_1$  not to be profitable:

$$\frac{\phi (\alpha - \bar{c})^2 (5\phi - 4)^2 \cdot (25\phi - 4\theta^2)}{(125\phi^2 - 40\phi\theta^2 - 100\phi + 16\theta^2 + 8)^2} \geq \frac{25\phi^3 (\alpha - \bar{c})^2 (5\phi - 4)^2 \cdot (25\phi - 9\theta^2)}{(625\phi^3 - 75\phi^2\theta^2 - 500\phi^2 - 60\phi\theta^2 + 40\phi + 24\theta^2)^2}$$

- For  $L_2$  not to be profitable:

$$\frac{\phi (\alpha - \bar{c})^2 (5\phi - 4)^2 \cdot (25\phi - 4\theta^2)}{(125\phi^2 - 40\phi\theta^2 - 100\phi + 16\theta^2 + 8)^2} \geq \frac{25\phi^3 (\alpha - \bar{c})^2 (5\phi - 4)^2 \cdot (25\phi - 9\theta^2)}{(625\phi^3 - 75\phi^2\theta^2 - 500\phi^2 - 60\phi\theta^2 + 40\phi + 24\theta^2)^2}$$

- Removing an edge between nodes  $H_2, L_1$ : **E3F** [2H2L]  $\rightarrow$  **E2F** [2H2L]

- For  $H_2$  not to be profitable:

$$\frac{\phi (\alpha - \bar{c})^2 (5\phi - 4)^2 \cdot (25\phi - 4)}{(125\phi^2 - 40\phi\theta^2 - 100\phi + 16\theta^2 + 8)^2} \geq \frac{25\phi^3 (\alpha - \bar{c})^2 (5\phi - 4)^2 (5\phi - 3\theta^2)^2 \cdot (25\phi - 9)}{(3125\phi^4 - 1625\phi^3\theta^2 - 3125\phi^3 + 1025\phi^2\theta^2 + 600\phi^2 + 180\phi\theta^4 - 72\theta^4)^2}$$

- For  $L_1$  not to be profitable:

$$\frac{\phi(\alpha - \bar{c})^2(5\phi - 4)^2 \cdot (25\phi - 4\theta^2)}{(125\phi^2 - 40\phi\theta^2 - 100\phi + 16\theta^2 + 8)^2} \geq \frac{25\phi^3(\alpha - \bar{c})^2(5\phi - 4)^2(5\phi - 3)^2 \cdot (25\phi - 9\theta^2)}{(3125\phi^4 - 1625\phi^3\theta^2 - 3125\phi^3 + 1025\phi^2\theta^2 + 600\phi^2 + 180\phi\theta^4 - 72\theta^4)^2}$$

### C.2.17 Stability conditions for: E3G [2H2L]

- Adding an edge between nodes  $H_1, H_2$ : E3G [2H2L]  $\rightarrow$  E4D [2H2L]

- For  $H_1$  not to be profitable:

$$\frac{25\phi^3(\alpha - \bar{c})^2(5\phi - 3\theta^2)^2 \cdot (25\phi - 9)}{(625\phi^3 - 250\phi^2\theta^2 - 75\phi^2 - 60\phi\theta^2 + 18\theta^4)^2} > \frac{25\phi^3(\alpha - \bar{c})^2(5\phi - 3\theta^2)^2 \cdot (25\phi - 4)}{(625\phi^3 - 250\phi^2\theta^2 - 200\phi^2 + 12\theta^4)^2}$$

- For  $H_2$  not to be profitable:

$$\frac{25\phi^3(\alpha - \bar{c})^2(5\phi - 3\theta^2)^2 \cdot (25\phi - 9)}{(625\phi^3 - 250\phi^2\theta^2 - 75\phi^2 - 60\phi\theta^2 + 18\theta^4)^2} > \frac{25\phi^3(\alpha - \bar{c})^2(5\phi - 3\theta^2)^2 \cdot (25\phi - 4)}{(625\phi^3 - 250\phi^2\theta^2 - 200\phi^2 + 12\theta^4)^2}$$

- Adding an edge between nodes  $H_1, L_1$ : E3G [2H2L]  $\rightarrow$  E4F [2H2L]

- For  $H_1$  not to be profitable:

$$\frac{25\phi^3(\alpha - \bar{c})^2(5\phi - 3\theta^2)^2 \cdot (25\phi - 9)}{(625\phi^3 - 250\phi^2\theta^2 - 75\phi^2 - 60\phi\theta^2 + 18\theta^4)^2} > \frac{25\phi^3(\alpha - \bar{c})^2(5\phi - 3)^2 \cdot (25\phi - 4)}{(625\phi^3 - 125\phi^2\theta^2 - 325\phi^2 + 6\theta^4 + 6\theta^2)^2}$$

- For  $L_1$  not to be profitable:

$$\frac{25\phi^3(\alpha - \bar{c})^2(5\phi - 3)^2 \cdot (25\phi - 9\theta^2)}{(625\phi^3 - 250\phi^2\theta^2 - 75\phi^2 - 60\phi\theta^2 + 18\theta^4)^2} > \frac{25\phi^3(\alpha - \bar{c})^2(5\phi - 3)^2 \cdot (25\phi - 4\theta^2)}{(625\phi^3 - 125\phi^2\theta^2 - 325\phi^2 + 6\theta^4 + 6\theta^2)^2}$$

- Removing an edge between nodes  $L_1, L_2$ : E3G [2H2L]  $\rightarrow$  E2C [2H2L]

- For  $L_1$  not to be profitable:

$$\frac{25\phi^3(\alpha - \bar{c})^2(5\phi - 3)^2 \cdot (25\phi - 9\theta^2)}{(625\phi^3 - 250\phi^2\theta^2 - 75\phi^2 - 60\phi\theta^2 + 18\theta^4)^2} \geq \frac{\phi(\alpha - \bar{c})^2 \cdot (25\phi - 16\theta^2)(25\phi^2 - 10\phi\theta^2 - 15\phi - 6\theta^2)^2}{(625\phi^3 - 500\phi^2\theta^2 - 75\phi^2 + 40\phi\theta^4 - 60\phi\theta^2 + 24\theta^4)^2}$$

- For  $L_2$  not to be profitable:

$$\frac{\phi(\alpha - \bar{c})^2 \cdot (25\phi - \theta^2)(25\phi^2 + 15\phi - 18\theta^2)^2}{(625\phi^3 - 250\phi^2\theta^2 - 75\phi^2 - 60\phi\theta^2 + 18\theta^4)^2} \geq \frac{\phi(\alpha - \bar{c})^2(5\phi + 3)^2(5\phi - 4\theta^2)^2 \cdot (25\phi - 4\theta^2)}{(625\phi^3 - 500\phi^2\theta^2 - 75\phi^2 + 40\phi\theta^4 - 60\phi\theta^2 + 24\theta^4)^2}$$

- Removing an edge between nodes  $H_1, L_2$ : E3G [2H2L]  $\rightarrow$  E2F [2H2L]

- For  $H_1$  not to be profitable:

$$\frac{25\phi^3(\alpha - \bar{c})^2(5\phi - 3\theta^2)^2 \cdot (25\phi - 9)}{(625\phi^3 - 250\phi^2\theta^2 - 75\phi^2 - 60\phi\theta^2 + 18\theta^4)^2} \geq \frac{\phi(\alpha - \bar{c})^2 \cdot (25\phi - 16)(125\phi^3 - 125\phi^2\theta^2 - 75\phi^2 + 45\phi\theta^2 + 18\theta^4)^2}{(3125\phi^4 - 1625\phi^3\theta^2 - 3125\phi^3 + 1025\phi^2\theta^2 + 600\phi^2 + 180\phi\theta^4 - 72\theta^4)^2}$$

- For  $L_2$  not to be profitable:

$$\frac{\phi(\alpha - \bar{c})^2 \cdot (25\phi - \theta^2)(25\phi^2 + 15\phi - 18\theta^2)^2}{(625\phi^3 - 250\phi^2\theta^2 - 75\phi^2 - 60\phi\theta^2 + 18\theta^4)^2} \geq \frac{\phi(\alpha - \bar{c})^2(5\phi - 4)^2(5\phi - 3\theta^2)^2(5\phi + 3\theta^2)^2 \cdot (25\phi - 4\theta^2)}{(3125\phi^4 - 1625\phi^3\theta^2 - 3125\phi^3 + 1025\phi^2\theta^2 + 600\phi^2 + 180\phi\theta^4 - 72\theta^4)^2}$$

### C.2.18 Stability conditions for: E3H [2H2L]

- Adding an edge between nodes  $H_1, H_2$ : E3H [2H2L]  $\rightarrow$  E4E [2H2L]

- For  $H_1$  not to be profitable:

$$\frac{\phi(\alpha - \bar{c})^2(5\phi - 2\theta^2)^2 \cdot (25\phi - 9)}{(125\phi^2 - 40\phi\theta^2 - 15\phi + 6\theta^2)^2} > \frac{\phi(\alpha - \bar{c})^2(5\phi - 2\theta^2)^2 \cdot (25\phi - 4)}{(125\phi^2 - 40\phi\theta^2 - 40\phi + 12\theta^2)^2}$$

- For  $H_2$  not to be profitable:

$$\frac{\phi(\alpha - \bar{c})^2(5\phi - 2\theta^2)^2 \cdot (25\phi - 9)}{(125\phi^2 - 40\phi\theta^2 - 15\phi + 6\theta^2)^2} > \frac{\phi(\alpha - \bar{c})^2(5\phi - 2\theta^2)^2 \cdot (25\phi - 4)}{(125\phi^2 - 40\phi\theta^2 - 40\phi + 12\theta^2)^2}$$

- Adding an edge between nodes  $H_1, L_1$ : [E3H \[2H2L\]](#) → [E4F \[2H2L\]](#)

– For  $H_1$  not to be profitable:

$$\frac{\phi(\alpha - \bar{c})^2 (5\phi - 2\theta^2)^2 \cdot (25\phi - 9)}{(125\phi^2 - 40\phi\theta^2 - 15\phi + 6\theta^2)^2} > \frac{25\phi^3 (\alpha - \bar{c})^2 (5\phi - 3)^2 \cdot (25\phi - 4)}{(625\phi^3 - 125\phi^2\theta^2 - 325\phi^2 + 6\theta^4 + 6\theta^2)^2}$$

– For  $L_1$  not to be profitable:

$$\frac{25\phi^3 (\alpha - \bar{c})^2 \cdot (25\phi - 4\theta^2)}{(125\phi^2 - 40\phi\theta^2 - 15\phi + 6\theta^2)^2} > \frac{\phi(\alpha - \bar{c})^2 \cdot (25\phi - \theta^2) (25\phi^2 - 6\theta^2 - 6)^2}{(625\phi^3 - 125\phi^2\theta^2 - 325\phi^2 + 6\theta^4 + 6\theta^2)^2}$$

- Removing an edge between nodes  $L_1, L_2$ : [E3H \[2H2L\]](#) → [E2D \[2H2L\]](#)

– For  $L_1$  not to be profitable:

$$\frac{25\phi^3 (\alpha - \bar{c})^2 \cdot (25\phi - 4\theta^2)}{(125\phi^2 - 40\phi\theta^2 - 15\phi + 6\theta^2)^2} \geq \frac{\phi(\alpha - \bar{c})^2 \cdot (25\phi - 9\theta^2)}{(25\phi - 3\theta^2 - 3)^2}$$

– For  $L_2$  not to be profitable:

$$\frac{25\phi^3 (\alpha - \bar{c})^2 \cdot (25\phi - 4\theta^2)}{(125\phi^2 - 40\phi\theta^2 - 15\phi + 6\theta^2)^2} \geq \frac{\phi(\alpha - \bar{c})^2 \cdot (25\phi - 9\theta^2)}{(25\phi - 3\theta^2 - 3)^2}$$

- Removing an edge between nodes  $H_1, L_2$ : [E3H \[2H2L\]](#) → [E2F \[2H2L\]](#)

– For  $H_1$  not to be profitable:

$$\frac{\phi(\alpha - \bar{c})^2 (5\phi - 2\theta^2)^2 \cdot (25\phi - 9)}{(125\phi^2 - 40\phi\theta^2 - 15\phi + 6\theta^2)^2} \geq \frac{\phi(\alpha - \bar{c})^2 \cdot (25\phi - 16) (125\phi^3 - 125\phi^2\theta^2 - 75\phi^2 + 45\phi\theta^2 + 18\theta^4)^2}{(3125\phi^4 - 1625\phi^3\theta^2 - 3125\phi^3 + 1025\phi^2\theta^2 + 600\phi^2 + 180\phi\theta^4 - 72\theta^4)^2}$$

– For  $L_2$  not to be profitable:

$$\frac{25\phi^3 (\alpha - \bar{c})^2 \cdot (25\phi - 4\theta^2)}{(125\phi^2 - 40\phi\theta^2 - 15\phi + 6\theta^2)^2} \geq \frac{25\phi^3 (\alpha - \bar{c})^2 (5\phi - 4)^2 (5\phi - 3)^2 \cdot (25\phi - 9\theta^2)}{(3125\phi^4 - 1625\phi^3\theta^2 - 3125\phi^3 + 1025\phi^2\theta^2 + 600\phi^2 + 180\phi\theta^4 - 72\theta^4)^2}$$

### C.2.19 Stability conditions for: [E4A \[2H2L\]](#)

- Adding an edge between nodes  $H_1, L_1$ : [E4A \[2H2L\]](#) → [E5A \[2H2L\]](#)

– For  $H_1$  not to be profitable:

$$\frac{25\phi^3 (\alpha - \bar{c})^2 (5\phi - 3\theta^2)^2 \cdot (25\phi - 4)}{(625\phi^3 - 325\phi^2\theta^2 - 125\phi^2 + 6\theta^4 + 6\theta^2)^2} > \frac{\phi(\alpha - \bar{c})^2 (5\phi + 2\theta^2)^2 \cdot (25\phi - 1)}{(125\phi^2 + 10\phi\theta^2 - 10\phi - 4\theta^2)^2}$$

– For  $L_1$  not to be profitable:

$$\frac{25\phi^3 (\alpha - \bar{c})^2 \cdot (25\phi - 9\theta^2) (5\phi - 2\theta^2 - 2)^2}{(625\phi^3 - 325\phi^2\theta^2 - 125\phi^2 + 6\theta^4 + 6\theta^2)^2} > \frac{25\phi^3 (\alpha - \bar{c})^2 \cdot (25\phi - 4\theta^2)}{(125\phi^2 + 10\phi\theta^2 - 10\phi - 4\theta^2)^2}$$

- Adding an edge between nodes  $L_1, L_2$ : [E4A \[2H2L\]](#) → [E5B \[2H2L\]](#)

– For  $L_1$  not to be profitable:

$$\frac{25\phi^3 (\alpha - \bar{c})^2 \cdot (25\phi - 9\theta^2) (5\phi - 2\theta^2 - 2)^2}{(625\phi^3 - 325\phi^2\theta^2 - 125\phi^2 + 6\theta^4 + 6\theta^2)^2} > \frac{25\phi^3 (\alpha - \bar{c})^2 (5\phi - 2)^2 \cdot (25\phi - 4\theta^2)}{(125\phi^2 - 4\theta^2)^2 (5\phi - \theta^2 - 1)^2}$$

– For  $L_2$  not to be profitable:

$$\frac{25\phi^3 (\alpha - \bar{c})^2 (5\phi - 3\theta^2)^2 \cdot (25\phi - 4\theta^2)}{(625\phi^3 - 325\phi^2\theta^2 - 125\phi^2 + 6\theta^4 + 6\theta^2)^2} > \frac{\phi(\alpha - \bar{c})^2 (5\phi - 2\theta)^2 (5\phi + 2\theta)^2 \cdot (25\phi - \theta^2)}{(125\phi^2 - 4\theta^2)^2 (5\phi - \theta^2 - 1)^2}$$

- Removing an edge between nodes  $H_1, L_2$ : [E4A \[2H2L\]](#) → [E3A \[2H2L\]](#)

– For  $H_1$  not to be profitable:

$$\frac{25\phi^3 (\alpha - \bar{c})^2 (5\phi - 3\theta^2)^2 \cdot (25\phi - 4)}{(625\phi^3 - 325\phi^2\theta^2 - 125\phi^2 + 6\theta^4 + 6\theta^2)^2} \geq \frac{25\phi^3 (\alpha - \bar{c})^2 (5\phi - 3\theta^2)^2 \cdot (25\phi - 9)}{(625\phi^3 - 75\phi^2\theta^2 - 250\phi^2 - 60\phi\theta^2 + 18\theta^2)^2}$$

- For  $L_2$  not to be profitable:

$$\frac{25\phi^3 (\alpha - \bar{c})^2 (5\phi - 3\theta^2)^2 \cdot (25\phi - 4\theta^2)}{(625\phi^3 - 325\phi^2\theta^2 - 125\phi^2 + 6\theta^4 + 6\theta^2)^2} \geq \frac{25\phi^3 (\alpha - \bar{c})^2 (5\phi - 3)^2 \cdot (25\phi - 9\theta^2)}{(625\phi^3 - 75\phi^2\theta^2 - 250\phi^2 - 60\phi\theta^2 + 18\theta^2)^2}$$

- Removing an edge between nodes  $H_2$ ,  $L_1$ : [E4A \[2H2L\]](#) → [E3B \[2H2L\]](#)

- For  $H_2$  not to be profitable:

$$\frac{\phi (\alpha - \bar{c})^2 \cdot (25\phi - 1) (25\phi^2 - 6\theta^4 - 6\theta^2)^2}{(625\phi^3 - 325\phi^2\theta^2 - 125\phi^2 + 6\theta^4 + 6\theta^2)^2} \geq \frac{\phi (\alpha - \bar{c})^2 (5\phi - 4\theta^2)^2 \cdot (25\phi - 4)}{(125\phi^2 - 100\phi\theta^2 - 40\phi + 8\theta^4 + 16\theta^2)^2}$$

- For  $L_1$  not to be profitable:

$$\frac{25\phi^3 (\alpha - \bar{c})^2 \cdot (25\phi - 9\theta^2) (5\phi - 2\theta^2 - 2)^2}{(625\phi^3 - 325\phi^2\theta^2 - 125\phi^2 + 6\theta^4 + 6\theta^2)^2} \geq \frac{\phi (\alpha - \bar{c})^2 \cdot (25\phi - 16\theta^2) (5\phi - 2\theta^2 - 4)^2}{(125\phi^2 - 100\phi\theta^2 - 40\phi + 8\theta^4 + 16\theta^2)^2}$$

- Removing an edge between nodes  $H_2$ ,  $L_2$ : [E4A \[2H2L\]](#) → [E3C \[2H2L\]](#)

- For  $H_2$  not to be profitable:

$$\frac{\phi (\alpha - \bar{c})^2 \cdot (25\phi - 1) (25\phi^2 - 6\theta^4 - 6\theta^2)^2}{(625\phi^3 - 325\phi^2\theta^2 - 125\phi^2 + 6\theta^4 + 6\theta^2)^2} \geq \frac{25\phi^3 (\alpha - \bar{c})^2 \cdot (25\phi - 4)}{(125\phi^2 - 15\phi\theta^2 - 40\phi + 6\theta^2)^2}$$

- For  $L_2$  not to be profitable:

$$\frac{25\phi^3 (\alpha - \bar{c})^2 (5\phi - 3\theta^2)^2 \cdot (25\phi - 4\theta^2)}{(625\phi^3 - 325\phi^2\theta^2 - 125\phi^2 + 6\theta^4 + 6\theta^2)^2} \geq \frac{\phi (\alpha - \bar{c})^2 (5\phi - 2)^2 \cdot (25\phi - 9\theta^2)}{(125\phi^2 - 15\phi\theta^2 - 40\phi + 6\theta^2)^2}$$

- Removing an edge between nodes  $H_1$ ,  $H_2$ : [E4A \[2H2L\]](#) → [E3D \[2H2L\]](#)

- For  $H_1$  not to be profitable:

$$\frac{25\phi^3 (\alpha - \bar{c})^2 (5\phi - 3\theta^2)^2 \cdot (25\phi - 4)}{(625\phi^3 - 325\phi^2\theta^2 - 125\phi^2 + 6\theta^4 + 6\theta^2)^2} \geq \frac{\phi (\alpha - \bar{c})^2 \cdot (25\phi - 9) (125\phi^3 - 75\phi^2\theta^2 - 50\phi^2 + 12\theta^4)^2}{(3125\phi^4 - 1625\phi^3\theta^2 - 1625\phi^3 + 225\phi^2\theta^2 + 120\phi\theta^4 + 120\phi\theta^2 - 36\theta^4)^2}$$

- For  $H_2$  not to be profitable:

$$\frac{\phi (\alpha - \bar{c})^2 \cdot (25\phi - 1) (25\phi^2 - 6\theta^4 - 6\theta^2)^2}{(625\phi^3 - 325\phi^2\theta^2 - 125\phi^2 + 6\theta^4 + 6\theta^2)^2} \geq \frac{25\phi^3 (\alpha - \bar{c})^2 \cdot (25\phi - 4) (25\phi^2 - 15\phi - 6\theta^4)^2}{(3125\phi^4 - 1625\phi^3\theta^2 - 1625\phi^3 + 225\phi^2\theta^2 + 120\phi\theta^4 + 120\phi\theta^2 - 36\theta^4)^2}$$

### C.2.20 Stability conditions for: [E4B \[2H2L\]](#)

- Adding an edge between nodes  $H_1$ ,  $H_2$ : [E4B \[2H2L\]](#) → [E5A \[2H2L\]](#)

- For  $H_1$  not to be profitable:

$$\frac{\phi (\alpha - \bar{c})^2 (5\phi + 2\theta^2)^2 \cdot (25\phi - 4)}{(125\phi^2 + 10\phi\theta^2 + 10\phi - 12\theta^2)^2} > \frac{\phi (\alpha - \bar{c})^2 (5\phi + 2\theta^2)^2 \cdot (25\phi - 1)}{(125\phi^2 + 10\phi\theta^2 - 10\phi - 4\theta^2)^2}$$

- For  $H_2$  not to be profitable:

$$\frac{\phi (\alpha - \bar{c})^2 (5\phi + 2\theta^2)^2 \cdot (25\phi - 4)}{(125\phi^2 + 10\phi\theta^2 + 10\phi - 12\theta^2)^2} > \frac{\phi (\alpha - \bar{c})^2 (5\phi + 2\theta^2)^2 \cdot (25\phi - 1)}{(125\phi^2 + 10\phi\theta^2 - 10\phi - 4\theta^2)^2}$$

- Adding an edge between nodes  $L_1$ ,  $L_2$ : [E4B \[2H2L\]](#) → [E5C \[2H2L\]](#)

- For  $L_1$  not to be profitable:

$$\frac{\phi (\alpha - \bar{c})^2 (5\phi + 2)^2 \cdot (25\phi - 4\theta^2)}{(125\phi^2 + 10\phi\theta^2 + 10\phi - 12\theta^2)^2} > \frac{\phi (\alpha - \bar{c})^2 (5\phi + 2)^2 \cdot (25\phi - \theta^2)}{(125\phi^2 - 10\phi\theta^2 + 10\phi - 4\theta^2)^2}$$

- For  $L_2$  not to be profitable:

$$\frac{\phi (\alpha - \bar{c})^2 (5\phi + 2)^2 \cdot (25\phi - 4\theta^2)}{(125\phi^2 + 10\phi\theta^2 + 10\phi - 12\theta^2)^2} > \frac{\phi (\alpha - \bar{c})^2 (5\phi + 2)^2 \cdot (25\phi - \theta^2)}{(125\phi^2 - 10\phi\theta^2 + 10\phi - 4\theta^2)^2}$$

- Removing an edge between nodes  $H_1$ ,  $L_1$ : [E4B \[2H2L\]](#) → [E3D \[2H2L\]](#)

- For  $H_1$  not to be profitable:

$$\frac{\phi(\alpha - \bar{c})^2 (5\phi + 2\theta^2)^2 \cdot (25\phi - 4)}{(125\phi^2 + 10\phi\theta^2 + 10\phi - 12\theta^2)^2} \geq \frac{\phi(\alpha - \bar{c})^2 \cdot (25\phi - 9) (125\phi^3 - 75\phi^2\theta^2 - 50\phi^2 + 12\theta^4)^2}{(3125\phi^4 - 1625\phi^3\theta^2 - 1625\phi^3 + 225\phi^2\theta^2 + 120\phi\theta^4 + 120\phi\theta^2 - 36\theta^4)^2}$$

- For  $L_1$  not to be profitable:

$$\frac{\phi(\alpha - \bar{c})^2 (5\phi + 2)^2 \cdot (25\phi - 4\theta^2)}{(125\phi^2 + 10\phi\theta^2 + 10\phi - 12\theta^2)^2} \geq \frac{\phi(\alpha - \bar{c})^2 \cdot (25\phi - 9\theta^2) (125\phi^3 - 50\phi^2\theta^2 - 75\phi^2 + 12\theta^2)^2}{(3125\phi^4 - 1625\phi^3\theta^2 - 1625\phi^3 + 225\phi^2\theta^2 + 120\phi\theta^4 + 120\phi\theta^2 - 36\theta^4)^2}$$

### C.2.21 Stability conditions for: E4C [2H2L]

- Adding an edge between nodes  $H_1$ ,  $L_1$ : E4C [2H2L]  $\rightarrow$  E5B [2H2L]

- For  $H_1$  not to be profitable:

$$\frac{25\phi^3 (\alpha - \bar{c})^2 (5\phi - 4\theta^2)^2 \cdot (25\phi - 9)}{(625\phi^3 - 200\phi^2\theta^2 - 250\phi^2 + 12\theta^2)^2} > \frac{25\phi^3 (\alpha - \bar{c})^2 (5\phi - 2\theta^2)^2 \cdot (25\phi - 4)}{(125\phi^2 - 4\theta^2)^2 (5\phi - \theta^2 - 1)^2}$$

- For  $L_1$  not to be profitable:

$$\frac{25\phi^3 (\alpha - \bar{c})^2 (5\phi - 3)^2 \cdot (25\phi - 4\theta^2)}{(625\phi^3 - 200\phi^2\theta^2 - 250\phi^2 + 12\theta^2)^2} > \frac{\phi(\alpha - \bar{c})^2 (5\phi - 2\theta)^2 (5\phi + 2\theta)^2 \cdot (25\phi - \theta^2)}{(125\phi^2 - 4\theta^2)^2 (5\phi - \theta^2 - 1)^2}$$

- Removing an edge between nodes  $L_1$ ,  $L_2$ : E4C [2H2L]  $\rightarrow$  E3A [2H2L]

- For  $L_1$  not to be profitable:

$$\frac{25\phi^3 (\alpha - \bar{c})^2 (5\phi - 3)^2 \cdot (25\phi - 4\theta^2)}{(625\phi^3 - 200\phi^2\theta^2 - 250\phi^2 + 12\theta^2)^2} \geq \frac{25\phi^3 (\alpha - \bar{c})^2 (5\phi - 3)^2 \cdot (25\phi - 9\theta^2)}{(625\phi^3 - 75\phi^2\theta^2 - 250\phi^2 - 60\phi\theta^2 + 18\theta^2)^2}$$

- For  $L_2$  not to be profitable:

$$\frac{25\phi^3 (\alpha - \bar{c})^2 (5\phi - 3)^2 \cdot (25\phi - 4\theta^2)}{(625\phi^3 - 200\phi^2\theta^2 - 250\phi^2 + 12\theta^2)^2} \geq \frac{25\phi^3 (\alpha - \bar{c})^2 (5\phi - 3)^2 \cdot (25\phi - 9\theta^2)}{(625\phi^3 - 75\phi^2\theta^2 - 250\phi^2 - 60\phi\theta^2 + 18\theta^2)^2}$$

- Removing an edge between nodes  $H_2$ ,  $L_1$ : E4C [2H2L]  $\rightarrow$  E3E [2H2L]

- For  $H_2$  not to be profitable:

$$\frac{\phi(\alpha - \bar{c})^2 \cdot (25\phi - 1) (25\phi^2 - 12\theta^2)^2}{(625\phi^3 - 200\phi^2\theta^2 - 250\phi^2 + 12\theta^2)^2} \geq \frac{25\phi^3 (\alpha - \bar{c})^2 \cdot (25\phi - 4) (25\phi^2 - 15\phi\theta^2 - 6\theta^2)^2}{(3125\phi^4 - 1625\phi^3\theta^2 - 1625\phi^3 + 225\phi^2\theta^2 + 120\phi\theta^4 + 120\phi\theta^2 - 36\theta^4)^2}$$

- For  $L_1$  not to be profitable:

$$\frac{25\phi^3 (\alpha - \bar{c})^2 (5\phi - 3)^2 \cdot (25\phi - 4\theta^2)}{(625\phi^3 - 200\phi^2\theta^2 - 250\phi^2 + 12\theta^2)^2} \geq \frac{\phi(\alpha - \bar{c})^2 \cdot (25\phi - 9\theta^2) (125\phi^3 - 125\phi^2 + 12\theta^2)^2}{(3125\phi^4 - 1625\phi^3\theta^2 - 1625\phi^3 + 225\phi^2\theta^2 + 120\phi\theta^4 + 120\phi\theta^2 - 36\theta^4)^2}$$

- Removing an edge between nodes  $H_1$ ,  $H_2$ : E4C [2H2L]  $\rightarrow$  E3F [2H2L]

- For  $H_1$  not to be profitable:

$$\frac{25\phi^3 (\alpha - \bar{c})^2 (5\phi - 4\theta^2)^2 \cdot (25\phi - 9)}{(625\phi^3 - 200\phi^2\theta^2 - 250\phi^2 + 12\theta^2)^2} \geq \frac{\phi(\alpha - \bar{c})^2 \cdot (25\phi - 16) (5\phi - 4\theta^2 - 2)^2}{(125\phi^2 - 40\phi\theta^2 - 100\phi + 16\theta^2 + 8)^2}$$

- For  $H_2$  not to be profitable:

$$\frac{\phi(\alpha - \bar{c})^2 \cdot (25\phi - 1) (25\phi^2 - 12\theta^2)^2}{(625\phi^3 - 200\phi^2\theta^2 - 250\phi^2 + 12\theta^2)^2} \geq \frac{\phi(\alpha - \bar{c})^2 (5\phi - 4)^2 \cdot (25\phi - 4)}{(125\phi^2 - 40\phi\theta^2 - 100\phi + 16\theta^2 + 8)^2}$$

### C.2.22 Stability conditions for: E4D [2H2L]

- Adding an edge between nodes  $H_1$ ,  $L_1$ : E4D [2H2L]  $\rightarrow$  E5B [2H2L]

- For  $H_1$  not to be profitable:

$$\frac{25\phi^3 (\alpha - \bar{c})^2 (5\phi - 3\theta^2)^2 \cdot (25\phi - 4)}{(625\phi^3 - 250\phi^2\theta^2 - 200\phi^2 + 12\theta^4)^2} > \frac{\phi (\alpha - \bar{c})^2 (5\phi - 2\theta)^2 (5\phi + 2\theta)^2 \cdot (25\phi - 1)}{(125\phi^2 - 4\theta^2)^2 (5\phi - \theta^2 - 1)^2}$$

– For  $L_1$  not to be profitable:

$$\frac{25\phi^3 (\alpha - \bar{c})^2 (5\phi - 4)^2 \cdot (25\phi - 9\theta^2)}{(625\phi^3 - 250\phi^2\theta^2 - 200\phi^2 + 12\theta^4)^2} > \frac{25\phi^3 (\alpha - \bar{c})^2 (5\phi - 2)^2 \cdot (25\phi - 4\theta^2)}{(125\phi^2 - 4\theta^2)^2 (5\phi - \theta^2 - 1)^2}$$

- Removing an edge between nodes  $L_1, L_2$ : **E4D** [2H2L]  $\rightarrow$  **E3B** [2H2L]

– For  $L_1$  not to be profitable:

$$\frac{25\phi^3 (\alpha - \bar{c})^2 (5\phi - 4)^2 \cdot (25\phi - 9\theta^2)}{(625\phi^3 - 250\phi^2\theta^2 - 200\phi^2 + 12\theta^4)^2} \geq \frac{\phi (\alpha - \bar{c})^2 \cdot (25\phi - 16\theta^2) (5\phi - 2\theta^2 - 4)^2}{(125\phi^2 - 100\phi\theta^2 - 40\phi + 8\theta^4 + 16\theta^2)^2}$$

– For  $L_2$  not to be profitable:

$$\frac{\phi (\alpha - \bar{c})^2 \cdot (25\phi - \theta^2) (25\phi^2 - 12\theta^2)^2}{(625\phi^3 - 250\phi^2\theta^2 - 200\phi^2 + 12\theta^4)^2} \geq \frac{\phi (\alpha - \bar{c})^2 (5\phi - 4\theta^2)^2 \cdot (25\phi - 4\theta^2)}{(125\phi^2 - 100\phi\theta^2 - 40\phi + 8\theta^4 + 16\theta^2)^2}$$

- Removing an edge between nodes  $H_1, L_2$ : **E4D** [2H2L]  $\rightarrow$  **E3E** [2H2L]

– For  $H_1$  not to be profitable:

$$\frac{25\phi^3 (\alpha - \bar{c})^2 (5\phi - 3\theta^2)^2 \cdot (25\phi - 4)}{(625\phi^3 - 250\phi^2\theta^2 - 200\phi^2 + 12\theta^4)^2} \geq \frac{\phi (\alpha - \bar{c})^2 \cdot (25\phi - 9) (125\phi^3 - 125\phi^2\theta^2 + 12\theta^4)^2}{(3125\phi^4 - 1625\phi^3\theta^2 - 1625\phi^3 + 225\phi^2\theta^2 + 120\phi\theta^4 + 120\phi\theta^2 - 36\theta^4)^2}$$

– For  $L_2$  not to be profitable:

$$\frac{\phi (\alpha - \bar{c})^2 \cdot (25\phi - \theta^2) (25\phi^2 - 12\theta^2)^2}{(625\phi^3 - 250\phi^2\theta^2 - 200\phi^2 + 12\theta^4)^2} \geq \frac{25\phi^3 (\alpha - \bar{c})^2 \cdot (25\phi - 4\theta^2) (25\phi^2 - 15\phi - 6\theta^2)^2}{(3125\phi^4 - 1625\phi^3\theta^2 - 1625\phi^3 + 225\phi^2\theta^2 + 120\phi\theta^4 + 120\phi\theta^2 - 36\theta^4)^2}$$

- Removing an edge between nodes  $H_1, H_2$ : **E4D** [2H2L]  $\rightarrow$  **E3G** [2H2L]

– For  $H_1$  not to be profitable:

$$\frac{25\phi^3 (\alpha - \bar{c})^2 (5\phi - 3\theta^2)^2 \cdot (25\phi - 4)}{(625\phi^3 - 250\phi^2\theta^2 - 200\phi^2 + 12\theta^4)^2} \geq \frac{25\phi^3 (\alpha - \bar{c})^2 (5\phi - 3\theta^2)^2 \cdot (25\phi - 9)}{(625\phi^3 - 250\phi^2\theta^2 - 75\phi^2 - 60\phi\theta^2 + 18\theta^4)^2}$$

– For  $H_2$  not to be profitable:

$$\frac{25\phi^3 (\alpha - \bar{c})^2 (5\phi - 3\theta^2)^2 \cdot (25\phi - 4)}{(625\phi^3 - 250\phi^2\theta^2 - 200\phi^2 + 12\theta^4)^2} \geq \frac{25\phi^3 (\alpha - \bar{c})^2 (5\phi - 3\theta^2)^2 \cdot (25\phi - 9)}{(625\phi^3 - 250\phi^2\theta^2 - 75\phi^2 - 60\phi\theta^2 + 18\theta^4)^2}$$

### C.2.23 Stability conditions for: **E4E** [2H2L]

- Adding an edge between nodes  $H_1, L_1$ : **E4E** [2H2L]  $\rightarrow$  **E5B** [2H2L]

– For  $H_1$  not to be profitable:

$$\frac{\phi (\alpha - \bar{c})^2 (5\phi - 2\theta^2)^2 \cdot (25\phi - 4)}{(125\phi^2 - 40\phi\theta^2 - 40\phi + 12\theta^2)^2} > \frac{\phi (\alpha - \bar{c})^2 (5\phi - 2\theta)^2 (5\phi + 2\theta)^2 \cdot (25\phi - 1)}{(125\phi^2 - 4\theta^2)^2 (5\phi - \theta^2 - 1)^2}$$

– For  $L_1$  not to be profitable:

$$\frac{\phi (\alpha - \bar{c})^2 (5\phi - 2)^2 \cdot (25\phi - 4\theta^2)}{(125\phi^2 - 40\phi\theta^2 - 40\phi + 12\theta^2)^2} > \frac{\phi (\alpha - \bar{c})^2 (5\phi - 2\theta)^2 (5\phi + 2\theta)^2 \cdot (25\phi - \theta^2)}{(125\phi^2 - 4\theta^2)^2 (5\phi - \theta^2 - 1)^2}$$

- Removing an edge between nodes  $L_1, L_2$ : **E4E** [2H2L]  $\rightarrow$  **E3C** [2H2L]

– For  $L_1$  not to be profitable:

$$\frac{\phi (\alpha - \bar{c})^2 (5\phi - 2)^2 \cdot (25\phi - 4\theta^2)}{(125\phi^2 - 40\phi\theta^2 - 40\phi + 12\theta^2)^2} \geq \frac{\phi (\alpha - \bar{c})^2 (5\phi - 2)^2 \cdot (25\phi - 9\theta^2)}{(125\phi^2 - 15\phi\theta^2 - 40\phi + 6\theta^2)^2}$$

– For  $L_2$  not to be profitable:

$$\frac{\phi(\alpha - \bar{c})^2 (5\phi - 2)^2 \cdot (25\phi - 4\theta^2)}{(125\phi^2 - 40\phi\theta^2 - 40\phi + 12\theta^2)^2} \geq \frac{\phi(\alpha - \bar{c})^2 (5\phi - 2)^2 \cdot (25\phi - 9\theta^2)}{(125\phi^2 - 15\phi\theta^2 - 40\phi + 6\theta^2)^2}$$

- Removing an edge between nodes  $H_1, L_2$ : [E4E \[2H2L\]](#)  $\rightarrow$  [E3E \[2H2L\]](#)

– For  $H_1$  not to be profitable:

$$\frac{\phi(\alpha - \bar{c})^2 (5\phi - 2\theta^2)^2 \cdot (25\phi - 4)}{(125\phi^2 - 40\phi\theta^2 - 40\phi + 12\theta^2)^2} \geq \frac{\phi(\alpha - \bar{c})^2 \cdot (25\phi - 9) (125\phi^3 - 125\phi^2\theta^2 + 12\theta^4)^2}{(3125\phi^4 - 1625\phi^3\theta^2 - 1625\phi^3 + 225\phi^2\theta^2 + 120\phi\theta^4 + 120\phi\theta^2 - 36\theta^4)^2}$$

– For  $L_2$  not to be profitable:

$$\frac{\phi(\alpha - \bar{c})^2 (5\phi - 2)^2 \cdot (25\phi - 4\theta^2)}{(125\phi^2 - 40\phi\theta^2 - 40\phi + 12\theta^2)^2} \geq \frac{\phi(\alpha - \bar{c})^2 \cdot (25\phi - 9\theta^2) (125\phi^3 - 125\phi^2 + 12\theta^2)^2}{(3125\phi^4 - 1625\phi^3\theta^2 - 1625\phi^3 + 225\phi^2\theta^2 + 120\phi\theta^4 + 120\phi\theta^2 - 36\theta^4)^2}$$

- Removing an edge between nodes  $H_1, H_2$ : [E4E \[2H2L\]](#)  $\rightarrow$  [E3H \[2H2L\]](#)

– For  $H_1$  not to be profitable:

$$\frac{\phi(\alpha - \bar{c})^2 (5\phi - 2\theta^2)^2 \cdot (25\phi - 4)}{(125\phi^2 - 40\phi\theta^2 - 40\phi + 12\theta^2)^2} \geq \frac{\phi(\alpha - \bar{c})^2 (5\phi - 2\theta^2)^2 \cdot (25\phi - 9)}{(125\phi^2 - 40\phi\theta^2 - 15\phi + 6\theta^2)^2}$$

– For  $H_2$  not to be profitable:

$$\frac{\phi(\alpha - \bar{c})^2 (5\phi - 2\theta^2)^2 \cdot (25\phi - 4)}{(125\phi^2 - 40\phi\theta^2 - 40\phi + 12\theta^2)^2} \geq \frac{\phi(\alpha - \bar{c})^2 (5\phi - 2\theta^2)^2 \cdot (25\phi - 9)}{(125\phi^2 - 40\phi\theta^2 - 15\phi + 6\theta^2)^2}$$

#### C.2.24 Stability conditions for: [E4F \[2H2L\]](#)

- Adding an edge between nodes  $H_1, H_2$ : [E4F \[2H2L\]](#)  $\rightarrow$  [E5B \[2H2L\]](#)

– For  $H_1$  not to be profitable:

$$\frac{25\phi^3 (\alpha - \bar{c})^2 \cdot (25\phi - 9) (5\phi - 2\theta^2 - 2)^2}{(625\phi^3 - 125\phi^2\theta^2 - 325\phi^2 + 6\theta^4 + 6\theta^2)^2} > \frac{25\phi^3 (\alpha - \bar{c})^2 (5\phi - 2\theta^2)^2 \cdot (25\phi - 4)}{(125\phi^2 - 4\theta^2)^2 (5\phi - \theta^2 - 1)^2}$$

– For  $H_2$  not to be profitable:

$$\frac{25\phi^3 (\alpha - \bar{c})^2 (5\phi - 3)^2 \cdot (25\phi - 4)}{(625\phi^3 - 125\phi^2\theta^2 - 325\phi^2 + 6\theta^4 + 6\theta^2)^2} > \frac{\phi(\alpha - \bar{c})^2 (5\phi - 2\theta)^2 (5\phi + 2\theta)^2 \cdot (25\phi - 1)}{(125\phi^2 - 4\theta^2)^2 (5\phi - \theta^2 - 1)^2}$$

- Adding an edge between nodes  $H_1, L_1$ : [E4F \[2H2L\]](#)  $\rightarrow$  [E5C \[2H2L\]](#)

– For  $H_1$  not to be profitable:

$$\frac{25\phi^3 (\alpha - \bar{c})^2 \cdot (25\phi - 9) (5\phi - 2\theta^2 - 2)^2}{(625\phi^3 - 125\phi^2\theta^2 - 325\phi^2 + 6\theta^4 + 6\theta^2)^2} > \frac{25\phi^3 (\alpha - \bar{c})^2 \cdot (25\phi - 4)}{(125\phi^2 - 10\phi\theta^2 + 10\phi - 4\theta^2)^2}$$

– For  $L_1$  not to be profitable:

$$\frac{25\phi^3 (\alpha - \bar{c})^2 (5\phi - 3)^2 \cdot (25\phi - 4\theta^2)}{(625\phi^3 - 125\phi^2\theta^2 - 325\phi^2 + 6\theta^4 + 6\theta^2)^2} > \frac{\phi(\alpha - \bar{c})^2 (5\phi + 2)^2 \cdot (25\phi - \theta^2)}{(125\phi^2 - 10\phi\theta^2 + 10\phi - 4\theta^2)^2}$$

- Removing an edge between nodes  $L_1, L_2$ : [E4F \[2H2L\]](#)  $\rightarrow$  [E3D \[2H2L\]](#)

– For  $L_1$  not to be profitable:

$$\frac{25\phi^3 (\alpha - \bar{c})^2 (5\phi - 3)^2 \cdot (25\phi - 4\theta^2)}{(625\phi^3 - 125\phi^2\theta^2 - 325\phi^2 + 6\theta^4 + 6\theta^2)^2} \geq \frac{\phi(\alpha - \bar{c})^2 \cdot (25\phi - 9\theta^2) (125\phi^3 - 50\phi^2\theta^2 - 75\phi^2 + 12\theta^2)^2}{(3125\phi^4 - 1625\phi^3\theta^2 - 1625\phi^3 + 225\phi^2\theta^2 + 120\phi\theta^4 + 120\phi\theta^2 - 36\theta^4)^2}$$

– For  $L_2$  not to be profitable:

$$\frac{\phi(\alpha - \bar{c})^2 \cdot (25\phi - \theta^2) (25\phi^2 - 6\theta^2 - 6)^2}{(625\phi^3 - 125\phi^2\theta^2 - 325\phi^2 + 6\theta^4 + 6\theta^2)^2} \geq \frac{25\phi^3 (\alpha - \bar{c})^2 \cdot (25\phi - 4\theta^2) (25\phi^2 - 15\phi\theta^2 - 6)^2}{(3125\phi^4 - 1625\phi^3\theta^2 - 1625\phi^3 + 225\phi^2\theta^2 + 120\phi\theta^4 + 120\phi\theta^2 - 36\theta^4)^2}$$

- Removing an edge between nodes  $H_1, L_2$ : [E4F \[2H2L\]](#)  $\rightarrow$  [E3F \[2H2L\]](#)

– For  $H_1$  not to be profitable:



$$\frac{25\phi^3 (\alpha - \bar{c})^2 \cdot (25\phi - 9) (5\phi - 2\theta^2 - 2)^2}{(625\phi^3 - 125\phi^2\theta^2 - 325\phi^2 + 6\theta^4 + 6\theta^2)^2} \geq \frac{\phi (\alpha - \bar{c})^2 \cdot (25\phi - 16) (5\phi - 4\theta^2 - 2)^2}{(125\phi^2 - 40\phi\theta^2 - 100\phi + 16\theta^2 + 8)^2}$$

– For  $L_2$  not to be profitable:

$$\frac{\phi (\alpha - \bar{c})^2 \cdot (25\phi - \theta^2) (25\phi^2 - 6\theta^2 - 6)^2}{(625\phi^3 - 125\phi^2\theta^2 - 325\phi^2 + 6\theta^4 + 6\theta^2)^2} \geq \frac{\phi (\alpha - \bar{c})^2 (5\phi - 4)^2 \cdot (25\phi - 4\theta^2)}{(125\phi^2 - 40\phi\theta^2 - 100\phi + 16\theta^2 + 8)^2}$$

- Removing an edge between nodes  $H_2$ ,  $L_1$ : [E4F \[2H2L\]](#) → [E3G \[2H2L\]](#)

– For  $H_2$  not to be profitable:

$$\frac{25\phi^3 (\alpha - \bar{c})^2 (5\phi - 3)^2 \cdot (25\phi - 4)}{(625\phi^3 - 125\phi^2\theta^2 - 325\phi^2 + 6\theta^4 + 6\theta^2)^2} \geq \frac{25\phi^3 (\alpha - \bar{c})^2 (5\phi - 3\theta^2)^2 \cdot (25\phi - 9)}{(625\phi^3 - 250\phi^2\theta^2 - 75\phi^2 - 60\phi\theta^2 + 18\theta^4)^2}$$

– For  $L_1$  not to be profitable:

$$\frac{25\phi^3 (\alpha - \bar{c})^2 (5\phi - 3)^2 \cdot (25\phi - 4\theta^2)}{(625\phi^3 - 125\phi^2\theta^2 - 325\phi^2 + 6\theta^4 + 6\theta^2)^2} \geq \frac{25\phi^3 (\alpha - \bar{c})^2 (5\phi - 3)^2 \cdot (25\phi - 9\theta^2)}{(625\phi^3 - 250\phi^2\theta^2 - 75\phi^2 - 60\phi\theta^2 + 18\theta^4)^2}$$

- Removing an edge between nodes  $H_2$ ,  $L_2$ : [E4F \[2H2L\]](#) → [E3H \[2H2L\]](#)

– For  $H_2$  not to be profitable:

$$\frac{25\phi^3 (\alpha - \bar{c})^2 (5\phi - 3)^2 \cdot (25\phi - 4)}{(625\phi^3 - 125\phi^2\theta^2 - 325\phi^2 + 6\theta^4 + 6\theta^2)^2} \geq \frac{\phi (\alpha - \bar{c})^2 (5\phi - 2\theta^2)^2 \cdot (25\phi - 9)}{(125\phi^2 - 40\phi\theta^2 - 15\phi + 6\theta^2)^2}$$

– For  $L_2$  not to be profitable:

$$\frac{\phi (\alpha - \bar{c})^2 \cdot (25\phi - \theta^2) (25\phi^2 - 6\theta^2 - 6)^2}{(625\phi^3 - 125\phi^2\theta^2 - 325\phi^2 + 6\theta^4 + 6\theta^2)^2} \geq \frac{25\phi^3 (\alpha - \bar{c})^2 \cdot (25\phi - 4\theta^2)}{(125\phi^2 - 40\phi\theta^2 - 15\phi + 6\theta^2)^2}$$

### C.2.25 Stability conditions for: [E5A \[2H2L\]](#)

- Adding an edge between nodes  $L_1$ ,  $L_2$ : [E5A \[2H2L\]](#) → [E6A \[2H2L\]](#)

– For  $L_1$  not to be profitable:

$$\frac{25\phi^3 (\alpha - \bar{c})^2 \cdot (25\phi - 4\theta^2)}{(125\phi^2 + 10\phi\theta^2 - 10\phi - 4\theta^2)^2} > \frac{\phi (\alpha - \bar{c})^2 \cdot (25\phi - \theta^2)}{(25\phi - 2\theta^2 - 2)^2}$$

– For  $L_2$  not to be profitable:

$$\frac{25\phi^3 (\alpha - \bar{c})^2 \cdot (25\phi - 4\theta^2)}{(125\phi^2 + 10\phi\theta^2 - 10\phi - 4\theta^2)^2} > \frac{\phi (\alpha - \bar{c})^2 \cdot (25\phi - \theta^2)}{(25\phi - 2\theta^2 - 2)^2}$$

- Removing an edge between nodes  $H_1$ ,  $L_1$ : [E5A \[2H2L\]](#) → [E4A \[2H2L\]](#)

– For  $H_1$  not to be profitable:

$$\frac{\phi (\alpha - \bar{c})^2 (5\phi + 2\theta^2)^2 \cdot (25\phi - 1)}{(125\phi^2 + 10\phi\theta^2 - 10\phi - 4\theta^2)^2} \geq \frac{25\phi^3 (\alpha - \bar{c})^2 (5\phi - 3\theta^2)^2 \cdot (25\phi - 4)}{(625\phi^3 - 325\phi^2\theta^2 - 125\phi^2 + 6\theta^4 + 6\theta^2)^2}$$

– For  $L_1$  not to be profitable:

$$\frac{25\phi^3 (\alpha - \bar{c})^2 \cdot (25\phi - 4\theta^2)}{(125\phi^2 + 10\phi\theta^2 - 10\phi - 4\theta^2)^2} \geq \frac{25\phi^3 (\alpha - \bar{c})^2 \cdot (25\phi - 9\theta^2) (5\phi - 2\theta^2 - 2)^2}{(625\phi^3 - 325\phi^2\theta^2 - 125\phi^2 + 6\theta^4 + 6\theta^2)^2}$$

- Removing an edge between nodes  $H_1$ ,  $H_2$ : [E5A \[2H2L\]](#) → [E4B \[2H2L\]](#)

– For  $H_1$  not to be profitable:

$$\frac{\phi (\alpha - \bar{c})^2 (5\phi + 2\theta^2)^2 \cdot (25\phi - 1)}{(125\phi^2 + 10\phi\theta^2 - 10\phi - 4\theta^2)^2} \geq \frac{\phi (\alpha - \bar{c})^2 (5\phi + 2\theta^2)^2 \cdot (25\phi - 4)}{(125\phi^2 + 10\phi\theta^2 + 10\phi - 12\theta^2)^2}$$

– For  $H_2$  not to be profitable:

$$\frac{\phi (\alpha - \bar{c})^2 (5\phi + 2\theta^2)^2 \cdot (25\phi - 1)}{(125\phi^2 + 10\phi\theta^2 - 10\phi - 4\theta^2)^2} \geq \frac{\phi (\alpha - \bar{c})^2 (5\phi + 2\theta^2)^2 \cdot (25\phi - 4)}{(125\phi^2 + 10\phi\theta^2 + 10\phi - 12\theta^2)^2}$$

### C.2.26 Stability conditions for: E5B [2H2L]

- Adding an edge between nodes  $H_1, L_1$ : E5B [2H2L]  $\rightarrow$  E6A [2H2L]

– For  $H_1$  not to be profitable:

$$\frac{25\phi^3 (\alpha - \bar{c})^2 (5\phi - 2\theta^2)^2 \cdot (25\phi - 4)}{(125\phi^2 - 4\theta^2)^2 (5\phi - \theta^2 - 1)^2} > \frac{\phi (\alpha - \bar{c})^2 \cdot (25\phi - 1)}{(25\phi - 2\theta^2 - 2)^2}$$

– For  $L_1$  not to be profitable:

$$\frac{25\phi^3 (\alpha - \bar{c})^2 (5\phi - 2)^2 \cdot (25\phi - 4\theta^2)}{(125\phi^2 - 4\theta^2)^2 (5\phi - \theta^2 - 1)^2} > \frac{\phi (\alpha - \bar{c})^2 \cdot (25\phi - \theta^2)}{(25\phi - 2\theta^2 - 2)^2}$$

- Removing an edge between nodes  $L_1, L_2$ : E5B [2H2L]  $\rightarrow$  E4A [2H2L]

– For  $L_1$  not to be profitable:

$$\frac{25\phi^3 (\alpha - \bar{c})^2 (5\phi - 2)^2 \cdot (25\phi - 4\theta^2)}{(125\phi^2 - 4\theta^2)^2 (5\phi - \theta^2 - 1)^2} \geq \frac{25\phi^3 (\alpha - \bar{c})^2 \cdot (25\phi - 9\theta^2) (5\phi - 2\theta^2 - 2)^2}{(625\phi^3 - 325\phi^2\theta^2 - 125\phi^2 + 6\theta^4 + 6\theta^2)^2}$$

– For  $L_2$  not to be profitable:

$$\frac{\phi (\alpha - \bar{c})^2 (5\phi - 2\theta)^2 (5\phi + 2\theta)^2 \cdot (25\phi - \theta^2)}{(125\phi^2 - 4\theta^2)^2 (5\phi - \theta^2 - 1)^2} \geq \frac{25\phi^3 (\alpha - \bar{c})^2 (5\phi - 3\theta^2)^2 \cdot (25\phi - 4\theta^2)}{(625\phi^3 - 325\phi^2\theta^2 - 125\phi^2 + 6\theta^4 + 6\theta^2)^2}$$

- Removing an edge between nodes  $H_1, L_2$ : E5B [2H2L]  $\rightarrow$  E4C [2H2L]

– For  $H_1$  not to be profitable:

$$\frac{25\phi^3 (\alpha - \bar{c})^2 (5\phi - 2\theta^2)^2 \cdot (25\phi - 4)}{(125\phi^2 - 4\theta^2)^2 (5\phi - \theta^2 - 1)^2} \geq \frac{25\phi^3 (\alpha - \bar{c})^2 (5\phi - 4\theta^2)^2 \cdot (25\phi - 9)}{(625\phi^3 - 200\phi^2\theta^2 - 250\phi^2 + 12\theta^2)^2}$$

– For  $L_2$  not to be profitable:

$$\frac{\phi (\alpha - \bar{c})^2 (5\phi - 2\theta)^2 (5\phi + 2\theta)^2 \cdot (25\phi - \theta^2)}{(125\phi^2 - 4\theta^2)^2 (5\phi - \theta^2 - 1)^2} \geq \frac{25\phi^3 (\alpha - \bar{c})^2 (5\phi - 3)^2 \cdot (25\phi - 4\theta^2)}{(625\phi^3 - 200\phi^2\theta^2 - 250\phi^2 + 12\theta^2)^2}$$

- Removing an edge between nodes  $H_2, L_1$ : E5B [2H2L]  $\rightarrow$  E4D [2H2L]

– For  $H_2$  not to be profitable:

$$\frac{\phi (\alpha - \bar{c})^2 (5\phi - 2\theta)^2 (5\phi + 2\theta)^2 \cdot (25\phi - 1)}{(125\phi^2 - 4\theta^2)^2 (5\phi - \theta^2 - 1)^2} \geq \frac{25\phi^3 (\alpha - \bar{c})^2 (5\phi - 3\theta^2)^2 \cdot (25\phi - 4)}{(625\phi^3 - 250\phi^2\theta^2 - 200\phi^2 + 12\theta^4)^2}$$

– For  $L_1$  not to be profitable:

$$\frac{25\phi^3 (\alpha - \bar{c})^2 (5\phi - 2)^2 \cdot (25\phi - 4\theta^2)}{(125\phi^2 - 4\theta^2)^2 (5\phi - \theta^2 - 1)^2} \geq \frac{25\phi^3 (\alpha - \bar{c})^2 (5\phi - 4)^2 \cdot (25\phi - 9\theta^2)}{(625\phi^3 - 250\phi^2\theta^2 - 200\phi^2 + 12\theta^4)^2}$$

- Removing an edge between nodes  $H_2, L_2$ : E5B [2H2L]  $\rightarrow$  E4E [2H2L]

– For  $H_2$  not to be profitable:

$$\frac{\phi (\alpha - \bar{c})^2 (5\phi - 2\theta)^2 (5\phi + 2\theta)^2 \cdot (25\phi - 1)}{(125\phi^2 - 4\theta^2)^2 (5\phi - \theta^2 - 1)^2} \geq \frac{\phi (\alpha - \bar{c})^2 (5\phi - 2\theta^2)^2 \cdot (25\phi - 4)}{(125\phi^2 - 40\phi\theta^2 - 40\phi + 12\theta^2)^2}$$

– For  $L_2$  not to be profitable:

$$\frac{\phi (\alpha - \bar{c})^2 (5\phi - 2\theta)^2 (5\phi + 2\theta)^2 \cdot (25\phi - \theta^2)}{(125\phi^2 - 4\theta^2)^2 (5\phi - \theta^2 - 1)^2} \geq \frac{\phi (\alpha - \bar{c})^2 (5\phi - 2)^2 \cdot (25\phi - 4\theta^2)}{(125\phi^2 - 40\phi\theta^2 - 40\phi + 12\theta^2)^2}$$

- Removing an edge between nodes  $H_1, H_2$ : E5B [2H2L]  $\rightarrow$  E4F [2H2L]

– For  $H_1$  not to be profitable:

$$\frac{25\phi^3 (\alpha - \bar{c})^2 (5\phi - 2\theta^2)^2 \cdot (25\phi - 4)}{(125\phi^2 - 4\theta^2)^2 (5\phi - \theta^2 - 1)^2} \geq \frac{25\phi^3 (\alpha - \bar{c})^2 \cdot (25\phi - 9) (5\phi - 2\theta^2 - 2)^2}{(625\phi^3 - 125\phi^2\theta^2 - 325\phi^2 + 6\theta^4 + 6\theta^2)^2}$$

– For  $H_2$  not to be profitable:

$$\frac{\phi(\alpha - \bar{c})^2 (5\phi - 2\theta)^2 (5\phi + 2\theta)^2 \cdot (25\phi - 1)}{(125\phi^2 - 4\theta^2)^2 (5\phi - \theta^2 - 1)^2} \geq \frac{25\phi^3 (\alpha - \bar{c})^2 (5\phi - 3)^2 \cdot (25\phi - 4)}{(625\phi^3 - 125\phi^2\theta^2 - 325\phi^2 + 6\theta^4 + 6\theta^2)^2}$$

### C.2.27 Stability conditions for: E5C [2H2L]

- Adding an edge between nodes  $H_1, H_2$ : E5C [2H2L]  $\rightarrow$  E6A [2H2L]

– For  $H_1$  not to be profitable:

$$\frac{25\phi^3 (\alpha - \bar{c})^2 \cdot (25\phi - 4)}{(125\phi^2 - 10\phi\theta^2 + 10\phi - 4\theta^2)^2} > \frac{\phi(\alpha - \bar{c})^2 \cdot (25\phi - 1)}{(25\phi - 2\theta^2 - 2)^2}$$

– For  $H_2$  not to be profitable:

$$\frac{25\phi^3 (\alpha - \bar{c})^2 \cdot (25\phi - 4)}{(125\phi^2 - 10\phi\theta^2 + 10\phi - 4\theta^2)^2} > \frac{\phi(\alpha - \bar{c})^2 \cdot (25\phi - 1)}{(25\phi - 2\theta^2 - 2)^2}$$

- Removing an edge between nodes  $L_1, L_2$ : E5C [2H2L]  $\rightarrow$  E4B [2H2L]

– For  $L_1$  not to be profitable:

$$\frac{\phi(\alpha - \bar{c})^2 (5\phi + 2)^2 \cdot (25\phi - \theta^2)}{(125\phi^2 - 10\phi\theta^2 + 10\phi - 4\theta^2)^2} \geq \frac{\phi(\alpha - \bar{c})^2 (5\phi + 2)^2 \cdot (25\phi - 4\theta^2)}{(125\phi^2 + 10\phi\theta^2 + 10\phi - 12\theta^2)^2}$$

– For  $L_2$  not to be profitable:

$$\frac{\phi(\alpha - \bar{c})^2 (5\phi + 2)^2 \cdot (25\phi - \theta^2)}{(125\phi^2 - 10\phi\theta^2 + 10\phi - 4\theta^2)^2} \geq \frac{\phi(\alpha - \bar{c})^2 (5\phi + 2)^2 \cdot (25\phi - 4\theta^2)}{(125\phi^2 + 10\phi\theta^2 + 10\phi - 12\theta^2)^2}$$

- Removing an edge between nodes  $H_1, L_1$ : E5C [2H2L]  $\rightarrow$  E4F [2H2L]

– For  $H_1$  not to be profitable:

$$\frac{25\phi^3 (\alpha - \bar{c})^2 \cdot (25\phi - 4)}{(125\phi^2 - 10\phi\theta^2 + 10\phi - 4\theta^2)^2} \geq \frac{25\phi^3 (\alpha - \bar{c})^2 \cdot (25\phi - 9) (5\phi - 2\theta^2 - 2)^2}{(625\phi^3 - 125\phi^2\theta^2 - 325\phi^2 + 6\theta^4 + 6\theta^2)^2}$$

– For  $L_1$  not to be profitable:

$$\frac{\phi(\alpha - \bar{c})^2 (5\phi + 2)^2 \cdot (25\phi - \theta^2)}{(125\phi^2 - 10\phi\theta^2 + 10\phi - 4\theta^2)^2} \geq \frac{25\phi^3 (\alpha - \bar{c})^2 (5\phi - 3)^2 \cdot (25\phi - 4\theta^2)}{(625\phi^3 - 125\phi^2\theta^2 - 325\phi^2 + 6\theta^4 + 6\theta^2)^2}$$

### C.2.28 Stability conditions for: E6A [2H2L]

- Removing an edge between nodes  $L_1, L_2$ : E6A [2H2L]  $\rightarrow$  E5A [2H2L]

– For  $L_1$  not to be profitable:

$$\frac{\phi(\alpha - \bar{c})^2 \cdot (25\phi - \theta^2)}{(25\phi - 2\theta^2 - 2)^2} \geq \frac{25\phi^3 (\alpha - \bar{c})^2 \cdot (25\phi - 4\theta^2)}{(125\phi^2 + 10\phi\theta^2 - 10\phi - 4\theta^2)^2}$$

– For  $L_2$  not to be profitable:

$$\frac{\phi(\alpha - \bar{c})^2 \cdot (25\phi - \theta^2)}{(25\phi - 2\theta^2 - 2)^2} \geq \frac{25\phi^3 (\alpha - \bar{c})^2 \cdot (25\phi - 4\theta^2)}{(125\phi^2 + 10\phi\theta^2 - 10\phi - 4\theta^2)^2}$$

- Removing an edge between nodes  $H_1, L_1$ : E6A [2H2L]  $\rightarrow$  E5B [2H2L]

– For  $H_1$  not to be profitable:

$$\frac{\phi(\alpha - \bar{c})^2 \cdot (25\phi - 1)}{(25\phi - 2\theta^2 - 2)^2} \geq \frac{25\phi^3 (\alpha - \bar{c})^2 (5\phi - 2\theta^2)^2 \cdot (25\phi - 4)}{(125\phi^2 - 4\theta^2)^2 (5\phi - \theta^2 - 1)^2}$$

– For  $L_1$  not to be profitable:

$$\frac{\phi(\alpha - \bar{c})^2 \cdot (25\phi - \theta^2)}{(25\phi - 2\theta^2 - 2)^2} \geq \frac{25\phi^3 (\alpha - \bar{c})^2 (5\phi - 2)^2 \cdot (25\phi - 4\theta^2)}{(125\phi^2 - 4\theta^2)^2 (5\phi - \theta^2 - 1)^2}$$

- Removing an edge between nodes  $H_1, H_2$ : E6A [2H2L]  $\rightarrow$  E5C [2H2L]

– For  $H_1$  not to be profitable:

$$\frac{\phi (\alpha - \bar{c})^2 \cdot (25\phi - 1)}{(25\phi - 2\theta^2 - 2)^2} \geq \frac{25\phi^3 (\alpha - \bar{c})^2 \cdot (25\phi - 4)}{(125\phi^2 - 10\phi\theta^2 + 10\phi - 4\theta^2)^2}$$

– For  $H_2$  not to be profitable:

$$\frac{\phi (\alpha - \bar{c})^2 \cdot (25\phi - 1)}{(25\phi - 2\theta^2 - 2)^2} \geq \frac{25\phi^3 (\alpha - \bar{c})^2 \cdot (25\phi - 4)}{(125\phi^2 - 10\phi\theta^2 + 10\phi - 4\theta^2)^2}$$

### C.3 Three *high*- and one *low*-productive firms (3H1L)

#### C.3.1 Stability conditions for: E0A [3H1L]

- Adding an edge between nodes  $H_1, H_2$ : E0A [3H1L]  $\rightarrow$  E1A [3H1L]

– For  $H_1$  not to be profitable:

$$\frac{\phi(\alpha - \bar{c})^2 (5\phi - 4\theta^2)^2 \cdot (25\phi - 16)}{(125\phi^2 - 80\phi\theta^2 - 40\phi + 16\theta^2)^2} > \frac{\phi(\alpha - \bar{c})^2 (5\phi - 4)^2 (5\phi - 4\theta^2)^2 \cdot (25\phi - 9)}{(625\phi^3 - 400\phi^2\theta^2 - 850\phi^2 + 480\phi\theta^2 + 240\phi - 96\theta^2)^2}$$

– For  $H_2$  not to be profitable:

$$\frac{\phi(\alpha - \bar{c})^2 (5\phi - 4\theta^2)^2 \cdot (25\phi - 16)}{(125\phi^2 - 80\phi\theta^2 - 40\phi + 16\theta^2)^2} > \frac{\phi(\alpha - \bar{c})^2 (5\phi - 4)^2 (5\phi - 4\theta^2)^2 \cdot (25\phi - 9)}{(625\phi^3 - 400\phi^2\theta^2 - 850\phi^2 + 480\phi\theta^2 + 240\phi - 96\theta^2)^2}$$

- Adding an edge between nodes  $H_1, L_1$ : E0A [3H1L]  $\rightarrow$  E1B [3H1L]

– For  $H_1$  not to be profitable:

$$\frac{\phi(\alpha - \bar{c})^2 (5\phi - 4\theta^2)^2 \cdot (25\phi - 16)}{(125\phi^2 - 80\phi\theta^2 - 40\phi + 16\theta^2)^2} > \frac{\phi(\alpha - \bar{c})^2 (5\phi - 4)^2 \cdot (25\phi - 9)}{(125\phi^2 - 45\phi\theta^2 - 105\phi + 12\theta^2 + 12)^2}$$

– For  $L_1$  not to be profitable:

$$\frac{\phi(\alpha - \bar{c})^2 (5\phi - 4)^2 \cdot (25\phi - 16\theta^2)}{(125\phi^2 - 80\phi\theta^2 - 40\phi + 16\theta^2)^2} > \frac{\phi(\alpha - \bar{c})^2 (5\phi - 4)^2 \cdot (25\phi - 9\theta^2)}{(125\phi^2 - 45\phi\theta^2 - 105\phi + 12\theta^2 + 12)^2}$$

#### C.3.2 Stability conditions for: E1A [3H1L]

- Adding an edge between nodes  $H_1, H_2$ : E1A [3H1L]  $\rightarrow$  E2A [3H1L]

– For  $H_1$  not to be profitable:

$$\frac{\phi(\alpha - \bar{c})^2 (5\phi - 6)^2 (5\phi - 4\theta^2)^2 \cdot (25\phi - 16)}{(625\phi^3 - 400\phi^2\theta^2 - 850\phi^2 + 480\phi\theta^2 + 240\phi - 96\theta^2)^2} > \frac{25\phi^3 (\alpha - \bar{c})^2 (5\phi - 4\theta^2)^2 \cdot (25\phi - 9)}{(5\phi + 1)^2 (125\phi^2 - 80\phi\theta^2 - 60\phi + 24\theta^2)^2}$$

– For  $H_2$  not to be profitable:

$$\frac{\phi(\alpha - \bar{c})^2 (5\phi - 4)^2 (5\phi - 4\theta^2)^2 \cdot (25\phi - 9)}{(625\phi^3 - 400\phi^2\theta^2 - 850\phi^2 + 480\phi\theta^2 + 240\phi - 96\theta^2)^2} > \frac{\phi(\alpha - \bar{c})^2 (5\phi + 3)^2 (5\phi - 4\theta^2)^2 \cdot (25\phi - 4)}{(5\phi + 1)^2 (125\phi^2 - 80\phi\theta^2 - 60\phi + 24\theta^2)^2}$$

- Adding an edge between nodes  $H_2, L_1$ : E1A [3H1L]  $\rightarrow$  E2B [3H1L]

– For  $H_2$  not to be profitable:

$$\frac{\phi(\alpha - \bar{c})^2 (5\phi - 4)^2 (5\phi - 4\theta^2)^2 \cdot (25\phi - 9)}{(625\phi^3 - 400\phi^2\theta^2 - 850\phi^2 + 480\phi\theta^2 + 240\phi - 96\theta^2)^2} > \frac{\phi(\alpha - \bar{c})^2 (5\phi - 4)^2 (5\phi - 3\theta^2)^2 (5\phi + 3\theta^2) \cdot (25\phi - 4)}{(3125\phi^4 - 1125\phi^3\theta^2 - 3625\phi^3 + 825\phi^2\theta^2 + 800\phi^2 + 180\phi\theta^2 - 72\theta^2)^2}$$

– For  $L_1$  not to be profitable:

$$\frac{\phi(\alpha - \bar{c})^2 (5\phi - 6)^2 (5\phi - 4)^2 \cdot (25\phi - 16\theta^2)}{(625\phi^3 - 400\phi^2\theta^2 - 850\phi^2 + 480\phi\theta^2 + 240\phi - 96\theta^2)^2} > \frac{25\phi^3 (\alpha - \bar{c})^2 (5\phi - 4)^2 (5\phi - 3)^2 \cdot (25\phi - 9\theta^2)}{(3125\phi^4 - 1125\phi^3\theta^2 - 3625\phi^3 + 825\phi^2\theta^2 + 800\phi^2 + 180\phi\theta^2 - 72\theta^2)^2}$$

- Adding an edge between nodes  $H_1, L_1$ : E1A [3H1L]  $\rightarrow$  E2C [3H1L]

– For  $H_1$  not to be profitable:

$$\frac{\phi(\alpha - \bar{c})^2 (5\phi - 6)^2 (5\phi - 4\theta^2)^2 \cdot (25\phi - 16)}{(625\phi^3 - 400\phi^2\theta^2 - 850\phi^2 + 480\phi\theta^2 + 240\phi - 96\theta^2)^2} > \frac{\phi(\alpha - \bar{c})^2 (5\phi - 6)^2 \cdot (25\phi - 9)}{(125\phi^2 - 45\phi\theta^2 - 135\phi + 18\theta^2 + 18)^2}$$

– For  $L_1$  not to be profitable:

$$\frac{\phi(\alpha - \bar{c})^2 (5\phi - 6)^2 (5\phi - 4)^2 \cdot (25\phi - 16\theta^2)}{(625\phi^3 - 400\phi^2\theta^2 - 850\phi^2 + 480\phi\theta^2 + 240\phi - 96\theta^2)^2} > \frac{\phi(\alpha - \bar{c})^2 (5\phi - 6)^2 \cdot (25\phi - 9\theta^2)}{(125\phi^2 - 45\phi\theta^2 - 135\phi + 18\theta^2 + 18)^2}$$

- Removing an edge between nodes  $H_2, H_3$ : E1A [3H1L]  $\rightarrow$  E0A [3H1L]

– For  $H_2$  not to be profitable:

$$\frac{\phi(\alpha - \bar{c})^2 (5\phi - 4)^2 (5\phi - 4\theta^2)^2 \cdot (25\phi - 9)}{(625\phi^3 - 400\phi^2\theta^2 - 850\phi^2 + 480\phi\theta^2 + 240\phi - 96\theta^2)^2} \geq \frac{\phi(\alpha - \bar{c})^2 (5\phi - 4\theta^2)^2 \cdot (25\phi - 16)}{(125\phi^2 - 80\phi\theta^2 - 40\phi + 16\theta^2)^2}$$

– For  $H_3$  not to be profitable:

$$\frac{\phi(\alpha - \bar{c})^2 (5\phi - 4)^2 (5\phi - 4\theta^2)^2 \cdot (25\phi - 9)}{(625\phi^3 - 400\phi^2\theta^2 - 850\phi^2 + 480\phi\theta^2 + 240\phi - 96\theta^2)^2} \geq \frac{\phi(\alpha - \bar{c})^2 (5\phi - 4\theta^2)^2 \cdot (25\phi - 16)}{(125\phi^2 - 80\phi\theta^2 - 40\phi + 16\theta^2)^2}$$

### C.3.3 Stability conditions for: E1B [3H1L]

- Adding an edge between nodes  $H_1, H_3$ : E1B [3H1L]  $\rightarrow$  E2B [3H1L]

– For  $H_1$  not to be profitable:

$$\frac{\phi(\alpha - \bar{c})^2 \cdot (25\phi - 16) (5\phi - 3\theta^2 - 3)^2}{(125\phi^2 - 45\phi\theta^2 - 105\phi + 12\theta^2 + 12)^2} > \frac{25\phi^3 (\alpha - \bar{c})^2 (5\phi - 4)^2 (5\phi - 3\theta^2)^2 \cdot (25\phi - 9)}{(3125\phi^4 - 1125\phi^3\theta^2 - 3625\phi^3 + 825\phi^2\theta^2 + 800\phi^2 + 180\phi\theta^2 - 72\theta^2)^2}$$

– For  $H_3$  not to be profitable:

$$\frac{\phi(\alpha - \bar{c})^2 (5\phi - 4)^2 \cdot (25\phi - 9)}{(125\phi^2 - 45\phi\theta^2 - 105\phi + 12\theta^2 + 12)^2} > \frac{\phi(\alpha - \bar{c})^2 (5\phi - 4)^2 (5\phi - 3\theta^2)^2 (5\phi + 3\theta^2)^2 \cdot (25\phi - 4)}{(3125\phi^4 - 1125\phi^3\theta^2 - 3625\phi^3 + 825\phi^2\theta^2 + 800\phi^2 + 180\phi\theta^2 - 72\theta^2)^2}$$

- Adding an edge between nodes  $H_1, H_2$ : E1B [3H1L]  $\rightarrow$  E2C [3H1L]

– For  $H_1$  not to be profitable:

$$\frac{\phi(\alpha - \bar{c})^2 \cdot (25\phi - 16) (5\phi - 3\theta^2 - 3)^2}{(125\phi^2 - 45\phi\theta^2 - 105\phi + 12\theta^2 + 12)^2} > \frac{\phi(\alpha - \bar{c})^2 \cdot (25\phi - 9) (5\phi - 3\theta^2 - 3)^2}{(125\phi^2 - 45\phi\theta^2 - 135\phi + 18\theta^2 + 18)^2}$$

– For  $H_2$  not to be profitable:

$$\frac{\phi(\alpha - \bar{c})^2 \cdot (25\phi - 16) (5\phi - 3\theta^2 - 3)^2}{(125\phi^2 - 45\phi\theta^2 - 105\phi + 12\theta^2 + 12)^2} > \frac{\phi(\alpha - \bar{c})^2 \cdot (25\phi - 9) (5\phi - 3\theta^2 - 3)^2}{(125\phi^2 - 45\phi\theta^2 - 135\phi + 18\theta^2 + 18)^2}$$

- Adding an edge between nodes  $H_1, L_1$ : E1B [3H1L]  $\rightarrow$  E2D [3H1L]

– For  $H_1$  not to be profitable:

$$\frac{\phi(\alpha - \bar{c})^2 \cdot (25\phi - 16) (5\phi - 3\theta^2 - 3)^2}{(125\phi^2 - 45\phi\theta^2 - 105\phi + 12\theta^2 + 12)^2} > \frac{25\phi^3 (\alpha - \bar{c})^2 (5\phi - 4)^2 \cdot (25\phi - 9)}{(625\phi^3 - 100\phi^2\theta^2 - 475\phi^2 - 20\phi\theta^2 + 24\theta^2)^2}$$

– For  $L_1$  not to be profitable:

$$\frac{\phi(\alpha - \bar{c})^2 (5\phi - 4)^2 \cdot (25\phi - 9\theta^2)}{(125\phi^2 - 45\phi\theta^2 - 105\phi + 12\theta^2 + 12)^2} > \frac{\phi(\alpha - \bar{c})^2 (5\phi - 4)^2 (5\phi + 3)^2 \cdot (25\phi - 4\theta^2)}{(625\phi^3 - 100\phi^2\theta^2 - 475\phi^2 - 20\phi\theta^2 + 24\theta^2)^2}$$

- Removing an edge between nodes  $H_3, L_1$ : E1B [3H1L]  $\rightarrow$  E0A [3H1L]

– For  $H_3$  not to be profitable:

$$\frac{\phi(\alpha - \bar{c})^2 (5\phi - 4)^2 \cdot (25\phi - 9)}{(125\phi^2 - 45\phi\theta^2 - 105\phi + 12\theta^2 + 12)^2} \geq \frac{\phi(\alpha - \bar{c})^2 (5\phi - 4\theta^2)^2 \cdot (25\phi - 16)}{(125\phi^2 - 80\phi\theta^2 - 40\phi + 16\theta^2)^2}$$

– For  $L_1$  not to be profitable:

$$\frac{\phi(\alpha - \bar{c})^2 (5\phi - 4)^2 \cdot (25\phi - 9\theta^2)}{(125\phi^2 - 45\phi\theta^2 - 105\phi + 12\theta^2 + 12)^2} \geq \frac{\phi(\alpha - \bar{c})^2 (5\phi - 4)^2 \cdot (25\phi - 16\theta^2)}{(125\phi^2 - 80\phi\theta^2 - 40\phi + 16\theta^2)^2}$$

### C.3.4 Stability conditions for: E2A [3H1L]

- Adding an edge between nodes  $H_3, L_1$ : E2A [3H1L]  $\rightarrow$  E3A [3H1L]

– For  $H_3$  not to be profitable:

$$\frac{\phi(\alpha - \bar{c})^2 (5\phi + 3)^2 (5\phi - 4\theta^2)^2 \cdot (25\phi - 4)}{(5\phi + 1)^2 (125\phi^2 - 80\phi\theta^2 - 60\phi + 24\theta^2)^2} > \frac{\phi(\alpha - \bar{c})^2 \cdot (25\phi - 1) (25\phi^2 + 15\phi - 18\theta^2)^2}{(625\phi^3 - 225\phi^2\theta^2 - 100\phi^2 - 45\phi\theta^2 - 15\phi + 18\theta^2)^2}$$

– For  $L_1$  not to be profitable:

$$\frac{\phi(\alpha - \bar{c})^2 (5\phi - 6)^2 \cdot (25\phi - 16\theta^2)}{(125\phi^2 - 80\phi\theta^2 - 60\phi + 24\theta^2)^2} > \frac{25\phi^3 (\alpha - \bar{c})^2 (5\phi - 3)^2 \cdot (25\phi - 9\theta^2)}{(625\phi^3 - 225\phi^2\theta^2 - 100\phi^2 - 45\phi\theta^2 - 15\phi + 18\theta^2)^2}$$

- Adding an edge between nodes  $H_1, H_2$ : **E2A** [3H1L]  $\rightarrow$  **E3B** [3H1L]

– For  $H_1$  not to be profitable:

$$\frac{25\phi^3 (\alpha - \bar{c})^2 (5\phi - 4\theta^2)^2 \cdot (25\phi - 9)}{(5\phi + 1)^2 (125\phi^2 - 80\phi\theta^2 - 60\phi + 24\theta^2)^2} > \frac{\phi(\alpha - \bar{c})^2 (5\phi - 4\theta^2)^2 \cdot (25\phi - 4)}{(125\phi^2 - 80\phi\theta^2 - 60\phi + 24\theta^2)^2}$$

– For  $H_2$  not to be profitable:

$$\frac{25\phi^3 (\alpha - \bar{c})^2 (5\phi - 4\theta^2)^2 \cdot (25\phi - 9)}{(5\phi + 1)^2 (125\phi^2 - 80\phi\theta^2 - 60\phi + 24\theta^2)^2} > \frac{\phi(\alpha - \bar{c})^2 (5\phi - 4\theta^2)^2 \cdot (25\phi - 4)}{(125\phi^2 - 80\phi\theta^2 - 60\phi + 24\theta^2)^2}$$

- Adding an edge between nodes  $H_1, L_1$ : **E2A** [3H1L]  $\rightarrow$  **E3C** [3H1L]

– For  $H_1$  not to be profitable:

$$\frac{25\phi^3 (\alpha - \bar{c})^2 (5\phi - 4\theta^2)^2 \cdot (25\phi - 9)}{(5\phi + 1)^2 (125\phi^2 - 80\phi\theta^2 - 60\phi + 24\theta^2)^2} > \frac{25\phi^3 (\alpha - \bar{c})^2 \cdot (25\phi - 4) (25\phi^2 - 15\phi - 6\theta^2)^2}{(3125\phi^4 - 1125\phi^3\theta^2 - 2125\phi^3 + 225\phi^2\theta^2 + 120\phi\theta^2 + 120\phi - 36\theta^2)^2}$$

– For  $L_1$  not to be profitable:

$$\frac{\phi(\alpha - \bar{c})^2 (5\phi - 6)^2 \cdot (25\phi - 16\theta^2)}{(125\phi^2 - 80\phi\theta^2 - 60\phi + 24\theta^2)^2} > \frac{\phi(\alpha - \bar{c})^2 (5\phi - 2)^2 \cdot (25\phi - 9\theta^2) (25\phi^2 - 15\phi - 6\theta^2)^2}{(3125\phi^4 - 1125\phi^3\theta^2 - 2125\phi^3 + 225\phi^2\theta^2 + 120\phi\theta^2 + 120\phi - 36\theta^2)^2}$$

- Removing an edge between nodes  $H_1, H_3$ : **E2A** [3H1L]  $\rightarrow$  **E1A** [3H1L]

– For  $H_1$  not to be profitable:

$$\frac{25\phi^3 (\alpha - \bar{c})^2 (5\phi - 4\theta^2)^2 \cdot (25\phi - 9)}{(5\phi + 1)^2 (125\phi^2 - 80\phi\theta^2 - 60\phi + 24\theta^2)^2} \geq \frac{\phi(\alpha - \bar{c})^2 (5\phi - 6)^2 (5\phi - 4\theta^2)^2 \cdot (25\phi - 16)}{(625\phi^3 - 400\phi^2\theta^2 - 850\phi^2 + 480\phi\theta^2 + 240\phi - 96\theta^2)^2}$$

– For  $H_3$  not to be profitable:

$$\frac{\phi(\alpha - \bar{c})^2 (5\phi + 3)^2 (5\phi - 4\theta^2)^2 \cdot (25\phi - 4)}{(5\phi + 1)^2 (125\phi^2 - 80\phi\theta^2 - 60\phi + 24\theta^2)^2} \geq \frac{\phi(\alpha - \bar{c})^2 (5\phi - 4)^2 (5\phi - 4\theta^2)^2 \cdot (25\phi - 9)}{(625\phi^3 - 400\phi^2\theta^2 - 850\phi^2 + 480\phi\theta^2 + 240\phi - 96\theta^2)^2}$$

### C.3.5 Stability conditions for: **E2B** [3H1L]

- Adding an edge between nodes  $H_1, H_3$ : **E2B** [3H1L]  $\rightarrow$  **E3A** [3H1L]

– For  $H_1$  not to be profitable:

$$\frac{\phi(\alpha - \bar{c})^2 \cdot (25\phi - 16) (125\phi^3 - 75\phi^2\theta^2 - 125\phi^2 + 45\phi\theta^2 + 18\theta^2)^2}{(3125\phi^4 - 1125\phi^3\theta^2 - 3625\phi^3 + 825\phi^2\theta^2 + 800\phi^2 + 180\phi\theta^2 - 72\theta^2)^2} > \frac{25\phi^3 (\alpha - \bar{c})^2 (5\phi - 3\theta^2)^2 \cdot (25\phi - 9)}{(625\phi^3 - 225\phi^2\theta^2 - 100\phi^2 - 45\phi\theta^2 - 15\phi + 18\theta^2)^2}$$

– For  $H_3$  not to be profitable:

$$\frac{\phi(\alpha - \bar{c})^2 (5\phi - 4)^2 (5\phi - 3\theta^2)^2 (5\phi + 3\theta^2)^2 \cdot (25\phi - 4)}{(3125\phi^4 - 1125\phi^3\theta^2 - 3625\phi^3 + 825\phi^2\theta^2 + 800\phi^2 + 180\phi\theta^2 - 72\theta^2)^2} > \frac{\phi(\alpha - \bar{c})^2 \cdot (25\phi - 1) (25\phi^2 + 15\phi - 18\theta^2)^2}{(625\phi^3 - 225\phi^2\theta^2 - 100\phi^2 - 45\phi\theta^2 - 15\phi + 18\theta^2)^2}$$

- Adding an edge between nodes  $H_1, H_2$ : **E2B** [3H1L]  $\rightarrow$  **E3C** [3H1L]

– For  $H_1$  not to be profitable:

$$\frac{\phi(\alpha - \bar{c})^2 \cdot (25\phi - 16) (125\phi^3 - 75\phi^2\theta^2 - 125\phi^2 + 45\phi\theta^2 + 18\theta^2)^2}{(3125\phi^4 - 1125\phi^3\theta^2 - 3625\phi^3 + 825\phi^2\theta^2 + 800\phi^2 + 180\phi\theta^2 - 72\theta^2)^2} > \frac{\phi(\alpha - \bar{c})^2 (5\phi - 2)^2 \cdot (25\phi - 9) (25\phi^2 - 15\phi\theta^2 - 6\theta^2)^2}{(3125\phi^4 - 1125\phi^3\theta^2 - 2125\phi^3 + 225\phi^2\theta^2 + 120\phi\theta^2 + 120\phi - 36\theta^2)^2}$$

– For  $H_2$  not to be profitable:

$$\frac{25\phi^3 (\alpha - \bar{c})^2 (5\phi - 4)^2 (5\phi - 3\theta^2)^2 \cdot (25\phi - 9)}{(3125\phi^4 - 1125\phi^3\theta^2 - 3625\phi^3 + 825\phi^2\theta^2 + 800\phi^2 + 180\phi\theta^2 - 72\theta^2)^2} > \frac{25\phi^3 (\alpha - \bar{c})^2 \cdot (25\phi - 4) (25\phi^2 - 15\phi\theta^2 - 6\theta^2)^2}{(3125\phi^4 - 1125\phi^3\theta^2 - 2125\phi^3 + 225\phi^2\theta^2 + 120\phi\theta^2 + 120\phi - 36\theta^2)^2}$$

- Adding an edge between nodes  $H_2, L_1$ : **E2B** [3H1L]  $\rightarrow$  **E3D** [3H1L]

- For  $H_2$  not to be profitable:

$$\frac{25\phi^3 (\alpha - \bar{c})^2 (5\phi - 4)^2 (5\phi - 3\theta^2)^2 \cdot (25\phi - 9)}{(3125\phi^4 - 1125\phi^3\theta^2 - 3625\phi^3 + 825\phi^2\theta^2 + 800\phi^2 + 180\phi\theta^2 - 72\theta^2)^2} > \frac{\phi (\alpha - \bar{c})^2 (5\phi - 4)^2 \cdot (25\phi - 4)}{(125\phi^2 - 20\phi\theta^2 - 120\phi + 8\theta^2 + 16)^2}$$

- For  $L_1$  not to be profitable:

$$\frac{25\phi^3 (\alpha - \bar{c})^2 (5\phi - 4)^2 (5\phi - 3)^2 \cdot (25\phi - 9\theta^2)}{(3125\phi^4 - 1125\phi^3\theta^2 - 3625\phi^3 + 825\phi^2\theta^2 + 800\phi^2 + 180\phi\theta^2 - 72\theta^2)^2} > \frac{\phi (\alpha - \bar{c})^2 (5\phi - 4)^2 \cdot (25\phi - 4\theta^2)}{(125\phi^2 - 20\phi\theta^2 - 120\phi + 8\theta^2 + 16)^2}$$

- Adding an edge between nodes  $H_1$ ,  $L_1$ : **E2B [3H1L]** → **E3E [3H1L]**

- For  $H_1$  not to be profitable:

$$\frac{\phi (\alpha - \bar{c})^2 \cdot (25\phi - 16) (125\phi^3 - 75\phi^2\theta^2 - 125\phi^2 + 45\phi\theta^2 + 18\theta^2)^2}{(3125\phi^4 - 1125\phi^3\theta^2 - 3625\phi^3 + 825\phi^2\theta^2 + 800\phi^2 + 180\phi\theta^2 - 72\theta^2)^2} > \frac{\phi (\alpha - \bar{c})^2 \cdot (25\phi - 9) (125\phi^3 - 125\phi^2 + 12\theta^2)^2}{(3125\phi^4 - 500\phi^3\theta^2 - 2750\phi^3 + 225\phi^2 + 240\phi\theta^2 - 36\theta^2)^2}$$

- For  $L_1$  not to be profitable:

$$\frac{25\phi^3 (\alpha - \bar{c})^2 (5\phi - 4)^2 (5\phi - 3)^2 \cdot (25\phi - 9\theta^2)}{(3125\phi^4 - 1125\phi^3\theta^2 - 3625\phi^3 + 825\phi^2\theta^2 + 800\phi^2 + 180\phi\theta^2 - 72\theta^2)^2} > \frac{25\phi^3 (\alpha - \bar{c})^2 \cdot (25\phi - 4\theta^2) (25\phi^2 - 15\phi - 6)^2}{(3125\phi^4 - 500\phi^3\theta^2 - 2750\phi^3 + 225\phi^2 + 240\phi\theta^2 - 36\theta^2)^2}$$

- Removing an edge between nodes  $H_3$ ,  $L_1$ : **E2B [3H1L]** → **E1A [3H1L]**

- For  $H_3$  not to be profitable:

$$\frac{\phi (\alpha - \bar{c})^2 (5\phi - 4)^2 (5\phi - 3\theta^2)^2 (5\phi + 3\theta^2)^2 \cdot (25\phi - 4)}{(3125\phi^4 - 1125\phi^3\theta^2 - 3625\phi^3 + 825\phi^2\theta^2 + 800\phi^2 + 180\phi\theta^2 - 72\theta^2)^2} \geq \frac{\phi (\alpha - \bar{c})^2 (5\phi - 4)^2 (5\phi - 4\theta^2)^2 \cdot (25\phi - 9)}{(625\phi^3 - 400\phi^2\theta^2 - 850\phi^2 + 480\phi\theta^2 + 240\phi - 96\theta^2)^2}$$

- For  $L_1$  not to be profitable:

$$\frac{25\phi^3 (\alpha - \bar{c})^2 (5\phi - 4)^2 (5\phi - 3)^2 \cdot (25\phi - 9\theta^2)}{(3125\phi^4 - 1125\phi^3\theta^2 - 3625\phi^3 + 825\phi^2\theta^2 + 800\phi^2 + 180\phi\theta^2 - 72\theta^2)^2} \geq \frac{\phi (\alpha - \bar{c})^2 (5\phi - 6)^2 (5\phi - 4)^2 \cdot (25\phi - 16\theta^2)}{(625\phi^3 - 400\phi^2\theta^2 - 850\phi^2 + 480\phi\theta^2 + 240\phi - 96\theta^2)^2}$$

- Removing an edge between nodes  $H_2$ ,  $H_3$ : **E2B [3H1L]** → **E1B [3H1L]**

- For  $H_2$  not to be profitable:

$$\frac{25\phi^3 (\alpha - \bar{c})^2 (5\phi - 4)^2 (5\phi - 3\theta^2)^2 \cdot (25\phi - 9)}{(3125\phi^4 - 1125\phi^3\theta^2 - 3625\phi^3 + 825\phi^2\theta^2 + 800\phi^2 + 180\phi\theta^2 - 72\theta^2)^2} \geq \frac{\phi (\alpha - \bar{c})^2 \cdot (25\phi - 16) (5\phi - 3\theta^2 - 3)^2}{(125\phi^2 - 45\phi\theta^2 - 105\phi + 12\theta^2 + 12)^2}$$

- For  $H_3$  not to be profitable:

$$\frac{\phi (\alpha - \bar{c})^2 (5\phi - 4)^2 (5\phi - 3\theta^2)^2 (5\phi + 3\theta^2)^2 \cdot (25\phi - 4)}{(3125\phi^4 - 1125\phi^3\theta^2 - 3625\phi^3 + 825\phi^2\theta^2 + 800\phi^2 + 180\phi\theta^2 - 72\theta^2)^2} \geq \frac{\phi (\alpha - \bar{c})^2 (5\phi - 4)^2 \cdot (25\phi - 9)}{(125\phi^2 - 45\phi\theta^2 - 105\phi + 12\theta^2 + 12)^2}$$

### C.3.6 Stability conditions for: **E2C [3H1L]**

- Adding an edge between nodes  $H_1$ ,  $H_2$ : **E2C [3H1L]** → **E3C [3H1L]**

- For  $H_1$  not to be profitable:

$$\frac{\phi (\alpha - \bar{c})^2 (5\phi - 6)^2 \cdot (25\phi - 9)}{(125\phi^2 - 45\phi\theta^2 - 135\phi + 18\theta^2 + 18)^2} > \frac{25\phi^3 (\alpha - \bar{c})^2 \cdot (25\phi - 4) (25\phi^2 - 15\phi - 6\theta^2)^2}{(3125\phi^4 - 1125\phi^3\theta^2 - 2125\phi^3 + 225\phi^2\theta^2 + 120\phi\theta^2 + 120\phi - 36\theta^2)^2}$$

- For  $H_2$  not to be profitable:

$$\frac{\phi (\alpha - \bar{c})^2 \cdot (25\phi - 9) (5\phi - 3\theta^2 - 3)^2}{(125\phi^2 - 45\phi\theta^2 - 135\phi + 18\theta^2 + 18)^2} > \frac{25\phi^3 (\alpha - \bar{c})^2 \cdot (25\phi - 4) (25\phi^2 - 15\phi\theta^2 - 6)^2}{(3125\phi^4 - 1125\phi^3\theta^2 - 2125\phi^3 + 225\phi^2\theta^2 + 120\phi\theta^2 + 120\phi - 36\theta^2)^2}$$

- Adding an edge between nodes  $H_2$ ,  $L_1$ : **E2C [3H1L]** → **E3E [3H1L]**

- For  $H_2$  not to be profitable:



$$\frac{\phi(\alpha - \bar{c})^2 \cdot (25\phi - 9)(5\phi - 3\theta^2 - 3)^2}{(125\phi^2 - 45\phi\theta^2 - 135\phi + 18\theta^2 + 18)^2} > \frac{25\phi^3(\alpha - \bar{c})^2 \cdot (25\phi - 4)(25\phi^2 - 15\phi - 6\theta^2)^2}{(3125\phi^4 - 500\phi^3\theta^2 - 2750\phi^3 + 225\phi^2 + 240\phi\theta^2 - 36\theta^2)^2}$$

– For  $L_1$  not to be profitable:

$$\frac{\phi(\alpha - \bar{c})^2(5\phi - 6)^2 \cdot (25\phi - 9\theta^2)}{(125\phi^2 - 45\phi\theta^2 - 135\phi + 18\theta^2 + 18)^2} > \frac{25\phi^3(\alpha - \bar{c})^2 \cdot (25\phi - 4\theta^2)(25\phi^2 - 15\phi - 6)^2}{(3125\phi^4 - 500\phi^3\theta^2 - 2750\phi^3 + 225\phi^2 + 240\phi\theta^2 - 36\theta^2)^2}$$

- Removing an edge between nodes  $H_1, L_1$ : **E2C** [3H1L]  $\rightarrow$  **E1A** [3H1L]

– For  $H_1$  not to be profitable:

$$\frac{\phi(\alpha - \bar{c})^2(5\phi - 6)^2 \cdot (25\phi - 9)}{(125\phi^2 - 45\phi\theta^2 - 135\phi + 18\theta^2 + 18)^2} \geq \frac{\phi(\alpha - \bar{c})^2(5\phi - 6)^2(5\phi - 4\theta^2)^2 \cdot (25\phi - 16)}{(625\phi^3 - 400\phi^2\theta^2 - 850\phi^2 + 480\phi\theta^2 + 240\phi - 96\theta^2)^2}$$

– For  $L_1$  not to be profitable:

$$\frac{\phi(\alpha - \bar{c})^2(5\phi - 6)^2 \cdot (25\phi - 9\theta^2)}{(125\phi^2 - 45\phi\theta^2 - 135\phi + 18\theta^2 + 18)^2} \geq \frac{\phi(\alpha - \bar{c})^2(5\phi - 6)^2(5\phi - 4)^2 \cdot (25\phi - 16\theta^2)}{(625\phi^3 - 400\phi^2\theta^2 - 850\phi^2 + 480\phi\theta^2 + 240\phi - 96\theta^2)^2}$$

- Removing an edge between nodes  $H_2, H_3$ : **E2C** [3H1L]  $\rightarrow$  **E1B** [3H1L]

– For  $H_2$  not to be profitable:

$$\frac{\phi(\alpha - \bar{c})^2 \cdot (25\phi - 9)(5\phi - 3\theta^2 - 3)^2}{(125\phi^2 - 45\phi\theta^2 - 135\phi + 18\theta^2 + 18)^2} \geq \frac{\phi(\alpha - \bar{c})^2 \cdot (25\phi - 16)(5\phi - 3\theta^2 - 3)^2}{(125\phi^2 - 45\phi\theta^2 - 105\phi + 12\theta^2 + 12)^2}$$

– For  $H_3$  not to be profitable:

$$\frac{\phi(\alpha - \bar{c})^2 \cdot (25\phi - 9)(5\phi - 3\theta^2 - 3)^2}{(125\phi^2 - 45\phi\theta^2 - 135\phi + 18\theta^2 + 18)^2} \geq \frac{\phi(\alpha - \bar{c})^2 \cdot (25\phi - 16)(5\phi - 3\theta^2 - 3)^2}{(125\phi^2 - 45\phi\theta^2 - 105\phi + 12\theta^2 + 12)^2}$$

### C.3.7 Stability conditions for: **E2D** [3H1L]

- Adding an edge between nodes  $H_2, H_3$ : **E2D** [3H1L]  $\rightarrow$  **E3D** [3H1L]

– For  $H_2$  not to be profitable:

$$\frac{25\phi^3(\alpha - \bar{c})^2(5\phi - 4)^2 \cdot (25\phi - 9)}{(625\phi^3 - 100\phi^2\theta^2 - 475\phi^2 - 20\phi\theta^2 + 24\theta^2)^2} > \frac{\phi(\alpha - \bar{c})^2(5\phi - 4)^2 \cdot (25\phi - 4)}{(125\phi^2 - 20\phi\theta^2 - 120\phi + 8\theta^2 + 16)^2}$$

– For  $H_3$  not to be profitable:

$$\frac{25\phi^3(\alpha - \bar{c})^2(5\phi - 4)^2 \cdot (25\phi - 9)}{(625\phi^3 - 100\phi^2\theta^2 - 475\phi^2 - 20\phi\theta^2 + 24\theta^2)^2} > \frac{\phi(\alpha - \bar{c})^2(5\phi - 4)^2 \cdot (25\phi - 4)}{(125\phi^2 - 20\phi\theta^2 - 120\phi + 8\theta^2 + 16)^2}$$

- Adding an edge between nodes  $H_1, H_2$ : **E2D** [3H1L]  $\rightarrow$  **E3E** [3H1L]

– For  $H_1$  not to be profitable:

$$\frac{\phi(\alpha - \bar{c})^2 \cdot (25\phi - 16)(25\phi^2 - 10\phi\theta^2 - 15\phi - 6\theta^2)^2}{(625\phi^3 - 100\phi^2\theta^2 - 475\phi^2 - 20\phi\theta^2 + 24\theta^2)^2} > \frac{\phi(\alpha - \bar{c})^2 \cdot (25\phi - 9)(125\phi^3 - 50\phi^2\theta^2 - 75\phi^2 + 12\theta^2)^2}{(3125\phi^4 - 500\phi^3\theta^2 - 2750\phi^3 + 225\phi^2 + 240\phi\theta^2 - 36\theta^2)^2}$$

– For  $H_2$  not to be profitable:

$$\frac{25\phi^3(\alpha - \bar{c})^2(5\phi - 4)^2 \cdot (25\phi - 9)}{(625\phi^3 - 100\phi^2\theta^2 - 475\phi^2 - 20\phi\theta^2 + 24\theta^2)^2} > \frac{25\phi^3(\alpha - \bar{c})^2 \cdot (25\phi - 4)(25\phi^2 - 15\phi - 6\theta^2)^2}{(3125\phi^4 - 500\phi^3\theta^2 - 2750\phi^3 + 225\phi^2 + 240\phi\theta^2 - 36\theta^2)^2}$$

- Adding an edge between nodes  $H_1, L_1$ : **E2D** [3H1L]  $\rightarrow$  **E3F** [3H1L]

– For  $H_1$  not to be profitable:

$$\frac{\phi(\alpha - \bar{c})^2 \cdot (25\phi - 16)(25\phi^2 - 10\phi\theta^2 - 15\phi - 6\theta^2)^2}{(625\phi^3 - 100\phi^2\theta^2 - 475\phi^2 - 20\phi\theta^2 + 24\theta^2)^2} > \frac{25\phi^3(\alpha - \bar{c})^2 \cdot (25\phi - 9)}{(125\phi^2 - 5\phi\theta^2 + 15\phi - 6\theta^2)^2}$$

– For  $L_1$  not to be profitable:

$$\frac{\phi(\alpha - \bar{c})^2(5\phi - 4)^2(5\phi + 3)^2 \cdot (25\phi - 4\theta^2)}{(625\phi^3 - 100\phi^2\theta^2 - 475\phi^2 - 20\phi\theta^2 + 24\theta^2)^2} > \frac{\phi(\alpha - \bar{c})^2(5\phi + 6)^2 \cdot (25\phi - \theta^2)}{(125\phi^2 - 5\phi\theta^2 + 15\phi - 6\theta^2)^2}$$

- Removing an edge between nodes  $H_2, L_1$ : **E2D** [3H1L]  $\rightarrow$  **E1B** [3H1L]

– For  $H_2$  not to be profitable:

$$\frac{25\phi^3 (\alpha - \bar{c})^2 (5\phi - 4)^2 \cdot (25\phi - 9)}{(625\phi^3 - 100\phi^2\theta^2 - 475\phi^2 - 20\phi\theta^2 + 24\theta^2)^2} \geq \frac{\phi (\alpha - \bar{c})^2 \cdot (25\phi - 16) (5\phi - 3\theta^2 - 3)^2}{(125\phi^2 - 45\phi\theta^2 - 105\phi + 12\theta^2 + 12)^2}$$

– For  $L_1$  not to be profitable:

$$\frac{\phi (\alpha - \bar{c})^2 (5\phi - 4)^2 (5\phi + 3)^2 \cdot (25\phi - 4\theta^2)}{(625\phi^3 - 100\phi^2\theta^2 - 475\phi^2 - 20\phi\theta^2 + 24\theta^2)^2} \geq \frac{\phi (\alpha - \bar{c})^2 (5\phi - 4)^2 \cdot (25\phi - 9\theta^2)}{(125\phi^2 - 45\phi\theta^2 - 105\phi + 12\theta^2 + 12)^2}$$

### C.3.8 Stability conditions for: **E3A** [3H1L]

- Adding an edge between nodes  $H_1, H_2$ : **E3A** [3H1L]  $\rightarrow$  **E4A** [3H1L]

– For  $H_1$  not to be profitable:

$$\frac{25\phi^3 (\alpha - \bar{c})^2 (5\phi - 3\theta^2)^2 \cdot (25\phi - 9)}{(625\phi^3 - 225\phi^2\theta^2 - 100\phi^2 - 45\phi\theta^2 - 15\phi + 18\theta^2)^2} > \frac{25\phi^3 (\alpha - \bar{c})^2 (5\phi - 3\theta^2)^2 \cdot (25\phi - 4)}{(625\phi^3 - 225\phi^2\theta^2 - 225\phi^2 + 12\theta^2)^2}$$

– For  $H_2$  not to be profitable:

$$\frac{25\phi^3 (\alpha - \bar{c})^2 (5\phi - 3\theta^2)^2 \cdot (25\phi - 9)}{(625\phi^3 - 225\phi^2\theta^2 - 100\phi^2 - 45\phi\theta^2 - 15\phi + 18\theta^2)^2} > \frac{25\phi^3 (\alpha - \bar{c})^2 (5\phi - 3\theta^2)^2 \cdot (25\phi - 4)}{(625\phi^3 - 225\phi^2\theta^2 - 225\phi^2 + 12\theta^2)^2}$$

- Adding an edge between nodes  $H_1, L_1$ : **E3A** [3H1L]  $\rightarrow$  **E4B** [3H1L]

– For  $H_1$  not to be profitable:

$$\frac{25\phi^3 (\alpha - \bar{c})^2 (5\phi - 3\theta^2)^2 \cdot (25\phi - 9)}{(625\phi^3 - 225\phi^2\theta^2 - 100\phi^2 - 45\phi\theta^2 - 15\phi + 18\theta^2)^2} > \frac{25\phi^3 (\alpha - \bar{c})^2 (5\phi - 3)^2 \cdot (25\phi - 4)}{(625\phi^3 - 100\phi^2\theta^2 - 350\phi^2 + 6\theta^2 + 6)^2}$$

– For  $L_1$  not to be profitable:

$$\frac{25\phi^3 (\alpha - \bar{c})^2 (5\phi - 3)^2 \cdot (25\phi - 9\theta^2)}{(625\phi^3 - 225\phi^2\theta^2 - 100\phi^2 - 45\phi\theta^2 - 15\phi + 18\theta^2)^2} > \frac{25\phi^3 (\alpha - \bar{c})^2 (5\phi - 3)^2 \cdot (25\phi - 4\theta^2)}{(625\phi^3 - 100\phi^2\theta^2 - 350\phi^2 + 6\theta^2 + 6)^2}$$

- Removing an edge between nodes  $H_3, L_1$ : **E3A** [3H1L]  $\rightarrow$  **E2A** [3H1L]

– For  $H_3$  not to be profitable:

$$\frac{\phi (\alpha - \bar{c})^2 \cdot (25\phi - 1) (25\phi^2 + 15\phi - 18\theta^2)^2}{(625\phi^3 - 225\phi^2\theta^2 - 100\phi^2 - 45\phi\theta^2 - 15\phi + 18\theta^2)^2} \geq \frac{\phi (\alpha - \bar{c})^2 (5\phi + 3)^2 (5\phi - 4\theta^2)^2 \cdot (25\phi - 4)}{(5\phi + 1)^2 (125\phi^2 - 80\phi\theta^2 - 60\phi + 24\theta^2)^2}$$

– For  $L_1$  not to be profitable:

$$\frac{25\phi^3 (\alpha - \bar{c})^2 (5\phi - 3)^2 \cdot (25\phi - 9\theta^2)}{(625\phi^3 - 225\phi^2\theta^2 - 100\phi^2 - 45\phi\theta^2 - 15\phi + 18\theta^2)^2} \geq \frac{\phi (\alpha - \bar{c})^2 (5\phi - 6)^2 \cdot (25\phi - 16\theta^2)}{(125\phi^2 - 80\phi\theta^2 - 60\phi + 24\theta^2)^2}$$

- Removing an edge between nodes  $H_1, H_3$ : **E3A** [3H1L]  $\rightarrow$  **E2B** [3H1L]

– For  $H_1$  not to be profitable:

$$\frac{25\phi^3 (\alpha - \bar{c})^2 (5\phi - 3\theta^2)^2 \cdot (25\phi - 9)}{(625\phi^3 - 225\phi^2\theta^2 - 100\phi^2 - 45\phi\theta^2 - 15\phi + 18\theta^2)^2} \geq \frac{\phi (\alpha - \bar{c})^2 \cdot (25\phi - 16) (125\phi^3 - 75\phi^2\theta^2 - 125\phi^2 + 45\phi\theta^2 + 18\theta^2)^2}{(3125\phi^4 - 1125\phi^3\theta^2 - 3625\phi^3 + 825\phi^2\theta^2 + 800\phi^2 + 180\phi\theta^2 - 72\theta^2)^2}$$

– For  $H_3$  not to be profitable:

$$\frac{\phi (\alpha - \bar{c})^2 \cdot (25\phi - 1) (25\phi^2 + 15\phi - 18\theta^2)^2}{(625\phi^3 - 225\phi^2\theta^2 - 100\phi^2 - 45\phi\theta^2 - 15\phi + 18\theta^2)^2} \geq \frac{\phi (\alpha - \bar{c})^2 (5\phi - 4)^2 (5\phi - 3\theta^2)^2 (5\phi + 3\theta^2)^2 \cdot (25\phi - 4)}{(3125\phi^4 - 1125\phi^3\theta^2 - 3625\phi^3 + 825\phi^2\theta^2 + 800\phi^2 + 180\phi\theta^2 - 72\theta^2)^2}$$

### C.3.9 Stability conditions for: **E3B** [3H1L]

- Adding an edge between nodes  $H_1, L_1$ : **E3B** [3H1L]  $\rightarrow$  **E4A** [3H1L]

– For  $H_1$  not to be profitable:

$$\frac{\phi(\alpha - \bar{c})^2 (5\phi - 4\theta^2)^2 \cdot (25\phi - 4)}{(125\phi^2 - 80\phi\theta^2 - 60\phi + 24\theta^2)^2} > \frac{\phi(\alpha - \bar{c})^2 \cdot (25\phi - 1) (25\phi^2 - 12\theta^2)^2}{(625\phi^3 - 225\phi^2\theta^2 - 225\phi^2 + 12\theta^2)^2}$$

– For  $L_1$  not to be profitable:

$$\frac{\phi(\alpha - \bar{c})^2 (5\phi - 6)^2 \cdot (25\phi - 16\theta^2)}{(125\phi^2 - 80\phi\theta^2 - 60\phi + 24\theta^2)^2} > \frac{25\phi^3 (\alpha - \bar{c})^2 (5\phi - 4)^2 \cdot (25\phi - 9\theta^2)}{(625\phi^3 - 225\phi^2\theta^2 - 225\phi^2 + 12\theta^2)^2}$$

- Removing an edge between nodes  $H_1, H_2$ : E3B [3H1L]  $\rightarrow$  E2A [3H1L]

– For  $H_1$  not to be profitable:

$$\frac{\phi(\alpha - \bar{c})^2 (5\phi - 4\theta^2)^2 \cdot (25\phi - 4)}{(125\phi^2 - 80\phi\theta^2 - 60\phi + 24\theta^2)^2} \geq \frac{25\phi^3 (\alpha - \bar{c})^2 (5\phi - 4\theta^2)^2 \cdot (25\phi - 9)}{(5\phi + 1)^2 (125\phi^2 - 80\phi\theta^2 - 60\phi + 24\theta^2)^2}$$

– For  $H_2$  not to be profitable:

$$\frac{\phi(\alpha - \bar{c})^2 (5\phi - 4\theta^2)^2 \cdot (25\phi - 4)}{(125\phi^2 - 80\phi\theta^2 - 60\phi + 24\theta^2)^2} \geq \frac{25\phi^3 (\alpha - \bar{c})^2 (5\phi - 4\theta^2)^2 \cdot (25\phi - 9)}{(5\phi + 1)^2 (125\phi^2 - 80\phi\theta^2 - 60\phi + 24\theta^2)^2}$$

### C.3.10 Stability conditions for: E3C [3H1L]

- Adding an edge between nodes  $H_1, H_2$ : E3C [3H1L]  $\rightarrow$  E4A [3H1L]

– For  $H_1$  not to be profitable:

$$\frac{\phi(\alpha - \bar{c})^2 (5\phi - 2)^2 \cdot (25\phi - 9) (25\phi^2 - 15\phi\theta^2 - 6\theta^2)^2}{(3125\phi^4 - 1125\phi^3\theta^2 - 2125\phi^3 + 225\phi^2\theta^2 + 120\phi\theta^2 + 120\phi - 36\theta^2)^2} > \frac{25\phi^3 (\alpha - \bar{c})^2 (5\phi - 3\theta^2)^2 \cdot (25\phi - 4)}{(625\phi^3 - 225\phi^2\theta^2 - 225\phi^2 + 12\theta^2)^2}$$

– For  $H_2$  not to be profitable:

$$\frac{25\phi^3 (\alpha - \bar{c})^2 \cdot (25\phi - 4) (25\phi^2 - 15\phi - 6\theta^2)^2}{(3125\phi^4 - 1125\phi^3\theta^2 - 2125\phi^3 + 225\phi^2\theta^2 + 120\phi\theta^2 + 120\phi - 36\theta^2)^2} > \frac{\phi(\alpha - \bar{c})^2 \cdot (25\phi - 1) (25\phi^2 - 12\theta^2)^2}{(625\phi^3 - 225\phi^2\theta^2 - 225\phi^2 + 12\theta^2)^2}$$

- Adding an edge between nodes  $H_3, L_1$ : E3C [3H1L]  $\rightarrow$  E4B [3H1L]

– For  $H_3$  not to be profitable:

$$\frac{25\phi^3 (\alpha - \bar{c})^2 \cdot (25\phi - 4) (25\phi^2 - 15\phi\theta^2 - 6)^2}{(3125\phi^4 - 1125\phi^3\theta^2 - 2125\phi^3 + 225\phi^2\theta^2 + 120\phi\theta^2 + 120\phi - 36\theta^2)^2} > \frac{\phi(\alpha - \bar{c})^2 \cdot (25\phi - 1) (25\phi^2 - 6\theta^2 - 6)^2}{(625\phi^3 - 100\phi^2\theta^2 - 350\phi^2 + 6\theta^2 + 6)^2}$$

– For  $L_1$  not to be profitable:

$$\frac{\phi(\alpha - \bar{c})^2 (5\phi - 2)^2 \cdot (25\phi - 9\theta^2) (25\phi^2 - 15\phi - 6)^2}{(3125\phi^4 - 1125\phi^3\theta^2 - 2125\phi^3 + 225\phi^2\theta^2 + 120\phi\theta^2 + 120\phi - 36\theta^2)^2} > \frac{25\phi^3 (\alpha - \bar{c})^2 (5\phi - 3)^2 \cdot (25\phi - 4\theta^2)}{(625\phi^3 - 100\phi^2\theta^2 - 350\phi^2 + 6\theta^2 + 6)^2}$$

- Adding an edge between nodes  $H_1, L_1$ : E3C [3H1L]  $\rightarrow$  E4C [3H1L]

– For  $H_1$  not to be profitable:

$$\frac{\phi(\alpha - \bar{c})^2 (5\phi - 2)^2 \cdot (25\phi - 9) (25\phi^2 - 15\phi\theta^2 - 6\theta^2)^2}{(3125\phi^4 - 1125\phi^3\theta^2 - 2125\phi^3 + 225\phi^2\theta^2 + 120\phi\theta^2 + 120\phi - 36\theta^2)^2} > \frac{\phi(\alpha - \bar{c})^2 (5\phi - 2\theta)^2 (5\phi + 2\theta)^2 \cdot (25\phi - 4)}{(625\phi^3 - 100\phi^2\theta^2 - 50\phi^2 - 60\phi\theta^2 - 40\phi + 24\theta^2)^2}$$

– For  $L_1$  not to be profitable:

$$\frac{\phi(\alpha - \bar{c})^2 (5\phi - 2)^2 \cdot (25\phi - 9\theta^2) (25\phi^2 - 15\phi - 6)^2}{(3125\phi^4 - 1125\phi^3\theta^2 - 2125\phi^3 + 225\phi^2\theta^2 + 120\phi\theta^2 + 120\phi - 36\theta^2)^2} > \frac{\phi(\alpha - \bar{c})^2 (5\phi - 2)^2 (5\phi + 2)^2 \cdot (25\phi - 4\theta^2)}{(625\phi^3 - 100\phi^2\theta^2 - 50\phi^2 - 60\phi\theta^2 - 40\phi + 24\theta^2)^2}$$

- Removing an edge between nodes  $H_2, L_1$ : E3C [3H1L]  $\rightarrow$  E2A [3H1L]

– For  $H_2$  not to be profitable:

$$\frac{25\phi^3 (\alpha - \bar{c})^2 \cdot (25\phi - 4) (25\phi^2 - 15\phi - 6\theta^2)^2}{(3125\phi^4 - 1125\phi^3\theta^2 - 2125\phi^3 + 225\phi^2\theta^2 + 120\phi\theta^2 + 120\phi - 36\theta^2)^2} \geq \frac{25\phi^3 (\alpha - \bar{c})^2 (5\phi - 4\theta^2)^2 \cdot (25\phi - 9)}{(5\phi + 1)^2 (125\phi^2 - 80\phi\theta^2 - 60\phi + 24\theta^2)^2}$$

- For  $L_1$  not to be profitable:

$$\frac{\phi(\alpha - \bar{c})^2 (5\phi - 2)^2 \cdot (25\phi - 9\theta^2) (25\phi^2 - 15\phi - 6)^2}{(3125\phi^4 - 1125\phi^3\theta^2 - 2125\phi^3 + 225\phi^2\theta^2 + 120\phi\theta^2 + 120\phi - 36\theta^2)^2} \geq \frac{\phi(\alpha - \bar{c})^2 (5\phi - 6)^2 \cdot (25\phi - 16\theta^2)}{(125\phi^2 - 80\phi\theta^2 - 60\phi + 24\theta^2)^2}$$

- Removing an edge between nodes  $H_1, H_3$ : **E3C** [3H1L]  $\rightarrow$  **E2B** [3H1L]

- For  $H_1$  not to be profitable:

$$\frac{\phi(\alpha - \bar{c})^2 (5\phi - 2)^2 \cdot (25\phi - 9) (25\phi^2 - 15\phi\theta^2 - 6\theta^2)^2}{(3125\phi^4 - 1125\phi^3\theta^2 - 2125\phi^3 + 225\phi^2\theta^2 + 120\phi\theta^2 + 120\phi - 36\theta^2)^2} \geq \frac{\phi(\alpha - \bar{c})^2 \cdot (25\phi - 16) (125\phi^3 - 75\phi^2\theta^2 - 125\phi^2 + 45\phi\theta^2 + 18\theta^2)^2}{(3125\phi^4 - 1125\phi^3\theta^2 - 3625\phi^3 + 825\phi^2\theta^2 + 800\phi^2 + 180\phi\theta^2 - 72\theta^2)^2}$$

- For  $H_3$  not to be profitable:

$$\frac{25\phi^3 (\alpha - \bar{c})^2 \cdot (25\phi - 4) (25\phi^2 - 15\phi\theta^2 - 6)^2}{(3125\phi^4 - 1125\phi^3\theta^2 - 2125\phi^3 + 225\phi^2\theta^2 + 120\phi\theta^2 + 120\phi - 36\theta^2)^2} \geq \frac{25\phi^3 (\alpha - \bar{c})^2 (5\phi - 4)^2 (5\phi - 3\theta^2)^2 \cdot (25\phi - 9)}{(3125\phi^4 - 1125\phi^3\theta^2 - 3625\phi^3 + 825\phi^2\theta^2 + 800\phi^2 + 180\phi\theta^2 - 72\theta^2)^2}$$

- Removing an edge between nodes  $H_2, H_3$ : **E3C** [3H1L]  $\rightarrow$  **E2C** [3H1L]

- For  $H_2$  not to be profitable:

$$\frac{25\phi^3 (\alpha - \bar{c})^2 \cdot (25\phi - 4) (25\phi^2 - 15\phi - 6\theta^2)^2}{(3125\phi^4 - 1125\phi^3\theta^2 - 2125\phi^3 + 225\phi^2\theta^2 + 120\phi\theta^2 + 120\phi - 36\theta^2)^2} \geq \frac{\phi(\alpha - \bar{c})^2 (5\phi - 6)^2 \cdot (25\phi - 9)}{(125\phi^2 - 45\phi\theta^2 - 135\phi + 18\theta^2 + 18)^2}$$

- For  $H_3$  not to be profitable:

$$\frac{25\phi^3 (\alpha - \bar{c})^2 \cdot (25\phi - 4) (25\phi^2 - 15\phi\theta^2 - 6)^2}{(3125\phi^4 - 1125\phi^3\theta^2 - 2125\phi^3 + 225\phi^2\theta^2 + 120\phi\theta^2 + 120\phi - 36\theta^2)^2} \geq \frac{\phi(\alpha - \bar{c})^2 \cdot (25\phi - 9) (5\phi - 3\theta^2 - 3)^2}{(125\phi^2 - 45\phi\theta^2 - 135\phi + 18\theta^2 + 18)^2}$$

### C.3.11 Stability conditions for: **E3D** [3H1L]

- Adding an edge between nodes  $H_1, H_2$ : **E3D** [3H1L]  $\rightarrow$  **E4B** [3H1L]

- For  $H_1$  not to be profitable:

$$\frac{\phi(\alpha - \bar{c})^2 \cdot (25\phi - 16) (5\phi - 2\theta^2 - 4)^2}{(125\phi^2 - 20\phi\theta^2 - 120\phi + 8\theta^2 + 16)^2} > \frac{25\phi^3 (\alpha - \bar{c})^2 \cdot (25\phi - 9) (5\phi - 2\theta^2 - 2)^2}{(625\phi^3 - 100\phi^2\theta^2 - 350\phi^2 + 6\theta^2 + 6)^2}$$

- For  $H_2$  not to be profitable:

$$\frac{\phi(\alpha - \bar{c})^2 (5\phi - 4)^2 \cdot (25\phi - 4)}{(125\phi^2 - 20\phi\theta^2 - 120\phi + 8\theta^2 + 16)^2} > \frac{\phi(\alpha - \bar{c})^2 \cdot (25\phi - 1) (25\phi^2 - 6\theta^2 - 6)^2}{(625\phi^3 - 100\phi^2\theta^2 - 350\phi^2 + 6\theta^2 + 6)^2}$$

- Adding an edge between nodes  $H_1, L_1$ : **E3D** [3H1L]  $\rightarrow$  **E4D** [3H1L]

- For  $H_1$  not to be profitable:

$$\frac{\phi(\alpha - \bar{c})^2 \cdot (25\phi - 16) (5\phi - 2\theta^2 - 4)^2}{(125\phi^2 - 20\phi\theta^2 - 120\phi + 8\theta^2 + 16)^2} > \frac{25\phi^3 (\alpha - \bar{c})^2 (5\phi - 4)^2 \cdot (25\phi - 9)}{(625\phi^3 - 25\phi^2\theta^2 - 425\phi^2 + 12\theta^2)^2}$$

- For  $L_1$  not to be profitable:

$$\frac{\phi(\alpha - \bar{c})^2 (5\phi - 4)^2 \cdot (25\phi - 4\theta^2)}{(125\phi^2 - 20\phi\theta^2 - 120\phi + 8\theta^2 + 16)^2} > \frac{\phi(\alpha - \bar{c})^2 \cdot (25\phi - \theta^2) (25\phi^2 - 12)^2}{(625\phi^3 - 25\phi^2\theta^2 - 425\phi^2 + 12\theta^2)^2}$$

- Removing an edge between nodes  $H_2, L_1$ : **E3D** [3H1L]  $\rightarrow$  **E2B** [3H1L]

- For  $H_2$  not to be profitable:

$$\frac{\phi(\alpha - \bar{c})^2 (5\phi - 4)^2 \cdot (25\phi - 4)}{(125\phi^2 - 20\phi\theta^2 - 120\phi + 8\theta^2 + 16)^2} \geq \frac{25\phi^3 (\alpha - \bar{c})^2 (5\phi - 4)^2 (5\phi - 3\theta^2)^2 \cdot (25\phi - 9)}{(3125\phi^4 - 1125\phi^3\theta^2 - 3625\phi^3 + 825\phi^2\theta^2 + 800\phi^2 + 180\phi\theta^2 - 72\theta^2)^2}$$

- For  $L_1$  not to be profitable:

$$\frac{\phi(\alpha - \bar{c})^2(5\phi - 4)^2 \cdot (25\phi - 4\theta^2)}{(125\phi^2 - 20\phi\theta^2 - 120\phi + 8\theta^2 + 16)^2} \geq \frac{25\phi^3(\alpha - \bar{c})^2(5\phi - 4)^2(5\phi - 3)^2 \cdot (25\phi - 9\theta^2)}{(3125\phi^4 - 1125\phi^3\theta^2 - 3625\phi^3 + 825\phi^2\theta^2 + 800\phi^2 + 180\phi\theta^2 - 72\theta^2)^2}$$

- Removing an edge between nodes  $H_2, H_3$ : [E3D \[3H1L\]](#)  $\rightarrow$  [E2D \[3H1L\]](#)

– For  $H_2$  not to be profitable:

$$\frac{\phi(\alpha - \bar{c})^2(5\phi - 4)^2 \cdot (25\phi - 4)}{(125\phi^2 - 20\phi\theta^2 - 120\phi + 8\theta^2 + 16)^2} \geq \frac{25\phi^3(\alpha - \bar{c})^2(5\phi - 4)^2 \cdot (25\phi - 9)}{(625\phi^3 - 100\phi^2\theta^2 - 475\phi^2 - 20\phi\theta^2 + 24\theta^2)^2}$$

– For  $H_3$  not to be profitable:

$$\frac{\phi(\alpha - \bar{c})^2(5\phi - 4)^2 \cdot (25\phi - 4)}{(125\phi^2 - 20\phi\theta^2 - 120\phi + 8\theta^2 + 16)^2} \geq \frac{25\phi^3(\alpha - \bar{c})^2(5\phi - 4)^2 \cdot (25\phi - 9)}{(625\phi^3 - 100\phi^2\theta^2 - 475\phi^2 - 20\phi\theta^2 + 24\theta^2)^2}$$

### C.3.12 Stability conditions for: [E3E \[3H1L\]](#)

- Adding an edge between nodes  $H_1, H_3$ : [E3E \[3H1L\]](#)  $\rightarrow$  [E4B \[3H1L\]](#)

– For  $H_1$  not to be profitable:

$$\frac{\phi(\alpha - \bar{c})^2 \cdot (25\phi - 9)(125\phi^3 - 125\phi^2 + 12\theta^2)^2}{(3125\phi^4 - 500\phi^3\theta^2 - 2750\phi^3 + 225\phi^2 + 240\phi\theta^2 - 36\theta^2)^2} > \frac{25\phi^3(\alpha - \bar{c})^2(5\phi - 3)^2 \cdot (25\phi - 4)}{(625\phi^3 - 100\phi^2\theta^2 - 350\phi^2 + 6\theta^2 + 6)^2}$$

– For  $H_3$  not to be profitable:

$$\frac{25\phi^3(\alpha - \bar{c})^2 \cdot (25\phi - 4)(25\phi^2 - 15\phi - 6\theta^2)^2}{(3125\phi^4 - 500\phi^3\theta^2 - 2750\phi^3 + 225\phi^2 + 240\phi\theta^2 - 36\theta^2)^2} > \frac{\phi(\alpha - \bar{c})^2 \cdot (25\phi - 1)(25\phi^2 - 6\theta^2 - 6)^2}{(625\phi^3 - 100\phi^2\theta^2 - 350\phi^2 + 6\theta^2 + 6)^2}$$

- Adding an edge between nodes  $H_1, H_2$ : [E3E \[3H1L\]](#)  $\rightarrow$  [E4C \[3H1L\]](#)

– For  $H_1$  not to be profitable:

$$\frac{\phi(\alpha - \bar{c})^2 \cdot (25\phi - 9)(125\phi^3 - 125\phi^2 + 12\theta^2)^2}{(3125\phi^4 - 500\phi^3\theta^2 - 2750\phi^3 + 225\phi^2 + 240\phi\theta^2 - 36\theta^2)^2} > \frac{\phi(\alpha - \bar{c})^2(5\phi - 2\theta^2)^2(5\phi + 2\theta^2)^2 \cdot (25\phi - 4)}{(625\phi^3 - 100\phi^2\theta^2 - 50\phi^2 - 60\phi\theta^2 - 40\phi + 24\theta^2)^2}$$

– For  $H_2$  not to be profitable:

$$\frac{\phi(\alpha - \bar{c})^2 \cdot (25\phi - 9)(125\phi^3 - 50\phi^2\theta^2 - 75\phi^2 + 12\theta^2)^2}{(3125\phi^4 - 500\phi^3\theta^2 - 2750\phi^3 + 225\phi^2 + 240\phi\theta^2 - 36\theta^2)^2} > \frac{\phi(\alpha - \bar{c})^2(5\phi + 2)^2(5\phi - 2\theta^2)^2 \cdot (25\phi - 4)}{(625\phi^3 - 100\phi^2\theta^2 - 50\phi^2 - 60\phi\theta^2 - 40\phi + 24\theta^2)^2}$$

- Adding an edge between nodes  $H_2, L_1$ : [E3E \[3H1L\]](#)  $\rightarrow$  [E4D \[3H1L\]](#)

– For  $H_2$  not to be profitable:

$$\frac{\phi(\alpha - \bar{c})^2 \cdot (25\phi - 9)(125\phi^3 - 50\phi^2\theta^2 - 75\phi^2 + 12\theta^2)^2}{(3125\phi^4 - 500\phi^3\theta^2 - 2750\phi^3 + 225\phi^2 + 240\phi\theta^2 - 36\theta^2)^2} > \frac{25\phi^3(\alpha - \bar{c})^2(5\phi - 3)^2 \cdot (25\phi - 4)}{(625\phi^3 - 25\phi^2\theta^2 - 425\phi^2 + 12\theta^2)^2}$$

– For  $L_1$  not to be profitable:

$$\frac{25\phi^3(\alpha - \bar{c})^2 \cdot (25\phi - 4\theta^2)(25\phi^2 - 15\phi - 6)^2}{(3125\phi^4 - 500\phi^3\theta^2 - 2750\phi^3 + 225\phi^2 + 240\phi\theta^2 - 36\theta^2)^2} > \frac{\phi(\alpha - \bar{c})^2 \cdot (25\phi - \theta^2)(25\phi^2 - 12)^2}{(625\phi^3 - 25\phi^2\theta^2 - 425\phi^2 + 12\theta^2)^2}$$

- Removing an edge between nodes  $H_1, L_1$ : [E3E \[3H1L\]](#)  $\rightarrow$  [E2B \[3H1L\]](#)

– For  $H_1$  not to be profitable:

$$\frac{\phi(\alpha - \bar{c})^2 \cdot (25\phi - 9)(125\phi^3 - 125\phi^2 + 12\theta^2)^2}{(3125\phi^4 - 500\phi^3\theta^2 - 2750\phi^3 + 225\phi^2 + 240\phi\theta^2 - 36\theta^2)^2} \geq \frac{\phi(\alpha - \bar{c})^2 \cdot (25\phi - 16)(125\phi^3 - 75\phi^2\theta^2 - 125\phi^2 + 45\phi\theta^2 + 18\theta^2)^2}{(3125\phi^4 - 1125\phi^3\theta^2 - 3625\phi^3 + 825\phi^2\theta^2 + 800\phi^2 + 180\phi\theta^2 - 72\theta^2)^2}$$

– For  $L_1$  not to be profitable:

$$\frac{25\phi^3(\alpha - \bar{c})^2 \cdot (25\phi - 4\theta^2)(25\phi^2 - 15\phi - 6)^2}{(3125\phi^4 - 500\phi^3\theta^2 - 2750\phi^3 + 225\phi^2 + 240\phi\theta^2 - 36\theta^2)^2} \geq \frac{25\phi^3(\alpha - \bar{c})^2(5\phi - 4)^2(5\phi - 3)^2 \cdot (25\phi - 9\theta^2)}{(3125\phi^4 - 1125\phi^3\theta^2 - 3625\phi^3 + 825\phi^2\theta^2 + 800\phi^2 + 180\phi\theta^2 - 72\theta^2)^2}$$

- Removing an edge between nodes  $H_3, L_1$ : [E3E \[3H1L\]](#)  $\rightarrow$  [E2C \[3H1L\]](#)

- For  $H_3$  not to be profitable:

$$\frac{25\phi^3 (\alpha - \bar{c})^2 \cdot (25\phi - 4) (25\phi^2 - 15\phi - 6\theta^2)^2}{(3125\phi^4 - 500\phi^3\theta^2 - 2750\phi^3 + 225\phi^2 + 240\phi\theta^2 - 36\theta^2)^2} \geq \frac{\phi (\alpha - \bar{c})^2 \cdot (25\phi - 9) (5\phi - 3\theta^2 - 3)^2}{(125\phi^2 - 45\phi\theta^2 - 135\phi + 18\theta^2 + 18)^2}$$

- For  $L_1$  not to be profitable:

$$\frac{25\phi^3 (\alpha - \bar{c})^2 \cdot (25\phi - 4\theta^2) (25\phi^2 - 15\phi - 6)^2}{(3125\phi^4 - 500\phi^3\theta^2 - 2750\phi^3 + 225\phi^2 + 240\phi\theta^2 - 36\theta^2)^2} \geq \frac{\phi (\alpha - \bar{c})^2 (5\phi - 6)^2 \cdot (25\phi - 9\theta^2)}{(125\phi^2 - 45\phi\theta^2 - 135\phi + 18\theta^2 + 18)^2}$$

- Removing an edge between nodes  $H_2, H_3$ : **E3E** [3H1L]  $\rightarrow$  **E2D** [3H1L]

- For  $H_2$  not to be profitable:

$$\frac{\phi (\alpha - \bar{c})^2 \cdot (25\phi - 9) (125\phi^3 - 50\phi^2\theta^2 - 75\phi^2 + 12\theta^2)^2}{(3125\phi^4 - 500\phi^3\theta^2 - 2750\phi^3 + 225\phi^2 + 240\phi\theta^2 - 36\theta^2)^2} \geq \frac{\phi (\alpha - \bar{c})^2 \cdot (25\phi - 16) (25\phi^2 - 10\phi\theta^2 - 15\phi - 6\theta^2)^2}{(625\phi^3 - 100\phi^2\theta^2 - 475\phi^2 - 20\phi\theta^2 + 24\theta^2)^2}$$

- For  $H_3$  not to be profitable:

$$\frac{25\phi^3 (\alpha - \bar{c})^2 \cdot (25\phi - 4) (25\phi^2 - 15\phi - 6\theta^2)^2}{(3125\phi^4 - 500\phi^3\theta^2 - 2750\phi^3 + 225\phi^2 + 240\phi\theta^2 - 36\theta^2)^2} \geq \frac{25\phi^3 (\alpha - \bar{c})^2 (5\phi - 4)^2 \cdot (25\phi - 9)}{(625\phi^3 - 100\phi^2\theta^2 - 475\phi^2 - 20\phi\theta^2 + 24\theta^2)^2}$$

### C.3.13 Stability conditions for: **E3F** [3H1L]

- Adding an edge between nodes  $H_1, H_2$ : **E3F** [3H1L]  $\rightarrow$  **E4D** [3H1L]

- For  $H_1$  not to be profitable:

$$\frac{25\phi^3 (\alpha - \bar{c})^2 \cdot (25\phi - 9)}{(125\phi^2 - 5\phi\theta^2 + 15\phi - 6\theta^2)^2} > \frac{25\phi^3 (\alpha - \bar{c})^2 (5\phi - 3)^2 \cdot (25\phi - 4)}{(625\phi^3 - 25\phi^2\theta^2 - 425\phi^2 + 12\theta^2)^2}$$

- For  $H_2$  not to be profitable:

$$\frac{25\phi^3 (\alpha - \bar{c})^2 \cdot (25\phi - 9)}{(125\phi^2 - 5\phi\theta^2 + 15\phi - 6\theta^2)^2} > \frac{25\phi^3 (\alpha - \bar{c})^2 (5\phi - 3)^2 \cdot (25\phi - 4)}{(625\phi^3 - 25\phi^2\theta^2 - 425\phi^2 + 12\theta^2)^2}$$

- Removing an edge between nodes  $H_1, L_1$ : **E3F** [3H1L]  $\rightarrow$  **E2D** [3H1L]

- For  $H_1$  not to be profitable:

$$\frac{25\phi^3 (\alpha - \bar{c})^2 \cdot (25\phi - 9)}{(125\phi^2 - 5\phi\theta^2 + 15\phi - 6\theta^2)^2} \geq \frac{\phi (\alpha - \bar{c})^2 \cdot (25\phi - 16) (25\phi^2 - 10\phi\theta^2 - 15\phi - 6\theta^2)^2}{(625\phi^3 - 100\phi^2\theta^2 - 475\phi^2 - 20\phi\theta^2 + 24\theta^2)^2}$$

- For  $L_1$  not to be profitable:

$$\frac{\phi (\alpha - \bar{c})^2 (5\phi + 6)^2 \cdot (25\phi - \theta^2)}{(125\phi^2 - 5\phi\theta^2 + 15\phi - 6\theta^2)^2} \geq \frac{\phi (\alpha - \bar{c})^2 (5\phi - 4)^2 (5\phi + 3)^2 \cdot (25\phi - 4\theta^2)}{(625\phi^3 - 100\phi^2\theta^2 - 475\phi^2 - 20\phi\theta^2 + 24\theta^2)^2}$$

### C.3.14 Stability conditions for: **E4A** [3H1L]

- Adding an edge between nodes  $H_1, L_1$ : **E4A** [3H1L]  $\rightarrow$  **E5A** [3H1L]

- For  $H_1$  not to be profitable:

$$\frac{25\phi^3 (\alpha - \bar{c})^2 (5\phi - 3\theta^2)^2 \cdot (25\phi - 4)}{(625\phi^3 - 225\phi^2\theta^2 - 225\phi^2 + 12\theta^2)^2} > \frac{\phi (\alpha - \bar{c})^2 (5\phi - 2\theta)^2 (5\phi + 2\theta)^2 \cdot (25\phi - 1)}{(625\phi^3 - 100\phi^2\theta^2 - 150\phi^2 - 20\phi\theta^2 + 8\theta^2)^2}$$

- For  $L_1$  not to be profitable:

$$\frac{25\phi^3 (\alpha - \bar{c})^2 (5\phi - 4)^2 \cdot (25\phi - 9\theta^2)}{(625\phi^3 - 225\phi^2\theta^2 - 225\phi^2 + 12\theta^2)^2} > \frac{25\phi^3 (\alpha - \bar{c})^2 (5\phi - 2)^2 \cdot (25\phi - 4\theta^2)}{(625\phi^3 - 100\phi^2\theta^2 - 150\phi^2 - 20\phi\theta^2 + 8\theta^2)^2}$$

- Removing an edge between nodes  $H_1, H_2$ : **E4A** [3H1L]  $\rightarrow$  **E3A** [3H1L]

- For  $H_1$  not to be profitable:

$$\frac{25\phi^3 (\alpha - \bar{c})^2 (5\phi - 3\theta^2)^2 \cdot (25\phi - 4)}{(625\phi^3 - 225\phi^2\theta^2 - 225\phi^2 + 12\theta^2)^2} \geq \frac{25\phi^3 (\alpha - \bar{c})^2 (5\phi - 3\theta^2)^2 \cdot (25\phi - 9)}{(625\phi^3 - 225\phi^2\theta^2 - 100\phi^2 - 45\phi\theta^2 - 15\phi + 18\theta^2)^2}$$

- For  $H_2$  not to be profitable:

$$\frac{25\phi^3 (\alpha - \bar{c})^2 (5\phi - 3\theta^2)^2 \cdot (25\phi - 4)}{(625\phi^3 - 225\phi^2\theta^2 - 225\phi^2 + 12\theta^2)^2} \geq \frac{25\phi^3 (\alpha - \bar{c})^2 (5\phi - 3\theta^2)^2 \cdot (25\phi - 9)}{(625\phi^3 - 225\phi^2\theta^2 - 100\phi^2 - 45\phi\theta^2 - 15\phi + 18\theta^2)^2}$$

- Removing an edge between nodes  $H_3$ ,  $L_1$ : [E4A \[3H1L\]](#) → [E3B \[3H1L\]](#)

- For  $H_3$  not to be profitable:

$$\frac{\phi (\alpha - \bar{c})^2 \cdot (25\phi - 1) (25\phi^2 - 12\theta^2)^2}{(625\phi^3 - 225\phi^2\theta^2 - 225\phi^2 + 12\theta^2)^2} \geq \frac{\phi (\alpha - \bar{c})^2 (5\phi - 4\theta^2)^2 \cdot (25\phi - 4)}{(125\phi^2 - 80\phi\theta^2 - 60\phi + 24\theta^2)^2}$$

- For  $L_1$  not to be profitable:

$$\frac{25\phi^3 (\alpha - \bar{c})^2 (5\phi - 4)^2 \cdot (25\phi - 9\theta^2)}{(625\phi^3 - 225\phi^2\theta^2 - 225\phi^2 + 12\theta^2)^2} \geq \frac{\phi (\alpha - \bar{c})^2 (5\phi - 6)^2 \cdot (25\phi - 16\theta^2)}{(125\phi^2 - 80\phi\theta^2 - 60\phi + 24\theta^2)^2}$$

- Removing an edge between nodes  $H_1$ ,  $H_3$ : [E4A \[3H1L\]](#) → [E3C \[3H1L\]](#)

- For  $H_1$  not to be profitable:

$$\frac{25\phi^3 (\alpha - \bar{c})^2 (5\phi - 3\theta^2)^2 \cdot (25\phi - 4)}{(625\phi^3 - 225\phi^2\theta^2 - 225\phi^2 + 12\theta^2)^2} \geq \frac{\phi (\alpha - \bar{c})^2 (5\phi - 2)^2 \cdot (25\phi - 9) (25\phi^2 - 15\phi\theta^2 - 6\theta^2)^2}{(3125\phi^4 - 1125\phi^3\theta^2 - 2125\phi^3 + 225\phi^2\theta^2 + 120\phi\theta^2 + 120\phi - 36\theta^2)^2}$$

- For  $H_3$  not to be profitable:

$$\frac{\phi (\alpha - \bar{c})^2 \cdot (25\phi - 1) (25\phi^2 - 12\theta^2)^2}{(625\phi^3 - 225\phi^2\theta^2 - 225\phi^2 + 12\theta^2)^2} \geq \frac{25\phi^3 (\alpha - \bar{c})^2 \cdot (25\phi - 4) (25\phi^2 - 15\phi - 6\theta^2)^2}{(3125\phi^4 - 1125\phi^3\theta^2 - 2125\phi^3 + 225\phi^2\theta^2 + 120\phi\theta^2 + 120\phi - 36\theta^2)^2}$$

### C.3.15 Stability conditions for: [E4B \[3H1L\]](#)

- Adding an edge between nodes  $H_1$ ,  $H_2$ : [E4B \[3H1L\]](#) → [E5A \[3H1L\]](#)

- For  $H_1$  not to be profitable:

$$\frac{25\phi^3 (\alpha - \bar{c})^2 \cdot (25\phi - 9) (5\phi - 2\theta^2 - 2)^2}{(625\phi^3 - 100\phi^2\theta^2 - 350\phi^2 + 6\theta^2 + 6)^2} > \frac{25\phi^3 (\alpha - \bar{c})^2 (5\phi - 2\theta^2)^2 \cdot (25\phi - 4)}{(625\phi^3 - 100\phi^2\theta^2 - 150\phi^2 - 20\phi\theta^2 + 8\theta^2)^2}$$

- For  $H_2$  not to be profitable:

$$\frac{25\phi^3 (\alpha - \bar{c})^2 (5\phi - 3)^2 \cdot (25\phi - 4)}{(625\phi^3 - 100\phi^2\theta^2 - 350\phi^2 + 6\theta^2 + 6)^2} > \frac{\phi (\alpha - \bar{c})^2 (5\phi - 2\theta)^2 (5\phi + 2\theta)^2 \cdot (25\phi - 1)}{(625\phi^3 - 100\phi^2\theta^2 - 150\phi^2 - 20\phi\theta^2 + 8\theta^2)^2}$$

- Adding an edge between nodes  $H_1$ ,  $L_1$ : [E4B \[3H1L\]](#) → [E5B \[3H1L\]](#)

- For  $H_1$  not to be profitable:

$$\frac{25\phi^3 (\alpha - \bar{c})^2 \cdot (25\phi - 9) (5\phi - 2\theta^2 - 2)^2}{(625\phi^3 - 100\phi^2\theta^2 - 350\phi^2 + 6\theta^2 + 6)^2} > \frac{25\phi^3 (\alpha - \bar{c})^2 \cdot (25\phi - 4)}{(125\phi^2 - 5\phi\theta^2 + 5\phi - 2\theta^2 - 2)^2}$$

- For  $L_1$  not to be profitable:

$$\frac{25\phi^3 (\alpha - \bar{c})^2 (5\phi - 3)^2 \cdot (25\phi - 4\theta^2)}{(625\phi^3 - 100\phi^2\theta^2 - 350\phi^2 + 6\theta^2 + 6)^2} > \frac{\phi (\alpha - \bar{c})^2 (5\phi + 2)^2 \cdot (25\phi - \theta^2)}{(125\phi^2 - 5\phi\theta^2 + 5\phi - 2\theta^2 - 2)^2}$$

- Removing an edge between nodes  $H_2$ ,  $L_1$ : [E4B \[3H1L\]](#) → [E3A \[3H1L\]](#)

- For  $H_2$  not to be profitable:

$$\frac{25\phi^3 (\alpha - \bar{c})^2 (5\phi - 3)^2 \cdot (25\phi - 4)}{(625\phi^3 - 100\phi^2\theta^2 - 350\phi^2 + 6\theta^2 + 6)^2} \geq \frac{25\phi^3 (\alpha - \bar{c})^2 (5\phi - 3\theta^2)^2 \cdot (25\phi - 9)}{(625\phi^3 - 225\phi^2\theta^2 - 100\phi^2 - 45\phi\theta^2 - 15\phi + 18\theta^2)^2}$$

- For  $L_1$  not to be profitable:

$$\frac{25\phi^3 (\alpha - \bar{c})^2 (5\phi - 3)^2 \cdot (25\phi - 4\theta^2)}{(625\phi^3 - 100\phi^2\theta^2 - 350\phi^2 + 6\theta^2 + 6)^2} \geq \frac{25\phi^3 (\alpha - \bar{c})^2 (5\phi - 3)^2 \cdot (25\phi - 9\theta^2)}{(625\phi^3 - 225\phi^2\theta^2 - 100\phi^2 - 45\phi\theta^2 - 15\phi + 18\theta^2)^2}$$

- Removing an edge between nodes  $H_3$ ,  $L_1$ : [E4B \[3H1L\]](#) → [E3C \[3H1L\]](#)

- For  $H_3$  not to be profitable:

$$\frac{\phi(\alpha - \bar{c})^2 \cdot (25\phi - 1)(25\phi^2 - 6\theta^2 - 6)^2}{(625\phi^3 - 100\phi^2\theta^2 - 350\phi^2 + 6\theta^2 + 6)^2} \geq \frac{25\phi^3(\alpha - \bar{c})^2 \cdot (25\phi - 4)(25\phi^2 - 15\phi\theta^2 - 6)^2}{(3125\phi^4 - 1125\phi^3\theta^2 - 2125\phi^3 + 225\phi^2\theta^2 + 120\phi\theta^2 + 120\phi - 36\theta^2)^2}$$

- For  $L_1$  not to be profitable:

$$\frac{25\phi^3(\alpha - \bar{c})^2(5\phi - 3)^2 \cdot (25\phi - 4\theta^2)}{(625\phi^3 - 100\phi^2\theta^2 - 350\phi^2 + 6\theta^2 + 6)^2} \geq \frac{\phi(\alpha - \bar{c})^2(5\phi - 2)^2 \cdot (25\phi - 9\theta^2)(25\phi^2 - 15\phi - 6)^2}{(3125\phi^4 - 1125\phi^3\theta^2 - 2125\phi^3 + 225\phi^2\theta^2 + 120\phi\theta^2 + 120\phi - 36\theta^2)^2}$$

- Removing an edge between nodes  $H_1, H_3$ : **E4B** [3H1L]  $\rightarrow$  **E3D** [3H1L]

- For  $H_1$  not to be profitable:

$$\frac{25\phi^3(\alpha - \bar{c})^2 \cdot (25\phi - 9)(5\phi - 2\theta^2 - 2)^2}{(625\phi^3 - 100\phi^2\theta^2 - 350\phi^2 + 6\theta^2 + 6)^2} \geq \frac{\phi(\alpha - \bar{c})^2 \cdot (25\phi - 16)(5\phi - 2\theta^2 - 4)^2}{(125\phi^2 - 20\phi\theta^2 - 120\phi + 8\theta^2 + 16)^2}$$

- For  $H_3$  not to be profitable:

$$\frac{\phi(\alpha - \bar{c})^2 \cdot (25\phi - 1)(25\phi^2 - 6\theta^2 - 6)^2}{(625\phi^3 - 100\phi^2\theta^2 - 350\phi^2 + 6\theta^2 + 6)^2} \geq \frac{\phi(\alpha - \bar{c})^2(5\phi - 4)^2 \cdot (25\phi - 4)}{(125\phi^2 - 20\phi\theta^2 - 120\phi + 8\theta^2 + 16)^2}$$

- Removing an edge between nodes  $H_2, H_3$ : **E4B** [3H1L]  $\rightarrow$  **E3E** [3H1L]

- For  $H_2$  not to be profitable:

$$\frac{25\phi^3(\alpha - \bar{c})^2(5\phi - 3)^2 \cdot (25\phi - 4)}{(625\phi^3 - 100\phi^2\theta^2 - 350\phi^2 + 6\theta^2 + 6)^2} \geq \frac{\phi(\alpha - \bar{c})^2 \cdot (25\phi - 9)(125\phi^3 - 125\phi^2 + 12\theta^2)^2}{(3125\phi^4 - 500\phi^3\theta^2 - 2750\phi^3 + 225\phi^2 + 240\phi\theta^2 - 36\theta^2)^2}$$

- For  $H_3$  not to be profitable:

$$\frac{\phi(\alpha - \bar{c})^2 \cdot (25\phi - 1)(25\phi^2 - 6\theta^2 - 6)^2}{(625\phi^3 - 100\phi^2\theta^2 - 350\phi^2 + 6\theta^2 + 6)^2} \geq \frac{25\phi^3(\alpha - \bar{c})^2 \cdot (25\phi - 4)(25\phi^2 - 15\phi - 6\theta^2)^2}{(3125\phi^4 - 500\phi^3\theta^2 - 2750\phi^3 + 225\phi^2 + 240\phi\theta^2 - 36\theta^2)^2}$$

### C.3.16 Stability conditions for: **E4C** [3H1L]

- Adding an edge between nodes  $H_1, H_2$ : **E4C** [3H1L]  $\rightarrow$  **E5A** [3H1L]

- For  $H_1$  not to be profitable:

$$\frac{\phi(\alpha - \bar{c})^2(5\phi - 2\theta)^2(5\phi + 2\theta)^2 \cdot (25\phi - 4)}{(625\phi^3 - 100\phi^2\theta^2 - 50\phi^2 - 60\phi\theta^2 - 40\phi + 24\theta^2)^2} > \frac{\phi(\alpha - \bar{c})^2(5\phi - 2\theta)^2(5\phi + 2\theta)^2 \cdot (25\phi - 1)}{(625\phi^3 - 100\phi^2\theta^2 - 150\phi^2 - 20\phi\theta^2 + 8\theta^2)^2}$$

- For  $H_2$  not to be profitable:

$$\frac{\phi(\alpha - \bar{c})^2(5\phi - 2\theta)^2(5\phi + 2\theta)^2 \cdot (25\phi - 4)}{(625\phi^3 - 100\phi^2\theta^2 - 50\phi^2 - 60\phi\theta^2 - 40\phi + 24\theta^2)^2} > \frac{\phi(\alpha - \bar{c})^2(5\phi - 2\theta)^2(5\phi + 2\theta)^2 \cdot (25\phi - 1)}{(625\phi^3 - 100\phi^2\theta^2 - 150\phi^2 - 20\phi\theta^2 + 8\theta^2)^2}$$

- Adding an edge between nodes  $H_3, L_1$ : **E4C** [3H1L]  $\rightarrow$  **E5B** [3H1L]

- For  $H_3$  not to be profitable:

$$\frac{\phi(\alpha - \bar{c})^2(5\phi + 2)^2(5\phi - 2\theta^2)^2 \cdot (25\phi - 4)}{(625\phi^3 - 100\phi^2\theta^2 - 50\phi^2 - 60\phi\theta^2 - 40\phi + 24\theta^2)^2} > \frac{\phi(\alpha - \bar{c})^2(5\phi + 2)^2 \cdot (25\phi - 1)}{(125\phi^2 - 5\phi\theta^2 + 5\phi - 2\theta^2 - 2)^2}$$

- For  $L_1$  not to be profitable:

$$\frac{\phi(\alpha - \bar{c})^2(5\phi - 2)^2(5\phi + 2)^2 \cdot (25\phi - 4\theta^2)}{(625\phi^3 - 100\phi^2\theta^2 - 50\phi^2 - 60\phi\theta^2 - 40\phi + 24\theta^2)^2} > \frac{\phi(\alpha - \bar{c})^2(5\phi + 2)^2 \cdot (25\phi - \theta^2)}{(125\phi^2 - 5\phi\theta^2 + 5\phi - 2\theta^2 - 2)^2}$$

- Removing an edge between nodes  $H_1, L_1$ : **E4C** [3H1L]  $\rightarrow$  **E3C** [3H1L]

- For  $H_1$  not to be profitable:

$$\frac{\phi(\alpha - \bar{c})^2(5\phi - 2\theta)^2(5\phi + 2\theta)^2 \cdot (25\phi - 4)}{(625\phi^3 - 100\phi^2\theta^2 - 50\phi^2 - 60\phi\theta^2 - 40\phi + 24\theta^2)^2} \geq \frac{\phi(\alpha - \bar{c})^2(5\phi - 2)^2 \cdot (25\phi - 9)(25\phi^2 - 15\phi\theta^2 - 6\theta^2)^2}{(3125\phi^4 - 1125\phi^3\theta^2 - 2125\phi^3 + 225\phi^2\theta^2 + 120\phi\theta^2 + 120\phi - 36\theta^2)^2}$$

- For  $L_1$  not to be profitable:



$$\frac{\phi(\alpha - \bar{c})^2 (5\phi - 2)^2 (5\phi + 2)^2 \cdot (25\phi - 4\theta^2)}{(625\phi^3 - 100\phi^2\theta^2 - 50\phi^2 - 60\phi\theta^2 - 40\phi + 24\theta^2)^2} \geq \frac{\phi(\alpha - \bar{c})^2 (5\phi - 2)^2 \cdot (25\phi - 9\theta^2) (25\phi^2 - 15\phi - 6)^2}{(3125\phi^4 - 1125\phi^3\theta^2 - 2125\phi^3 + 225\phi^2\theta^2 + 120\phi\theta^2 + 120\phi - 36\theta^2)^2}$$

- Removing an edge between nodes  $H_1, H_3$ : [E4C \[3H1L\]](#)  $\rightarrow$  [E3E \[3H1L\]](#)

– For  $H_1$  not to be profitable:

$$\frac{\phi(\alpha - \bar{c})^2 (5\phi - 2\theta)^2 (5\phi + 2\theta)^2 \cdot (25\phi - 4)}{(625\phi^3 - 100\phi^2\theta^2 - 50\phi^2 - 60\phi\theta^2 - 40\phi + 24\theta^2)^2} \geq \frac{\phi(\alpha - \bar{c})^2 \cdot (25\phi - 9) (125\phi^3 - 125\phi^2 + 12\theta^2)^2}{(3125\phi^4 - 500\phi^3\theta^2 - 2750\phi^3 + 225\phi^2 + 240\phi\theta^2 - 36\theta^2)^2}$$

– For  $H_3$  not to be profitable:

$$\frac{\phi(\alpha - \bar{c})^2 (5\phi + 2)^2 (5\phi - 2\theta^2)^2 \cdot (25\phi - 4)}{(625\phi^3 - 100\phi^2\theta^2 - 50\phi^2 - 60\phi\theta^2 - 40\phi + 24\theta^2)^2} \geq \frac{\phi(\alpha - \bar{c})^2 \cdot (25\phi - 9) (125\phi^3 - 50\phi^2\theta^2 - 75\phi^2 + 12\theta^2)^2}{(3125\phi^4 - 500\phi^3\theta^2 - 2750\phi^3 + 225\phi^2 + 240\phi\theta^2 - 36\theta^2)^2}$$

### C.3.17 Stability conditions for: [E4D \[3H1L\]](#)

- Adding an edge between nodes  $H_1, H_2$ : [E4D \[3H1L\]](#)  $\rightarrow$  [E5B \[3H1L\]](#)

– For  $H_1$  not to be profitable:

$$\frac{25\phi^3 (\alpha - \bar{c})^2 (5\phi - 4)^2 \cdot (25\phi - 9)}{(625\phi^3 - 25\phi^2\theta^2 - 425\phi^2 + 12\theta^2)^2} > \frac{25\phi^3 (\alpha - \bar{c})^2 \cdot (25\phi - 4)}{(125\phi^2 - 5\phi\theta^2 + 5\phi - 2\theta^2 - 2)^2}$$

– For  $H_2$  not to be profitable:

$$\frac{25\phi^3 (\alpha - \bar{c})^2 (5\phi - 3)^2 \cdot (25\phi - 4)}{(625\phi^3 - 25\phi^2\theta^2 - 425\phi^2 + 12\theta^2)^2} > \frac{\phi(\alpha - \bar{c})^2 (5\phi + 2)^2 \cdot (25\phi - 1)}{(125\phi^2 - 5\phi\theta^2 + 5\phi - 2\theta^2 - 2)^2}$$

- Removing an edge between nodes  $H_1, L_1$ : [E4D \[3H1L\]](#)  $\rightarrow$  [E3D \[3H1L\]](#)

– For  $H_1$  not to be profitable:

$$\frac{25\phi^3 (\alpha - \bar{c})^2 (5\phi - 4)^2 \cdot (25\phi - 9)}{(625\phi^3 - 25\phi^2\theta^2 - 425\phi^2 + 12\theta^2)^2} \geq \frac{\phi(\alpha - \bar{c})^2 \cdot (25\phi - 16) (5\phi - 2\theta^2 - 4)^2}{(125\phi^2 - 20\phi\theta^2 - 120\phi + 8\theta^2 + 16)^2}$$

– For  $L_1$  not to be profitable:

$$\frac{\phi(\alpha - \bar{c})^2 \cdot (25\phi - \theta^2) (25\phi^2 - 12)^2}{(625\phi^3 - 25\phi^2\theta^2 - 425\phi^2 + 12\theta^2)^2} \geq \frac{\phi(\alpha - \bar{c})^2 (5\phi - 4)^2 \cdot (25\phi - 4\theta^2)}{(125\phi^2 - 20\phi\theta^2 - 120\phi + 8\theta^2 + 16)^2}$$

- Removing an edge between nodes  $H_2, L_1$ : [E4D \[3H1L\]](#)  $\rightarrow$  [E3E \[3H1L\]](#)

– For  $H_2$  not to be profitable:

$$\frac{25\phi^3 (\alpha - \bar{c})^2 (5\phi - 3)^2 \cdot (25\phi - 4)}{(625\phi^3 - 25\phi^2\theta^2 - 425\phi^2 + 12\theta^2)^2} \geq \frac{\phi(\alpha - \bar{c})^2 \cdot (25\phi - 9) (125\phi^3 - 50\phi^2\theta^2 - 75\phi^2 + 12\theta^2)^2}{(3125\phi^4 - 500\phi^3\theta^2 - 2750\phi^3 + 225\phi^2 + 240\phi\theta^2 - 36\theta^2)^2}$$

– For  $L_1$  not to be profitable:

$$\frac{\phi(\alpha - \bar{c})^2 \cdot (25\phi - \theta^2) (25\phi^2 - 12)^2}{(625\phi^3 - 25\phi^2\theta^2 - 425\phi^2 + 12\theta^2)^2} \geq \frac{25\phi^3 (\alpha - \bar{c})^2 \cdot (25\phi - 4\theta^2) (25\phi^2 - 15\phi - 6)^2}{(3125\phi^4 - 500\phi^3\theta^2 - 2750\phi^3 + 225\phi^2 + 240\phi\theta^2 - 36\theta^2)^2}$$

- Removing an edge between nodes  $H_2, H_3$ : [E4D \[3H1L\]](#)  $\rightarrow$  [E3F \[3H1L\]](#)

– For  $H_2$  not to be profitable:

$$\frac{25\phi^3 (\alpha - \bar{c})^2 (5\phi - 3)^2 \cdot (25\phi - 4)}{(625\phi^3 - 25\phi^2\theta^2 - 425\phi^2 + 12\theta^2)^2} \geq \frac{25\phi^3 (\alpha - \bar{c})^2 \cdot (25\phi - 9)}{(125\phi^2 - 5\phi\theta^2 + 15\phi - 6\theta^2)^2}$$

– For  $H_3$  not to be profitable:

$$\frac{25\phi^3 (\alpha - \bar{c})^2 (5\phi - 3)^2 \cdot (25\phi - 4)}{(625\phi^3 - 25\phi^2\theta^2 - 425\phi^2 + 12\theta^2)^2} \geq \frac{25\phi^3 (\alpha - \bar{c})^2 \cdot (25\phi - 9)}{(125\phi^2 - 5\phi\theta^2 + 15\phi - 6\theta^2)^2}$$

### C.3.18 Stability conditions for: E5A [3H1L]

- Adding an edge between nodes  $H_1, L_1$ : E5A [3H1L]  $\rightarrow$  E6A [3H1L]

– For  $H_1$  not to be profitable:

$$\frac{25\phi^3 (\alpha - \bar{c})^2 (5\phi - 2\theta^2)^2 \cdot (25\phi - 4)}{(625\phi^3 - 100\phi^2\theta^2 - 150\phi^2 - 20\phi\theta^2 + 8\theta^2)^2} > \frac{\phi (\alpha - \bar{c})^2 \cdot (25\phi - 1)}{(25\phi - \theta^2 - 3)^2}$$

– For  $L_1$  not to be profitable:

$$\frac{25\phi^3 (\alpha - \bar{c})^2 (5\phi - 2)^2 \cdot (25\phi - 4\theta^2)}{(625\phi^3 - 100\phi^2\theta^2 - 150\phi^2 - 20\phi\theta^2 + 8\theta^2)^2} > \frac{\phi (\alpha - \bar{c})^2 \cdot (25\phi - \theta^2)}{(25\phi - \theta^2 - 3)^2}$$

- Removing an edge between nodes  $H_2, L_1$ : E5A [3H1L]  $\rightarrow$  E4A [3H1L]

– For  $H_2$  not to be profitable:

$$\frac{\phi (\alpha - \bar{c})^2 (5\phi - 2\theta)^2 (5\phi + 2\theta)^2 \cdot (25\phi - 1)}{(625\phi^3 - 100\phi^2\theta^2 - 150\phi^2 - 20\phi\theta^2 + 8\theta^2)^2} \geq \frac{25\phi^3 (\alpha - \bar{c})^2 (5\phi - 3\theta^2)^2 \cdot (25\phi - 4)}{(625\phi^3 - 225\phi^2\theta^2 - 225\phi^2 + 12\theta^2)^2}$$

– For  $L_1$  not to be profitable:

$$\frac{25\phi^3 (\alpha - \bar{c})^2 (5\phi - 2)^2 \cdot (25\phi - 4\theta^2)}{(625\phi^3 - 100\phi^2\theta^2 - 150\phi^2 - 20\phi\theta^2 + 8\theta^2)^2} \geq \frac{25\phi^3 (\alpha - \bar{c})^2 (5\phi - 4)^2 \cdot (25\phi - 9\theta^2)}{(625\phi^3 - 225\phi^2\theta^2 - 225\phi^2 + 12\theta^2)^2}$$

- Removing an edge between nodes  $H_1, H_2$ : E5A [3H1L]  $\rightarrow$  E4B [3H1L]

– For  $H_1$  not to be profitable:

$$\frac{25\phi^3 (\alpha - \bar{c})^2 (5\phi - 2\theta^2)^2 \cdot (25\phi - 4)}{(625\phi^3 - 100\phi^2\theta^2 - 150\phi^2 - 20\phi\theta^2 + 8\theta^2)^2} \geq \frac{25\phi^3 (\alpha - \bar{c})^2 \cdot (25\phi - 9) (5\phi - 2\theta^2 - 2)^2}{(625\phi^3 - 100\phi^2\theta^2 - 350\phi^2 + 6\theta^2 + 6)^2}$$

– For  $H_2$  not to be profitable:

$$\frac{\phi (\alpha - \bar{c})^2 (5\phi - 2\theta)^2 (5\phi + 2\theta)^2 \cdot (25\phi - 1)}{(625\phi^3 - 100\phi^2\theta^2 - 150\phi^2 - 20\phi\theta^2 + 8\theta^2)^2} \geq \frac{25\phi^3 (\alpha - \bar{c})^2 (5\phi - 3)^2 \cdot (25\phi - 4)}{(625\phi^3 - 100\phi^2\theta^2 - 350\phi^2 + 6\theta^2 + 6)^2}$$

- Removing an edge between nodes  $H_2, H_3$ : E5A [3H1L]  $\rightarrow$  E4C [3H1L]

– For  $H_2$  not to be profitable:

$$\frac{\phi (\alpha - \bar{c})^2 (5\phi - 2\theta)^2 (5\phi + 2\theta)^2 \cdot (25\phi - 1)}{(625\phi^3 - 100\phi^2\theta^2 - 150\phi^2 - 20\phi\theta^2 + 8\theta^2)^2} \geq \frac{\phi (\alpha - \bar{c})^2 (5\phi - 2\theta)^2 (5\phi + 2\theta)^2 \cdot (25\phi - 4)}{(625\phi^3 - 100\phi^2\theta^2 - 50\phi^2 - 60\phi\theta^2 - 40\phi + 24\theta^2)^2}$$

– For  $H_3$  not to be profitable:

$$\frac{\phi (\alpha - \bar{c})^2 (5\phi - 2\theta)^2 (5\phi + 2\theta)^2 \cdot (25\phi - 1)}{(625\phi^3 - 100\phi^2\theta^2 - 150\phi^2 - 20\phi\theta^2 + 8\theta^2)^2} \geq \frac{\phi (\alpha - \bar{c})^2 (5\phi - 2\theta)^2 (5\phi + 2\theta)^2 \cdot (25\phi - 4)}{(625\phi^3 - 100\phi^2\theta^2 - 50\phi^2 - 60\phi\theta^2 - 40\phi + 24\theta^2)^2}$$

### C.3.19 Stability conditions for: E5B [3H1L]

- Adding an edge between nodes  $H_1, H_2$ : E5B [3H1L]  $\rightarrow$  E6A [3H1L]

– For  $H_1$  not to be profitable:

$$\frac{25\phi^3 (\alpha - \bar{c})^2 \cdot (25\phi - 4)}{(125\phi^2 - 5\phi\theta^2 + 5\phi - 2\theta^2 - 2)^2} > \frac{\phi (\alpha - \bar{c})^2 \cdot (25\phi - 1)}{(25\phi - \theta^2 - 3)^2}$$

– For  $H_2$  not to be profitable:

$$\frac{25\phi^3 (\alpha - \bar{c})^2 \cdot (25\phi - 4)}{(125\phi^2 - 5\phi\theta^2 + 5\phi - 2\theta^2 - 2)^2} > \frac{\phi (\alpha - \bar{c})^2 \cdot (25\phi - 1)}{(25\phi - \theta^2 - 3)^2}$$

- Removing an edge between nodes  $H_1, L_1$ : E5B [3H1L]  $\rightarrow$  E4B [3H1L]

– For  $H_1$  not to be profitable:

$$\frac{25\phi^3 (\alpha - \bar{c})^2 \cdot (25\phi - 4)}{(125\phi^2 - 5\phi\theta^2 + 5\phi - 2\theta^2 - 2)^2} \geq \frac{25\phi^3 (\alpha - \bar{c})^2 \cdot (25\phi - 9) (5\phi - 2\theta^2 - 2)^2}{(625\phi^3 - 100\phi^2\theta^2 - 350\phi^2 + 6\theta^2 + 6)^2}$$

- For  $L_1$  not to be profitable:

$$\frac{\phi(\alpha - \bar{c})^2 (5\phi + 2)^2 \cdot (25\phi - \theta^2)}{(125\phi^2 - 5\phi\theta^2 + 5\phi - 2\theta^2 - 2)^2} \geq \frac{25\phi^3 (\alpha - \bar{c})^2 (5\phi - 3)^2 \cdot (25\phi - 4\theta^2)}{(625\phi^3 - 100\phi^2\theta^2 - 350\phi^2 + 6\theta^2 + 6)^2}$$

- Removing an edge between nodes  $H_3, L_1$ : [E5B \[3H1L\]](#) → [E4C \[3H1L\]](#)

- For  $H_3$  not to be profitable:

$$\frac{\phi(\alpha - \bar{c})^2 (5\phi + 2)^2 \cdot (25\phi - 1)}{(125\phi^2 - 5\phi\theta^2 + 5\phi - 2\theta^2 - 2)^2} \geq \frac{\phi(\alpha - \bar{c})^2 (5\phi + 2)^2 (5\phi - 2\theta^2)^2 \cdot (25\phi - 4)}{(625\phi^3 - 100\phi^2\theta^2 - 50\phi^2 - 60\phi\theta^2 - 40\phi + 24\theta^2)^2}$$

- For  $L_1$  not to be profitable:

$$\frac{\phi(\alpha - \bar{c})^2 (5\phi + 2)^2 \cdot (25\phi - \theta^2)}{(125\phi^2 - 5\phi\theta^2 + 5\phi - 2\theta^2 - 2)^2} \geq \frac{\phi(\alpha - \bar{c})^2 (5\phi - 2)^2 (5\phi + 2)^2 \cdot (25\phi - 4\theta^2)}{(625\phi^3 - 100\phi^2\theta^2 - 50\phi^2 - 60\phi\theta^2 - 40\phi + 24\theta^2)^2}$$

- Removing an edge between nodes  $H_1, H_3$ : [E5B \[3H1L\]](#) → [E4D \[3H1L\]](#)

- For  $H_1$  not to be profitable:

$$\frac{25\phi^3 (\alpha - \bar{c})^2 \cdot (25\phi - 4)}{(125\phi^2 - 5\phi\theta^2 + 5\phi - 2\theta^2 - 2)^2} \geq \frac{25\phi^3 (\alpha - \bar{c})^2 (5\phi - 4)^2 \cdot (25\phi - 9)}{(625\phi^3 - 25\phi^2\theta^2 - 425\phi^2 + 12\theta^2)^2}$$

- For  $H_3$  not to be profitable:

$$\frac{\phi(\alpha - \bar{c})^2 (5\phi + 2)^2 \cdot (25\phi - 1)}{(125\phi^2 - 5\phi\theta^2 + 5\phi - 2\theta^2 - 2)^2} \geq \frac{25\phi^3 (\alpha - \bar{c})^2 (5\phi - 3)^2 \cdot (25\phi - 4)}{(625\phi^3 - 25\phi^2\theta^2 - 425\phi^2 + 12\theta^2)^2}$$

### C.3.20 Stability conditions for: [E6A \[3H1L\]](#)

- Removing an edge between nodes  $H_1, L_1$ : [E6A \[3H1L\]](#) → [E5A \[3H1L\]](#)

- For  $H_1$  not to be profitable:

$$\frac{\phi(\alpha - \bar{c})^2 \cdot (25\phi - 1)}{(25\phi - \theta^2 - 3)^2} \geq \frac{25\phi^3 (\alpha - \bar{c})^2 (5\phi - 2\theta^2)^2 \cdot (25\phi - 4)}{(625\phi^3 - 100\phi^2\theta^2 - 150\phi^2 - 20\phi\theta^2 + 8\theta^2)^2}$$

- For  $L_1$  not to be profitable:

$$\frac{\phi(\alpha - \bar{c})^2 \cdot (25\phi - \theta^2)}{(25\phi - \theta^2 - 3)^2} \geq \frac{25\phi^3 (\alpha - \bar{c})^2 (5\phi - 2)^2 \cdot (25\phi - 4\theta^2)}{(625\phi^3 - 100\phi^2\theta^2 - 150\phi^2 - 20\phi\theta^2 + 8\theta^2)^2}$$

- Removing an edge between nodes  $H_1, H_2$ : [E6A \[3H1L\]](#) → [E5B \[3H1L\]](#)

- For  $H_1$  not to be profitable:

$$\frac{\phi(\alpha - \bar{c})^2 \cdot (25\phi - 1)}{(25\phi - \theta^2 - 3)^2} \geq \frac{25\phi^3 (\alpha - \bar{c})^2 \cdot (25\phi - 4)}{(125\phi^2 - 5\phi\theta^2 + 5\phi - 2\theta^2 - 2)^2}$$

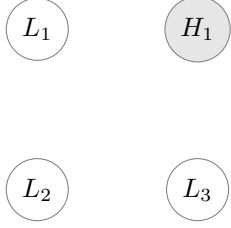
- For  $H_2$  not to be profitable:

$$\frac{\phi(\alpha - \bar{c})^2 \cdot (25\phi - 1)}{(25\phi - \theta^2 - 3)^2} \geq \frac{25\phi^3 (\alpha - \bar{c})^2 \cdot (25\phi - 4)}{(125\phi^2 - 5\phi\theta^2 + 5\phi - 2\theta^2 - 2)^2}$$

## D Solution of four-firm case

### D.1 One *high*- and three *low*-productive firms (1H3L)

#### D.1.1 E0A [1H3L]



$$\text{Equations : } \begin{cases} e_1^H \cdot \left( \frac{25\phi}{4} - 4 \right) = \alpha - e_1^L \theta - e_2^L \theta - e_3^L \theta - \bar{c} \\ e_1^L \cdot \left( \frac{25\phi}{4\theta} - 4\theta \right) = \alpha - e_1^H - e_2^L \theta - e_3^L \theta - \bar{c} \\ e_2^L \cdot \left( \frac{25\phi}{4\theta} - 4\theta \right) = \alpha - e_1^H - e_1^L \theta - e_3^L \theta - \bar{c} \\ e_3^L \cdot \left( \frac{25\phi}{4\theta} - 4\theta \right) = \alpha - e_1^H - e_1^L \theta - e_2^L \theta - \bar{c} \end{cases}$$

Optimal efforts:

$$\begin{cases} e_1^H &= \frac{4(\alpha - \bar{c})(5\phi - 4\theta^2)}{125\phi^2 - 40\phi\theta^2 - 80\phi + 16\theta^2} \\ e_1^L &= \frac{4\theta(\alpha - \bar{c})(5\phi - 4)}{125\phi^2 - 40\phi\theta^2 - 80\phi + 16\theta^2} \\ e_2^L &= \frac{4\theta(\alpha - \bar{c})(5\phi - 4)}{125\phi^2 - 40\phi\theta^2 - 80\phi + 16\theta^2} \\ e_3^L &= \frac{4\theta(\alpha - \bar{c})(5\phi - 4)}{125\phi^2 - 40\phi\theta^2 - 80\phi + 16\theta^2} \end{cases}$$

Production Costs:

$$\begin{cases} c_1^H &= -\frac{20\alpha\phi - 16\alpha\theta^2 - 125\bar{c}\phi^2 + 40\bar{c}\phi\theta^2 + 60\bar{c}\phi}{125\phi^2 - 40\phi\theta^2 - 80\phi + 16\theta^2} \\ c_1^L &= -\frac{20\alpha\phi\theta^2 - 16\alpha\theta^2 - 125\bar{c}\phi^2 + 20\bar{c}\phi\theta^2 + 80\bar{c}\phi}{125\phi^2 - 40\phi\theta^2 - 80\phi + 16\theta^2} \\ c_2^L &= -\frac{20\alpha\phi\theta^2 - 16\alpha\theta^2 - 125\bar{c}\phi^2 + 20\bar{c}\phi\theta^2 + 80\bar{c}\phi}{125\phi^2 - 40\phi\theta^2 - 80\phi + 16\theta^2} \\ c_3^L &= -\frac{20\alpha\phi\theta^2 - 16\alpha\theta^2 - 125\bar{c}\phi^2 + 20\bar{c}\phi\theta^2 + 80\bar{c}\phi}{125\phi^2 - 40\phi\theta^2 - 80\phi + 16\theta^2} \end{cases}$$

Production Quantities:

$$\begin{cases} q_1^H &= \frac{5\phi(\alpha - \bar{c})(5\phi - 4\theta^2)}{125\phi^2 - 40\phi\theta^2 - 80\phi + 16\theta^2} \\ q_1^L &= \frac{5\phi(\alpha - \bar{c})(5\phi - 4)}{125\phi^2 - 40\phi\theta^2 - 80\phi + 16\theta^2} \\ q_2^L &= \frac{5\phi(\alpha - \bar{c})(5\phi - 4)}{125\phi^2 - 40\phi\theta^2 - 80\phi + 16\theta^2} \\ q_3^L &= \frac{5\phi(\alpha - \bar{c})(5\phi - 4)}{125\phi^2 - 40\phi\theta^2 - 80\phi + 16\theta^2} \end{cases}$$

Profits:

$$\begin{cases} \pi_1^H &= \frac{\phi(\alpha - \bar{c})^2(5\phi - 4\theta^2)^2 \cdot (25\phi - 16)}{(125\phi^2 - 40\phi\theta^2 - 80\phi + 16\theta^2)^2} \\ \pi_1^L &= \frac{\phi(\alpha - \bar{c})^2(5\phi - 4)^2 \cdot (25\phi - 16\theta^2)}{(125\phi^2 - 40\phi\theta^2 - 80\phi + 16\theta^2)^2} \\ \pi_2^L &= \frac{\phi(\alpha - \bar{c})^2(5\phi - 4)^2 \cdot (25\phi - 16\theta^2)}{(125\phi^2 - 40\phi\theta^2 - 80\phi + 16\theta^2)^2} \\ \pi_3^L &= \frac{\phi(\alpha - \bar{c})^2(5\phi - 4)^2 \cdot (25\phi - 16\theta^2)}{(125\phi^2 - 40\phi\theta^2 - 80\phi + 16\theta^2)^2} \end{cases} \quad (32)$$

Total Production:

$$\frac{20\phi(\alpha - \bar{c})(5\phi - \theta^2 - 3)}{125\phi^2 - 40\phi\theta^2 - 80\phi + 16\theta^2}$$

Price:

$$\frac{25\alpha\phi^2 - 20\alpha\phi\theta^2 - 20\alpha\phi + 16\alpha\theta^2 + 100\bar{c}\phi^2 - 20\bar{c}\phi\theta^2 - 60\bar{c}\phi}{125\phi^2 - 40\phi\theta^2 - 80\phi + 16\theta^2}$$

Firm Surplus:

$$\frac{4\phi(\alpha - \bar{c})^2 \cdot (625\phi^3 - 550\phi^2\theta^2 - 850\phi^2 + 100\phi\theta^4 + 640\phi\theta^2 + 300\phi - 64\theta^4 - 192\theta^2)}{(125\phi^2 - 40\phi\theta^2 - 80\phi + 16\theta^2)^2}$$

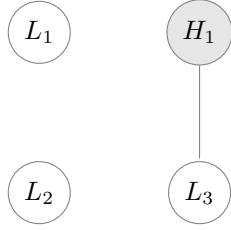
Consumer Surplus:

$$\frac{200\phi^2(\alpha - \bar{c})^2(5\phi - \theta^2 - 3)^2}{(125\phi^2 - 40\phi\theta^2 - 80\phi + 16\theta^2)^2}$$

Social Welfare:

$$\frac{4\phi(\alpha - \bar{c})^2 \cdot (1875\phi^3 - 1050\phi^2\theta^2 - 2350\phi^2 + 150\phi\theta^4 + 940\phi\theta^2 + 750\phi - 64\theta^4 - 192\theta^2)}{(125\phi^2 - 40\phi\theta^2 - 80\phi + 16\theta^2)^2}$$

### D.1.2 E1A [1H3L]



$$\text{Equations : } \begin{cases} e_1^H \cdot \left(\frac{25\phi}{3} - 3\right) = \alpha - e_1^L\theta - e_2^L\theta + 3e_3^L\theta - \bar{c} \\ e_1^L \cdot \left(\frac{25\phi}{4\theta} - 4\theta\right) = \alpha - 2e_1^H - e_2^L\theta - 2e_3^L\theta - \bar{c} \\ e_2^L \cdot \left(\frac{25\phi}{4\theta} - 4\theta\right) = \alpha - 2e_1^H - e_1^L\theta - 2e_3^L\theta - \bar{c} \\ e_3^L \cdot \left(\frac{25\phi}{3\theta} - 3\theta\right) = \alpha + 3e_1^H - e_1^L\theta - e_2^L\theta - \bar{c} \end{cases}$$

Optimal efforts:

$$\begin{cases} e_1^H = \frac{3(\alpha - \bar{c})(5\phi - 4\theta^2)}{125\phi^2 - 105\phi\theta^2 - 45\phi + 12\theta^4 + 12\theta^2} \\ e_1^L = \frac{4\theta(\alpha - \bar{c})(5\phi - 3\theta^2 - 3)}{125\phi^2 - 105\phi\theta^2 - 45\phi + 12\theta^4 + 12\theta^2} \\ e_2^L = \frac{4\theta(\alpha - \bar{c})(5\phi - 3\theta^2 - 3)}{125\phi^2 - 105\phi\theta^2 - 45\phi + 12\theta^4 + 12\theta^2} \\ e_3^L = \frac{3\theta(\alpha - \bar{c})(5\phi - 4\theta^2)}{125\phi^2 - 105\phi\theta^2 - 45\phi + 12\theta^4 + 12\theta^2} \end{cases}$$

Production Costs:

$$\begin{cases} c_1^H = -\frac{15\alpha\phi\theta^2 + 15\alpha\phi - 12\alpha\theta^4 - 12\alpha\theta^2 - 125\bar{c}\phi^2 + 90\bar{c}\phi\theta^2 + 30\bar{c}\phi}{125\phi^2 - 105\phi\theta^2 - 45\phi + 12\theta^4 + 12\theta^2} \\ c_1^L = -\frac{20\alpha\phi\theta^2 - 12\alpha\theta^4 - 12\alpha\theta^2 - 125\bar{c}\phi^2 + 85\bar{c}\phi\theta^2 + 45\bar{c}\phi}{125\phi^2 - 105\phi\theta^2 - 45\phi + 12\theta^4 + 12\theta^2} \\ c_2^L = -\frac{20\alpha\phi\theta^2 - 12\alpha\theta^4 - 12\alpha\theta^2 - 125\bar{c}\phi^2 + 85\bar{c}\phi\theta^2 + 45\bar{c}\phi}{125\phi^2 - 105\phi\theta^2 - 45\phi + 12\theta^4 + 12\theta^2} \\ c_3^L = -\frac{15\alpha\phi\theta^2 + 15\alpha\phi - 12\alpha\theta^4 - 12\alpha\theta^2 - 125\bar{c}\phi^2 + 90\bar{c}\phi\theta^2 + 30\bar{c}\phi}{125\phi^2 - 105\phi\theta^2 - 45\phi + 12\theta^4 + 12\theta^2} \end{cases}$$

Production Quantities:

$$\begin{cases} q_1^H = \frac{5\phi(\alpha - \bar{c})(5\phi - 4\theta^2)}{125\phi^2 - 105\phi\theta^2 - 45\phi + 12\theta^4 + 12\theta^2} \\ q_1^L = \frac{5\phi(\alpha - \bar{c})(5\phi - 3\theta^2 - 3)}{125\phi^2 - 105\phi\theta^2 - 45\phi + 12\theta^4 + 12\theta^2} \\ q_2^L = \frac{5\phi(\alpha - \bar{c})(5\phi - 3\theta^2 - 3)}{125\phi^2 - 105\phi\theta^2 - 45\phi + 12\theta^4 + 12\theta^2} \\ q_3^L = \frac{5\phi(\alpha - \bar{c})(5\phi - 4\theta^2)}{125\phi^2 - 105\phi\theta^2 - 45\phi + 12\theta^4 + 12\theta^2} \end{cases}$$

Profits:

$$\begin{cases} \pi_1^H = \frac{\phi(\alpha - \bar{c})^2(5\phi - 4\theta^2)^2 \cdot (25\phi - 9)}{(125\phi^2 - 105\phi\theta^2 - 45\phi + 12\theta^4 + 12\theta^2)^2} \\ \pi_1^L = \frac{\phi(\alpha - \bar{c})^2 \cdot (25\phi - 16\theta^2)(5\phi - 3\theta^2 - 3)^2}{(125\phi^2 - 105\phi\theta^2 - 45\phi + 12\theta^4 + 12\theta^2)^2} \\ \pi_2^L = \frac{\phi(\alpha - \bar{c})^2 \cdot (25\phi - 16\theta^2)(5\phi - 3\theta^2 - 3)^2}{(125\phi^2 - 105\phi\theta^2 - 45\phi + 12\theta^4 + 12\theta^2)^2} \\ \pi_3^L = \frac{\phi(\alpha - \bar{c})^2(5\phi - 4\theta^2)^2 \cdot (25\phi - 9\theta^2)}{(125\phi^2 - 105\phi\theta^2 - 45\phi + 12\theta^4 + 12\theta^2)^2} \end{cases} \quad (33)$$

Total Production:

$$\frac{10\phi(\alpha - \bar{c})(10\phi - 7\theta^2 - 3)}{125\phi^2 - 105\phi\theta^2 - 45\phi + 12\theta^4 + 12\theta^2}$$

Price:

$$\frac{25\alpha\phi^2 - 35\alpha\phi\theta^2 - 15\alpha\phi + 12\alpha\theta^4 + 12\alpha\theta^2 + 100\bar{c}\phi^2 - 70\bar{c}\phi\theta^2 - 30\bar{c}\phi}{125\phi^2 - 105\phi\theta^2 - 45\phi + 12\theta^4 + 12\theta^2}$$

Firm Surplus:

$$\frac{\phi(\alpha - \bar{c})^2 \cdot (2500\phi^3 - 4525\phi^2\theta^2 - 1725\phi^2 + 2570\phi\theta^4 + 2220\phi\theta^2 + 450\phi - 432\theta^6 - 720\theta^4 - 288\theta^2)}{(125\phi^2 - 105\phi\theta^2 - 45\phi + 12\theta^4 + 12\theta^2)^2}$$

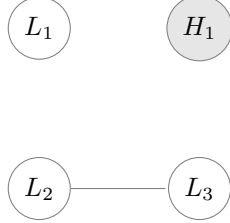
Consumer Surplus:

$$\frac{50\phi^2 (\alpha - \bar{c})^2 (10\phi - 7\theta^2 - 3)^2}{(125\phi^2 - 105\phi\theta^2 - 45\phi + 12\theta^4 + 12\theta^2)^2}$$

Social Welfare:

$$\frac{\phi (\alpha - \bar{c})^2 \cdot (7500\phi^3 - 11525\phi^2\theta^2 - 4725\phi^2 + 5020\phi\theta^4 + 4320\phi\theta^2 + 900\phi - 432\theta^6 - 720\theta^4 - 288\theta^2)}{(125\phi^2 - 105\phi\theta^2 - 45\phi + 12\theta^4 + 12\theta^2)^2}$$

### D.1.3 E1B [1H3L]



$$\text{Equations : } \begin{cases} e_1^H \cdot \left( \frac{25\phi}{4} - 4 \right) = \alpha - e_1^L\theta - 2e_2^L\theta - 2e_3^L\theta - \bar{c} \\ e_1^L \cdot \left( \frac{25\phi}{4\theta} - 4\theta \right) = \alpha - e_1^H - 2e_2^L\theta - 2e_3^L\theta - \bar{c} \\ e_2^L \cdot \left( \frac{25\phi}{3\theta} - 3\theta \right) = \alpha - e_1^H - e_1^L\theta + 3e_3^L\theta - \bar{c} \\ e_3^L \cdot \left( \frac{25\phi}{3\theta} - 3\theta \right) = \alpha - e_1^H - e_1^L\theta + 3e_2^L\theta - \bar{c} \end{cases}$$

Optimal efforts:

$$\begin{cases} e_1^H = \frac{4(\alpha - \bar{c})(5\phi - 6\theta^2)(5\phi - 4\theta^2)}{625\phi^3 - 850\phi^2\theta^2 - 400\phi^2 + 240\phi\theta^4 + 480\phi\theta^2 - 96\theta^4} \\ e_1^L = \frac{4\theta(\alpha - \bar{c})(5\phi - 4)(5\phi - 6\theta^2)}{625\phi^3 - 850\phi^2\theta^2 - 400\phi^2 + 240\phi\theta^4 + 480\phi\theta^2 - 96\theta^4} \\ e_2^L = \frac{3\theta(\alpha - \bar{c})(5\phi - 4)(5\phi - 4\theta^2)}{625\phi^3 - 850\phi^2\theta^2 - 400\phi^2 + 240\phi\theta^4 + 480\phi\theta^2 - 96\theta^4} \\ e_3^L = \frac{3\theta(\alpha - \bar{c})(5\phi - 4)(5\phi - 4\theta^2)}{625\phi^3 - 850\phi^2\theta^2 - 400\phi^2 + 240\phi\theta^4 + 480\phi\theta^2 - 96\theta^4} \end{cases}$$

Production Costs:

$$\begin{cases} c_1^H = -\frac{100\alpha\phi^2 - 200\alpha\phi\theta^2 + 96\alpha\theta^4 - 625\bar{c}\phi^3 + 850\bar{c}\phi^2\theta^2 + 300\bar{c}\phi^2 - 240\bar{c}\phi\theta^4 - 280\bar{c}\phi\theta^2}{625\phi^3 - 850\phi^2\theta^2 - 400\phi^2 + 240\phi\theta^4 + 480\phi\theta^2 - 96\theta^4} \\ c_1^L = -\frac{100\alpha\phi^2\theta^2 - 120\alpha\phi\theta^4 - 80\alpha\phi\theta^2 + 96\alpha\theta^4 - 625\bar{c}\phi^3 + 750\bar{c}\phi^2\theta^2 + 400\bar{c}\phi^2 - 120\bar{c}\phi\theta^4 - 400\bar{c}\phi\theta^2}{625\phi^3 - 850\phi^2\theta^2 - 400\phi^2 + 240\phi\theta^4 + 480\phi\theta^2 - 96\theta^4} \\ c_2^L = -\frac{150\alpha\phi^2\theta^2 - 120\alpha\phi\theta^4 - 120\alpha\phi\theta^2 + 96\alpha\theta^4 - 625\bar{c}\phi^3 + 700\bar{c}\phi^2\theta^2 + 400\bar{c}\phi^2 - 120\bar{c}\phi\theta^4 - 360\bar{c}\phi\theta^2}{625\phi^3 - 850\phi^2\theta^2 - 400\phi^2 + 240\phi\theta^4 + 480\phi\theta^2 - 96\theta^4} \\ c_3^L = -\frac{150\alpha\phi^2\theta^2 - 120\alpha\phi\theta^4 - 120\alpha\phi\theta^2 + 96\alpha\theta^4 - 625\bar{c}\phi^3 + 700\bar{c}\phi^2\theta^2 + 400\bar{c}\phi^2 - 120\bar{c}\phi\theta^4 - 360\bar{c}\phi\theta^2}{625\phi^3 - 850\phi^2\theta^2 - 400\phi^2 + 240\phi\theta^4 + 480\phi\theta^2 - 96\theta^4} \end{cases}$$

Production Quantities:

$$\begin{cases} q_1^H = \frac{5\phi(\alpha - \bar{c})(5\phi - 6\theta^2)(5\phi - 4\theta^2)}{625\phi^3 - 850\phi^2\theta^2 - 400\phi^2 + 240\phi\theta^4 + 480\phi\theta^2 - 96\theta^4} \\ q_1^L = \frac{5\phi(\alpha - \bar{c})(5\phi - 4)(5\phi - 6\theta^2)}{625\phi^3 - 850\phi^2\theta^2 - 400\phi^2 + 240\phi\theta^4 + 480\phi\theta^2 - 96\theta^4} \\ q_2^L = \frac{5\phi(\alpha - \bar{c})(5\phi - 4)(5\phi - 4\theta^2)}{625\phi^3 - 850\phi^2\theta^2 - 400\phi^2 + 240\phi\theta^4 + 480\phi\theta^2 - 96\theta^4} \\ q_3^L = \frac{5\phi(\alpha - \bar{c})(5\phi - 4)(5\phi - 4\theta^2)}{625\phi^3 - 850\phi^2\theta^2 - 400\phi^2 + 240\phi\theta^4 + 480\phi\theta^2 - 96\theta^4} \end{cases}$$

Profits:

$$\begin{cases} \pi_1^H = \frac{\phi(\alpha - \bar{c})^2(5\phi - 6\theta^2)^2(5\phi - 4\theta^2)^2 \cdot (25\phi - 16)}{(625\phi^3 - 850\phi^2\theta^2 - 400\phi^2 + 240\phi\theta^4 + 480\phi\theta^2 - 96\theta^4)^2} \\ \pi_1^L = \frac{\phi(\alpha - \bar{c})^2(5\phi - 4)^2(5\phi - 6\theta^2)^2 \cdot (25\phi - 16\theta^2)}{(625\phi^3 - 850\phi^2\theta^2 - 400\phi^2 + 240\phi\theta^4 + 480\phi\theta^2 - 96\theta^4)^2} \\ \pi_2^L = \frac{\phi(\alpha - \bar{c})^2(5\phi - 4)^2(5\phi - 4\theta^2)^2 \cdot (25\phi - 9\theta^2)}{(625\phi^3 - 850\phi^2\theta^2 - 400\phi^2 + 240\phi\theta^4 + 480\phi\theta^2 - 96\theta^4)^2} \\ \pi_3^L = \frac{\phi(\alpha - \bar{c})^2(5\phi - 4)^2(5\phi - 4\theta^2)^2 \cdot (25\phi - 9\theta^2)}{(625\phi^3 - 850\phi^2\theta^2 - 400\phi^2 + 240\phi\theta^4 + 480\phi\theta^2 - 96\theta^4)^2} \end{cases} \quad (34)$$

Total Production:

$$\frac{20\phi(\alpha - \bar{c})(25\phi^2 - 30\phi\theta^2 - 15\phi + 6\theta^4 + 14\theta^2)}{625\phi^3 - 850\phi^2\theta^2 - 400\phi^2 + 240\phi\theta^4 + 480\phi\theta^2 - 96\theta^4}$$

Price:

$$\frac{125\alpha\phi^3 - 250\alpha\phi^2\theta^2 - 100\alpha\phi^2 + 120\alpha\phi\theta^4 + 200\alpha\phi\theta^2 - 96\alpha\theta^4 + 500\bar{c}\phi^3 - 600\bar{c}\phi^2\theta^2 - 300\bar{c}\phi^2 + 120\bar{c}\phi\theta^4 + 280\bar{c}\phi\theta^2}{625\phi^3 - 850\phi^2\theta^2 - 400\phi^2 + 240\phi\theta^4 + 480\phi\theta^2 - 96\theta^4}$$

Firm Surplus:

$$\frac{2\phi(\alpha - \bar{c})^2 \cdot (31250\phi^5 - 85625\phi^4\theta^2 - 42500\phi^4 + 88500\phi^3\theta^4 + 107000\phi^3\theta^2 + 15000\phi^3 - 40800\phi^2\theta^6 - 97200\phi^2\theta^4 - 34800\phi^2\theta^2 + 7200\phi\theta^8 + 36480\phi\theta^6 + 27040\phi\theta^4 - 4608\theta^8 - 6912\theta^6)}{(625\phi^3 - 850\phi^2\theta^2 - 400\phi^2 + 240\phi\theta^4 + 480\phi\theta^2 - 96\theta^4)^2}$$

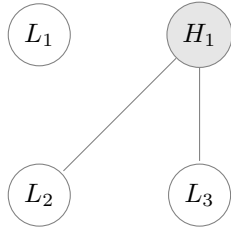
Consumer Surplus:

$$\frac{200\phi^2(\alpha - \bar{c})^2(25\phi^2 - 30\phi\theta^2 - 15\phi + 6\theta^4 + 14\theta^2)^2}{(625\phi^3 - 850\phi^2\theta^2 - 400\phi^2 + 240\phi\theta^4 + 480\phi\theta^2 - 96\theta^4)^2}$$

Social Welfare:

$$\frac{2\phi(\alpha - \bar{c})^2 \cdot (93750\phi^5 - 235625\phi^4\theta^2 - 117500\phi^4 + 208500\phi^3\theta^4 + 267000\phi^3\theta^2 + 375000\phi^3 - 76800\phi^2\theta^6 - 199200\phi^2\theta^4 - 76800\phi^2\theta^2 + 10800\phi\theta^8 + 53280\phi\theta^6 + 46640\phi\theta^4 - 4608\theta^8 - 6912\theta^6)}{(625\phi^3 - 850\phi^2\theta^2 - 400\phi^2 + 240\phi\theta^4 + 480\phi\theta^2 - 96\theta^4)^2}$$

#### D.1.4 E2A [1H3L]



$$\text{Equations : } \begin{cases} e_1^H \cdot \left(\frac{25\phi}{2} - 2\right) = \alpha - e_1^L\theta + 3e_2^L\theta + 3e_3^L\theta - \bar{c} \\ e_1^L \cdot \left(\frac{25\phi}{4\theta} - 4\theta\right) = \alpha - 3e_1^H - 2e_2^L\theta - 2e_3^L\theta - \bar{c} \\ e_2^L \cdot \left(\frac{25\phi}{3\theta} - 3\theta\right) = \alpha + 2e_1^H - e_1^L\theta - 2e_3^L\theta - \bar{c} \\ e_3^L \cdot \left(\frac{25\phi}{3\theta} - 3\theta\right) = \alpha + 2e_1^H - e_1^L\theta - 2e_2^L\theta - \bar{c} \end{cases}$$

Optimal efforts:

$$\begin{cases} e_1^H = \frac{2(\alpha - \bar{c})(5\phi - 4\theta^2)(5\phi + 3\theta^2)}{625\phi^3 - 475\phi^2\theta^2 - 100\phi^2 - 20\phi\theta^2 + 24\theta^4} \\ e_1^L = \frac{4\theta(\alpha - \bar{c})(25\phi^2 - 15\phi\theta^2 - 10\phi - 6\theta^2)}{625\phi^3 - 475\phi^2\theta^2 - 100\phi^2 - 20\phi\theta^2 + 24\theta^4} \\ e_2^L = \frac{15\phi\theta(\alpha - \bar{c})(5\phi - 4\theta^2)}{625\phi^3 - 475\phi^2\theta^2 - 100\phi^2 - 20\phi\theta^2 + 24\theta^4} \\ e_3^L = \frac{15\phi\theta(\alpha - \bar{c})(5\phi - 4\theta^2)}{625\phi^3 - 475\phi^2\theta^2 - 100\phi^2 - 20\phi\theta^2 + 24\theta^4} \end{cases}$$

Production Costs:

$$\begin{cases} c_1^H = -\frac{150\alpha\phi^2\theta^2 + 50\alpha\phi^2 - 120\alpha\phi\theta^4 - 10\alpha\phi\theta^2 - 24\alpha\theta^4 - 625\bar{c}\phi^3 + 325\bar{c}\phi^2\theta^2 + 50\bar{c}\phi^2 + 120\bar{c}\phi\theta^4 + 30\bar{c}\phi\theta^2}{625\phi^3 - 475\phi^2\theta^2 - 100\phi^2 - 20\phi\theta^2 + 24\theta^4} \\ c_1^L = -\frac{100\alpha\phi^2\theta^2 - 60\alpha\phi\theta^4 - 40\alpha\phi\theta^2 - 24\alpha\theta^4 - 625\bar{c}\phi^3 + 375\bar{c}\phi^2\theta^2 + 100\bar{c}\phi^2 + 60\bar{c}\phi\theta^4 + 60\bar{c}\phi\theta^2}{625\phi^3 - 475\phi^2\theta^2 - 100\phi^2 - 20\phi\theta^2 + 24\theta^4} \\ c_2^L = -\frac{75\alpha\phi^2\theta^2 + 50\alpha\phi^2 - 60\alpha\phi\theta^4 - 10\alpha\phi\theta^2 - 24\alpha\theta^4 - 625\bar{c}\phi^3 + 400\bar{c}\phi^2\theta^2 + 50\bar{c}\phi^2 + 60\bar{c}\phi\theta^4 + 30\bar{c}\phi\theta^2}{625\phi^3 - 475\phi^2\theta^2 - 100\phi^2 - 20\phi\theta^2 + 24\theta^4} \\ c_3^L = -\frac{75\alpha\phi^2\theta^2 + 50\alpha\phi^2 - 60\alpha\phi\theta^4 - 10\alpha\phi\theta^2 - 24\alpha\theta^4 - 625\bar{c}\phi^3 + 400\bar{c}\phi^2\theta^2 + 50\bar{c}\phi^2 + 60\bar{c}\phi\theta^4 + 30\bar{c}\phi\theta^2}{625\phi^3 - 475\phi^2\theta^2 - 100\phi^2 - 20\phi\theta^2 + 24\theta^4} \end{cases}$$

Production Quantities:

$$\begin{cases} q_1^H = \frac{5\phi(\alpha - \bar{c})(5\phi - 4\theta^2)(5\phi + 3\theta^2)}{625\phi^3 - 475\phi^2\theta^2 - 100\phi^2 - 20\phi\theta^2 + 24\theta^4} \\ q_1^L = \frac{5\phi(\alpha - \bar{c})(25\phi^2 - 15\phi\theta^2 - 10\phi - 6\theta^2)}{625\phi^3 - 475\phi^2\theta^2 - 100\phi^2 - 20\phi\theta^2 + 24\theta^4} \\ q_2^L = \frac{25\phi^2(\alpha - \bar{c})(5\phi - 4\theta^2)}{625\phi^3 - 475\phi^2\theta^2 - 100\phi^2 - 20\phi\theta^2 + 24\theta^4} \\ q_3^L = \frac{25\phi^2(\alpha - \bar{c})(5\phi - 4\theta^2)}{625\phi^3 - 475\phi^2\theta^2 - 100\phi^2 - 20\phi\theta^2 + 24\theta^4} \end{cases}$$

Profits:

$$\begin{cases} \pi_1^H = \frac{\phi(\alpha - \bar{c})^2(5\phi - 4\theta^2)^2(5\phi + 3\theta^2)^2 \cdot (25\phi - 4)}{(625\phi^3 - 475\phi^2\theta^2 - 100\phi^2 - 20\phi\theta^2 + 24\theta^4)^2} \\ \pi_1^L = \frac{\phi(\alpha - \bar{c})^2 \cdot (25\phi - 16\theta^2)(25\phi^2 - 15\phi\theta^2 - 10\phi - 6\theta^2)^2}{(625\phi^3 - 475\phi^2\theta^2 - 100\phi^2 - 20\phi\theta^2 + 24\theta^4)^2} \\ \pi_2^L = \frac{25\phi^3(\alpha - \bar{c})^2(5\phi - 4\theta^2)^2 \cdot (25\phi - 9\theta^2)}{(625\phi^3 - 475\phi^2\theta^2 - 100\phi^2 - 20\phi\theta^2 + 24\theta^4)^2} \\ \pi_3^L = \frac{25\phi^3(\alpha - \bar{c})^2(5\phi - 4\theta^2)^2 \cdot (25\phi - 9\theta^2)}{(625\phi^3 - 475\phi^2\theta^2 - 100\phi^2 - 20\phi\theta^2 + 24\theta^4)^2} \end{cases} \quad (35)$$

Total Production:

$$\frac{10\phi(\alpha - \bar{c})(50\phi^2 - 30\phi\theta^2 - 5\phi - 6\theta^4 - 3\theta^2)}{625\phi^3 - 475\phi^2\theta^2 - 100\phi^2 - 20\phi\theta^2 + 24\theta^4}$$

Price:

$$\frac{125\alpha\phi^3 - 175\alpha\phi^2\theta^2 - 50\alpha\phi^2 + 60\alpha\phi\theta^4 + 10\alpha\phi\theta^2 + 24\alpha\theta^4 + 500\bar{c}\phi^3 - 300\bar{c}\phi^2\theta^2 - 50\bar{c}\phi^2 - 60\bar{c}\phi\theta^4 - 30\bar{c}\phi\theta^2}{625\phi^3 - 475\phi^2\theta^2 - 100\phi^2 - 20\phi\theta^2 + 24\theta^4}$$

Firm Surplus:

$$\frac{2\phi(\alpha - \bar{c})^2 \cdot (31250\phi^5 - 48125\phi^4\theta^2 - 7500\phi^4 + 20625\phi^3\theta^4 + 4500\phi^3\theta^2 + 1250\phi^3 - 3900\phi^2\theta^6 + 3400\phi^2\theta^4 + 700\phi^2\theta^2 + 1800\phi\theta^8 - 1680\phi\theta^6 - 510\phi\theta^4 - 288\theta^8 - 288\theta^6)}{(625\phi^3 - 475\phi^2\theta^2 - 100\phi^2 - 20\phi\theta^2 + 24\theta^4)^2}$$

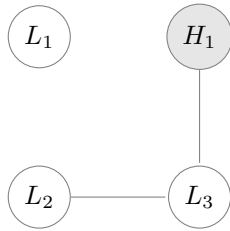
Consumer Surplus:

$$\frac{50\phi^2(\alpha - \bar{c})^2(50\phi^2 - 30\phi\theta^2 - 5\phi - 6\theta^4 - 3\theta^2)^2}{(625\phi^3 - 475\phi^2\theta^2 - 100\phi^2 - 20\phi\theta^2 + 24\theta^4)^2}$$

Social Welfare:

$$\frac{2\phi(\alpha - \bar{c})^2 \cdot (93750\phi^5 - 123125\phi^4\theta^2 - 20000\phi^4 + 28125\phi^3\theta^4 + 4500\phi^3\theta^2 + 1875\phi^3 + 5100\phi^2\theta^6 + 9400\phi^2\theta^4 + 1450\phi^2\theta^2 + 2700\phi\theta^8 - 780\phi\theta^6 - 285\phi\theta^4 - 288\theta^8 - 288\theta^6)}{(625\phi^3 - 475\phi^2\theta^2 - 100\phi^2 - 20\phi\theta^2 + 24\theta^4)^2}$$

### D.1.5 E2B [1H3L]



$$\text{Equations : } \begin{cases} e_1^H \cdot \left(\frac{25\phi}{3} - 3\right) = \alpha - e_1^L\theta - 2e_2^L\theta + 2e_3^L\theta - \bar{c} \\ e_1^L \cdot \left(\frac{25\phi}{4\theta} - 4\theta\right) = \alpha - 2e_1^H - 2e_2^H\theta - 3e_3^H\theta - \bar{c} \\ e_2^L \cdot \left(\frac{25\phi}{3\theta} - 3\theta\right) = \alpha - 2e_1^H - e_1^L\theta + 2e_3^L\theta - \bar{c} \\ e_3^L \cdot \left(\frac{25\phi}{2\theta} - 2\theta\right) = \alpha + 3e_1^H - e_1^L\theta + 3e_2^L\theta - \bar{c} \end{cases}$$

Optimal efforts:

$$\begin{cases} e_1^H = \frac{15\phi(\alpha - \bar{c})(5\phi - 4\theta^2)(5\phi - 3\theta^2)}{3125\phi^4 - 3625\phi^3\theta^2 - 1125\phi^3 + 800\phi^2\theta^4 + 825\phi^2\theta^2 + 180\phi\theta^4 - 72\theta^6} \\ e_1^L = \frac{4\theta(\alpha - \bar{c})(125\phi^3 - 125\phi^2\theta^2 - 75\phi^2 + 45\phi\theta^2 + 18\theta^4)}{3125\phi^4 - 3625\phi^3\theta^2 - 1125\phi^3 + 800\phi^2\theta^4 + 825\phi^2\theta^2 + 180\phi\theta^4 - 72\theta^6} \\ e_2^L = \frac{15\phi\theta(\alpha - \bar{c})(5\phi - 3)(5\phi - 4\theta^2)}{3125\phi^4 - 3625\phi^3\theta^2 - 1125\phi^3 + 800\phi^2\theta^4 + 825\phi^2\theta^2 + 180\phi\theta^4 - 72\theta^6} \\ e_3^L = \frac{2\theta(\alpha - \bar{c})(5\phi - 3\theta)(5\phi + 3\theta)(5\phi - 4\theta^2)}{3125\phi^4 - 3625\phi^3\theta^2 - 1125\phi^3 + 800\phi^2\theta^4 + 825\phi^2\theta^2 + 180\phi\theta^4 - 72\theta^6} \end{cases}$$

Production Costs:

$$\begin{cases} c_1^H = -\frac{250\alpha\phi^3\theta^2 + 375\alpha\phi^3 - 200\alpha\phi^2\theta^4 - 525\alpha\phi^2\theta^2 + 90\alpha\phi\theta^4 + 72\alpha\theta^6 - 3125\bar{c}\phi^4 + 3375\bar{c}\phi^3\theta^2 + 750\bar{c}\phi^3 - 600\bar{c}\phi^2\theta^4 - 300\bar{c}\phi^2\theta^2 - 270\bar{c}\phi\theta^4}{3125\phi^4 - 3625\phi^3\theta^2 - 1125\phi^3 + 800\phi^2\theta^4 + 825\phi^2\theta^2 + 180\phi\theta^4 - 72\theta^6} \\ c_1^L = -\frac{500\alpha\phi^3\theta^2 - 500\alpha\phi^2\theta^4 - 300\alpha\phi^2\theta^2 + 180\alpha\phi\theta^4 + 72\alpha\theta^6 - 3125\bar{c}\phi^4 + 3125\bar{c}\phi^3\theta^2 + 1125\bar{c}\phi^3 - 300\bar{c}\phi^2\theta^4 - 525\bar{c}\phi^2\theta^2 - 360\bar{c}\phi\theta^4}{3125\phi^4 - 3625\phi^3\theta^2 - 1125\phi^3 + 800\phi^2\theta^4 + 825\phi^2\theta^2 + 180\phi\theta^4 - 72\theta^6} \\ c_2^L = -\frac{625\alpha\phi^3\theta^2 - 500\alpha\phi^2\theta^4 - 225\alpha\phi^2\theta^2 + 90\alpha\phi\theta^4 + 72\alpha\theta^6 - 3125\bar{c}\phi^4 + 3000\bar{c}\phi^3\theta^2 + 1125\bar{c}\phi^3 - 300\bar{c}\phi^2\theta^4 - 600\bar{c}\phi^2\theta^2 - 270\bar{c}\phi\theta^4}{3125\phi^4 - 3625\phi^3\theta^2 - 1125\phi^3 + 800\phi^2\theta^4 + 825\phi^2\theta^2 + 180\phi\theta^4 - 72\theta^6} \\ c_3^L = -\frac{625\alpha\phi^3\theta^2 + 375\alpha\phi^3 - 500\alpha\phi^2\theta^4 - 750\alpha\phi^2\theta^2 + 270\alpha\phi\theta^4 + 72\alpha\theta^6 - 3125\bar{c}\phi^4 + 3000\bar{c}\phi^3\theta^2 + 750\bar{c}\phi^3 - 300\bar{c}\phi^2\theta^4 - 75\bar{c}\phi^2\theta^2 - 450\bar{c}\phi\theta^4}{3125\phi^4 - 3625\phi^3\theta^2 - 1125\phi^3 + 800\phi^2\theta^4 + 825\phi^2\theta^2 + 180\phi\theta^4 - 72\theta^6} \end{cases}$$

Production Quantities:

$$\begin{cases} q_1^H = \frac{25\phi^2(\alpha - \bar{c})(5\phi - 4\theta^2)(5\phi - 3\theta^2)}{3125\phi^4 - 3625\phi^3\theta^2 - 1125\phi^3 + 800\phi^2\theta^4 + 825\phi^2\theta^2 + 180\phi\theta^4 - 72\theta^6} \\ q_1^L = \frac{5\phi(\alpha - \bar{c})(125\phi^3 - 125\phi^2\theta^2 - 75\phi^2 + 45\phi\theta^2 + 18\theta^4)}{3125\phi^4 - 3625\phi^3\theta^2 - 1125\phi^3 + 800\phi^2\theta^4 + 825\phi^2\theta^2 + 180\phi\theta^4 - 72\theta^6} \\ q_2^L = \frac{25\phi^2(\alpha - \bar{c})(5\phi - 3)(5\phi - 4\theta^2)}{3125\phi^4 - 3625\phi^3\theta^2 - 1125\phi^3 + 800\phi^2\theta^4 + 825\phi^2\theta^2 + 180\phi\theta^4 - 72\theta^6} \\ q_3^L = \frac{5\phi(\alpha - \bar{c})(5\phi - 3\theta)(5\phi + 3\theta)(5\phi - 4\theta^2)}{3125\phi^4 - 3625\phi^3\theta^2 - 1125\phi^3 + 800\phi^2\theta^4 + 825\phi^2\theta^2 + 180\phi\theta^4 - 72\theta^6} \end{cases}$$

Profits:

$$\begin{cases} \pi_1^H = \frac{25\phi^3(\alpha - \bar{c})^2(5\phi - 4\theta^2)^2(5\phi - 3\theta^2)^2 \cdot (25\phi - 9)}{(3125\phi^4 - 3625\phi^3\theta^2 - 1125\phi^3 + 800\phi^2\theta^4 + 825\phi^2\theta^2 + 180\phi\theta^4 - 72\theta^6)^2} \\ \pi_1^L = \frac{\phi(\alpha - \bar{c})^2 \cdot (25\phi - 16\theta^2)(125\phi^3 - 125\phi^2\theta^2 - 75\phi^2 + 45\phi\theta^2 + 18\theta^4)^2}{(3125\phi^4 - 3625\phi^3\theta^2 - 1125\phi^3 + 800\phi^2\theta^4 + 825\phi^2\theta^2 + 180\phi\theta^4 - 72\theta^6)^2} \\ \pi_2^L = \frac{25\phi^3(\alpha - \bar{c})^2(5\phi - 3)^2(5\phi - 4\theta^2)^2 \cdot (25\phi - 9\theta^2)}{(3125\phi^4 - 3625\phi^3\theta^2 - 1125\phi^3 + 800\phi^2\theta^4 + 825\phi^2\theta^2 + 180\phi\theta^4 - 72\theta^6)^2} \\ \pi_3^L = \frac{\phi(\alpha - \bar{c})^2(5\phi - 3\theta)^2(5\phi + 3\theta)^2(5\phi - 4\theta^2)^2 \cdot (25\phi - 4\theta^2)}{(3125\phi^4 - 3625\phi^3\theta^2 - 1125\phi^3 + 800\phi^2\theta^4 + 825\phi^2\theta^2 + 180\phi\theta^4 - 72\theta^6)^2} \end{cases} \quad (36)$$



Total Production:

$$\frac{10\phi(\alpha - \bar{c})(250\phi^3 - 250\phi^2\theta^2 - 75\phi^2 + 30\phi\theta^4 + 30\phi\theta^2 + 27\theta^4)}{3125\phi^4 - 3625\phi^3\theta^2 - 1125\phi^3 + 800\phi^2\theta^4 + 825\phi^2\theta^2 + 180\phi\theta^4 - 72\theta^6}$$

Price:

$$\frac{625\alpha\phi^4 - 1125\alpha\phi^3\theta^2 - 375\alpha\phi^3 + 500\alpha\phi^2\theta^4 + 525\alpha\phi^2\theta^2 - 90\alpha\phi\theta^4 - 72\alpha\theta^6 + 2500\bar{c}\phi^4 - 2500\bar{c}\phi^3\theta^2 - 750\bar{c}\phi^3 + 300\bar{c}\phi^2\theta^4 + 300\bar{c}\phi^2\theta^2 + 270\bar{c}\phi\theta^4}{3125\phi^4 - 3625\phi^3\theta^2 - 1125\phi^3 + 800\phi^2\theta^4 + 825\phi^2\theta^2 + 180\phi\theta^4 - 72\theta^6}$$

Firm Surplus:

$$\frac{\phi(\alpha - \bar{c})^2 \cdot (1562500\phi^7 - 3578125\phi^6\theta^2 - 1078125\phi^6 + 2856250\phi^5\theta^4 + 2081250\phi^5\theta^2 + 281250\phi^5 - 905000\phi^4\theta^6 - 1134375\phi^4\theta^4 - 534375\phi^4\theta^2 + 90000\phi^3\theta^8 + 40500\phi^3\theta^6 + 312750\phi^3\theta^4 + 68400\phi^2\theta^8 - 70200\phi^2\theta^6 + 27540\phi\theta^8 - 10368\theta^{10})}{(3125\phi^4 - 3625\phi^3\theta^2 - 1125\phi^3 + 800\phi^2\theta^4 + 825\phi^2\theta^2 + 180\phi\theta^4 - 72\theta^6)^2}$$

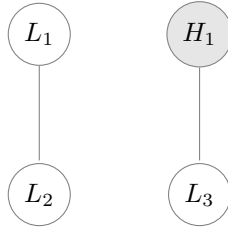
Consumer Surplus:

$$\frac{50\phi^2(\alpha - \bar{c})^2(250\phi^3 - 250\phi^2\theta^2 - 75\phi^2 + 30\phi\theta^4 + 30\phi\theta^2 + 27\theta^4)^2}{(3125\phi^4 - 3625\phi^3\theta^2 - 1125\phi^3 + 800\phi^2\theta^4 + 825\phi^2\theta^2 + 180\phi\theta^4 - 72\theta^6)^2}$$

Social Welfare:

$$\frac{\phi(\alpha - \bar{c})^2 \cdot (4687500\phi^7 - 9828125\phi^6\theta^2 - 2953125\phi^6 + 6731250\phi^5\theta^4 + 4706250\phi^5\theta^2 + 562500\phi^5 - 1655000\phi^4\theta^6 - 1434375\phi^4\theta^4 - 759375\phi^4\theta^2 + 135000\phi^3\theta^8 - 544500\phi^3\theta^6 + 155250\phi^3\theta^4 + 149400\phi^2\theta^8 + 10800\phi^2\theta^6 + 63990\phi\theta^8 - 10368\theta^{10})}{(3125\phi^4 - 3625\phi^3\theta^2 - 1125\phi^3 + 800\phi^2\theta^4 + 825\phi^2\theta^2 + 180\phi\theta^4 - 72\theta^6)^2}$$

### D.1.6 E2C [1H3L]



$$\text{Equations : } \begin{cases} e_1^H \cdot \left(\frac{25\phi}{3} - 3\right) = \alpha - 2e_1^L\theta - 2e_2^L\theta + 3e_3^L\theta - \bar{c} \\ e_1^L \cdot \left(\frac{25\phi}{3\theta} - 3\theta\right) = \alpha - 2e_1^H + 3e_2^L\theta - 2e_3^L\theta - \bar{c} \\ e_2^L \cdot \left(\frac{25\phi}{3\theta} - 3\theta\right) = \alpha - 2e_1^H + 3e_1^L\theta - 2e_3^L\theta - \bar{c} \\ e_3^L \cdot \left(\frac{25\phi}{3\theta} - 3\theta\right) = \alpha + 3e_1^H - 2e_1^L\theta - 2e_2^L\theta - \bar{c} \end{cases}$$

Optimal efforts:

$$\begin{cases} e_1^H = \frac{3(\alpha - \bar{c})(5\phi - 6\theta^2)}{125\phi^2 - 135\phi\theta^2 - 45\phi + 18\theta^4 + 18\theta^2} \\ e_1^L = \frac{3\theta(\alpha - \bar{c})(5\phi - 3\theta^2 - 3)}{125\phi^2 - 135\phi\theta^2 - 45\phi + 18\theta^4 + 18\theta^2} \\ e_2^L = \frac{3\theta(\alpha - \bar{c})(5\phi - 3\theta^2 - 3)}{125\phi^2 - 135\phi\theta^2 - 45\phi + 18\theta^4 + 18\theta^2} \\ e_3^L = \frac{3\theta(\alpha - \bar{c})(5\phi - 6\theta^2)}{125\phi^2 - 135\phi\theta^2 - 45\phi + 18\theta^4 + 18\theta^2} \end{cases}$$

Production Costs:

$$\begin{cases} c_1^H = -\frac{15\alpha\phi\theta^2 + 15\alpha\phi - 18\alpha\theta^4 - 18\alpha\theta^2 - 125\bar{c}\phi^2 + 120\bar{c}\phi\theta^2 + 30\bar{c}\phi}{125\phi^2 - 135\phi\theta^2 - 45\phi + 18\theta^4 + 18\theta^2} \\ c_1^L = -\frac{30\alpha\phi\theta^2 - 18\alpha\theta^4 - 18\alpha\theta^2 - 125\bar{c}\phi^2 + 105\bar{c}\phi\theta^2 + 45\bar{c}\phi}{125\phi^2 - 135\phi\theta^2 - 45\phi + 18\theta^4 + 18\theta^2} \\ c_2^L = -\frac{30\alpha\phi\theta^2 - 18\alpha\theta^4 - 18\alpha\theta^2 - 125\bar{c}\phi^2 + 105\bar{c}\phi\theta^2 + 45\bar{c}\phi}{125\phi^2 - 135\phi\theta^2 - 45\phi + 18\theta^4 + 18\theta^2} \\ c_3^L = -\frac{15\alpha\phi\theta^2 + 15\alpha\phi - 18\alpha\theta^4 - 18\alpha\theta^2 - 125\bar{c}\phi^2 + 120\bar{c}\phi\theta^2 + 30\bar{c}\phi}{125\phi^2 - 135\phi\theta^2 - 45\phi + 18\theta^4 + 18\theta^2} \end{cases}$$

Production Quantities:

$$\begin{cases} q_1^H = \frac{5\phi(\alpha - \bar{c})(5\phi - 6\theta^2)}{125\phi^2 - 135\phi\theta^2 - 45\phi + 18\theta^4 + 18\theta^2} \\ q_1^L = \frac{5\phi(\alpha - \bar{c})(5\phi - 3\theta^2 - 3)}{125\phi^2 - 135\phi\theta^2 - 45\phi + 18\theta^4 + 18\theta^2} \\ q_2^L = \frac{5\phi(\alpha - \bar{c})(5\phi - 3\theta^2 - 3)}{125\phi^2 - 135\phi\theta^2 - 45\phi + 18\theta^4 + 18\theta^2} \\ q_3^L = \frac{5\phi(\alpha - \bar{c})(5\phi - 6\theta^2)}{125\phi^2 - 135\phi\theta^2 - 45\phi + 18\theta^4 + 18\theta^2} \end{cases}$$

Profits:

$$\begin{cases} \pi_1^H &= \frac{\phi(\alpha-\bar{c})^2(5\phi-6\theta^2)^2 \cdot (25\phi-9)}{(125\phi^2-135\phi\theta^2-45\phi+18\theta^4+18\theta^2)^2} \\ \pi_1^L &= \frac{\phi(\alpha-\bar{c})^2 \cdot (25\phi-9\theta^2)(5\phi-3\theta^2-3)^2}{(125\phi^2-135\phi\theta^2-45\phi+18\theta^4+18\theta^2)^2} \\ \pi_2^L &= \frac{\phi(\alpha-\bar{c})^2 \cdot (25\phi-9\theta^2)(5\phi-3\theta^2-3)^2}{(125\phi^2-135\phi\theta^2-45\phi+18\theta^4+18\theta^2)^2} \\ \pi_3^L &= \frac{\phi(\alpha-\bar{c})^2(5\phi-6\theta^2)^2 \cdot (25\phi-9\theta^2)}{(125\phi^2-135\phi\theta^2-45\phi+18\theta^4+18\theta^2)^2} \end{cases} \quad (37)$$

Total Production:

$$\frac{10\phi(\alpha-\bar{c})(10\phi-9\theta^2-3)}{125\phi^2-135\phi\theta^2-45\phi+18\theta^4+18\theta^2}$$

Price:

$$\frac{25\alpha\phi^2-45\alpha\phi\theta^2-15\alpha\phi+18\alpha\theta^4+18\alpha\theta^2+100\bar{c}\phi^2-90\bar{c}\phi\theta^2-30\bar{c}\phi}{125\phi^2-135\phi\theta^2-45\phi+18\theta^4+18\theta^2}$$

Firm Surplus:

$$\frac{\phi(\alpha-\bar{c})^2 \cdot (2500\phi^3-5175\phi^2\theta^2-1725\phi^2+3330\phi\theta^4+1980\phi\theta^2+450\phi-486\theta^6-648\theta^4-162\theta^2)}{(125\phi^2-135\phi\theta^2-45\phi+18\theta^4+18\theta^2)^2}$$

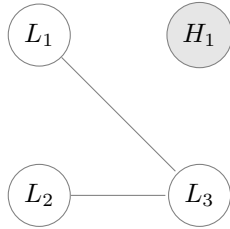
Consumer Surplus:

$$\frac{50\phi^2(\alpha-\bar{c})^2(10\phi-9\theta^2-3)^2}{(125\phi^2-135\phi\theta^2-45\phi+18\theta^4+18\theta^2)^2}$$

Social Welfare:

$$\frac{3\phi(\alpha-\bar{c})^2 \cdot (2500\phi^3-4725\phi^2\theta^2-1575\phi^2+2460\phi\theta^4+1560\phi\theta^2+300\phi-162\theta^6-216\theta^4-54\theta^2)}{(125\phi^2-135\phi\theta^2-45\phi+18\theta^4+18\theta^2)^2}$$

#### D.1.7 E2D [1H3L]



$$\text{Equations : } \begin{cases} e_1^H \cdot \left(\frac{25\phi}{4}-4\right) = \alpha - 2e_1^L\theta - 2e_2^L\theta - 3e_3^L\theta - \bar{c} \\ e_1^L \cdot \left(\frac{25\phi}{3\theta}-3\theta\right) = \alpha - e_1^H - 2e_2^L\theta + 2e_3^L\theta - \bar{c} \\ e_2^L \cdot \left(\frac{25\phi}{3\theta}-3\theta\right) = \alpha - e_1^H - 2e_1^L\theta + 2e_3^L\theta - \bar{c} \\ e_3^L \cdot \left(\frac{25\phi}{2\theta}-2\theta\right) = \alpha - e_1^H + 3e_1^L\theta + 3e_2^L\theta - \bar{c} \end{cases}$$

Optimal efforts:

$$\begin{cases} e_1^H &= \frac{4(\alpha-\bar{c})(5\phi-6\theta^2)}{125\phi^2-60\phi\theta^2-80\phi+24\theta^2} \\ e_1^L &= \frac{15\phi\theta(\alpha-\bar{c})(5\phi-4)}{(5\phi+\theta^2)(125\phi^2-60\phi\theta^2-80\phi+24\theta^2)} \\ e_2^L &= \frac{15\phi\theta(\alpha-\bar{c})(5\phi-4)}{(5\phi+\theta^2)(125\phi^2-60\phi\theta^2-80\phi+24\theta^2)} \\ e_3^L &= \frac{2\theta(\alpha-\bar{c})(5\phi-4)(5\phi+3\theta^2)}{(5\phi+\theta^2)(125\phi^2-60\phi\theta^2-80\phi+24\theta^2)} \end{cases}$$

Production Costs:

$$\begin{cases} c_1^H &= -\frac{20\alpha\phi-24\alpha\theta^2-125\bar{c}\phi^2+60\bar{c}\phi\theta^2+60\bar{c}\phi}{125\phi^2-60\phi\theta^2-80\phi+24\theta^2} \\ c_1^L &= -\frac{125\alpha\phi^2\theta^2+30\alpha\phi\theta^4-100\alpha\phi\theta^2-24\alpha\theta^4-625\bar{c}\phi^3+50\bar{c}\phi^2\theta^2+400\bar{c}\phi^2+30\bar{c}\phi\theta^4+60\bar{c}\phi\theta^2}{(5\phi+\theta^2)(125\phi^2-60\phi\theta^2-80\phi+24\theta^2)} \\ c_2^L &= -\frac{125\alpha\phi^2\theta^2+30\alpha\phi\theta^4-100\alpha\phi\theta^2-24\alpha\theta^4-625\bar{c}\phi^3+50\bar{c}\phi^2\theta^2+400\bar{c}\phi^2+30\bar{c}\phi\theta^4+60\bar{c}\phi\theta^2}{(5\phi+\theta^2)(125\phi^2-60\phi\theta^2-80\phi+24\theta^2)} \\ c_3^L &= -\frac{200\alpha\phi^2\theta^2+30\alpha\phi\theta^4-160\alpha\phi\theta^2-24\alpha\theta^4-625\bar{c}\phi^3-25\bar{c}\phi^2\theta^2+400\bar{c}\phi^2+30\bar{c}\phi\theta^4+120\bar{c}\phi\theta^2}{(5\phi+\theta^2)(125\phi^2-60\phi\theta^2-80\phi+24\theta^2)} \end{cases}$$

Production Quantities:

$$\begin{cases} q_1^H &= \frac{5\phi(\alpha-\bar{c})(5\phi-6\theta^2)}{125\phi^2-60\phi\theta^2-80\phi+24\theta^2} \\ q_1^L &= \frac{25\phi^2(\alpha-\bar{c})(5\phi-4)}{(5\phi+\theta^2)(125\phi^2-60\phi\theta^2-80\phi+24\theta^2)} \\ q_2^L &= \frac{25\phi^2(\alpha-\bar{c})(5\phi-4)}{(5\phi+\theta^2)(125\phi^2-60\phi\theta^2-80\phi+24\theta^2)} \\ q_3^L &= \frac{5\phi(\alpha-\bar{c})(5\phi-4)(5\phi+3\theta^2)}{(5\phi+\theta^2)(125\phi^2-60\phi\theta^2-80\phi+24\theta^2)} \end{cases}$$

Profits:

$$\begin{cases} \pi_1^H &= \frac{\phi(\alpha-\bar{c})^2(5\phi-6\theta^2)^2 \cdot (25\phi-16)}{(125\phi^2-60\phi\theta^2-80\phi+24\theta^2)^2} \\ \pi_1^L &= \frac{25\phi^3(\alpha-\bar{c})^2(5\phi-4)^2 \cdot (25\phi-9\theta^2)}{(5\phi+\theta^2)^2(125\phi^2-60\phi\theta^2-80\phi+24\theta^2)^2} \\ \pi_2^L &= \frac{25\phi^3(\alpha-\bar{c})^2(5\phi-4)^2 \cdot (25\phi-9\theta^2)}{(5\phi+\theta^2)^2(125\phi^2-60\phi\theta^2-80\phi+24\theta^2)^2} \\ \pi_3^L &= \frac{\phi(\alpha-\bar{c})^2(5\phi-4)^2(5\phi+3\theta^2)^2 \cdot (25\phi-4\theta^2)}{(5\phi+\theta^2)^2(125\phi^2-60\phi\theta^2-80\phi+24\theta^2)^2} \end{cases} \quad (38)$$

Total Production:

$$\frac{10\phi(\alpha-\bar{c})(10\phi-3\theta^2-6)}{125\phi^2-60\phi\theta^2-80\phi+24\theta^2}$$

Price:

$$\frac{25\alpha\phi^2-30\alpha\phi\theta^2-20\alpha\phi+24\alpha\theta^2+100\bar{c}\phi^2-30\bar{c}\phi\theta^2-60\bar{c}\phi}{125\phi^2-60\phi\theta^2-80\phi+24\theta^2}$$

Firm Surplus:

$$\frac{2\phi(\alpha-\bar{c})^2 \cdot (31250\phi^5-13125\phi^4\theta^2-42500\phi^4+5375\phi^3\theta^4+6000\phi^3\theta^2+15000\phi^3+3300\phi^2\theta^6-4700\phi^2\theta^4+1600\phi^2\theta^2+450\phi\theta^8-1680\phi\theta^6+840\phi\theta^4-288\theta^8-288\theta^6)}{(5\phi+\theta^2)^2(125\phi^2-60\phi\theta^2-80\phi+24\theta^2)^2}$$

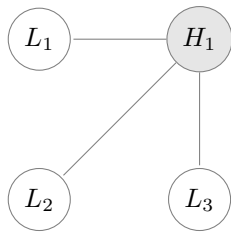
Consumer Surplus:

$$\frac{50\phi^2(\alpha-\bar{c})^2(10\phi-3\theta^2-6)^2}{(125\phi^2-60\phi\theta^2-80\phi+24\theta^2)^2}$$

Social Welfare:

$$\frac{2\phi(\alpha-\bar{c})^2 \cdot (93750\phi^5-25625\phi^4\theta^2-117500\phi^4-1500\phi^3\theta^4-1500\phi^3\theta^2+37500\phi^3+4050\phi^2\theta^6+1300\phi^2\theta^4+10600\phi^2\theta^2+675\phi\theta^8-780\phi\theta^6+1740\phi\theta^4-288\theta^8-288\theta^6)}{(5\phi+\theta^2)^2(125\phi^2-60\phi\theta^2-80\phi+24\theta^2)^2}$$

#### D.1.8 E3A [1H3L]



$$\text{Equations : } \begin{cases} e_1^H \cdot (25\phi-1) = \alpha + 3e_1^L\theta + 3e_2^L\theta + 3e_3^L\theta - \bar{c} \\ e_1^L \cdot \left(\frac{25\phi}{3\theta} - 3\theta\right) = \alpha + e_1^H - 2e_2^L\theta - 2e_3^L\theta - \bar{c} \\ e_2^L \cdot \left(\frac{25\phi}{3\theta} - 3\theta\right) = \alpha + e_1^H - 2e_1^L\theta - 2e_3^L\theta - \bar{c} \\ e_3^L \cdot \left(\frac{25\phi}{3\theta} - 3\theta\right) = \alpha + e_1^H - 2e_1^L\theta - 2e_2^L\theta - \bar{c} \end{cases}$$

Optimal efforts:

$$\begin{cases} e_1^H &= \frac{(\alpha-\bar{c})(5\phi+6\theta^2)}{125\phi^2+15\phi\theta^2-5\phi-6\theta^2} \\ e_1^L &= \frac{15\phi\theta(\alpha-\bar{c})}{125\phi^2+15\phi\theta^2-5\phi-6\theta^2} \\ e_2^L &= \frac{15\phi\theta(\alpha-\bar{c})}{125\phi^2+15\phi\theta^2-5\phi-6\theta^2} \\ e_3^L &= \frac{15\phi\theta(\alpha-\bar{c})}{125\phi^2+15\phi\theta^2-5\phi-6\theta^2} \end{cases}$$

Production Costs:

$$\begin{cases} c_1^H &= -\frac{45\alpha\phi\theta^2+5\alpha\phi+6\alpha\theta^2-125\bar{c}\phi^2-60\bar{c}\phi\theta^2}{125\phi^2+15\phi\theta^2-5\phi-6\theta^2} \\ c_1^L &= -\frac{15\alpha\phi\theta^2+5\alpha\phi+6\alpha\theta^2-125\bar{c}\phi^2-30\bar{c}\phi\theta^2}{125\phi^2+15\phi\theta^2-5\phi-6\theta^2} \\ c_2^L &= -\frac{15\alpha\phi\theta^2+5\alpha\phi+6\alpha\theta^2-125\bar{c}\phi^2-30\bar{c}\phi\theta^2}{125\phi^2+15\phi\theta^2-5\phi-6\theta^2} \\ c_3^L &= -\frac{15\alpha\phi\theta^2+5\alpha\phi+6\alpha\theta^2-125\bar{c}\phi^2-30\bar{c}\phi\theta^2}{125\phi^2+15\phi\theta^2-5\phi-6\theta^2} \end{cases}$$

Production Quantities:

$$\begin{cases} q_1^H &= \frac{5\phi(\alpha-\bar{c})(5\phi+6\theta^2)}{125\phi^2+15\phi\theta^2-5\phi-6\theta^2} \\ q_1^L &= \frac{25\phi^2(\alpha-\bar{c})}{125\phi^2+15\phi\theta^2-5\phi-6\theta^2} \\ q_2^L &= \frac{25\phi^2(\alpha-\bar{c})}{125\phi^2+15\phi\theta^2-5\phi-6\theta^2} \\ q_3^L &= \frac{25\phi^2(\alpha-\bar{c})}{125\phi^2+15\phi\theta^2-5\phi-6\theta^2} \end{cases}$$

Profits:

$$\begin{cases} \pi_1^H &= \frac{\phi(\alpha-\bar{c})^2(5\phi+6\theta^2)^2 \cdot (25\phi-1)}{(125\phi^2+15\phi\theta^2-5\phi-6\theta^2)^2} \\ \pi_1^L &= \frac{25\phi^3(\alpha-\bar{c})^2 \cdot (25\phi-9\theta^2)}{(125\phi^2+15\phi\theta^2-5\phi-6\theta^2)^2} \\ \pi_2^L &= \frac{25\phi^3(\alpha-\bar{c})^2 \cdot (25\phi-9\theta^2)}{(125\phi^2+15\phi\theta^2-5\phi-6\theta^2)^2} \\ \pi_3^L &= \frac{25\phi^3(\alpha-\bar{c})^2 \cdot (25\phi-9\theta^2)}{(125\phi^2+15\phi\theta^2-5\phi-6\theta^2)^2} \end{cases} \quad (39)$$

Total Production:

$$\frac{10\phi(\alpha-\bar{c})(10\phi+3\theta^2)}{125\phi^2+15\phi\theta^2-5\phi-6\theta^2}$$

Price:

$$\frac{25\alpha\phi^2-15\alpha\phi\theta^2-5\alpha\phi-6\alpha\theta^2+100\bar{c}\phi^2+30\bar{c}\phi\theta^2}{125\phi^2+15\phi\theta^2-5\phi-6\theta^2}$$

Firm Surplus:

$$\frac{\phi(\alpha-\bar{c})^2 \cdot (2500\phi^3+825\phi^2\theta^2-25\phi^2+900\phi\theta^4-60\phi\theta^2-36\theta^4)}{(125\phi^2+15\phi\theta^2-5\phi-6\theta^2)^2}$$

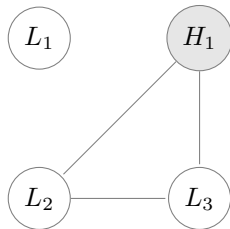
Consumer Surplus:

$$\frac{50\phi^2(\alpha-\bar{c})^2(10\phi+3\theta^2)^2}{(125\phi^2+15\phi\theta^2-5\phi-6\theta^2)^2}$$

Social Welfare:

$$\frac{\phi(\alpha-\bar{c})^2 \cdot (7500\phi^3+3825\phi^2\theta^2-25\phi^2+1350\phi\theta^4-60\phi\theta^2-36\theta^4)}{(125\phi^2+15\phi\theta^2-5\phi-6\theta^2)^2}$$

#### D.1.9 E3B [1H3L]



$$\text{Equations : } \begin{cases} e_1^H \cdot \left(\frac{25\phi}{2} - 2\right) = \alpha - e_1^L\theta + 2e_2^L\theta + 2e_3^L\theta - \bar{c} \\ e_1^L \cdot \left(\frac{25\phi}{4\theta} - 4\theta\right) = \alpha - 3e_1^H - 3e_2^L\theta - 3e_3^L\theta - \bar{c} \\ e_2^L \cdot \left(\frac{25\phi}{2\theta} - 2\theta\right) = \alpha + 2e_1^H - e_1^L\theta + 2e_3^L\theta - \bar{c} \\ e_3^L \cdot \left(\frac{25\phi}{2\theta} - 2\theta\right) = \alpha + 2e_1^H - e_1^L\theta + 2e_2^L\theta - \bar{c} \end{cases}$$

Optimal efforts:

$$\begin{cases} e_1^H &= \frac{2(\alpha - \bar{c})(5\phi - 4\theta^2)}{125\phi^2 - 120\phi\theta^2 - 20\phi + 16\theta^4 + 8\theta^2} \\ e_1^L &= \frac{4\theta(\alpha - \bar{c})(5\phi - 4\theta^2 - 2)}{125\phi^2 - 120\phi\theta^2 - 20\phi + 16\theta^4 + 8\theta^2} \\ e_2^L &= \frac{2\theta(\alpha - \bar{c})(5\phi - 4\theta^2)}{125\phi^2 - 120\phi\theta^2 - 20\phi + 16\theta^4 + 8\theta^2} \\ e_3^L &= \frac{2\theta(\alpha - \bar{c})(5\phi - 4\theta^2)}{125\phi^2 - 120\phi\theta^2 - 20\phi + 16\theta^4 + 8\theta^2} \end{cases}$$

Production Costs:

$$\begin{cases} c_1^H &= -\frac{20\alpha\phi\theta^2 + 10\alpha\phi - 16\alpha\theta^4 - 8\alpha\theta^2 - 125\bar{c}\phi^2 + 100\bar{c}\phi\theta^2 + 10\bar{c}\phi}{125\phi^2 - 120\phi\theta^2 - 20\phi + 16\theta^4 + 8\theta^2} \\ c_1^L &= -\frac{20\alpha\phi\theta^2 - 16\alpha\theta^4 - 8\alpha\theta^2 - 125\bar{c}\phi^2 + 100\bar{c}\phi\theta^2 + 20\bar{c}\phi}{125\phi^2 - 120\phi\theta^2 - 20\phi + 16\theta^4 + 8\theta^2} \\ c_2^L &= -\frac{20\alpha\phi\theta^2 + 10\alpha\phi - 16\alpha\theta^4 - 8\alpha\theta^2 - 125\bar{c}\phi^2 + 100\bar{c}\phi\theta^2 + 10\bar{c}\phi}{125\phi^2 - 120\phi\theta^2 - 20\phi + 16\theta^4 + 8\theta^2} \\ c_3^L &= -\frac{20\alpha\phi\theta^2 + 10\alpha\phi - 16\alpha\theta^4 - 8\alpha\theta^2 - 125\bar{c}\phi^2 + 100\bar{c}\phi\theta^2 + 10\bar{c}\phi}{125\phi^2 - 120\phi\theta^2 - 20\phi + 16\theta^4 + 8\theta^2} \end{cases}$$

Production Quantities:

$$\begin{cases} q_1^H &= \frac{5\phi(\alpha - \bar{c})(5\phi - 4\theta^2)}{125\phi^2 - 120\phi\theta^2 - 20\phi + 16\theta^4 + 8\theta^2} \\ q_1^L &= \frac{5\phi(\alpha - \bar{c})(5\phi - 4\theta^2 - 2)}{125\phi^2 - 120\phi\theta^2 - 20\phi + 16\theta^4 + 8\theta^2} \\ q_2^L &= \frac{5\phi(\alpha - \bar{c})(5\phi - 4\theta^2)}{125\phi^2 - 120\phi\theta^2 - 20\phi + 16\theta^4 + 8\theta^2} \\ q_3^L &= \frac{5\phi(\alpha - \bar{c})(5\phi - 4\theta^2)}{125\phi^2 - 120\phi\theta^2 - 20\phi + 16\theta^4 + 8\theta^2} \end{cases}$$

Profits:

$$\begin{cases} \pi_1^H &= \frac{\phi(\alpha - \bar{c})^2(5\phi - 4\theta^2)^2 \cdot (25\phi - 4)}{(125\phi^2 - 120\phi\theta^2 - 20\phi + 16\theta^4 + 8\theta^2)^2} \\ \pi_1^L &= \frac{\phi(\alpha - \bar{c})^2 \cdot (25\phi - 16\theta^2)(5\phi - 4\theta^2 - 2)^2}{(125\phi^2 - 120\phi\theta^2 - 20\phi + 16\theta^4 + 8\theta^2)^2} \\ \pi_2^L &= \frac{\phi(\alpha - \bar{c})^2(5\phi - 4\theta^2)^2 \cdot (25\phi - 4\theta^2)}{(125\phi^2 - 120\phi\theta^2 - 20\phi + 16\theta^4 + 8\theta^2)^2} \\ \pi_3^L &= \frac{\phi(\alpha - \bar{c})^2(5\phi - 4\theta^2)^2 \cdot (25\phi - 4\theta^2)}{(125\phi^2 - 120\phi\theta^2 - 20\phi + 16\theta^4 + 8\theta^2)^2} \end{cases} \quad (40)$$

Total Production:

$$\frac{10\phi(\alpha - \bar{c})(10\phi - 8\theta^2 - 1)}{125\phi^2 - 120\phi\theta^2 - 20\phi + 16\theta^4 + 8\theta^2}$$

Price:

$$\frac{25\alpha\phi^2 - 40\alpha\phi\theta^2 - 10\alpha\phi + 16\alpha\theta^4 + 8\alpha\theta^2 + 100\bar{c}\phi^2 - 80\bar{c}\phi\theta^2 - 10\bar{c}\phi}{125\phi^2 - 120\phi\theta^2 - 20\phi + 16\theta^4 + 8\theta^2}$$

Firm Surplus:

$$\frac{4\phi(\alpha - \bar{c})^2 \cdot (625\phi^3 - 1150\phi^2\theta^2 - 150\phi^2 + 640\phi\theta^4 + 220\phi\theta^2 + 25\phi - 96\theta^6 - 80\theta^4 - 16\theta^2)}{(125\phi^2 - 120\phi\theta^2 - 20\phi + 16\theta^4 + 8\theta^2)^2}$$

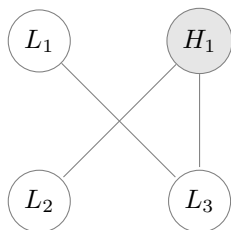
Consumer Surplus:

$$\frac{50\phi^2(\alpha - \bar{c})^2(10\phi - 8\theta^2 - 1)^2}{(125\phi^2 - 120\phi\theta^2 - 20\phi + 16\theta^4 + 8\theta^2)^2}$$

Social Welfare:

$$\frac{2\phi(\alpha - \bar{c})^2 \cdot (3750\phi^3 - 6300\phi^2\theta^2 - 800\phi^2 + 2880\phi\theta^4 + 840\phi\theta^2 + 75\phi - 192\theta^6 - 160\theta^4 - 32\theta^2)}{(125\phi^2 - 120\phi\theta^2 - 20\phi + 16\theta^4 + 8\theta^2)^2}$$

#### D.1.10 E3C [1H3L]



$$\text{Equations : } \begin{cases} e_1^H \cdot \left(\frac{25\phi}{2} - 2\right) = \alpha - 2e_1^L\theta + 3e_2^L\theta + 2e_3^L\theta - \bar{c} \\ e_1^L \cdot \left(\frac{25\phi}{3\theta} - 3\theta\right) = \alpha - 3e_1^H - 2e_2^L\theta + 2e_3^L\theta - \bar{c} \\ e_2^L \cdot \left(\frac{25\phi}{3\theta} - 3\theta\right) = \alpha + 2e_1^H - 2e_1^L\theta - 3e_3^L\theta - \bar{c} \\ e_3^L \cdot \left(\frac{25\phi}{2\theta} - 2\theta\right) = \alpha + 2e_1^H + 3e_1^L\theta - 2e_2^L\theta - \bar{c} \end{cases}$$

Optimal efforts:

$$\begin{cases} e_1^H &= \frac{10\phi(\alpha-\bar{c})(25\phi^2-15\phi\theta^2-6\theta^4)}{3125\phi^4-2750\phi^3\theta^2-500\phi^3+225\phi^2\theta^4+240\phi\theta^4-36\theta^6} \\ e_1^L &= \frac{3\theta(\alpha-\bar{c})(125\phi^3-75\phi^2\theta^2-50\phi^2+12\theta^4)}{3125\phi^4-2750\phi^3\theta^2-500\phi^3+225\phi^2\theta^4+240\phi\theta^4-36\theta^6} \\ e_2^L &= \frac{3\theta(\alpha-\bar{c})(125\phi^3-125\phi^2\theta^2+12\theta^4)}{3125\phi^4-2750\phi^3\theta^2-500\phi^3+225\phi^2\theta^4+240\phi\theta^4-36\theta^6} \\ e_3^L &= \frac{10\phi\theta(\alpha-\bar{c})(25\phi^2-15\phi\theta^2-6\theta^2)}{3125\phi^4-2750\phi^3\theta^2-500\phi^3+225\phi^2\theta^4+240\phi\theta^4-36\theta^6} \end{cases}$$

Production Costs:

$$\begin{cases} c_1^H &= -\frac{625\alpha\phi^3\theta^2+250\alpha\phi^3-525\alpha\phi^2\theta^4-150\alpha\phi^2\theta^2-120\alpha\phi\theta^4+36\alpha\theta^6-3125\bar{c}\phi^4+2125\bar{c}\phi^3\theta^2+250\bar{c}\phi^3+300\bar{c}\phi^2\theta^4+150\bar{c}\phi^2\theta^2-120\bar{c}\phi\theta^4}{3125\phi^4-2750\phi^3\theta^2-500\phi^3+225\phi^2\theta^4+240\phi\theta^4-36\theta^6} \\ c_1^L &= -\frac{625\alpha\phi^3\theta^2-375\alpha\phi^2\theta^4-150\alpha\phi^2\theta^2-60\alpha\phi\theta^4+36\alpha\theta^6-3125\bar{c}\phi^4+2125\bar{c}\phi^3\theta^2+500\bar{c}\phi^3+150\bar{c}\phi^2\theta^4+150\bar{c}\phi^2\theta^2-180\bar{c}\phi\theta^4}{3125\phi^4-2750\phi^3\theta^2-500\phi^3+225\phi^2\theta^4+240\phi\theta^4-36\theta^6} \\ c_2^L &= -\frac{375\alpha\phi^3\theta^2+250\alpha\phi^3-375\alpha\phi^2\theta^4-150\alpha\phi^2\theta^2-60\alpha\phi\theta^4+36\alpha\theta^6-3125\bar{c}\phi^4+2375\bar{c}\phi^3\theta^2+250\bar{c}\phi^3+150\bar{c}\phi^2\theta^4+150\bar{c}\phi^2\theta^2-180\bar{c}\phi\theta^4}{3125\phi^4-2750\phi^3\theta^2-500\phi^3+225\phi^2\theta^4+240\phi\theta^4-36\theta^6} \\ c_3^L &= -\frac{625\alpha\phi^3\theta^2+250\alpha\phi^3-375\alpha\phi^2\theta^4-300\alpha\phi^2\theta^2-120\alpha\phi\theta^4+36\alpha\theta^6-3125\bar{c}\phi^4+2125\bar{c}\phi^3\theta^2+250\bar{c}\phi^3+150\bar{c}\phi^2\theta^4+300\bar{c}\phi^2\theta^2-120\bar{c}\phi\theta^4}{3125\phi^4-2750\phi^3\theta^2-500\phi^3+225\phi^2\theta^4+240\phi\theta^4-36\theta^6} \end{cases}$$

Production Quantities:

$$\begin{cases} q_1^H &= \frac{25\phi^2(\alpha-\bar{c})(25\phi^2-15\phi\theta^2-6\theta^4)}{3125\phi^4-2750\phi^3\theta^2-500\phi^3+225\phi^2\theta^4+240\phi\theta^4-36\theta^6} \\ q_1^L &= \frac{5\phi(\alpha-\bar{c})(125\phi^3-75\phi^2\theta^2-50\phi^2+12\theta^4)}{3125\phi^4-2750\phi^3\theta^2-500\phi^3+225\phi^2\theta^4+240\phi\theta^4-36\theta^6} \\ q_2^L &= \frac{5\phi(\alpha-\bar{c})(125\phi^3-125\phi^2\theta^2+12\theta^4)}{3125\phi^4-2750\phi^3\theta^2-500\phi^3+225\phi^2\theta^4+240\phi\theta^4-36\theta^6} \\ q_3^L &= \frac{25\phi^2(\alpha-\bar{c})(25\phi^2-15\phi\theta^2-6\theta^2)}{3125\phi^4-2750\phi^3\theta^2-500\phi^3+225\phi^2\theta^4+240\phi\theta^4-36\theta^6} \end{cases}$$

Profits:

$$\begin{cases} \pi_1^H &= \frac{25\phi^3(\alpha-\bar{c})^2 \cdot (25\phi-4)(25\phi^2-15\phi\theta^2-6\theta^4)^2}{(3125\phi^4-2750\phi^3\theta^2-500\phi^3+225\phi^2\theta^4+240\phi\theta^4-36\theta^6)^2} \\ \pi_1^L &= \frac{\phi(\alpha-\bar{c})^2 \cdot (25\phi-9\theta^2)(125\phi^3-75\phi^2\theta^2-50\phi^2+12\theta^4)^2}{(3125\phi^4-2750\phi^3\theta^2-500\phi^3+225\phi^2\theta^4+240\phi\theta^4-36\theta^6)^2} \\ \pi_2^L &= \frac{\phi(\alpha-\bar{c})^2 \cdot (25\phi-9\theta^2)(125\phi^3-125\phi^2\theta^2+12\theta^4)^2}{(3125\phi^4-2750\phi^3\theta^2-500\phi^3+225\phi^2\theta^4+240\phi\theta^4-36\theta^6)^2} \\ \pi_3^L &= \frac{25\phi^3(\alpha-\bar{c})^2 \cdot (25\phi-4\theta^2)(25\phi^2-15\phi\theta^2-6\theta^2)^2}{(3125\phi^4-2750\phi^3\theta^2-500\phi^3+225\phi^2\theta^4+240\phi\theta^4-36\theta^6)^2} \end{cases} \quad (41)$$

Total Production:

$$\frac{10\phi(\alpha-\bar{c})(250\phi^3-175\phi^2\theta^2-25\phi^2-15\phi\theta^4-15\phi\theta^2+12\theta^4)}{3125\phi^4-2750\phi^3\theta^2-500\phi^3+225\phi^2\theta^4+240\phi\theta^4-36\theta^6}$$

Price:

$$\frac{625\alpha\phi^4-1000\alpha\phi^3\theta^2-250\alpha\phi^3+375\alpha\phi^2\theta^4+150\alpha\phi^2\theta^2+120\alpha\phi\theta^4-36\alpha\theta^6+2500\bar{c}\phi^4-1750\bar{c}\phi^3\theta^2-250\bar{c}\phi^3-150\bar{c}\phi^2\theta^4-150\bar{c}\phi^2\theta^2+120\bar{c}\phi\theta^4}{3125\phi^4-2750\phi^3\theta^2-500\phi^3+225\phi^2\theta^4+240\phi\theta^4-36\theta^6}$$

Firm Surplus:

$$\frac{2\phi(\alpha-\bar{c})^2 \cdot (781250\phi^7-1265625\phi^6\theta^2-187500\phi^6+575000\phi^5\theta^4+93750\phi^5\theta^2+31250\phi^5-50625\phi^4\theta^6+116250\phi^4\theta^4-11250\phi^4\theta^2+11250\phi^3\theta^8-105000\phi^3\theta^6-3750\phi^3\theta^4+19800\phi^2\theta^8+3600\phi^2\theta^6+3600\phi\theta^8-1296\theta^{10})}{(3125\phi^4-2750\phi^3\theta^2-500\phi^3+225\phi^2\theta^4+240\phi\theta^4-36\theta^6)^2}$$

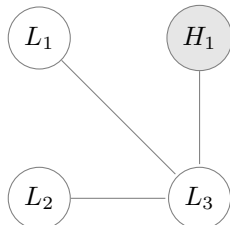
Consumer Surplus:

$$\frac{50\phi^2(\alpha-\bar{c})^2(250\phi^3-175\phi^2\theta^2-25\phi^2-15\phi\theta^4-15\phi\theta^2+12\theta^4)^2}{(3125\phi^4-2750\phi^3\theta^2-500\phi^3+225\phi^2\theta^4+240\phi\theta^4-36\theta^6)^2}$$

Social Welfare:

$$\frac{2\phi(\alpha-\bar{c})^2 \cdot (2343750\phi^7-3453125\phi^6\theta^2-500000\phi^6+1153125\phi^5\theta^4+125000\phi^5\theta^2+46875\phi^5+80625\phi^4\theta^6+416250\phi^4\theta^4+7500\phi^4\theta^2+16875\phi^3\theta^8-198750\phi^3\theta^6-13125\phi^3\theta^4+10800\phi^2\theta^8-5400\phi^2\theta^6+7200\phi\theta^8-1296\theta^{10})}{(3125\phi^4-2750\phi^3\theta^2-500\phi^3+225\phi^2\theta^4+240\phi\theta^4-36\theta^6)^2}$$

#### D.1.11 E3D [1H3L]



$$\text{Equations : } \begin{cases} e_1^H \cdot \left( \frac{25\phi}{3} - 3 \right) = \alpha - 2e_1^L\theta - 2e_2^L\theta + e_3^L\theta - \bar{c} \\ e_1^L \cdot \left( \frac{25\phi}{3\theta} - 3\theta \right) = \alpha - 2e_1^H - 2e_2^L\theta + e_3^L\theta - \bar{c} \\ e_2^L \cdot \left( \frac{25\phi}{3\theta} - 3\theta \right) = \alpha - 2e_1^H - 2e_1^L\theta + e_3^L\theta - \bar{c} \\ e_3^L \cdot \left( \frac{25\phi}{\theta} - \theta \right) = \alpha + 3e_1^H + 3e_1^L\theta + 3e_2^L\theta - \bar{c} \end{cases}$$

Optimal efforts:

$$\begin{cases} e_1^H &= \frac{15\phi(\alpha - \bar{c})(5\phi - 3\theta^2)}{625\phi^3 - 100\phi^2\theta^2 - 225\phi^2 - 15\phi\theta^4 - 45\phi\theta^2 + 18\theta^4} \\ e_1^L &= \frac{15\phi\theta(\alpha - \bar{c})(5\phi - 3)}{625\phi^3 - 100\phi^2\theta^2 - 225\phi^2 - 15\phi\theta^4 - 45\phi\theta^2 + 18\theta^4} \\ e_2^L &= \frac{15\phi\theta(\alpha - \bar{c})(5\phi - 3)}{625\phi^3 - 100\phi^2\theta^2 - 225\phi^2 - 15\phi\theta^4 - 45\phi\theta^2 + 18\theta^4} \\ e_3^L &= \frac{\theta(\alpha - \bar{c})(25\phi^2 + 15\phi\theta^2 - 18\theta^2)}{625\phi^3 - 100\phi^2\theta^2 - 225\phi^2 - 15\phi\theta^4 - 45\phi\theta^2 + 18\theta^4} \end{cases}$$

Production Costs:

$$\begin{cases} c_1^H &= -\frac{25\alpha\phi^2\theta^2 + 75\alpha\phi^2 + 15\alpha\phi\theta^4 - 45\alpha\phi\theta^2 - 18\alpha\theta^4 - 625\bar{c}\phi^3 + 75\bar{c}\phi^2\theta^2 + 150\bar{c}\phi^2 + 90\bar{c}\phi\theta^2}{625\phi^3 - 100\phi^2\theta^2 - 225\phi^2 - 15\phi\theta^4 - 45\phi\theta^2 + 18\theta^4} \\ c_1^L &= -\frac{100\alpha\phi^2\theta^2 + 15\alpha\phi\theta^4 - 45\alpha\phi\theta^2 - 18\alpha\theta^4 - 625\bar{c}\phi^3 + 225\bar{c}\phi^2 + 90\bar{c}\phi\theta^2}{625\phi^3 - 100\phi^2\theta^2 - 225\phi^2 - 15\phi\theta^4 - 45\phi\theta^2 + 18\theta^4} \\ c_2^L &= -\frac{100\alpha\phi^2\theta^2 + 15\alpha\phi\theta^4 - 45\alpha\phi\theta^2 - 18\alpha\theta^4 - 625\bar{c}\phi^3 + 225\bar{c}\phi^2 + 90\bar{c}\phi\theta^2}{625\phi^3 - 100\phi^2\theta^2 - 225\phi^2 - 15\phi\theta^4 - 45\phi\theta^2 + 18\theta^4} \\ c_3^L &= -\frac{175\alpha\phi^2\theta^2 + 75\alpha\phi^2 + 15\alpha\phi\theta^4 - 135\alpha\phi\theta^2 - 18\alpha\theta^4 - 625\bar{c}\phi^3 - 75\bar{c}\phi^2\theta^2 + 150\bar{c}\phi^2 + 180\bar{c}\phi\theta^2}{625\phi^3 - 100\phi^2\theta^2 - 225\phi^2 - 15\phi\theta^4 - 45\phi\theta^2 + 18\theta^4} \end{cases}$$

Production Quantities:

$$\begin{cases} q_1^H &= \frac{25\phi^2(\alpha - \bar{c})(5\phi - 3\theta^2)}{625\phi^3 - 100\phi^2\theta^2 - 225\phi^2 - 15\phi\theta^4 - 45\phi\theta^2 + 18\theta^4} \\ q_1^L &= \frac{25\phi^2(\alpha - \bar{c})(5\phi - 3)}{625\phi^3 - 100\phi^2\theta^2 - 225\phi^2 - 15\phi\theta^4 - 45\phi\theta^2 + 18\theta^4} \\ q_2^L &= \frac{25\phi^2(\alpha - \bar{c})(5\phi - 3)}{625\phi^3 - 100\phi^2\theta^2 - 225\phi^2 - 15\phi\theta^4 - 45\phi\theta^2 + 18\theta^4} \\ q_3^L &= \frac{5\phi(\alpha - \bar{c})(25\phi^2 + 15\phi\theta^2 - 18\theta^2)}{625\phi^3 - 100\phi^2\theta^2 - 225\phi^2 - 15\phi\theta^4 - 45\phi\theta^2 + 18\theta^4} \end{cases}$$

Profits:

$$\begin{cases} \pi_1^H &= \frac{25\phi^3(\alpha - \bar{c})^2(5\phi - 3\theta^2)^2 \cdot (25\phi - 9)}{(625\phi^3 - 100\phi^2\theta^2 - 225\phi^2 - 15\phi\theta^4 - 45\phi\theta^2 + 18\theta^4)^2} \\ \pi_1^L &= \frac{25\phi^3(\alpha - \bar{c})^2(5\phi - 3)^2 \cdot (25\phi - 9\theta^2)}{(625\phi^3 - 100\phi^2\theta^2 - 225\phi^2 - 15\phi\theta^4 - 45\phi\theta^2 + 18\theta^4)^2} \\ \pi_2^L &= \frac{25\phi^3(\alpha - \bar{c})^2(5\phi - 3)^2 \cdot (25\phi - 9\theta^2)}{(625\phi^3 - 100\phi^2\theta^2 - 225\phi^2 - 15\phi\theta^4 - 45\phi\theta^2 + 18\theta^4)^2} \\ \pi_3^L &= \frac{\phi(\alpha - \bar{c})^2 \cdot (25\phi - \theta^2)(25\phi^2 + 15\phi\theta^2 - 18\theta^2)^2}{(625\phi^3 - 100\phi^2\theta^2 - 225\phi^2 - 15\phi\theta^4 - 45\phi\theta^2 + 18\theta^4)^2} \end{cases} \quad (42)$$

Total Production:

$$\frac{10\phi(\alpha - \bar{c})(50\phi^2 - 15\phi - 9\theta^2)}{625\phi^3 - 100\phi^2\theta^2 - 225\phi^2 - 15\phi\theta^4 - 45\phi\theta^2 + 18\theta^4}$$

Price:

$$\frac{125\alpha\phi^3 - 100\alpha\phi^2\theta^2 - 75\alpha\phi^2 - 15\alpha\phi\theta^4 + 45\alpha\phi\theta^2 + 18\alpha\theta^4 + 500\bar{c}\phi^3 - 150\bar{c}\phi^2 - 90\bar{c}\phi\theta^2}{625\phi^3 - 100\phi^2\theta^2 - 225\phi^2 - 15\phi\theta^4 - 45\phi\theta^2 + 18\theta^4}$$

Firm Surplus:

$$\frac{\phi(\alpha - \bar{c})^2 \cdot (62500\phi^5 - 11875\phi^4\theta^2 - 43125\phi^4 + 10500\phi^3\theta^4 - 2250\phi^3\theta^2 + 11250\phi^3 - 225\phi^2\theta^6 - 14625\phi^2\theta^4 - 4050\phi^2\theta^2 + 540\phi\theta^6 + 8100\phi\theta^4 - 324\theta^6)}{(625\phi^3 - 100\phi^2\theta^2 - 225\phi^2 - 15\phi\theta^4 - 45\phi\theta^2 + 18\theta^4)^2}$$

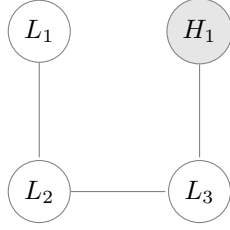
Consumer Surplus:

$$\frac{50\phi^2(\alpha - \bar{c})^2(50\phi^2 - 15\phi - 9\theta^2)^2}{(625\phi^3 - 100\phi^2\theta^2 - 225\phi^2 - 15\phi\theta^4 - 45\phi\theta^2 + 18\theta^4)^2}$$

Social Welfare:

$$\frac{\phi(\alpha - \bar{c})^2 \cdot (187500\phi^5 - 11875\phi^4\theta^2 - 118125\phi^4 + 10500\phi^3\theta^4 - 47250\phi^3\theta^2 + 22500\phi^3 - 225\phi^2\theta^6 - 14625\phi^2\theta^4 + 9450\phi^2\theta^2 + 540\phi\theta^6 + 12150\phi\theta^4 - 324\theta^6)}{(625\phi^3 - 100\phi^2\theta^2 - 225\phi^2 - 15\phi\theta^4 - 45\phi\theta^2 + 18\theta^4)^2}$$

### D.1.12 E3E [1H3L]



$$\text{Equations : } \begin{cases} e_1^H \cdot \left( \frac{25\phi}{3} - 3 \right) = \alpha - 2e_1^L\theta - 3e_2^L\theta + 2e_3^L\theta - \bar{c} \\ e_1^L \cdot \left( \frac{25\phi}{3\theta} - 3\theta \right) = \alpha - 2e_1^H + 2e_2^H\theta - 3e_3^H\theta - \bar{c} \\ e_2^L \cdot \left( \frac{25\phi}{2\theta} - 2\theta \right) = \alpha - 2e_1^H + 3e_1^L\theta + 2e_3^L\theta - \bar{c} \\ e_3^L \cdot \left( \frac{25\phi}{2\theta} - 2\theta \right) = \alpha + 3e_1^H - 2e_1^L\theta + 2e_2^L\theta - \bar{c} \end{cases}$$

Optimal efforts:

$$\begin{cases} e_1^H = \frac{3(\alpha - \bar{c})(5\phi - 2\theta^2)(25\phi^2 - 15\phi\theta^2 - 6\theta^4)}{3125\phi^4 - 2125\phi^3\theta^2 - 1125\phi^3 + 225\phi^2\theta^2 + 120\phi\theta^6 + 120\phi\theta^4 - 36\theta^6} \\ e_1^L = \frac{3\theta(\alpha - \bar{c})(5\phi - 2\theta^2)(25\phi^2 - 15\phi - 6\theta^2)}{3125\phi^4 - 2125\phi^3\theta^2 - 1125\phi^3 + 225\phi^2\theta^2 + 120\phi\theta^6 + 120\phi\theta^4 - 36\theta^6} \\ e_2^L = \frac{10\phi\theta(\alpha - \bar{c})(25\phi^2 - 15\phi - 6\theta^4)}{3125\phi^4 - 2125\phi^3\theta^2 - 1125\phi^3 + 225\phi^2\theta^2 + 120\phi\theta^6 + 120\phi\theta^4 - 36\theta^6} \\ e_3^L = \frac{10\phi\theta(\alpha - \bar{c})(25\phi^2 - 15\phi\theta^2 - 6\theta^2)}{3125\phi^4 - 2125\phi^3\theta^2 - 1125\phi^3 + 225\phi^2\theta^2 + 120\phi\theta^6 + 120\phi\theta^4 - 36\theta^6} \end{cases}$$

Production Costs:

$$\begin{cases} c_1^H = -\frac{250\alpha\phi^3\theta^2 + 375\alpha\phi^3 - 150\alpha\phi^2\theta^4 - 375\alpha\phi^2\theta^2 - 60\alpha\phi\theta^4 + 36\alpha\theta^6 - 3125\bar{c}\phi^4 + 1875\bar{c}\phi^3\theta^2 + 750\bar{c}\phi^3 + 150\bar{c}\phi^2\theta^4 + 150\bar{c}\phi^2\theta^2 - 120\bar{c}\phi\theta^6 - 60\bar{c}\phi\theta^4}{3125\phi^4 - 2125\phi^3\theta^2 - 1125\phi^3 + 225\phi^2\theta^2 + 120\phi\theta^6 + 120\phi\theta^4 - 36\theta^6} \\ c_1^L = -\frac{625\alpha\phi^3\theta^2 - 150\alpha\phi^2\theta^4 - 375\alpha\phi^2\theta^2 - 60\alpha\phi\theta^6 + 36\alpha\theta^6 - 3125\bar{c}\phi^4 + 1500\bar{c}\phi^3\theta^2 + 1125\bar{c}\phi^3 + 150\bar{c}\phi^2\theta^4 + 150\bar{c}\phi^2\theta^2 - 60\bar{c}\phi\theta^6 - 120\bar{c}\phi\theta^4}{3125\phi^4 - 2125\phi^3\theta^2 - 1125\phi^3 + 225\phi^2\theta^2 + 120\phi\theta^6 + 120\phi\theta^4 - 36\theta^6} \\ c_2^L = -\frac{875\alpha\phi^3\theta^2 - 300\alpha\phi^2\theta^4 - 375\alpha\phi^2\theta^2 - 60\alpha\phi\theta^6 - 60\alpha\phi\theta^4 + 36\alpha\theta^6 - 3125\bar{c}\phi^4 + 1250\bar{c}\phi^3\theta^2 + 1125\bar{c}\phi^3 + 300\bar{c}\phi^2\theta^4 + 150\bar{c}\phi^2\theta^2 - 60\bar{c}\phi\theta^6 - 60\bar{c}\phi\theta^4}{3125\phi^4 - 2125\phi^3\theta^2 - 1125\phi^3 + 225\phi^2\theta^2 + 120\phi\theta^6 + 120\phi\theta^4 - 36\theta^6} \\ c_3^L = -\frac{500\alpha\phi^3\theta^2 + 375\alpha\phi^3 - 150\alpha\phi^2\theta^4 - 525\alpha\phi^2\theta^2 - 60\alpha\phi\theta^6 - 60\alpha\phi\theta^4 + 36\alpha\theta^6 - 3125\bar{c}\phi^4 + 1625\bar{c}\phi^3\theta^2 + 750\bar{c}\phi^3 + 150\bar{c}\phi^2\theta^4 + 300\bar{c}\phi^2\theta^2 - 60\bar{c}\phi\theta^6 - 60\bar{c}\phi\theta^4}{3125\phi^4 - 2125\phi^3\theta^2 - 1125\phi^3 + 225\phi^2\theta^2 + 120\phi\theta^6 + 120\phi\theta^4 - 36\theta^6} \end{cases}$$

Production Quantities:

$$\begin{cases} q_1^H = \frac{5\phi(\alpha - \bar{c})(5\phi - 2\theta^2)(25\phi^2 - 15\phi\theta^2 - 6\theta^4)}{3125\phi^4 - 2125\phi^3\theta^2 - 1125\phi^3 + 225\phi^2\theta^2 + 120\phi\theta^6 + 120\phi\theta^4 - 36\theta^6} \\ q_1^L = \frac{5\phi(\alpha - \bar{c})(5\phi - 2\theta^2)(25\phi^2 - 15\phi - 6\theta^2)}{3125\phi^4 - 2125\phi^3\theta^2 - 1125\phi^3 + 225\phi^2\theta^2 + 120\phi\theta^6 + 120\phi\theta^4 - 36\theta^6} \\ q_2^L = \frac{25\phi^2(\alpha - \bar{c})(25\phi^2 - 15\phi - 6\theta^4)}{3125\phi^4 - 2125\phi^3\theta^2 - 1125\phi^3 + 225\phi^2\theta^2 + 120\phi\theta^6 + 120\phi\theta^4 - 36\theta^6} \\ q_3^L = \frac{25\phi^2(\alpha - \bar{c})(25\phi^2 - 15\phi\theta^2 - 6\theta^2)}{3125\phi^4 - 2125\phi^3\theta^2 - 1125\phi^3 + 225\phi^2\theta^2 + 120\phi\theta^6 + 120\phi\theta^4 - 36\theta^6} \end{cases}$$

Profits:

$$\begin{cases} \pi_1^H = \frac{\phi(\alpha - \bar{c})^2(5\phi - 2\theta^2)^2 \cdot (25\phi - 9)(25\phi^2 - 15\phi\theta^2 - 6\theta^4)^2}{(3125\phi^4 - 2125\phi^3\theta^2 - 1125\phi^3 + 225\phi^2\theta^2 + 120\phi\theta^6 + 120\phi\theta^4 - 36\theta^6)^2} \\ \pi_1^L = \frac{\phi(\alpha - \bar{c})^2(5\phi - 2\theta^2)^2 \cdot (25\phi - 9\theta^2)(25\phi^2 - 15\phi - 6\theta^2)^2}{(3125\phi^4 - 2125\phi^3\theta^2 - 1125\phi^3 + 225\phi^2\theta^2 + 120\phi\theta^6 + 120\phi\theta^4 - 36\theta^6)^2} \\ \pi_2^L = \frac{25\phi^3(\alpha - \bar{c})^2 \cdot (25\phi - 4\theta^2)(25\phi^2 - 15\phi - 6\theta^4)^2}{(3125\phi^4 - 2125\phi^3\theta^2 - 1125\phi^3 + 225\phi^2\theta^2 + 120\phi\theta^6 + 120\phi\theta^4 - 36\theta^6)^2} \\ \pi_3^L = \frac{25\phi^3(\alpha - \bar{c})^2 \cdot (25\phi - 4\theta^2)(25\phi^2 - 15\phi\theta^2 - 6\theta^2)^2}{(3125\phi^4 - 2125\phi^3\theta^2 - 1125\phi^3 + 225\phi^2\theta^2 + 120\phi\theta^6 + 120\phi\theta^4 - 36\theta^6)^2} \end{cases} \quad (43)$$

Total Production:

$$\frac{10\phi(\alpha - \bar{c})(5\phi - \theta^2)(50\phi^2 - 15\phi\theta^2 - 15\phi - 6\theta^4 - 6\theta^2)}{3125\phi^4 - 2125\phi^3\theta^2 - 1125\phi^3 + 225\phi^2\theta^2 + 120\phi\theta^6 + 120\phi\theta^4 - 36\theta^6}$$

Price:

$$\frac{625\alpha\phi^4 - 875\alpha\phi^3\theta^2 - 375\alpha\phi^3 + 150\alpha\phi^2\theta^4 + 375\alpha\phi^2\theta^2 + 60\alpha\phi\theta^6 + 60\alpha\phi\theta^4 - 36\alpha\theta^6 + 2500\bar{c}\phi^4 - 1250\bar{c}\phi^3\theta^2 - 750\bar{c}\phi^3 - 150\bar{c}\phi^2\theta^4 - 150\bar{c}\phi^2\theta^2 + 60\bar{c}\phi\theta^6 + 60\bar{c}\phi\theta^4}{3125\phi^4 - 2125\phi^3\theta^2 - 1125\phi^3 + 225\phi^2\theta^2 + 120\phi\theta^6 + 120\phi\theta^4 - 36\theta^6}$$

Firm Surplus:

$$\frac{\phi(\alpha - \bar{c})^2 \cdot (1562500\phi^7 - 1828125\phi^6\theta^2 - 1078125\phi^6 + 593750\phi^5\theta^4 + 525000\phi^5\theta^2 + 281250\phi^5 + 600000\phi^4\theta^6 + 121875\phi^4\theta^4 - 73125\phi^4\theta^2 - 52500\phi^4\theta^0 - 120000\phi^3\theta^6 - 22500\phi^3\theta^4 - 3600\phi^3\theta^2 + 37800\phi^3\theta^0 + 12600\phi^2\theta^6 + 3600\phi^2\theta^4 + 3600\phi^2\theta^2 - 1296\phi^2 - 1296\phi^{10})}{(3125\phi^4 - 2125\phi^3\theta^2 - 1125\phi^3 + 225\phi^2\theta^2 + 120\phi\theta^6 + 120\phi\theta^4 - 36\theta^6)^2}$$

Consumer Surplus:

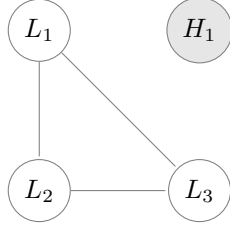
$$\frac{50\phi^2(\alpha - \bar{c})^2(5\phi - \theta^2)^2(50\phi^2 - 15\phi\theta^2 - 15\phi - 6\theta^4 - 6\theta^2)^2}{(3125\phi^4 - 2125\phi^3\theta^2 - 1125\phi^3 + 225\phi^2\theta^2 + 120\phi\theta^6 + 120\phi\theta^4 - 36\theta^6)^2}$$

Social Welfare:



$$\frac{\phi(\alpha - \bar{c})^2 \cdot (4687500\phi^7 - 4953125\phi^6\theta^2 - 2953125\phi^6 + 1000000\phi^5\theta^4 + 1087500\phi^5\theta^2 + 562500\phi^5 + 397500\phi^4\theta^6 + 571875\phi^4\theta^4 + 39375\phi^4\theta^2 - 116250\phi^3\theta^6 - 217500\phi^3\theta^4 - 56250\phi^3\theta^2 - 12600\phi^2\theta^{10} + 19800\phi^2\theta^8 + 3600\phi^2\theta^6 + 5400\phi\theta^{12} + 3600\phi\theta^{10} + 5400\phi\theta^8 - 1296\theta^{12} - 1296\theta^{10})}{(3125\phi^4 - 2125\phi^3\theta^2 - 1125\phi^3 + 225\phi^2\theta^2 + 120\phi\theta^6 + 120\phi\theta^4 - 36\theta^6)^2}$$

### D.1.13 E3F [1H3L]



$$\text{Equations : } \begin{cases} e_1^H \cdot \left( \frac{25\phi}{4} - 4 \right) = \alpha - 3e_1^L\theta - 3e_2^L\theta - 3e_3^L\theta - \bar{c} \\ e_1^L \cdot \left( \frac{25\phi}{2\theta} - 2\theta \right) = \alpha - e_1^H + 2e_2^L\theta + 2e_3^L\theta - \bar{c} \\ e_2^L \cdot \left( \frac{25\phi}{2\theta} - 2\theta \right) = \alpha - e_1^H + 2e_1^L\theta + 2e_3^L\theta - \bar{c} \\ e_3^L \cdot \left( \frac{25\phi}{2\theta} - 2\theta \right) = \alpha - e_1^H + 2e_1^L\theta + 2e_2^L\theta - \bar{c} \end{cases}$$

Optimal efforts:

$$\begin{cases} e_1^H = \frac{4(\alpha - \bar{c})(5\phi - 6\theta^2)}{125\phi^2 - 60\phi\theta^2 - 80\phi + 24\theta^2} \\ e_1^L = \frac{2\theta(\alpha - \bar{c})(5\phi - 4)}{125\phi^2 - 60\phi\theta^2 - 80\phi + 24\theta^2} \\ e_2^L = \frac{2\theta(\alpha - \bar{c})(5\phi - 4)}{125\phi^2 - 60\phi\theta^2 - 80\phi + 24\theta^2} \\ e_3^L = \frac{2\theta(\alpha - \bar{c})(5\phi - 4)}{125\phi^2 - 60\phi\theta^2 - 80\phi + 24\theta^2} \end{cases}$$

Production Costs:

$$\begin{cases} c_1^H = -\frac{20\alpha\phi - 24\alpha\theta^2 - 125\bar{c}\phi^2 + 60\bar{c}\phi\theta^2 + 60\bar{c}\phi}{125\phi^2 - 60\phi\theta^2 - 80\phi + 24\theta^2} \\ c_1^L = -\frac{30\alpha\phi\theta^2 - 24\alpha\theta^2 - 125\bar{c}\phi^2 + 30\bar{c}\phi\theta^2 + 80\bar{c}\phi}{125\phi^2 - 60\phi\theta^2 - 80\phi + 24\theta^2} \\ c_2^L = -\frac{30\alpha\phi\theta^2 - 24\alpha\theta^2 - 125\bar{c}\phi^2 + 30\bar{c}\phi\theta^2 + 80\bar{c}\phi}{125\phi^2 - 60\phi\theta^2 - 80\phi + 24\theta^2} \\ c_3^L = -\frac{30\alpha\phi\theta^2 - 24\alpha\theta^2 - 125\bar{c}\phi^2 + 30\bar{c}\phi\theta^2 + 80\bar{c}\phi}{125\phi^2 - 60\phi\theta^2 - 80\phi + 24\theta^2} \end{cases}$$

Production Quantities:

$$\begin{cases} q_1^H = \frac{5\phi(\alpha - \bar{c})(5\phi - 6\theta^2)}{125\phi^2 - 60\phi\theta^2 - 80\phi + 24\theta^2} \\ q_1^L = \frac{5\phi(\alpha - \bar{c})(5\phi - 4)}{125\phi^2 - 60\phi\theta^2 - 80\phi + 24\theta^2} \\ q_2^L = \frac{5\phi(\alpha - \bar{c})(5\phi - 4)}{125\phi^2 - 60\phi\theta^2 - 80\phi + 24\theta^2} \\ q_3^L = \frac{5\phi(\alpha - \bar{c})(5\phi - 4)}{125\phi^2 - 60\phi\theta^2 - 80\phi + 24\theta^2} \end{cases}$$

Profits:

$$\begin{cases} \pi_1^H = \frac{\phi(\alpha - \bar{c})^2(5\phi - 6\theta^2)^2 \cdot (25\phi - 16)}{(125\phi^2 - 60\phi\theta^2 - 80\phi + 24\theta^2)^2} \\ \pi_1^L = \frac{\phi(\alpha - \bar{c})^2(5\phi - 4)^2 \cdot (25\phi - 4\theta^2)}{(125\phi^2 - 60\phi\theta^2 - 80\phi + 24\theta^2)^2} \\ \pi_2^L = \frac{\phi(\alpha - \bar{c})^2(5\phi - 4)^2 \cdot (25\phi - 4\theta^2)}{(125\phi^2 - 60\phi\theta^2 - 80\phi + 24\theta^2)^2} \\ \pi_3^L = \frac{\phi(\alpha - \bar{c})^2(5\phi - 4)^2 \cdot (25\phi - 4\theta^2)}{(125\phi^2 - 60\phi\theta^2 - 80\phi + 24\theta^2)^2} \end{cases} \quad (44)$$

Total Production:

$$\frac{10\phi(\alpha - \bar{c})(10\phi - 3\theta^2 - 6)}{125\phi^2 - 60\phi\theta^2 - 80\phi + 24\theta^2}$$

Price:

$$\frac{25\alpha\phi^2 - 30\alpha\phi\theta^2 - 20\alpha\phi + 24\alpha\theta^2 + 100\bar{c}\phi^2 - 30\bar{c}\phi\theta^2 - 60\bar{c}\phi}{125\phi^2 - 60\phi\theta^2 - 80\phi + 24\theta^2}$$

Firm Surplus:

$$\frac{4\phi(\alpha - \bar{c})^2 \cdot (625\phi^3 - 450\phi^2\theta^2 - 850\phi^2 + 225\phi\theta^4 + 360\phi\theta^2 + 300\phi - 144\theta^4 - 48\theta^2)}{(125\phi^2 - 60\phi\theta^2 - 80\phi + 24\theta^2)^2}$$

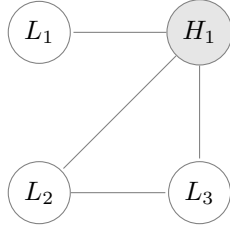
Consumer Surplus:

$$\frac{50\phi^2(\alpha - \bar{c})^2(10\phi - 3\theta^2 - 6)^2}{(125\phi^2 - 60\phi\theta^2 - 80\phi + 24\theta^2)^2}$$

Social Welfare:

$$\frac{2\phi(\alpha - \bar{c})^2 \cdot (3750\phi^3 - 2400\phi^2\theta^2 - 4700\phi^2 + 675\phi\theta^4 + 1620\phi\theta^2 + 1500\phi - 288\theta^4 - 96\theta^2)}{(125\phi^2 - 60\phi\theta^2 - 80\phi + 24\theta^2)^2}$$

#### D.1.14 E4A [1H3L]



$$\text{Equations : } \begin{cases} e_1^H \cdot (25\phi - 1) = \alpha + 3e_1^L\theta + 2e_2^L\theta + 2e_3^L\theta - \bar{c} \\ e_1^L \cdot \left(\frac{25\phi}{3\theta} - 3\theta\right) = \alpha + e_1^H - 3e_2^L\theta - 3e_3^L\theta - \bar{c} \\ e_2^L \cdot \left(\frac{25\phi}{2\theta} - 2\theta\right) = \alpha + e_1^H - 2e_1^L\theta + 2e_3^L\theta - \bar{c} \\ e_3^L \cdot \left(\frac{25\phi}{2\theta} - 2\theta\right) = \alpha + e_1^H - 2e_1^L\theta + 2e_2^L\theta - \bar{c} \end{cases}$$

Optimal efforts:

$$\begin{cases} e_1^H = \frac{(\alpha - \bar{c})(25\phi^2 - 12\theta^4)}{625\phi^3 - 425\phi^2\theta^2 - 25\phi^2 + 12\theta^4} \\ e_1^L = \frac{15\phi\theta(\alpha - \bar{c})(5\phi - 4\theta^2)}{625\phi^3 - 425\phi^2\theta^2 - 25\phi^2 + 12\theta^4} \\ e_2^L = \frac{10\phi\theta(\alpha - \bar{c})(5\phi - 3\theta^2)}{625\phi^3 - 425\phi^2\theta^2 - 25\phi^2 + 12\theta^4} \\ e_3^L = \frac{10\phi\theta(\alpha - \bar{c})(5\phi - 3\theta^2)}{625\phi^3 - 425\phi^2\theta^2 - 25\phi^2 + 12\theta^4} \end{cases}$$

Production Costs:

$$\begin{cases} c_1^H = -\frac{175\alpha\phi^2\theta^2 + 25\alpha\phi^2 - 120\alpha\phi\theta^4 - 12\alpha\theta^4 - 625\bar{c}\phi^3 + 250\bar{c}\phi^2\theta^2 + 120\bar{c}\phi\theta^4}{625\phi^3 - 425\phi^2\theta^2 - 25\phi^2 + 12\theta^4} \\ c_1^L = -\frac{75\alpha\phi^2\theta^2 + 25\alpha\phi^2 - 60\alpha\phi\theta^4 - 12\alpha\theta^4 - 625\bar{c}\phi^3 + 350\bar{c}\phi^2\theta^2 + 60\bar{c}\phi\theta^4}{625\phi^3 - 425\phi^2\theta^2 - 25\phi^2 + 12\theta^4} \\ c_2^L = -\frac{100\alpha\phi^2\theta^2 + 25\alpha\phi^2 - 60\alpha\phi\theta^4 - 12\alpha\theta^4 - 625\bar{c}\phi^3 + 325\bar{c}\phi^2\theta^2 + 60\bar{c}\phi\theta^4}{625\phi^3 - 425\phi^2\theta^2 - 25\phi^2 + 12\theta^4} \\ c_3^L = -\frac{100\alpha\phi^2\theta^2 + 25\alpha\phi^2 - 60\alpha\phi\theta^4 - 12\alpha\theta^4 - 625\bar{c}\phi^3 + 325\bar{c}\phi^2\theta^2 + 60\bar{c}\phi\theta^4}{625\phi^3 - 425\phi^2\theta^2 - 25\phi^2 + 12\theta^4} \end{cases}$$

Production Quantities:

$$\begin{cases} q_1^H = \frac{5\phi(\alpha - \bar{c})(25\phi^2 - 12\theta^4)}{625\phi^3 - 425\phi^2\theta^2 - 25\phi^2 + 12\theta^4} \\ q_1^L = \frac{25\phi^2(\alpha - \bar{c})(5\phi - 4\theta^2)}{625\phi^3 - 425\phi^2\theta^2 - 25\phi^2 + 12\theta^4} \\ q_2^L = \frac{25\phi^2(\alpha - \bar{c})(5\phi - 3\theta^2)}{625\phi^3 - 425\phi^2\theta^2 - 25\phi^2 + 12\theta^4} \\ q_3^L = \frac{25\phi^2(\alpha - \bar{c})(5\phi - 3\theta^2)}{625\phi^3 - 425\phi^2\theta^2 - 25\phi^2 + 12\theta^4} \end{cases}$$

Profits:

$$\begin{cases} \pi_1^H = \frac{\phi(\alpha - \bar{c})^2 \cdot (25\phi - 1)(25\phi^2 - 12\theta^4)^2}{(625\phi^3 - 425\phi^2\theta^2 - 25\phi^2 + 12\theta^4)^2} \\ \pi_1^L = \frac{25\phi^3(\alpha - \bar{c})^2(5\phi - 4\theta^2)^2 \cdot (25\phi - 9\theta^2)}{(625\phi^3 - 425\phi^2\theta^2 - 25\phi^2 + 12\theta^4)^2} \\ \pi_2^L = \frac{25\phi^3(\alpha - \bar{c})^2(5\phi - 3\theta^2)^2 \cdot (25\phi - 4\theta^2)}{(625\phi^3 - 425\phi^2\theta^2 - 25\phi^2 + 12\theta^4)^2} \\ \pi_3^L = \frac{25\phi^3(\alpha - \bar{c})^2(5\phi - 3\theta^2)^2 \cdot (25\phi - 4\theta^2)}{(625\phi^3 - 425\phi^2\theta^2 - 25\phi^2 + 12\theta^4)^2} \end{cases} \quad (45)$$

Total Production:

$$\frac{10\phi(\alpha - \bar{c})(50\phi^2 - 25\phi\theta^2 - 6\theta^4)}{625\phi^3 - 425\phi^2\theta^2 - 25\phi^2 + 12\theta^4}$$

Price:

$$\frac{125\alpha\phi^3 - 175\alpha\phi^2\theta^2 - 25\alpha\phi^2 + 60\alpha\phi\theta^4 + 12\alpha\theta^4 + 500\bar{c}\phi^3 - 250\bar{c}\phi^2\theta^2 - 60\bar{c}\phi\theta^4}{625\phi^3 - 425\phi^2\theta^2 - 25\phi^2 + 12\theta^4}$$

Firm Surplus:

$$\frac{\phi(\alpha - \bar{c})^2 \cdot (62500\phi^5 - 73125\phi^4\theta^2 - 625\phi^4 + 21250\phi^3\theta^4 - 5400\phi^2\theta^6 + 600\phi^2\theta^4 + 3600\phi\theta^8 - 144\theta^8)}{(625\phi^3 - 425\phi^2\theta^2 - 25\phi^2 + 12\theta^4)^2}$$

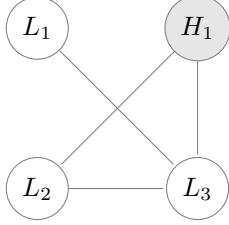
Consumer Surplus:

$$\frac{50\phi^2 (\alpha - \bar{c})^2 (50\phi^2 - 25\phi\theta^2 - 6\theta^4)^2}{(625\phi^3 - 425\phi^2\theta^2 - 25\phi^2 + 12\theta^4)^2}$$

Social Welfare:

$$\frac{\phi (\alpha - \bar{c})^2 \cdot (187500\phi^5 - 198125\phi^4\theta^2 - 625\phi^4 + 22500\phi^3\theta^4 + 9600\phi^2\theta^6 + 600\phi^2\theta^4 + 5400\phi\theta^8 - 144\theta^8)}{(625\phi^3 - 425\phi^2\theta^2 - 25\phi^2 + 12\theta^4)^2}$$

#### D.1.15 E4B [1H3L]



$$\text{Equations : } \begin{cases} e_1^H \cdot \left( \frac{25\phi}{2} - 2 \right) = \alpha - 2e_1^L\theta + 2e_2^L\theta + e_3^L\theta - \bar{c} \\ e_1^L \cdot \left( \frac{25\phi}{3\theta} - 3\theta \right) = \alpha - 3e_1^H - 3e_2^L\theta + e_3^L\theta - \bar{c} \\ e_2^L \cdot \left( \frac{25\phi}{2\theta} - 2\theta \right) = \alpha + 2e_1^H - 2e_1^L\theta + e_3^L\theta - \bar{c} \\ e_3^L \cdot \left( \frac{25\phi}{\theta} - \theta \right) = \alpha + 2e_1^H + 3e_1^L\theta + 2e_2^L\theta - \bar{c} \end{cases}$$

Optimal efforts:

$$\begin{cases} e_1^H = \frac{10\phi(\alpha - \bar{c})(5\phi - 3\theta^2)}{625\phi^3 - 350\phi^2\theta^2 - 100\phi^2 + 6\theta^6 + 6\theta^4} \\ e_1^L = \frac{15\phi\theta(\alpha - \bar{c})(5\phi - 2\theta^2 - 2)}{625\phi^3 - 350\phi^2\theta^2 - 100\phi^2 + 6\theta^6 + 6\theta^4} \\ e_2^L = \frac{10\phi\theta(\alpha - \bar{c})(5\phi - 3\theta^2)}{625\phi^3 - 350\phi^2\theta^2 - 100\phi^2 + 6\theta^6 + 6\theta^4} \\ e_3^L = \frac{\theta(\alpha - \bar{c})(25\phi^2 - 6\theta^4 - 6\theta^2)}{625\phi^3 - 350\phi^2\theta^2 - 100\phi^2 + 6\theta^6 + 6\theta^4} \end{cases}$$

Production Costs:

$$\begin{cases} c_1^H = -\frac{75\alpha\phi^2\theta^2 + 50\alpha\phi^2 - 30\alpha\phi\theta^4 - 30\alpha\phi\theta^2 - 6\alpha\theta^6 - 6\alpha\theta^4 - 625\bar{c}\phi^3 + 275\bar{c}\phi^2\theta^2 + 50\bar{c}\phi^2 + 30\bar{c}\phi\theta^4 + 30\bar{c}\phi\theta^2}{625\phi^3 - 350\phi^2\theta^2 - 100\phi^2 + 6\theta^6 + 6\theta^4} \\ c_1^L = -\frac{100\alpha\phi^2\theta^2 - 30\alpha\phi\theta^4 - 30\alpha\phi\theta^2 - 6\alpha\theta^6 - 6\alpha\theta^4 - 625\bar{c}\phi^3 + 250\bar{c}\phi^2\theta^2 + 100\bar{c}\phi^2 + 30\bar{c}\phi\theta^4 + 30\bar{c}\phi\theta^2}{625\phi^3 - 350\phi^2\theta^2 - 100\phi^2 + 6\theta^6 + 6\theta^4} \\ c_2^L = -\frac{75\alpha\phi^2\theta^2 + 50\alpha\phi^2 - 30\alpha\phi\theta^4 - 30\alpha\phi\theta^2 - 6\alpha\theta^6 - 6\alpha\theta^4 - 625\bar{c}\phi^3 + 275\bar{c}\phi^2\theta^2 + 50\bar{c}\phi^2 + 30\bar{c}\phi\theta^4 + 30\bar{c}\phi\theta^2}{625\phi^3 - 350\phi^2\theta^2 - 100\phi^2 + 6\theta^6 + 6\theta^4} \\ c_3^L = -\frac{150\alpha\phi^2\theta^2 + 50\alpha\phi^2 - 60\alpha\phi\theta^4 - 60\alpha\phi\theta^2 - 6\alpha\theta^6 - 6\alpha\theta^4 - 625\bar{c}\phi^3 + 200\bar{c}\phi^2\theta^2 + 50\bar{c}\phi^2 + 60\bar{c}\phi\theta^4 + 60\bar{c}\phi\theta^2}{625\phi^3 - 350\phi^2\theta^2 - 100\phi^2 + 6\theta^6 + 6\theta^4} \end{cases}$$

Production Quantities:

$$\begin{cases} q_1^H = \frac{25\phi^2(\alpha - \bar{c})(5\phi - 3\theta^2)}{625\phi^3 - 350\phi^2\theta^2 - 100\phi^2 + 6\theta^6 + 6\theta^4} \\ q_1^L = \frac{25\phi^2(\alpha - \bar{c})(5\phi - 2\theta^2 - 2)}{625\phi^3 - 350\phi^2\theta^2 - 100\phi^2 + 6\theta^6 + 6\theta^4} \\ q_2^L = \frac{25\phi^2(\alpha - \bar{c})(5\phi - 3\theta^2)}{625\phi^3 - 350\phi^2\theta^2 - 100\phi^2 + 6\theta^6 + 6\theta^4} \\ q_3^L = \frac{5\phi(\alpha - \bar{c})(25\phi^2 - 6\theta^4 - 6\theta^2)}{625\phi^3 - 350\phi^2\theta^2 - 100\phi^2 + 6\theta^6 + 6\theta^4} \end{cases}$$

Profits:

$$\begin{cases} \pi_1^H = \frac{25\phi^3(\alpha - \bar{c})^2(5\phi - 3\theta^2)^2 \cdot (25\phi - 4)}{(625\phi^3 - 350\phi^2\theta^2 - 100\phi^2 + 6\theta^6 + 6\theta^4)^2} \\ \pi_1^L = \frac{25\phi^3(\alpha - \bar{c})^2 \cdot (25\phi - 9\theta^2)(5\phi - 2\theta^2 - 2)^2}{(625\phi^3 - 350\phi^2\theta^2 - 100\phi^2 + 6\theta^6 + 6\theta^4)^2} \\ \pi_2^L = \frac{25\phi^3(\alpha - \bar{c})^2(5\phi - 3\theta^2)^2 \cdot (25\phi - 4\theta^2)}{(625\phi^3 - 350\phi^2\theta^2 - 100\phi^2 + 6\theta^6 + 6\theta^4)^2} \\ \pi_3^L = \frac{\phi(\alpha - \bar{c})^2 \cdot (25\phi - \theta^2)(25\phi^2 - 6\theta^4 - 6\theta^2)^2}{(625\phi^3 - 350\phi^2\theta^2 - 100\phi^2 + 6\theta^6 + 6\theta^4)^2} \end{cases} \quad (46)$$

Total Production:

$$\frac{10\phi(\alpha - \bar{c})(50\phi^2 - 20\phi\theta^2 - 5\phi - 3\theta^4 - 3\theta^2)}{625\phi^3 - 350\phi^2\theta^2 - 100\phi^2 + 6\theta^6 + 6\theta^4}$$

Price:

$$\frac{125\alpha\phi^3 - 150\alpha\phi^2\theta^2 - 50\alpha\phi^2 + 30\alpha\phi\theta^4 + 30\alpha\phi\theta^2 + 6\alpha\theta^6 + 6\alpha\theta^4 + 500\bar{c}\phi^3 - 200\bar{c}\phi^2\theta^2 - 50\bar{c}\phi^2 - 30\bar{c}\phi\theta^4 - 30\bar{c}\phi\theta^2}{625\phi^3 - 350\phi^2\theta^2 - 100\phi^2 + 6\theta^6 + 6\theta^4}$$

Firm Surplus:

$$\frac{2\phi(\alpha - \bar{c})^2 \cdot (31250\phi^5 - 29375\phi^4\theta^2 - 7500\phi^4 + 6875\phi^3\theta^4 + 2500\phi^3\theta^2 + 1250\phi^3 - 750\phi^2\theta^6 - 1200\phi^2\theta^4 - 450\phi^2\theta^2 + 450\phi\theta^8 + 900\phi\theta^6 + 450\phi\theta^4 - 18\theta^{10} - 36\theta^8 - 18\theta^6)}{(625\phi^3 - 350\phi^2\theta^2 - 100\phi^2 + 6\theta^6 + 6\theta^4)^2}$$

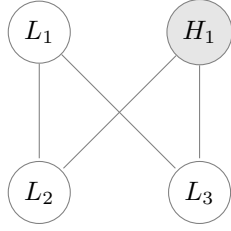
Consumer Surplus:

$$\frac{50\phi^2(\alpha - \bar{c})^2(50\phi^2 - 20\phi\theta^2 - 5\phi - 3\theta^4 - 3\theta^2)^2}{(625\phi^3 - 350\phi^2\theta^2 - 100\phi^2 + 6\theta^6 + 6\theta^4)^2}$$

Social Welfare:

$$\frac{2\phi(\alpha - \bar{c})^2 \cdot (93750\phi^5 - 79375\phi^4\theta^2 - 20000\phi^4 + 9375\phi^3\theta^4 + 1875\phi^3 + 2250\phi^2\theta^6 + 2550\phi^2\theta^4 + 300\phi^2\theta^2 + 675\phi\theta^8 + 1350\phi\theta^6 + 675\phi\theta^4 - 18\theta^{10} - 36\theta^8 - 18\theta^6)}{(625\phi^3 - 350\phi^2\theta^2 - 100\phi^2 + 6\theta^6 + 6\theta^4)^2}$$

#### D.1.16 E4C [1H3L]



$$\text{Equations : } \begin{cases} e_1^H \cdot \left(\frac{25\phi}{2} - 2\right) = \alpha - 3e_1^L\theta + 2e_2^L\theta + 2e_3^L\theta - \bar{c} \\ e_1^L \cdot \left(\frac{25\phi}{2\theta} - 2\theta\right) = \alpha - 3e_1^H + 2e_2^H\theta + 2e_3^H\theta - \bar{c} \\ e_2^L \cdot \left(\frac{25\phi}{2\theta} - 2\theta\right) = \alpha + 2e_1^H + 2e_1^L\theta - 3e_3^L\theta - \bar{c} \\ e_3^L \cdot \left(\frac{25\phi}{2\theta} - 2\theta\right) = \alpha + 2e_1^H + 2e_1^L\theta - 3e_2^L\theta - \bar{c} \end{cases}$$

Optimal efforts:

$$\begin{cases} e_1^H = \frac{2(\alpha - \bar{c})(5\phi - 2\theta^2)(5\phi + 2\theta^2)}{625\phi^3 - 50\phi^2\theta^2 - 100\phi^2 - 40\phi\theta^4 - 60\phi\theta^2 + 24\theta^4} \\ e_1^L = \frac{2\theta(\alpha - \bar{c})(5\phi - 2)(5\phi + 2\theta^2)}{625\phi^3 - 50\phi^2\theta^2 - 100\phi^2 - 40\phi\theta^4 - 60\phi\theta^2 + 24\theta^4} \\ e_2^L = \frac{2\theta(\alpha - \bar{c})(5\phi - 2\theta)(5\phi + 2\theta)}{625\phi^3 - 50\phi^2\theta^2 - 100\phi^2 - 40\phi\theta^4 - 60\phi\theta^2 + 24\theta^4} \\ e_3^L = \frac{2\theta(\alpha - \bar{c})(5\phi - 2\theta)(5\phi + 2\theta)}{625\phi^3 - 50\phi^2\theta^2 - 100\phi^2 - 40\phi\theta^4 - 60\phi\theta^2 + 24\theta^4} \end{cases}$$

Production Costs:

$$\begin{cases} c_1^H = -\frac{100\alpha\phi^2\theta^2 + 50\alpha\phi^2 - 24\alpha\theta^4 - 625\bar{c}\phi^3 - 50\bar{c}\phi^2\theta^2 + 50\bar{c}\phi^2 + 40\bar{c}\phi\theta^4 + 60\bar{c}\phi\theta^2}{625\phi^3 - 50\phi^2\theta^2 - 100\phi^2 - 40\phi\theta^4 - 60\phi\theta^2 + 24\theta^4} \\ c_1^L = -\frac{150\alpha\phi^2\theta^2 + 20\alpha\phi\theta^4 - 20\alpha\phi\theta^2 - 24\alpha\theta^4 - 625\bar{c}\phi^3 - 100\bar{c}\phi^2\theta^2 + 100\bar{c}\phi^2 + 20\bar{c}\phi\theta^4 + 80\bar{c}\phi\theta^2}{625\phi^3 - 50\phi^2\theta^2 - 100\phi^2 - 40\phi\theta^4 - 60\phi\theta^2 + 24\theta^4} \\ c_2^L = -\frac{100\alpha\phi^2\theta^2 + 50\alpha\phi^2 + 20\alpha\phi\theta^4 - 20\alpha\phi\theta^2 - 24\alpha\theta^4 - 625\bar{c}\phi^3 - 50\bar{c}\phi^2\theta^2 + 50\bar{c}\phi^2 + 20\bar{c}\phi\theta^4 + 80\bar{c}\phi\theta^2}{625\phi^3 - 50\phi^2\theta^2 - 100\phi^2 - 40\phi\theta^4 - 60\phi\theta^2 + 24\theta^4} \\ c_3^L = -\frac{100\alpha\phi^2\theta^2 + 50\alpha\phi^2 + 20\alpha\phi\theta^4 - 20\alpha\phi\theta^2 - 24\alpha\theta^4 - 625\bar{c}\phi^3 - 50\bar{c}\phi^2\theta^2 + 50\bar{c}\phi^2 + 20\bar{c}\phi\theta^4 + 80\bar{c}\phi\theta^2}{625\phi^3 - 50\phi^2\theta^2 - 100\phi^2 - 40\phi\theta^4 - 60\phi\theta^2 + 24\theta^4} \end{cases}$$

Production Quantities:

$$\begin{cases} q_1^H = \frac{5\phi(\alpha - \bar{c})(5\phi - 2\theta^2)(5\phi + 2\theta^2)}{625\phi^3 - 50\phi^2\theta^2 - 100\phi^2 - 40\phi\theta^4 - 60\phi\theta^2 + 24\theta^4} \\ q_1^L = \frac{5\phi(\alpha - \bar{c})(5\phi - 2)(5\phi + 2\theta^2)}{625\phi^3 - 50\phi^2\theta^2 - 100\phi^2 - 40\phi\theta^4 - 60\phi\theta^2 + 24\theta^4} \\ q_2^L = \frac{5\phi(\alpha - \bar{c})(5\phi - 2\theta)(5\phi + 2\theta)}{625\phi^3 - 50\phi^2\theta^2 - 100\phi^2 - 40\phi\theta^4 - 60\phi\theta^2 + 24\theta^4} \\ q_3^L = \frac{5\phi(\alpha - \bar{c})(5\phi - 2\theta)(5\phi + 2\theta)}{625\phi^3 - 50\phi^2\theta^2 - 100\phi^2 - 40\phi\theta^4 - 60\phi\theta^2 + 24\theta^4} \end{cases}$$

Profits:

$$\begin{cases} \pi_1^H = \frac{\phi(\alpha - \bar{c})^2(5\phi - 2\theta^2)^2(5\phi + 2\theta^2)^2 \cdot (25\phi - 4)}{(625\phi^3 - 50\phi^2\theta^2 - 100\phi^2 - 40\phi\theta^4 - 60\phi\theta^2 + 24\theta^4)^2} \\ \pi_1^L = \frac{\phi(\alpha - \bar{c})^2(5\phi - 2)^2(5\phi + 2\theta^2)^2 \cdot (25\phi - 4\theta^2)}{(625\phi^3 - 50\phi^2\theta^2 - 100\phi^2 - 40\phi\theta^4 - 60\phi\theta^2 + 24\theta^4)^2} \\ \pi_2^L = \frac{\phi(\alpha - \bar{c})^2(5\phi - 2\theta)^2(5\phi + 2\theta)^2 \cdot (25\phi - 4\theta^2)}{(625\phi^3 - 50\phi^2\theta^2 - 100\phi^2 - 40\phi\theta^4 - 60\phi\theta^2 + 24\theta^4)^2} \\ \pi_3^L = \frac{\phi(\alpha - \bar{c})^2(5\phi - 2\theta)^2(5\phi + 2\theta)^2 \cdot (25\phi - 4\theta^2)}{(625\phi^3 - 50\phi^2\theta^2 - 100\phi^2 - 40\phi\theta^4 - 60\phi\theta^2 + 24\theta^4)^2} \end{cases} \quad (47)$$

Total Production:

$$\frac{10\phi(\alpha - \bar{c})(50\phi^2 + 5\phi\theta^2 - 5\phi - 2\theta^4 - 6\theta^2)}{625\phi^3 - 50\phi^2\theta^2 - 100\phi^2 - 40\phi\theta^4 - 60\phi\theta^2 + 24\theta^4}$$

Price:

$$\frac{125\alpha\phi^3 - 100\alpha\phi^2\theta^2 - 50\alpha\phi^2 - 20\alpha\phi\theta^4 + 24\alpha\theta^4 + 500\bar{c}\phi^3 + 50\bar{c}\phi^2\theta^2 - 50\bar{c}\phi^2 - 20\bar{c}\phi\theta^4 - 60\bar{c}\phi\theta^2}{625\phi^3 - 50\phi^2\theta^2 - 100\phi^2 - 40\phi\theta^4 - 60\phi\theta^2 + 24\theta^4}$$

Firm Surplus:

$$\frac{4\phi(\alpha - \bar{c})^2 \cdot (15625\phi^5 + 1250\phi^4\theta^2 - 3750\phi^4 - 1125\phi^3\theta^4 - 4500\phi^3\theta^2 + 625\phi^3 - 100\phi^2\theta^6 + 500\phi^2\theta^4 + 400\phi^2\theta^2 + 100\phi\theta^8 + 80\phi\theta^6 + 220\phi\theta^4 - 16\theta^8 - 48\theta^6)}{(625\phi^3 - 50\phi^2\theta^2 - 100\phi^2 - 40\phi\theta^4 - 60\phi\theta^2 + 24\theta^4)^2}$$

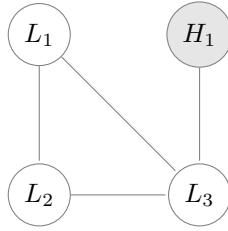
Consumer Surplus:

$$\frac{50\phi^2(\alpha - \bar{c})^2(50\phi^2 + 5\phi\theta^2 - 5\phi - 2\theta^4 - 6\theta^2)^2}{(625\phi^3 - 50\phi^2\theta^2 - 100\phi^2 - 40\phi\theta^4 - 60\phi\theta^2 + 24\theta^4)^2}$$

Social Welfare:

$$\frac{2\phi(\alpha - \bar{c})^2 \cdot (93750\phi^5 + 15000\phi^4\theta^2 - 20000\phi^4 - 6625\phi^3\theta^4 - 25250\phi^3\theta^2 + 1875\phi^3 - 700\phi^2\theta^6 + 2300\phi^2\theta^2 + 300\phi\theta^8 + 760\phi\theta^6 + 1340\phi\theta^4 - 32\theta^8 - 96\theta^6)}{(625\phi^3 - 50\phi^2\theta^2 - 100\phi^2 - 40\phi\theta^4 - 60\phi\theta^2 + 24\theta^4)^2}$$

#### D.1.17 E4D [1H3L]



$$\text{Equations : } \begin{cases} e_1^H \cdot \left(\frac{25\phi}{3} - 3\right) = \alpha - 3e_1^L\theta - 3e_2^L\theta + e_3^L\theta - \bar{c} \\ e_1^L \cdot \left(\frac{25\phi}{2\theta} - 2\theta\right) = \alpha - 2e_1^H + 2e_2^L\theta + e_3^L\theta - \bar{c} \\ e_2^L \cdot \left(\frac{25\phi}{2\theta} - 2\theta\right) = \alpha - 2e_1^H + 2e_1^L\theta + e_3^L\theta - \bar{c} \\ e_3^L \cdot \left(\frac{25\phi}{\theta} - \theta\right) = \alpha + 3e_1^H + 2e_1^L\theta + 2e_2^L\theta - \bar{c} \end{cases}$$

Optimal efforts:

$$\begin{cases} e_1^H = \frac{15\phi(\alpha - \bar{c})(5\phi - 4\theta^2)}{625\phi^3 - 225\phi^2\theta^2 - 225\phi^2 + 12\theta^4} \\ e_1^L = \frac{10\phi\theta(\alpha - \bar{c})(5\phi - 3)}{625\phi^3 - 225\phi^2\theta^2 - 225\phi^2 + 12\theta^4} \\ e_2^L = \frac{10\phi\theta(\alpha - \bar{c})(5\phi - 3)}{625\phi^3 - 225\phi^2\theta^2 - 225\phi^2 + 12\theta^4} \\ e_3^L = \frac{\theta(\alpha - \bar{c})(25\phi^2 - 12\theta^2)}{625\phi^3 - 225\phi^2\theta^2 - 225\phi^2 + 12\theta^4} \end{cases}$$

Production Costs:

$$\begin{cases} c_1^H = -\frac{25\alpha\phi^2\theta^2 + 75\alpha\phi^2 - 60\alpha\phi\theta^2 - 12\alpha\theta^4 - 625\bar{c}\phi^3 + 200\bar{c}\phi^2\theta^2 + 150\bar{c}\phi^2 + 60\bar{c}\phi\theta^2}{625\phi^3 - 225\phi^2\theta^2 - 225\phi^2 + 12\theta^4} \\ c_1^L = -\frac{125\alpha\phi^2\theta^2 - 60\alpha\phi\theta^2 - 12\alpha\theta^4 - 625\bar{c}\phi^3 + 100\bar{c}\phi^2\theta^2 + 225\bar{c}\phi^2 + 60\bar{c}\phi\theta^2}{625\phi^3 - 225\phi^2\theta^2 - 225\phi^2 + 12\theta^4} \\ c_2^L = -\frac{125\alpha\phi^2\theta^2 - 60\alpha\phi\theta^2 - 12\alpha\theta^4 - 625\bar{c}\phi^3 + 100\bar{c}\phi^2\theta^2 + 225\bar{c}\phi^2 + 60\bar{c}\phi\theta^2}{625\phi^3 - 225\phi^2\theta^2 - 225\phi^2 + 12\theta^4} \\ c_3^L = -\frac{125\alpha\phi^2\theta^2 + 75\alpha\phi^2 - 120\alpha\phi\theta^2 - 12\alpha\theta^4 - 625\bar{c}\phi^3 + 100\bar{c}\phi^2\theta^2 + 150\bar{c}\phi^2 + 120\bar{c}\phi\theta^2}{625\phi^3 - 225\phi^2\theta^2 - 225\phi^2 + 12\theta^4} \end{cases}$$

Production Quantities:

$$\begin{cases} q_1^H = \frac{25\phi^2(\alpha - \bar{c})(5\phi - 4\theta^2)}{625\phi^3 - 225\phi^2\theta^2 - 225\phi^2 + 12\theta^4} \\ q_1^L = \frac{25\phi^2(\alpha - \bar{c})(5\phi - 3)}{625\phi^3 - 225\phi^2\theta^2 - 225\phi^2 + 12\theta^4} \\ q_2^L = \frac{25\phi^2(\alpha - \bar{c})(5\phi - 3)}{625\phi^3 - 225\phi^2\theta^2 - 225\phi^2 + 12\theta^4} \\ q_3^L = \frac{5\phi(\alpha - \bar{c})(25\phi^2 - 12\theta^2)}{625\phi^3 - 225\phi^2\theta^2 - 225\phi^2 + 12\theta^4} \end{cases}$$

Profits:

$$\begin{cases} \pi_1^H = \frac{25\phi^3(\alpha - \bar{c})^2(5\phi - 4\theta^2)^2 \cdot (25\phi - 9)}{(625\phi^3 - 225\phi^2\theta^2 - 225\phi^2 + 12\theta^4)^2} \\ \pi_1^L = \frac{25\phi^3(\alpha - \bar{c})^2(5\phi - 3)^2 \cdot (25\phi - 4\theta^2)}{(625\phi^3 - 225\phi^2\theta^2 - 225\phi^2 + 12\theta^4)^2} \\ \pi_2^L = \frac{25\phi^3(\alpha - \bar{c})^2(5\phi - 3)^2 \cdot (25\phi - 4\theta^2)}{(625\phi^3 - 225\phi^2\theta^2 - 225\phi^2 + 12\theta^4)^2} \\ \pi_3^L = \frac{\phi(\alpha - \bar{c})^2 \cdot (25\phi - \theta^2)(25\phi^2 - 12\theta^2)^2}{(625\phi^3 - 225\phi^2\theta^2 - 225\phi^2 + 12\theta^4)^2} \end{cases} \quad (48)$$

Total Production:

$$\frac{10\phi(\alpha - \bar{c})(50\phi^2 - 10\phi\theta^2 - 15\phi - 6\theta^2)}{625\phi^3 - 225\phi^2\theta^2 - 225\phi^2 + 12\theta^4}$$

Price:

$$\frac{125\alpha\phi^3 - 125\alpha\phi^2\theta^2 - 75\alpha\phi^2 + 60\alpha\phi\theta^2 + 12\alpha\theta^4 + 500\bar{c}\phi^3 - 100\bar{c}\phi^2\theta^2 - 150\bar{c}\phi^2 - 60\bar{c}\phi\theta^2}{625\phi^3 - 225\phi^2\theta^2 - 225\phi^2 + 12\theta^4}$$

Firm Surplus:

$$\frac{\phi(\alpha - \bar{c})^2 \cdot (62500\phi^5 - 30625\phi^4\theta^2 - 43125\phi^4 + 10000\phi^3\theta^4 + 11250\phi^3 - 3000\phi^2\theta^4 - 1800\phi^2\theta^2 + 3600\phi\theta^4 - 144\theta^6)}{(625\phi^3 - 225\phi^2\theta^2 - 225\phi^2 + 12\theta^4)^2}$$

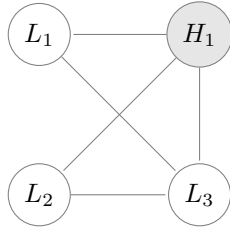
Consumer Surplus:

$$\frac{50\phi^2(\alpha - \bar{c})^2(50\phi^2 - 10\phi\theta^2 - 15\phi - 6\theta^2)^2}{(625\phi^3 - 225\phi^2\theta^2 - 225\phi^2 + 12\theta^4)^2}$$

Social Welfare:

$$\frac{3\phi(\alpha - \bar{c})^2 \cdot (62500\phi^5 - 26875\phi^4\theta^2 - 39375\phi^4 + 5000\phi^3\theta^4 - 5000\phi^3\theta^2 + 7500\phi^3 + 1000\phi^2\theta^4 + 2400\phi^2\theta^2 + 1800\phi\theta^4 - 48\theta^6)}{(625\phi^3 - 225\phi^2\theta^2 - 225\phi^2 + 12\theta^4)^2}$$

#### D.1.18 E5A [1H3L]



$$\text{Equations : } \begin{cases} e_1^H \cdot (25\phi - 1) = \alpha + 2e_1^L\theta + 2e_2^L\theta + e_3^L\theta - \bar{c} \\ e_1^L \cdot \left(\frac{25\phi}{2\theta} - 2\theta\right) = \alpha + e_1^H - 3e_2^L\theta + e_3^L\theta - \bar{c} \\ e_2^L \cdot \left(\frac{25\phi}{2\theta} - 2\theta\right) = \alpha + e_1^H - 3e_1^L\theta + e_3^L\theta - \bar{c} \\ e_3^L \cdot \left(\frac{25\phi}{\theta} - \theta\right) = \alpha + e_1^H + 2e_1^L\theta + 2e_2^L\theta - \bar{c} \end{cases}$$

Optimal efforts:

$$\begin{cases} e_1^H = \frac{(\alpha - \bar{c})(5\phi + 2\theta^2)}{125\phi^2 + 5\phi\theta^2 - 5\phi - 2\theta^4 - 2\theta^2} \\ e_1^L = \frac{10\phi\theta(\alpha - \bar{c})}{125\phi^2 + 5\phi\theta^2 - 5\phi - 2\theta^4 - 2\theta^2} \\ e_2^L = \frac{10\phi\theta(\alpha - \bar{c})}{125\phi^2 + 5\phi\theta^2 - 5\phi - 2\theta^4 - 2\theta^2} \\ e_3^L = \frac{\theta(\alpha - \bar{c})(5\phi + 2\theta^2)}{125\phi^2 + 5\phi\theta^2 - 5\phi - 2\theta^4 - 2\theta^2} \end{cases}$$

Production Costs:

$$\begin{cases} c_1^H = -\frac{25\alpha\phi\theta^2 + 5\alpha\phi + 2\alpha\theta^4 + 2\alpha\theta^2 - 125\bar{c}\phi^2 - 30\bar{c}\phi\theta^2}{125\phi^2 + 5\phi\theta^2 - 5\phi - 2\theta^4 - 2\theta^2} \\ c_1^L = -\frac{15\alpha\phi\theta^2 + 5\alpha\phi + 2\alpha\theta^4 + 2\alpha\theta^2 - 125\bar{c}\phi^2 - 20\bar{c}\phi\theta^2}{125\phi^2 + 5\phi\theta^2 - 5\phi - 2\theta^4 - 2\theta^2} \\ c_2^L = -\frac{15\alpha\phi\theta^2 + 5\alpha\phi + 2\alpha\theta^4 + 2\alpha\theta^2 - 125\bar{c}\phi^2 - 20\bar{c}\phi\theta^2}{125\phi^2 + 5\phi\theta^2 - 5\phi - 2\theta^4 - 2\theta^2} \\ c_3^L = -\frac{25\alpha\phi\theta^2 + 5\alpha\phi + 2\alpha\theta^4 + 2\alpha\theta^2 - 125\bar{c}\phi^2 - 30\bar{c}\phi\theta^2}{125\phi^2 + 5\phi\theta^2 - 5\phi - 2\theta^4 - 2\theta^2} \end{cases}$$

Production Quantities:

$$\begin{cases} q_1^H = \frac{5\phi(\alpha - \bar{c})(5\phi + 2\theta^2)}{125\phi^2 + 5\phi\theta^2 - 5\phi - 2\theta^4 - 2\theta^2} \\ q_1^L = \frac{25\phi^2(\alpha - \bar{c})}{125\phi^2 + 5\phi\theta^2 - 5\phi - 2\theta^4 - 2\theta^2} \\ q_2^L = \frac{25\phi^2(\alpha - \bar{c})}{125\phi^2 + 5\phi\theta^2 - 5\phi - 2\theta^4 - 2\theta^2} \\ q_3^L = \frac{5\phi(\alpha - \bar{c})(5\phi + 2\theta^2)}{125\phi^2 + 5\phi\theta^2 - 5\phi - 2\theta^4 - 2\theta^2} \end{cases}$$

Profits:

$$\begin{cases} \pi_1^H &= \frac{\phi(\alpha-\bar{c})^2(5\phi+2\theta^2)^2 \cdot (25\phi-1)}{(125\phi^2+5\phi\theta^2-5\phi-2\theta^4-2\theta^2)^2} \\ \pi_1^L &= \frac{25\phi^3(\alpha-\bar{c})^2 \cdot (25\phi-4\theta^2)}{(125\phi^2+5\phi\theta^2-5\phi-2\theta^4-2\theta^2)^2} \\ \pi_2^L &= \frac{25\phi^3(\alpha-\bar{c})^2 \cdot (25\phi-4\theta^2)}{(125\phi^2+5\phi\theta^2-5\phi-2\theta^4-2\theta^2)^2} \\ \pi_3^L &= \frac{\phi(\alpha-\bar{c})^2(5\phi+2\theta^2)^2 \cdot (25\phi-\theta^2)}{(125\phi^2+5\phi\theta^2-5\phi-2\theta^4-2\theta^2)^2} \end{cases} \quad (49)$$

Total Production:

$$\frac{20\phi(\alpha-\bar{c})(5\phi+\theta^2)}{125\phi^2+5\phi\theta^2-5\phi-2\theta^4-2\theta^2}$$

Price:

$$\frac{25\alpha\phi^2-15\alpha\phi\theta^2-5\alpha\phi-2\alpha\theta^4-2\alpha\theta^2+100\bar{c}\phi^2+20\bar{c}\phi\theta^2}{125\phi^2+5\phi\theta^2-5\phi-2\theta^4-2\theta^2}$$

Firm Surplus:

$$\frac{\phi(\alpha-\bar{c})^2 \cdot (2500\phi^3+775\phi^2\theta^2-25\phi^2+180\phi\theta^4-20\phi\theta^2-4\theta^6-4\theta^4)}{(125\phi^2+5\phi\theta^2-5\phi-2\theta^4-2\theta^2)^2}$$

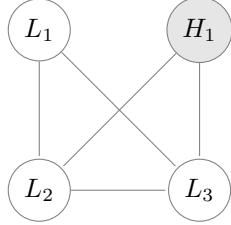
Consumer Surplus:

$$\frac{200\phi^2(\alpha-\bar{c})^2(5\phi+\theta^2)^2}{(125\phi^2+5\phi\theta^2-5\phi-2\theta^4-2\theta^2)^2}$$

Social Welfare:

$$\frac{\phi(\alpha-\bar{c})^2 \cdot (7500\phi^3+2775\phi^2\theta^2-25\phi^2+380\phi\theta^4-20\phi\theta^2-4\theta^6-4\theta^4)}{(125\phi^2+5\phi\theta^2-5\phi-2\theta^4-2\theta^2)^2}$$

#### D.1.19 E5B [1H3L]



$$\text{Equations : } \begin{cases} e_1^H \cdot \left(\frac{25\phi}{2} - 2\right) = \alpha - 3e_1^L\theta + e_2^L\theta + e_3^L\theta - \bar{c} \\ e_1^L \cdot \left(\frac{25\phi}{2\theta} - 2\theta\right) = \alpha - 3e_1^H + e_2^H\theta + e_3^L\theta - \bar{c} \\ e_2^L \cdot \left(\frac{25\phi}{\theta} - \theta\right) = \alpha + 2e_1^H + 2e_1^L\theta + e_3^L\theta - \bar{c} \\ e_3^L \cdot \left(\frac{25\phi}{\theta} - \theta\right) = \alpha + 2e_1^H + 2e_1^L\theta + e_2^L\theta - \bar{c} \end{cases}$$

Optimal efforts:

$$\begin{cases} e_1^H &= \frac{10\phi(\alpha-\bar{c})(5\phi-2\theta^2)}{625\phi^3-150\phi^2\theta^2-100\phi^2-20\phi\theta^2+8\theta^4} \\ e_1^L &= \frac{10\phi\theta(\alpha-\bar{c})(5\phi-2)}{625\phi^3-150\phi^2\theta^2-100\phi^2-20\phi\theta^2+8\theta^4} \\ e_2^L &= \frac{\theta(\alpha-\bar{c})(5\phi-2\theta)(5\phi+2\theta)}{625\phi^3-150\phi^2\theta^2-100\phi^2-20\phi\theta^2+8\theta^4} \\ e_3^L &= \frac{\theta(\alpha-\bar{c})(5\phi-2\theta)(5\phi+2\theta)}{625\phi^3-150\phi^2\theta^2-100\phi^2-20\phi\theta^2+8\theta^4} \end{cases}$$

Production Costs:

$$\begin{cases} c_1^H &= -\frac{50\alpha\phi^2\theta^2+50\alpha\phi^2-20\alpha\phi\theta^2-8\alpha\theta^4-625\bar{c}\phi^3+100\bar{c}\phi^2\theta^2+50\bar{c}\phi^2+40\bar{c}\phi\theta^2}{625\phi^3-150\phi^2\theta^2-100\phi^2-20\phi\theta^2+8\theta^4} \\ c_1^L &= -\frac{100\alpha\phi^2\theta^2-20\alpha\phi\theta^2-8\alpha\theta^4-625\bar{c}\phi^3+50\bar{c}\phi^2\theta^2+100\bar{c}\phi^2+40\bar{c}\phi\theta^2}{625\phi^3-150\phi^2\theta^2-100\phi^2-20\phi\theta^2+8\theta^4} \\ c_2^L &= -\frac{100\alpha\phi^2\theta^2+50\alpha\phi^2-40\alpha\phi\theta^2-8\alpha\theta^4-625\bar{c}\phi^3+50\bar{c}\phi^2\theta^2+50\bar{c}\phi^2+60\bar{c}\phi\theta^2}{625\phi^3-150\phi^2\theta^2-100\phi^2-20\phi\theta^2+8\theta^4} \\ c_3^L &= -\frac{100\alpha\phi^2\theta^2+50\alpha\phi^2-40\alpha\phi\theta^2-8\alpha\theta^4-625\bar{c}\phi^3+50\bar{c}\phi^2\theta^2+50\bar{c}\phi^2+60\bar{c}\phi\theta^2}{625\phi^3-150\phi^2\theta^2-100\phi^2-20\phi\theta^2+8\theta^4} \end{cases}$$

Production Quantities:

$$\begin{cases} q_1^H &= \frac{25\phi^2(\alpha-\bar{c})(5\phi-2\theta^2)}{625\phi^3-150\phi^2\theta^2-100\phi^2-20\phi\theta^2+8\theta^4} \\ q_1^L &= \frac{25\phi^2(\alpha-\bar{c})(5\phi-2)}{625\phi^3-150\phi^2\theta^2-100\phi^2-20\phi\theta^2+8\theta^4} \\ q_2^L &= \frac{5\phi(\alpha-\bar{c})(5\phi-2\theta)(5\phi+2\theta)}{625\phi^3-150\phi^2\theta^2-100\phi^2-20\phi\theta^2+8\theta^4} \\ q_3^L &= \frac{5\phi(\alpha-\bar{c})(5\phi-2\theta)(5\phi+2\theta)}{625\phi^3-150\phi^2\theta^2-100\phi^2-20\phi\theta^2+8\theta^4} \end{cases}$$

Profits:

$$\begin{cases} \pi_1^H &= \frac{25\phi^3(\alpha-\bar{c})^2(5\phi-2\theta^2)^2 \cdot (25\phi-4)}{(625\phi^3-150\phi^2\theta^2-100\phi^2-20\phi\theta^2+8\theta^4)^2} \\ \pi_1^L &= \frac{25\phi^3(\alpha-\bar{c})^2(5\phi-2)^2 \cdot (25\phi-4\theta^2)}{(625\phi^3-150\phi^2\theta^2-100\phi^2-20\phi\theta^2+8\theta^4)^2} \\ \pi_2^L &= \frac{\phi(\alpha-\bar{c})^2(5\phi-2\theta)^2(5\phi+2\theta)^2 \cdot (25\phi-\theta^2)}{(625\phi^3-150\phi^2\theta^2-100\phi^2-20\phi\theta^2+8\theta^4)^2} \\ \pi_3^L &= \frac{\phi(\alpha-\bar{c})^2(5\phi-2\theta)^2(5\phi+2\theta)^2 \cdot (25\phi-\theta^2)}{(625\phi^3-150\phi^2\theta^2-100\phi^2-20\phi\theta^2+8\theta^4)^2} \end{cases} \quad (50)$$

Total Production:

$$\frac{10\phi(\alpha-\bar{c})(50\phi^2-5\phi\theta^2-5\phi-4\theta^2)}{625\phi^3-150\phi^2\theta^2-100\phi^2-20\phi\theta^2+8\theta^4}$$

Price:

$$\frac{125\alpha\phi^3-100\alpha\phi^2\theta^2-50\alpha\phi^2+20\alpha\phi\theta^2+8\alpha\theta^4+500\bar{c}\phi^3-50\bar{c}\phi^2\theta^2-50\bar{c}\phi^2-40\bar{c}\phi\theta^2}{625\phi^3-150\phi^2\theta^2-100\phi^2-20\phi\theta^2+8\theta^4}$$

Firm Surplus:

$$\frac{2\phi(\alpha-\bar{c})^2 \cdot (31250\phi^5-8125\phi^4\theta^2-7500\phi^4+1250\phi^3\theta^4-3000\phi^3\theta^2+1250\phi^3-200\phi^2\theta^2+400\phi\theta^4-16\theta^6)}{(625\phi^3-150\phi^2\theta^2-100\phi^2-20\phi\theta^2+8\theta^4)^2}$$

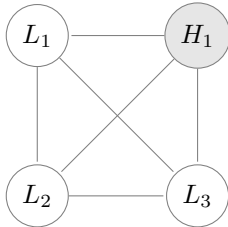
Consumer Surplus:

$$\frac{50\phi^2(\alpha-\bar{c})^2(50\phi^2-5\phi\theta^2-5\phi-4\theta^2)^2}{(625\phi^3-150\phi^2\theta^2-100\phi^2-20\phi\theta^2+8\theta^4)^2}$$

Social Welfare:

$$\frac{2\phi(\alpha-\bar{c})^2 \cdot (93750\phi^5-20625\phi^4\theta^2-20000\phi^4+1875\phi^3\theta^4-11750\phi^3\theta^2+1875\phi^3+1000\phi^2\theta^4+800\phi^2\theta^2+800\phi\theta^4-16\theta^6)}{(625\phi^3-150\phi^2\theta^2-100\phi^2-20\phi\theta^2+8\theta^4)^2}$$

#### D.1.20 E6A [1H3L]



$$\text{Equations : } \begin{cases} e_1^H \cdot (25\phi-1) = \alpha + e_1^L\theta + e_2^L\theta + e_3^L\theta - \bar{c} \\ e_1^L \cdot \left(\frac{25\phi}{\theta} - \theta\right) = \alpha + e_1^H + e_2^L\theta + e_3^L\theta - \bar{c} \\ e_2^L \cdot \left(\frac{25\phi}{\theta} - \theta\right) = \alpha + e_1^H + e_1^L\theta + e_3^L\theta - \bar{c} \\ e_3^L \cdot \left(\frac{25\phi}{\theta} - \theta\right) = \alpha + e_1^H + e_1^L\theta + e_2^L\theta - \bar{c} \end{cases}$$

Optimal efforts:

$$\begin{cases} e_1^H &= \frac{\alpha-\bar{c}}{25\phi-3\theta^2-1} \\ e_1^L &= \frac{\theta(\alpha-\bar{c})}{25\phi-3\theta^2-1} \\ e_2^L &= \frac{\theta(\alpha-\bar{c})}{25\phi-3\theta^2-1} \\ e_3^L &= \frac{\theta(\alpha-\bar{c})}{25\phi-3\theta^2-1} \end{cases}$$

Production Costs:

$$\begin{cases} c_1^H &= -\frac{3\alpha\theta^2+\alpha-25\bar{c}\phi}{25\phi-3\theta^2-1} \\ c_1^L &= -\frac{3\alpha\theta^2+\alpha-25\bar{c}\phi}{25\phi-3\theta^2-1} \\ c_2^L &= -\frac{3\alpha\theta^2+\alpha-25\bar{c}\phi}{25\phi-3\theta^2-1} \\ c_3^L &= -\frac{3\alpha\theta^2+\alpha-25\bar{c}\phi}{25\phi-3\theta^2-1} \end{cases}$$

Production Quantities:



$$\begin{cases} q_1^H &= \frac{5\phi(\alpha-\bar{c})}{25\phi-3\theta^2-1} \\ q_1^L &= \frac{5\phi(\alpha-\bar{c})}{25\phi-3\theta^2-1} \\ q_2^L &= \frac{5\phi(\alpha-\bar{c})}{25\phi-3\theta^2-1} \\ q_3^L &= \frac{5\phi(\alpha-\bar{c})}{25\phi-3\theta^2-1} \end{cases}$$

Profits:

$$\begin{cases} \pi_1^H &= \frac{\phi(\alpha-\bar{c})^2 \cdot (25\phi-1)}{(25\phi-3\theta^2-1)^2} \\ \pi_1^L &= \frac{\phi(\alpha-\bar{c})^2 \cdot (25\phi-\theta^2)}{(25\phi-3\theta^2-1)^2} \\ \pi_2^L &= \frac{\phi(\alpha-\bar{c})^2 \cdot (25\phi-\theta^2)}{(25\phi-3\theta^2-1)^2} \\ \pi_3^L &= \frac{\phi(\alpha-\bar{c})^2 \cdot (25\phi-\theta^2)}{(25\phi-3\theta^2-1)^2} \end{cases} \quad (51)$$

Total Production:

$$\frac{20\phi(\alpha-\bar{c})}{25\phi-3\theta^2-1}$$

Price:

$$\frac{5\alpha\phi-3\alpha\theta^2-\alpha+20\bar{c}\phi}{25\phi-3\theta^2-1}$$

Firm Surplus:

$$\frac{\phi(\alpha-\bar{c})^2 \cdot (100\phi-3\theta^2-1)}{(25\phi-3\theta^2-1)^2}$$

Consumer Surplus:

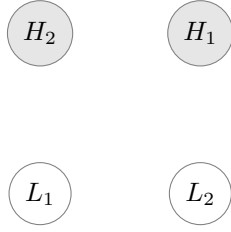
$$\frac{200\phi^2(\alpha-\bar{c})^2}{(25\phi-3\theta^2-1)^2}$$

Social Welfare:

$$\frac{\phi(\alpha-\bar{c})^2 \cdot (300\phi-3\theta^2-1)}{(25\phi-3\theta^2-1)^2}$$

## D.2 Two *high*- and two *low*-productive firms (2H2L)

### D.2.1 E0A [2H2L]



$$\text{Equations : } \begin{cases} e_1^H \cdot \left( \frac{25\phi}{4} - 4 \right) = \alpha - e_2^H - e_1^L \theta - e_2^L \theta - \bar{c} \\ e_2^H \cdot \left( \frac{25\phi}{4} - 4 \right) = \alpha - e_1^H - e_1^L \theta - e_2^L \theta - \bar{c} \\ e_1^L \cdot \left( \frac{25\phi}{4\theta} - 4\theta \right) = \alpha - e_1^H - e_2^H - e_2^L \theta - \bar{c} \\ e_2^L \cdot \left( \frac{25\phi}{4\theta} - 4\theta \right) = \alpha - e_1^H - e_2^H - e_1^L \theta - \bar{c} \end{cases}$$

Optimal efforts:

$$\begin{cases} e_1^H = \frac{4(\alpha - \bar{c})(5\phi - 4\theta^2)}{125\phi^2 - 60\phi\theta^2 - 60\phi + 16\theta^2} \\ e_2^H = \frac{4(\alpha - \bar{c})(5\phi - 4\theta^2)}{125\phi^2 - 60\phi\theta^2 - 60\phi + 16\theta^2} \\ e_1^L = \frac{4\theta(\alpha - \bar{c})(5\phi - 4)}{125\phi^2 - 60\phi\theta^2 - 60\phi + 16\theta^2} \\ e_2^L = \frac{4\theta(\alpha - \bar{c})(5\phi - 4)}{125\phi^2 - 60\phi\theta^2 - 60\phi + 16\theta^2} \end{cases}$$

Production Costs:

$$\begin{cases} c_1^H = -\frac{20\alpha\phi - 16\alpha\theta^2 - 125\bar{c}\phi^2 + 60\bar{c}\phi\theta^2 + 40\bar{c}\phi}{125\phi^2 - 60\phi\theta^2 - 60\phi + 16\theta^2} \\ c_2^H = -\frac{20\alpha\phi - 16\alpha\theta^2 - 125\bar{c}\phi^2 + 60\bar{c}\phi\theta^2 + 40\bar{c}\phi}{125\phi^2 - 60\phi\theta^2 - 60\phi + 16\theta^2} \\ c_1^L = -\frac{20\alpha\phi\theta^2 - 16\alpha\theta^2 - 125\bar{c}\phi^2 + 40\bar{c}\phi\theta^2 + 60\bar{c}\phi}{125\phi^2 - 60\phi\theta^2 - 60\phi + 16\theta^2} \\ c_2^L = -\frac{20\alpha\phi\theta^2 - 16\alpha\theta^2 - 125\bar{c}\phi^2 + 40\bar{c}\phi\theta^2 + 60\bar{c}\phi}{125\phi^2 - 60\phi\theta^2 - 60\phi + 16\theta^2} \end{cases}$$

Production Quantities:

$$\begin{cases} q_1^H = \frac{5\phi(\alpha - \bar{c})(5\phi - 4\theta^2)}{125\phi^2 - 60\phi\theta^2 - 60\phi + 16\theta^2} \\ q_2^H = \frac{5\phi(\alpha - \bar{c})(5\phi - 4\theta^2)}{125\phi^2 - 60\phi\theta^2 - 60\phi + 16\theta^2} \\ q_1^L = \frac{5\phi(\alpha - \bar{c})(5\phi - 4)}{125\phi^2 - 60\phi\theta^2 - 60\phi + 16\theta^2} \\ q_2^L = \frac{5\phi(\alpha - \bar{c})(5\phi - 4)}{125\phi^2 - 60\phi\theta^2 - 60\phi + 16\theta^2} \end{cases}$$

Profits:

$$\begin{cases} \pi_1^H = \frac{\phi(\alpha - \bar{c})^2(5\phi - 4\theta^2)^2 \cdot (25\phi - 16)}{(125\phi^2 - 60\phi\theta^2 - 60\phi + 16\theta^2)^2} \\ \pi_2^H = \frac{\phi(\alpha - \bar{c})^2(5\phi - 4\theta^2)^2 \cdot (25\phi - 16)}{(125\phi^2 - 60\phi\theta^2 - 60\phi + 16\theta^2)^2} \\ \pi_1^L = \frac{\phi(\alpha - \bar{c})^2(5\phi - 4)^2 \cdot (25\phi - 16\theta^2)}{(125\phi^2 - 60\phi\theta^2 - 60\phi + 16\theta^2)^2} \\ \pi_2^L = \frac{\phi(\alpha - \bar{c})^2(5\phi - 4)^2 \cdot (25\phi - 16\theta^2)}{(125\phi^2 - 60\phi\theta^2 - 60\phi + 16\theta^2)^2} \end{cases} \quad (52)$$

Total Production:

$$\frac{20\phi(\alpha - \bar{c})(5\phi - 2\theta^2 - 2)}{125\phi^2 - 60\phi\theta^2 - 60\phi + 16\theta^2}$$

Price:

$$\frac{25\alpha\phi^2 - 20\alpha\phi\theta^2 - 20\alpha\phi + 16\alpha\theta^2 + 100\bar{c}\phi^2 - 40\bar{c}\phi\theta^2 - 40\bar{c}\phi}{125\phi^2 - 60\phi\theta^2 - 60\phi + 16\theta^2}$$

Firm Surplus:

$$\frac{4\phi(\alpha - \bar{c})^2 \cdot (625\phi^3 - 700\phi^2\theta^2 - 700\phi^2 + 200\phi\theta^4 + 640\phi\theta^2 + 200\phi - 128\theta^4 - 128\theta^2)}{(125\phi^2 - 60\phi\theta^2 - 60\phi + 16\theta^2)^2}$$

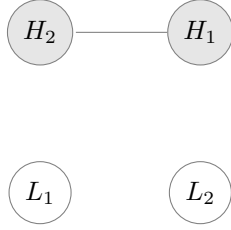
Consumer Surplus:

$$\frac{200\phi^2(\alpha - \bar{c})^2(5\phi - 2\theta^2 - 2)^2}{(125\phi^2 - 60\phi\theta^2 - 60\phi + 16\theta^2)^2}$$

Social Welfare:

$$\frac{4\phi(\alpha - \bar{c})^2 \cdot (1875\phi^3 - 1700\phi^2\theta^2 - 1700\phi^2 + 400\phi\theta^4 + 1040\phi\theta^2 + 400\phi - 128\theta^4 - 128\theta^2)}{(125\phi^2 - 60\phi\theta^2 - 60\phi + 16\theta^2)^2}$$

### D.2.2 E1A [2H2L]



$$\text{Equations : } \begin{cases} e_1^H \cdot \left(\frac{25\phi}{3} - 3\right) = \alpha + 3e_2^H - e_1^L\theta - e_2^L\theta - \bar{c} \\ e_2^H \cdot \left(\frac{25\phi}{3} - 3\right) = \alpha + 3e_1^H - e_1^L\theta - e_2^L\theta - \bar{c} \\ e_1^L \cdot \left(\frac{25\phi}{4\theta} - 4\theta\right) = \alpha - 2e_1^H - 2e_2^H - e_2^L\theta - \bar{c} \\ e_2^L \cdot \left(\frac{25\phi}{4\theta} - 4\theta\right) = \alpha - 2e_1^H - 2e_2^H - e_1^L\theta - \bar{c} \end{cases}$$

Optimal efforts:

$$\begin{cases} e_1^H = \frac{3(\alpha - \bar{c})(5\phi - 4\theta^2)}{125\phi^2 - 60\phi\theta^2 - 90\phi + 24\theta^2} \\ e_2^H = \frac{3(\alpha - \bar{c})(5\phi - 4\theta^2)}{125\phi^2 - 60\phi\theta^2 - 90\phi + 24\theta^2} \\ e_1^L = \frac{4\theta(\alpha - \bar{c})(5\phi - 6)}{125\phi^2 - 60\phi\theta^2 - 90\phi + 24\theta^2} \\ e_2^L = \frac{4\theta(\alpha - \bar{c})(5\phi - 6)}{125\phi^2 - 60\phi\theta^2 - 90\phi + 24\theta^2} \end{cases}$$

Production Costs:

$$\begin{cases} c_1^H = -\frac{30\alpha\phi - 24\alpha\theta^2 - 125\bar{c}\phi^2 + 60\bar{c}\phi\theta^2 + 60\bar{c}\phi}{125\phi^2 - 60\phi\theta^2 - 90\phi + 24\theta^2} \\ c_2^H = -\frac{30\alpha\phi - 24\alpha\theta^2 - 125\bar{c}\phi^2 + 60\bar{c}\phi\theta^2 + 60\bar{c}\phi}{125\phi^2 - 60\phi\theta^2 - 90\phi + 24\theta^2} \\ c_1^L = -\frac{20\alpha\phi\theta^2 - 24\alpha\theta^2 - 125\bar{c}\phi^2 + 40\bar{c}\phi\theta^2 + 90\bar{c}\phi}{125\phi^2 - 60\phi\theta^2 - 90\phi + 24\theta^2} \\ c_2^L = -\frac{20\alpha\phi\theta^2 - 24\alpha\theta^2 - 125\bar{c}\phi^2 + 40\bar{c}\phi\theta^2 + 90\bar{c}\phi}{125\phi^2 - 60\phi\theta^2 - 90\phi + 24\theta^2} \end{cases}$$

Production Quantities:

$$\begin{cases} q_1^H = \frac{5\phi(\alpha - \bar{c})(5\phi - 4\theta^2)}{125\phi^2 - 60\phi\theta^2 - 90\phi + 24\theta^2} \\ q_2^H = \frac{5\phi(\alpha - \bar{c})(5\phi - 4\theta^2)}{125\phi^2 - 60\phi\theta^2 - 90\phi + 24\theta^2} \\ q_1^L = \frac{5\phi(\alpha - \bar{c})(5\phi - 6)}{125\phi^2 - 60\phi\theta^2 - 90\phi + 24\theta^2} \\ q_2^L = \frac{5\phi(\alpha - \bar{c})(5\phi - 6)}{125\phi^2 - 60\phi\theta^2 - 90\phi + 24\theta^2} \end{cases}$$

Profits:

$$\begin{cases} \pi_1^H = \frac{\phi(\alpha - \bar{c})^2(5\phi - 4\theta^2)^2 \cdot (25\phi - 9)}{(125\phi^2 - 60\phi\theta^2 - 90\phi + 24\theta^2)^2} \\ \pi_2^H = \frac{\phi(\alpha - \bar{c})^2(5\phi - 4\theta^2)^2 \cdot (25\phi - 9)}{(125\phi^2 - 60\phi\theta^2 - 90\phi + 24\theta^2)^2} \\ \pi_1^L = \frac{\phi(\alpha - \bar{c})^2(5\phi - 6)^2 \cdot (25\phi - 16\theta^2)}{(125\phi^2 - 60\phi\theta^2 - 90\phi + 24\theta^2)^2} \\ \pi_2^L = \frac{\phi(\alpha - \bar{c})^2(5\phi - 6)^2 \cdot (25\phi - 16\theta^2)}{(125\phi^2 - 60\phi\theta^2 - 90\phi + 24\theta^2)^2} \end{cases} \quad (53)$$

Total Production:

$$\frac{20\phi(\alpha - \bar{c})(5\phi - 2\theta^2 - 3)}{125\phi^2 - 60\phi\theta^2 - 90\phi + 24\theta^2}$$

Price:

$$\frac{25\alpha\phi^2 - 20\alpha\phi\theta^2 - 30\alpha\phi + 24\alpha\theta^2 + 100\bar{c}\phi^2 - 40\bar{c}\phi\theta^2 - 60\bar{c}\phi}{125\phi^2 - 60\phi\theta^2 - 90\phi + 24\theta^2}$$

Firm Surplus:

$$\frac{2\phi(\alpha - \bar{c})^2 \cdot (1250\phi^3 - 1400\phi^2\theta^2 - 1725\phi^2 + 400\phi\theta^4 + 1320\phi\theta^2 + 900\phi - 144\theta^4 - 576\theta^2)}{(125\phi^2 - 60\phi\theta^2 - 90\phi + 24\theta^2)^2}$$

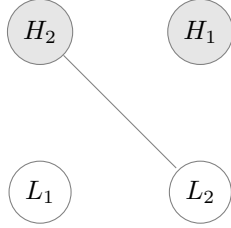
Consumer Surplus:

$$\frac{200\phi^2 (\alpha - \bar{c})^2 (5\phi - 2\theta^2 - 3)^2}{(125\phi^2 - 60\phi\theta^2 - 90\phi + 24\theta^2)^2}$$

Social Welfare:

$$\frac{2\phi (\alpha - \bar{c})^2 \cdot (3750\phi^3 - 3400\phi^2\theta^2 - 4725\phi^2 + 800\phi\theta^4 + 2520\phi\theta^2 + 1800\phi - 144\theta^4 - 576\theta^2)}{(125\phi^2 - 60\phi\theta^2 - 90\phi + 24\theta^2)^2}$$

### D.2.3 E1B [2H2L]



$$\text{Equations : } \begin{cases} e_1^H \cdot \left( \frac{25\phi}{4} - 4 \right) = \alpha - 2e_2^H - e_1^L\theta - 2e_2^L\theta - \bar{c} \\ e_2^H \cdot \left( \frac{25\phi}{3} - 3 \right) = \alpha - e_1^H - e_1^L\theta + 3e_2^L\theta - \bar{c} \\ e_1^L \cdot \left( \frac{25\phi}{4\theta} - 4\theta \right) = \alpha - e_1^H - 2e_2^H - 2e_2^L\theta - \bar{c} \\ e_2^L \cdot \left( \frac{25\phi}{3\theta} - 3\theta \right) = \alpha - e_1^H + 3e_2^H - e_1^L\theta - \bar{c} \end{cases}$$

Optimal efforts:

$$\begin{cases} e_1^H = \frac{4(\alpha - \bar{c})(5\phi - 4\theta^2)(5\phi - 3\theta^2 - 3)}{625\phi^3 - 625\phi^2\theta^2 - 625\phi^2 + 120\phi\theta^4 + 480\phi\theta^2 + 120\phi - 48\theta^4 - 48\theta^2} \\ e_2^H = \frac{3(\alpha - \bar{c})(5\phi - 4)(5\phi - 4\theta^2)}{625\phi^3 - 625\phi^2\theta^2 - 625\phi^2 + 120\phi\theta^4 + 480\phi\theta^2 + 120\phi - 48\theta^4 - 48\theta^2} \\ e_1^L = \frac{4\theta(\alpha - \bar{c})(5\phi - 4)(5\phi - 3\theta^2 - 3)}{625\phi^3 - 625\phi^2\theta^2 - 625\phi^2 + 120\phi\theta^4 + 480\phi\theta^2 + 120\phi - 48\theta^4 - 48\theta^2} \\ e_2^L = \frac{3\theta(\alpha - \bar{c})(5\phi - 4)(5\phi - 4\theta^2)}{625\phi^3 - 625\phi^2\theta^2 - 625\phi^2 + 120\phi\theta^4 + 480\phi\theta^2 + 120\phi - 48\theta^4 - 48\theta^2} \end{cases}$$

Production Costs:

$$\begin{cases} C_1^H = -\frac{100\alpha\phi^2 - 140\alpha\phi\theta^2 - 60\alpha\phi + 48\alpha\theta^4 + 48\alpha\theta^2 - 625\bar{c}\phi^3 + 625\bar{c}\phi^2\theta^2 + 525\bar{c}\phi^2 - 120\bar{c}\phi\theta^4 - 340\bar{c}\phi\theta^2 - 60\bar{c}\phi}{625\phi^3 - 625\phi^2\theta^2 - 625\phi^2 + 120\phi\theta^4 + 480\phi\theta^2 + 120\phi - 48\theta^4 - 48\theta^2} \\ C_2^H = -\frac{75\alpha\phi^2\theta^2 + 75\alpha\phi^2 - 60\alpha\phi\theta^4 - 120\alpha\phi\theta^2 - 60\alpha\phi + 48\alpha\theta^4 + 48\alpha\theta^2 - 625\bar{c}\phi^3 + 550\bar{c}\phi^2\theta^2 + 550\bar{c}\phi^2 - 60\bar{c}\phi\theta^4 - 360\bar{c}\phi\theta^2 - 60\bar{c}\phi}{625\phi^3 - 625\phi^2\theta^2 - 625\phi^2 + 120\phi\theta^4 + 480\phi\theta^2 + 120\phi - 48\theta^4 - 48\theta^2} \\ C_1^L = -\frac{100\alpha\phi^2\theta^2 - 60\alpha\phi\theta^4 - 140\alpha\phi\theta^2 + 48\alpha\theta^4 + 48\alpha\theta^2 - 625\bar{c}\phi^3 + 525\bar{c}\phi^2\theta^2 + 625\bar{c}\phi^2 - 60\bar{c}\phi\theta^4 - 340\bar{c}\phi\theta^2 - 120\bar{c}\phi}{625\phi^3 - 625\phi^2\theta^2 - 625\phi^2 + 120\phi\theta^4 + 480\phi\theta^2 + 120\phi - 48\theta^4 - 48\theta^2} \\ C_2^L = -\frac{75\alpha\phi^2\theta^2 + 75\alpha\phi^2 - 60\alpha\phi\theta^4 - 120\alpha\phi\theta^2 - 60\alpha\phi + 48\alpha\theta^4 + 48\alpha\theta^2 - 625\bar{c}\phi^3 + 550\bar{c}\phi^2\theta^2 + 550\bar{c}\phi^2 - 60\bar{c}\phi\theta^4 - 360\bar{c}\phi\theta^2 - 60\bar{c}\phi}{625\phi^3 - 625\phi^2\theta^2 - 625\phi^2 + 120\phi\theta^4 + 480\phi\theta^2 + 120\phi - 48\theta^4 - 48\theta^2} \end{cases}$$

Production Quantities:

$$\begin{cases} q_1^H = \frac{5\phi(\alpha - \bar{c})(5\phi - 4\theta^2)(5\phi - 3\theta^2 - 3)}{625\phi^3 - 625\phi^2\theta^2 - 625\phi^2 + 120\phi\theta^4 + 480\phi\theta^2 + 120\phi - 48\theta^4 - 48\theta^2} \\ q_2^H = \frac{5\phi(\alpha - \bar{c})(5\phi - 4)(5\phi - 4\theta^2)}{625\phi^3 - 625\phi^2\theta^2 - 625\phi^2 + 120\phi\theta^4 + 480\phi\theta^2 + 120\phi - 48\theta^4 - 48\theta^2} \\ q_1^L = \frac{5\phi(\alpha - \bar{c})(5\phi - 4)(5\phi - 3\theta^2 - 3)}{625\phi^3 - 625\phi^2\theta^2 - 625\phi^2 + 120\phi\theta^4 + 480\phi\theta^2 + 120\phi - 48\theta^4 - 48\theta^2} \\ q_2^L = \frac{5\phi(\alpha - \bar{c})(5\phi - 4)(5\phi - 4\theta^2)}{625\phi^3 - 625\phi^2\theta^2 - 625\phi^2 + 120\phi\theta^4 + 480\phi\theta^2 + 120\phi - 48\theta^4 - 48\theta^2} \end{cases}$$

Profits:

$$\begin{cases} \pi_1^H = \frac{\phi(\alpha - \bar{c})^2 (5\phi - 4\theta^2)^2 \cdot (25\phi - 16)(5\phi - 3\theta^2 - 3)^2}{(625\phi^3 - 625\phi^2\theta^2 - 625\phi^2 + 120\phi\theta^4 + 480\phi\theta^2 + 120\phi - 48\theta^4 - 48\theta^2)^2} \\ \pi_2^H = \frac{\phi(\alpha - \bar{c})^2 (5\phi - 4)^2 (5\phi - 4\theta^2)^2 \cdot (25\phi - 9)}{(625\phi^3 - 625\phi^2\theta^2 - 625\phi^2 + 120\phi\theta^4 + 480\phi\theta^2 + 120\phi - 48\theta^4 - 48\theta^2)^2} \\ \pi_1^L = \frac{\phi(\alpha - \bar{c})^2 (5\phi - 4)^2 \cdot (25\phi - 16\theta^2)(5\phi - 3\theta^2 - 3)^2}{(625\phi^3 - 625\phi^2\theta^2 - 625\phi^2 + 120\phi\theta^4 + 480\phi\theta^2 + 120\phi - 48\theta^4 - 48\theta^2)^2} \\ \pi_2^L = \frac{\phi(\alpha - \bar{c})^2 (5\phi - 4)^2 (5\phi - 4\theta^2)^2 \cdot (25\phi - 9\theta^2)}{(625\phi^3 - 625\phi^2\theta^2 - 625\phi^2 + 120\phi\theta^4 + 480\phi\theta^2 + 120\phi - 48\theta^4 - 48\theta^2)^2} \end{cases} \quad (54)$$

Total Production:

$$\frac{10\phi(\alpha - \bar{c})(50\phi^2 - 45\phi\theta^2 - 45\phi + 6\theta^4 + 28\theta^2 + 6)}{625\phi^3 - 625\phi^2\theta^2 - 625\phi^2 + 120\phi\theta^4 + 480\phi\theta^2 + 120\phi - 48\theta^4 - 48\theta^2}$$

Price:

$$\frac{125\alpha\phi^3 - 175\alpha\phi^2\theta^2 - 175\alpha\phi^2 + 60\alpha\phi\theta^4 + 200\alpha\phi\theta^2 + 60\alpha\phi - 48\alpha\theta^4 - 48\alpha\theta^2 + 500\bar{c}\phi^3 - 450\bar{c}\phi^2\theta^2 - 450\bar{c}\phi^2 + 60\bar{c}\phi\theta^4 + 280\bar{c}\phi\theta^2 + 60\bar{c}\phi}{625\phi^3 - 625\phi^2\theta^2 - 625\phi^2 + 120\phi\theta^4 + 480\phi\theta^2 + 120\phi - 48\theta^4 - 48\theta^2}$$

Firm Surplus:

$$\frac{\phi(\alpha - \bar{c})^2 \cdot (62500\phi^5 - 128125\phi^4\theta^2 - 128125\phi^4 + 92250\phi^3\theta^4 + 236500\phi^3\theta^2 + 92250\phi^3 - 28200\phi^2\theta^6 - 144600\phi^2\theta^4 - 144600\phi^2\theta^2 - 28200\phi^2 + 3600\phi\theta^8 + 32160\phi\theta^6 + 69920\phi\theta^4 + 32160\phi\theta^2 + 3600\phi - 2304\theta^8 - 9216\theta^6 - 9216\theta^4 - 2304\theta^2)}{(625\phi^3 - 625\phi^2\theta^2 - 625\phi^2 + 120\phi\theta^4 + 480\phi\theta^2 + 120\phi - 48\theta^4 - 48\theta^2)^2}$$

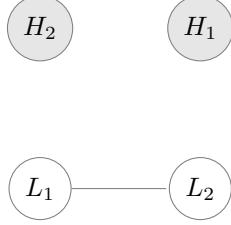
Consumer Surplus:

$$\frac{50\phi^2(\alpha - \bar{c})^2(50\phi^2 - 45\phi\theta^2 - 45\phi + 6\theta^4 + 28\theta^2 + 6)^2}{(625\phi^3 - 625\phi^2\theta^2 - 625\phi^2 + 120\phi\theta^4 + 480\phi\theta^2 + 120\phi - 48\theta^4 - 48\theta^2)^2}$$

Social Welfare:

$$\frac{\phi(\alpha - \bar{c})^2 \cdot (187500\phi^5 - 353125\phi^4\theta^2 - 353125\phi^4 + 223500\phi^3\theta^4 + 579000\phi^3\theta^2 + 223500\phi^3 - 55200\phi^2\theta^6 - 297600\phi^2\theta^4 - 297600\phi^2\theta^2 - 55200\phi^2 + 5400\phi\theta^8 + 48960\phi\theta^6 + 112720\phi\theta^4 + 48960\phi\theta^2 + 5400\phi - 2304\theta^8 - 9216\theta^6 - 9216\theta^4 - 2304\theta^2)}{(625\phi^3 - 625\phi^2\theta^2 - 625\phi^2 + 120\phi\theta^4 + 480\phi\theta^2 + 120\phi - 48\theta^4 - 48\theta^2)^2}$$

## D.2.4 E1C [2H2L]



$$\text{Equations : } \begin{cases} e_1^H \cdot \left(\frac{25\phi}{4} - 4\right) = \alpha - e_2^H - 2e_1^L\theta - 2e_2^L\theta - \bar{c} \\ e_2^H \cdot \left(\frac{25\phi}{4} - 4\right) = \alpha - e_1^H - 2e_1^L\theta - 2e_2^L\theta - \bar{c} \\ e_1^L \cdot \left(\frac{25\phi}{3\theta} - 3\theta\right) = \alpha - e_1^H - e_2^H + 3e_2^L\theta - \bar{c} \\ e_2^L \cdot \left(\frac{25\phi}{3\theta} - 3\theta\right) = \alpha - e_1^H - e_2^H + 3e_1^L\theta - \bar{c} \end{cases}$$

Optimal efforts:

$$\begin{cases} e_1^H = \frac{4(\alpha - \bar{c})(5\phi - 6\theta^2)}{125\phi^2 - 90\phi\theta^2 - 60\phi + 24\theta^2} \\ e_2^H = \frac{4(\alpha - \bar{c})(5\phi - 6\theta^2)}{125\phi^2 - 90\phi\theta^2 - 60\phi + 24\theta^2} \\ e_1^L = \frac{3\theta(\alpha - \bar{c})(5\phi - 4)}{125\phi^2 - 90\phi\theta^2 - 60\phi + 24\theta^2} \\ e_2^L = \frac{3\theta(\alpha - \bar{c})(5\phi - 4)}{125\phi^2 - 90\phi\theta^2 - 60\phi + 24\theta^2} \end{cases}$$

Production Costs:

$$\begin{cases} c_1^H = -\frac{20\alpha\phi - 24\alpha\theta^2 - 125\bar{c}\phi^2 + 90\bar{c}\phi\theta^2 + 40\bar{c}\phi}{125\phi^2 - 90\phi\theta^2 - 60\phi + 24\theta^2} \\ c_2^H = -\frac{20\alpha\phi - 24\alpha\theta^2 - 125\bar{c}\phi^2 + 90\bar{c}\phi\theta^2 + 40\bar{c}\phi}{125\phi^2 - 90\phi\theta^2 - 60\phi + 24\theta^2} \\ c_1^L = -\frac{30\alpha\phi\theta^2 - 24\alpha\theta^2 - 125\bar{c}\phi^2 + 60\bar{c}\phi\theta^2 + 60\bar{c}\phi}{125\phi^2 - 90\phi\theta^2 - 60\phi + 24\theta^2} \\ c_2^L = -\frac{30\alpha\phi\theta^2 - 24\alpha\theta^2 - 125\bar{c}\phi^2 + 60\bar{c}\phi\theta^2 + 60\bar{c}\phi}{125\phi^2 - 90\phi\theta^2 - 60\phi + 24\theta^2} \end{cases}$$

Production Quantities:

$$\begin{cases} q_1^H = \frac{5\phi(\alpha - \bar{c})(5\phi - 6\theta^2)}{125\phi^2 - 90\phi\theta^2 - 60\phi + 24\theta^2} \\ q_2^H = \frac{5\phi(\alpha - \bar{c})(5\phi - 6\theta^2)}{125\phi^2 - 90\phi\theta^2 - 60\phi + 24\theta^2} \\ q_1^L = \frac{5\phi(\alpha - \bar{c})(5\phi - 4)}{125\phi^2 - 90\phi\theta^2 - 60\phi + 24\theta^2} \\ q_2^L = \frac{5\phi(\alpha - \bar{c})(5\phi - 4)}{125\phi^2 - 90\phi\theta^2 - 60\phi + 24\theta^2} \end{cases}$$

Profits:

$$\begin{cases} \pi_1^H = \frac{\phi(\alpha - \bar{c})^2(5\phi - 6\theta^2)^2 \cdot (25\phi - 16)}{(125\phi^2 - 90\phi\theta^2 - 60\phi + 24\theta^2)^2} \\ \pi_2^H = \frac{\phi(\alpha - \bar{c})^2(5\phi - 6\theta^2)^2 \cdot (25\phi - 16)}{(125\phi^2 - 90\phi\theta^2 - 60\phi + 24\theta^2)^2} \\ \pi_1^L = \frac{\phi(\alpha - \bar{c})^2(5\phi - 4)^2 \cdot (25\phi - 9\theta^2)}{(125\phi^2 - 90\phi\theta^2 - 60\phi + 24\theta^2)^2} \\ \pi_2^L = \frac{\phi(\alpha - \bar{c})^2(5\phi - 4)^2 \cdot (25\phi - 9\theta^2)}{(125\phi^2 - 90\phi\theta^2 - 60\phi + 24\theta^2)^2} \end{cases} \quad (55)$$

Total Production:

$$\frac{20\phi(\alpha - \bar{c})(5\phi - 3\theta^2 - 2)}{125\phi^2 - 90\phi\theta^2 - 60\phi + 24\theta^2}$$

Price:

$$\frac{25\alpha\phi^2 - 30\alpha\phi\theta^2 - 20\alpha\phi + 24\alpha\theta^2 + 100\bar{c}\phi^2 - 60\bar{c}\phi\theta^2 - 40\bar{c}\phi}{125\phi^2 - 90\phi\theta^2 - 60\phi + 24\theta^2}$$

Firm Surplus:

$$\frac{2\phi(\alpha - \bar{c})^2 \cdot (1250\phi^3 - 1725\phi^2\theta^2 - 1400\phi^2 + 900\phi\theta^4 + 1320\phi\theta^2 + 400\phi - 576\theta^4 - 144\theta^2)}{(125\phi^2 - 90\phi\theta^2 - 60\phi + 24\theta^2)^2}$$

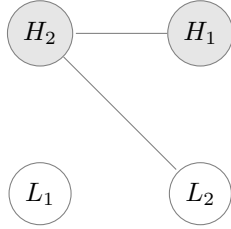
Consumer Surplus:

$$\frac{200\phi^2(\alpha - \bar{c})^2(5\phi - 3\theta^2 - 2)^2}{(125\phi^2 - 90\phi\theta^2 - 60\phi + 24\theta^2)^2}$$

Social Welfare:

$$\frac{2\phi(\alpha - \bar{c})^2 \cdot (3750\phi^3 - 4725\phi^2\theta^2 - 3400\phi^2 + 1800\phi\theta^4 + 2520\phi\theta^2 + 800\phi - 576\theta^4 - 144\theta^2)}{(125\phi^2 - 90\phi\theta^2 - 60\phi + 24\theta^2)^2}$$

## D.2.5 E2A [2H2L]



$$\text{Equations : } \begin{cases} e_1^H \cdot \left(\frac{25\phi}{3} - 3\right) = \alpha + 2e_2^H - e_1^L\theta - 2e_2^L\theta - \bar{c} \\ e_2^H \cdot \left(\frac{25\phi}{2} - 2\right) = \alpha + 3e_1^H - e_1^L\theta + 3e_2^L\theta - \bar{c} \\ e_1^L \cdot \left(\frac{25\phi}{4\theta} - 4\theta\right) = \alpha - 2e_1^H - 3e_2^H - 2e_2^L\theta - \bar{c} \\ e_2^L \cdot \left(\frac{25\phi}{3\theta} - 3\theta\right) = \alpha - 2e_1^H + 2e_2^H - e_1^L\theta - \bar{c} \end{cases}$$

Optimal efforts:

$$\begin{cases} e_1^H = \frac{15\phi(\alpha - \bar{c})(5\phi - 4\theta^2)(5\phi - 3\theta^2)}{3125\phi^4 - 3125\phi^3\theta^2 - 1625\phi^3 + 600\phi^2\theta^4 + 1025\phi^2\theta^2 + 180\phi\theta^2 - 72\theta^4} \\ e_2^H = \frac{2(\alpha - \bar{c})(5\phi - 3\theta)(5\phi + 3\theta)(5\phi - 4\theta^2)}{3125\phi^4 - 3125\phi^3\theta^2 - 1625\phi^3 + 600\phi^2\theta^4 + 1025\phi^2\theta^2 + 180\phi\theta^2 - 72\theta^4} \\ e_1^L = \frac{4\theta(\alpha - \bar{c})(125\phi^3 - 75\phi^2\theta^2 - 125\phi^2 + 45\phi\theta^2 + 18\theta^2)}{3125\phi^4 - 3125\phi^3\theta^2 - 1625\phi^3 + 600\phi^2\theta^4 + 1025\phi^2\theta^2 + 180\phi\theta^2 - 72\theta^4} \\ e_2^L = \frac{15\phi\theta(\alpha - \bar{c})(5\phi - 3)(5\phi - 4\theta^2)}{3125\phi^4 - 3125\phi^3\theta^2 - 1625\phi^3 + 600\phi^2\theta^4 + 1025\phi^2\theta^2 + 180\phi\theta^2 - 72\theta^4} \end{cases}$$

Production Costs:

$$\begin{cases} c_1^H = -\frac{625\alpha\phi^3 - 725\alpha\phi^2\theta^2 + 180\alpha\phi\theta^4 - 90\alpha\phi\theta^2 + 72\alpha\theta^4 - 3125\bar{c}\phi^4 + 3125\bar{c}\phi^3\theta^2 + 1000\bar{c}\phi^3 - 600\bar{c}\phi^2\theta^4 - 300\bar{c}\phi^2\theta^2 - 180\bar{c}\phi\theta^4 - 90\bar{c}\phi\theta^2}{3125\phi^4 - 3125\phi^3\theta^2 - 1625\phi^3 + 600\phi^2\theta^4 + 1025\phi^2\theta^2 + 180\phi\theta^2 - 72\theta^4} \\ c_2^H = -\frac{375\alpha\phi^3\theta^2 + 625\alpha\phi^3 - 300\alpha\phi^2\theta^4 - 950\alpha\phi^2\theta^2 + 360\alpha\phi\theta^4 - 90\alpha\phi\theta^2 + 72\alpha\theta^4 - 3125\bar{c}\phi^4 + 2750\bar{c}\phi^3\theta^2 + 1000\bar{c}\phi^3 - 300\bar{c}\phi^2\theta^4 - 75\bar{c}\phi^2\theta^2 - 360\bar{c}\phi\theta^4 - 90\bar{c}\phi\theta^2}{3125\phi^4 - 3125\phi^3\theta^2 - 1625\phi^3 + 600\phi^2\theta^4 + 1025\phi^2\theta^2 + 180\phi\theta^2 - 72\theta^4} \\ c_1^L = -\frac{500\alpha\phi^3\theta^2 - 300\alpha\phi^2\theta^4 - 500\alpha\phi^2\theta^2 + 180\alpha\phi\theta^4 + 72\alpha\theta^4 - 3125\bar{c}\phi^4 + 2625\bar{c}\phi^3\theta^2 + 1625\bar{c}\phi^3 - 300\bar{c}\phi^2\theta^4 - 525\bar{c}\phi^2\theta^2 - 180\bar{c}\phi\theta^4 - 180\bar{c}\phi\theta^2}{3125\phi^4 - 3125\phi^3\theta^2 - 1625\phi^3 + 600\phi^2\theta^4 + 1025\phi^2\theta^2 + 180\phi\theta^2 - 72\theta^4} \\ c_2^L = -\frac{375\alpha\phi^3\theta^2 + 250\alpha\phi^3 - 300\alpha\phi^2\theta^4 - 425\alpha\phi^2\theta^2 + 180\alpha\phi\theta^4 - 90\alpha\phi\theta^2 + 72\alpha\theta^4 - 3125\bar{c}\phi^4 + 2750\bar{c}\phi^3\theta^2 + 1375\bar{c}\phi^3 - 300\bar{c}\phi^2\theta^4 - 600\bar{c}\phi^2\theta^2 - 180\bar{c}\phi\theta^4 - 90\bar{c}\phi\theta^2}{3125\phi^4 - 3125\phi^3\theta^2 - 1625\phi^3 + 600\phi^2\theta^4 + 1025\phi^2\theta^2 + 180\phi\theta^2 - 72\theta^4} \end{cases}$$

Production Quantities:

$$\begin{cases} q_1^H = \frac{25\phi^2(\alpha - \bar{c})(5\phi - 4\theta^2)(5\phi - 3\theta^2)}{3125\phi^4 - 3125\phi^3\theta^2 - 1625\phi^3 + 600\phi^2\theta^4 + 1025\phi^2\theta^2 + 180\phi\theta^2 - 72\theta^4} \\ q_2^H = \frac{5\phi(\alpha - \bar{c})(5\phi - 3\theta)(5\phi + 3\theta)(5\phi - 4\theta^2)}{3125\phi^4 - 3125\phi^3\theta^2 - 1625\phi^3 + 600\phi^2\theta^4 + 1025\phi^2\theta^2 + 180\phi\theta^2 - 72\theta^4} \\ q_1^L = \frac{5\phi(\alpha - \bar{c})(125\phi^3 - 75\phi^2\theta^2 - 125\phi^2 + 45\phi\theta^2 + 18\theta^2)}{3125\phi^4 - 3125\phi^3\theta^2 - 1625\phi^3 + 600\phi^2\theta^4 + 1025\phi^2\theta^2 + 180\phi\theta^2 - 72\theta^4} \\ q_2^L = \frac{25\phi^2(\alpha - \bar{c})(5\phi - 3)(5\phi - 4\theta^2)}{3125\phi^4 - 3125\phi^3\theta^2 - 1625\phi^3 + 600\phi^2\theta^4 + 1025\phi^2\theta^2 + 180\phi\theta^2 - 72\theta^4} \end{cases}$$

Profits:

$$\begin{cases} \pi_1^H = \frac{25\phi^3(\alpha - \bar{c})^2(5\phi - 4\theta^2)^2(5\phi - 3\theta^2)^2 \cdot (25\phi - 9)}{(3125\phi^4 - 3125\phi^3\theta^2 - 1625\phi^3 + 600\phi^2\theta^4 + 1025\phi^2\theta^2 + 180\phi\theta^2 - 72\theta^4)^2} \\ \pi_2^H = \frac{\phi(\alpha - \bar{c})^2(5\phi - 3\theta)^2(5\phi + 3\theta)^2(5\phi - 4\theta^2)^2 \cdot (25\phi - 4)}{(3125\phi^4 - 3125\phi^3\theta^2 - 1625\phi^3 + 600\phi^2\theta^4 + 1025\phi^2\theta^2 + 180\phi\theta^2 - 72\theta^4)^2} \\ \pi_1^L = \frac{\phi(\alpha - \bar{c})^2 \cdot (25\phi - 16\theta^2)(125\phi^3 - 75\phi^2\theta^2 - 125\phi^2 + 45\phi\theta^2 + 18\theta^2)^2}{(3125\phi^4 - 3125\phi^3\theta^2 - 1625\phi^3 + 600\phi^2\theta^4 + 1025\phi^2\theta^2 + 180\phi\theta^2 - 72\theta^4)^2} \\ \pi_2^L = \frac{25\phi^3(\alpha - \bar{c})^2(5\phi - 3)^2(5\phi - 4\theta^2)^2 \cdot (25\phi - 9\theta^2)}{(3125\phi^4 - 3125\phi^3\theta^2 - 1625\phi^3 + 600\phi^2\theta^4 + 1025\phi^2\theta^2 + 180\phi\theta^2 - 72\theta^4)^2} \end{cases} \quad (56)$$

Total Production:

$$\frac{10\phi(\alpha - \bar{c})(250\phi^3 - 225\phi^2\theta^2 - 100\phi^2 + 30\phi\theta^4 + 30\phi\theta^2 + 18\theta^4 + 9\theta^2)}{3125\phi^4 - 3125\phi^3\theta^2 - 1625\phi^3 + 600\phi^2\theta^4 + 1025\phi^2\theta^2 + 180\phi\theta^2 - 72\theta^4}$$

Price:

$$\frac{625\alpha\phi^4 - 875\alpha\phi^3\theta^2 - 625\alpha\phi^3 + 300\alpha\phi^2\theta^4 + 725\alpha\phi^2\theta^2 - 180\alpha\phi\theta^4 + 90\alpha\phi\theta^2 - 72\alpha\theta^4 + 2500\bar{c}\phi^4 - 2250\bar{c}\phi^3\theta^2 - 1000\bar{c}\phi^3 + 300\bar{c}\phi^2\theta^4 + 300\bar{c}\phi^2\theta^2 + 180\bar{c}\phi\theta^4 + 90\bar{c}\phi\theta^2}{3125\phi^4 - 3125\phi^3\theta^2 - 1625\phi^3 + 600\phi^2\theta^4 + 1025\phi^2\theta^2 + 180\phi\theta^2 - 72\theta^4}$$

Firm Surplus:

$$\frac{\phi(\alpha - \bar{c})^2 \cdot (1562500\phi^7 - 3203125\phi^6\theta^2 - 1453125\phi^6 + 2306250\phi^5\theta^4 + 2381250\phi^5\theta^2 + 531250\phi^5 - 705000\phi^4\theta^6 - 1219375\phi^4\theta^4 - 649375\phi^4\theta^2 + 90000\phi^3\theta^6 + 225000\phi^3\theta^4 + 240750\phi^3\theta^2 - 112500\phi^3\theta^2 - 32400\phi^2\theta^6 - 73800\phi^2\theta^4 + 104400\phi^2\theta^4 + 32400\phi\theta^6 - 12960\phi\theta^4 + 8100\phi\theta^4 - 5184\theta^6 - 5184\theta^6)}{(3125\phi^4 - 3125\phi^3\theta^2 - 1625\phi^3 + 600\phi^2\theta^4 + 1025\phi^2\theta^2 + 180\phi\theta^2 - 72\theta^4)^2}$$

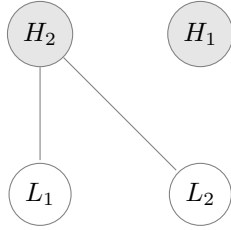
Consumer Surplus:

$$\frac{50\phi^2(\alpha - \bar{c})^2(250\phi^3 - 225\phi^2\theta^2 - 100\phi^2 + 30\phi\theta^4 + 30\phi\theta^2 + 18\theta^4 + 9\theta^2)^2}{(3125\phi^4 - 3125\phi^3\theta^2 - 1625\phi^3 + 600\phi^2\theta^4 + 1025\phi^2\theta^2 + 180\phi\theta^2 - 72\theta^4)^2}$$

Social Welfare:

$$\frac{\phi(\alpha - \bar{c})^2 \cdot (4687500\phi^7 - 8828125\phi^6\theta^2 - 3953125\phi^6 + 5587500\phi^5\theta^4 + 5381250\phi^5\theta^2 + 1031250\phi^5 - 1380000\phi^4\theta^6 - 1744375\phi^4\theta^4 - 724375\phi^4\theta^2 + 135000\phi^3\theta^6 - 90000\phi^3\theta^4 - 96750\phi^3\theta^2 - 202500\phi^3\theta^2 + 21600\phi^2\theta^6 + 7200\phi^2\theta^4 + 131400\phi^2\theta^4 + 48600\phi\theta^6 + 3240\phi\theta^6 + 12150\phi\theta^4 - 5184\theta^6 - 5184\theta^6)}{(3125\phi^4 - 3125\phi^3\theta^2 - 1625\phi^3 + 600\phi^2\theta^4 + 1025\phi^2\theta^2 + 180\phi\theta^2 - 72\theta^4)^2}$$

## D.2.6 E2B [2H2L]



$$\text{Equations : } \begin{cases} e_1^H \cdot \left(\frac{25\phi}{4} - 4\right) = \alpha - 3e_2^H - 2e_1^L\theta - 2e_2^L\theta - \bar{c} \\ e_2^H \cdot \left(\frac{25\phi}{2} - 2\right) = \alpha - e_1^H + 3e_1^L\theta + 3e_2^L\theta - \bar{c} \\ e_1^L \cdot \left(\frac{25\phi}{3\theta} - 3\theta\right) = \alpha - e_1^H + 2e_2^H - 2e_2^L\theta - \bar{c} \\ e_2^L \cdot \left(\frac{25\phi}{3\theta} - 3\theta\right) = \alpha - e_1^H + 2e_2^H - 2e_1^L\theta - \bar{c} \end{cases}$$

Optimal efforts:

$$\begin{cases} e_1^H = \frac{4(\alpha - \bar{c})(25\phi^2 - 15\phi\theta^2 - 10\phi - 6\theta^2)}{625\phi^3 - 75\phi^2\theta^2 - 500\phi^2 - 60\phi\theta^2 + 40\phi + 24\theta^2} \\ e_2^H = \frac{2(\alpha - \bar{c})(5\phi - 4)(5\phi + 3\theta^2)}{625\phi^3 - 75\phi^2\theta^2 - 500\phi^2 - 60\phi\theta^2 + 40\phi + 24\theta^2} \\ e_1^L = \frac{15\phi\theta(\alpha - \bar{c})(5\phi - 4)}{625\phi^3 - 75\phi^2\theta^2 - 500\phi^2 - 60\phi\theta^2 + 40\phi + 24\theta^2} \\ e_2^L = \frac{15\phi\theta(\alpha - \bar{c})(5\phi - 4)}{625\phi^3 - 75\phi^2\theta^2 - 500\phi^2 - 60\phi\theta^2 + 40\phi + 24\theta^2} \end{cases}$$

Production Costs:

$$\begin{cases} c_1^H = -\frac{100\alpha\phi^2 - 60\alpha\phi\theta^2 - 40\alpha\phi - 24\alpha\theta^2 - 625\bar{c}\phi^3 + 75\bar{c}\phi^2\theta^2 + 400\bar{c}\phi^2 + 120\bar{c}\phi\theta^2}{625\phi^3 - 75\phi^2\theta^2 - 500\phi^2 - 60\phi\theta^2 + 40\phi + 24\theta^2} \\ c_2^H = -\frac{150\alpha\phi^2\theta^2 + 50\alpha\phi^2 - 90\alpha\phi\theta^2 - 40\alpha\phi - 24\alpha\theta^2 - 625\bar{c}\phi^3 - 75\bar{c}\phi^2\theta^2 + 450\bar{c}\phi^2 + 150\bar{c}\phi\theta^2}{625\phi^3 - 75\phi^2\theta^2 - 500\phi^2 - 60\phi\theta^2 + 40\phi + 24\theta^2} \\ c_1^L = -\frac{75\alpha\phi^2\theta^2 + 50\alpha\phi^2 - 30\alpha\phi\theta^2 - 40\alpha\phi - 24\alpha\theta^2 - 625\bar{c}\phi^3 + 450\bar{c}\phi^2 + 90\bar{c}\phi\theta^2}{625\phi^3 - 75\phi^2\theta^2 - 500\phi^2 - 60\phi\theta^2 + 40\phi + 24\theta^2} \\ c_2^L = -\frac{75\alpha\phi^2\theta^2 + 50\alpha\phi^2 - 30\alpha\phi\theta^2 - 40\alpha\phi - 24\alpha\theta^2 - 625\bar{c}\phi^3 + 450\bar{c}\phi^2 + 90\bar{c}\phi\theta^2}{625\phi^3 - 75\phi^2\theta^2 - 500\phi^2 - 60\phi\theta^2 + 40\phi + 24\theta^2} \end{cases}$$

Production Quantities:

$$\begin{cases} q_1^H = \frac{5\phi(\alpha - \bar{c})(25\phi^2 - 15\phi\theta^2 - 10\phi - 6\theta^2)}{625\phi^3 - 75\phi^2\theta^2 - 500\phi^2 - 60\phi\theta^2 + 40\phi + 24\theta^2} \\ q_2^H = \frac{5\phi(\alpha - \bar{c})(5\phi - 4)(5\phi + 3\theta^2)}{625\phi^3 - 75\phi^2\theta^2 - 500\phi^2 - 60\phi\theta^2 + 40\phi + 24\theta^2} \\ q_1^L = \frac{25\phi^2(\alpha - \bar{c})(5\phi - 4)}{625\phi^3 - 75\phi^2\theta^2 - 500\phi^2 - 60\phi\theta^2 + 40\phi + 24\theta^2} \\ q_2^L = \frac{25\phi^2(\alpha - \bar{c})(5\phi - 4)}{625\phi^3 - 75\phi^2\theta^2 - 500\phi^2 - 60\phi\theta^2 + 40\phi + 24\theta^2} \end{cases}$$

Profits:

$$\begin{cases} \pi_1^H = \frac{\phi(\alpha - \bar{c})^2 \cdot (25\phi - 16)(25\phi^2 - 15\phi\theta^2 - 10\phi - 6\theta^2)^2}{(625\phi^3 - 75\phi^2\theta^2 - 500\phi^2 - 60\phi\theta^2 + 40\phi + 24\theta^2)^2} \\ \pi_2^H = \frac{\phi(\alpha - \bar{c})^2(5\phi - 4)^2(5\phi + 3\theta^2)^2 \cdot (25\phi - 4)}{(625\phi^3 - 75\phi^2\theta^2 - 500\phi^2 - 60\phi\theta^2 + 40\phi + 24\theta^2)^2} \\ \pi_1^L = \frac{25\phi^3(\alpha - \bar{c})^2(5\phi - 4)^2 \cdot (25\phi - 9\theta^2)}{(625\phi^3 - 75\phi^2\theta^2 - 500\phi^2 - 60\phi\theta^2 + 40\phi + 24\theta^2)^2} \\ \pi_2^L = \frac{25\phi^3(\alpha - \bar{c})^2(5\phi - 4)^2 \cdot (25\phi - 9\theta^2)}{(625\phi^3 - 75\phi^2\theta^2 - 500\phi^2 - 60\phi\theta^2 + 40\phi + 24\theta^2)^2} \end{cases} \quad (57)$$

Total Production:

$$\frac{10\phi(\alpha - \bar{c})(50\phi^2 - 35\phi - 9\theta^2)}{625\phi^3 - 75\phi^2\theta^2 - 500\phi^2 - 60\phi\theta^2 + 40\phi + 24\theta^2}$$

Price:

$$\frac{125\alpha\phi^3 - 75\alpha\phi^2\theta^2 - 150\alpha\phi^2 + 30\alpha\phi\theta^2 + 40\alpha\phi + 24\alpha\theta^2 + 500\bar{c}\phi^3 - 350\bar{c}\phi^2 - 90\bar{c}\phi\theta^2}{625\phi^3 - 75\phi^2\theta^2 - 500\phi^2 - 60\phi\theta^2 + 40\phi + 24\theta^2}$$

Firm Surplus:

$$\frac{2\phi(\alpha - \bar{c})^2 \cdot (31250\phi^5 - 5625\phi^4\theta^2 - 50000\phi^4 + 5625\phi^3\theta^4 - 1500\phi^3\theta^2 + 22250\phi^3 - 4500\phi^2\theta^4 + 6300\phi^2\theta^2 - 1600\phi^2 + 1530\phi\theta^4 - 1920\phi\theta^2 - 576\theta^4)}{(625\phi^3 - 75\phi^2\theta^2 - 500\phi^2 - 60\phi\theta^2 + 40\phi + 24\theta^2)^2}$$

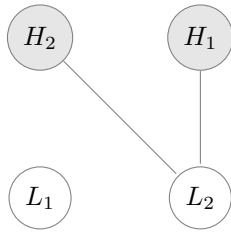
Consumer Surplus:

$$\frac{50\phi^2(\alpha - \bar{c})^2(50\phi^2 - 35\phi - 9\theta^2)^2}{(625\phi^3 - 75\phi^2\theta^2 - 500\phi^2 - 60\phi\theta^2 + 40\phi + 24\theta^2)^2}$$

Social Welfare:

$$\frac{2\phi(\alpha - \bar{c})^2 \cdot (93750\phi^5 - 5625\phi^4\theta^2 - 137500\phi^4 + 5625\phi^3\theta^4 - 24000\phi^3\theta^2 + 52875\phi^3 - 4500\phi^2\theta^4 + 22050\phi^2\theta^2 - 1600\phi^2 + 3555\phi\theta^4 - 1920\phi\theta^2 - 576\theta^4)}{(625\phi^3 - 75\phi^2\theta^2 - 500\phi^2 - 60\phi\theta^2 + 40\phi + 24\theta^2)^2}$$

## D.2.7 E2C [2H2L]



$$\text{Equations : } \begin{cases} e_1^H \cdot \left(\frac{25\phi}{3} - 3\right) = \alpha - 2e_2^H - e_1^L\theta + 2e_2^L\theta - \bar{c} \\ e_2^H \cdot \left(\frac{25\phi}{3} - 3\right) = \alpha - 2e_1^H - e_1^L\theta + 2e_2^L\theta - \bar{c} \\ e_1^L \cdot \left(\frac{25\phi}{4\theta} - 4\theta\right) = \alpha - 2e_1^H - 2e_2^H - 3e_2^L\theta - \bar{c} \\ e_2^L \cdot \left(\frac{25\phi}{2\theta} - 2\theta\right) = \alpha + 3e_1^H + 3e_2^H - e_1^L\theta - \bar{c} \end{cases}$$

Optimal efforts:

$$\begin{cases} e_1^H = \frac{15\phi(\alpha - \bar{c})(5\phi - 4\theta^2)}{625\phi^3 - 500\phi^2\theta^2 - 75\phi^2 + 40\phi\theta^4 - 60\phi\theta^2 + 24\theta^4} \\ e_2^H = \frac{15\phi(\alpha - \bar{c})(5\phi - 4\theta^2)}{625\phi^3 - 500\phi^2\theta^2 - 75\phi^2 + 40\phi\theta^4 - 60\phi\theta^2 + 24\theta^4} \\ e_1^L = \frac{4\theta(\alpha - \bar{c})(25\phi^2 - 10\phi\theta^2 - 15\phi - 6\theta^2)}{625\phi^3 - 500\phi^2\theta^2 - 75\phi^2 + 40\phi\theta^4 - 60\phi\theta^2 + 24\theta^4} \\ e_2^L = \frac{2\theta(\alpha - \bar{c})(5\phi + 3)(5\phi - 4\theta^2)}{625\phi^3 - 500\phi^2\theta^2 - 75\phi^2 + 40\phi\theta^4 - 60\phi\theta^2 + 24\theta^4} \end{cases}$$

Production Costs:

$$\begin{cases} c_1^H = -\frac{50\alpha\phi^2\theta^2 + 75\alpha\phi^2 - 40\alpha\phi\theta^4 - 30\alpha\phi\theta^2 - 24\alpha\theta^4 - 625\bar{c}\phi^3 + 450\bar{c}\phi^2\theta^2 + 90\bar{c}\phi\theta^2}{625\phi^3 - 500\phi^2\theta^2 - 75\phi^2 + 40\phi\theta^4 - 60\phi\theta^2 + 24\theta^4} \\ c_2^H = -\frac{50\alpha\phi^2\theta^2 + 75\alpha\phi^2 - 40\alpha\phi\theta^4 - 30\alpha\phi\theta^2 - 24\alpha\theta^4 - 625\bar{c}\phi^3 + 450\bar{c}\phi^2\theta^2 + 90\bar{c}\phi\theta^2}{625\phi^3 - 500\phi^2\theta^2 - 75\phi^2 + 40\phi\theta^4 - 60\phi\theta^2 + 24\theta^4} \\ c_1^L = -\frac{100\alpha\phi^2\theta^2 - 40\alpha\phi\theta^4 - 60\alpha\phi\theta^2 - 24\alpha\theta^4 - 625\bar{c}\phi^3 + 400\bar{c}\phi^2\theta^2 + 75\bar{c}\phi^2 + 120\bar{c}\phi\theta^2}{625\phi^3 - 500\phi^2\theta^2 - 75\phi^2 + 40\phi\theta^4 - 60\phi\theta^2 + 24\theta^4} \\ c_2^L = -\frac{50\alpha\phi^2\theta^2 + 150\alpha\phi^2 - 40\alpha\phi\theta^4 - 90\alpha\phi\theta^2 - 24\alpha\theta^4 - 625\bar{c}\phi^3 + 450\bar{c}\phi^2\theta^2 - 75\bar{c}\phi^2 + 150\bar{c}\phi\theta^2}{625\phi^3 - 500\phi^2\theta^2 - 75\phi^2 + 40\phi\theta^4 - 60\phi\theta^2 + 24\theta^4} \end{cases}$$

Production Quantities:

$$\begin{cases} q_1^H = \frac{25\phi^2(\alpha - \bar{c})(5\phi - 4\theta^2)}{625\phi^3 - 500\phi^2\theta^2 - 75\phi^2 + 40\phi\theta^4 - 60\phi\theta^2 + 24\theta^4} \\ q_2^H = \frac{25\phi^2(\alpha - \bar{c})(5\phi - 4\theta^2)}{625\phi^3 - 500\phi^2\theta^2 - 75\phi^2 + 40\phi\theta^4 - 60\phi\theta^2 + 24\theta^4} \\ q_1^L = \frac{5\phi(\alpha - \bar{c})(25\phi^2 - 10\phi\theta^2 - 15\phi - 6\theta^2)}{625\phi^3 - 500\phi^2\theta^2 - 75\phi^2 + 40\phi\theta^4 - 60\phi\theta^2 + 24\theta^4} \\ q_2^L = \frac{5\phi(\alpha - \bar{c})(5\phi + 3)(5\phi - 4\theta^2)}{625\phi^3 - 500\phi^2\theta^2 - 75\phi^2 + 40\phi\theta^4 - 60\phi\theta^2 + 24\theta^4} \end{cases}$$

Profits:

$$\begin{cases} \pi_1^H = \frac{25\phi^3(\alpha - \bar{c})^2(5\phi - 4\theta^2)^2 \cdot (25\phi - 9)}{(625\phi^3 - 500\phi^2\theta^2 - 75\phi^2 + 40\phi\theta^4 - 60\phi\theta^2 + 24\theta^4)^2} \\ \pi_2^H = \frac{25\phi^3(\alpha - \bar{c})^2(5\phi - 4\theta^2)^2 \cdot (25\phi - 9)}{(625\phi^3 - 500\phi^2\theta^2 - 75\phi^2 + 40\phi\theta^4 - 60\phi\theta^2 + 24\theta^4)^2} \\ \pi_1^L = \frac{\phi(\alpha - \bar{c})^2 \cdot (25\phi - 16\theta^2)(25\phi^2 - 10\phi\theta^2 - 15\phi - 6\theta^2)^2}{(625\phi^3 - 500\phi^2\theta^2 - 75\phi^2 + 40\phi\theta^4 - 60\phi\theta^2 + 24\theta^4)^2} \\ \pi_2^L = \frac{\phi(\alpha - \bar{c})^2(5\phi + 3)^2(5\phi - 4\theta^2)^2 \cdot (25\phi - 4\theta^2)}{(625\phi^3 - 500\phi^2\theta^2 - 75\phi^2 + 40\phi\theta^4 - 60\phi\theta^2 + 24\theta^4)^2} \end{cases} \quad (58)$$

Total Production:



$$\frac{10\phi(\alpha - \bar{c})(50\phi^2 - 35\phi\theta^2 - 9\theta^2)}{625\phi^3 - 500\phi^2\theta^2 - 75\phi^2 + 40\phi\theta^4 - 60\phi\theta^2 + 24\theta^4}$$

Price:

$$\frac{125\alpha\phi^3 - 150\alpha\phi^2\theta^2 - 75\alpha\phi^2 + 40\alpha\phi\theta^4 + 30\alpha\phi\theta^2 + 24\alpha\theta^4 + 500\bar{c}\phi^3 - 350\bar{c}\phi^2\theta^2 - 90\bar{c}\phi\theta^2}{625\phi^3 - 500\phi^2\theta^2 - 75\phi^2 + 40\phi\theta^4 - 60\phi\theta^2 + 24\theta^4}$$

Firm Surplus:

$$\frac{2\phi(\alpha - \bar{c})^2 \cdot (31250\phi^5 - 50000\phi^4\theta^2 - 5625\phi^4 + 22250\phi^3\theta^4 - 1500\phi^3\theta^2 + 5625\phi^3 - 1600\phi^2\theta^6 + 6300\phi^2\theta^4 - 4500\phi^2\theta^2 - 1920\phi\theta^6 + 1530\phi\theta^4 - 576\theta^6)}{(625\phi^3 - 500\phi^2\theta^2 - 75\phi^2 + 40\phi\theta^4 - 60\phi\theta^2 + 24\theta^4)^2}$$

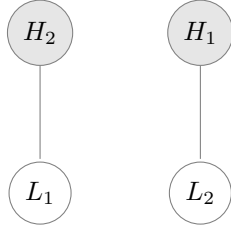
Consumer Surplus:

$$\frac{50\phi^2(\alpha - \bar{c})^2(50\phi^2 - 35\phi\theta^2 - 9\theta^2)^2}{(625\phi^3 - 500\phi^2\theta^2 - 75\phi^2 + 40\phi\theta^4 - 60\phi\theta^2 + 24\theta^4)^2}$$

Social Welfare:

$$\frac{2\phi(\alpha - \bar{c})^2 \cdot (93750\phi^5 - 137500\phi^4\theta^2 - 5625\phi^4 + 52875\phi^3\theta^4 - 24000\phi^3\theta^2 + 5625\phi^3 - 1600\phi^2\theta^6 + 22050\phi^2\theta^4 - 4500\phi^2\theta^2 - 1920\phi\theta^6 + 3555\phi\theta^4 - 576\theta^6)}{(625\phi^3 - 500\phi^2\theta^2 - 75\phi^2 + 40\phi\theta^4 - 60\phi\theta^2 + 24\theta^4)^2}$$

### D.2.8 E2D [2H2L]



$$\text{Equations : } \begin{cases} e_1^H \cdot \left( \frac{25\phi}{3} - 3 \right) = \alpha - 2e_2^H - 2e_1^L\theta + 3e_2^L\theta - \bar{c} \\ e_2^H \cdot \left( \frac{25\phi}{3} - 3 \right) = \alpha - 2e_1^H + 3e_1^L\theta - 2e_2^L\theta - \bar{c} \\ e_1^L \cdot \left( \frac{25\phi}{3\theta} - 3\theta \right) = \alpha - 2e_1^H + 3e_2^H - 2e_2^L\theta - \bar{c} \\ e_2^L \cdot \left( \frac{25\phi}{3\theta} - 3\theta \right) = \alpha + 3e_1^H - 2e_2^H - 2e_1^L\theta - \bar{c} \end{cases}$$

Optimal efforts:

$$\begin{cases} e_1^H = \frac{3(\alpha - \bar{c})}{25\phi - 3\theta^2 - 3} \\ e_2^H = \frac{3(\alpha - \bar{c})}{25\phi - 3\theta^2 - 3} \\ e_1^L = \frac{3\theta(\alpha - \bar{c})}{25\phi - 3\theta^2 - 3} \\ e_2^L = \frac{3\theta(\alpha - \bar{c})}{25\phi - 3\theta^2 - 3} \end{cases}$$

Production Costs:

$$\begin{cases} c_1^H = -\frac{3\alpha\theta^2 + 3\alpha - 25\bar{c}\phi}{25\phi - 3\theta^2 - 3} \\ c_2^H = -\frac{3\alpha\theta^2 + 3\alpha - 25\bar{c}\phi}{25\phi - 3\theta^2 - 3} \\ c_1^L = -\frac{3\alpha\theta^2 + 3\alpha - 25\bar{c}\phi}{25\phi - 3\theta^2 - 3} \\ c_2^L = -\frac{3\alpha\theta^2 + 3\alpha - 25\bar{c}\phi}{25\phi - 3\theta^2 - 3} \end{cases}$$

Production Quantities:

$$\begin{cases} q_1^H = \frac{5\phi(\alpha - \bar{c})}{25\phi - 3\theta^2 - 3} \\ q_2^H = \frac{5\phi(\alpha - \bar{c})}{25\phi - 3\theta^2 - 3} \\ q_1^L = \frac{5\phi(\alpha - \bar{c})}{25\phi - 3\theta^2 - 3} \\ q_2^L = \frac{5\phi(\alpha - \bar{c})}{25\phi - 3\theta^2 - 3} \end{cases}$$

Profits:

$$\begin{cases} \pi_1^H = \frac{\phi(\alpha - \bar{c})^2 \cdot (25\phi - 9)}{(25\phi - 3\theta^2 - 3)^2} \\ \pi_2^H = \frac{\phi(\alpha - \bar{c})^2 \cdot (25\phi - 9)}{(25\phi - 3\theta^2 - 3)^2} \\ \pi_1^L = \frac{\phi(\alpha - \bar{c})^2 \cdot (25\phi - 9\theta^2)}{(25\phi - 3\theta^2 - 3)^2} \\ \pi_2^L = \frac{\phi(\alpha - \bar{c})^2 \cdot (25\phi - 9\theta^2)}{(25\phi - 3\theta^2 - 3)^2} \end{cases} \quad (59)$$

Total Production:

$$\frac{20\phi(\alpha - \bar{c})}{25\phi - 3\theta^2 - 3}$$

Price:

$$\frac{5\alpha\phi - 3\alpha\theta^2 - 3\alpha + 20\bar{c}\phi}{25\phi - 3\theta^2 - 3}$$

Firm Surplus:

$$\frac{2\phi(\alpha - \bar{c})^2 \cdot (50\phi - 9\theta^2 - 9)}{(25\phi - 3\theta^2 - 3)^2}$$

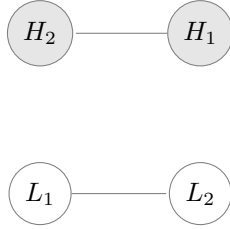
Consumer Surplus:

$$\frac{200\phi^2(\alpha - \bar{c})^2}{(25\phi - 3\theta^2 - 3)^2}$$

Social Welfare:

$$\frac{6\phi(\alpha - \bar{c})^2 \cdot (50\phi - 3\theta^2 - 3)}{(25\phi - 3\theta^2 - 3)^2}$$

## D.2.9 E2E [2H2L]



$$\text{Equations : } \begin{cases} e_1^H \cdot \left( \frac{25\phi}{3} - 3 \right) = \alpha + 3e_2^H - 2e_1^L\theta - 2e_2^L\theta - \bar{c} \\ e_2^H \cdot \left( \frac{25\phi}{3} - 3 \right) = \alpha + 3e_1^H - 2e_1^L\theta - 2e_2^L\theta - \bar{c} \\ e_1^L \cdot \left( \frac{25\phi}{3\theta} - 3\theta \right) = \alpha - 2e_1^H - 2e_2^H + 3e_2^L\theta - \bar{c} \\ e_2^L \cdot \left( \frac{25\phi}{3\theta} - 3\theta \right) = \alpha - 2e_1^H - 2e_2^H + 3e_1^L\theta - \bar{c} \end{cases}$$

Optimal efforts:

$$\begin{cases} e_1^H = \frac{3(\alpha - \bar{c})(5\phi - 6\theta^2)}{125\phi^2 - 90\phi\theta^2 - 90\phi + 36\theta^2} \\ e_2^H = \frac{3(\alpha - \bar{c})(5\phi - 6\theta^2)}{125\phi^2 - 90\phi\theta^2 - 90\phi + 36\theta^2} \\ e_1^L = \frac{3\theta(\alpha - \bar{c})(5\phi - 6)}{125\phi^2 - 90\phi\theta^2 - 90\phi + 36\theta^2} \\ e_2^L = \frac{3\theta(\alpha - \bar{c})(5\phi - 6)}{125\phi^2 - 90\phi\theta^2 - 90\phi + 36\theta^2} \end{cases}$$

Production Costs:

$$\begin{cases} c_1^H = -\frac{30\alpha\phi - 36\alpha\theta^2 - 125\bar{c}\phi^2 + 90\bar{c}\phi\theta^2 + 60\bar{c}\phi}{125\phi^2 - 90\phi\theta^2 - 90\phi + 36\theta^2} \\ c_2^H = -\frac{30\alpha\phi - 36\alpha\theta^2 - 125\bar{c}\phi^2 + 90\bar{c}\phi\theta^2 + 60\bar{c}\phi}{125\phi^2 - 90\phi\theta^2 - 90\phi + 36\theta^2} \\ c_1^L = -\frac{30\alpha\phi\theta^2 - 36\alpha\theta^2 - 125\bar{c}\phi^2 + 60\bar{c}\phi\theta^2 + 90\bar{c}\phi}{125\phi^2 - 90\phi\theta^2 - 90\phi + 36\theta^2} \\ c_2^L = -\frac{30\alpha\phi\theta^2 - 36\alpha\theta^2 - 125\bar{c}\phi^2 + 60\bar{c}\phi\theta^2 + 90\bar{c}\phi}{125\phi^2 - 90\phi\theta^2 - 90\phi + 36\theta^2} \end{cases}$$

Production Quantities:

$$\begin{cases} q_1^H = \frac{5\phi(\alpha - \bar{c})(5\phi - 6\theta^2)}{125\phi^2 - 90\phi\theta^2 - 90\phi + 36\theta^2} \\ q_2^H = \frac{5\phi(\alpha - \bar{c})(5\phi - 6\theta^2)}{125\phi^2 - 90\phi\theta^2 - 90\phi + 36\theta^2} \\ q_1^L = \frac{5\phi(\alpha - \bar{c})(5\phi - 6)}{125\phi^2 - 90\phi\theta^2 - 90\phi + 36\theta^2} \\ q_2^L = \frac{5\phi(\alpha - \bar{c})(5\phi - 6)}{125\phi^2 - 90\phi\theta^2 - 90\phi + 36\theta^2} \end{cases}$$

Profits:

$$\begin{cases} \pi_1^H &= \frac{\phi(\alpha-\bar{c})^2(5\phi-6\theta^2)^2 \cdot (25\phi-9)}{(125\phi^2-90\phi\theta^2-90\phi+36\theta^2)^2} \\ \pi_2^H &= \frac{\phi(\alpha-\bar{c})^2(5\phi-6\theta^2)^2 \cdot (25\phi-9)}{(125\phi^2-90\phi\theta^2-90\phi+36\theta^2)^2} \\ \pi_1^L &= \frac{\phi(\alpha-\bar{c})^2(5\phi-6)^2 \cdot (25\phi-9\theta^2)}{(125\phi^2-90\phi\theta^2-90\phi+36\theta^2)^2} \\ \pi_2^L &= \frac{\phi(\alpha-\bar{c})^2(5\phi-6)^2 \cdot (25\phi-9\theta^2)}{(125\phi^2-90\phi\theta^2-90\phi+36\theta^2)^2} \end{cases} \quad (60)$$

Total Production:

$$\frac{20\phi(\alpha-\bar{c})(5\phi-3\theta^2-3)}{125\phi^2-90\phi\theta^2-90\phi+36\theta^2}$$

Price:

$$\frac{25\alpha\phi^2-30\alpha\phi\theta^2-30\alpha\phi+36\alpha\theta^2+100\bar{c}\phi^2-60\bar{c}\phi\theta^2-60\bar{c}\phi}{125\phi^2-90\phi\theta^2-90\phi+36\theta^2}$$

Firm Surplus:

$$\frac{2\phi(\alpha-\bar{c})^2 \cdot (1250\phi^3-1725\phi^2\theta^2-1725\phi^2+900\phi\theta^4+1080\phi\theta^2+900\phi-324\theta^4-324\theta^2)}{(125\phi^2-90\phi\theta^2-90\phi+36\theta^2)^2}$$

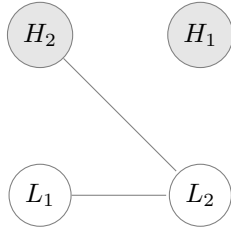
Consumer Surplus:

$$\frac{200\phi^2(\alpha-\bar{c})^2(5\phi-3\theta^2-3)^2}{(125\phi^2-90\phi\theta^2-90\phi+36\theta^2)^2}$$

Social Welfare:

$$\frac{6\phi(\alpha-\bar{c})^2 \cdot (1250\phi^3-1575\phi^2\theta^2-1575\phi^2+600\phi\theta^4+960\phi\theta^2+600\phi-108\theta^4-108\theta^2)}{(125\phi^2-90\phi\theta^2-90\phi+36\theta^2)^2}$$

#### D.2.10 E2F [2H2L]



$$\text{Equations : } \begin{cases} e_1^H \cdot \left(\frac{25\phi}{4} - 4\right) = \alpha - 2e_2^H - 2e_1^L\theta - 3e_2^L\theta - \bar{c} \\ e_2^H \cdot \left(\frac{25\phi}{3} - 3\right) = \alpha - e_1^H - 2e_1^L\theta + 2e_2^L\theta - \bar{c} \\ e_1^L \cdot \left(\frac{25\phi}{3\theta} - 3\theta\right) = \alpha - e_1^H - 2e_2^H + 2e_2^L\theta - \bar{c} \\ e_2^L \cdot \left(\frac{25\phi}{2\theta} - 2\theta\right) = \alpha - e_1^H + 3e_2^H + 3e_1^L\theta - \bar{c} \end{cases}$$

Optimal efforts:

$$\begin{cases} e_1^H &= \frac{4(\alpha-\bar{c})(125\phi^3-125\phi^2\theta^2-75\phi^2+45\phi\theta^2+18\theta^4)}{3125\phi^4-1625\phi^3\theta^2-3125\phi^3+1025\phi^2\theta^2+600\phi^2+180\phi\theta^4-72\theta^4} \\ e_2^H &= \frac{15\phi(\alpha-\bar{c})(5\phi-4)(5\phi-3\theta^2)}{3125\phi^4-1625\phi^3\theta^2-3125\phi^3+1025\phi^2\theta^2+600\phi^2+180\phi\theta^4-72\theta^4} \\ e_1^L &= \frac{15\phi\theta(\alpha-\bar{c})(5\phi-4)(5\phi-3)}{3125\phi^4-1625\phi^3\theta^2-3125\phi^3+1025\phi^2\theta^2+600\phi^2+180\phi\theta^4-72\theta^4} \\ e_2^L &= \frac{2\theta(\alpha-\bar{c})(5\phi-4)(5\phi-3\theta)(5\phi+3\theta)}{3125\phi^4-1625\phi^3\theta^2-3125\phi^3+1025\phi^2\theta^2+600\phi^2+180\phi\theta^4-72\theta^4} \end{cases}$$

Production Costs:

$$\begin{cases} c_1^H &= -\frac{500\alpha\phi^3-500\alpha\phi^2\theta^2-300\alpha\phi^2+180\alpha\phi\theta^2+72\alpha\theta^4-3125\bar{c}\phi^4+1625\bar{c}\phi^3\theta^2+2625\bar{c}\phi^3-525\bar{c}\phi^2\theta^2-300\bar{c}\phi^2-180\bar{c}\phi\theta^4-180\bar{c}\phi\theta^2}{3125\phi^4-1625\phi^3\theta^2-3125\phi^3+1025\phi^2\theta^2+600\phi^2+180\phi\theta^4-72\theta^4} \\ c_2^H &= -\frac{250\alpha\phi^3\theta^2+375\alpha\phi^3-425\alpha\phi^2\theta^2-300\alpha\phi^2-90\alpha\phi\theta^4+180\alpha\phi\theta^2+72\alpha\theta^4-3125\bar{c}\phi^4+1375\bar{c}\phi^3\theta^2+2750\bar{c}\phi^3-600\bar{c}\phi^2\theta^2-300\bar{c}\phi^2-90\bar{c}\phi\theta^4-180\bar{c}\phi\theta^2}{3125\phi^4-1625\phi^3\theta^2-3125\phi^3+1025\phi^2\theta^2+600\phi^2+180\phi\theta^4-72\theta^4} \\ c_1^L &= -\frac{625\alpha\phi^3\theta^2-725\alpha\phi^2\theta^2-90\alpha\phi\theta^4+180\alpha\phi\theta^2+72\alpha\theta^4-3125\bar{c}\phi^4+1000\bar{c}\phi^3\theta^2+3125\bar{c}\phi^3-300\bar{c}\phi^2\theta^2-600\bar{c}\phi^2-90\bar{c}\phi\theta^4-180\bar{c}\phi\theta^2}{3125\phi^4-1625\phi^3\theta^2-3125\phi^3+1025\phi^2\theta^2+600\phi^2+180\phi\theta^4-72\theta^4} \\ c_2^L &= -\frac{625\alpha\phi^3\theta^2+375\alpha\phi^3-950\alpha\phi^2\theta^2-300\alpha\phi^2-90\alpha\phi\theta^4+360\alpha\phi\theta^2+72\alpha\theta^4-3125\bar{c}\phi^4+1000\bar{c}\phi^3\theta^2+2750\bar{c}\phi^3-75\bar{c}\phi^2\theta^2-300\bar{c}\phi^2-90\bar{c}\phi\theta^4-360\bar{c}\phi\theta^2}{3125\phi^4-1625\phi^3\theta^2-3125\phi^3+1025\phi^2\theta^2+600\phi^2+180\phi\theta^4-72\theta^4} \end{cases}$$

Production Quantities:

$$\begin{cases} q_1^H &= \frac{5\phi(\alpha-\bar{c})(125\phi^3-125\phi^2\theta^2-75\phi^2+45\phi\theta^2+18\theta^4)}{3125\phi^4-1625\phi^3\theta^2-3125\phi^3+1025\phi^2\theta^2+600\phi^2+180\phi\theta^4-72\theta^4} \\ q_2^H &= \frac{25\phi^2(\alpha-\bar{c})(5\phi-4)(5\phi-3\theta^2)}{3125\phi^4-1625\phi^3\theta^2-3125\phi^3+1025\phi^2\theta^2+600\phi^2+180\phi\theta^4-72\theta^4} \\ q_1^L &= \frac{25\phi^2(\alpha-\bar{c})(5\phi-4)(5\phi-3)}{3125\phi^4-1625\phi^3\theta^2-3125\phi^3+1025\phi^2\theta^2+600\phi^2+180\phi\theta^4-72\theta^4} \\ q_2^L &= \frac{5\phi(\alpha-\bar{c})(5\phi-4)(5\phi-3\theta)(5\phi+3\theta)}{3125\phi^4-1625\phi^3\theta^2-3125\phi^3+1025\phi^2\theta^2+600\phi^2+180\phi\theta^4-72\theta^4} \end{cases}$$

Profits:

$$\begin{cases} \pi_1^H &= \frac{\phi(\alpha-\bar{c})^2 \cdot (25\phi-16)(125\phi^3-125\phi^2\theta^2-75\phi^2+45\phi\theta^2+18\theta^4)^2}{(3125\phi^4-1625\phi^3\theta^2-3125\phi^3+1025\phi^2\theta^2+600\phi^2+180\phi\theta^4-72\theta^4)^2} \\ \pi_2^H &= \frac{25\phi^3(\alpha-\bar{c})^2(5\phi-4)^2(5\phi-3\theta^2)^2 \cdot (25\phi-9)}{(3125\phi^4-1625\phi^3\theta^2-3125\phi^3+1025\phi^2\theta^2+600\phi^2+180\phi\theta^4-72\theta^4)^2} \\ \pi_1^L &= \frac{25\phi^3(\alpha-\bar{c})^2(5\phi-4)^2(5\phi-3)^2 \cdot (25\phi-9\theta^2)}{(3125\phi^4-1625\phi^3\theta^2-3125\phi^3+1025\phi^2\theta^2+600\phi^2+180\phi\theta^4-72\theta^4)^2} \\ \pi_2^L &= \frac{\phi(\alpha-\bar{c})^2(5\phi-4)^2(5\phi-3\theta)^2(5\phi+3\theta)^2 \cdot (25\phi-4\theta^2)}{(3125\phi^4-1625\phi^3\theta^2-3125\phi^3+1025\phi^2\theta^2+600\phi^2+180\phi\theta^4-72\theta^4)^2} \end{cases} \quad (61)$$

Total Production:

$$\frac{10\phi(\alpha-\bar{c})(250\phi^3-100\phi^2\theta^2-225\phi^2+30\phi\theta^2+30\phi+9\theta^4+18\theta^2)}{3125\phi^4-1625\phi^3\theta^2-3125\phi^3+1025\phi^2\theta^2+600\phi^2+180\phi\theta^4-72\theta^4}$$

Price:

$$\frac{625\alpha\phi^4-625\alpha\phi^3\theta^2-875\alpha\phi^3+725\alpha\phi^2\theta^2+300\alpha\phi^2+90\alpha\phi\theta^4-180\alpha\phi\theta^2-72\alpha\theta^4+2500\bar{c}\phi^4-1000\bar{c}\phi^3\theta^2-2250\bar{c}\phi^3+300\bar{c}\phi^2\theta^2+300\bar{c}\phi^2+90\bar{c}\phi\theta^4+180\bar{c}\phi\theta^2}{3125\phi^4-1625\phi^3\theta^2-3125\phi^3+1025\phi^2\theta^2+600\phi^2+180\phi\theta^4-72\theta^4}$$

Firm Surplus:

$$\frac{\phi(\alpha-\bar{c})^2 \cdot (1562500\phi^7-1453125\phi^6\theta^2-3203125\phi^6+531250\phi^5\theta^4+2381250\phi^5\theta^2+2306250\phi^5-649375\phi^4\theta^4-1219375\phi^4\theta^2-705000\phi^4-112500\phi^3\theta^6+240750\phi^3\theta^4+225000\phi^3\theta^2+90000\phi^3+104400\phi^2\theta^6-73800\phi^2\theta^4-32400\phi^2\theta^2+8100\phi\theta^6-129600\phi\theta^4+32400\phi\theta^2-5184\theta^6-5184\theta^4)}{(3125\phi^4-1625\phi^3\theta^2-3125\phi^3+1025\phi^2\theta^2+600\phi^2+180\phi\theta^4-72\theta^4)^2}$$

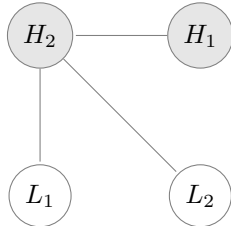
Consumer Surplus:

$$\frac{50\phi^2(\alpha-\bar{c})^2(250\phi^3-100\phi^2\theta^2-225\phi^2+30\phi\theta^2+30\phi+9\theta^4+18\theta^2)^2}{(3125\phi^4-1625\phi^3\theta^2-3125\phi^3+1025\phi^2\theta^2+600\phi^2+180\phi\theta^4-72\theta^4)^2}$$

Social Welfare:

$$\frac{\phi(\alpha-\bar{c})^2 \cdot (4687500\phi^7-3953125\phi^6\theta^2-8828125\phi^6+1031250\phi^5\theta^4+5381250\phi^5\theta^2+5587500\phi^5-724375\phi^4\theta^4-1744375\phi^4\theta^2-1380000\phi^4-202500\phi^3\theta^6-96750\phi^3\theta^4-90000\phi^3\theta^2+135000\phi^3+131400\phi^2\theta^6+7200\phi^2\theta^4+21600\phi^2\theta^2+12150\phi\theta^6+3240\phi\theta^4+48600\phi\theta^2-5184\theta^6-5184\theta^4)}{(3125\phi^4-1625\phi^3\theta^2-3125\phi^3+1025\phi^2\theta^2+600\phi^2+180\phi\theta^4-72\theta^4)^2}$$

## D.2.11 E3A [2H2L]



$$\text{Equations : } \begin{cases} e_1^H \cdot \left(\frac{25\phi}{3} - 3\right) = \alpha + e_2^H - 2e_1^L\theta - 2e_2^L\theta - \bar{c} \\ e_2^H \cdot (25\phi - 1) = \alpha + 3e_1^H + 3e_1^L\theta + 3e_2^L\theta - \bar{c} \\ e_1^L \cdot \left(\frac{25\phi}{3\theta} - 3\theta\right) = \alpha - 2e_1^H + e_2^H - 2e_2^L\theta - \bar{c} \\ e_2^L \cdot \left(\frac{25\phi}{3\theta} - 3\theta\right) = \alpha - 2e_1^H + e_2^H - 2e_1^L\theta - \bar{c} \end{cases}$$

Optimal efforts:

$$\begin{cases} e_1^H &= \frac{15\phi(\alpha-\bar{c})(5\phi-3\theta^2)}{625\phi^3-75\phi^2\theta^2-250\phi^2-60\phi\theta^2+18\theta^2} \\ e_2^H &= \frac{(\alpha-\bar{c})(25\phi^2+15\phi\theta^2-18\theta^2)}{625\phi^3-75\phi^2\theta^2-250\phi^2-60\phi\theta^2+18\theta^2} \\ e_1^L &= \frac{15\phi\theta(\alpha-\bar{c})(5\phi-3)}{625\phi^3-75\phi^2\theta^2-250\phi^2-60\phi\theta^2+18\theta^2} \\ e_2^L &= \frac{15\phi\theta(\alpha-\bar{c})(5\phi-3)}{625\phi^3-75\phi^2\theta^2-250\phi^2-60\phi\theta^2+18\theta^2} \end{cases}$$

Production Costs:

$$\begin{cases} c_1^H &= -\frac{100\alpha\phi^2-30\alpha\phi\theta^2-18\alpha\theta^2-625\bar{c}\phi^3+75\bar{c}\phi^2\theta^2+150\bar{c}\phi^2+90\bar{c}\phi\theta^2}{625\phi^3-75\phi^2\theta^2-250\phi^2-60\phi\theta^2+18\theta^2} \\ c_2^H &= -\frac{150\alpha\phi^2\theta^2+100\alpha\phi^2-120\alpha\phi\theta^2-18\alpha\theta^2-625\bar{c}\phi^3-75\bar{c}\phi^2\theta^2+150\bar{c}\phi^2+180\bar{c}\phi\theta^2}{625\phi^3-75\phi^2\theta^2-250\phi^2-60\phi\theta^2+18\theta^2} \\ c_1^L &= -\frac{75\alpha\phi^2\theta^2+25\alpha\phi^2-30\alpha\phi\theta^2-18\alpha\theta^2-625\bar{c}\phi^3+225\bar{c}\phi^2+90\bar{c}\phi\theta^2}{625\phi^3-75\phi^2\theta^2-250\phi^2-60\phi\theta^2+18\theta^2} \\ c_2^L &= -\frac{75\alpha\phi^2\theta^2+25\alpha\phi^2-30\alpha\phi\theta^2-18\alpha\theta^2-625\bar{c}\phi^3+225\bar{c}\phi^2+90\bar{c}\phi\theta^2}{625\phi^3-75\phi^2\theta^2-250\phi^2-60\phi\theta^2+18\theta^2} \end{cases}$$

Production Quantities:

$$\begin{cases} q_1^H &= \frac{25\phi^2(\alpha-\bar{c})(5\phi-3\theta^2)}{625\phi^3-75\phi^2\theta^2-250\phi^2-60\phi\theta^2+18\theta^2} \\ q_2^H &= \frac{5\phi(\alpha-\bar{c})(25\phi^2+15\phi\theta^2-18\theta^2)}{625\phi^3-75\phi^2\theta^2-250\phi^2-60\phi\theta^2+18\theta^2} \\ q_1^L &= \frac{25\phi^2(\alpha-\bar{c})(5\phi-3)}{625\phi^3-75\phi^2\theta^2-250\phi^2-60\phi\theta^2+18\theta^2} \\ q_2^L &= \frac{25\phi^2(\alpha-\bar{c})(5\phi-3)}{625\phi^3-75\phi^2\theta^2-250\phi^2-60\phi\theta^2+18\theta^2} \end{cases}$$

Profits:

$$\begin{cases} \pi_1^H &= \frac{25\phi^3(\alpha-\bar{c})^2(5\phi-3\theta^2)^2 \cdot (25\phi-9)}{(625\phi^3-75\phi^2\theta^2-250\phi^2-60\phi\theta^2+18\theta^2)^2} \\ \pi_2^H &= \frac{\phi(\alpha-\bar{c})^2 \cdot (25\phi-1)(25\phi^2+15\phi\theta^2-18\theta^2)^2}{(625\phi^3-75\phi^2\theta^2-250\phi^2-60\phi\theta^2+18\theta^2)^2} \\ \pi_1^L &= \frac{25\phi^3(\alpha-\bar{c})^2(5\phi-3)^2 \cdot (25\phi-9\theta^2)}{(625\phi^3-75\phi^2\theta^2-250\phi^2-60\phi\theta^2+18\theta^2)^2} \\ \pi_2^L &= \frac{25\phi^3(\alpha-\bar{c})^2(5\phi-3)^2 \cdot (25\phi-9\theta^2)}{(625\phi^3-75\phi^2\theta^2-250\phi^2-60\phi\theta^2+18\theta^2)^2} \end{cases} \quad (62)$$

Total Production:

$$\frac{10\phi(\alpha-\bar{c})(50\phi^2-15\phi-9\theta^2)}{625\phi^3-75\phi^2\theta^2-250\phi^2-60\phi\theta^2+18\theta^2}$$

Price:

$$\frac{125\alpha\phi^3-75\alpha\phi^2\theta^2-100\alpha\phi^2+30\alpha\phi\theta^2+18\alpha\theta^2+500\bar{c}\phi^3-150\bar{c}\phi^2-90\bar{c}\phi\theta^2}{625\phi^3-75\phi^2\theta^2-250\phi^2-60\phi\theta^2+18\theta^2}$$

Firm Surplus:

$$\frac{2\phi(\alpha-\bar{c})^2 \cdot (31250\phi^5-5625\phi^4\theta^2-21875\phi^4+5625\phi^3\theta^4-1500\phi^3\theta^2+5625\phi^3-7875\phi^2\theta^4-1575\phi^2\theta^2+4320\phi\theta^4-162\theta^4)}{(625\phi^3-75\phi^2\theta^2-250\phi^2-60\phi\theta^2+18\theta^2)^2}$$

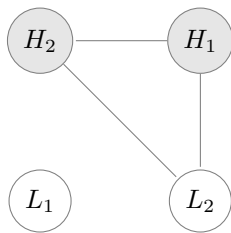
Consumer Surplus:

$$\frac{50\phi^2(\alpha-\bar{c})^2(50\phi^2-15\phi-9\theta^2)^2}{(625\phi^3-75\phi^2\theta^2-250\phi^2-60\phi\theta^2+18\theta^2)^2}$$

Social Welfare:

$$\frac{2\phi(\alpha-\bar{c})^2 \cdot (93750\phi^5-5625\phi^4\theta^2-59375\phi^4+5625\phi^3\theta^4-24000\phi^3\theta^2+11250\phi^3-7875\phi^2\theta^4+5175\phi^2\theta^2+6345\phi\theta^4-162\theta^4)}{(625\phi^3-75\phi^2\theta^2-250\phi^2-60\phi\theta^2+18\theta^2)^2}$$

#### D.2.12 E3B [2H2L]



$$\text{Equations : } \begin{cases} e_1^H \cdot \left(\frac{25\phi}{2} - 2\right) = \alpha + 2e_2^H - e_1^L\theta + 2e_2^L\theta - \bar{c} \\ e_2^H \cdot \left(\frac{25\phi}{2} - 2\right) = \alpha + 2e_1^H - e_1^L\theta + 2e_2^L\theta - \bar{c} \\ e_1^L \cdot \left(\frac{25\phi}{4\theta} - 4\theta\right) = \alpha - 3e_1^H - 3e_2^H - 3e_2^L\theta - \bar{c} \\ e_2^L \cdot \left(\frac{25\phi}{2\theta} - 2\theta\right) = \alpha + 2e_1^H + 2e_2^H - e_1^L\theta - \bar{c} \end{cases}$$

Optimal efforts:

$$\begin{cases} e_1^H &= \frac{2(\alpha-\bar{c})(5\phi-4\theta^2)}{125\phi^2-100\phi\theta^2-40\phi+8\theta^4+16\theta^2} \\ e_2^H &= \frac{2(\alpha-\bar{c})(5\phi-4\theta^2)}{125\phi^2-100\phi\theta^2-40\phi+8\theta^4+16\theta^2} \\ e_1^L &= \frac{4\theta(\alpha-\bar{c})(5\phi-2\theta^2-4)}{125\phi^2-100\phi\theta^2-40\phi+8\theta^4+16\theta^2} \\ e_2^L &= \frac{2\theta(\alpha-\bar{c})(5\phi-4\theta^2)}{125\phi^2-100\phi\theta^2-40\phi+8\theta^4+16\theta^2} \end{cases}$$

Production Costs:

$$\begin{cases} c_1^H &= -\frac{10\alpha\phi\theta^2+20\alpha\phi-8\alpha\theta^4-16\alpha\theta^2-125\bar{c}\phi^2+90\bar{c}\phi\theta^2+20\bar{c}\phi}{125\phi^2-100\phi\theta^2-40\phi+8\theta^4+16\theta^2} \\ c_2^H &= -\frac{10\alpha\phi\theta^2+20\alpha\phi-8\alpha\theta^4-16\alpha\theta^2-125\bar{c}\phi^2+90\bar{c}\phi\theta^2+20\bar{c}\phi}{125\phi^2-100\phi\theta^2-40\phi+8\theta^4+16\theta^2} \\ c_1^L &= -\frac{20\alpha\phi\theta^2-8\alpha\theta^4-16\alpha\theta^2-125\bar{c}\phi^2+80\bar{c}\phi\theta^2+40\bar{c}\phi}{125\phi^2-100\phi\theta^2-40\phi+8\theta^4+16\theta^2} \\ c_2^L &= -\frac{10\alpha\phi\theta^2+20\alpha\phi-8\alpha\theta^4-16\alpha\theta^2-125\bar{c}\phi^2+90\bar{c}\phi\theta^2+20\bar{c}\phi}{125\phi^2-100\phi\theta^2-40\phi+8\theta^4+16\theta^2} \end{cases}$$

Production Quantities:

$$\begin{cases} q_1^H &= \frac{5\phi(\alpha-\bar{c})(5\phi-4\theta^2)}{125\phi^2-100\phi\theta^2-40\phi+8\theta^4+16\theta^2} \\ q_2^H &= \frac{5\phi(\alpha-\bar{c})(5\phi-4\theta^2)}{125\phi^2-100\phi\theta^2-40\phi+8\theta^4+16\theta^2} \\ q_1^L &= \frac{5\phi(\alpha-\bar{c})(5\phi-2\theta^2-4)}{125\phi^2-100\phi\theta^2-40\phi+8\theta^4+16\theta^2} \\ q_2^L &= \frac{5\phi(\alpha-\bar{c})(5\phi-4\theta^2)}{125\phi^2-100\phi\theta^2-40\phi+8\theta^4+16\theta^2} \end{cases}$$

Profits:

$$\begin{cases} \pi_1^H &= \frac{\phi(\alpha-\bar{c})^2(5\phi-4\theta^2)^2 \cdot (25\phi-4)}{(125\phi^2-100\phi\theta^2-40\phi+8\theta^4+16\theta^2)^2} \\ \pi_2^H &= \frac{\phi(\alpha-\bar{c})^2(5\phi-4\theta^2)^2 \cdot (25\phi-4)}{(125\phi^2-100\phi\theta^2-40\phi+8\theta^4+16\theta^2)^2} \\ \pi_1^L &= \frac{\phi(\alpha-\bar{c})^2 \cdot (25\phi-16\theta^2)(5\phi-2\theta^2-4)^2}{(125\phi^2-100\phi\theta^2-40\phi+8\theta^4+16\theta^2)^2} \\ \pi_2^L &= \frac{\phi(\alpha-\bar{c})^2(5\phi-4\theta^2)^2 \cdot (25\phi-4\theta^2)}{(125\phi^2-100\phi\theta^2-40\phi+8\theta^4+16\theta^2)^2} \end{cases} \quad (63)$$

Total Production:

$$\frac{10\phi(\alpha-\bar{c})(10\phi-7\theta^2-2)}{125\phi^2-100\phi\theta^2-40\phi+8\theta^4+16\theta^2}$$

Price:

$$\frac{25\alpha\phi^2-30\alpha\phi\theta^2-20\alpha\phi+8\alpha\theta^4+16\alpha\theta^2+100\bar{c}\phi^2-70\bar{c}\phi\theta^2-20\bar{c}\phi}{125\phi^2-100\phi\theta^2-40\phi+8\theta^4+16\theta^2}$$

Firm Surplus:

$$\frac{4\phi(\alpha-\bar{c})^2 \cdot (625\phi^3-1000\phi^2\theta^2-300\phi^2+445\phi\theta^4+340\phi\theta^2+100\phi-32\theta^6-96\theta^4-64\theta^2)}{(125\phi^2-100\phi\theta^2-40\phi+8\theta^4+16\theta^2)^2}$$

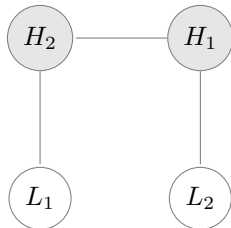
Consumer Surplus:

$$\frac{50\phi^2(\alpha-\bar{c})^2(10\phi-7\theta^2-2)^2}{(125\phi^2-100\phi\theta^2-40\phi+8\theta^4+16\theta^2)^2}$$

Social Welfare:

$$\frac{2\phi(\alpha-\bar{c})^2 \cdot (3750\phi^3-5500\phi^2\theta^2-1600\phi^2+2115\phi\theta^4+1380\phi\theta^2+300\phi-64\theta^6-192\theta^4-128\theta^2)}{(125\phi^2-100\phi\theta^2-40\phi+8\theta^4+16\theta^2)^2}$$

#### D.2.13 E3C [2H2L]



$$\text{Equations : } \begin{cases} e_1^H \cdot \left(\frac{25\phi}{2} - 2\right) = \alpha + 2e_2^H - 2e_1^L\theta + 3e_2^L\theta - \bar{c} \\ e_2^H \cdot \left(\frac{25\phi}{2} - 2\right) = \alpha + 2e_1^H + 3e_1^L\theta - 2e_2^L\theta - \bar{c} \\ e_1^L \cdot \left(\frac{25\phi}{3\theta} - 3\theta\right) = \alpha - 3e_1^H + 2e_2^H - 2e_2^L\theta - \bar{c} \\ e_2^L \cdot \left(\frac{25\phi}{3\theta} - 3\theta\right) = \alpha + 2e_1^H - 3e_2^H - 2e_1^L\theta - \bar{c} \end{cases}$$

Optimal efforts:

$$\begin{cases} e_1^H &= \frac{10\phi(\alpha-\bar{c})}{125\phi^2-15\phi\theta^2-40\phi+6\theta^2} \\ e_2^H &= \frac{10\phi(\alpha-\bar{c})}{125\phi^2-15\phi\theta^2-40\phi+6\theta^2} \\ e_1^L &= \frac{3\theta(\alpha-\bar{c})(5\phi-2)}{125\phi^2-15\phi\theta^2-40\phi+6\theta^2} \\ e_2^L &= \frac{3\theta(\alpha-\bar{c})(5\phi-2)}{125\phi^2-15\phi\theta^2-40\phi+6\theta^2} \end{cases}$$

Production Costs:

$$\begin{cases} c_1^H &= -\frac{15\alpha\phi\theta^2+20\alpha\phi-6\alpha\theta^2-125\bar{c}\phi^2+20\bar{c}\phi}{125\phi^2-15\phi\theta^2-40\phi+6\theta^2} \\ c_2^H &= -\frac{15\alpha\phi\theta^2+20\alpha\phi-6\alpha\theta^2-125\bar{c}\phi^2+20\bar{c}\phi}{125\phi^2-15\phi\theta^2-40\phi+6\theta^2} \\ c_1^L &= -\frac{15\alpha\phi\theta^2+10\alpha\phi-6\alpha\theta^2-125\bar{c}\phi^2+30\bar{c}\phi}{125\phi^2-15\phi\theta^2-40\phi+6\theta^2} \\ c_2^L &= -\frac{15\alpha\phi\theta^2+10\alpha\phi-6\alpha\theta^2-125\bar{c}\phi^2+30\bar{c}\phi}{125\phi^2-15\phi\theta^2-40\phi+6\theta^2} \end{cases}$$

Production Quantities:

$$\begin{cases} q_1^H &= \frac{25\phi^2(\alpha-\bar{c})}{125\phi^2-15\phi\theta^2-40\phi+6\theta^2} \\ q_2^H &= \frac{25\phi^2(\alpha-\bar{c})}{125\phi^2-15\phi\theta^2-40\phi+6\theta^2} \\ q_1^L &= \frac{5\phi(\alpha-\bar{c})(5\phi-2)}{125\phi^2-15\phi\theta^2-40\phi+6\theta^2} \\ q_2^L &= \frac{5\phi(\alpha-\bar{c})(5\phi-2)}{125\phi^2-15\phi\theta^2-40\phi+6\theta^2} \end{cases}$$

Profits:

$$\begin{cases} \pi_1^H &= \frac{25\phi^3(\alpha-\bar{c})^2 \cdot (25\phi-4)}{(125\phi^2-15\phi\theta^2-40\phi+6\theta^2)^2} \\ \pi_2^H &= \frac{25\phi^3(\alpha-\bar{c})^2 \cdot (25\phi-4)}{(125\phi^2-15\phi\theta^2-40\phi+6\theta^2)^2} \\ \pi_1^L &= \frac{\phi(\alpha-\bar{c})^2(5\phi-2)^2 \cdot (25\phi-9\theta^2)}{(125\phi^2-15\phi\theta^2-40\phi+6\theta^2)^2} \\ \pi_2^L &= \frac{\phi(\alpha-\bar{c})^2(5\phi-2)^2 \cdot (25\phi-9\theta^2)}{(125\phi^2-15\phi\theta^2-40\phi+6\theta^2)^2} \end{cases} \quad (64)$$

Total Production:

$$\frac{20\phi(\alpha-\bar{c})(5\phi-1)}{125\phi^2-15\phi\theta^2-40\phi+6\theta^2}$$

Price:

$$\frac{25\alpha\phi^2-15\alpha\phi\theta^2-20\alpha\phi+6\alpha\theta^2+100\bar{c}\phi^2-20\bar{c}\phi}{125\phi^2-15\phi\theta^2-40\phi+6\theta^2}$$

Firm Surplus:

$$\frac{2\phi(\alpha-\bar{c})^2 \cdot (1250\phi^3-225\phi^2\theta^2-600\phi^2+180\phi\theta^2+100\phi-36\theta^2)}{(125\phi^2-15\phi\theta^2-40\phi+6\theta^2)^2}$$

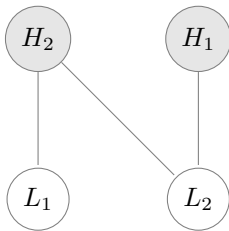
Consumer Surplus:

$$\frac{200\phi^2(\alpha-\bar{c})^2(5\phi-1)^2}{(125\phi^2-15\phi\theta^2-40\phi+6\theta^2)^2}$$

Social Welfare:

$$\frac{2\phi(\alpha-\bar{c})^2 \cdot (3750\phi^3-225\phi^2\theta^2-1600\phi^2+180\phi\theta^2+200\phi-36\theta^2)}{(125\phi^2-15\phi\theta^2-40\phi+6\theta^2)^2}$$

#### D.2.14 E3D [2H2L]



$$\text{Equations : } \begin{cases} e_1^H \cdot \left(\frac{25\phi}{3} - 3\right) = \alpha - 3e_2^H - 2e_1^L\theta + 2e_2^L\theta - \bar{c} \\ e_2^H \cdot \left(\frac{25\phi}{2} - 2\right) = \alpha - 2e_1^H + 3e_1^L\theta + 2e_2^L\theta - \bar{c} \\ e_1^L \cdot \left(\frac{25\phi}{3\theta} - 3\theta\right) = \alpha - 2e_1^H + 2e_2^H - 3e_2^L\theta - \bar{c} \\ e_2^L \cdot \left(\frac{25\phi}{2\theta} - 2\theta\right) = \alpha + 3e_1^H + 2e_2^H - 2e_1^L\theta - \bar{c} \end{cases}$$

Optimal efforts:

$$\begin{cases} e_1^H = \frac{3(\alpha - \bar{c})(125\phi^3 - 75\phi^2\theta^2 - 50\phi^2 + 12\theta^4)}{3125\phi^4 - 1625\phi^3\theta^2 - 1625\phi^3 + 225\phi^2\theta^2 + 120\phi\theta^4 + 120\phi\theta^2 - 36\theta^4} \\ e_2^H = \frac{10\phi(\alpha - \bar{c})(25\phi^2 - 15\phi - 6\theta^4)}{3125\phi^4 - 1625\phi^3\theta^2 - 1625\phi^3 + 225\phi^2\theta^2 + 120\phi\theta^4 + 120\phi\theta^2 - 36\theta^4} \\ e_1^L = \frac{3\theta(\alpha - \bar{c})(125\phi^3 - 50\phi^2\theta^2 - 75\phi^2 + 12\theta^2)}{3125\phi^4 - 1625\phi^3\theta^2 - 1625\phi^3 + 225\phi^2\theta^2 + 120\phi\theta^4 + 120\phi\theta^2 - 36\theta^4} \\ e_2^L = \frac{10\phi\theta(\alpha - \bar{c})(25\phi^2 - 15\phi\theta^2 - 6)}{3125\phi^4 - 1625\phi^3\theta^2 - 1625\phi^3 + 225\phi^2\theta^2 + 120\phi\theta^4 + 120\phi\theta^2 - 36\theta^4} \end{cases}$$

Production Costs:

$$\begin{cases} c_1^H = -\frac{250\alpha\phi^3\theta^2 + 375\alpha\phi^3 - 150\alpha\phi^2\theta^4 - 225\alpha\phi^2\theta^2 - 150\alpha\phi^2 - 60\alpha\phi\theta^2 + 36\alpha\theta^4 - 3125\bar{c}\phi^4 + 1375\bar{c}\phi^3\theta^2 + 1250\bar{c}\phi^3 + 150\bar{c}\phi^2\theta^4 + 150\bar{c}\phi^2 - 120\bar{c}\phi\theta^4 - 60\bar{c}\phi\theta^2}{3125\phi^4 - 1625\phi^3\theta^2 - 1625\phi^3 + 225\phi^2\theta^2 + 120\phi\theta^4 + 120\phi\theta^2 - 36\theta^4} \\ c_2^H = -\frac{625\alpha\phi^3\theta^2 + 250\alpha\phi^3 - 300\alpha\phi^2\theta^4 - 225\alpha\phi^2\theta^2 - 150\alpha\phi^2 - 60\alpha\phi\theta^4 - 60\alpha\phi\theta^2 + 36\alpha\theta^4 - 3125\bar{c}\phi^4 + 1000\bar{c}\phi^3\theta^2 + 1375\bar{c}\phi^3 + 300\bar{c}\phi^2\theta^4 + 150\bar{c}\phi^2 - 60\bar{c}\phi\theta^4 - 60\bar{c}\phi\theta^2}{3125\phi^4 - 1625\phi^3\theta^2 - 1625\phi^3 + 225\phi^2\theta^2 + 120\phi\theta^4 + 120\phi\theta^2 - 36\theta^4} \\ c_1^L = -\frac{375\alpha\phi^3\theta^2 + 250\alpha\phi^3 - 150\alpha\phi^2\theta^4 - 225\alpha\phi^2\theta^2 - 150\alpha\phi^2 - 60\alpha\phi\theta^4 + 36\alpha\theta^4 - 3125\bar{c}\phi^4 + 1250\bar{c}\phi^3\theta^2 + 1375\bar{c}\phi^3 + 150\bar{c}\phi^2\theta^4 + 150\bar{c}\phi^2 - 60\bar{c}\phi\theta^4 - 120\bar{c}\phi\theta^2}{3125\phi^4 - 1625\phi^3\theta^2 - 1625\phi^3 + 225\phi^2\theta^2 + 120\phi\theta^4 + 120\phi\theta^2 - 36\theta^4} \\ c_2^L = -\frac{250\alpha\phi^3\theta^2 + 625\alpha\phi^3 - 150\alpha\phi^2\theta^4 - 225\alpha\phi^2\theta^2 - 300\alpha\phi^2 - 60\alpha\phi\theta^4 - 60\alpha\phi\theta^2 + 36\alpha\theta^4 - 3125\bar{c}\phi^4 + 1375\bar{c}\phi^3\theta^2 + 1000\bar{c}\phi^3 + 150\bar{c}\phi^2\theta^4 + 300\bar{c}\phi^2 - 60\bar{c}\phi\theta^4 - 60\bar{c}\phi\theta^2}{3125\phi^4 - 1625\phi^3\theta^2 - 1625\phi^3 + 225\phi^2\theta^2 + 120\phi\theta^4 + 120\phi\theta^2 - 36\theta^4} \end{cases}$$

Production Quantities:

$$\begin{cases} q_1^H = \frac{5\phi(\alpha - \bar{c})(125\phi^3 - 75\phi^2\theta^2 - 50\phi^2 + 12\theta^4)}{3125\phi^4 - 1625\phi^3\theta^2 - 1625\phi^3 + 225\phi^2\theta^2 + 120\phi\theta^4 + 120\phi\theta^2 - 36\theta^4} \\ q_2^H = \frac{25\phi^2(\alpha - \bar{c})(25\phi^2 - 15\phi - 6\theta^4)}{3125\phi^4 - 1625\phi^3\theta^2 - 1625\phi^3 + 225\phi^2\theta^2 + 120\phi\theta^4 + 120\phi\theta^2 - 36\theta^4} \\ q_1^L = \frac{5\phi(\alpha - \bar{c})(125\phi^3 - 50\phi^2\theta^2 - 75\phi^2 + 12\theta^2)}{3125\phi^4 - 1625\phi^3\theta^2 - 1625\phi^3 + 225\phi^2\theta^2 + 120\phi\theta^4 + 120\phi\theta^2 - 36\theta^4} \\ q_2^L = \frac{25\phi^2(\alpha - \bar{c})(25\phi^2 - 15\phi\theta^2 - 6)}{3125\phi^4 - 1625\phi^3\theta^2 - 1625\phi^3 + 225\phi^2\theta^2 + 120\phi\theta^4 + 120\phi\theta^2 - 36\theta^4} \end{cases}$$

Profits:

$$\begin{cases} \pi_1^H = \frac{\phi(\alpha - \bar{c})^2 \cdot (25\phi - 9)(125\phi^3 - 75\phi^2\theta^2 - 50\phi^2 + 12\theta^4)^2}{(3125\phi^4 - 1625\phi^3\theta^2 - 1625\phi^3 + 225\phi^2\theta^2 + 120\phi\theta^4 + 120\phi\theta^2 - 36\theta^4)^2} \\ \pi_2^H = \frac{25\phi^3(\alpha - \bar{c})^2 \cdot (25\phi - 4)(25\phi^2 - 15\phi - 6\theta^4)^2}{(3125\phi^4 - 1625\phi^3\theta^2 - 1625\phi^3 + 225\phi^2\theta^2 + 120\phi\theta^4 + 120\phi\theta^2 - 36\theta^4)^2} \\ \pi_1^L = \frac{\phi(\alpha - \bar{c})^2 \cdot (25\phi - 9\theta^2)(125\phi^3 - 50\phi^2\theta^2 - 75\phi^2 + 12\theta^2)^2}{(3125\phi^4 - 1625\phi^3\theta^2 - 1625\phi^3 + 225\phi^2\theta^2 + 120\phi\theta^4 + 120\phi\theta^2 - 36\theta^4)^2} \\ \pi_2^L = \frac{25\phi^3(\alpha - \bar{c})^2 \cdot (25\phi - 4\theta^2)(25\phi^2 - 15\phi\theta^2 - 6)^2}{(3125\phi^4 - 1625\phi^3\theta^2 - 1625\phi^3 + 225\phi^2\theta^2 + 120\phi\theta^4 + 120\phi\theta^2 - 36\theta^4)^2} \end{cases} \quad (65)$$

Total Production:

$$\frac{10\phi(\alpha - \bar{c})(250\phi^3 - 100\phi^2\theta^2 - 100\phi^2 - 15\phi\theta^4 - 15\phi + 6\theta^4 + 6\theta^2)}{3125\phi^4 - 1625\phi^3\theta^2 - 1625\phi^3 + 225\phi^2\theta^2 + 120\phi\theta^4 + 120\phi\theta^2 - 36\theta^4}$$

Price:

$$\frac{625\alpha\phi^4 - 625\alpha\phi^3\theta^2 - 625\alpha\phi^3 + 150\alpha\phi^2\theta^4 + 225\alpha\phi^2\theta^2 + 150\alpha\phi^2 + 60\alpha\phi\theta^4 + 60\alpha\phi\theta^2 - 36\alpha\theta^4 + 2500\bar{c}\phi^4 - 1000\bar{c}\phi^3\theta^2 - 1000\bar{c}\phi^3 - 150\bar{c}\phi^2\theta^4 - 150\bar{c}\phi^2 + 60\bar{c}\phi\theta^4 + 60\bar{c}\phi\theta^2}{3125\phi^4 - 1625\phi^3\theta^2 - 1625\phi^3 + 225\phi^2\theta^2 + 120\phi\theta^4 + 120\phi\theta^2 - 36\theta^4}$$

Firm Surplus:

$$\frac{\phi(\alpha - \bar{c})^2 \cdot (1562500\phi^7 - 1453125\phi^6\theta^2 - 1453125\phi^6 + 343750\phi^5\theta^4 + 712500\phi^5\theta^2 + 343750\phi^5 - 45000\phi^4\theta^6 + 99375\phi^4\theta^4 + 99375\phi^4\theta^2 - 45000\phi^4 + 22500\phi^3\theta^6 - 45000\phi^3\theta^4 - 150000\phi^3\theta^2 + 22500\phi^3 - 3600\phi^2\theta^6 + 27000\phi^2\theta^4 + 27000\phi^2\theta^2 - 3600\phi^2 + 3600\phi\theta^6 + 3600\phi\theta^4 - 1296\theta^6 - 1296\theta^4)}{(3125\phi^4 - 1625\phi^3\theta^2 - 1625\phi^3 + 225\phi^2\theta^2 + 120\phi\theta^4 + 120\phi\theta^2 - 36\theta^4)^2}$$

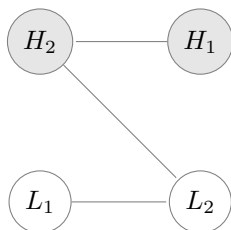
Consumer Surplus:

$$\frac{50\phi^2(\alpha - \bar{c})^2(250\phi^3 - 100\phi^2\theta^2 - 100\phi^2 - 15\phi\theta^4 - 15\phi + 6\theta^4 + 6\theta^2)^2}{(3125\phi^4 - 1625\phi^3\theta^2 - 1625\phi^3 + 225\phi^2\theta^2 + 120\phi\theta^4 + 120\phi\theta^2 - 36\theta^4)^2}$$

Social Welfare:

$$\frac{\phi(\alpha - \bar{c})^2 \cdot (4687500\phi^7 - 3953125\phi^6\theta^2 - 3953125\phi^6 + 468750\phi^5\theta^4 + 1712500\phi^5\theta^2 + 468750\phi^5 + 105000\phi^4\theta^6 + 399375\phi^4\theta^4 + 399375\phi^4\theta^2 + 105000\phi^4 + 33750\phi^3\theta^6 - 105000\phi^3\theta^4 - 247500\phi^3\theta^2 - 105000\phi^3 + 33750\phi^2 - 12600\phi^2\theta^6 + 18000\phi^2\theta^4 + 18000\phi^2\theta^2 - 12600\phi^2 + 5400\phi\theta^6 + 3600\phi\theta^4 + 5400\phi\theta^2 - 1296\theta^6 - 1296\theta^4)}{(3125\phi^4 - 1625\phi^3\theta^2 - 1625\phi^3 + 225\phi^2\theta^2 + 120\phi\theta^4 + 120\phi\theta^2 - 36\theta^4)^2}$$

## D.2.15 E3E [2H2L]



$$\text{Equations : } \begin{cases} e_1^H \cdot \left(\frac{25\phi}{3} - 3\right) = \alpha + 2e_2^H - 2e_1^L\theta - 3e_2^L\theta - \bar{c} \\ e_2^H \cdot \left(\frac{25\phi}{2} - 2\right) = \alpha + 3e_1^H - 2e_1^L\theta + 2e_2^L\theta - \bar{c} \\ e_1^L \cdot \left(\frac{25\phi}{3\theta} - 3\theta\right) = \alpha - 2e_1^H - 3e_2^H + 2e_2^L\theta - \bar{c} \\ e_2^L \cdot \left(\frac{25\phi}{2\theta} - 2\theta\right) = \alpha - 2e_1^H + 2e_2^H + 3e_1^L\theta - \bar{c} \end{cases}$$



Optimal efforts:

$$\begin{cases} e_1^H &= \frac{3(\alpha - \bar{c})(125\phi^3 - 125\phi^2\theta^2 + 12\theta^4)}{3125\phi^4 - 1625\phi^3\theta^2 - 1625\phi^3 + 225\phi^2\theta^2 + 120\phi\theta^4 + 120\phi\theta^2 - 36\theta^4} \\ e_2^H &= \frac{10\phi(\alpha - \bar{c})(25\phi^2 - 15\phi\theta^2 - 6\theta^2)}{3125\phi^4 - 1625\phi^3\theta^2 - 1625\phi^3 + 225\phi^2\theta^2 + 120\phi\theta^4 + 120\phi\theta^2 - 36\theta^4} \\ e_1^L &= \frac{3\theta(\alpha - \bar{c})(125\phi^3 - 125\phi^2 + 12\theta^2)}{3125\phi^4 - 1625\phi^3\theta^2 - 1625\phi^3 + 225\phi^2\theta^2 + 120\phi\theta^4 + 120\phi\theta^2 - 36\theta^4} \\ e_2^L &= \frac{10\phi\theta(\alpha - \bar{c})(25\phi^2 - 15\phi - 6\theta^2)}{3125\phi^4 - 1625\phi^3\theta^2 - 1625\phi^3 + 225\phi^2\theta^2 + 120\phi\theta^4 + 120\phi\theta^2 - 36\theta^4} \end{cases}$$

Production Costs:

$$\begin{cases} c_1^H &= -\frac{625\alpha\phi^3 - 525\alpha\phi^2\theta^2 - 60\alpha\phi\theta^4 + 36\alpha\theta^4 - 3125\bar{c}\phi^4 + 1625\bar{c}\phi^3\theta^2 + 1000\bar{c}\phi^3 + 300\bar{c}\phi^2\theta^2 - 120\bar{c}\phi\theta^4 - 60\bar{c}\phi\theta^2}{3125\phi^4 - 1625\phi^3\theta^2 - 1625\phi^3 + 225\phi^2\theta^2 + 120\phi\theta^4 + 120\phi\theta^2 - 36\theta^4} \\ c_2^H &= -\frac{250\alpha\phi^3\theta^2 + 625\alpha\phi^3 - 675\alpha\phi^2\theta^2 - 60\alpha\phi\theta^4 - 60\alpha\phi\theta^2 + 36\alpha\theta^4 - 3125\bar{c}\phi^4 + 1375\bar{c}\phi^3\theta^2 + 1000\bar{c}\phi^3 + 450\bar{c}\phi^2\theta^2 - 60\bar{c}\phi\theta^4 - 60\bar{c}\phi\theta^2}{3125\phi^4 - 1625\phi^3\theta^2 - 1625\phi^3 + 225\phi^2\theta^2 + 120\phi\theta^4 + 120\phi\theta^2 - 36\theta^4} \\ c_1^L &= -\frac{625\alpha\phi^3\theta^2 - 525\alpha\phi^2\theta^2 - 60\alpha\phi\theta^4 + 36\alpha\theta^4 - 3125\bar{c}\phi^4 + 1000\bar{c}\phi^3\theta^2 + 1625\bar{c}\phi^3 + 300\bar{c}\phi^2\theta^2 - 60\bar{c}\phi\theta^4 - 120\bar{c}\phi\theta^2}{3125\phi^4 - 1625\phi^3\theta^2 - 1625\phi^3 + 225\phi^2\theta^2 + 120\phi\theta^4 + 120\phi\theta^2 - 36\theta^4} \\ c_2^L &= -\frac{625\alpha\phi^3\theta^2 + 250\alpha\phi^3 - 675\alpha\phi^2\theta^2 - 60\alpha\phi\theta^4 - 60\alpha\phi\theta^2 + 36\alpha\theta^4 - 3125\bar{c}\phi^4 + 1000\bar{c}\phi^3\theta^2 + 1375\bar{c}\phi^3 + 450\bar{c}\phi^2\theta^2 - 60\bar{c}\phi\theta^4 - 60\bar{c}\phi\theta^2}{3125\phi^4 - 1625\phi^3\theta^2 - 1625\phi^3 + 225\phi^2\theta^2 + 120\phi\theta^4 + 120\phi\theta^2 - 36\theta^4} \end{cases}$$

Production Quantities:

$$\begin{cases} q_1^H &= \frac{5\phi(\alpha - \bar{c})(125\phi^3 - 125\phi^2\theta^2 + 12\theta^4)}{3125\phi^4 - 1625\phi^3\theta^2 - 1625\phi^3 + 225\phi^2\theta^2 + 120\phi\theta^4 + 120\phi\theta^2 - 36\theta^4} \\ q_2^H &= \frac{25\phi^2(\alpha - \bar{c})(25\phi^2 - 15\phi\theta^2 - 6\theta^2)}{3125\phi^4 - 1625\phi^3\theta^2 - 1625\phi^3 + 225\phi^2\theta^2 + 120\phi\theta^4 + 120\phi\theta^2 - 36\theta^4} \\ q_1^L &= \frac{5\phi(\alpha - \bar{c})(125\phi^3 - 125\phi^2 + 12\theta^2)}{3125\phi^4 - 1625\phi^3\theta^2 - 1625\phi^3 + 225\phi^2\theta^2 + 120\phi\theta^4 + 120\phi\theta^2 - 36\theta^4} \\ q_2^L &= \frac{25\phi^2(\alpha - \bar{c})(25\phi^2 - 15\phi - 6\theta^2)}{3125\phi^4 - 1625\phi^3\theta^2 - 1625\phi^3 + 225\phi^2\theta^2 + 120\phi\theta^4 + 120\phi\theta^2 - 36\theta^4} \end{cases}$$

Profits:

$$\begin{cases} \pi_1^H &= \frac{\phi(\alpha - \bar{c})^2 \cdot (25\phi - 9)(125\phi^3 - 125\phi^2\theta^2 + 12\theta^4)^2}{(3125\phi^4 - 1625\phi^3\theta^2 - 1625\phi^3 + 225\phi^2\theta^2 + 120\phi\theta^4 + 120\phi\theta^2 - 36\theta^4)^2} \\ \pi_2^H &= \frac{25\phi^3(\alpha - \bar{c})^2 \cdot (25\phi - 4)(25\phi^2 - 15\phi\theta^2 - 6\theta^2)^2}{(3125\phi^4 - 1625\phi^3\theta^2 - 1625\phi^3 + 225\phi^2\theta^2 + 120\phi\theta^4 + 120\phi\theta^2 - 36\theta^4)^2} \\ \pi_1^L &= \frac{\phi(\alpha - \bar{c})^2 \cdot (25\phi - 9\theta^2)(125\phi^3 - 125\phi^2 + 12\theta^2)^2}{(3125\phi^4 - 1625\phi^3\theta^2 - 1625\phi^3 + 225\phi^2\theta^2 + 120\phi\theta^4 + 120\phi\theta^2 - 36\theta^4)^2} \\ \pi_2^L &= \frac{25\phi^3(\alpha - \bar{c})^2 \cdot (25\phi - 4\theta^2)(25\phi^2 - 15\phi - 6\theta^2)^2}{(3125\phi^4 - 1625\phi^3\theta^2 - 1625\phi^3 + 225\phi^2\theta^2 + 120\phi\theta^4 + 120\phi\theta^2 - 36\theta^4)^2} \end{cases} \quad (66)$$

Total Production:

$$\frac{20\phi(\alpha - \bar{c})(125\phi^3 - 50\phi^2\theta^2 - 50\phi^2 - 15\phi\theta^2 + 3\theta^4 + 3\theta^2)}{3125\phi^4 - 1625\phi^3\theta^2 - 1625\phi^3 + 225\phi^2\theta^2 + 120\phi\theta^4 + 120\phi\theta^2 - 36\theta^4}$$

Price:

$$\frac{625\alpha\phi^4 - 625\alpha\phi^3\theta^2 - 625\alpha\phi^3 + 525\alpha\phi^2\theta^2 + 60\alpha\phi\theta^4 + 60\alpha\phi\theta^2 - 36\alpha\theta^4 + 2500\bar{c}\phi^4 - 1000\bar{c}\phi^3\theta^2 - 1000\bar{c}\phi^3 - 300\bar{c}\phi^2\theta^2 + 60\bar{c}\phi\theta^4 + 60\bar{c}\phi\theta^2}{3125\phi^4 - 1625\phi^3\theta^2 - 1625\phi^3 + 225\phi^2\theta^2 + 120\phi\theta^4 + 120\phi\theta^2 - 36\theta^4}$$

Firm Surplus:

$$\frac{\phi(\alpha - \bar{c})^2 \cdot (1562500\phi^7 - 1453125\phi^6\theta^2 - 1453125\phi^6 + 531250\phi^5\theta^4 + 337500\phi^5\theta^2 + 531250\phi^5 + 54375\phi^4\theta^4 + 54375\phi^4\theta^2 - 75000\phi^3\theta^6 - 45000\phi^3\theta^4 - 75000\phi^3\theta^2 + 23400\phi^2\theta^6 + 23400\phi^2\theta^4 + 3600\phi\theta^8 + 3600\phi\theta^4 - 1296\theta^8 - 1296\theta^6)}{(3125\phi^4 - 1625\phi^3\theta^2 - 1625\phi^3 + 225\phi^2\theta^2 + 120\phi\theta^4 + 120\phi\theta^2 - 36\theta^4)^2}$$

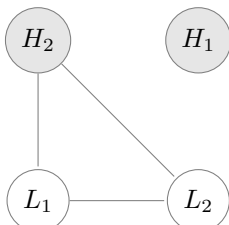
Consumer Surplus:

$$\frac{200\phi^2(\alpha - \bar{c})^2(125\phi^3 - 50\phi^2\theta^2 - 50\phi^2 - 15\phi\theta^2 + 3\theta^4 + 3\theta^2)^2}{(3125\phi^4 - 1625\phi^3\theta^2 - 1625\phi^3 + 225\phi^2\theta^2 + 120\phi\theta^4 + 120\phi\theta^2 - 36\theta^4)^2}$$

Social Welfare:

$$\frac{\phi(\alpha - \bar{c})^2 \cdot (4687500\phi^7 - 3953125\phi^6\theta^2 - 3953125\phi^6 + 1031250\phi^5\theta^4 + 587500\phi^5\theta^2 + 1031250\phi^5 + 504375\phi^4\theta^4 + 504375\phi^4\theta^2 - 135000\phi^3\theta^6 - 120000\phi^3\theta^4 - 135000\phi^3\theta^2 + 5400\phi^2\theta^6 + 5400\phi^2\theta^4 + 5400\phi\theta^8 + 3600\phi\theta^6 + 5400\phi\theta^4 - 1296\theta^8 - 1296\theta^6)}{(3125\phi^4 - 1625\phi^3\theta^2 - 1625\phi^3 + 225\phi^2\theta^2 + 120\phi\theta^4 + 120\phi\theta^2 - 36\theta^4)^2}$$

## D.2.16 E3F [2H2L]



$$\text{Equations : } \begin{cases} e_1^H \cdot \left( \frac{25\phi}{4} - 4 \right) = \alpha - 3e_2^H - 3e_1^L\theta - 3e_2^L\theta - \bar{c} \\ e_2^H \cdot \left( \frac{25\phi}{2} - 2 \right) = \alpha - e_1^H + 2e_1^L\theta + 2e_2^L\theta - \bar{c} \\ e_1^L \cdot \left( \frac{25\phi}{2\theta} - 2\theta \right) = \alpha - e_1^H + 2e_2^H + 2e_2^L\theta - \bar{c} \\ e_2^L \cdot \left( \frac{25\phi}{2\theta} - 2\theta \right) = \alpha - e_1^H + 2e_2^H + 2e_1^L\theta - \bar{c} \end{cases}$$

Optimal efforts:

$$\begin{cases} e_1^H &= \frac{4(\alpha - \bar{c})(5\phi - 4\theta^2 - 2)}{125\phi^2 - 40\phi\theta^2 - 100\phi + 16\theta^2 + 8} \\ e_2^H &= \frac{2(\alpha - \bar{c})(5\phi - 4)}{125\phi^2 - 40\phi\theta^2 - 100\phi + 16\theta^2 + 8} \\ e_1^L &= \frac{2\theta(\alpha - \bar{c})(5\phi - 4)}{125\phi^2 - 40\phi\theta^2 - 100\phi + 16\theta^2 + 8} \\ e_2^L &= \frac{2\theta(\alpha - \bar{c})(5\phi - 4)}{125\phi^2 - 40\phi\theta^2 - 100\phi + 16\theta^2 + 8} \end{cases}$$

Production Costs:

$$\begin{cases} c_1^H &= -\frac{20\alpha\phi - 16\alpha\theta^2 - 8\alpha - 125\bar{c}\phi^2 + 40\bar{c}\phi\theta^2 + 80\bar{c}\phi}{125\phi^2 - 40\phi\theta^2 - 100\phi + 16\theta^2 + 8} \\ c_2^H &= -\frac{20\alpha\phi\theta^2 + 10\alpha\phi - 16\alpha\theta^2 - 8\alpha - 125\bar{c}\phi^2 + 20\bar{c}\phi\theta^2 + 90\bar{c}\phi}{125\phi^2 - 40\phi\theta^2 - 100\phi + 16\theta^2 + 8} \\ c_1^L &= -\frac{20\alpha\phi\theta^2 + 10\alpha\phi - 16\alpha\theta^2 - 8\alpha - 125\bar{c}\phi^2 + 20\bar{c}\phi\theta^2 + 90\bar{c}\phi}{125\phi^2 - 40\phi\theta^2 - 100\phi + 16\theta^2 + 8} \\ c_2^L &= -\frac{20\alpha\phi\theta^2 + 10\alpha\phi - 16\alpha\theta^2 - 8\alpha - 125\bar{c}\phi^2 + 20\bar{c}\phi\theta^2 + 90\bar{c}\phi}{125\phi^2 - 40\phi\theta^2 - 100\phi + 16\theta^2 + 8} \end{cases}$$

Production Quantities:

$$\begin{cases} q_1^H &= \frac{5\phi(\alpha - \bar{c})(5\phi - 4\theta^2 - 2)}{125\phi^2 - 40\phi\theta^2 - 100\phi + 16\theta^2 + 8} \\ q_2^H &= \frac{5\phi(\alpha - \bar{c})(5\phi - 4)}{125\phi^2 - 40\phi\theta^2 - 100\phi + 16\theta^2 + 8} \\ q_1^L &= \frac{5\phi(\alpha - \bar{c})(5\phi - 4)}{125\phi^2 - 40\phi\theta^2 - 100\phi + 16\theta^2 + 8} \\ q_2^L &= \frac{5\phi(\alpha - \bar{c})(5\phi - 4)}{125\phi^2 - 40\phi\theta^2 - 100\phi + 16\theta^2 + 8} \end{cases}$$

Profits:

$$\begin{cases} \pi_1^H &= \frac{\phi(\alpha - \bar{c})^2 \cdot (25\phi - 16)(5\phi - 4\theta^2 - 2)^2}{(125\phi^2 - 40\phi\theta^2 - 100\phi + 16\theta^2 + 8)^2} \\ \pi_2^H &= \frac{\phi(\alpha - \bar{c})^2 (5\phi - 4)^2 \cdot (25\phi - 4)}{(125\phi^2 - 40\phi\theta^2 - 100\phi + 16\theta^2 + 8)^2} \\ \pi_1^L &= \frac{\phi(\alpha - \bar{c})^2 (5\phi - 4)^2 \cdot (25\phi - 4\theta^2)}{(125\phi^2 - 40\phi\theta^2 - 100\phi + 16\theta^2 + 8)^2} \\ \pi_2^L &= \frac{\phi(\alpha - \bar{c})^2 (5\phi - 4)^2 \cdot (25\phi - 4\theta^2)}{(125\phi^2 - 40\phi\theta^2 - 100\phi + 16\theta^2 + 8)^2} \end{cases} \quad (67)$$

Total Production:

$$\frac{10\phi(\alpha - \bar{c})(10\phi - 2\theta^2 - 7)}{125\phi^2 - 40\phi\theta^2 - 100\phi + 16\theta^2 + 8}$$

Price:

$$\frac{25\alpha\phi^2 - 20\alpha\phi\theta^2 - 30\alpha\phi + 16\alpha\theta^2 + 8\alpha + 100\bar{c}\phi^2 - 20\bar{c}\phi\theta^2 - 70\bar{c}\phi}{125\phi^2 - 40\phi\theta^2 - 100\phi + 16\theta^2 + 8}$$

Firm Surplus:

$$\frac{4\phi(\alpha - \bar{c})^2 \cdot (625\phi^3 - 300\phi^2\theta^2 - 1000\phi^2 + 100\phi\theta^4 + 340\phi\theta^2 + 445\phi - 64\theta^4 - 96\theta^2 - 32)}{(125\phi^2 - 40\phi\theta^2 - 100\phi + 16\theta^2 + 8)^2}$$

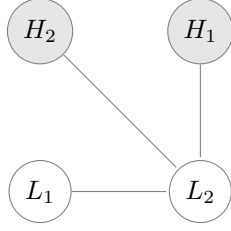
Consumer Surplus:

$$\frac{50\phi^2(\alpha - \bar{c})^2(10\phi - 2\theta^2 - 7)^2}{(125\phi^2 - 40\phi\theta^2 - 100\phi + 16\theta^2 + 8)^2}$$

Social Welfare:

$$\frac{2\phi(\alpha - \bar{c})^2 \cdot (3750\phi^3 - 1600\phi^2\theta^2 - 5500\phi^2 + 300\phi\theta^4 + 1380\phi\theta^2 + 2115\phi - 128\theta^4 - 192\theta^2 - 64)}{(125\phi^2 - 40\phi\theta^2 - 100\phi + 16\theta^2 + 8)^2}$$

## D.2.17 E3G [2H2L]



$$\text{Equations : } \begin{cases} e_1^H \cdot \left( \frac{25\phi}{3} - 3 \right) = \alpha - 2e_2^H - 2e_1^L\theta + e_2^L\theta - \bar{c} \\ e_2^H \cdot \left( \frac{25\phi}{3} - 3 \right) = \alpha - 2e_1^H - 2e_1^L\theta + e_2^L\theta - \bar{c} \\ e_1^L \cdot \left( \frac{25\phi}{3\theta} - 3\theta \right) = \alpha - 2e_1^H - 2e_2^H + e_2^L\theta - \bar{c} \\ e_2^L \cdot \left( \frac{25\phi}{\theta} - \theta \right) = \alpha + 3e_1^H + 3e_2^H + 3e_1^L\theta - \bar{c} \end{cases}$$

Optimal efforts:

$$\begin{cases} e_1^H = \frac{15\phi(\alpha - \bar{c})(5\phi - 3\theta^2)}{625\phi^3 - 250\phi^2\theta^2 - 75\phi^2 - 60\phi\theta^2 + 18\theta^4} \\ e_2^H = \frac{15\phi(\alpha - \bar{c})(5\phi - 3\theta^2)}{625\phi^3 - 250\phi^2\theta^2 - 75\phi^2 - 60\phi\theta^2 + 18\theta^4} \\ e_1^L = \frac{15\phi\theta(\alpha - \bar{c})(5\phi - 3)}{625\phi^3 - 250\phi^2\theta^2 - 75\phi^2 - 60\phi\theta^2 + 18\theta^4} \\ e_2^L = \frac{\theta(\alpha - \bar{c})(25\phi^2 + 15\phi - 18\theta^2)}{625\phi^3 - 250\phi^2\theta^2 - 75\phi^2 - 60\phi\theta^2 + 18\theta^4} \end{cases}$$

Production Costs:

$$\begin{cases} c_1^H = -\frac{25\alpha\phi^2\theta^2 + 75\alpha\phi^2 - 30\alpha\phi\theta^2 - 18\alpha\theta^4 - 625\bar{c}\phi^3 + 225\bar{c}\phi^2\theta^2 + 90\bar{c}\phi\theta^2}{625\phi^3 - 250\phi^2\theta^2 - 75\phi^2 - 60\phi\theta^2 + 18\theta^4} \\ c_2^H = -\frac{25\alpha\phi^2\theta^2 + 75\alpha\phi^2 - 30\alpha\phi\theta^2 - 18\alpha\theta^4 - 625\bar{c}\phi^3 + 225\bar{c}\phi^2\theta^2 + 90\bar{c}\phi\theta^2}{625\phi^3 - 250\phi^2\theta^2 - 75\phi^2 - 60\phi\theta^2 + 18\theta^4} \\ c_1^L = -\frac{100\alpha\phi^2\theta^2 - 30\alpha\phi\theta^2 - 18\alpha\theta^4 - 625\bar{c}\phi^3 + 150\bar{c}\phi^2\theta^2 + 75\bar{c}\phi\theta^2 + 90\bar{c}\phi\theta^2}{625\phi^3 - 250\phi^2\theta^2 - 75\phi^2 - 60\phi\theta^2 + 18\theta^4} \\ c_2^L = -\frac{100\alpha\phi^2\theta^2 + 150\alpha\phi^2 - 120\alpha\phi\theta^2 - 18\alpha\theta^4 - 625\bar{c}\phi^3 + 150\bar{c}\phi^2\theta^2 - 75\bar{c}\phi\theta^2 + 180\bar{c}\phi\theta^2}{625\phi^3 - 250\phi^2\theta^2 - 75\phi^2 - 60\phi\theta^2 + 18\theta^4} \end{cases}$$

Production Quantities:

$$\begin{cases} q_1^H = \frac{25\phi^2(\alpha - \bar{c})(5\phi - 3\theta^2)}{625\phi^3 - 250\phi^2\theta^2 - 75\phi^2 - 60\phi\theta^2 + 18\theta^4} \\ q_2^H = \frac{25\phi^2(\alpha - \bar{c})(5\phi - 3\theta^2)}{625\phi^3 - 250\phi^2\theta^2 - 75\phi^2 - 60\phi\theta^2 + 18\theta^4} \\ q_1^L = \frac{25\phi^2(\alpha - \bar{c})(5\phi - 3)}{625\phi^3 - 250\phi^2\theta^2 - 75\phi^2 - 60\phi\theta^2 + 18\theta^4} \\ q_2^L = \frac{5\phi(\alpha - \bar{c})(25\phi^2 + 15\phi - 18\theta^2)}{625\phi^3 - 250\phi^2\theta^2 - 75\phi^2 - 60\phi\theta^2 + 18\theta^4} \end{cases}$$

Profits:

$$\begin{cases} \pi_1^H = \frac{25\phi^3(\alpha - \bar{c})^2(5\phi - 3\theta^2)^2 \cdot (25\phi - 9)}{(625\phi^3 - 250\phi^2\theta^2 - 75\phi^2 - 60\phi\theta^2 + 18\theta^4)^2} \\ \pi_2^H = \frac{25\phi^3(\alpha - \bar{c})^2(5\phi - 3\theta^2)^2 \cdot (25\phi - 9)}{(625\phi^3 - 250\phi^2\theta^2 - 75\phi^2 - 60\phi\theta^2 + 18\theta^4)^2} \\ \pi_1^L = \frac{25\phi^3(\alpha - \bar{c})^2(5\phi - 3)^2 \cdot (25\phi - 9\theta^2)}{(625\phi^3 - 250\phi^2\theta^2 - 75\phi^2 - 60\phi\theta^2 + 18\theta^4)^2} \\ \pi_2^L = \frac{\phi(\alpha - \bar{c})^2 \cdot (25\phi - \theta^2)(25\phi^2 + 15\phi - 18\theta^2)^2}{(625\phi^3 - 250\phi^2\theta^2 - 75\phi^2 - 60\phi\theta^2 + 18\theta^4)^2} \end{cases} \quad (68)$$

Total Production:

$$\frac{10\phi(\alpha - \bar{c})(50\phi^2 - 15\phi\theta^2 - 9\theta^2)}{625\phi^3 - 250\phi^2\theta^2 - 75\phi^2 - 60\phi\theta^2 + 18\theta^4}$$

Price:

$$\frac{125\alpha\phi^3 - 100\alpha\phi^2\theta^2 - 75\alpha\phi^2 + 30\alpha\phi\theta^2 + 18\alpha\theta^4 + 500\bar{c}\phi^3 - 150\bar{c}\phi^2\theta^2 - 90\bar{c}\phi\theta^2}{625\phi^3 - 250\phi^2\theta^2 - 75\phi^2 - 60\phi\theta^2 + 18\theta^4}$$

Firm Surplus:

$$\frac{2\phi(\alpha - \bar{c})^2 \cdot (31250\phi^5 - 21875\phi^4\theta^2 - 5625\phi^4 + 5625\phi^3\theta^4 - 1500\phi^3\theta^2 + 5625\phi^3 - 1575\phi^2\theta^4 - 7875\phi^2\theta^2 + 4320\phi\theta^4 - 162\theta^6)}{(625\phi^3 - 250\phi^2\theta^2 - 75\phi^2 - 60\phi\theta^2 + 18\theta^4)^2}$$

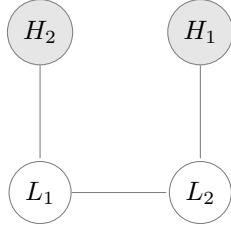
Consumer Surplus:

$$\frac{50\phi^2(\alpha - \bar{c})^2(50\phi^2 - 15\phi\theta^2 - 9\theta^2)^2}{(625\phi^3 - 250\phi^2\theta^2 - 75\phi^2 - 60\phi\theta^2 + 18\theta^4)^2}$$

Social Welfare:

$$\frac{2\phi(\alpha - \bar{c})^2 \cdot (93750\phi^5 - 59375\phi^4\theta^2 - 5625\phi^4 + 11250\phi^3\theta^4 - 24000\phi^3\theta^2 + 5625\phi^3 + 5175\phi^2\theta^4 - 7875\phi^2\theta^2 + 6345\phi\theta^4 - 162\theta^6)}{(625\phi^3 - 250\phi^2\theta^2 - 75\phi^2 - 60\phi\theta^2 + 18\theta^4)^2}$$

#### D.2.18 E3H [2H2L]



$$\text{Equations : } \begin{cases} e_1^H \cdot \left( \frac{25\phi}{3} - 3 \right) = \alpha - 2e_2^H - 3e_1^L\theta + 2e_2^L\theta - \bar{c} \\ e_2^H \cdot \left( \frac{25\phi}{3} - 3 \right) = \alpha - 2e_1^H + 2e_1^L\theta - 3e_2^L\theta - \bar{c} \\ e_1^L \cdot \left( \frac{25\phi}{2\theta} - 2\theta \right) = \alpha - 2e_1^H + 3e_2^H + 2e_2^L\theta - \bar{c} \\ e_2^L \cdot \left( \frac{25\phi}{2\theta} - 2\theta \right) = \alpha + 3e_1^H - 2e_2^H + 2e_1^L\theta - \bar{c} \end{cases}$$

Optimal efforts:

$$\begin{cases} e_1^H = \frac{3(\alpha - \bar{c})(5\phi - 2\theta^2)}{125\phi^2 - 40\phi\theta^2 - 15\phi + 6\theta^2} \\ e_2^H = \frac{3(\alpha - \bar{c})(5\phi - 2\theta^2)}{125\phi^2 - 40\phi\theta^2 - 15\phi + 6\theta^2} \\ e_1^L = \frac{10\phi\theta(\alpha - \bar{c})}{125\phi^2 - 40\phi\theta^2 - 15\phi + 6\theta^2} \\ e_2^L = \frac{10\phi\theta(\alpha - \bar{c})}{125\phi^2 - 40\phi\theta^2 - 15\phi + 6\theta^2} \end{cases}$$

Production Costs:

$$\begin{cases} c_1^H = -\frac{10\alpha\phi\theta^2 + 15\alpha\phi - 6\alpha\theta^2 - 125\bar{c}\phi^2 + 30\bar{c}\phi\theta^2}{125\phi^2 - 40\phi\theta^2 - 15\phi + 6\theta^2} \\ c_2^H = -\frac{10\alpha\phi\theta^2 + 15\alpha\phi - 6\alpha\theta^2 - 125\bar{c}\phi^2 + 30\bar{c}\phi\theta^2}{125\phi^2 - 40\phi\theta^2 - 15\phi + 6\theta^2} \\ c_1^L = -\frac{20\alpha\phi\theta^2 + 15\alpha\phi - 6\alpha\theta^2 - 125\bar{c}\phi^2 + 20\bar{c}\phi\theta^2}{125\phi^2 - 40\phi\theta^2 - 15\phi + 6\theta^2} \\ c_2^L = -\frac{20\alpha\phi\theta^2 + 15\alpha\phi - 6\alpha\theta^2 - 125\bar{c}\phi^2 + 20\bar{c}\phi\theta^2}{125\phi^2 - 40\phi\theta^2 - 15\phi + 6\theta^2} \end{cases}$$

Production Quantities:

$$\begin{cases} q_1^H = \frac{5\phi(\alpha - \bar{c})(5\phi - 2\theta^2)}{125\phi^2 - 40\phi\theta^2 - 15\phi + 6\theta^2} \\ q_2^H = \frac{5\phi(\alpha - \bar{c})(5\phi - 2\theta^2)}{125\phi^2 - 40\phi\theta^2 - 15\phi + 6\theta^2} \\ q_1^L = \frac{25\phi^2(\alpha - \bar{c})}{125\phi^2 - 40\phi\theta^2 - 15\phi + 6\theta^2} \\ q_2^L = \frac{25\phi^2(\alpha - \bar{c})}{125\phi^2 - 40\phi\theta^2 - 15\phi + 6\theta^2} \end{cases}$$

Profits:

$$\begin{cases} \pi_1^H = \frac{\phi(\alpha - \bar{c})^2(5\phi - 2\theta^2)^2 \cdot (25\phi - 9)}{(125\phi^2 - 40\phi\theta^2 - 15\phi + 6\theta^2)^2} \\ \pi_2^H = \frac{\phi(\alpha - \bar{c})^2(5\phi - 2\theta^2)^2 \cdot (25\phi - 9)}{(125\phi^2 - 40\phi\theta^2 - 15\phi + 6\theta^2)^2} \\ \pi_1^L = \frac{25\phi^3(\alpha - \bar{c})^2 \cdot (25\phi - 4\theta^2)}{(125\phi^2 - 40\phi\theta^2 - 15\phi + 6\theta^2)^2} \\ \pi_2^L = \frac{25\phi^3(\alpha - \bar{c})^2 \cdot (25\phi - 4\theta^2)}{(125\phi^2 - 40\phi\theta^2 - 15\phi + 6\theta^2)^2} \end{cases} \quad (69)$$

Total Production:

$$\frac{20\phi(\alpha - \bar{c})(5\phi - \theta^2)}{125\phi^2 - 40\phi\theta^2 - 15\phi + 6\theta^2}$$

Price:

$$\frac{25\alpha\phi^2 - 20\alpha\phi\theta^2 - 15\alpha\phi + 6\alpha\theta^2 + 100\bar{c}\phi^2 - 20\bar{c}\phi\theta^2}{125\phi^2 - 40\phi\theta^2 - 15\phi + 6\theta^2}$$

Firm Surplus:

$$\frac{2\phi(\alpha - \bar{c})^2 \cdot (1250\phi^3 - 600\phi^2\theta^2 - 225\phi^2 + 100\phi\theta^4 + 180\phi\theta^2 - 36\theta^4)}{(125\phi^2 - 40\phi\theta^2 - 15\phi + 6\theta^2)^2}$$

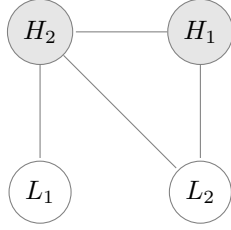
Consumer Surplus:

$$\frac{200\phi^2 (\alpha - \bar{c})^2 (5\phi - \theta^2)^2}{(125\phi^2 - 40\phi\theta^2 - 15\phi + 6\theta^2)^2}$$

Social Welfare:

$$\frac{2\phi (\alpha - \bar{c})^2 \cdot (3750\phi^3 - 1600\phi^2\theta^2 - 225\phi^2 + 200\phi\theta^4 + 180\phi\theta^2 - 36\theta^4)}{(125\phi^2 - 40\phi\theta^2 - 15\phi + 6\theta^2)^2}$$

#### D.2.19 E4A [2H2L]



$$\text{Equations : } \begin{cases} e_1^H \cdot \left(\frac{25\phi}{2} - 2\right) = \alpha + e_2^H - 2e_1^L\theta + 2e_2^L\theta - \bar{c} \\ e_2^H \cdot (25\phi - 1) = \alpha + 2e_1^H + 3e_1^L\theta + 2e_2^L\theta - \bar{c} \\ e_1^L \cdot \left(\frac{25\phi}{3\theta} - 3\theta\right) = \alpha - 3e_1^H + e_2^H - 3e_2^L\theta - \bar{c} \\ e_2^L \cdot \left(\frac{25\phi}{2\theta} - 2\theta\right) = \alpha + 2e_1^H + e_2^H - 2e_1^L\theta - \bar{c} \end{cases}$$

Optimal efforts:

$$\begin{cases} e_1^H = \frac{10\phi(\alpha - \bar{c})(5\phi - 3\theta^2)}{625\phi^3 - 325\phi^2\theta^2 - 125\phi^2 + 6\theta^4 + 6\theta^2} \\ e_2^H = \frac{(\alpha - \bar{c})(25\phi^2 - 6\theta^4 - 6\theta^2)}{625\phi^3 - 325\phi^2\theta^2 - 125\phi^2 + 6\theta^4 + 6\theta^2} \\ e_1^L = \frac{15\phi\theta(\alpha - \bar{c})(5\phi - 2\theta^2 - 2)}{625\phi^3 - 325\phi^2\theta^2 - 125\phi^2 + 6\theta^4 + 6\theta^2} \\ e_2^L = \frac{10\phi\theta(\alpha - \bar{c})(5\phi - 3\theta^2)}{625\phi^3 - 325\phi^2\theta^2 - 125\phi^2 + 6\theta^4 + 6\theta^2} \end{cases}$$

Production Costs:

$$\begin{cases} c_1^H = -\frac{50\alpha\phi^2\theta^2 + 75\alpha\phi^2 - 30\alpha\phi\theta^4 - 30\alpha\phi\theta^2 - 6\alpha\theta^4 - 6\alpha\theta^2 - 625\bar{c}\phi^3 + 275\bar{c}\phi^2\theta^2 + 50\bar{c}\phi^2 + 30\bar{c}\phi\theta^4 + 30\bar{c}\phi\theta^2}{625\phi^3 - 325\phi^2\theta^2 - 125\phi^2 + 6\theta^4 + 6\theta^2} \\ c_2^H = -\frac{125\alpha\phi^2\theta^2 + 75\alpha\phi^2 - 60\alpha\phi\theta^4 - 60\alpha\phi\theta^2 - 6\alpha\theta^4 - 6\alpha\theta^2 - 625\bar{c}\phi^3 + 200\bar{c}\phi^2\theta^2 + 50\bar{c}\phi^2 + 60\bar{c}\phi\theta^4 + 60\bar{c}\phi\theta^2}{625\phi^3 - 325\phi^2\theta^2 - 125\phi^2 + 6\theta^4 + 6\theta^2} \\ c_1^L = -\frac{75\alpha\phi^2\theta^2 + 25\alpha\phi^2 - 30\alpha\phi\theta^4 - 30\alpha\phi\theta^2 - 6\alpha\theta^4 - 6\alpha\theta^2 - 625\bar{c}\phi^3 + 250\bar{c}\phi^2\theta^2 + 100\bar{c}\phi^2 + 30\bar{c}\phi\theta^4 + 30\bar{c}\phi\theta^2}{625\phi^3 - 325\phi^2\theta^2 - 125\phi^2 + 6\theta^4 + 6\theta^2} \\ c_2^L = -\frac{50\alpha\phi^2\theta^2 + 75\alpha\phi^2 - 30\alpha\phi\theta^4 - 30\alpha\phi\theta^2 - 6\alpha\theta^4 - 6\alpha\theta^2 - 625\bar{c}\phi^3 + 275\bar{c}\phi^2\theta^2 + 50\bar{c}\phi^2 + 30\bar{c}\phi\theta^4 + 30\bar{c}\phi\theta^2}{625\phi^3 - 325\phi^2\theta^2 - 125\phi^2 + 6\theta^4 + 6\theta^2} \end{cases}$$

Production Quantities:

$$\begin{cases} q_1^H = \frac{25\phi^2(\alpha - \bar{c})(5\phi - 3\theta^2)}{625\phi^3 - 325\phi^2\theta^2 - 125\phi^2 + 6\theta^4 + 6\theta^2} \\ q_2^H = \frac{5\phi(\alpha - \bar{c})(25\phi^2 - 6\theta^4 - 6\theta^2)}{625\phi^3 - 325\phi^2\theta^2 - 125\phi^2 + 6\theta^4 + 6\theta^2} \\ q_1^L = \frac{25\phi^2(\alpha - \bar{c})(5\phi - 2\theta^2 - 2)}{625\phi^3 - 325\phi^2\theta^2 - 125\phi^2 + 6\theta^4 + 6\theta^2} \\ q_2^L = \frac{25\phi^2(\alpha - \bar{c})(5\phi - 3\theta^2)}{625\phi^3 - 325\phi^2\theta^2 - 125\phi^2 + 6\theta^4 + 6\theta^2} \end{cases}$$

Profits:

$$\begin{cases} \pi_1^H = \frac{25\phi^3(\alpha - \bar{c})^2(5\phi - 3\theta^2)^2 \cdot (25\phi - 4)}{(625\phi^3 - 325\phi^2\theta^2 - 125\phi^2 + 6\theta^4 + 6\theta^2)^2} \\ \pi_2^H = \frac{\phi(\alpha - \bar{c})^2 \cdot (25\phi - 1)(25\phi^2 - 6\theta^4 - 6\theta^2)^2}{(625\phi^3 - 325\phi^2\theta^2 - 125\phi^2 + 6\theta^4 + 6\theta^2)^2} \\ \pi_1^L = \frac{25\phi^3(\alpha - \bar{c})^2 \cdot (25\phi - 9\theta^2)(5\phi - 2\theta^2 - 2)^2}{(625\phi^3 - 325\phi^2\theta^2 - 125\phi^2 + 6\theta^4 + 6\theta^2)^2} \\ \pi_2^L = \frac{25\phi^3(\alpha - \bar{c})^2(5\phi - 3\theta^2)^2 \cdot (25\phi - 4\theta^2)}{(625\phi^3 - 325\phi^2\theta^2 - 125\phi^2 + 6\theta^4 + 6\theta^2)^2} \end{cases} \quad (70)$$

Total Production:

$$\frac{10\phi(\alpha - \bar{c})(50\phi^2 - 20\phi\theta^2 - 5\phi - 3\theta^4 - 3\theta^2)}{625\phi^3 - 325\phi^2\theta^2 - 125\phi^2 + 6\theta^4 + 6\theta^2}$$

Price:

$$\frac{125\alpha\phi^3 - 125\alpha\phi^2\theta^2 - 75\alpha\phi^2 + 30\alpha\phi\theta^4 + 30\alpha\phi\theta^2 + 6\alpha\theta^4 + 6\alpha\theta^2 + 500\bar{c}\phi^3 - 200\bar{c}\phi^2\theta^2 - 50\bar{c}\phi^2 - 30\bar{c}\phi\theta^4 - 30\bar{c}\phi\theta^2}{625\phi^3 - 325\phi^2\theta^2 - 125\phi^2 + 6\theta^4 + 6\theta^2}$$

Firm Surplus:

$$\frac{\phi(\alpha - \bar{c})^2 \cdot (62500\phi^5 - 58125\phi^4\theta^2 - 15625\phi^4 + 13750\phi^3\theta^4 + 5000\phi^3\theta^2 + 2500\phi^3 - 1800\phi^2\theta^6 - 2400\phi^2\theta^4 - 600\phi^2\theta^2 + 900\phi\theta^8 + 1800\phi\theta^6 + 900\phi\theta^4 - 36\theta^8 - 72\theta^6 - 36\theta^4)}{(625\phi^3 - 325\phi^2\theta^2 - 125\phi^2 + 6\theta^4 + 6\theta^2)^2}$$

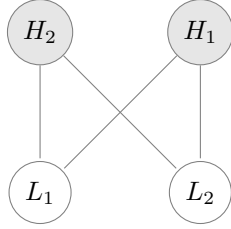
Consumer Surplus:

$$\frac{50\phi^2(\alpha - \bar{c})^2(50\phi^2 - 20\phi\theta^2 - 5\phi - 3\theta^4 - 3\theta^2)^2}{(625\phi^3 - 325\phi^2\theta^2 - 125\phi^2 + 6\theta^4 + 6\theta^2)^2}$$

Social Welfare:

$$\frac{\phi(\alpha - \bar{c})^2 \cdot (187500\phi^5 - 158125\phi^4\theta^2 - 40625\phi^4 + 18750\phi^3\theta^4 + 3750\phi^3 + 4200\phi^2\theta^6 + 5100\phi^2\theta^4 + 900\phi^2\theta^2 + 1350\phi\theta^8 + 2700\phi\theta^6 + 1350\phi\theta^4 - 36\theta^8 - 72\theta^6 - 36\theta^4)}{(625\phi^3 - 325\phi^2\theta^2 - 125\phi^2 + 6\theta^4 + 6\theta^2)^2}$$

## D.2.20 E4B [2H2L]



$$\text{Equations : } \begin{cases} e_1^H \cdot \left( \frac{25\phi}{2} - 2 \right) = \alpha - 3e_2^H + 2e_1^L\theta + 2e_2^L\theta - \bar{c} \\ e_2^H \cdot \left( \frac{25\phi}{2} - 2 \right) = \alpha - 3e_1^H + 2e_1^L\theta + 2e_2^L\theta - \bar{c} \\ e_1^L \cdot \left( \frac{25\phi}{2\theta} - 2\theta \right) = \alpha + 2e_1^H + 2e_2^H - 3e_2^L\theta - \bar{c} \\ e_2^L \cdot \left( \frac{25\phi}{2\theta} - 2\theta \right) = \alpha + 2e_1^H + 2e_2^H - 3e_1^L\theta - \bar{c} \end{cases}$$

Optimal efforts:

$$\begin{cases} e_1^H = \frac{2(\alpha - \bar{c})(5\phi + 2\theta^2)}{125\phi^2 + 10\phi\theta^2 + 10\phi - 12\theta^2} \\ e_2^H = \frac{2(\alpha - \bar{c})(5\phi + 2\theta^2)}{125\phi^2 + 10\phi\theta^2 + 10\phi - 12\theta^2} \\ e_1^L = \frac{2\theta(\alpha - \bar{c})(5\phi + 2)}{125\phi^2 + 10\phi\theta^2 + 10\phi - 12\theta^2} \\ e_2^L = \frac{2\theta(\alpha - \bar{c})(5\phi + 2)}{125\phi^2 + 10\phi\theta^2 + 10\phi - 12\theta^2} \end{cases}$$

Production Costs:

$$\begin{cases} c_1^H = -\frac{20\alpha\phi\theta^2 + 10\alpha\phi + 12\alpha\theta^2 - 125\bar{c}\phi^2 - 30\bar{c}\phi\theta^2 - 20\bar{c}\phi}{125\phi^2 + 10\phi\theta^2 + 10\phi - 12\theta^2} \\ c_2^H = -\frac{20\alpha\phi\theta^2 + 10\alpha\phi + 12\alpha\theta^2 - 125\bar{c}\phi^2 - 30\bar{c}\phi\theta^2 - 20\bar{c}\phi}{125\phi^2 + 10\phi\theta^2 + 10\phi - 12\theta^2} \\ c_1^L = -\frac{10\alpha\phi\theta^2 + 20\alpha\phi + 12\alpha\theta^2 - 125\bar{c}\phi^2 - 20\bar{c}\phi\theta^2 - 30\bar{c}\phi}{125\phi^2 + 10\phi\theta^2 + 10\phi - 12\theta^2} \\ c_2^L = -\frac{10\alpha\phi\theta^2 + 20\alpha\phi + 12\alpha\theta^2 - 125\bar{c}\phi^2 - 20\bar{c}\phi\theta^2 - 30\bar{c}\phi}{125\phi^2 + 10\phi\theta^2 + 10\phi - 12\theta^2} \end{cases}$$

Production Quantities:

$$\begin{cases} q_1^H = \frac{5\phi(\alpha - \bar{c})(5\phi + 2\theta^2)}{125\phi^2 + 10\phi\theta^2 + 10\phi - 12\theta^2} \\ q_2^H = \frac{5\phi(\alpha - \bar{c})(5\phi + 2\theta^2)}{125\phi^2 + 10\phi\theta^2 + 10\phi - 12\theta^2} \\ q_1^L = \frac{5\phi(\alpha - \bar{c})(5\phi + 2)}{125\phi^2 + 10\phi\theta^2 + 10\phi - 12\theta^2} \\ q_2^L = \frac{5\phi(\alpha - \bar{c})(5\phi + 2)}{125\phi^2 + 10\phi\theta^2 + 10\phi - 12\theta^2} \end{cases}$$

Profits:

$$\begin{cases} \pi_1^H = \frac{\phi(\alpha - \bar{c})^2(5\phi + 2\theta^2)^2 \cdot (25\phi - 4)}{(125\phi^2 + 10\phi\theta^2 + 10\phi - 12\theta^2)^2} \\ \pi_2^H = \frac{\phi(\alpha - \bar{c})^2(5\phi + 2\theta^2)^2 \cdot (25\phi - 4)}{(125\phi^2 + 10\phi\theta^2 + 10\phi - 12\theta^2)^2} \\ \pi_1^L = \frac{\phi(\alpha - \bar{c})^2(5\phi + 2)^2 \cdot (25\phi - 4\theta^2)}{(125\phi^2 + 10\phi\theta^2 + 10\phi - 12\theta^2)^2} \\ \pi_2^L = \frac{\phi(\alpha - \bar{c})^2(5\phi + 2)^2 \cdot (25\phi - 4\theta^2)}{(125\phi^2 + 10\phi\theta^2 + 10\phi - 12\theta^2)^2} \end{cases} \quad (71)$$

Total Production:

$$\frac{20\phi(\alpha - \bar{c})(5\phi + \theta^2 + 1)}{125\phi^2 + 10\phi\theta^2 + 10\phi - 12\theta^2}$$

Price:

$$\frac{25\alpha\phi^2 - 10\alpha\phi\theta^2 - 10\alpha\phi - 12\alpha\theta^2 + 100\bar{c}\phi^2 + 20\bar{c}\phi\theta^2 + 20\bar{c}\phi}{125\phi^2 + 10\phi\theta^2 + 10\phi - 12\theta^2}$$

Firm Surplus:

$$\frac{4\phi(\alpha - \bar{c})^2 \cdot (625\phi^3 + 200\phi^2\theta^2 + 200\phi^2 + 50\phi\theta^4 - 80\phi\theta^2 + 50\phi - 8\theta^4 - 8\theta^2)}{(125\phi^2 + 10\phi\theta^2 + 10\phi - 12\theta^2)^2}$$

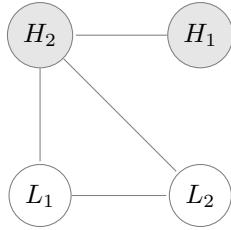
Consumer Surplus:

$$\frac{200\phi^2(\alpha - \bar{c})^2(5\phi + \theta^2 + 1)^2}{(125\phi^2 + 10\phi\theta^2 + 10\phi - 12\theta^2)^2}$$

Social Welfare:

$$\frac{4\phi(\alpha - \bar{c})^2 \cdot (1875\phi^3 + 700\phi^2\theta^2 + 700\phi^2 + 100\phi\theta^4 + 20\phi\theta^2 + 100\phi - 8\theta^4 - 8\theta^2)}{(125\phi^2 + 10\phi\theta^2 + 10\phi - 12\theta^2)^2}$$

#### D.2.21 E4C [2H2L]



$$\text{Equations : } \begin{cases} e_1^H \cdot \left(\frac{25\phi}{3} - 3\right) = \alpha + e_2^H - 3e_1^L\theta - 3e_2^L\theta - \bar{c} \\ e_2^H \cdot (25\phi - 1) = \alpha + 3e_1^H + 2e_1^L\theta + 2e_2^L\theta - \bar{c} \\ e_1^L \cdot \left(\frac{25\phi}{2\theta} - 2\theta\right) = \alpha - 2e_1^H + e_2^H + 2e_2^L\theta - \bar{c} \\ e_2^L \cdot \left(\frac{25\phi}{2\theta} - 2\theta\right) = \alpha - 2e_1^H + e_2^H + 2e_1^L\theta - \bar{c} \end{cases}$$

Optimal efforts:

$$\begin{cases} e_1^H = \frac{15\phi(\alpha - \bar{c})(5\phi - 4\theta^2)}{625\phi^3 - 200\phi^2\theta^2 - 250\phi^2 + 12\theta^2} \\ e_2^H = \frac{(\alpha - \bar{c})(25\phi^2 - 12\theta^2)}{625\phi^3 - 200\phi^2\theta^2 - 250\phi^2 + 12\theta^2} \\ e_1^L = \frac{10\phi\theta(\alpha - \bar{c})(5\phi - 3)}{625\phi^3 - 200\phi^2\theta^2 - 250\phi^2 + 12\theta^2} \\ e_2^L = \frac{10\phi\theta(\alpha - \bar{c})(5\phi - 3)}{625\phi^3 - 200\phi^2\theta^2 - 250\phi^2 + 12\theta^2} \end{cases}$$

Production Costs:

$$\begin{cases} c_1^H = -\frac{100\alpha\phi^2 - 60\alpha\phi\theta^2 - 12\alpha\theta^2 - 625\bar{c}\phi^3 + 200\bar{c}\phi^2\theta^2 + 150\bar{c}\phi^2 + 60\bar{c}\phi\theta^2}{625\phi^3 - 200\phi^2\theta^2 - 250\phi^2 + 12\theta^2} \\ c_2^H = -\frac{100\alpha\phi^2\theta^2 + 100\alpha\phi^2 - 120\alpha\phi\theta^2 - 12\alpha\theta^2 - 625\bar{c}\phi^3 + 100\bar{c}\phi^2\theta^2 + 150\bar{c}\phi^2 + 120\bar{c}\phi\theta^2}{625\phi^3 - 200\phi^2\theta^2 - 250\phi^2 + 12\theta^2} \\ c_1^L = -\frac{100\alpha\phi^2\theta^2 + 25\alpha\phi^2 - 60\alpha\phi\theta^2 - 12\alpha\theta^2 - 625\bar{c}\phi^3 + 100\bar{c}\phi^2\theta^2 + 225\bar{c}\phi^2 + 60\bar{c}\phi\theta^2}{625\phi^3 - 200\phi^2\theta^2 - 250\phi^2 + 12\theta^2} \\ c_2^L = -\frac{100\alpha\phi^2\theta^2 + 25\alpha\phi^2 - 60\alpha\phi\theta^2 - 12\alpha\theta^2 - 625\bar{c}\phi^3 + 100\bar{c}\phi^2\theta^2 + 225\bar{c}\phi^2 + 60\bar{c}\phi\theta^2}{625\phi^3 - 200\phi^2\theta^2 - 250\phi^2 + 12\theta^2} \end{cases}$$

Production Quantities:

$$\begin{cases} q_1^H = \frac{25\phi^2(\alpha - \bar{c})(5\phi - 4\theta^2)}{625\phi^3 - 200\phi^2\theta^2 - 250\phi^2 + 12\theta^2} \\ q_2^H = \frac{5\phi(\alpha - \bar{c})(25\phi^2 - 12\theta^2)}{625\phi^3 - 200\phi^2\theta^2 - 250\phi^2 + 12\theta^2} \\ q_1^L = \frac{25\phi^2(\alpha - \bar{c})(5\phi - 3)}{625\phi^3 - 200\phi^2\theta^2 - 250\phi^2 + 12\theta^2} \\ q_2^L = \frac{25\phi^2(\alpha - \bar{c})(5\phi - 3)}{625\phi^3 - 200\phi^2\theta^2 - 250\phi^2 + 12\theta^2} \end{cases}$$

Profits:

$$\begin{cases} \pi_1^H = \frac{25\phi^3(\alpha - \bar{c})^2(5\phi - 4\theta^2)^2 \cdot (25\phi - 9)}{(625\phi^3 - 200\phi^2\theta^2 - 250\phi^2 + 12\theta^2)^2} \\ \pi_2^H = \frac{\phi(\alpha - \bar{c})^2 \cdot (25\phi - 1)(25\phi^2 - 12\theta^2)^2}{(625\phi^3 - 200\phi^2\theta^2 - 250\phi^2 + 12\theta^2)^2} \\ \pi_1^L = \frac{25\phi^3(\alpha - \bar{c})^2(5\phi - 3)^2 \cdot (25\phi - 4\theta^2)}{(625\phi^3 - 200\phi^2\theta^2 - 250\phi^2 + 12\theta^2)^2} \\ \pi_2^L = \frac{25\phi^3(\alpha - \bar{c})^2(5\phi - 3)^2 \cdot (25\phi - 4\theta^2)}{(625\phi^3 - 200\phi^2\theta^2 - 250\phi^2 + 12\theta^2)^2} \end{cases} \quad (72)$$

Total Production:

$$\frac{10\phi(\alpha - \bar{c})(50\phi^2 - 10\phi\theta^2 - 15\phi - 6\theta^2)}{625\phi^3 - 200\phi^2\theta^2 - 250\phi^2 + 12\theta^2}$$

Price:

$$\frac{125\alpha\phi^3 - 100\alpha\phi^2\theta^2 - 100\alpha\phi^2 + 60\alpha\phi\theta^2 + 12\alpha\theta^2 + 500\bar{c}\phi^3 - 100\bar{c}\phi^2\theta^2 - 150\bar{c}\phi^2 - 60\bar{c}\phi\theta^2}{625\phi^3 - 200\phi^2\theta^2 - 250\phi^2 + 12\theta^2}$$

Firm Surplus:

$$\frac{2\phi(\alpha - \bar{c})^2 \cdot (31250\phi^5 - 15000\phi^4\theta^2 - 21875\phi^4 + 5000\phi^3\theta^4 + 5625\phi^3 - 1800\phi^2\theta^4 - 600\phi^2\theta^2 + 1800\phi\theta^4 - 72\theta^4)}{(625\phi^3 - 200\phi^2\theta^2 - 250\phi^2 + 12\theta^2)^2}$$

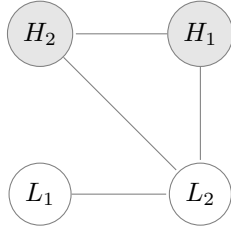
Consumer Surplus:

$$\frac{50\phi^2(\alpha - \bar{c})^2(50\phi^2 - 10\phi\theta^2 - 15\phi - 6\theta^2)^2}{(625\phi^3 - 200\phi^2\theta^2 - 250\phi^2 + 12\theta^2)^2}$$

Social Welfare:

$$\frac{2\phi(\alpha - \bar{c})^2 \cdot (93750\phi^5 - 40000\phi^4\theta^2 - 59375\phi^4 + 7500\phi^3\theta^4 - 7500\phi^3\theta^2 + 11250\phi^3 + 1200\phi^2\theta^4 + 3900\phi^2\theta^2 + 2700\phi\theta^4 - 72\theta^4)}{(625\phi^3 - 200\phi^2\theta^2 - 250\phi^2 + 12\theta^2)^2}$$

#### D.2.22 E4D [2H2L]



$$\text{Equations : } \begin{cases} e_1^H \cdot \left(\frac{25\phi}{2} - 2\right) = \alpha + 2e_2^H - 2e_1^L\theta + e_2^L\theta - \bar{c} \\ e_2^H \cdot \left(\frac{25\phi}{2} - 2\right) = \alpha + 2e_1^H - 2e_1^L\theta + e_2^L\theta - \bar{c} \\ e_1^L \cdot \left(\frac{25\phi}{3\theta} - 3\theta\right) = \alpha - 3e_1^H - 3e_2^H + e_2^L\theta - \bar{c} \\ e_2^L \cdot \left(\frac{25\phi}{\theta} - \theta\right) = \alpha + 2e_1^H + 2e_2^H + 3e_1^L\theta - \bar{c} \end{cases}$$

Optimal efforts:

$$\begin{cases} e_1^H = \frac{10\phi(\alpha - \bar{c})(5\phi - 3\theta^2)}{625\phi^3 - 250\phi^2\theta^2 - 200\phi^2 + 12\theta^4} \\ e_2^H = \frac{10\phi(\alpha - \bar{c})(5\phi - 3\theta^2)}{625\phi^3 - 250\phi^2\theta^2 - 200\phi^2 + 12\theta^4} \\ e_1^L = \frac{15\phi\theta(\alpha - \bar{c})(5\phi - 4)}{625\phi^3 - 250\phi^2\theta^2 - 200\phi^2 + 12\theta^4} \\ e_2^L = \frac{\theta(\alpha - \bar{c})(25\phi^2 - 12\theta^2)}{625\phi^3 - 250\phi^2\theta^2 - 200\phi^2 + 12\theta^4} \end{cases}$$

Production Costs:

$$\begin{cases} c_1^H = -\frac{25\alpha\phi^2\theta^2 + 100\alpha\phi^2 - 60\alpha\phi\theta^2 - 12\alpha\theta^4 - 625\bar{c}\phi^3 + 225\bar{c}\phi^2\theta^2 + 100\bar{c}\phi^2 + 60\bar{c}\phi\theta^2}{625\phi^3 - 250\phi^2\theta^2 - 200\phi^2 + 12\theta^4} \\ c_2^H = -\frac{25\alpha\phi^2\theta^2 + 100\alpha\phi^2 - 60\alpha\phi\theta^2 - 12\alpha\theta^4 - 625\bar{c}\phi^3 + 225\bar{c}\phi^2\theta^2 + 100\bar{c}\phi^2 + 60\bar{c}\phi\theta^2}{625\phi^3 - 250\phi^2\theta^2 - 200\phi^2 + 12\theta^4} \\ c_1^L = -\frac{100\alpha\phi^2\theta^2 - 60\alpha\phi\theta^2 - 12\alpha\theta^4 - 625\bar{c}\phi^3 + 150\bar{c}\phi^2\theta^2 + 200\bar{c}\phi^2 + 60\bar{c}\phi\theta^2}{625\phi^3 - 250\phi^2\theta^2 - 200\phi^2 + 12\theta^4} \\ c_2^L = -\frac{100\alpha\phi^2\theta^2 + 100\alpha\phi^2 - 120\alpha\phi\theta^2 - 12\alpha\theta^4 - 625\bar{c}\phi^3 + 150\bar{c}\phi^2\theta^2 + 100\bar{c}\phi^2 + 120\bar{c}\phi\theta^2}{625\phi^3 - 250\phi^2\theta^2 - 200\phi^2 + 12\theta^4} \end{cases}$$

Production Quantities:

$$\begin{cases} q_1^H = \frac{25\phi^2(\alpha - \bar{c})(5\phi - 3\theta^2)}{625\phi^3 - 250\phi^2\theta^2 - 200\phi^2 + 12\theta^4} \\ q_2^H = \frac{25\phi^2(\alpha - \bar{c})(5\phi - 3\theta^2)}{625\phi^3 - 250\phi^2\theta^2 - 200\phi^2 + 12\theta^4} \\ q_1^L = \frac{25\phi^2(\alpha - \bar{c})(5\phi - 4)}{625\phi^3 - 250\phi^2\theta^2 - 200\phi^2 + 12\theta^4} \\ q_2^L = \frac{5\phi(\alpha - \bar{c})(25\phi^2 - 12\theta^2)}{625\phi^3 - 250\phi^2\theta^2 - 200\phi^2 + 12\theta^4} \end{cases}$$

Profits:



$$\begin{cases} \pi_1^H &= \frac{25\phi^3(\alpha-\bar{c})^2(5\phi-3\theta^2)^2 \cdot (25\phi-4)}{(625\phi^3-250\phi^2\theta^2-200\phi^2+12\theta^4)^2} \\ \pi_2^H &= \frac{25\phi^3(\alpha-\bar{c})^2(5\phi-3\theta^2)^2 \cdot (25\phi-4)}{(625\phi^3-250\phi^2\theta^2-200\phi^2+12\theta^4)^2} \\ \pi_1^L &= \frac{25\phi^3(\alpha-\bar{c})^2(5\phi-4)^2 \cdot (25\phi-9\theta^2)}{(625\phi^3-250\phi^2\theta^2-200\phi^2+12\theta^4)^2} \\ \pi_2^L &= \frac{\phi(\alpha-\bar{c})^2 \cdot (25\phi-\theta^2)(25\phi-12\theta^2)^2}{(625\phi^3-250\phi^2\theta^2-200\phi^2+12\theta^4)^2} \end{cases} \quad (73)$$

Total Production:

$$\frac{10\phi(\alpha-\bar{c})(50\phi^2-15\phi\theta^2-10\phi-6\theta^2)}{625\phi^3-250\phi^2\theta^2-200\phi^2+12\theta^4}$$

Price:

$$\frac{125\alpha\phi^3-100\alpha\phi^2\theta^2-100\alpha\phi^2+60\alpha\phi\theta^2+12\alpha\theta^4+500\bar{c}\phi^3-150\bar{c}\phi^2\theta^2-100\bar{c}\phi^2-60\bar{c}\phi\theta^2}{625\phi^3-250\phi^2\theta^2-200\phi^2+12\theta^4}$$

Firm Surplus:

$$\frac{2\phi(\alpha-\bar{c})^2 \cdot (31250\phi^5-21875\phi^4\theta^2-15000\phi^4+5625\phi^3\theta^4+5000\phi^3-600\phi^2\theta^4-1800\phi^2\theta^2+1800\phi\theta^4-72\theta^6)}{(625\phi^3-250\phi^2\theta^2-200\phi^2+12\theta^4)^2}$$

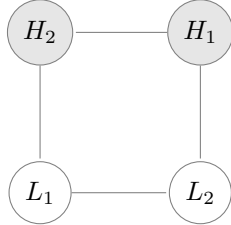
Consumer Surplus:

$$\frac{50\phi^2(\alpha-\bar{c})^2(50\phi^2-15\phi\theta^2-10\phi-6\theta^2)^2}{(625\phi^3-250\phi^2\theta^2-200\phi^2+12\theta^4)^2}$$

Social Welfare:

$$\frac{2\phi(\alpha-\bar{c})^2 \cdot (93750\phi^5-59375\phi^4\theta^2-40000\phi^4+11250\phi^3\theta^4-7500\phi^3\theta^2+7500\phi^3+3900\phi^2\theta^4+1200\phi^2\theta^2+2700\phi\theta^4-72\theta^6)}{(625\phi^3-250\phi^2\theta^2-200\phi^2+12\theta^4)^2}$$

#### D.2.23 E4E [2H2L]



$$\text{Equations : } \begin{cases} e_1^H \cdot \left(\frac{25\phi}{2} - 2\right) = \alpha + 2e_2^H - 3e_1^L\theta + 2e_2^L\theta - \bar{c} \\ e_2^H \cdot \left(\frac{25\phi}{2} - 2\right) = \alpha + 2e_1^H + 2e_1^L\theta - 3e_2^L\theta - \bar{c} \\ e_1^L \cdot \left(\frac{25\phi}{2\theta} - 2\theta\right) = \alpha - 3e_1^H + 2e_2^H + 2e_2^L\theta - \bar{c} \\ e_2^L \cdot \left(\frac{25\phi}{2\theta} - 2\theta\right) = \alpha + 2e_1^H - 3e_2^H + 2e_1^L\theta - \bar{c} \end{cases}$$

Optimal efforts:

$$\begin{cases} e_1^H &= \frac{2(\alpha-\bar{c})(5\phi-2\theta^2)}{125\phi^2-40\phi\theta^2-40\phi+12\theta^2} \\ e_2^H &= \frac{2(\alpha-\bar{c})(5\phi-2\theta^2)}{125\phi^2-40\phi\theta^2-40\phi+12\theta^2} \\ e_1^L &= \frac{2\theta(\alpha-\bar{c})(5\phi-2)}{125\phi^2-40\phi\theta^2-40\phi+12\theta^2} \\ e_2^L &= \frac{2\theta(\alpha-\bar{c})(5\phi-2)}{125\phi^2-40\phi\theta^2-40\phi+12\theta^2} \end{cases}$$

Production Costs:

$$\begin{cases} c_1^H &= -\frac{10\alpha\phi\theta^2+20\alpha\phi-12\alpha\theta^2-125\bar{c}\phi^2+30\bar{c}\phi\theta^2+20\bar{c}\phi}{125\phi^2-40\phi\theta^2-40\phi+12\theta^2} \\ c_2^H &= -\frac{10\alpha\phi\theta^2+20\alpha\phi-12\alpha\theta^2-125\bar{c}\phi^2+30\bar{c}\phi\theta^2+20\bar{c}\phi}{125\phi^2-40\phi\theta^2-40\phi+12\theta^2} \\ c_1^L &= -\frac{20\alpha\phi\theta^2+10\alpha\phi-12\alpha\theta^2-125\bar{c}\phi^2+20\bar{c}\phi\theta^2+30\bar{c}\phi}{125\phi^2-40\phi\theta^2-40\phi+12\theta^2} \\ c_2^L &= -\frac{20\alpha\phi\theta^2+10\alpha\phi-12\alpha\theta^2-125\bar{c}\phi^2+20\bar{c}\phi\theta^2+30\bar{c}\phi}{125\phi^2-40\phi\theta^2-40\phi+12\theta^2} \end{cases}$$

Production Quantities:

$$\begin{cases} q_1^H &= \frac{5\phi(\alpha-\bar{c})(5\phi-2\theta^2)}{125\phi^2-40\phi\theta^2-40\phi+12\theta^2} \\ q_2^H &= \frac{5\phi(\alpha-\bar{c})(5\phi-2\theta^2)}{125\phi^2-40\phi\theta^2-40\phi+12\theta^2} \\ q_1^L &= \frac{5\phi(\alpha-\bar{c})(5\phi-2)}{125\phi^2-40\phi\theta^2-40\phi+12\theta^2} \\ q_2^L &= \frac{5\phi(\alpha-\bar{c})(5\phi-2)}{125\phi^2-40\phi\theta^2-40\phi+12\theta^2} \end{cases}$$

Profits:

$$\begin{cases} \pi_1^H &= \frac{\phi(\alpha-\bar{c})^2(5\phi-2\theta^2)^2 \cdot (25\phi-4)}{(125\phi^2-40\phi\theta^2-40\phi+12\theta^2)^2} \\ \pi_2^H &= \frac{\phi(\alpha-\bar{c})^2(5\phi-2\theta^2)^2 \cdot (25\phi-4)}{(125\phi^2-40\phi\theta^2-40\phi+12\theta^2)^2} \\ \pi_1^L &= \frac{\phi(\alpha-\bar{c})^2(5\phi-2)^2 \cdot (25\phi-4\theta^2)}{(125\phi^2-40\phi\theta^2-40\phi+12\theta^2)^2} \\ \pi_2^L &= \frac{\phi(\alpha-\bar{c})^2(5\phi-2)^2 \cdot (25\phi-4\theta^2)}{(125\phi^2-40\phi\theta^2-40\phi+12\theta^2)^2} \end{cases} \quad (74)$$

Total Production:

$$\frac{20\phi(\alpha-\bar{c})(5\phi-\theta^2-1)}{125\phi^2-40\phi\theta^2-40\phi+12\theta^2}$$

Price:

$$\frac{25\alpha\phi^2-20\alpha\phi\theta^2-20\alpha\phi+12\alpha\theta^2+100\bar{c}\phi^2-20\bar{c}\phi\theta^2-20\bar{c}\phi}{125\phi^2-40\phi\theta^2-40\phi+12\theta^2}$$

Firm Surplus:

$$\frac{4\phi(\alpha-\bar{c})^2 \cdot (625\phi^3-300\phi^2\theta^2-300\phi^2+50\phi\theta^4+80\phi\theta^2+50\phi-8\theta^4-8\theta^2)}{(125\phi^2-40\phi\theta^2-40\phi+12\theta^2)^2}$$

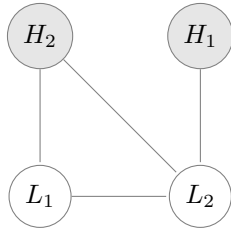
Consumer Surplus:

$$\frac{200\phi^2(\alpha-\bar{c})^2(5\phi-\theta^2-1)^2}{(125\phi^2-40\phi\theta^2-40\phi+12\theta^2)^2}$$

Social Welfare:

$$\frac{4\phi(\alpha-\bar{c})^2 \cdot (1875\phi^3-800\phi^2\theta^2-800\phi^2+100\phi\theta^4+180\phi\theta^2+100\phi-8\theta^4-8\theta^2)}{(125\phi^2-40\phi\theta^2-40\phi+12\theta^2)^2}$$

#### D.2.24 E4F [2H2L]



$$\text{Equations : } \begin{cases} e_1^H \cdot \left(\frac{25\phi}{3}-3\right) = \alpha - 3e_2^H - 3e_1^L\theta + e_2^L\theta - \bar{c} \\ e_2^H \cdot \left(\frac{25\phi}{2}-2\right) = \alpha - 2e_1^H + 2e_1^L\theta + e_2^L\theta - \bar{c} \\ e_1^L \cdot \left(\frac{25\phi}{2\theta}-2\theta\right) = \alpha - 2e_1^H + 2e_2^H + e_2^L\theta - \bar{c} \\ e_2^L \cdot \left(\frac{25\phi}{\theta}-\theta\right) = \alpha + 3e_1^H + 2e_2^H + 2e_1^L\theta - \bar{c} \end{cases}$$

Optimal efforts:

$$\begin{cases} e_1^H &= \frac{15\phi(\alpha-\bar{c})(5\phi-2\theta^2-2)}{625\phi^3-125\phi^2\theta^2-325\phi^2+6\theta^4+6\theta^2} \\ e_2^H &= \frac{10\phi(\alpha-\bar{c})(5\phi-3)}{625\phi^3-125\phi^2\theta^2-325\phi^2+6\theta^4+6\theta^2} \\ e_1^L &= \frac{10\phi\theta(\alpha-\bar{c})(5\phi-3)}{625\phi^3-125\phi^2\theta^2-325\phi^2+6\theta^4+6\theta^2} \\ e_2^L &= \frac{\theta(\alpha-\bar{c})(25\phi^2-6\theta^2-6)}{625\phi^3-125\phi^2\theta^2-325\phi^2+6\theta^4+6\theta^2} \end{cases}$$

Production Costs:

$$\begin{cases} c_1^H &= -\frac{25\alpha\phi^2\theta^2+75\alpha\phi^2-30\alpha\phi\theta^2-30\alpha\phi-6\alpha\theta^4-6\alpha\theta^2-625\bar{c}\phi^3+100\bar{c}\phi^2\theta^2+250\bar{c}\phi^2+30\bar{c}\phi\theta^2+30\bar{c}\phi}{625\phi^3-125\phi^2\theta^2-325\phi^2+6\theta^4+6\theta^2} \\ c_2^H &= -\frac{75\alpha\phi^2\theta^2+50\alpha\phi^2-30\alpha\phi\theta^2-30\alpha\phi-6\alpha\theta^4-6\alpha\theta^2-625\bar{c}\phi^3+50\bar{c}\phi^2\theta^2+275\bar{c}\phi^2+30\bar{c}\phi\theta^2+30\bar{c}\phi}{625\phi^3-125\phi^2\theta^2-325\phi^2+6\theta^4+6\theta^2} \\ c_1^L &= -\frac{75\alpha\phi^2\theta^2+50\alpha\phi^2-30\alpha\phi\theta^2-30\alpha\phi-6\alpha\theta^4-6\alpha\theta^2-625\bar{c}\phi^3+50\bar{c}\phi^2\theta^2+275\bar{c}\phi^2+30\bar{c}\phi\theta^2+30\bar{c}\phi}{625\phi^3-125\phi^2\theta^2-325\phi^2+6\theta^4+6\theta^2} \\ c_2^L &= -\frac{75\alpha\phi^2\theta^2+125\alpha\phi^2-60\alpha\phi\theta^2-60\alpha\phi-6\alpha\theta^4-6\alpha\theta^2-625\bar{c}\phi^3+50\bar{c}\phi^2\theta^2+200\bar{c}\phi^2+60\bar{c}\phi\theta^2+60\bar{c}\phi}{625\phi^3-125\phi^2\theta^2-325\phi^2+6\theta^4+6\theta^2} \end{cases}$$

Production Quantities:

$$\begin{cases} q_1^H &= \frac{25\phi^2(\alpha-\bar{c})(5\phi-2\theta^2-2)}{625\phi^3-125\phi^2\theta^2-325\phi^2+6\theta^4+6\theta^2} \\ q_2^H &= \frac{25\phi^2(\alpha-\bar{c})(5\phi-3)}{625\phi^3-125\phi^2\theta^2-325\phi^2+6\theta^4+6\theta^2} \\ q_1^L &= \frac{25\phi^2(\alpha-\bar{c})(5\phi-3)}{625\phi^3-125\phi^2\theta^2-325\phi^2+6\theta^4+6\theta^2} \\ q_2^L &= \frac{5\phi(\alpha-\bar{c})(25\phi^2-6\theta^2-6)}{625\phi^3-125\phi^2\theta^2-325\phi^2+6\theta^4+6\theta^2} \end{cases}$$

Profits:

$$\begin{cases} \pi_1^H &= \frac{25\phi^3(\alpha-\bar{c})^2 \cdot (25\phi-9)(5\phi-2\theta^2-2)^2}{(625\phi^3-125\phi^2\theta^2-325\phi^2+6\theta^4+6\theta^2)^2} \\ \pi_2^H &= \frac{25\phi^3(\alpha-\bar{c})^2(5\phi-3)^2 \cdot (25\phi-4)}{(625\phi^3-125\phi^2\theta^2-325\phi^2+6\theta^4+6\theta^2)^2} \\ \pi_1^L &= \frac{25\phi^3(\alpha-\bar{c})^2(5\phi-3)^2 \cdot (25\phi-4\theta^2)}{(625\phi^3-125\phi^2\theta^2-325\phi^2+6\theta^4+6\theta^2)^2} \\ \pi_2^L &= \frac{\phi(\alpha-\bar{c})^2 \cdot (25\phi-\theta^2)(25\phi^2-6\theta^2-6)^2}{(625\phi^3-125\phi^2\theta^2-325\phi^2+6\theta^4+6\theta^2)^2} \end{cases} \quad (75)$$

Total Production:

$$\frac{10\phi(\alpha-\bar{c})(50\phi^2-5\phi\theta^2-20\phi-3\theta^2-3)}{625\phi^3-125\phi^2\theta^2-325\phi^2+6\theta^4+6\theta^2}$$

Price:

$$\frac{125\alpha\phi^3-75\alpha\phi^2\theta^2-125\alpha\phi^2+30\alpha\phi\theta^2+30\alpha\phi+6\alpha\theta^4+6\alpha\theta^2+500\bar{c}\phi^3-50\bar{c}\phi^2\theta^2-200\bar{c}\phi^2-30\bar{c}\phi\theta^2-30\bar{c}\phi}{625\phi^3-125\phi^2\theta^2-325\phi^2+6\theta^4+6\theta^2}$$

Firm Surplus:

$$\frac{\phi(\alpha-\bar{c})^2 \cdot (62500\phi^5-15625\phi^4\theta^2-58125\phi^4+2500\phi^3\theta^4+5000\phi^3\theta^2+13750\phi^3-600\phi^2\theta^4-2400\phi^2\theta^2-1800\phi^2+900\phi\theta^4+1800\phi\theta^2+900\phi-36\theta^6-72\theta^4-36\theta^2)}{(625\phi^3-125\phi^2\theta^2-325\phi^2+6\theta^4+6\theta^2)^2}$$

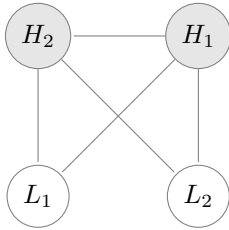
Consumer Surplus:

$$\frac{50\phi^2(\alpha-\bar{c})^2(50\phi^2-5\phi\theta^2-20\phi-3\theta^2-3)^2}{(625\phi^3-125\phi^2\theta^2-325\phi^2+6\theta^4+6\theta^2)^2}$$

Social Welfare:

$$\frac{\phi(\alpha-\bar{c})^2 \cdot (187500\phi^5-40625\phi^4\theta^2-158125\phi^4+3750\phi^3\theta^4+18750\phi^3+900\phi^2\theta^4+5100\phi^2\theta^2+4200\phi^2+1350\phi\theta^4+2700\phi\theta^2+1350\phi-36\theta^6-72\theta^4-36\theta^2)}{(625\phi^3-125\phi^2\theta^2-325\phi^2+6\theta^4+6\theta^2)^2}$$

## D.2.25 E5A [2H2L]



$$\text{Equations : } \begin{cases} e_1^H \cdot (25\phi-1) = \alpha + e_2^H + 2e_1^L\theta + 2e_2^L\theta - \bar{c} \\ e_2^H \cdot (25\phi-1) = \alpha + e_1^H + 2e_1^L\theta + 2e_2^L\theta - \bar{c} \\ e_1^L \cdot \left(\frac{25\phi}{2\theta} - 2\theta\right) = \alpha + e_1^H + e_2^H - 3e_2^L\theta - \bar{c} \\ e_2^L \cdot \left(\frac{25\phi}{2\theta} - 2\theta\right) = \alpha + e_1^H + e_2^H - 3e_1^L\theta - \bar{c} \end{cases}$$

Optimal efforts:

$$\begin{cases} e_1^H &= \frac{(\alpha-\bar{c})(5\phi+2\theta^2)}{125\phi^2+10\phi\theta^2-10\phi-4\theta^2} \\ e_2^H &= \frac{(\alpha-\bar{c})(5\phi+2\theta^2)}{125\phi^2+10\phi\theta^2-10\phi-4\theta^2} \\ e_1^L &= \frac{10\phi\theta(\alpha-\bar{c})}{125\phi^2+10\phi\theta^2-10\phi-4\theta^2} \\ e_2^L &= \frac{10\phi\theta(\alpha-\bar{c})}{125\phi^2+10\phi\theta^2-10\phi-4\theta^2} \end{cases}$$

Production Costs:

$$\begin{cases} c_1^H &= -\frac{20\alpha\phi\theta^2+10\alpha\phi+4\alpha\theta^2-125\bar{c}\phi^2-30\bar{c}\phi\theta^2}{125\phi^2+10\phi\theta^2-10\phi-4\theta^2} \\ c_2^H &= -\frac{20\alpha\phi\theta^2+10\alpha\phi+4\alpha\theta^2-125\bar{c}\phi^2-30\bar{c}\phi\theta^2}{125\phi^2+10\phi\theta^2-10\phi-4\theta^2} \\ c_1^L &= -\frac{10\alpha\phi\theta^2+10\alpha\phi+4\alpha\theta^2-125\bar{c}\phi^2-20\bar{c}\phi\theta^2}{125\phi^2+10\phi\theta^2-10\phi-4\theta^2} \\ c_2^L &= -\frac{10\alpha\phi\theta^2+10\alpha\phi+4\alpha\theta^2-125\bar{c}\phi^2-20\bar{c}\phi\theta^2}{125\phi^2+10\phi\theta^2-10\phi-4\theta^2} \end{cases}$$

Production Quantities:

$$\begin{cases} q_1^H &= \frac{5\phi(\alpha-\bar{c})(5\phi+2\theta^2)}{125\phi^2+10\phi\theta^2-10\phi-4\theta^2} \\ q_2^H &= \frac{5\phi(\alpha-\bar{c})(5\phi+2\theta^2)}{125\phi^2+10\phi\theta^2-10\phi-4\theta^2} \\ q_1^L &= \frac{25\phi^2(\alpha-\bar{c})}{125\phi^2+10\phi\theta^2-10\phi-4\theta^2} \\ q_2^L &= \frac{25\phi^2(\alpha-\bar{c})}{125\phi^2+10\phi\theta^2-10\phi-4\theta^2} \end{cases}$$

Profits:

$$\begin{cases} \pi_1^H &= \frac{\phi(\alpha-\bar{c})^2(5\phi+2\theta^2)^2 \cdot (25\phi-1)}{(125\phi^2+10\phi\theta^2-10\phi-4\theta^2)^2} \\ \pi_2^H &= \frac{\phi(\alpha-\bar{c})^2(5\phi+2\theta^2)^2 \cdot (25\phi-1)}{(125\phi^2+10\phi\theta^2-10\phi-4\theta^2)^2} \\ \pi_1^L &= \frac{25\phi^3(\alpha-\bar{c})^2 \cdot (25\phi-4\theta^2)}{(125\phi^2+10\phi\theta^2-10\phi-4\theta^2)^2} \\ \pi_2^L &= \frac{25\phi^3(\alpha-\bar{c})^2 \cdot (25\phi-4\theta^2)}{(125\phi^2+10\phi\theta^2-10\phi-4\theta^2)^2} \end{cases} \quad (76)$$

Total Production:

$$\frac{20\phi(\alpha-\bar{c})(5\phi+\theta^2)}{125\phi^2+10\phi\theta^2-10\phi-4\theta^2}$$

Price:

$$\frac{25\alpha\phi^2-10\alpha\phi\theta^2-10\alpha\phi-4\alpha\theta^2+100\bar{c}\phi^2+20\bar{c}\phi\theta^2}{125\phi^2+10\phi\theta^2-10\phi-4\theta^2}$$

Firm Surplus:

$$\frac{2\phi(\alpha-\bar{c})^2 \cdot (1250\phi^3+400\phi^2\theta^2-25\phi^2+100\phi\theta^4-20\phi\theta^2-4\theta^4)}{(125\phi^2+10\phi\theta^2-10\phi-4\theta^2)^2}$$

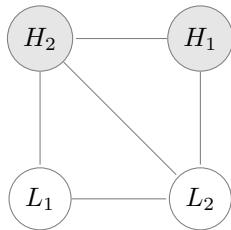
Consumer Surplus:

$$\frac{200\phi^2(\alpha-\bar{c})^2(5\phi+\theta^2)^2}{(125\phi^2+10\phi\theta^2-10\phi-4\theta^2)^2}$$

Social Welfare:

$$\frac{2\phi(\alpha-\bar{c})^2 \cdot (3750\phi^3+1400\phi^2\theta^2-25\phi^2+200\phi\theta^4-20\phi\theta^2-4\theta^4)}{(125\phi^2+10\phi\theta^2-10\phi-4\theta^2)^2}$$

#### D.2.26 E5B [2H2L]



$$\text{Equations : } \begin{cases} e_1^H \cdot \left(\frac{25\phi}{2} - 2\right) = \alpha + e_2^H - 3e_1^L\theta + e_2^L\theta - \bar{c} \\ e_2^H \cdot (25\phi - 1) = \alpha + 2e_1^H + 2e_1^L\theta + e_2^L\theta - \bar{c} \\ e_1^L \cdot \left(\frac{25\phi}{2\theta} - 2\theta\right) = \alpha - 3e_1^H + e_2^H + e_2^L\theta - \bar{c} \\ e_2^L \cdot \left(\frac{25\phi}{\theta} - \theta\right) = \alpha + 2e_1^H + e_2^H + 2e_1^L\theta - \bar{c} \end{cases}$$

Optimal efforts:

$$\begin{cases} e_1^H &= \frac{10\phi(\alpha-\bar{c})(5\phi-2\theta^2)}{(125\phi^2-4\theta^2)(5\phi-\theta^2-1)} \\ e_2^H &= \frac{(\alpha-\bar{c})(5\phi-2\theta)(5\phi+2\theta)}{(125\phi^2-4\theta^2)(5\phi-\theta^2-1)} \\ e_1^L &= \frac{10\phi\theta(\alpha-\bar{c})(5\phi-2)}{(125\phi^2-4\theta^2)(5\phi-\theta^2-1)} \\ e_2^L &= \frac{\theta(\alpha-\bar{c})(5\phi-2\theta)(5\phi+2\theta)}{(125\phi^2-4\theta^2)(5\phi-\theta^2-1)} \end{cases}$$

Production Costs:

$$\begin{cases} c_1^H &= -\frac{25\alpha\phi^2\theta^2+75\alpha\phi^2-20\alpha\phi\theta^2-4\alpha\theta^4-4\alpha\theta^2-625\bar{c}\phi^3+100\bar{c}\phi^2\theta^2+50\bar{c}\phi^2+40\bar{c}\phi\theta^2}{(125\phi^2-4\theta^2)(5\phi-\theta^2-1)} \\ c_2^H &= -\frac{75\alpha\phi^2\theta^2+75\alpha\phi^2-40\alpha\phi\theta^2-4\alpha\theta^4-4\alpha\theta^2-625\bar{c}\phi^3+50\bar{c}\phi^2\theta^2+50\bar{c}\phi^2+60\bar{c}\phi\theta^2}{(125\phi^2-4\theta^2)(5\phi-\theta^2-1)} \\ c_1^L &= -\frac{75\alpha\phi^2\theta^2+25\alpha\phi^2-20\alpha\phi\theta^2-4\alpha\theta^4-4\alpha\theta^2-625\bar{c}\phi^3+50\bar{c}\phi^2\theta^2+100\bar{c}\phi^2+40\bar{c}\phi\theta^2}{(125\phi^2-4\theta^2)(5\phi-\theta^2-1)} \\ c_2^L &= -\frac{75\alpha\phi^2\theta^2+75\alpha\phi^2-40\alpha\phi\theta^2-4\alpha\theta^4-4\alpha\theta^2-625\bar{c}\phi^3+50\bar{c}\phi^2\theta^2+50\bar{c}\phi^2+60\bar{c}\phi\theta^2}{(125\phi^2-4\theta^2)(5\phi-\theta^2-1)} \end{cases}$$

Production Quantities:

$$\begin{cases} q_1^H &= \frac{25\phi^2(\alpha-\bar{c})(5\phi-2\theta^2)}{(125\phi^2-4\theta^2)(5\phi-\theta^2-1)} \\ q_2^H &= \frac{5\phi(\alpha-\bar{c})(5\phi-2\theta)(5\phi+2\theta)}{(125\phi^2-4\theta^2)(5\phi-\theta^2-1)} \\ q_1^L &= \frac{25\phi^2(\alpha-\bar{c})(5\phi-2)}{(125\phi^2-4\theta^2)(5\phi-\theta^2-1)} \\ q_2^L &= \frac{5\phi(\alpha-\bar{c})(5\phi-2\theta)(5\phi+2\theta)}{(125\phi^2-4\theta^2)(5\phi-\theta^2-1)} \end{cases}$$

Profits:

$$\begin{cases} \pi_1^H &= \frac{25\phi^3(\alpha-\bar{c})^2(5\phi-2\theta^2)^2 \cdot (25\phi-4)}{(125\phi^2-4\theta^2)^2(5\phi-\theta^2-1)^2} \\ \pi_2^H &= \frac{\phi(\alpha-\bar{c})^2(5\phi-2\theta)^2(5\phi+2\theta)^2 \cdot (25\phi-1)}{(125\phi^2-4\theta^2)^2(5\phi-\theta^2-1)^2} \\ \pi_1^L &= \frac{25\phi^3(\alpha-\bar{c})^2(5\phi-2)^2 \cdot (25\phi-4\theta^2)}{(125\phi^2-4\theta^2)^2(5\phi-\theta^2-1)^2} \\ \pi_2^L &= \frac{\phi(\alpha-\bar{c})^2(5\phi-2\theta)^2(5\phi+2\theta)^2 \cdot (25\phi-\theta^2)}{(125\phi^2-4\theta^2)^2(5\phi-\theta^2-1)^2} \end{cases} \quad (77)$$

Total Production:

$$\frac{10\phi(\alpha-\bar{c})(50\phi^2-5\phi\theta^2-5\phi-4\theta^2)}{(125\phi^2-4\theta^2)(5\phi-\theta^2-1)}$$

Price:

$$\frac{125\alpha\phi^3-75\alpha\phi^2\theta^2-75\alpha\phi^2+20\alpha\phi\theta^2+4\alpha\theta^4+4\alpha\theta^2+500\bar{c}\phi^3-50\bar{c}\phi^2\theta^2-50\bar{c}\phi^2-40\bar{c}\phi\theta^2}{(125\phi^2-4\theta^2)(5\phi-\theta^2-1)}$$

Firm Surplus:

$$\frac{\phi(\alpha-\bar{c})^2 \cdot (62500\phi^5-15625\phi^4\theta^2-15625\phi^4+2500\phi^3\theta^4-6000\phi^3\theta^2+2500\phi^3-200\phi^2\theta^4-200\phi^2\theta^2+800\phi\theta^4-16\theta^6-16\theta^4)}{(125\phi^2-4\theta^2)^2(5\phi-\theta^2-1)^2}$$

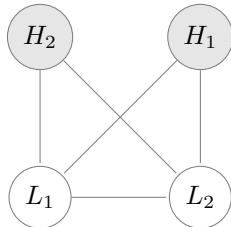
Consumer Surplus:

$$\frac{50\phi^2(\alpha-\bar{c})^2(50\phi^2-5\phi\theta^2-5\phi-4\theta^2)^2}{(125\phi^2-4\theta^2)^2(5\phi-\theta^2-1)^2}$$

Social Welfare:

$$\frac{\phi(\alpha-\bar{c})^2 \cdot (187500\phi^5-40625\phi^4\theta^2-40625\phi^4+3750\phi^3\theta^4-23500\phi^3\theta^2+3750\phi^3+1800\phi^2\theta^4+1800\phi^2\theta^2+1600\phi\theta^4-16\theta^6-16\theta^4)}{(125\phi^2-4\theta^2)^2(5\phi-\theta^2-1)^2}$$

#### D.2.27 E5C [2H2L]



$$\text{Equations : } \begin{cases} e_1^H \cdot \left(\frac{25\phi}{2} - 2\right) = \alpha - 3e_2^H + e_1^L\theta + e_2^L\theta - \bar{c} \\ e_2^H \cdot \left(\frac{25\phi}{2} - 2\right) = \alpha - 3e_1^H + e_1^L\theta + e_2^L\theta - \bar{c} \\ e_1^L \cdot \left(\frac{25\phi}{\theta} - \theta\right) = \alpha + 2e_1^H + 2e_2^H + e_2^L\theta - \bar{c} \\ e_2^L \cdot \left(\frac{25\phi}{\theta} - \theta\right) = \alpha + 2e_1^H + 2e_2^H + e_1^L\theta - \bar{c} \end{cases}$$

Optimal efforts:

$$\begin{cases} e_1^H &= \frac{10\phi(\alpha-\bar{c})}{125\phi^2-10\phi\theta^2+10\phi-4\theta^2} \\ e_2^H &= \frac{10\phi(\alpha-\bar{c})}{125\phi^2-10\phi\theta^2+10\phi-4\theta^2} \\ e_1^L &= \frac{\theta(\alpha-\bar{c})(5\phi+2)}{125\phi^2-10\phi\theta^2+10\phi-4\theta^2} \\ e_2^L &= \frac{\theta(\alpha-\bar{c})(5\phi+2)}{125\phi^2-10\phi\theta^2+10\phi-4\theta^2} \end{cases}$$

Production Costs:

$$\begin{cases} c_1^H &= -\frac{10\alpha\phi\theta^2+10\alpha\phi+4\alpha\theta^2-125\bar{c}\phi^2-20\bar{c}\phi}{125\phi^2-10\phi\theta^2+10\phi-4\theta^2} \\ c_2^H &= -\frac{10\alpha\phi\theta^2+10\alpha\phi+4\alpha\theta^2-125\bar{c}\phi^2-20\bar{c}\phi}{125\phi^2-10\phi\theta^2+10\phi-4\theta^2} \\ c_1^L &= -\frac{10\alpha\phi\theta^2+20\alpha\phi+4\alpha\theta^2-125\bar{c}\phi^2-30\bar{c}\phi}{125\phi^2-10\phi\theta^2+10\phi-4\theta^2} \\ c_2^L &= -\frac{10\alpha\phi\theta^2+20\alpha\phi+4\alpha\theta^2-125\bar{c}\phi^2-30\bar{c}\phi}{125\phi^2-10\phi\theta^2+10\phi-4\theta^2} \end{cases}$$

Production Quantities:

$$\begin{cases} q_1^H &= \frac{25\phi^2(\alpha-\bar{c})}{125\phi^2-10\phi\theta^2+10\phi-4\theta^2} \\ q_2^H &= \frac{25\phi^2(\alpha-\bar{c})}{125\phi^2-10\phi\theta^2+10\phi-4\theta^2} \\ q_1^L &= \frac{5\phi(\alpha-\bar{c})(5\phi+2)}{125\phi^2-10\phi\theta^2+10\phi-4\theta^2} \\ q_2^L &= \frac{5\phi(\alpha-\bar{c})(5\phi+2)}{125\phi^2-10\phi\theta^2+10\phi-4\theta^2} \end{cases}$$

Profits:

$$\begin{cases} \pi_1^H &= \frac{25\phi^3(\alpha-\bar{c})^2 \cdot (25\phi-4)}{(125\phi^2-10\phi\theta^2+10\phi-4\theta^2)^2} \\ \pi_2^H &= \frac{25\phi^3(\alpha-\bar{c})^2 \cdot (25\phi-4)}{(125\phi^2-10\phi\theta^2+10\phi-4\theta^2)^2} \\ \pi_1^L &= \frac{\phi(\alpha-\bar{c})^2(5\phi+2)^2 \cdot (25\phi-\theta^2)}{(125\phi^2-10\phi\theta^2+10\phi-4\theta^2)^2} \\ \pi_2^L &= \frac{\phi(\alpha-\bar{c})^2(5\phi+2)^2 \cdot (25\phi-\theta^2)}{(125\phi^2-10\phi\theta^2+10\phi-4\theta^2)^2} \end{cases} \quad (78)$$

Total Production:

$$\frac{20\phi(\alpha-\bar{c})(5\phi+1)}{125\phi^2-10\phi\theta^2+10\phi-4\theta^2}$$

Price:

$$\frac{25\alpha\phi^2-10\alpha\phi\theta^2-10\alpha\phi-4\alpha\theta^2+100\bar{c}\phi^2+20\bar{c}\phi}{125\phi^2-10\phi\theta^2+10\phi-4\theta^2}$$

Firm Surplus:

$$\frac{2\phi(\alpha-\bar{c})^2 \cdot (1250\phi^3-25\phi^2\theta^2+400\phi^2-20\phi\theta^2+100\phi-4\theta^2)}{(125\phi^2-10\phi\theta^2+10\phi-4\theta^2)^2}$$

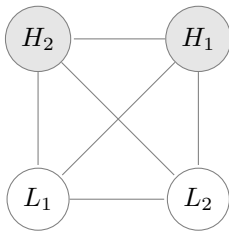
Consumer Surplus:

$$\frac{200\phi^2(\alpha-\bar{c})^2(5\phi+1)^2}{(125\phi^2-10\phi\theta^2+10\phi-4\theta^2)^2}$$

Social Welfare:

$$\frac{2\phi(\alpha-\bar{c})^2 \cdot (3750\phi^3-25\phi^2\theta^2+1400\phi^2-20\phi\theta^2+200\phi-4\theta^2)}{(125\phi^2-10\phi\theta^2+10\phi-4\theta^2)^2}$$

#### D.2.28 E6A [2H2L]



$$\text{Equations : } \begin{cases} e_1^H \cdot (25\phi-1) = \alpha + e_2^H + e_1^L\theta + e_2^L\theta - \bar{c} \\ e_2^H \cdot (25\phi-1) = \alpha + e_1^H + e_1^L\theta + e_2^L\theta - \bar{c} \\ e_1^L \cdot \left(\frac{25\phi}{\theta} - \theta\right) = \alpha + e_1^H + e_2^H + e_2^L\theta - \bar{c} \\ e_2^L \cdot \left(\frac{25\phi}{\theta} - \theta\right) = \alpha + e_1^H + e_2^H + e_1^L\theta - \bar{c} \end{cases}$$

Optimal efforts:

$$\begin{cases} e_1^H &= \frac{\alpha - \bar{c}}{25\phi - 2\theta^2 - 2} \\ e_2^H &= \frac{\alpha - \bar{c}}{25\phi - 2\theta^2 - 2} \\ e_1^L &= \frac{\theta(\alpha - \bar{c})}{25\phi - 2\theta^2 - 2} \\ e_2^L &= \frac{\theta(\alpha - \bar{c})}{25\phi - 2\theta^2 - 2} \end{cases}$$

Production Costs:

$$\begin{cases} c_1^H &= -\frac{2\alpha\theta^2 + 2\alpha - 25\bar{c}\phi}{25\phi - 2\theta^2 - 2} \\ c_2^H &= -\frac{2\alpha\theta^2 + 2\alpha - 25\bar{c}\phi}{25\phi - 2\theta^2 - 2} \\ c_1^L &= -\frac{2\alpha\theta^2 + 2\alpha - 25\bar{c}\phi}{25\phi - 2\theta^2 - 2} \\ c_2^L &= -\frac{2\alpha\theta^2 + 2\alpha - 25\bar{c}\phi}{25\phi - 2\theta^2 - 2} \end{cases}$$

Production Quantities:

$$\begin{cases} q_1^H &= \frac{5\phi(\alpha - \bar{c})}{25\phi - 2\theta^2 - 2} \\ q_2^H &= \frac{5\phi(\alpha - \bar{c})}{25\phi - 2\theta^2 - 2} \\ q_1^L &= \frac{5\phi(\alpha - \bar{c})}{25\phi - 2\theta^2 - 2} \\ q_2^L &= \frac{5\phi(\alpha - \bar{c})}{25\phi - 2\theta^2 - 2} \end{cases}$$

Profits:

$$\begin{cases} \pi_1^H &= \frac{\phi(\alpha - \bar{c})^2 \cdot (25\phi - 1)}{(25\phi - 2\theta^2 - 2)^2} \\ \pi_2^H &= \frac{\phi(\alpha - \bar{c})^2 \cdot (25\phi - 1)}{(25\phi - 2\theta^2 - 2)^2} \\ \pi_1^L &= \frac{\phi(\alpha - \bar{c})^2 \cdot (25\phi - \theta^2)}{(25\phi - 2\theta^2 - 2)^2} \\ \pi_2^L &= \frac{\phi(\alpha - \bar{c})^2 \cdot (25\phi - \theta^2)}{(25\phi - 2\theta^2 - 2)^2} \end{cases} \quad (79)$$

Total Production:

$$\frac{20\phi(\alpha - \bar{c})}{25\phi - 2\theta^2 - 2}$$

Price:

$$\frac{5\alpha\phi - 2\alpha\theta^2 - 2\alpha + 20\bar{c}\phi}{25\phi - 2\theta^2 - 2}$$

Firm Surplus:

$$\frac{2\phi(\alpha - \bar{c})^2 \cdot (50\phi - \theta^2 - 1)}{(25\phi - 2\theta^2 - 2)^2}$$

Consumer Surplus:

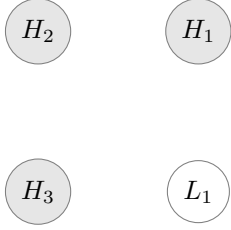
$$\frac{200\phi^2(\alpha - \bar{c})^2}{(25\phi - 2\theta^2 - 2)^2}$$

Social Welfare:

$$\frac{2\phi(\alpha - \bar{c})^2 \cdot (150\phi - \theta^2 - 1)}{(25\phi - 2\theta^2 - 2)^2}$$

### D.3 Three *high*- and one *low*-productive firms (3H1L)

#### D.3.1 E0A [3H1L]



$$\text{Equations : } \begin{cases} e_1^H \cdot \left( \frac{25\phi}{4} - 4 \right) = \alpha - e_2^H - e_3^H - e_1^L \theta - \bar{c} \\ e_2^H \cdot \left( \frac{25\phi}{4} - 4 \right) = \alpha - e_1^H - e_3^H - e_1^L \theta - \bar{c} \\ e_3^H \cdot \left( \frac{25\phi}{4} - 4 \right) = \alpha - e_1^H - e_2^H - e_1^L \theta - \bar{c} \\ e_1^L \cdot \left( \frac{25\phi}{4\theta} - 4\theta \right) = \alpha - e_1^H - e_2^H - e_3^H - \bar{c} \end{cases}$$

Optimal efforts:

$$\begin{cases} e_1^H = \frac{4(\alpha - \bar{c})(5\phi - 4\theta^2)}{125\phi^2 - 80\phi\theta^2 - 40\phi + 16\theta^2} \\ e_2^H = \frac{4(\alpha - \bar{c})(5\phi - 4\theta^2)}{125\phi^2 - 80\phi\theta^2 - 40\phi + 16\theta^2} \\ e_3^H = \frac{4(\alpha - \bar{c})(5\phi - 4\theta^2)}{125\phi^2 - 80\phi\theta^2 - 40\phi + 16\theta^2} \\ e_1^L = \frac{4\theta(\alpha - \bar{c})(5\phi - 4)}{125\phi^2 - 80\phi\theta^2 - 40\phi + 16\theta^2} \end{cases}$$

Production Costs:

$$\begin{cases} c_1^H = -\frac{20\alpha\phi - 16\alpha\theta^2 - 125\bar{c}\phi^2 + 80\bar{c}\phi\theta^2 + 20\bar{c}\phi}{125\phi^2 - 80\phi\theta^2 - 40\phi + 16\theta^2} \\ c_2^H = -\frac{20\alpha\phi - 16\alpha\theta^2 - 125\bar{c}\phi^2 + 80\bar{c}\phi\theta^2 + 20\bar{c}\phi}{125\phi^2 - 80\phi\theta^2 - 40\phi + 16\theta^2} \\ c_3^H = -\frac{20\alpha\phi - 16\alpha\theta^2 - 125\bar{c}\phi^2 + 80\bar{c}\phi\theta^2 + 20\bar{c}\phi}{125\phi^2 - 80\phi\theta^2 - 40\phi + 16\theta^2} \\ c_1^L = -\frac{20\alpha\phi\theta^2 - 16\alpha\theta^2 - 125\bar{c}\phi^2 + 60\bar{c}\phi\theta^2 + 40\bar{c}\phi}{125\phi^2 - 80\phi\theta^2 - 40\phi + 16\theta^2} \end{cases}$$

Production Quantities:

$$\begin{cases} q_1^H = \frac{5\phi(\alpha - \bar{c})(5\phi - 4\theta^2)}{125\phi^2 - 80\phi\theta^2 - 40\phi + 16\theta^2} \\ q_2^H = \frac{5\phi(\alpha - \bar{c})(5\phi - 4\theta^2)}{125\phi^2 - 80\phi\theta^2 - 40\phi + 16\theta^2} \\ q_3^H = \frac{5\phi(\alpha - \bar{c})(5\phi - 4\theta^2)}{125\phi^2 - 80\phi\theta^2 - 40\phi + 16\theta^2} \\ q_1^L = \frac{5\phi(\alpha - \bar{c})(5\phi - 4)}{125\phi^2 - 80\phi\theta^2 - 40\phi + 16\theta^2} \end{cases}$$

Profits:

$$\begin{cases} \pi_1^H = \frac{\phi(\alpha - \bar{c})^2(5\phi - 4\theta^2)^2 \cdot (25\phi - 16)}{(125\phi^2 - 80\phi\theta^2 - 40\phi + 16\theta^2)^2} \\ \pi_2^H = \frac{\phi(\alpha - \bar{c})^2(5\phi - 4\theta^2)^2 \cdot (25\phi - 16)}{(125\phi^2 - 80\phi\theta^2 - 40\phi + 16\theta^2)^2} \\ \pi_3^H = \frac{\phi(\alpha - \bar{c})^2(5\phi - 4\theta^2)^2 \cdot (25\phi - 16)}{(125\phi^2 - 80\phi\theta^2 - 40\phi + 16\theta^2)^2} \\ \pi_1^L = \frac{\phi(\alpha - \bar{c})^2(5\phi - 4) \cdot (25\phi - 16\theta^2)}{(125\phi^2 - 80\phi\theta^2 - 40\phi + 16\theta^2)^2} \end{cases} \quad (80)$$

Total Production:

$$\frac{20\phi(\alpha - \bar{c})(5\phi - 3\theta^2 - 1)}{125\phi^2 - 80\phi\theta^2 - 40\phi + 16\theta^2}$$

Price:

$$\frac{25\alpha\phi^2 - 20\alpha\phi\theta^2 - 20\alpha\phi + 16\alpha\theta^2 + 100\bar{c}\phi^2 - 60\bar{c}\phi\theta^2 - 20\bar{c}\phi}{125\phi^2 - 80\phi\theta^2 - 40\phi + 16\theta^2}$$

Firm Surplus:

$$\frac{4\phi(\alpha - \bar{c})^2 \cdot (625\phi^3 - 850\phi^2\theta^2 - 550\phi^2 + 300\phi\theta^4 + 640\phi\theta^2 + 100\phi - 192\theta^4 - 64\theta^2)}{(125\phi^2 - 80\phi\theta^2 - 40\phi + 16\theta^2)^2}$$

Consumer Surplus:

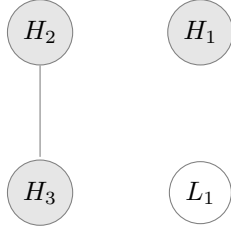
$$\frac{200\phi^2(\alpha - \bar{c})^2(5\phi - 3\theta^2 - 1)^2}{(125\phi^2 - 80\phi\theta^2 - 40\phi + 16\theta^2)^2}$$



Social Welfare:

$$\frac{4\phi(\alpha - \bar{c})^2 \cdot (1875\phi^3 - 2350\phi^2\theta^2 - 1050\phi^2 + 750\phi\theta^4 + 940\phi\theta^2 + 150\phi - 192\theta^4 - 64\theta^2)}{(125\phi^2 - 80\phi\theta^2 - 40\phi + 16\theta^2)^2}$$

### D.3.2 E1A [3H1L]



$$\text{Equations : } \begin{cases} e_1^H \cdot \left(\frac{25\phi}{4} - 4\right) = \alpha - 2e_2^H - 2e_3^H - e_1^L\theta - \bar{c} \\ e_2^H \cdot \left(\frac{25\phi}{3} - 3\right) = \alpha - e_1^H + 3e_3^H - e_1^L\theta - \bar{c} \\ e_3^H \cdot \left(\frac{25\phi}{3} - 3\right) = \alpha - e_1^H + 3e_2^H - e_1^L\theta - \bar{c} \\ e_1^L \cdot \left(\frac{25\phi}{4\theta} - 4\theta\right) = \alpha - e_1^H - 2e_2^H - 2e_3^H - \bar{c} \end{cases}$$

Optimal efforts:

$$\begin{cases} e_1^H = \frac{4(\alpha - \bar{c})(5\phi - 6)(5\phi - 4\theta^2)}{625\phi^3 - 400\phi^2\theta^2 - 850\phi^2 + 480\phi\theta^2 + 240\phi - 96\theta^2} \\ e_2^H = \frac{3(\alpha - \bar{c})(5\phi - 4)(5\phi - 4\theta^2)}{625\phi^3 - 400\phi^2\theta^2 - 850\phi^2 + 480\phi\theta^2 + 240\phi - 96\theta^2} \\ e_3^H = \frac{3(\alpha - \bar{c})(5\phi - 4)(5\phi - 4\theta^2)}{625\phi^3 - 400\phi^2\theta^2 - 850\phi^2 + 480\phi\theta^2 + 240\phi - 96\theta^2} \\ e_1^L = \frac{4\theta(\alpha - \bar{c})(5\phi - 6)(5\phi - 4)}{625\phi^3 - 400\phi^2\theta^2 - 850\phi^2 + 480\phi\theta^2 + 240\phi - 96\theta^2} \end{cases}$$

Production Costs:

$$\begin{cases} c_1^H = -\frac{100\alpha\phi^2 - 80\alpha\phi\theta^2 - 120\alpha\phi + 96\alpha\theta^2 - 625\bar{c}\phi^3 + 400\bar{c}\phi^2\theta^2 + 750\bar{c}\phi^2 - 400\bar{c}\phi\theta^2 - 120\bar{c}\phi}{625\phi^3 - 400\phi^2\theta^2 - 850\phi^2 + 480\phi\theta^2 + 240\phi - 96\theta^2} \\ c_2^H = -\frac{150\alpha\phi^2 - 120\alpha\phi\theta^2 - 120\alpha\phi + 96\alpha\theta^2 - 625\bar{c}\phi^3 + 400\bar{c}\phi^2\theta^2 + 700\bar{c}\phi^2 - 360\bar{c}\phi\theta^2 - 120\bar{c}\phi}{625\phi^3 - 400\phi^2\theta^2 - 850\phi^2 + 480\phi\theta^2 + 240\phi - 96\theta^2} \\ c_3^H = -\frac{150\alpha\phi^2 - 120\alpha\phi\theta^2 - 120\alpha\phi + 96\alpha\theta^2 - 625\bar{c}\phi^3 + 400\bar{c}\phi^2\theta^2 + 700\bar{c}\phi^2 - 360\bar{c}\phi\theta^2 - 120\bar{c}\phi}{625\phi^3 - 400\phi^2\theta^2 - 850\phi^2 + 480\phi\theta^2 + 240\phi - 96\theta^2} \\ c_1^L = -\frac{100\alpha\phi^2\theta^2 - 200\alpha\phi\theta^2 + 96\alpha\theta^2 - 625\bar{c}\phi^3 + 300\bar{c}\phi^2\theta^2 + 850\bar{c}\phi^2 - 280\bar{c}\phi\theta^2 - 240\bar{c}\phi}{625\phi^3 - 400\phi^2\theta^2 - 850\phi^2 + 480\phi\theta^2 + 240\phi - 96\theta^2} \end{cases}$$

Production Quantities:

$$\begin{cases} q_1^H = \frac{5\phi(\alpha - \bar{c})(5\phi - 6)(5\phi - 4\theta^2)}{625\phi^3 - 400\phi^2\theta^2 - 850\phi^2 + 480\phi\theta^2 + 240\phi - 96\theta^2} \\ q_2^H = \frac{5\phi(\alpha - \bar{c})(5\phi - 4)(5\phi - 4\theta^2)}{625\phi^3 - 400\phi^2\theta^2 - 850\phi^2 + 480\phi\theta^2 + 240\phi - 96\theta^2} \\ q_3^H = \frac{5\phi(\alpha - \bar{c})(5\phi - 4)(5\phi - 4\theta^2)}{625\phi^3 - 400\phi^2\theta^2 - 850\phi^2 + 480\phi\theta^2 + 240\phi - 96\theta^2} \\ q_1^L = \frac{5\phi(\alpha - \bar{c})(5\phi - 6)(5\phi - 4)}{625\phi^3 - 400\phi^2\theta^2 - 850\phi^2 + 480\phi\theta^2 + 240\phi - 96\theta^2} \end{cases}$$

Profits:

$$\begin{cases} \pi_1^H = \frac{\phi(\alpha - \bar{c})^2(5\phi - 6)^2(5\phi - 4\theta^2)^2 \cdot (25\phi - 16)}{(625\phi^3 - 400\phi^2\theta^2 - 850\phi^2 + 480\phi\theta^2 + 240\phi - 96\theta^2)^2} \\ \pi_2^H = \frac{\phi(\alpha - \bar{c})^2(5\phi - 4)^2(5\phi - 4\theta^2)^2 \cdot (25\phi - 9)}{(625\phi^3 - 400\phi^2\theta^2 - 850\phi^2 + 480\phi\theta^2 + 240\phi - 96\theta^2)^2} \\ \pi_3^H = \frac{\phi(\alpha - \bar{c})^2(5\phi - 4)^2(5\phi - 4\theta^2)^2 \cdot (25\phi - 9)}{(625\phi^3 - 400\phi^2\theta^2 - 850\phi^2 + 480\phi\theta^2 + 240\phi - 96\theta^2)^2} \\ \pi_1^L = \frac{\phi(\alpha - \bar{c})^2(5\phi - 6)^2(5\phi - 4)^2 \cdot (25\phi - 16\theta^2)}{(625\phi^3 - 400\phi^2\theta^2 - 850\phi^2 + 480\phi\theta^2 + 240\phi - 96\theta^2)^2} \end{cases} \quad (81)$$

Total Production:

$$\frac{20\phi(\alpha - \bar{c})(25\phi^2 - 15\phi\theta^2 - 30\phi + 14\theta^2 + 6)}{625\phi^3 - 400\phi^2\theta^2 - 850\phi^2 + 480\phi\theta^2 + 240\phi - 96\theta^2}$$

Price:

$$\frac{125\alpha\phi^3 - 100\alpha\phi^2\theta^2 - 250\alpha\phi^2 + 200\alpha\phi\theta^2 + 120\alpha\phi - 96\alpha\theta^2 + 500\bar{c}\phi^3 - 300\bar{c}\phi^2\theta^2 - 600\bar{c}\phi^2 + 280\bar{c}\phi\theta^2 + 120\bar{c}\phi}{625\phi^3 - 400\phi^2\theta^2 - 850\phi^2 + 480\phi\theta^2 + 240\phi - 96\theta^2}$$

Firm Surplus:

$$\frac{2\phi(\alpha - \bar{c})^2 \cdot (31250\phi^5 - 42500\phi^4\theta^2 - 85625\phi^4 + 15000\phi^3\theta^4 + 107000\phi^3\theta^2 + 88500\phi^3 - 34800\phi^2\theta^4 - 97200\phi^2\theta^2 - 40800\phi^2 + 27040\phi\theta^4 + 36480\phi\theta^2 + 7200\phi - 6912\theta^4 - 4608\theta^2)}{(625\phi^3 - 400\phi^2\theta^2 - 850\phi^2 + 480\phi\theta^2 + 240\phi - 96\theta^2)^2}$$

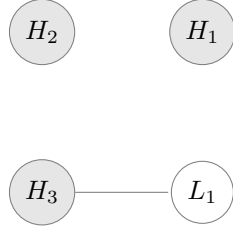
Consumer Surplus:

$$\frac{200\phi^2 (\alpha - \bar{c})^2 (25\phi^2 - 15\phi\theta^2 - 30\phi + 14\theta^2 + 6)^2}{(625\phi^3 - 400\phi^2\theta^2 - 850\phi^2 + 480\phi\theta^2 + 240\phi - 96\theta^2)^2}$$

Social Welfare:

$$\frac{2\phi (\alpha - \bar{c})^2 \cdot (93750\phi^5 - 117500\phi^4\theta^2 - 235625\phi^4 + 37500\phi^3\theta^4 + 267000\phi^3\theta^2 + 208500\phi^3 - 76800\phi^2\theta^4 - 199200\phi^2\theta^2 - 76800\phi^2 + 46640\phi\theta^4 + 53280\phi\theta^2 + 10800\phi - 6912\theta^4 - 4608\theta^2)}{(625\phi^3 - 400\phi^2\theta^2 - 850\phi^2 + 480\phi\theta^2 + 240\phi - 96\theta^2)^2}$$

### D.3.3 E1B [3H1L]



$$\text{Equations : } \begin{cases} e_1^H \cdot \left(\frac{25\phi}{4} - 4\right) = \alpha - e_2^H - 2e_3^H - 2e_1^L\theta - \bar{c} \\ e_2^H \cdot \left(\frac{25\phi}{4} - 4\right) = \alpha - e_1^H - 2e_3^H - 2e_1^L\theta - \bar{c} \\ e_3^H \cdot \left(\frac{25\phi}{3} - 3\right) = \alpha - e_1^H - e_2^H + 3e_1^L\theta - \bar{c} \\ e_1^L \cdot \left(\frac{25\phi}{3\theta} - 3\theta\right) = \alpha - e_1^H - e_2^H + 3e_3^H - \bar{c} \end{cases}$$

Optimal efforts:

$$\begin{cases} e_1^H = \frac{4(\alpha - \bar{c})(5\phi - 3\theta^2 - 3)}{125\phi^2 - 45\phi\theta^2 - 105\phi + 12\theta^2 + 12} \\ e_2^H = \frac{4(\alpha - \bar{c})(5\phi - 3\theta^2 - 3)}{125\phi^2 - 45\phi\theta^2 - 105\phi + 12\theta^2 + 12} \\ e_3^H = \frac{3(\alpha - \bar{c})(5\phi - 4)}{125\phi^2 - 45\phi\theta^2 - 105\phi + 12\theta^2 + 12} \\ e_1^L = \frac{3\theta(\alpha - \bar{c})(5\phi - 4)}{125\phi^2 - 45\phi\theta^2 - 105\phi + 12\theta^2 + 12} \end{cases}$$

Production Costs:

$$\begin{cases} c_1^H = -\frac{20\alpha\phi - 12\alpha\theta^2 - 12\alpha - 125\bar{c}\phi^2 + 45\bar{c}\phi\theta^2 + 85\bar{c}\phi}{125\phi^2 - 45\phi\theta^2 - 105\phi + 12\theta^2 + 12} \\ c_2^H = -\frac{20\alpha\phi - 12\alpha\theta^2 - 12\alpha - 125\bar{c}\phi^2 + 45\bar{c}\phi\theta^2 + 85\bar{c}\phi}{125\phi^2 - 45\phi\theta^2 - 105\phi + 12\theta^2 + 12} \\ c_3^H = -\frac{15\alpha\phi\theta^2 + 15\alpha\phi - 12\alpha\theta^2 - 12\alpha - 125\bar{c}\phi^2 + 30\bar{c}\phi\theta^2 + 90\bar{c}\phi}{125\phi^2 - 45\phi\theta^2 - 105\phi + 12\theta^2 + 12} \\ c_1^L = -\frac{15\alpha\phi\theta^2 + 15\alpha\phi - 12\alpha\theta^2 - 12\alpha - 125\bar{c}\phi^2 + 30\bar{c}\phi\theta^2 + 90\bar{c}\phi}{125\phi^2 - 45\phi\theta^2 - 105\phi + 12\theta^2 + 12} \end{cases}$$

Production Quantities:

$$\begin{cases} q_1^H = \frac{5\phi(\alpha - \bar{c})(5\phi - 3\theta^2 - 3)}{125\phi^2 - 45\phi\theta^2 - 105\phi + 12\theta^2 + 12} \\ q_2^H = \frac{5\phi(\alpha - \bar{c})(5\phi - 3\theta^2 - 3)}{125\phi^2 - 45\phi\theta^2 - 105\phi + 12\theta^2 + 12} \\ q_3^H = \frac{5\phi(\alpha - \bar{c})(5\phi - 4)}{125\phi^2 - 45\phi\theta^2 - 105\phi + 12\theta^2 + 12} \\ q_1^L = \frac{5\phi(\alpha - \bar{c})(5\phi - 4)}{125\phi^2 - 45\phi\theta^2 - 105\phi + 12\theta^2 + 12} \end{cases}$$

Profits:

$$\begin{cases} \pi_1^H = \frac{\phi(\alpha - \bar{c})^2 \cdot (25\phi - 16)(5\phi - 3\theta^2 - 3)^2}{(125\phi^2 - 45\phi\theta^2 - 105\phi + 12\theta^2 + 12)^2} \\ \pi_2^H = \frac{\phi(\alpha - \bar{c})^2 \cdot (25\phi - 16)(5\phi - 3\theta^2 - 3)^2}{(125\phi^2 - 45\phi\theta^2 - 105\phi + 12\theta^2 + 12)^2} \\ \pi_3^H = \frac{\phi(\alpha - \bar{c})^2 (5\phi - 4)^2 \cdot (25\phi - 9)}{(125\phi^2 - 45\phi\theta^2 - 105\phi + 12\theta^2 + 12)^2} \\ \pi_1^L = \frac{\phi(\alpha - \bar{c})^2 (5\phi - 4)^2 \cdot (25\phi - 9\theta^2)}{(125\phi^2 - 45\phi\theta^2 - 105\phi + 12\theta^2 + 12)^2} \end{cases} \quad (82)$$

Total Production:

$$\frac{10\phi(\alpha - \bar{c})(10\phi - 3\theta^2 - 7)}{125\phi^2 - 45\phi\theta^2 - 105\phi + 12\theta^2 + 12}$$

Price:

$$\frac{25\alpha\phi^2 - 15\alpha\phi\theta^2 - 35\alpha\phi + 12\alpha\theta^2 + 12\alpha + 100\bar{c}\phi^2 - 30\bar{c}\phi\theta^2 - 70\bar{c}\phi}{125\phi^2 - 45\phi\theta^2 - 105\phi + 12\theta^2 + 12}$$

Firm Surplus:

$$\frac{\phi(\alpha - \bar{c})^2 \cdot (2500\phi^3 - 1725\phi^2\theta^2 - 4525\phi^2 + 450\phi\theta^4 + 2220\phi\theta^2 + 2570\phi - 288\theta^4 - 720\theta^2 - 432)}{(125\phi^2 - 45\phi\theta^2 - 105\phi + 12\theta^2 + 12)^2}$$

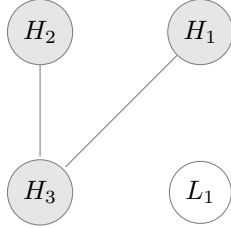
Consumer Surplus:

$$\frac{50\phi^2(\alpha - \bar{c})^2(10\phi - 3\theta^2 - 7)^2}{(125\phi^2 - 45\phi\theta^2 - 105\phi + 12\theta^2 + 12)^2}$$

Social Welfare:

$$\frac{\phi(\alpha - \bar{c})^2 \cdot (7500\phi^3 - 4725\phi^2\theta^2 - 11525\phi^2 + 900\phi\theta^4 + 4320\phi\theta^2 + 5020\phi - 288\theta^4 - 720\theta^2 - 432)}{(125\phi^2 - 45\phi\theta^2 - 105\phi + 12\theta^2 + 12)^2}$$

#### D.3.4 E2A [3H1L]



$$\text{Equations : } \begin{cases} e_1^H \cdot \left(\frac{25\phi}{3} - 3\right) = \alpha - 2e_2^H + 2e_3^H - e_1^L\theta - \bar{c} \\ e_2^H \cdot \left(\frac{25\phi}{3} - 3\right) = \alpha - 2e_1^H + 2e_3^H - e_1^L\theta - \bar{c} \\ e_3^H \cdot \left(\frac{25\phi}{2} - 2\right) = \alpha + 3e_1^H + 3e_2^H - e_1^L\theta - \bar{c} \\ e_1^L \cdot \left(\frac{25\phi}{4\theta} - 4\theta\right) = \alpha - 2e_1^H - 2e_2^H - 3e_3^H - \bar{c} \end{cases}$$

Optimal efforts:

$$\begin{cases} e_1^H = \frac{15\phi(\alpha - \bar{c})(5\phi - 4\theta^2)}{(5\phi + 1)(125\phi^2 - 80\phi\theta^2 - 60\phi + 24\theta^2)} \\ e_2^H = \frac{15\phi(\alpha - \bar{c})(5\phi - 4\theta^2)}{(5\phi + 1)(125\phi^2 - 80\phi\theta^2 - 60\phi + 24\theta^2)} \\ e_3^H = \frac{2(\alpha - \bar{c})(5\phi + 3)(5\phi - 4\theta^2)}{(5\phi + 1)(125\phi^2 - 80\phi\theta^2 - 60\phi + 24\theta^2)} \\ e_1^L = \frac{4\theta(\alpha - \bar{c})(5\phi - 6)}{125\phi^2 - 80\phi\theta^2 - 60\phi + 24\theta^2} \end{cases}$$

Production Costs:

$$\begin{cases} c_1^H = -\frac{125\alpha\phi^2 - 100\alpha\phi\theta^2 + 30\alpha\phi - 24\alpha\theta^2 - 625\bar{c}\phi^3 + 400\bar{c}\phi^2\theta^2 + 50\bar{c}\phi^2 + 60\bar{c}\phi\theta^2 + 30\bar{c}\phi}{(5\phi + 1)(125\phi^2 - 80\phi\theta^2 - 60\phi + 24\theta^2)} \\ c_2^H = -\frac{125\alpha\phi^2 - 100\alpha\phi\theta^2 + 30\alpha\phi - 24\alpha\theta^2 - 625\bar{c}\phi^3 + 400\bar{c}\phi^2\theta^2 + 50\bar{c}\phi^2 + 60\bar{c}\phi\theta^2 + 30\bar{c}\phi}{(5\phi + 1)(125\phi^2 - 80\phi\theta^2 - 60\phi + 24\theta^2)} \\ c_3^H = -\frac{200\alpha\phi^2 - 160\alpha\phi\theta^2 + 30\alpha\phi - 24\alpha\theta^2 - 625\bar{c}\phi^3 + 400\bar{c}\phi^2\theta^2 - 25\bar{c}\phi^2 + 120\bar{c}\phi\theta^2 + 30\bar{c}\phi}{(5\phi + 1)(125\phi^2 - 80\phi\theta^2 - 60\phi + 24\theta^2)} \\ c_1^L = -\frac{20\alpha\phi\theta^2 - 24\alpha\theta^2 - 125\bar{c}\phi^2 + 60\bar{c}\phi\theta^2 + 60\bar{c}\phi}{125\phi^2 - 80\phi\theta^2 - 60\phi + 24\theta^2} \end{cases}$$

Production Quantities:

$$\begin{cases} q_1^H = \frac{25\phi^2(\alpha - \bar{c})(5\phi - 4\theta^2)}{(5\phi + 1)(125\phi^2 - 80\phi\theta^2 - 60\phi + 24\theta^2)} \\ q_2^H = \frac{25\phi^2(\alpha - \bar{c})(5\phi - 4\theta^2)}{(5\phi + 1)(125\phi^2 - 80\phi\theta^2 - 60\phi + 24\theta^2)} \\ q_3^H = \frac{5\phi(\alpha - \bar{c})(5\phi + 3)(5\phi - 4\theta^2)}{(5\phi + 1)(125\phi^2 - 80\phi\theta^2 - 60\phi + 24\theta^2)} \\ q_1^L = \frac{5\phi(\alpha - \bar{c})(5\phi - 6)}{125\phi^2 - 80\phi\theta^2 - 60\phi + 24\theta^2} \end{cases}$$

Profits:

$$\begin{cases} \pi_1^H = \frac{25\phi^3(\alpha - \bar{c})^2(5\phi - 4\theta^2)^2 \cdot (25\phi - 9)}{(5\phi + 1)^2(125\phi^2 - 80\phi\theta^2 - 60\phi + 24\theta^2)^2} \\ \pi_2^H = \frac{25\phi^3(\alpha - \bar{c})^2(5\phi - 4\theta^2)^2 \cdot (25\phi - 9)}{(5\phi + 1)^2(125\phi^2 - 80\phi\theta^2 - 60\phi + 24\theta^2)^2} \\ \pi_3^H = \frac{\phi(\alpha - \bar{c})^2(5\phi + 3)^2(5\phi - 4\theta^2)^2 \cdot (25\phi - 4)}{(5\phi + 1)^2(125\phi^2 - 80\phi\theta^2 - 60\phi + 24\theta^2)^2} \\ \pi_1^L = \frac{\phi(\alpha - \bar{c})^2(5\phi - 6)^2 \cdot (25\phi - 16\theta^2)}{(125\phi^2 - 80\phi\theta^2 - 60\phi + 24\theta^2)^2} \end{cases} \quad (83)$$

Total Production:

$$\frac{10\phi(\alpha - \bar{c})(10\phi - 6\theta^2 - 3)}{125\phi^2 - 80\phi\theta^2 - 60\phi + 24\theta^2}$$

Price:

$$\frac{25\alpha\phi^2 - 20\alpha\phi\theta^2 - 30\alpha\phi + 24\alpha\theta^2 + 100\bar{c}\phi^2 - 60\bar{c}\phi\theta^2 - 30\bar{c}\phi}{125\phi^2 - 80\phi\theta^2 - 60\phi + 24\theta^2}$$

Firm Surplus:

$$\frac{2\phi(\alpha - \bar{c})^2 \cdot (93750\phi^5 - 42500\phi^4\theta^2 - 13125\phi^4 + 15000\phi^3\theta^4 + 6000\phi^3\theta^2 + 5375\phi^3 + 1600\phi^2\theta^4 - 4700\phi^2\theta^2 + 3300\phi^2 + 840\phi\theta^4 - 1680\phi\theta^2 + 450\phi - 288\theta^4 - 288\theta^2)}{(5\phi + 1)^2 (125\phi^2 - 80\phi\theta^2 - 60\phi + 24\theta^2)^2}$$

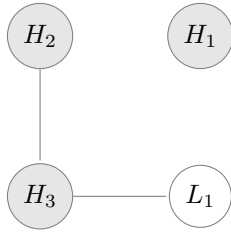
Consumer Surplus:

$$\frac{50\phi^2 (\alpha - \bar{c})^2 (10\phi - 6\theta^2 - 3)^2}{(125\phi^2 - 80\phi\theta^2 - 60\phi + 24\theta^2)^2}$$

Social Welfare:

$$\frac{2\phi(\alpha - \bar{c})^2 \cdot (93750\phi^5 - 117500\phi^4\theta^2 - 25625\phi^4 + 37500\phi^3\theta^4 - 15000\phi^3\theta^2 - 1500\phi^3 + 10600\phi^2\theta^4 + 1300\phi^2\theta^2 + 4050\phi^2 + 1740\phi\theta^4 - 780\phi\theta^2 + 675\phi - 288\theta^4 - 288\theta^2)}{(5\phi + 1)^2 (125\phi^2 - 80\phi\theta^2 - 60\phi + 24\theta^2)^2}$$

### D.3.5 E2B [3H1L]



$$\text{Equations : } \begin{cases} e_1^H \cdot \left( \frac{25\phi}{4} - 4 \right) = \alpha - 2e_2^H - 3e_3^H - 2e_1^L\theta - \bar{c} \\ e_2^H \cdot \left( \frac{25\phi}{3} - 3 \right) = \alpha - e_1^H + 2e_3^H - 2e_1^L\theta - \bar{c} \\ e_3^H \cdot \left( \frac{25\phi}{2} - 2 \right) = \alpha - e_1^H + 3e_2^H + 3e_1^L\theta - \bar{c} \\ e_1^L \cdot \left( \frac{25\phi}{3\theta} - 3\theta \right) = \alpha - e_1^H - 2e_2^H + 2e_3^H - \bar{c} \end{cases}$$

Optimal efforts:

$$\begin{cases} e_1^H = \frac{4(\alpha - \bar{c})(125\phi^3 - 75\phi^2\theta^2 - 125\phi^2 + 45\phi\theta^2 + 18\theta^2)}{3125\phi^4 - 1125\phi^3\theta^2 - 3625\phi^3 + 825\phi^2\theta^2 + 800\phi^2 + 180\phi\theta^2 - 72\theta^2} \\ e_2^H = \frac{15\phi(\alpha - \bar{c})(5\phi - 4)(5\phi - 3\theta^2)}{3125\phi^4 - 1125\phi^3\theta^2 - 3625\phi^3 + 825\phi^2\theta^2 + 800\phi^2 + 180\phi\theta^2 - 72\theta^2} \\ e_3^H = \frac{2(\alpha - \bar{c})(5\phi - 4)(5\phi - 3\theta)(5\phi + 3\theta)}{3125\phi^4 - 1125\phi^3\theta^2 - 3625\phi^3 + 825\phi^2\theta^2 + 800\phi^2 + 180\phi\theta^2 - 72\theta^2} \\ e_1^L = \frac{15\phi\theta(\alpha - \bar{c})(5\phi - 4)(5\phi - 3)}{3125\phi^4 - 1125\phi^3\theta^2 - 3625\phi^3 + 825\phi^2\theta^2 + 800\phi^2 + 180\phi\theta^2 - 72\theta^2} \end{cases}$$

Production Costs:

$$\begin{cases} c_1^H = -\frac{500\alpha\phi^3 - 300\alpha\phi^2\theta^2 - 500\alpha\phi^2 + 180\alpha\phi\theta^2 + 72\alpha\theta^2 - 3125\bar{c}\phi^4 + 1125\bar{c}\phi^3\theta^2 + 3125\bar{c}\phi^3 - 525\bar{c}\phi^2\theta^2 - 300\bar{c}\phi^2 - 360\bar{c}\phi\theta^2}{3125\phi^4 - 1125\phi^3\theta^2 - 3625\phi^3 + 825\phi^2\theta^2 + 800\phi^2 + 180\phi\theta^2 - 72\theta^2} \\ c_2^H = -\frac{625\alpha\phi^3 - 225\alpha\phi^2\theta^2 - 500\alpha\phi^2 + 90\alpha\phi\theta^2 + 72\alpha\theta^2 - 3125\bar{c}\phi^4 + 1125\bar{c}\phi^3\theta^2 + 3000\bar{c}\phi^3 - 600\bar{c}\phi^2\theta^2 - 300\bar{c}\phi^2 - 270\bar{c}\phi\theta^2}{3125\phi^4 - 1125\phi^3\theta^2 - 3625\phi^3 + 825\phi^2\theta^2 + 800\phi^2 + 180\phi\theta^2 - 72\theta^2} \\ c_3^H = -\frac{375\alpha\phi^3\theta^2 + 625\alpha\phi^3 - 750\alpha\phi^2\theta^2 - 500\alpha\phi^2 + 270\alpha\phi\theta^2 + 72\alpha\theta^2 - 3125\bar{c}\phi^4 + 750\bar{c}\phi^3\theta^2 + 3000\bar{c}\phi^3 - 75\bar{c}\phi^2\theta^2 - 300\bar{c}\phi^2 - 450\bar{c}\phi\theta^2}{3125\phi^4 - 1125\phi^3\theta^2 - 3625\phi^3 + 825\phi^2\theta^2 + 800\phi^2 + 180\phi\theta^2 - 72\theta^2} \\ c_1^L = -\frac{375\alpha\phi^3\theta^2 + 250\alpha\phi^3 - 525\alpha\phi^2\theta^2 - 200\alpha\phi^2 + 90\alpha\phi\theta^2 + 72\alpha\theta^2 - 3125\bar{c}\phi^4 + 750\bar{c}\phi^3\theta^2 + 3375\bar{c}\phi^3 - 300\bar{c}\phi^2\theta^2 - 600\bar{c}\phi^2 - 270\bar{c}\phi\theta^2}{3125\phi^4 - 1125\phi^3\theta^2 - 3625\phi^3 + 825\phi^2\theta^2 + 800\phi^2 + 180\phi\theta^2 - 72\theta^2} \end{cases}$$

Production Quantities:

$$\begin{cases} q_1^H = \frac{5\phi(\alpha - \bar{c})(125\phi^3 - 75\phi^2\theta^2 - 125\phi^2 + 45\phi\theta^2 + 18\theta^2)}{3125\phi^4 - 1125\phi^3\theta^2 - 3625\phi^3 + 825\phi^2\theta^2 + 800\phi^2 + 180\phi\theta^2 - 72\theta^2} \\ q_2^H = \frac{25\phi^2(\alpha - \bar{c})(5\phi - 4)(5\phi - 3\theta^2)}{3125\phi^4 - 1125\phi^3\theta^2 - 3625\phi^3 + 825\phi^2\theta^2 + 800\phi^2 + 180\phi\theta^2 - 72\theta^2} \\ q_3^H = \frac{5\phi(\alpha - \bar{c})(5\phi - 4)(5\phi - 3\theta)(5\phi + 3\theta)}{3125\phi^4 - 1125\phi^3\theta^2 - 3625\phi^3 + 825\phi^2\theta^2 + 800\phi^2 + 180\phi\theta^2 - 72\theta^2} \\ q_1^L = \frac{25\phi^2(\alpha - \bar{c})(5\phi - 4)(5\phi - 3)}{3125\phi^4 - 1125\phi^3\theta^2 - 3625\phi^3 + 825\phi^2\theta^2 + 800\phi^2 + 180\phi\theta^2 - 72\theta^2} \end{cases}$$

Profits:

$$\begin{cases} \pi_1^H = \frac{\phi(\alpha - \bar{c})^2 \cdot (25\phi - 16)(125\phi^3 - 75\phi^2\theta^2 - 125\phi^2 + 45\phi\theta^2 + 18\theta^2)^2}{(3125\phi^4 - 1125\phi^3\theta^2 - 3625\phi^3 + 825\phi^2\theta^2 + 800\phi^2 + 180\phi\theta^2 - 72\theta^2)^2} \\ \pi_2^H = \frac{25\phi^3(\alpha - \bar{c})^2(5\phi - 4)^2(5\phi - 3\theta^2)^2 \cdot (25\phi - 9)}{(3125\phi^4 - 1125\phi^3\theta^2 - 3625\phi^3 + 825\phi^2\theta^2 + 800\phi^2 + 180\phi\theta^2 - 72\theta^2)^2} \\ \pi_3^H = \frac{\phi(\alpha - \bar{c})^2(5\phi - 4)^2(5\phi - 3\theta)^2(5\phi + 3\theta)^2 \cdot (25\phi - 4)}{(3125\phi^4 - 1125\phi^3\theta^2 - 3625\phi^3 + 825\phi^2\theta^2 + 800\phi^2 + 180\phi\theta^2 - 72\theta^2)^2} \\ \pi_1^L = \frac{25\phi^3(\alpha - \bar{c})^2(5\phi - 4)^2(5\phi - 3)^2 \cdot (25\phi - 9\theta^2)}{(3125\phi^4 - 1125\phi^3\theta^2 - 3625\phi^3 + 825\phi^2\theta^2 + 800\phi^2 + 180\phi\theta^2 - 72\theta^2)^2} \end{cases} \quad (84)$$

Total Production:

$$\frac{10\phi(\alpha - \bar{c})(250\phi^3 - 75\phi^2\theta^2 - 250\phi^2 + 30\phi\theta^2 + 30\phi + 27\theta^2)}{3125\phi^4 - 1125\phi^3\theta^2 - 3625\phi^3 + 825\phi^2\theta^2 + 800\phi^2 + 180\phi\theta^2 - 72\theta^2}$$

Price:

$$\frac{625\alpha\phi^4 - 375\alpha\phi^3\theta^2 - 1125\alpha\phi^3 + 525\alpha\phi^2\theta^2 + 500\alpha\phi^2 - 90\alpha\phi\theta^2 - 72\alpha\theta^2 + 2500\bar{c}\phi^4 - 750\bar{c}\phi^3\theta^2 - 2500\bar{c}\phi^3 + 300\bar{c}\phi^2\theta^2 + 300\bar{c}\phi^2 + 270\bar{c}\phi\theta^2}{3125\phi^4 - 1125\phi^3\theta^2 - 3625\phi^3 + 825\phi^2\theta^2 + 800\phi^2 + 180\phi\theta^2 - 72\theta^2}$$

Firm Surplus:

$$\frac{\phi(\alpha - \bar{c})^2 \cdot (1562500\phi^7 - 1078125\phi^6\theta^2 - 3578125\phi^6 + 281250\phi^5\theta^4 + 2081250\phi^5\theta^2 + 2856250\phi^5 - 534375\phi^4\theta^4 - 1134375\phi^4\theta^2 - 905000\phi^4 + 312750\phi^3\theta^4 + 40500\phi^3\theta^2 + 90000\phi^3 - 70200\phi^2\theta^4 + 68400\phi^2\theta^2 + 27540\phi\theta^4 - 10368\theta^4)}{(3125\phi^4 - 1125\phi^3\theta^2 - 3625\phi^3 + 825\phi^2\theta^2 + 800\phi^2 + 180\phi\theta^2 - 72\theta^2)^2}$$

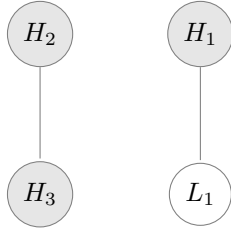
Consumer Surplus:

$$\frac{50\phi^2(\alpha - \bar{c})^2(250\phi^3 - 75\phi^2\theta^2 - 250\phi^2 + 30\phi\theta^2 + 30\phi + 27\theta^2)^2}{(3125\phi^4 - 1125\phi^3\theta^2 - 3625\phi^3 + 825\phi^2\theta^2 + 800\phi^2 + 180\phi\theta^2 - 72\theta^2)^2}$$

Social Welfare:

$$\frac{\phi(\alpha - \bar{c})^2 \cdot (4687500\phi^7 - 2953125\phi^6\theta^2 - 9828125\phi^6 + 562500\phi^5\theta^4 + 4706250\phi^5\theta^2 + 6731250\phi^5 - 759375\phi^4\theta^4 - 1434375\phi^4\theta^2 - 1655000\phi^4 + 155250\phi^3\theta^4 - 544500\phi^3\theta^2 + 135000\phi^3 + 10800\phi^2\theta^4 + 149400\phi^2\theta^2 + 63990\phi\theta^4 - 10368\theta^4)}{(3125\phi^4 - 1125\phi^3\theta^2 - 3625\phi^3 + 825\phi^2\theta^2 + 800\phi^2 + 180\phi\theta^2 - 72\theta^2)^2}$$

### D.3.6 E2C [3H1L]



$$\text{Equations : } \begin{cases} e_1^H \cdot \left(\frac{25\phi}{3} - 3\right) = \alpha - 2e_2^H - 2e_3^H + 3e_1^L\theta - \bar{c} \\ e_2^H \cdot \left(\frac{25\phi}{3} - 3\right) = \alpha - 2e_1^H + 3e_3^H - 2e_1^L\theta - \bar{c} \\ e_3^H \cdot \left(\frac{25\phi}{3} - 3\right) = \alpha - 2e_1^H + 3e_2^H - 2e_1^L\theta - \bar{c} \\ e_1^L \cdot \left(\frac{25\phi}{3\theta} - 3\theta\right) = \alpha + 3e_1^H - 2e_2^H - 2e_3^H - \bar{c} \end{cases}$$

Optimal efforts:

$$\begin{cases} e_1^H = \frac{3(\alpha - \bar{c})(5\phi - 6)}{125\phi^2 - 45\phi\theta^2 - 135\phi + 18\theta^2 + 18} \\ e_2^H = \frac{3(\alpha - \bar{c})(5\phi - 3\theta^2 - 3)}{125\phi^2 - 45\phi\theta^2 - 135\phi + 18\theta^2 + 18} \\ e_3^H = \frac{3(\alpha - \bar{c})(5\phi - 3\theta^2 - 3)}{125\phi^2 - 45\phi\theta^2 - 135\phi + 18\theta^2 + 18} \\ e_1^L = \frac{3\theta(\alpha - \bar{c})(5\phi - 6)}{125\phi^2 - 45\phi\theta^2 - 135\phi + 18\theta^2 + 18} \end{cases}$$

Production Costs:

$$\begin{cases} c_1^H = -\frac{15\alpha\phi\theta^2 + 15\alpha\phi - 18\alpha\theta^2 - 18\alpha - 125\bar{c}\phi^2 + 30\bar{c}\phi\theta^2 + 120\bar{c}\phi}{125\phi^2 - 45\phi\theta^2 - 135\phi + 18\theta^2 + 18} \\ c_2^H = -\frac{30\alpha\phi - 18\alpha\theta^2 - 18\alpha - 125\bar{c}\phi^2 + 45\bar{c}\phi\theta^2 + 105\bar{c}\phi}{125\phi^2 - 45\phi\theta^2 - 135\phi + 18\theta^2 + 18} \\ c_3^H = -\frac{30\alpha\phi - 18\alpha\theta^2 - 18\alpha - 125\bar{c}\phi^2 + 45\bar{c}\phi\theta^2 + 105\bar{c}\phi}{125\phi^2 - 45\phi\theta^2 - 135\phi + 18\theta^2 + 18} \\ c_1^L = -\frac{15\alpha\phi\theta^2 + 15\alpha\phi - 18\alpha\theta^2 - 18\alpha - 125\bar{c}\phi^2 + 30\bar{c}\phi\theta^2 + 120\bar{c}\phi}{125\phi^2 - 45\phi\theta^2 - 135\phi + 18\theta^2 + 18} \end{cases}$$

Production Quantities:

$$\begin{cases} q_1^H = \frac{5\phi(\alpha - \bar{c})(5\phi - 6)}{125\phi^2 - 45\phi\theta^2 - 135\phi + 18\theta^2 + 18} \\ q_2^H = \frac{5\phi(\alpha - \bar{c})(5\phi - 3\theta^2 - 3)}{125\phi^2 - 45\phi\theta^2 - 135\phi + 18\theta^2 + 18} \\ q_3^H = \frac{5\phi(\alpha - \bar{c})(5\phi - 3\theta^2 - 3)}{125\phi^2 - 45\phi\theta^2 - 135\phi + 18\theta^2 + 18} \\ q_1^L = \frac{5\phi(\alpha - \bar{c})(5\phi - 6)}{125\phi^2 - 45\phi\theta^2 - 135\phi + 18\theta^2 + 18} \end{cases}$$

Profits:

$$\begin{cases} \pi_1^H &= \frac{\phi(\alpha-\bar{c})^2(5\phi-6)^2 \cdot (25\phi-9)}{(125\phi^2-45\phi\theta^2-135\phi+18\theta^2+18)^2} \\ \pi_2^H &= \frac{\phi(\alpha-\bar{c})^2 \cdot (25\phi-9)(5\phi-3\theta^2-3)^2}{(125\phi^2-45\phi\theta^2-135\phi+18\theta^2+18)^2} \\ \pi_3^H &= \frac{\phi(\alpha-\bar{c})^2 \cdot (25\phi-9)(5\phi-3\theta^2-3)^2}{(125\phi^2-45\phi\theta^2-135\phi+18\theta^2+18)^2} \\ \pi_1^L &= \frac{\phi(\alpha-\bar{c})^2(5\phi-6)^2 \cdot (25\phi-9\theta^2)}{(125\phi^2-45\phi\theta^2-135\phi+18\theta^2+18)^2} \end{cases} \quad (85)$$

Total Production:

$$\frac{10\phi(\alpha-\bar{c})(10\phi-3\theta^2-9)}{125\phi^2-45\phi\theta^2-135\phi+18\theta^2+18}$$

Price:

$$\frac{25\alpha\phi^2-15\alpha\phi\theta^2-45\alpha\phi+18\alpha\theta^2+18\alpha+100\bar{c}\phi^2-30\bar{c}\phi\theta^2-90\bar{c}\phi}{125\phi^2-45\phi\theta^2-135\phi+18\theta^2+18}$$

Firm Surplus:

$$\frac{\phi(\alpha-\bar{c})^2 \cdot (2500\phi^3-1725\phi^2\theta^2-5175\phi^2+450\phi\theta^4+1980\phi\theta^2+3330\phi-162\theta^4-648\theta^2-486)}{(125\phi^2-45\phi\theta^2-135\phi+18\theta^2+18)^2}$$

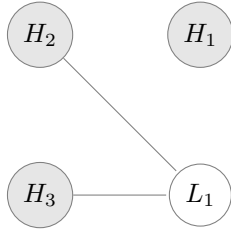
Consumer Surplus:

$$\frac{50\phi^2(\alpha-\bar{c})^2(10\phi-3\theta^2-9)^2}{(125\phi^2-45\phi\theta^2-135\phi+18\theta^2+18)^2}$$

Social Welfare:

$$\frac{3\phi(\alpha-\bar{c})^2 \cdot (2500\phi^3-1575\phi^2\theta^2-4725\phi^2+300\phi\theta^4+1560\phi\theta^2+2460\phi-54\theta^4-216\theta^2-162)}{(125\phi^2-45\phi\theta^2-135\phi+18\theta^2+18)^2}$$

### D.3.7 E2D [3H1L]



$$\text{Equations : } \begin{cases} e_1^H \cdot \left(\frac{25\phi}{4}-4\right) = \alpha - 2e_2^H - 2e_3^H - 3e_1^L\theta - \bar{c} \\ e_2^H \cdot \left(\frac{25\phi}{3}-3\right) = \alpha - e_1^H - 2e_3^H + 2e_1^L\theta - \bar{c} \\ e_3^H \cdot \left(\frac{25\phi}{3}-3\right) = \alpha - e_1^H - 2e_2^H + 2e_1^L\theta - \bar{c} \\ e_1^L \cdot \left(\frac{25\phi}{2\theta}-2\theta\right) = \alpha - e_1^H + 3e_2^H + 3e_3^H - \bar{c} \end{cases}$$

Optimal efforts:

$$\begin{cases} e_1^H &= \frac{4(\alpha-\bar{c})(25\phi^2-10\phi\theta^2-15\phi-6\theta^2)}{625\phi^3-100\phi^2\theta^2-475\phi^2-20\phi\theta^2+24\theta^2} \\ e_2^H &= \frac{15\phi(\alpha-\bar{c})(5\phi-4)}{625\phi^3-100\phi^2\theta^2-475\phi^2-20\phi\theta^2+24\theta^2} \\ e_3^H &= \frac{15\phi(\alpha-\bar{c})(5\phi-4)}{625\phi^3-100\phi^2\theta^2-475\phi^2-20\phi\theta^2+24\theta^2} \\ e_1^L &= \frac{2\theta(\alpha-\bar{c})(5\phi-4)(5\phi+3)}{625\phi^3-100\phi^2\theta^2-475\phi^2-20\phi\theta^2+24\theta^2} \end{cases}$$

Production Costs:

$$\begin{cases} c_1^H &= -\frac{100\alpha\phi^2-40\alpha\phi\theta^2-60\alpha\phi-24\alpha\theta^2-625\bar{c}\phi^3+100\bar{c}\phi^2\theta^2+375\bar{c}\phi^2+60\bar{c}\phi\theta^2+60\bar{c}\phi}{625\phi^3-100\phi^2\theta^2-475\phi^2-20\phi\theta^2+24\theta^2} \\ c_2^H &= -\frac{50\alpha\phi^2\theta^2+75\alpha\phi^2-10\alpha\phi\theta^2-60\alpha\phi-24\alpha\theta^2-625\bar{c}\phi^3+50\bar{c}\phi^2\theta^2+400\bar{c}\phi^2+30\bar{c}\phi\theta^2+60\bar{c}\phi}{625\phi^3-100\phi^2\theta^2-475\phi^2-20\phi\theta^2+24\theta^2} \\ c_3^H &= -\frac{50\alpha\phi^2\theta^2+75\alpha\phi^2-10\alpha\phi\theta^2-60\alpha\phi-24\alpha\theta^2-625\bar{c}\phi^3+50\bar{c}\phi^2\theta^2+400\bar{c}\phi^2+30\bar{c}\phi\theta^2+60\bar{c}\phi}{625\phi^3-100\phi^2\theta^2-475\phi^2-20\phi\theta^2+24\theta^2} \\ c_1^L &= -\frac{50\alpha\phi^2\theta^2+150\alpha\phi^2-10\alpha\phi\theta^2-120\alpha\phi-24\alpha\theta^2-625\bar{c}\phi^3+50\bar{c}\phi^2\theta^2+325\bar{c}\phi^2+30\bar{c}\phi\theta^2+120\bar{c}\phi}{625\phi^3-100\phi^2\theta^2-475\phi^2-20\phi\theta^2+24\theta^2} \end{cases}$$

Production Quantities:

$$\begin{cases} q_1^H &= \frac{5\phi(\alpha-\bar{c})(25\phi^2-10\phi\theta^2-15\phi-6\theta^2)}{625\phi^3-100\phi^2\theta^2-475\phi^2-20\phi\theta^2+24\theta^2} \\ q_2^H &= \frac{25\phi^2(\alpha-\bar{c})(5\phi-4)}{625\phi^3-100\phi^2\theta^2-475\phi^2-20\phi\theta^2+24\theta^2} \\ q_3^H &= \frac{25\phi^2(\alpha-\bar{c})(5\phi-4)}{625\phi^3-100\phi^2\theta^2-475\phi^2-20\phi\theta^2+24\theta^2} \\ q_1^L &= \frac{5\phi(\alpha-\bar{c})(5\phi-4)(5\phi+3)}{625\phi^3-100\phi^2\theta^2-475\phi^2-20\phi\theta^2+24\theta^2} \end{cases}$$

Profits:

$$\begin{cases} \pi_1^H = \frac{\phi(\alpha-\bar{c})^2 \cdot (25\phi-16)(25\phi^2-10\phi\theta^2-15\phi-6\theta^2)^2}{(625\phi^3-100\phi^2\theta^2-475\phi^2-20\phi\theta^2+24\theta^2)^2} \\ \pi_2^H = \frac{25\phi^3(\alpha-\bar{c})^2(5\phi-4)^2 \cdot (25\phi-9)}{(625\phi^3-100\phi^2\theta^2-475\phi^2-20\phi\theta^2+24\theta^2)^2} \\ \pi_3^H = \frac{25\phi^3(\alpha-\bar{c})^2(5\phi-4)^2 \cdot (25\phi-9)}{(625\phi^3-100\phi^2\theta^2-475\phi^2-20\phi\theta^2+24\theta^2)^2} \\ \pi_1^L = \frac{\phi(\alpha-\bar{c})^2(5\phi-4)^2(5\phi+3)^2 \cdot (25\phi-4\theta^2)}{(625\phi^3-100\phi^2\theta^2-475\phi^2-20\phi\theta^2+24\theta^2)^2} \end{cases} \quad (86)$$

Total Production:

$$\frac{10\phi(\alpha-\bar{c})(50\phi^2-5\phi\theta^2-30\phi-3\theta^2-6)}{625\phi^3-100\phi^2\theta^2-475\phi^2-20\phi\theta^2+24\theta^2}$$

Price:

$$\frac{125\alpha\phi^3-50\alpha\phi^2\theta^2-175\alpha\phi^2+10\alpha\phi\theta^2+60\alpha\phi+24\alpha\theta^2+500\bar{c}\phi^3-50\bar{c}\phi^2\theta^2-300\bar{c}\phi^2-30\bar{c}\phi\theta^2-60\bar{c}\phi}{625\phi^3-100\phi^2\theta^2-475\phi^2-20\phi\theta^2+24\theta^2}$$

Firm Surplus:

$$\frac{2\phi(\alpha-\bar{c})^2 \cdot (31250\phi^5-7500\phi^4\theta^2-48125\phi^4+1250\phi^3\theta^4+4500\phi^3\theta^2+20625\phi^3+700\phi^2\theta^4+3400\phi^2\theta^2-3900\phi^2-510\phi\theta^4-1680\phi\theta^2+1800\phi-288\theta^4-288\theta^2)}{(625\phi^3-100\phi^2\theta^2-475\phi^2-20\phi\theta^2+24\theta^2)^2}$$

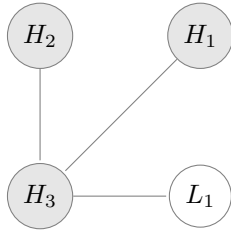
Consumer Surplus:

$$\frac{50\phi^2(\alpha-\bar{c})^2(50\phi^2-5\phi\theta^2-30\phi-3\theta^2-6)^2}{(625\phi^3-100\phi^2\theta^2-475\phi^2-20\phi\theta^2+24\theta^2)^2}$$

Social Welfare:

$$\frac{2\phi(\alpha-\bar{c})^2 \cdot (93750\phi^5-20000\phi^4\theta^2-123125\phi^4+1875\phi^3\theta^4+4500\phi^3\theta^2+28125\phi^3+1450\phi^2\theta^4+9400\phi^2\theta^2+5100\phi^2-285\phi\theta^4-780\phi\theta^2+2700\phi-288\theta^4-288\theta^2)}{(625\phi^3-100\phi^2\theta^2-475\phi^2-20\phi\theta^2+24\theta^2)^2}$$

### D.3.8 E3A [3H1L]



$$\text{Equations : } \begin{cases} e_1^H \cdot \left(\frac{25\phi}{3} - 3\right) = \alpha - 2e_2^H + e_3^H - 2e_1^L\theta - \bar{c} \\ e_2^H \cdot \left(\frac{25\phi}{3} - 3\right) = \alpha - 2e_1^H + e_3^H - 2e_1^L\theta - \bar{c} \\ e_3^H \cdot (25\phi - 1) = \alpha + 3e_1^H + 3e_2^H + 3e_1^L\theta - \bar{c} \\ e_1^L \cdot \left(\frac{25\phi}{3\theta} - 3\theta\right) = \alpha - 2e_1^H - 2e_2^H + e_3^H - \bar{c} \end{cases}$$

Optimal efforts:

$$\begin{cases} e_1^H = \frac{15\phi(\alpha-\bar{c})(5\phi-3\theta^2)}{625\phi^3-225\phi^2\theta^2-100\phi^2-45\phi\theta^2-15\phi+18\theta^2} \\ e_2^H = \frac{15\phi(\alpha-\bar{c})(5\phi-3\theta^2)}{625\phi^3-225\phi^2\theta^2-100\phi^2-45\phi\theta^2-15\phi+18\theta^2} \\ e_3^H = \frac{(\alpha-\bar{c})(25\phi^2+15\phi-18\theta^2)}{625\phi^3-225\phi^2\theta^2-100\phi^2-45\phi\theta^2-15\phi+18\theta^2} \\ e_1^L = \frac{15\phi\theta(\alpha-\bar{c})(5\phi-3)}{625\phi^3-225\phi^2\theta^2-100\phi^2-45\phi\theta^2-15\phi+18\theta^2} \end{cases}$$

Production Costs:

$$\begin{cases} c_1^H = -\frac{100\alpha\phi^2-45\alpha\phi\theta^2+15\alpha\phi-18\alpha\theta^2-625\bar{c}\phi^3+225\bar{c}\phi^2\theta^2+90\bar{c}\phi\theta^2}{625\phi^3-225\phi^2\theta^2-100\phi^2-45\phi\theta^2-15\phi+18\theta^2} \\ c_2^H = -\frac{100\alpha\phi^2-45\alpha\phi\theta^2+15\alpha\phi-18\alpha\theta^2-625\bar{c}\phi^3+225\bar{c}\phi^2\theta^2+90\bar{c}\phi\theta^2}{625\phi^3-225\phi^2\theta^2-100\phi^2-45\phi\theta^2-15\phi+18\theta^2} \\ c_3^H = -\frac{75\alpha\phi^2\theta^2+175\alpha\phi^2-135\alpha\phi\theta^2+15\alpha\phi-18\alpha\theta^2-625\bar{c}\phi^3+150\bar{c}\phi^2\theta^2-75\bar{c}\phi^2+180\bar{c}\phi\theta^2}{625\phi^3-225\phi^2\theta^2-100\phi^2-45\phi\theta^2-15\phi+18\theta^2} \\ c_1^L = -\frac{75\alpha\phi^2\theta^2+25\alpha\phi^2-45\alpha\phi\theta^2+15\alpha\phi-18\alpha\theta^2-625\bar{c}\phi^3+150\bar{c}\phi^2\theta^2+75\bar{c}\phi^2+90\bar{c}\phi\theta^2}{625\phi^3-225\phi^2\theta^2-100\phi^2-45\phi\theta^2-15\phi+18\theta^2} \end{cases}$$

Production Quantities:

$$\begin{cases} q_1^H &= \frac{25\phi^2(\alpha-\bar{c})(5\phi-3\theta^2)}{625\phi^3-225\phi^2\theta^2-100\phi^2-45\phi\theta^2-15\phi+18\theta^2} \\ q_2^H &= \frac{25\phi^2(\alpha-\bar{c})(5\phi-3\theta^2)}{625\phi^3-225\phi^2\theta^2-100\phi^2-45\phi\theta^2-15\phi+18\theta^2} \\ q_3^H &= \frac{5\phi(\alpha-\bar{c})(25\phi^2+15\phi-18\theta^2)}{625\phi^3-225\phi^2\theta^2-100\phi^2-45\phi\theta^2-15\phi+18\theta^2} \\ q_1^L &= \frac{25\phi^2(\alpha-\bar{c})(5\phi-3)}{625\phi^3-225\phi^2\theta^2-100\phi^2-45\phi\theta^2-15\phi+18\theta^2} \end{cases}$$

Profits:

$$\begin{cases} \pi_1^H &= \frac{25\phi^3(\alpha-\bar{c})^2(5\phi-3\theta^2)^2 \cdot (25\phi-9)}{(625\phi^3-225\phi^2\theta^2-100\phi^2-45\phi\theta^2-15\phi+18\theta^2)^2} \\ \pi_2^H &= \frac{25\phi^3(\alpha-\bar{c})^2(5\phi-3\theta^2)^2 \cdot (25\phi-9)}{(625\phi^3-225\phi^2\theta^2-100\phi^2-45\phi\theta^2-15\phi+18\theta^2)^2} \\ \pi_3^H &= \frac{\phi(\alpha-\bar{c})^2 \cdot (25\phi-1)(25\phi^2+15\phi-18\theta^2)^2}{(625\phi^3-225\phi^2\theta^2-100\phi^2-45\phi\theta^2-15\phi+18\theta^2)^2} \\ \pi_1^L &= \frac{25\phi^3(\alpha-\bar{c})^2(5\phi-3)^2 \cdot (25\phi-9\theta^2)}{(625\phi^3-225\phi^2\theta^2-100\phi^2-45\phi\theta^2-15\phi+18\theta^2)^2} \end{cases} \quad (87)$$

Total Production:

$$\frac{10\phi(\alpha-\bar{c})(50\phi^2-15\phi\theta^2-9\theta^2)}{625\phi^3-225\phi^2\theta^2-100\phi^2-45\phi\theta^2-15\phi+18\theta^2}$$

Price:

$$\frac{125\alpha\phi^3-75\alpha\phi^2\theta^2-100\alpha\phi^2+45\alpha\phi\theta^2-15\alpha\phi+18\alpha\theta^2+500\bar{c}\phi^3-150\bar{c}\phi^2\theta^2-90\bar{c}\phi\theta^2}{625\phi^3-225\phi^2\theta^2-100\phi^2-45\phi\theta^2-15\phi+18\theta^2}$$

Firm Surplus:

$$\frac{\phi(\alpha-\bar{c})^2 \cdot (62500\phi^5-43125\phi^4\theta^2-11875\phi^4+11250\phi^3\theta^4-2250\phi^3\theta^2+10500\phi^3-4050\phi^2\theta^4-14625\phi^2\theta^2-225\phi^2+8100\phi\theta^4+540\phi\theta^2-324\theta^4)}{(625\phi^3-225\phi^2\theta^2-100\phi^2-45\phi\theta^2-15\phi+18\theta^2)^2}$$

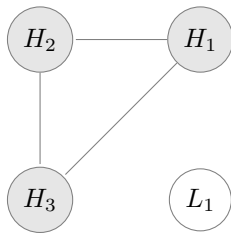
Consumer Surplus:

$$\frac{50\phi^2(\alpha-\bar{c})^2(50\phi^2-15\phi\theta^2-9\theta^2)^2}{(625\phi^3-225\phi^2\theta^2-100\phi^2-45\phi\theta^2-15\phi+18\theta^2)^2}$$

Social Welfare:

$$\frac{\phi(\alpha-\bar{c})^2 \cdot (187500\phi^5-118125\phi^4\theta^2-11875\phi^4+22500\phi^3\theta^4-47250\phi^3\theta^2+10500\phi^3+9450\phi^2\theta^4-14625\phi^2\theta^2-225\phi^2+12150\phi\theta^4+540\phi\theta^2-324\theta^4)}{(625\phi^3-225\phi^2\theta^2-100\phi^2-45\phi\theta^2-15\phi+18\theta^2)^2}$$

### D.3.9 E3B [3H1L]



$$\text{Equations : } \begin{cases} e_1^H \cdot \left(\frac{25\phi}{2} - 2\right) = \alpha + 2e_2^H + 2e_3^H - e_1^L\theta - \bar{c} \\ e_2^H \cdot \left(\frac{25\phi}{2} - 2\right) = \alpha + 2e_1^H + 2e_3^H - e_1^L\theta - \bar{c} \\ e_3^H \cdot \left(\frac{25\phi}{2} - 2\right) = \alpha + 2e_1^H + 2e_2^H - e_1^L\theta - \bar{c} \\ e_1^L \cdot \left(\frac{25\phi}{4\theta} - 4\theta\right) = \alpha - 3e_1^H - 3e_2^H - 3e_3^H - \bar{c} \end{cases}$$

Optimal efforts:

$$\begin{cases} e_1^H &= \frac{2(\alpha-\bar{c})(5\phi-4\theta^2)}{125\phi^2-80\phi\theta^2-60\phi+24\theta^2} \\ e_2^H &= \frac{2(\alpha-\bar{c})(5\phi-4\theta^2)}{125\phi^2-80\phi\theta^2-60\phi+24\theta^2} \\ e_3^H &= \frac{2(\alpha-\bar{c})(5\phi-4\theta^2)}{125\phi^2-80\phi\theta^2-60\phi+24\theta^2} \\ e_1^L &= \frac{4\theta(\alpha-\bar{c})(5\phi-6)}{125\phi^2-80\phi\theta^2-60\phi+24\theta^2} \end{cases}$$

Production Costs:



$$\begin{cases} c_1^H &= -\frac{30\alpha\phi-24\alpha\theta^2-125\bar{c}\phi^2+80\bar{c}\phi\theta^2+30\bar{c}\phi}{125\phi^2-80\phi\theta^2-60\phi+24\theta^2} \\ c_2^H &= -\frac{30\alpha\phi-24\alpha\theta^2-125\bar{c}\phi^2+80\bar{c}\phi\theta^2+30\bar{c}\phi}{125\phi^2-80\phi\theta^2-60\phi+24\theta^2} \\ c_3^H &= -\frac{30\alpha\phi-24\alpha\theta^2-125\bar{c}\phi^2+80\bar{c}\phi\theta^2+30\bar{c}\phi}{125\phi^2-80\phi\theta^2-60\phi+24\theta^2} \\ c_1^L &= -\frac{20\alpha\phi\theta^2-24\alpha\theta^2-125\bar{c}\phi^2+60\bar{c}\phi\theta^2+60\bar{c}\phi}{125\phi^2-80\phi\theta^2-60\phi+24\theta^2} \end{cases}$$

Production Quantities:

$$\begin{cases} q_1^H &= \frac{5\phi(\alpha-\bar{c})(5\phi-4\theta^2)}{125\phi^2-80\phi\theta^2-60\phi+24\theta^2} \\ q_2^H &= \frac{5\phi(\alpha-\bar{c})(5\phi-4\theta^2)}{125\phi^2-80\phi\theta^2-60\phi+24\theta^2} \\ q_3^H &= \frac{5\phi(\alpha-\bar{c})(5\phi-4\theta^2)}{125\phi^2-80\phi\theta^2-60\phi+24\theta^2} \\ q_1^L &= \frac{5\phi(\alpha-\bar{c})(5\phi-6)}{125\phi^2-80\phi\theta^2-60\phi+24\theta^2} \end{cases}$$

Profits:

$$\begin{cases} \pi_1^H &= \frac{\phi(\alpha-\bar{c})^2(5\phi-4\theta^2)^2 \cdot (25\phi-4)}{(125\phi^2-80\phi\theta^2-60\phi+24\theta^2)^2} \\ \pi_2^H &= \frac{\phi(\alpha-\bar{c})^2(5\phi-4\theta^2)^2 \cdot (25\phi-4)}{(125\phi^2-80\phi\theta^2-60\phi+24\theta^2)^2} \\ \pi_3^H &= \frac{\phi(\alpha-\bar{c})^2(5\phi-4\theta^2)^2 \cdot (25\phi-4)}{(125\phi^2-80\phi\theta^2-60\phi+24\theta^2)^2} \\ \pi_1^L &= \frac{\phi(\alpha-\bar{c})^2(5\phi-6)^2 \cdot (25\phi-16\theta^2)}{(125\phi^2-80\phi\theta^2-60\phi+24\theta^2)^2} \end{cases} \quad (88)$$

Total Production:

$$\frac{10\phi(\alpha-\bar{c})(10\phi-6\theta^2-3)}{125\phi^2-80\phi\theta^2-60\phi+24\theta^2}$$

Price:

$$\frac{25\alpha\phi^2-20\alpha\phi\theta^2-30\alpha\phi+24\alpha\theta^2+100\bar{c}\phi^2-60\bar{c}\phi\theta^2-30\bar{c}\phi}{125\phi^2-80\phi\theta^2-60\phi+24\theta^2}$$

Firm Surplus:

$$\frac{4\phi(\alpha-\bar{c})^2 \cdot (625\phi^3-850\phi^2\theta^2-450\phi^2+300\phi\theta^4+360\phi\theta^2+225\phi-48\theta^4-144\theta^2)}{(125\phi^2-80\phi\theta^2-60\phi+24\theta^2)^2}$$

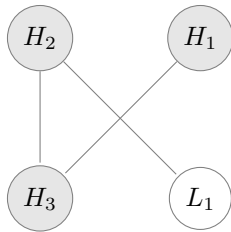
Consumer Surplus:

$$\frac{50\phi^2(\alpha-\bar{c})^2(10\phi-6\theta^2-3)^2}{(125\phi^2-80\phi\theta^2-60\phi+24\theta^2)^2}$$

Social Welfare:

$$\frac{2\phi(\alpha-\bar{c})^2 \cdot (3750\phi^3-4700\phi^2\theta^2-2400\phi^2+1500\phi\theta^4+1620\phi\theta^2+675\phi-96\theta^4-288\theta^2)}{(125\phi^2-80\phi\theta^2-60\phi+24\theta^2)^2}$$

### D.3.10 E3C [3H1L]



$$\text{Equations : } \begin{cases} e_1^H \cdot \left(\frac{25\phi}{3} - 3\right) = \alpha - 3e_2^H + 2e_3^H - 2e_1^L\theta - \bar{c} \\ e_2^H \cdot \left(\frac{25\phi}{2} - 2\right) = \alpha - 2e_1^H + 2e_3^H + 3e_1^L\theta - \bar{c} \\ e_3^H \cdot \left(\frac{25\phi}{2} - 2\right) = \alpha + 3e_1^H + 2e_2^H - 2e_1^L\theta - \bar{c} \\ e_1^L \cdot \left(\frac{25\phi}{3\theta} - 3\theta\right) = \alpha - 2e_1^H + 2e_2^H - 3e_3^H - \bar{c} \end{cases}$$

Optimal efforts:

$$\begin{cases} e_1^H &= \frac{3(\alpha - \bar{c})(5\phi - 2)(25\phi^2 - 15\phi\theta^2 - 6\theta^2)}{3125\phi^4 - 1125\phi^3\theta^2 - 2125\phi^3 + 225\phi^2\theta^2 + 120\phi\theta^2 + 120\phi - 36\theta^2} \\ e_2^H &= \frac{10\phi(\alpha - \bar{c})(25\phi^2 - 15\phi - 6\theta^2)}{3125\phi^4 - 1125\phi^3\theta^2 - 2125\phi^3 + 225\phi^2\theta^2 + 120\phi\theta^2 + 120\phi - 36\theta^2} \\ e_3^H &= \frac{10\phi(\alpha - \bar{c})(25\phi^2 - 15\phi\theta^2 - 6)}{3125\phi^4 - 1125\phi^3\theta^2 - 2125\phi^3 + 225\phi^2\theta^2 + 120\phi\theta^2 + 120\phi - 36\theta^2} \\ e_1^L &= \frac{3\theta(\alpha - \bar{c})(5\phi - 2)(25\phi^2 - 15\phi - 6)}{3125\phi^4 - 1125\phi^3\theta^2 - 2125\phi^3 + 225\phi^2\theta^2 + 120\phi\theta^2 + 120\phi - 36\theta^2} \end{cases}$$

Production Costs:

$$\begin{cases} c_1^H &= -\frac{625\alpha\phi^3 - 375\alpha\phi^2\theta^2 - 150\alpha\phi^2 - 60\alpha\phi + 36\alpha\theta^2 - 3125\bar{c}\phi^4 + 1125\bar{c}\phi^3\theta^2 + 1500\bar{c}\phi^3 + 150\bar{c}\phi^2\theta^2 + 150\bar{c}\phi^2 - 120\bar{c}\phi\theta^2 - 60\bar{c}\phi}{3125\phi^4 - 1125\phi^3\theta^2 - 2125\phi^3 + 225\phi^2\theta^2 + 120\phi\theta^2 + 120\phi - 36\theta^2} \\ c_2^H &= -\frac{375\alpha\phi^3\theta^2 + 500\alpha\phi^3 - 525\alpha\phi^2\theta^2 - 150\alpha\phi^2 - 60\alpha\phi\theta^2 - 60\alpha\phi + 36\alpha\theta^2 - 3125\bar{c}\phi^4 + 750\bar{c}\phi^3\theta^2 + 1625\bar{c}\phi^3 + 300\bar{c}\phi^2\theta^2 + 150\bar{c}\phi^2 - 60\bar{c}\phi\theta^2 - 60\bar{c}\phi}{3125\phi^4 - 1125\phi^3\theta^2 - 2125\phi^3 + 225\phi^2\theta^2 + 120\phi\theta^2 + 120\phi - 36\theta^2} \\ c_3^H &= -\frac{875\alpha\phi^3 - 375\alpha\phi^2\theta^2 - 300\alpha\phi^2 - 60\alpha\phi\theta^2 - 60\alpha\phi + 36\alpha\theta^2 - 3125\bar{c}\phi^4 + 1125\bar{c}\phi^3\theta^2 + 1250\bar{c}\phi^3 + 150\bar{c}\phi^2\theta^2 + 300\bar{c}\phi^2 - 60\bar{c}\phi\theta^2 - 60\bar{c}\phi}{3125\phi^4 - 1125\phi^3\theta^2 - 2125\phi^3 + 225\phi^2\theta^2 + 120\phi\theta^2 + 120\phi - 36\theta^2} \\ c_1^L &= -\frac{375\alpha\phi^3\theta^2 + 250\alpha\phi^3 - 375\alpha\phi^2\theta^2 - 150\alpha\phi^2 - 60\alpha\phi\theta^2 + 36\alpha\theta^2 - 3125\bar{c}\phi^4 + 750\bar{c}\phi^3\theta^2 + 1875\bar{c}\phi^3 + 150\bar{c}\phi^2\theta^2 + 150\bar{c}\phi^2 - 60\bar{c}\phi\theta^2 - 120\bar{c}\phi}{3125\phi^4 - 1125\phi^3\theta^2 - 2125\phi^3 + 225\phi^2\theta^2 + 120\phi\theta^2 + 120\phi - 36\theta^2} \end{cases}$$

Production Quantities:

$$\begin{cases} q_1^H &= \frac{5\phi(\alpha - \bar{c})(5\phi - 2)(25\phi^2 - 15\phi\theta^2 - 6\theta^2)}{3125\phi^4 - 1125\phi^3\theta^2 - 2125\phi^3 + 225\phi^2\theta^2 + 120\phi\theta^2 + 120\phi - 36\theta^2} \\ q_2^H &= \frac{25\phi^2(\alpha - \bar{c})(25\phi^2 - 15\phi - 6\theta^2)}{3125\phi^4 - 1125\phi^3\theta^2 - 2125\phi^3 + 225\phi^2\theta^2 + 120\phi\theta^2 + 120\phi - 36\theta^2} \\ q_3^H &= \frac{25\phi^2(\alpha - \bar{c})(25\phi^2 - 15\phi\theta^2 - 6)}{3125\phi^4 - 1125\phi^3\theta^2 - 2125\phi^3 + 225\phi^2\theta^2 + 120\phi\theta^2 + 120\phi - 36\theta^2} \\ q_1^L &= \frac{5\phi(\alpha - \bar{c})(5\phi - 2)(25\phi^2 - 15\phi - 6)}{3125\phi^4 - 1125\phi^3\theta^2 - 2125\phi^3 + 225\phi^2\theta^2 + 120\phi\theta^2 + 120\phi - 36\theta^2} \end{cases}$$

Profits:

$$\begin{cases} \pi_1^H &= \frac{\phi(\alpha - \bar{c})^2(5\phi - 2)^2 \cdot (25\phi - 9)(25\phi^2 - 15\phi\theta^2 - 6\theta^2)^2}{(3125\phi^4 - 1125\phi^3\theta^2 - 2125\phi^3 + 225\phi^2\theta^2 + 120\phi\theta^2 + 120\phi - 36\theta^2)^2} \\ \pi_2^H &= \frac{25\phi^3(\alpha - \bar{c})^2 \cdot (25\phi - 4)(25\phi^2 - 15\phi - 6\theta^2)^2}{(3125\phi^4 - 1125\phi^3\theta^2 - 2125\phi^3 + 225\phi^2\theta^2 + 120\phi\theta^2 + 120\phi - 36\theta^2)^2} \\ \pi_3^H &= \frac{25\phi^3(\alpha - \bar{c})^2 \cdot (25\phi - 4)(25\phi^2 - 15\phi\theta^2 - 6)^2}{(3125\phi^4 - 1125\phi^3\theta^2 - 2125\phi^3 + 225\phi^2\theta^2 + 120\phi\theta^2 + 120\phi - 36\theta^2)^2} \\ \pi_1^L &= \frac{\phi(\alpha - \bar{c})^2(5\phi - 2)^2 \cdot (25\phi - 9\theta^2)(25\phi^2 - 15\phi - 6)^2}{(3125\phi^4 - 1125\phi^3\theta^2 - 2125\phi^3 + 225\phi^2\theta^2 + 120\phi\theta^2 + 120\phi - 36\theta^2)^2} \end{cases} \quad (89)$$

Total Production:

$$\frac{10\phi(\alpha - \bar{c})(5\phi - 1)(50\phi^2 - 15\phi\theta^2 - 15\phi - 6\theta^2 - 6)}{3125\phi^4 - 1125\phi^3\theta^2 - 2125\phi^3 + 225\phi^2\theta^2 + 120\phi\theta^2 + 120\phi - 36\theta^2}$$

Price:

$$\frac{625\alpha\phi^4 - 375\alpha\phi^3\theta^2 - 875\alpha\phi^3 + 375\alpha\phi^2\theta^2 + 150\alpha\phi^2 + 60\alpha\phi\theta^2 + 60\alpha\phi - 36\alpha\theta^2 + 2500\bar{c}\phi^4 - 750\bar{c}\phi^3\theta^2 - 1250\bar{c}\phi^3 - 150\bar{c}\phi^2\theta^2 - 150\bar{c}\phi^2 + 60\bar{c}\phi\theta^2 + 60\bar{c}\phi}{3125\phi^4 - 1125\phi^3\theta^2 - 2125\phi^3 + 225\phi^2\theta^2 + 120\phi\theta^2 + 120\phi - 36\theta^2}$$

Firm Surplus:

$$\frac{\phi(\alpha - \bar{c})^2 \cdot (1562500\phi^7 - 1078125\phi^6\theta^2 - 1828125\phi^6 + 281250\phi^5\theta^4 + 525000\phi^5\theta^2 + 593750\phi^5 - 73125\phi^4\theta^4 + 121875\phi^4\theta^2 + 60000\phi^4 - 22500\phi^3\theta^4 - 120000\phi^3\theta^2 - 52500\phi^3 + 12600\phi^2\theta^4 + 37800\phi^2\theta^2 - 3600\phi^2 + 3600\phi\theta^4 + 3600\phi - 1296\theta^4 - 1296\theta^2)}{(3125\phi^4 - 1125\phi^3\theta^2 - 2125\phi^3 + 225\phi^2\theta^2 + 120\phi\theta^2 + 120\phi - 36\theta^2)^2}$$

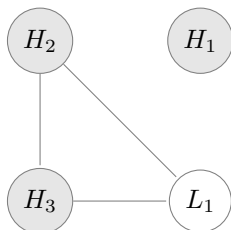
Consumer Surplus:

$$\frac{50\phi^2(\alpha - \bar{c})^2(5\phi - 1)^2(50\phi^2 - 15\phi\theta^2 - 15\phi - 6\theta^2 - 6)^2}{(3125\phi^4 - 1125\phi^3\theta^2 - 2125\phi^3 + 225\phi^2\theta^2 + 120\phi\theta^2 + 120\phi - 36\theta^2)^2}$$

Social Welfare:

$$\frac{\phi(\alpha - \bar{c})^2 \cdot (4687500\phi^7 - 2953125\phi^6\theta^2 - 4953125\phi^6 + 562500\phi^5\theta^4 + 1087500\phi^5\theta^2 + 1000000\phi^5 + 39375\phi^4\theta^4 + 571875\phi^4\theta^2 + 397500\phi^4 - 56250\phi^3\theta^4 - 217500\phi^3\theta^2 - 116250\phi^3 + 3600\phi^2\theta^4 + 19800\phi^2\theta^2 - 12600\phi^2 + 5400\phi\theta^4 + 3600\phi\theta^2 + 5400\phi - 1296\theta^4 - 1296\theta^2)}{(3125\phi^4 - 1125\phi^3\theta^2 - 2125\phi^3 + 225\phi^2\theta^2 + 120\phi\theta^2 + 120\phi - 36\theta^2)^2}$$

### D.3.11 E3D [3H1L]



$$\text{Equations : } \begin{cases} e_1^H \cdot \left( \frac{25\phi}{4} - 4 \right) = \alpha - 3e_2^H - 3e_3^H - 3e_1^L\theta - \bar{c} \\ e_2^H \cdot \left( \frac{25\phi}{2} - 2 \right) = \alpha - e_1^H + 2e_3^H + 2e_1^L\theta - \bar{c} \\ e_3^H \cdot \left( \frac{25\phi}{2} - 2 \right) = \alpha - e_1^H + 2e_2^H + 2e_1^L\theta - \bar{c} \\ e_1^L \cdot \left( \frac{25\phi}{2\theta} - 2\theta \right) = \alpha - e_1^H + 2e_2^H + 2e_3^H - \bar{c} \end{cases}$$

Optimal efforts:

$$\begin{cases} e_1^H &= \frac{4(\alpha - \bar{c})(5\phi - 2\theta^2 - 4)}{125\phi^2 - 20\phi\theta^2 - 120\phi + 8\theta^2 + 16} \\ e_2^H &= \frac{2(\alpha - \bar{c})(5\phi - 4)}{125\phi^2 - 20\phi\theta^2 - 120\phi + 8\theta^2 + 16} \\ e_3^H &= \frac{2(\alpha - \bar{c})(5\phi - 4)}{125\phi^2 - 20\phi\theta^2 - 120\phi + 8\theta^2 + 16} \\ e_1^L &= \frac{2\theta(\alpha - \bar{c})(5\phi - 4)}{125\phi^2 - 20\phi\theta^2 - 120\phi + 8\theta^2 + 16} \end{cases}$$

Production Costs:

$$\begin{cases} c_1^H &= -\frac{20\alpha\phi - 8\alpha\theta^2 - 16\alpha - 125\bar{c}\phi^2 + 20\bar{c}\phi\theta^2 + 100\bar{c}\phi}{125\phi^2 - 20\phi\theta^2 - 120\phi + 8\theta^2 + 16} \\ c_2^H &= -\frac{10\alpha\phi\theta^2 + 20\alpha\phi - 8\alpha\theta^2 - 16\alpha - 125\bar{c}\phi^2 + 10\bar{c}\phi\theta^2 + 100\bar{c}\phi}{125\phi^2 - 20\phi\theta^2 - 120\phi + 8\theta^2 + 16} \\ c_3^H &= -\frac{10\alpha\phi\theta^2 + 20\alpha\phi - 8\alpha\theta^2 - 16\alpha - 125\bar{c}\phi^2 + 10\bar{c}\phi\theta^2 + 100\bar{c}\phi}{125\phi^2 - 20\phi\theta^2 - 120\phi + 8\theta^2 + 16} \\ c_1^L &= -\frac{10\alpha\phi\theta^2 + 20\alpha\phi - 8\alpha\theta^2 - 16\alpha - 125\bar{c}\phi^2 + 10\bar{c}\phi\theta^2 + 100\bar{c}\phi}{125\phi^2 - 20\phi\theta^2 - 120\phi + 8\theta^2 + 16} \end{cases}$$

Production Quantities:

$$\begin{cases} q_1^H &= \frac{5\phi(\alpha - \bar{c})(5\phi - 2\theta^2 - 4)}{125\phi^2 - 20\phi\theta^2 - 120\phi + 8\theta^2 + 16} \\ q_2^H &= \frac{5\phi(\alpha - \bar{c})(5\phi - 4)}{125\phi^2 - 20\phi\theta^2 - 120\phi + 8\theta^2 + 16} \\ q_3^H &= \frac{5\phi(\alpha - \bar{c})(5\phi - 4)}{125\phi^2 - 20\phi\theta^2 - 120\phi + 8\theta^2 + 16} \\ q_1^L &= \frac{5\phi(\alpha - \bar{c})(5\phi - 4)}{125\phi^2 - 20\phi\theta^2 - 120\phi + 8\theta^2 + 16} \end{cases}$$

Profits:

$$\begin{cases} \pi_1^H &= \frac{\phi(\alpha - \bar{c})^2 \cdot (25\phi - 16)(5\phi - 2\theta^2 - 4)^2}{(125\phi^2 - 20\phi\theta^2 - 120\phi + 8\theta^2 + 16)^2} \\ \pi_2^H &= \frac{\phi(\alpha - \bar{c})^2 (5\phi - 4)^2 \cdot (25\phi - 4)}{(125\phi^2 - 20\phi\theta^2 - 120\phi + 8\theta^2 + 16)^2} \\ \pi_3^H &= \frac{\phi(\alpha - \bar{c})^2 (5\phi - 4)^2 \cdot (25\phi - 4)}{(125\phi^2 - 20\phi\theta^2 - 120\phi + 8\theta^2 + 16)^2} \\ \pi_1^L &= \frac{\phi(\alpha - \bar{c})^2 (5\phi - 4)^2 \cdot (25\phi - 4\theta^2)}{(125\phi^2 - 20\phi\theta^2 - 120\phi + 8\theta^2 + 16)^2} \end{cases} \quad (90)$$

Total Production:

$$\frac{10\phi(\alpha - \bar{c})(10\phi - \theta^2 - 8)}{125\phi^2 - 20\phi\theta^2 - 120\phi + 8\theta^2 + 16}$$

Price:

$$\frac{25\alpha\phi^2 - 10\alpha\phi\theta^2 - 40\alpha\phi + 8\alpha\theta^2 + 16\alpha + 100\bar{c}\phi^2 - 10\bar{c}\phi\theta^2 - 80\bar{c}\phi}{125\phi^2 - 20\phi\theta^2 - 120\phi + 8\theta^2 + 16}$$

Firm Surplus:

$$\frac{4\phi(\alpha - \bar{c})^2 \cdot (625\phi^3 - 150\phi^2\theta^2 - 1150\phi^2 + 25\phi\theta^4 + 220\phi\theta^2 + 640\phi - 16\theta^4 - 80\theta^2 - 96)}{(125\phi^2 - 20\phi\theta^2 - 120\phi + 8\theta^2 + 16)^2}$$

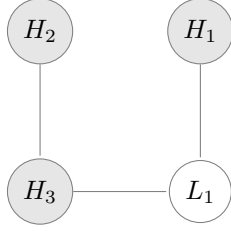
Consumer Surplus:

$$\frac{50\phi^2(\alpha - \bar{c})^2(10\phi - \theta^2 - 8)^2}{(125\phi^2 - 20\phi\theta^2 - 120\phi + 8\theta^2 + 16)^2}$$

Social Welfare:

$$\frac{2\phi(\alpha - \bar{c})^2 \cdot (3750\phi^3 - 800\phi^2\theta^2 - 6300\phi^2 + 75\phi\theta^4 + 840\phi\theta^2 + 2880\phi - 32\theta^4 - 160\theta^2 - 192)}{(125\phi^2 - 20\phi\theta^2 - 120\phi + 8\theta^2 + 16)^2}$$

**D.3.12 E3E [3H1L]**



$$\text{Equations : } \begin{cases} e_1^H \cdot \left( \frac{25\phi}{3} - 3 \right) = \alpha - 2e_2^H - 3e_3^H + 2e_1^L\theta - \bar{c} \\ e_2^H \cdot \left( \frac{25\phi}{3} - 3 \right) = \alpha - 2e_1^H + 2e_3^H - 3e_1^L\theta - \bar{c} \\ e_3^H \cdot \left( \frac{25\phi}{2} - 2 \right) = \alpha - 2e_1^H + 3e_2^H + 2e_1^L\theta - \bar{c} \\ e_1^L \cdot \left( \frac{25\phi}{2\theta} - 2\theta \right) = \alpha + 3e_1^H - 2e_2^H + 2e_3^H - \bar{c} \end{cases}$$

Optimal efforts:

$$\begin{cases} e_1^H = \frac{3(\alpha - \bar{c})(125\phi^3 - 125\phi^2 + 12\theta^2)}{3125\phi^4 - 500\phi^3\theta^2 - 2750\phi^3 + 225\phi^2 + 240\phi\theta^2 - 36\theta^2} \\ e_2^H = \frac{3(\alpha - \bar{c})(125\phi^3 - 50\phi^2\theta^2 - 75\phi^2 + 12\theta^2)}{3125\phi^4 - 500\phi^3\theta^2 - 2750\phi^3 + 225\phi^2 + 240\phi\theta^2 - 36\theta^2} \\ e_3^H = \frac{10\phi(\alpha - \bar{c})(25\phi^2 - 15\phi - 6\theta^2)}{3125\phi^4 - 500\phi^3\theta^2 - 2750\phi^3 + 225\phi^2 + 240\phi\theta^2 - 36\theta^2} \\ e_1^L = \frac{10\phi\theta(\alpha - \bar{c})(25\phi^2 - 15\phi - 6)}{3125\phi^4 - 500\phi^3\theta^2 - 2750\phi^3 + 225\phi^2 + 240\phi\theta^2 - 36\theta^2} \end{cases}$$

Production Costs:

$$\begin{cases} c_1^H = -\frac{250\alpha\phi^3\theta^2 + 375\alpha\phi^3 - 150\alpha\phi^2\theta^2 - 375\alpha\phi^2 - 60\alpha\phi\theta^2 + 36\alpha\theta^2 - 3125\bar{c}\phi^4 + 250\bar{c}\phi^3\theta^2 + 2375\bar{c}\phi^3 + 150\bar{c}\phi^2\theta^2 + 150\bar{c}\phi^2 - 180\bar{c}\phi\theta^2}{3125\phi^4 - 500\phi^3\theta^2 - 2750\phi^3 + 225\phi^2 + 240\phi\theta^2 - 36\theta^2} \\ c_2^H = -\frac{625\alpha\phi^3 - 150\alpha\phi^2\theta^2 - 375\alpha\phi^2 - 60\alpha\phi\theta^2 + 36\alpha\theta^2 - 3125\bar{c}\phi^4 + 500\bar{c}\phi^3\theta^2 + 2125\bar{c}\phi^3 + 150\bar{c}\phi^2\theta^2 + 150\bar{c}\phi^2 - 180\bar{c}\phi\theta^2}{3125\phi^4 - 500\phi^3\theta^2 - 2750\phi^3 + 225\phi^2 + 240\phi\theta^2 - 36\theta^2} \\ c_3^H = -\frac{250\alpha\phi^3\theta^2 + 625\alpha\phi^3 - 300\alpha\phi^2\theta^2 - 375\alpha\phi^2 - 120\alpha\phi\theta^2 + 36\alpha\theta^2 - 3125\bar{c}\phi^4 + 250\bar{c}\phi^3\theta^2 + 2125\bar{c}\phi^3 + 300\bar{c}\phi^2\theta^2 + 150\bar{c}\phi^2 - 120\bar{c}\phi\theta^2}{3125\phi^4 - 500\phi^3\theta^2 - 2750\phi^3 + 225\phi^2 + 240\phi\theta^2 - 36\theta^2} \\ c_1^L = -\frac{250\alpha\phi^3\theta^2 + 625\alpha\phi^3 - 150\alpha\phi^2\theta^2 - 525\alpha\phi^2 - 120\alpha\phi\theta^2 + 36\alpha\theta^2 - 3125\bar{c}\phi^4 + 250\bar{c}\phi^3\theta^2 + 2125\bar{c}\phi^3 + 150\bar{c}\phi^2\theta^2 + 300\bar{c}\phi^2 - 120\bar{c}\phi\theta^2}{3125\phi^4 - 500\phi^3\theta^2 - 2750\phi^3 + 225\phi^2 + 240\phi\theta^2 - 36\theta^2} \end{cases}$$

Production Quantities:

$$\begin{cases} q_1^H = \frac{5\phi(\alpha - \bar{c})(125\phi^3 - 125\phi^2 + 12\theta^2)}{3125\phi^4 - 500\phi^3\theta^2 - 2750\phi^3 + 225\phi^2 + 240\phi\theta^2 - 36\theta^2} \\ q_2^H = \frac{5\phi(\alpha - \bar{c})(125\phi^3 - 50\phi^2\theta^2 - 75\phi^2 + 12\theta^2)}{3125\phi^4 - 500\phi^3\theta^2 - 2750\phi^3 + 225\phi^2 + 240\phi\theta^2 - 36\theta^2} \\ q_3^H = \frac{25\phi^2(\alpha - \bar{c})(25\phi^2 - 15\phi - 6\theta^2)}{3125\phi^4 - 500\phi^3\theta^2 - 2750\phi^3 + 225\phi^2 + 240\phi\theta^2 - 36\theta^2} \\ q_1^L = \frac{25\phi^2(\alpha - \bar{c})(25\phi^2 - 15\phi - 6)}{3125\phi^4 - 500\phi^3\theta^2 - 2750\phi^3 + 225\phi^2 + 240\phi\theta^2 - 36\theta^2} \end{cases}$$

Profits:

$$\begin{cases} \pi_1^H = \frac{\phi(\alpha - \bar{c})^2 \cdot (25\phi - 9)(125\phi^3 - 125\phi^2 + 12\theta^2)^2}{(3125\phi^4 - 500\phi^3\theta^2 - 2750\phi^3 + 225\phi^2 + 240\phi\theta^2 - 36\theta^2)^2} \\ \pi_2^H = \frac{\phi(\alpha - \bar{c})^2 \cdot (25\phi - 9)(125\phi^3 - 50\phi^2\theta^2 - 75\phi^2 + 12\theta^2)^2}{(3125\phi^4 - 500\phi^3\theta^2 - 2750\phi^3 + 225\phi^2 + 240\phi\theta^2 - 36\theta^2)^2} \\ \pi_3^H = \frac{25\phi^3(\alpha - \bar{c})^2 \cdot (25\phi - 4)(25\phi^2 - 15\phi - 6\theta^2)^2}{(3125\phi^4 - 500\phi^3\theta^2 - 2750\phi^3 + 225\phi^2 + 240\phi\theta^2 - 36\theta^2)^2} \\ \pi_1^L = \frac{25\phi^3(\alpha - \bar{c})^2 \cdot (25\phi - 4\theta^2)(25\phi^2 - 15\phi - 6)^2}{(3125\phi^4 - 500\phi^3\theta^2 - 2750\phi^3 + 225\phi^2 + 240\phi\theta^2 - 36\theta^2)^2} \end{cases} \quad (91)$$

Total Production:

$$\frac{10\phi(\alpha - \bar{c})(250\phi^3 - 25\phi^2\theta^2 - 175\phi^2 - 15\phi\theta^2 - 15\phi + 12\theta^2)}{3125\phi^4 - 500\phi^3\theta^2 - 2750\phi^3 + 225\phi^2 + 240\phi\theta^2 - 36\theta^2}$$

Price:

$$\frac{625\alpha\phi^4 - 250\alpha\phi^3\theta^2 - 1000\alpha\phi^3 + 150\alpha\phi^2\theta^2 + 375\alpha\phi^2 + 120\alpha\phi\theta^2 - 36\alpha\theta^2 + 2500\bar{c}\phi^4 - 250\bar{c}\phi^3\theta^2 - 1750\bar{c}\phi^3 - 150\bar{c}\phi^2\theta^2 - 150\bar{c}\phi^2 + 120\bar{c}\phi\theta^2}{3125\phi^4 - 500\phi^3\theta^2 - 2750\phi^3 + 225\phi^2 + 240\phi\theta^2 - 36\theta^2}$$

Firm Surplus:

$$\frac{2\phi(\alpha - \bar{c})^2 \cdot (781250\phi^7 - 187500\phi^6\theta^2 - 1265625\phi^6 + 31250\phi^5\theta^4 + 93750\phi^5\theta^2 + 575000\phi^5 - 11250\phi^4\theta^4 + 116250\phi^4\theta^2 - 50625\phi^4 - 3750\phi^3\theta^4 - 105000\phi^3\theta^2 + 11250\phi^3 + 3600\phi^2\theta^4 + 19800\phi^2\theta^2 + 3600\phi\theta^4 - 1296\theta^4)}{(3125\phi^4 - 500\phi^3\theta^2 - 2750\phi^3 + 225\phi^2 + 240\phi\theta^2 - 36\theta^2)^2}$$

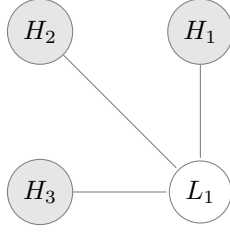
Consumer Surplus:

$$\frac{50\phi^2(\alpha - \bar{c})^2(250\phi^3 - 25\phi^2\theta^2 - 175\phi^2 - 15\phi\theta^2 - 15\phi + 12\theta^2)^2}{(3125\phi^4 - 500\phi^3\theta^2 - 2750\phi^3 + 225\phi^2 + 240\phi\theta^2 - 36\theta^2)^2}$$

Social Welfare:

$$\frac{2\phi(\alpha - \bar{c})^2 \cdot (2343750\phi^7 - 500000\phi^6\theta^2 - 3453125\phi^6 + 46875\phi^5\theta^4 + 125000\phi^5\theta^2 + 1153125\phi^5 + 7500\phi^4\theta^4 + 416250\phi^4\theta^2 + 80625\phi^4 - 13125\phi^3\theta^4 - 198750\phi^3\theta^2 + 16875\phi^3 - 5400\phi^2\theta^4 + 10800\phi^2\theta^2 + 7200\phi\theta^4 - 1296\theta^4)}{(3125\phi^4 - 500\phi^3\theta^2 - 2750\phi^3 + 225\phi^2 + 240\phi\theta^2 - 36\theta^2)^2}$$

### D.3.13 E3F [3H1L]



$$\text{Equations : } \begin{cases} e_1^H \cdot \left(\frac{25\phi}{3} - 3\right) = \alpha - 2e_2^H - 2e_3^H + e_1^L\theta - \bar{c} \\ e_2^H \cdot \left(\frac{25\phi}{3} - 3\right) = \alpha - 2e_1^H - 2e_3^H + e_1^L\theta - \bar{c} \\ e_3^H \cdot \left(\frac{25\phi}{3} - 3\right) = \alpha - 2e_1^H - 2e_2^H + e_1^L\theta - \bar{c} \\ e_1^L \cdot \left(\frac{25\phi}{\theta} - \theta\right) = \alpha + 3e_1^H + 3e_2^H + 3e_3^H - \bar{c} \end{cases}$$

Optimal efforts:

$$\begin{cases} e_1^H = \frac{15\phi(\alpha - \bar{c})}{125\phi^2 - 5\phi\theta^2 + 15\phi - 6\theta^2} \\ e_2^H = \frac{15\phi(\alpha - \bar{c})}{125\phi^2 - 5\phi\theta^2 + 15\phi - 6\theta^2} \\ e_3^H = \frac{15\phi(\alpha - \bar{c})}{125\phi^2 - 5\phi\theta^2 + 15\phi - 6\theta^2} \\ e_1^L = \frac{\theta(\alpha - \bar{c})(5\phi + 6)}{125\phi^2 - 5\phi\theta^2 + 15\phi - 6\theta^2} \end{cases}$$

Production Costs:

$$\begin{cases} c_1^H = -\frac{5\alpha\phi\theta^2 + 15\alpha\phi + 6\alpha\theta^2 - 125\bar{c}\phi^2 - 30\bar{c}\phi}{125\phi^2 - 5\phi\theta^2 + 15\phi - 6\theta^2} \\ c_2^H = -\frac{5\alpha\phi\theta^2 + 15\alpha\phi + 6\alpha\theta^2 - 125\bar{c}\phi^2 - 30\bar{c}\phi}{125\phi^2 - 5\phi\theta^2 + 15\phi - 6\theta^2} \\ c_3^H = -\frac{5\alpha\phi\theta^2 + 15\alpha\phi + 6\alpha\theta^2 - 125\bar{c}\phi^2 - 30\bar{c}\phi}{125\phi^2 - 5\phi\theta^2 + 15\phi - 6\theta^2} \\ c_1^L = -\frac{5\alpha\phi\theta^2 + 45\alpha\phi + 6\alpha\theta^2 - 125\bar{c}\phi^2 - 60\bar{c}\phi}{125\phi^2 - 5\phi\theta^2 + 15\phi - 6\theta^2} \end{cases}$$

Production Quantities:

$$\begin{cases} q_1^H = \frac{25\phi^2(\alpha - \bar{c})}{125\phi^2 - 5\phi\theta^2 + 15\phi - 6\theta^2} \\ q_2^H = \frac{25\phi^2(\alpha - \bar{c})}{125\phi^2 - 5\phi\theta^2 + 15\phi - 6\theta^2} \\ q_3^H = \frac{25\phi^2(\alpha - \bar{c})}{125\phi^2 - 5\phi\theta^2 + 15\phi - 6\theta^2} \\ q_1^L = \frac{5\phi(\alpha - \bar{c})(5\phi + 6)}{125\phi^2 - 5\phi\theta^2 + 15\phi - 6\theta^2} \end{cases}$$

Profits:

$$\begin{cases} \pi_1^H = \frac{25\phi^3(\alpha - \bar{c})^2 \cdot (25\phi - 9)}{(125\phi^2 - 5\phi\theta^2 + 15\phi - 6\theta^2)^2} \\ \pi_2^H = \frac{25\phi^3(\alpha - \bar{c})^2 \cdot (25\phi - 9)}{(125\phi^2 - 5\phi\theta^2 + 15\phi - 6\theta^2)^2} \\ \pi_3^H = \frac{25\phi^3(\alpha - \bar{c})^2 \cdot (25\phi - 9)}{(125\phi^2 - 5\phi\theta^2 + 15\phi - 6\theta^2)^2} \\ \pi_1^L = \frac{\phi(\alpha - \bar{c})^2(5\phi + 6) \cdot (25\phi - \theta^2)}{(125\phi^2 - 5\phi\theta^2 + 15\phi - 6\theta^2)^2} \end{cases} \quad (92)$$

Total Production:

$$\frac{10\phi(\alpha - \bar{c})(10\phi + 3)}{125\phi^2 - 5\phi\theta^2 + 15\phi - 6\theta^2}$$

Price:

$$\frac{25\alpha\phi^2 - 5\alpha\phi\theta^2 - 15\alpha\phi - 6\alpha\theta^2 + 100\bar{c}\phi^2 + 30\bar{c}\phi}{125\phi^2 - 5\phi\theta^2 + 15\phi - 6\theta^2}$$

Firm Surplus:

$$\frac{\phi(\alpha - \bar{c})^2 \cdot (2500\phi^3 - 25\phi^2\theta^2 + 825\phi^2 - 60\phi\theta^2 + 900\phi - 36\theta^2)}{(125\phi^2 - 5\phi\theta^2 + 15\phi - 6\theta^2)^2}$$

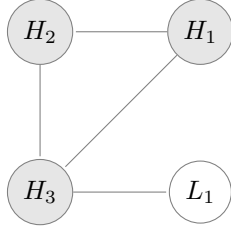
Consumer Surplus:

$$\frac{50\phi^2(\alpha - \bar{c})^2(10\phi + 3)^2}{(125\phi^2 - 5\phi\theta^2 + 15\phi - 6\theta^2)^2}$$

Social Welfare:

$$\frac{\phi(\alpha - \bar{c})^2 \cdot (7500\phi^3 - 25\phi^2\theta^2 + 3825\phi^2 - 60\phi\theta^2 + 1350\phi - 36\theta^2)}{(125\phi^2 - 5\phi\theta^2 + 15\phi - 6\theta^2)^2}$$

#### D.3.14 E4A [3H1L]



$$\text{Equations : } \begin{cases} e_1^H \cdot \left(\frac{25\phi}{2} - 2\right) = \alpha + 2e_2^H + e_3^H - 2e_1^L\theta - \bar{c} \\ e_2^H \cdot \left(\frac{25\phi}{2} - 2\right) = \alpha + 2e_1^H + e_3^H - 2e_1^L\theta - \bar{c} \\ e_3^H \cdot (25\phi - 1) = \alpha + 2e_1^H + 2e_2^H + 3e_1^L\theta - \bar{c} \\ e_1^L \cdot \left(\frac{25\phi}{3\theta} - 3\theta\right) = \alpha - 3e_1^H - 3e_2^H + e_3^H - \bar{c} \end{cases}$$

Optimal efforts:

$$\begin{cases} e_1^H = \frac{10\phi(\alpha - \bar{c})(5\phi - 3\theta^2)}{625\phi^3 - 225\phi^2\theta^2 - 225\phi^2 + 12\theta^2} \\ e_2^H = \frac{10\phi(\alpha - \bar{c})(5\phi - 3\theta^2)}{625\phi^3 - 225\phi^2\theta^2 - 225\phi^2 + 12\theta^2} \\ e_3^H = \frac{(\alpha - \bar{c})(25\phi^2 - 12\theta^2)}{625\phi^3 - 225\phi^2\theta^2 - 225\phi^2 + 12\theta^2} \\ e_1^L = \frac{15\phi\theta(\alpha - \bar{c})(5\phi - 4)}{625\phi^3 - 225\phi^2\theta^2 - 225\phi^2 + 12\theta^2} \end{cases}$$

Production Costs:

$$\begin{cases} c_1^H = -\frac{125\alpha\phi^2 - 60\alpha\phi\theta^2 - 12\alpha\theta^2 - 625\bar{c}\phi^3 + 225\bar{c}\phi^2\theta^2 + 100\bar{c}\phi^2 + 60\bar{c}\phi\theta^2}{625\phi^3 - 225\phi^2\theta^2 - 225\phi^2 + 12\theta^2} \\ c_2^H = -\frac{125\alpha\phi^2 - 60\alpha\phi\theta^2 - 12\alpha\theta^2 - 625\bar{c}\phi^3 + 225\bar{c}\phi^2\theta^2 + 100\bar{c}\phi^2 + 60\bar{c}\phi\theta^2}{625\phi^3 - 225\phi^2\theta^2 - 225\phi^2 + 12\theta^2} \\ c_3^H = -\frac{75\alpha\phi^2\theta^2 + 125\alpha\phi^2 - 120\alpha\phi\theta^2 - 12\alpha\theta^2 - 625\bar{c}\phi^3 + 150\bar{c}\phi^2\theta^2 + 100\bar{c}\phi^2 + 120\bar{c}\phi\theta^2}{625\phi^3 - 225\phi^2\theta^2 - 225\phi^2 + 12\theta^2} \\ c_1^L = -\frac{75\alpha\phi^2\theta^2 + 25\alpha\phi^2 - 60\alpha\phi\theta^2 - 12\alpha\theta^2 - 625\bar{c}\phi^3 + 150\bar{c}\phi^2\theta^2 + 200\bar{c}\phi^2 + 60\bar{c}\phi\theta^2}{625\phi^3 - 225\phi^2\theta^2 - 225\phi^2 + 12\theta^2} \end{cases}$$

Production Quantities:

$$\begin{cases} q_1^H = \frac{25\phi^2(\alpha - \bar{c})(5\phi - 3\theta^2)}{625\phi^3 - 225\phi^2\theta^2 - 225\phi^2 + 12\theta^2} \\ q_2^H = \frac{25\phi^2(\alpha - \bar{c})(5\phi - 3\theta^2)}{625\phi^3 - 225\phi^2\theta^2 - 225\phi^2 + 12\theta^2} \\ q_3^H = \frac{5\phi(\alpha - \bar{c})(25\phi^2 - 12\theta^2)}{625\phi^3 - 225\phi^2\theta^2 - 225\phi^2 + 12\theta^2} \\ q_1^L = \frac{25\phi^2(\alpha - \bar{c})(5\phi - 4)}{625\phi^3 - 225\phi^2\theta^2 - 225\phi^2 + 12\theta^2} \end{cases}$$

Profits:

$$\begin{cases} \pi_1^H = \frac{25\phi^3(\alpha - \bar{c})^2(5\phi - 3\theta^2)^2 \cdot (25\phi - 4)}{(625\phi^3 - 225\phi^2\theta^2 - 225\phi^2 + 12\theta^2)^2} \\ \pi_2^H = \frac{25\phi^3(\alpha - \bar{c})^2(5\phi - 3\theta^2)^2 \cdot (25\phi - 4)}{(625\phi^3 - 225\phi^2\theta^2 - 225\phi^2 + 12\theta^2)^2} \\ \pi_3^H = \frac{\phi(\alpha - \bar{c})^2 \cdot (25\phi - 1)(25\phi^2 - 12\theta^2)^2}{(625\phi^3 - 225\phi^2\theta^2 - 225\phi^2 + 12\theta^2)^2} \\ \pi_1^L = \frac{25\phi^3(\alpha - \bar{c})^2(5\phi - 4)^2 \cdot (25\phi - 9\theta^2)}{(625\phi^3 - 225\phi^2\theta^2 - 225\phi^2 + 12\theta^2)^2} \end{cases} \quad (93)$$

Total Production:

$$\frac{10\phi(\alpha - \bar{c})(50\phi^2 - 15\phi\theta^2 - 10\phi - 6\theta^2)}{625\phi^3 - 225\phi^2\theta^2 - 225\phi^2 + 12\theta^2}$$

Price:

$$\frac{125\alpha\phi^3 - 75\alpha\phi^2\theta^2 - 125\alpha\phi^2 + 60\alpha\phi\theta^2 + 12\alpha\theta^2 + 500\bar{c}\phi^3 - 150\bar{c}\phi^2\theta^2 - 100\bar{c}\phi^2 - 60\bar{c}\phi\theta^2}{625\phi^3 - 225\phi^2\theta^2 - 225\phi^2 + 12\theta^2}$$

Firm Surplus:

$$\frac{\phi(\alpha - \bar{c})^2 \cdot (62500\phi^5 - 43125\phi^4\theta^2 - 30625\phi^4 + 11250\phi^3\theta^4 + 10000\phi^3 - 1800\phi^2\theta^4 - 3000\phi^2\theta^2 + 3600\phi\theta^4 - 144\theta^4)}{(625\phi^3 - 225\phi^2\theta^2 - 225\phi^2 + 12\theta^2)^2}$$

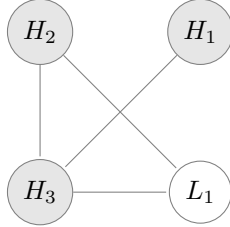
Consumer Surplus:

$$\frac{50\phi^2 (\alpha - \bar{c})^2 (50\phi^2 - 15\phi\theta^2 - 10\phi - 6\theta^2)^2}{(625\phi^3 - 225\phi^2\theta^2 - 225\phi^2 + 12\theta^2)^2}$$

Social Welfare:

$$\frac{3\phi (\alpha - \bar{c})^2 \cdot (62500\phi^5 - 39375\phi^4\theta^2 - 26875\phi^4 + 7500\phi^3\theta^4 - 5000\phi^3\theta^2 + 5000\phi^3 + 2400\phi^2\theta^4 + 1000\phi^2\theta^2 + 1800\phi\theta^4 - 48\theta^4)}{(625\phi^3 - 225\phi^2\theta^2 - 225\phi^2 + 12\theta^2)^2}$$

### D.3.15 E4B [3H1L]



$$\text{Equations : } \begin{cases} e_1^H \cdot \left(\frac{25\phi}{3} - 3\right) = \alpha - 3e_2^H + e_3^H - 3e_1^L\theta - \bar{c} \\ e_2^H \cdot \left(\frac{25\phi}{2} - 2\right) = \alpha - 2e_1^H + e_3^H + 2e_1^L\theta - \bar{c} \\ e_3^H \cdot (25\phi - 1) = \alpha + 3e_1^H + 2e_2^H + 2e_1^L\theta - \bar{c} \\ e_1^L \cdot \left(\frac{25\phi}{2\theta} - 2\theta\right) = \alpha - 2e_1^H + 2e_2^H + e_3^H - \bar{c} \end{cases}$$

Optimal efforts:

$$\begin{cases} e_1^H = \frac{15\phi(\alpha - \bar{c})(5\phi - 2\theta^2 - 2)}{625\phi^3 - 100\phi^2\theta^2 - 350\phi^2 + 6\theta^2 + 6} \\ e_2^H = \frac{10\phi(\alpha - \bar{c})(5\phi - 3)}{625\phi^3 - 100\phi^2\theta^2 - 350\phi^2 + 6\theta^2 + 6} \\ e_3^H = \frac{(\alpha - \bar{c})(25\phi^2 - 6\theta^2 - 6)}{625\phi^3 - 100\phi^2\theta^2 - 350\phi^2 + 6\theta^2 + 6} \\ e_1^L = \frac{10\phi\theta(\alpha - \bar{c})(5\phi - 3)}{625\phi^3 - 100\phi^2\theta^2 - 350\phi^2 + 6\theta^2 + 6} \end{cases}$$

Production Costs:

$$\begin{cases} c_1^H = -\frac{100\alpha\phi^2 - 30\alpha\phi\theta^2 - 30\alpha\phi - 6\alpha\theta^2 - 6\alpha - 625\bar{c}\phi^3 + 100\bar{c}\phi^2\theta^2 + 250\bar{c}\phi^2 + 30\bar{c}\phi\theta^2 + 30\bar{c}\phi}{625\phi^3 - 100\phi^2\theta^2 - 350\phi^2 + 6\theta^2 + 6} \\ c_2^H = -\frac{50\alpha\phi^2\theta^2 + 75\alpha\phi^2 - 30\alpha\phi\theta^2 - 30\alpha\phi - 6\alpha\theta^2 - 6\alpha - 625\bar{c}\phi^3 + 50\bar{c}\phi^2\theta^2 + 275\bar{c}\phi^2 + 30\bar{c}\phi\theta^2 + 30\bar{c}\phi}{625\phi^3 - 100\phi^2\theta^2 - 350\phi^2 + 6\theta^2 + 6} \\ c_3^H = -\frac{50\alpha\phi^2\theta^2 + 150\alpha\phi^2 - 60\alpha\phi\theta^2 - 60\alpha\phi - 6\alpha\theta^2 - 6\alpha - 625\bar{c}\phi^3 + 50\bar{c}\phi^2\theta^2 + 200\bar{c}\phi^2 + 60\bar{c}\phi\theta^2 + 60\bar{c}\phi}{625\phi^3 - 100\phi^2\theta^2 - 350\phi^2 + 6\theta^2 + 6} \\ c_1^L = -\frac{50\alpha\phi^2\theta^2 + 75\alpha\phi^2 - 30\alpha\phi\theta^2 - 30\alpha\phi - 6\alpha\theta^2 - 6\alpha - 625\bar{c}\phi^3 + 50\bar{c}\phi^2\theta^2 + 275\bar{c}\phi^2 + 30\bar{c}\phi\theta^2 + 30\bar{c}\phi}{625\phi^3 - 100\phi^2\theta^2 - 350\phi^2 + 6\theta^2 + 6} \end{cases}$$

Production Quantities:

$$\begin{cases} q_1^H = \frac{25\phi^2(\alpha - \bar{c})(5\phi - 2\theta^2 - 2)}{625\phi^3 - 100\phi^2\theta^2 - 350\phi^2 + 6\theta^2 + 6} \\ q_2^H = \frac{25\phi^2(\alpha - \bar{c})(5\phi - 3)}{625\phi^3 - 100\phi^2\theta^2 - 350\phi^2 + 6\theta^2 + 6} \\ q_3^H = \frac{5\phi(\alpha - \bar{c})(25\phi^2 - 6\theta^2 - 6)}{625\phi^3 - 100\phi^2\theta^2 - 350\phi^2 + 6\theta^2 + 6} \\ q_1^L = \frac{25\phi^2(\alpha - \bar{c})(5\phi - 3)}{625\phi^3 - 100\phi^2\theta^2 - 350\phi^2 + 6\theta^2 + 6} \end{cases}$$

Profits:

$$\begin{cases} \pi_1^H = \frac{25\phi^3(\alpha - \bar{c})^2 \cdot (25\phi - 9)(5\phi - 2\theta^2 - 2)^2}{(625\phi^3 - 100\phi^2\theta^2 - 350\phi^2 + 6\theta^2 + 6)^2} \\ \pi_2^H = \frac{25\phi^3(\alpha - \bar{c})^2(5\phi - 3)^2 \cdot (25\phi - 4)}{(625\phi^3 - 100\phi^2\theta^2 - 350\phi^2 + 6\theta^2 + 6)^2} \\ \pi_3^H = \frac{\phi(\alpha - \bar{c})^2 \cdot (25\phi - 1)(25\phi^2 - 6\theta^2 - 6)^2}{(625\phi^3 - 100\phi^2\theta^2 - 350\phi^2 + 6\theta^2 + 6)^2} \\ \pi_1^L = \frac{25\phi^3(\alpha - \bar{c})^2(5\phi - 3)^2 \cdot (25\phi - 4\theta^2)}{(625\phi^3 - 100\phi^2\theta^2 - 350\phi^2 + 6\theta^2 + 6)^2} \end{cases} \quad (94)$$

Total Production:

$$\frac{10\phi(\alpha - \bar{c})(50\phi^2 - 5\phi\theta^2 - 20\phi - 3\theta^2 - 3)}{625\phi^3 - 100\phi^2\theta^2 - 350\phi^2 + 6\theta^2 + 6}$$

Price:

$$\frac{125\alpha\phi^3 - 50\alpha\phi^2\theta^2 - 150\alpha\phi^2 + 30\alpha\phi\theta^2 + 30\alpha\phi + 6\alpha\theta^2 + 6\alpha + 500\bar{c}\phi^3 - 50\bar{c}\phi^2\theta^2 - 200\bar{c}\phi^2 - 30\bar{c}\phi\theta^2 - 30\bar{c}\phi}{625\phi^3 - 100\phi^2\theta^2 - 350\phi^2 + 6\theta^2 + 6}$$

Firm Surplus:

$$\frac{2\phi(\alpha - \bar{c})^2 \cdot (31250\phi^5 - 7500\phi^4\theta^2 - 29375\phi^4 + 1250\phi^3\theta^4 + 2500\phi^3\theta^2 + 6875\phi^3 - 450\phi^2\theta^4 - 1200\phi^2\theta^2 - 750\phi^2 + 450\phi\theta^4 + 900\phi\theta^2 + 450\phi - 18\theta^4 - 36\theta^2 - 18)}{(625\phi^3 - 100\phi^2\theta^2 - 350\phi^2 + 6\theta^2 + 6)^2}$$

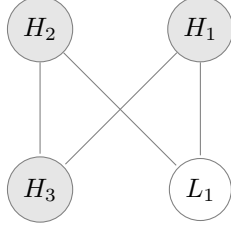
Consumer Surplus:

$$\frac{50\phi^2(\alpha - \bar{c})^2(50\phi^2 - 5\phi\theta^2 - 20\phi - 3\theta^2 - 3)^2}{(625\phi^3 - 100\phi^2\theta^2 - 350\phi^2 + 6\theta^2 + 6)^2}$$

Social Welfare:

$$\frac{2\phi(\alpha - \bar{c})^2 \cdot (93750\phi^5 - 20000\phi^4\theta^2 - 79375\phi^4 + 1875\phi^3\theta^4 + 9375\phi^3 + 300\phi^2\theta^4 + 2550\phi^2\theta^2 + 2250\phi^2 + 675\phi\theta^4 + 1350\phi\theta^2 + 675\phi - 18\theta^4 - 36\theta^2 - 18)}{(625\phi^3 - 100\phi^2\theta^2 - 350\phi^2 + 6\theta^2 + 6)^2}$$

### D.3.16 E4C [3H1L]



$$\text{Equations : } \begin{cases} e_1^H \cdot \left(\frac{25\phi}{2} - 2\right) = \alpha - 3e_2^H + 2e_3^H + 2e_1^L\theta - \bar{c} \\ e_2^H \cdot \left(\frac{25\phi}{2} - 2\right) = \alpha - 3e_1^H + 2e_3^H + 2e_1^L\theta - \bar{c} \\ e_3^H \cdot \left(\frac{25\phi}{2} - 2\right) = \alpha + 2e_1^H + 2e_2^H - 3e_1^L\theta - \bar{c} \\ e_1^L \cdot \left(\frac{25\phi}{2\theta} - 2\theta\right) = \alpha + 2e_1^H + 2e_2^H - 3e_3^H - \bar{c} \end{cases}$$

Optimal efforts:

$$\begin{cases} e_1^H = \frac{2(\alpha - \bar{c})(5\phi - 2\theta)(5\phi + 2\theta)}{625\phi^3 - 100\phi^2\theta^2 - 50\phi^2 - 60\phi\theta^2 - 40\phi + 24\theta^2} \\ e_2^H = \frac{2(\alpha - \bar{c})(5\phi - 2\theta)(5\phi + 2\theta)}{625\phi^3 - 100\phi^2\theta^2 - 50\phi^2 - 60\phi\theta^2 - 40\phi + 24\theta^2} \\ e_3^H = \frac{2(\alpha - \bar{c})(5\phi + 2)(5\phi - 2\theta^2)}{625\phi^3 - 100\phi^2\theta^2 - 50\phi^2 - 60\phi\theta^2 - 40\phi + 24\theta^2} \\ e_1^L = \frac{2\theta(\alpha - \bar{c})(5\phi - 2)(5\phi + 2)}{625\phi^3 - 100\phi^2\theta^2 - 50\phi^2 - 60\phi\theta^2 - 40\phi + 24\theta^2} \end{cases}$$

Production Costs:

$$\begin{cases} c_1^H = -\frac{50\alpha\phi^2\theta^2 + 100\alpha\phi^2 - 20\alpha\phi\theta^2 + 20\alpha\phi - 24\alpha\theta^2 - 625\bar{c}\phi^3 + 50\bar{c}\phi^2\theta^2 - 50\bar{c}\phi^2 + 80\bar{c}\phi\theta^2 + 20\bar{c}\phi}{625\phi^3 - 100\phi^2\theta^2 - 50\phi^2 - 60\phi\theta^2 - 40\phi + 24\theta^2} \\ c_2^H = -\frac{50\alpha\phi^2\theta^2 + 100\alpha\phi^2 - 20\alpha\phi\theta^2 + 20\alpha\phi - 24\alpha\theta^2 - 625\bar{c}\phi^3 + 50\bar{c}\phi^2\theta^2 - 50\bar{c}\phi^2 + 80\bar{c}\phi\theta^2 + 20\bar{c}\phi}{625\phi^3 - 100\phi^2\theta^2 - 50\phi^2 - 60\phi\theta^2 - 40\phi + 24\theta^2} \\ c_3^H = -\frac{150\alpha\phi^2 - 20\alpha\phi\theta^2 + 20\alpha\phi - 24\alpha\theta^2 - 625\bar{c}\phi^3 + 100\bar{c}\phi^2\theta^2 - 100\bar{c}\phi^2 + 80\bar{c}\phi\theta^2 + 20\bar{c}\phi}{625\phi^3 - 100\phi^2\theta^2 - 50\phi^2 - 60\phi\theta^2 - 40\phi + 24\theta^2} \\ c_1^L = -\frac{50\alpha\phi^2\theta^2 + 100\alpha\phi^2 - 24\alpha\theta^2 - 625\bar{c}\phi^3 + 50\bar{c}\phi^2\theta^2 - 50\bar{c}\phi^2 + 60\bar{c}\phi\theta^2 + 40\bar{c}\phi}{625\phi^3 - 100\phi^2\theta^2 - 50\phi^2 - 60\phi\theta^2 - 40\phi + 24\theta^2} \end{cases}$$

Production Quantities:

$$\begin{cases} q_1^H = \frac{5\phi(\alpha - \bar{c})(5\phi - 2\theta)(5\phi + 2\theta)}{625\phi^3 - 100\phi^2\theta^2 - 50\phi^2 - 60\phi\theta^2 - 40\phi + 24\theta^2} \\ q_2^H = \frac{5\phi(\alpha - \bar{c})(5\phi - 2\theta)(5\phi + 2\theta)}{625\phi^3 - 100\phi^2\theta^2 - 50\phi^2 - 60\phi\theta^2 - 40\phi + 24\theta^2} \\ q_3^H = \frac{5\phi(\alpha - \bar{c})(5\phi + 2)(5\phi - 2\theta^2)}{625\phi^3 - 100\phi^2\theta^2 - 50\phi^2 - 60\phi\theta^2 - 40\phi + 24\theta^2} \\ q_1^L = \frac{5\phi(\alpha - \bar{c})(5\phi - 2)(5\phi + 2)}{625\phi^3 - 100\phi^2\theta^2 - 50\phi^2 - 60\phi\theta^2 - 40\phi + 24\theta^2} \end{cases}$$

Profits:

$$\begin{cases} \pi_1^H = \frac{\phi(\alpha - \bar{c})^2(5\phi - 2\theta)^2(5\phi + 2\theta)^2 \cdot (25\phi - 4)}{(625\phi^3 - 100\phi^2\theta^2 - 50\phi^2 - 60\phi\theta^2 - 40\phi + 24\theta^2)^2} \\ \pi_2^H = \frac{\phi(\alpha - \bar{c})^2(5\phi - 2\theta)^2(5\phi + 2\theta)^2 \cdot (25\phi - 4)}{(625\phi^3 - 100\phi^2\theta^2 - 50\phi^2 - 60\phi\theta^2 - 40\phi + 24\theta^2)^2} \\ \pi_3^H = \frac{\phi(\alpha - \bar{c})^2(5\phi + 2)^2(5\phi - 2\theta^2)^2 \cdot (25\phi - 4)}{(625\phi^3 - 100\phi^2\theta^2 - 50\phi^2 - 60\phi\theta^2 - 40\phi + 24\theta^2)^2} \\ \pi_1^L = \frac{\phi(\alpha - \bar{c})^2(5\phi - 2)^2(5\phi + 2)^2 \cdot (25\phi - 4\theta^2)}{(625\phi^3 - 100\phi^2\theta^2 - 50\phi^2 - 60\phi\theta^2 - 40\phi + 24\theta^2)^2} \end{cases} \quad (95)$$

Total Production:

$$\frac{10\phi(\alpha - \bar{c})(50\phi^2 - 5\phi\theta^2 + 5\phi - 6\theta^2 - 2)}{625\phi^3 - 100\phi^2\theta^2 - 50\phi^2 - 60\phi\theta^2 - 40\phi + 24\theta^2}$$

Price:



$$\frac{125\alpha\phi^3 - 50\alpha\phi^2\theta^2 - 100\alpha\phi^2 - 20\alpha\phi + 24\alpha\theta^2 + 500\bar{c}\phi^3 - 50\bar{c}\phi^2\theta^2 + 50\bar{c}\phi^2 - 60\bar{c}\phi\theta^2 - 20\bar{c}\phi}{625\phi^3 - 100\phi^2\theta^2 - 50\phi^2 - 60\phi\theta^2 - 40\phi + 24\theta^2}$$

Firm Surplus:

$$\frac{4\phi(\alpha - \bar{c})^2 \cdot (15625\phi^5 - 3750\phi^4\theta^2 + 1250\phi^4 + 625\phi^3\theta^4 - 4500\phi^3\theta^2 - 1125\phi^3 + 400\phi^2\theta^4 + 500\phi^2\theta^2 - 100\phi^2 + 220\phi\theta^4 + 80\phi\theta^2 + 100\phi - 48\theta^4 - 16\theta^2)}{(625\phi^3 - 100\phi^2\theta^2 - 50\phi^2 - 60\phi\theta^2 - 40\phi + 24\theta^2)^2}$$

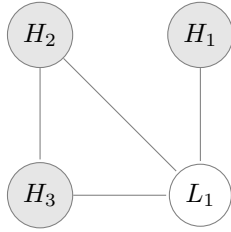
Consumer Surplus:

$$\frac{50\phi^2(\alpha - \bar{c})^2(50\phi^2 - 5\phi\theta^2 + 5\phi - 6\theta^2 - 2)^2}{(625\phi^3 - 100\phi^2\theta^2 - 50\phi^2 - 60\phi\theta^2 - 40\phi + 24\theta^2)^2}$$

Social Welfare:

$$\frac{2\phi(\alpha - \bar{c})^2 \cdot (93750\phi^5 - 20000\phi^4\theta^2 + 15000\phi^4 + 1875\phi^3\theta^4 - 25250\phi^3\theta^2 - 6625\phi^3 + 2300\phi^2\theta^4 - 700\phi^2 + 1340\phi\theta^4 + 760\phi\theta^2 + 300\phi - 96\theta^4 - 32\theta^2)}{(625\phi^3 - 100\phi^2\theta^2 - 50\phi^2 - 60\phi\theta^2 - 40\phi + 24\theta^2)^2}$$

### D.3.17 E4D [3H1L]



$$\text{Equations : } \begin{cases} e_1^H \cdot \left(\frac{25\phi}{3} - 3\right) = \alpha - 3e_2^H - 3e_3^H + e_1^L\theta - \bar{c} \\ e_2^H \cdot \left(\frac{25\phi}{2} - 2\right) = \alpha - 2e_1^H + 2e_3^H + e_1^L\theta - \bar{c} \\ e_3^H \cdot \left(\frac{25\phi}{2} - 2\right) = \alpha - 2e_1^H + 2e_2^H + e_1^L\theta - \bar{c} \\ e_1^L \cdot \left(\frac{25\phi}{\theta} - \theta\right) = \alpha + 3e_1^H + 2e_2^H + 2e_3^H - \bar{c} \end{cases}$$

Optimal efforts:

$$\begin{cases} e_1^H = \frac{15\phi(\alpha - \bar{c})(5\phi - 4)}{625\phi^3 - 25\phi^2\theta^2 - 425\phi^2 + 12\theta^2} \\ e_2^H = \frac{10\phi(\alpha - \bar{c})(5\phi - 3)}{625\phi^3 - 25\phi^2\theta^2 - 425\phi^2 + 12\theta^2} \\ e_3^H = \frac{10\phi(\alpha - \bar{c})(5\phi - 3)}{625\phi^3 - 25\phi^2\theta^2 - 425\phi^2 + 12\theta^2} \\ e_1^L = \frac{\theta(\alpha - \bar{c})(25\phi^2 - 12)}{625\phi^3 - 25\phi^2\theta^2 - 425\phi^2 + 12\theta^2} \end{cases}$$

Production Costs:

$$\begin{cases} c_1^H = -\frac{25\alpha\phi^2\theta^2 + 75\alpha\phi^2 - 60\alpha\phi - 12\alpha\theta^2 - 625\bar{c}\phi^3 + 350\bar{c}\phi^2 + 60\bar{c}\phi}{625\phi^3 - 25\phi^2\theta^2 - 425\phi^2 + 12\theta^2} \\ c_2^H = -\frac{25\alpha\phi^2\theta^2 + 100\alpha\phi^2 - 60\alpha\phi - 12\alpha\theta^2 - 625\bar{c}\phi^3 + 325\bar{c}\phi^2 + 60\bar{c}\phi}{625\phi^3 - 25\phi^2\theta^2 - 425\phi^2 + 12\theta^2} \\ c_3^H = -\frac{25\alpha\phi^2\theta^2 + 100\alpha\phi^2 - 60\alpha\phi - 12\alpha\theta^2 - 625\bar{c}\phi^3 + 325\bar{c}\phi^2 + 60\bar{c}\phi}{625\phi^3 - 25\phi^2\theta^2 - 425\phi^2 + 12\theta^2} \\ c_1^L = -\frac{25\alpha\phi^2\theta^2 + 175\alpha\phi^2 - 120\alpha\phi - 12\alpha\theta^2 - 625\bar{c}\phi^3 + 250\bar{c}\phi^2 + 120\bar{c}\phi}{625\phi^3 - 25\phi^2\theta^2 - 425\phi^2 + 12\theta^2} \end{cases}$$

Production Quantities:

$$\begin{cases} q_1^H = \frac{25\phi^2(\alpha - \bar{c})(5\phi - 4)}{625\phi^3 - 25\phi^2\theta^2 - 425\phi^2 + 12\theta^2} \\ q_2^H = \frac{25\phi^2(\alpha - \bar{c})(5\phi - 3)}{625\phi^3 - 25\phi^2\theta^2 - 425\phi^2 + 12\theta^2} \\ q_3^H = \frac{25\phi^2(\alpha - \bar{c})(5\phi - 3)}{625\phi^3 - 25\phi^2\theta^2 - 425\phi^2 + 12\theta^2} \\ q_1^L = \frac{5\phi(\alpha - \bar{c})(25\phi^2 - 12)}{625\phi^3 - 25\phi^2\theta^2 - 425\phi^2 + 12\theta^2} \end{cases}$$

Profits:

$$\begin{cases} \pi_1^H = \frac{25\phi^3(\alpha - \bar{c})^2(5\phi - 4)^2 \cdot (25\phi - 9)}{(625\phi^3 - 25\phi^2\theta^2 - 425\phi^2 + 12\theta^2)^2} \\ \pi_2^H = \frac{25\phi^3(\alpha - \bar{c})^2(5\phi - 3)^2 \cdot (25\phi - 4)}{(625\phi^3 - 25\phi^2\theta^2 - 425\phi^2 + 12\theta^2)^2} \\ \pi_3^H = \frac{25\phi^3(\alpha - \bar{c})^2(5\phi - 3)^2 \cdot (25\phi - 4)}{(625\phi^3 - 25\phi^2\theta^2 - 425\phi^2 + 12\theta^2)^2} \\ \pi_1^L = \frac{\phi(\alpha - \bar{c})^2 \cdot (25\phi - \theta^2)(25\phi^2 - 12)^2}{(625\phi^3 - 25\phi^2\theta^2 - 425\phi^2 + 12\theta^2)^2} \end{cases} \quad (96)$$

Total Production:

$$\frac{10\phi(\alpha - \bar{c})(50\phi^2 - 25\phi - 6)}{625\phi^3 - 25\phi^2\theta^2 - 425\phi^2 + 12\theta^2}$$

Price:

$$\frac{125\alpha\phi^3 - 25\alpha\phi^2\theta^2 - 175\alpha\phi^2 + 60\alpha\phi + 12\alpha\theta^2 + 500\bar{c}\phi^3 - 250\bar{c}\phi^2 - 60\bar{c}\phi}{625\phi^3 - 25\phi^2\theta^2 - 425\phi^2 + 12\theta^2}$$

Firm Surplus:

$$\frac{\phi(\alpha - \bar{c})^2 \cdot (62500\phi^5 - 625\phi^4\theta^2 - 73125\phi^4 + 21250\phi^3 + 600\phi^2\theta^2 - 5400\phi^2 + 3600\phi - 144\theta^2)}{(625\phi^3 - 25\phi^2\theta^2 - 425\phi^2 + 12\theta^2)^2}$$

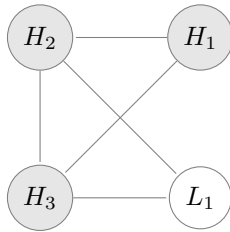
Consumer Surplus:

$$\frac{50\phi^2(\alpha - \bar{c})^2(50\phi^2 - 25\phi - 6)^2}{(625\phi^3 - 25\phi^2\theta^2 - 425\phi^2 + 12\theta^2)^2}$$

Social Welfare:

$$\frac{\phi(\alpha - \bar{c})^2 \cdot (187500\phi^5 - 625\phi^4\theta^2 - 198125\phi^4 + 22500\phi^3 + 600\phi^2\theta^2 + 9600\phi^2 + 5400\phi - 144\theta^2)}{(625\phi^3 - 25\phi^2\theta^2 - 425\phi^2 + 12\theta^2)^2}$$

### D.3.18 E5A [3H1L]



$$\text{Equations : } \begin{cases} e_1^H \cdot \left(\frac{25\phi}{2} - 2\right) = \alpha + e_2^H + e_3^H - 3e_1^L\theta - \bar{c} \\ e_2^H \cdot (25\phi - 1) = \alpha + 2e_1^H + e_3^H + 2e_1^L\theta - \bar{c} \\ e_3^H \cdot (25\phi - 1) = \alpha + 2e_1^H + e_2^H + 2e_1^L\theta - \bar{c} \\ e_1^L \cdot \left(\frac{25\phi}{2\theta} - 2\theta\right) = \alpha - 3e_1^H + e_2^H + e_3^H - \bar{c} \end{cases}$$

Optimal efforts:

$$\begin{cases} e_1^H = \frac{10\phi(\alpha - \bar{c})(5\phi - 2\theta^2)}{625\phi^3 - 100\phi^2\theta^2 - 150\phi^2 - 20\phi\theta^2 + 8\theta^2} \\ e_2^H = \frac{(\alpha - \bar{c})(5\phi - 2\theta)(5\phi + 2\theta)}{625\phi^3 - 100\phi^2\theta^2 - 150\phi^2 - 20\phi\theta^2 + 8\theta^2} \\ e_3^H = \frac{(\alpha - \bar{c})(5\phi - 2\theta)(5\phi + 2\theta)}{625\phi^3 - 100\phi^2\theta^2 - 150\phi^2 - 20\phi\theta^2 + 8\theta^2} \\ e_1^L = \frac{10\phi\theta(\alpha - \bar{c})(5\phi - 2)}{625\phi^3 - 100\phi^2\theta^2 - 150\phi^2 - 20\phi\theta^2 + 8\theta^2} \end{cases}$$

Production Costs:

$$\begin{cases} c_1^H = -\frac{100\alpha\phi^2 - 20\alpha\phi\theta^2 - 8\alpha\theta^2 - 625\bar{c}\phi^3 + 100\bar{c}\phi^2\theta^2 + 50\bar{c}\phi^2 + 40\bar{c}\phi\theta^2}{625\phi^3 - 100\phi^2\theta^2 - 150\phi^2 - 20\phi\theta^2 + 8\theta^2} \\ c_2^H = -\frac{50\alpha\phi^2\theta^2 + 100\alpha\phi^2 - 40\alpha\phi\theta^2 - 8\alpha\theta^2 - 625\bar{c}\phi^3 + 50\bar{c}\phi^2\theta^2 + 50\bar{c}\phi^2 + 60\bar{c}\phi\theta^2}{625\phi^3 - 100\phi^2\theta^2 - 150\phi^2 - 20\phi\theta^2 + 8\theta^2} \\ c_3^H = -\frac{50\alpha\phi^2\theta^2 + 100\alpha\phi^2 - 40\alpha\phi\theta^2 - 8\alpha\theta^2 - 625\bar{c}\phi^3 + 50\bar{c}\phi^2\theta^2 + 50\bar{c}\phi^2 + 60\bar{c}\phi\theta^2}{625\phi^3 - 100\phi^2\theta^2 - 150\phi^2 - 20\phi\theta^2 + 8\theta^2} \\ c_1^L = -\frac{50\alpha\phi^2\theta^2 + 50\alpha\phi^2 - 20\alpha\phi\theta^2 - 8\alpha\theta^2 - 625\bar{c}\phi^3 + 50\bar{c}\phi^2\theta^2 + 100\bar{c}\phi^2 + 40\bar{c}\phi\theta^2}{625\phi^3 - 100\phi^2\theta^2 - 150\phi^2 - 20\phi\theta^2 + 8\theta^2} \end{cases}$$

Production Quantities:

$$\begin{cases} q_1^H = \frac{25\phi^2(\alpha - \bar{c})(5\phi - 2\theta^2)}{625\phi^3 - 100\phi^2\theta^2 - 150\phi^2 - 20\phi\theta^2 + 8\theta^2} \\ q_2^H = \frac{5\phi(\alpha - \bar{c})(5\phi - 2\theta)(5\phi + 2\theta)}{625\phi^3 - 100\phi^2\theta^2 - 150\phi^2 - 20\phi\theta^2 + 8\theta^2} \\ q_3^H = \frac{5\phi(\alpha - \bar{c})(5\phi - 2\theta)(5\phi + 2\theta)}{625\phi^3 - 100\phi^2\theta^2 - 150\phi^2 - 20\phi\theta^2 + 8\theta^2} \\ q_1^L = \frac{25\phi^2(\alpha - \bar{c})(5\phi - 2)}{625\phi^3 - 100\phi^2\theta^2 - 150\phi^2 - 20\phi\theta^2 + 8\theta^2} \end{cases}$$

Profits:

$$\begin{cases} \pi_1^H = \frac{25\phi^3(\alpha - \bar{c})^2(5\phi - 2\theta^2)^2 \cdot (25\phi - 4)}{(625\phi^3 - 100\phi^2\theta^2 - 150\phi^2 - 20\phi\theta^2 + 8\theta^2)^2} \\ \pi_2^H = \frac{\phi(\alpha - \bar{c})^2(5\phi - 2\theta)^2(5\phi + 2\theta)^2 \cdot (25\phi - 1)}{(625\phi^3 - 100\phi^2\theta^2 - 150\phi^2 - 20\phi\theta^2 + 8\theta^2)^2} \\ \pi_3^H = \frac{\phi(\alpha - \bar{c})^2(5\phi - 2\theta)^2(5\phi + 2\theta)^2 \cdot (25\phi - 1)}{(625\phi^3 - 100\phi^2\theta^2 - 150\phi^2 - 20\phi\theta^2 + 8\theta^2)^2} \\ \pi_1^L = \frac{25\phi^3(\alpha - \bar{c})^2(5\phi - 2)^2 \cdot (25\phi - 4\theta^2)}{(625\phi^3 - 100\phi^2\theta^2 - 150\phi^2 - 20\phi\theta^2 + 8\theta^2)^2} \end{cases} \quad (97)$$

Total Production:

$$\frac{10\phi(\alpha - \bar{c})(50\phi^2 - 5\phi\theta^2 - 5\phi - 4\theta^2)}{625\phi^3 - 100\phi^2\theta^2 - 150\phi^2 - 20\phi\theta^2 + 8\theta^2}$$

Price:

$$\frac{125\alpha\phi^3 - 50\alpha\phi^2\theta^2 - 100\alpha\phi^2 + 20\alpha\phi\theta^2 + 8\alpha\theta^2 + 500\bar{c}\phi^3 - 50\bar{c}\phi^2\theta^2 - 50\bar{c}\phi^2 - 40\bar{c}\phi\theta^2}{625\phi^3 - 100\phi^2\theta^2 - 150\phi^2 - 20\phi\theta^2 + 8\theta^2}$$

Firm Surplus:

$$\frac{2\phi(\alpha - \bar{c})^2 \cdot (31250\phi^5 - 7500\phi^4\theta^2 - 8125\phi^4 + 1250\phi^3\theta^4 - 3000\phi^3\theta^2 + 1250\phi^3 - 200\phi^2\theta^4 + 400\phi\theta^4 - 16\theta^4)}{(625\phi^3 - 100\phi^2\theta^2 - 150\phi^2 - 20\phi\theta^2 + 8\theta^2)^2}$$

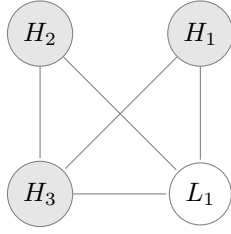
Consumer Surplus:

$$\frac{50\phi^2(\alpha - \bar{c})^2(50\phi^2 - 5\phi\theta^2 - 5\phi - 4\theta^2)^2}{(625\phi^3 - 100\phi^2\theta^2 - 150\phi^2 - 20\phi\theta^2 + 8\theta^2)^2}$$

Social Welfare:

$$\frac{2\phi(\alpha - \bar{c})^2 \cdot (93750\phi^5 - 20000\phi^4\theta^2 - 20625\phi^4 + 1875\phi^3\theta^4 - 11750\phi^3\theta^2 + 1875\phi^3 + 800\phi^2\theta^4 + 1000\phi^2\theta^2 + 800\phi\theta^4 - 16\theta^4)}{(625\phi^3 - 100\phi^2\theta^2 - 150\phi^2 - 20\phi\theta^2 + 8\theta^2)^2}$$

### D.3.19 E5B [3H1L]



$$\text{Equations : } \begin{cases} e_1^H \cdot \left(\frac{25\phi}{2} - 2\right) = \alpha - 3e_2^H + e_3^H + e_1^L\theta - \bar{c} \\ e_2^H \cdot \left(\frac{25\phi}{2} - 2\right) = \alpha - 3e_1^H + e_3^H + e_1^L\theta - \bar{c} \\ e_3^H \cdot (25\phi - 1) = \alpha + 2e_1^H + 2e_2^H + e_1^L\theta - \bar{c} \\ e_1^L \cdot \left(\frac{25\phi}{\theta} - \theta\right) = \alpha + 2e_1^H + 2e_2^H + e_3^H - \bar{c} \end{cases}$$

Optimal efforts:

$$\begin{cases} e_1^H = \frac{10\phi(\alpha - \bar{c})}{125\phi^2 - 5\phi\theta^2 + 5\phi - 2\theta^2 - 2} \\ e_2^H = \frac{10\phi(\alpha - \bar{c})}{125\phi^2 - 5\phi\theta^2 + 5\phi - 2\theta^2 - 2} \\ e_3^H = \frac{(\alpha - \bar{c})(5\phi + 2)}{125\phi^2 - 5\phi\theta^2 + 5\phi - 2\theta^2 - 2} \\ e_1^L = \frac{\theta(\alpha - \bar{c})(5\phi + 2)}{125\phi^2 - 5\phi\theta^2 + 5\phi - 2\theta^2 - 2} \end{cases}$$

Production Costs:

$$\begin{cases} c_1^H = -\frac{5\alpha\phi\theta^2 + 15\alpha\phi + 2\alpha\theta^2 + 2\alpha - 125\bar{c}\phi^2 - 20\bar{c}\phi}{125\phi^2 - 5\phi\theta^2 + 5\phi - 2\theta^2 - 2} \\ c_2^H = -\frac{5\alpha\phi\theta^2 + 15\alpha\phi + 2\alpha\theta^2 + 2\alpha - 125\bar{c}\phi^2 - 20\bar{c}\phi}{125\phi^2 - 5\phi\theta^2 + 5\phi - 2\theta^2 - 2} \\ c_3^H = -\frac{5\alpha\phi\theta^2 + 25\alpha\phi + 2\alpha\theta^2 + 2\alpha - 125\bar{c}\phi^2 - 30\bar{c}\phi}{125\phi^2 - 5\phi\theta^2 + 5\phi - 2\theta^2 - 2} \\ c_1^L = -\frac{5\alpha\phi\theta^2 + 25\alpha\phi + 2\alpha\theta^2 + 2\alpha - 125\bar{c}\phi^2 - 30\bar{c}\phi}{125\phi^2 - 5\phi\theta^2 + 5\phi - 2\theta^2 - 2} \end{cases}$$

Production Quantities:

$$\begin{cases} q_1^H = \frac{25\phi^2(\alpha - \bar{c})}{125\phi^2 - 5\phi\theta^2 + 5\phi - 2\theta^2 - 2} \\ q_2^H = \frac{25\phi^2(\alpha - \bar{c})}{125\phi^2 - 5\phi\theta^2 + 5\phi - 2\theta^2 - 2} \\ q_3^H = \frac{5\phi(\alpha - \bar{c})(5\phi + 2)}{125\phi^2 - 5\phi\theta^2 + 5\phi - 2\theta^2 - 2} \\ q_1^L = \frac{5\phi(\alpha - \bar{c})(5\phi + 2)}{125\phi^2 - 5\phi\theta^2 + 5\phi - 2\theta^2 - 2} \end{cases}$$

Profits:

$$\begin{cases} \pi_1^H &= \frac{25\phi^3(\alpha-\bar{c})^2 \cdot (25\phi-4)}{(125\phi^2-5\phi\theta^2+5\phi-2\theta^2-2)^2} \\ \pi_2^H &= \frac{25\phi^3(\alpha-\bar{c})^2 \cdot (25\phi-4)}{(125\phi^2-5\phi\theta^2+5\phi-2\theta^2-2)^2} \\ \pi_3^H &= \frac{\phi(\alpha-\bar{c})^2(5\phi+2)^2 \cdot (25\phi-1)}{(125\phi^2-5\phi\theta^2+5\phi-2\theta^2-2)^2} \\ \pi_1^L &= \frac{\phi(\alpha-\bar{c})^2(5\phi+2)^2 \cdot (25\phi-\theta^2)}{(125\phi^2-5\phi\theta^2+5\phi-2\theta^2-2)^2} \end{cases} \quad (98)$$

Total Production:

$$\frac{20\phi(\alpha-\bar{c})(5\phi+1)}{125\phi^2-5\phi\theta^2+5\phi-2\theta^2-2}$$

Price:

$$\frac{25\alpha\phi^2-5\alpha\phi\theta^2-15\alpha\phi-2\alpha\theta^2-2\alpha+100\bar{c}\phi^2+20\bar{c}\phi}{125\phi^2-5\phi\theta^2+5\phi-2\theta^2-2}$$

Firm Surplus:

$$\frac{\phi(\alpha-\bar{c})^2 \cdot (2500\phi^3-25\phi^2\theta^2+775\phi^2-20\phi\theta^2+180\phi-4\theta^2-4)}{(125\phi^2-5\phi\theta^2+5\phi-2\theta^2-2)^2}$$

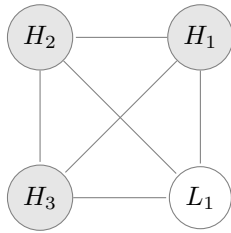
Consumer Surplus:

$$\frac{200\phi^2(\alpha-\bar{c})^2(5\phi+1)^2}{(125\phi^2-5\phi\theta^2+5\phi-2\theta^2-2)^2}$$

Social Welfare:

$$\frac{\phi(\alpha-\bar{c})^2 \cdot (7500\phi^3-25\phi^2\theta^2+2775\phi^2-20\phi\theta^2+380\phi-4\theta^2-4)}{(125\phi^2-5\phi\theta^2+5\phi-2\theta^2-2)^2}$$

### D.3.20 E6A [3H1L]



$$\text{Equations : } \begin{cases} e_1^H \cdot (25\phi-1) = \alpha + e_2^H + e_3^H + e_1^L\theta - \bar{c} \\ e_2^H \cdot (25\phi-1) = \alpha + e_1^H + e_3^H + e_1^L\theta - \bar{c} \\ e_3^H \cdot (25\phi-1) = \alpha + e_1^H + e_2^H + e_1^L\theta - \bar{c} \\ e_1^L \cdot \left(\frac{25\phi}{\theta} - \theta\right) = \alpha + e_1^H + e_2^H + e_3^H - \bar{c} \end{cases}$$

Optimal efforts:

$$\begin{cases} e_1^H &= \frac{\alpha-\bar{c}}{25\phi-\theta^2-3} \\ e_2^H &= \frac{\alpha-\bar{c}}{25\phi-\theta^2-3} \\ e_3^H &= \frac{\alpha-\bar{c}}{25\phi-\theta^2-3} \\ e_1^L &= \frac{\theta(\alpha-\bar{c})}{25\phi-\theta^2-3} \end{cases}$$

Production Costs:

$$\begin{cases} c_1^H &= -\frac{\alpha\theta^2+3\alpha-25\bar{c}\phi}{25\phi-\theta^2-3} \\ c_2^H &= -\frac{\alpha\theta^2+3\alpha-25\bar{c}\phi}{25\phi-\theta^2-3} \\ c_3^H &= -\frac{\alpha\theta^2+3\alpha-25\bar{c}\phi}{25\phi-\theta^2-3} \\ c_1^L &= -\frac{\alpha\theta^2+3\alpha-25\bar{c}\phi}{25\phi-\theta^2-3} \end{cases}$$

Production Quantities:

$$\begin{cases} q_1^H &= \frac{5\phi(\alpha-\bar{c})}{25\phi-\theta^2-3} \\ q_2^H &= \frac{5\phi(\alpha-\bar{c})}{25\phi-\theta^2-3} \\ q_3^H &= \frac{5\phi(\alpha-\bar{c})}{25\phi-\theta^2-3} \\ q_1^L &= \frac{5\phi(\alpha-\bar{c})}{25\phi-\theta^2-3} \end{cases}$$

Profits:

$$\begin{cases} \pi_1^H &= \frac{\phi(\alpha-\bar{c})^2 \cdot (25\phi-1)}{(25\phi-\theta^2-3)^2} \\ \pi_2^H &= \frac{\phi(\alpha-\bar{c})^2 \cdot (25\phi-1)}{(25\phi-\theta^2-3)^2} \\ \pi_3^H &= \frac{\phi(\alpha-\bar{c})^2 \cdot (25\phi-1)}{(25\phi-\theta^2-3)^2} \\ \pi_1^L &= \frac{\phi(\alpha-\bar{c})^2 \cdot (25\phi-\theta^2)}{(25\phi-\theta^2-3)^2} \end{cases} \quad (99)$$

Total Production:

$$\frac{20\phi(\alpha-\bar{c})}{25\phi-\theta^2-3}$$

Price:

$$\frac{5\alpha\phi-\alpha\theta^2-3\alpha+20\bar{c}\phi}{25\phi-\theta^2-3}$$

Firm Surplus:

$$\frac{\phi(\alpha-\bar{c})^2 \cdot (100\phi-\theta^2-3)}{(25\phi-\theta^2-3)^2}$$

Consumer Surplus:

$$\frac{200\phi^2(\alpha-\bar{c})^2}{(25\phi-\theta^2-3)^2}$$

Social Welfare:

$$\frac{\phi(\alpha-\bar{c})^2 \cdot (300\phi-\theta^2-3)}{(25\phi-\theta^2-3)^2}$$