R&D Networks under Heterogeneous Firm Productivities

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Motivation

Innovation as the Engine of Growth

- Innovation is a fundamental driver of economic growth (Romer, 1990; Jones, 1995).
- A key engine of innovation is the R&D investment by firms (Aghion and Howitt, 1992).
- Firms' R&D have increasingly included collaborative efforts (Calero et al., 2007).
- R&D collaborations allow firms to improve efficiency while competing in the market.

The Gap between Theory and Reality

- Standard R&D network models typically assume firms are homogeneous in their ability to innovate (e.g. Goyal and Joshi, 2003).
- Empirical studies show vast heterogeneity in innovation efficiency (e.g. Lentz and Mortensen, 2008).

Question: How does productivity heterogeneity shape R&D network formation, firms' profits, and the overall welfare?

This Paper: Contribution & Main Results

We generalize the framework of Goyal and Moraga-Gonzalez (2001) by introducing heterogeneous firm productivities to transform R&D effort into technological improvements.

Key Mechanisms & Findings:

1. **Asymmetric Incentives:** Heterogeneity creates asymmetric gains. Less-productive firms benefit disproportionately from linking to more-productive ones, while higher-productive firms bear higher R&D effort and cost.
2. **Structural Instability:** The benchmark **Complete Network**, stable under homogeneity, becomes unstable when productivity gaps are large.
3. **New Stable Configurations:** A **Positive Assortative (PA)** network (clustering by type) emerges as the stable structure and is typically stable where the complete is not.
4. **Welfare Paradox:** Increasing the average productivity of firms in the economy may **reduce** the overall welfare—through changing the stable structure and crowding out effect.

Related Literature

1. **Endogenous R&D Networks (Homogeneous)**
   - Goyal and Moraga-Gonzalez (2001); Goyal and Joshi (2003); König et al. (2019).
   - *Focus:* Symmetric environments, complete/star equilibrium networks.
   - *Our Advance:* We relax the symmetry assumption to allow asymmetric benefits.

2. **Heterogeneity in Networks**
   - Zirulia (2012); Billand et al. (2019).
   - *Focus:* Partner-specific spillovers or linking costs.
   - *Our Advance:* We model heterogeneity in the **productivity** of R&D itself. Both connectivity and the marginal value of R&D effort become asymmetric.

3. **Dynamic Networks**
   - Hsieh et al. (2024); Bischi and Lamantia (2012).
   - We focus on the static equilibrium characterization to fully isolate the effect of heterogeneity.

Notation

- $\mathcal{N}$ — set of firms (agents)
$$\mathcal{N} = \{1, 2, \ldots, n\},$$
- $\mathbf{G}$ — adjacency matrix (network)
$$\mathbf{G} \in \mathcal{G}^n,$$
$$\mathcal{G}^n \equiv \left\{ \mathbf{M} \in \{0,1\}^{n \times n} : \begin{array}{ll} \mathbf{M}_{ii} = 0 & \forall i \in \mathcal{N}, \\ \mathbf{M}_{ij} = \mathbf{M}_{ji} & \forall i, j \in \mathcal{N} \end{array} \right\}$$
(undirected network, no self-loops)
- G_{ij} — indicator of a link between i and j
$$G_{ij} = \begin{cases} 1, & \text{if firms } i \text{ and } j \text{ are linked,} \\ 0, & \text{otherwise.} \end{cases}$$
Note: $G_{ij} = G_{ji}$ and $G_{ii} = 0$.

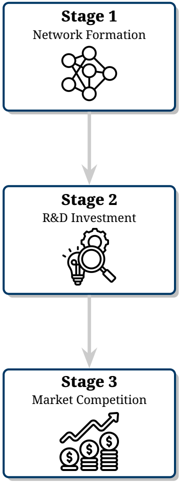
- $\mathcal{N}_i(\mathbf{G})$ — neighbours of firm i
$$\mathcal{N}_i(\mathbf{G}) = \{\, j \in \mathcal{N} \mid G_{ij} = 1 \,\}$$
- $\mathcal{N}_{-i}(\mathbf{G})$ — non-neighbours of firm i
$$\mathcal{N}_{-i}(\mathbf{G}) = \{\, j \in \mathcal{N}/\{i\} \mid G_{ij} = 0 \,\}$$
- $d_i(\mathbf{G})$ — degree of firm i
$$d_i(\mathbf{G}) = |\mathcal{N}_i(\mathbf{G})| = \sum_{j \in \mathcal{N}} G_{ij}$$
- $\eta_i(\mathbf{G})$ — sparsity coefficient of firm i
(share of firms not linked to i, normalized)
$$\eta_i(\mathbf{G}) = \frac{n - d_i(\mathbf{G})}{n+1}$$

Model Overview

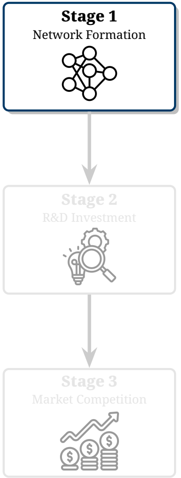
We consider a **three-stage game** with complete information:

1. **Network Formation:** Firms strategically form bilateral R&D collaborations.
2. **R&D Investment:** Firms choose costly effort levels ($\tilde{e}_i$) to reduce marginal cost of production.
3. **Market Competition:** Firms compete *à la Cournot* in a homogeneous product market.

Solution Concept: Backward Induction (Subgame Perfect Nash Equilibrium), utilizing Pairwise Stability for the network stage.



Stage 1: Network Formation



Action: Firms choose to form bilateral links $G_{ij} \in \{0, 1\}$.

Pairwise Stability (Jackson and Wolinsky, 1996): A network **G** is pairwise stable if:

1. **No severing:** For any existing link $G_{ij} = 1$, neither firm i nor j benefits from breaking it.
2. **No new links:** For any non-existing link $G_{ij} = 0$, no pair (i, j) strictly benefits from adding it.

Stage 2: R&D Investment

Given the network **G**, firms choose R&D effort $\tilde{e}_i$ to reduce marginal cost.

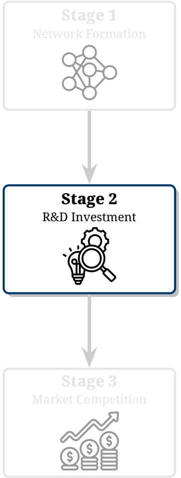
Heterogeneity: Firms differ in their R&D efficiency. The cost of effort is quadratic:

$$C(\tilde{e}_i) = \phi_i \cdot \tilde{e}_i^2$$

- ϕ_i: The **inverse of productivity**.
- A higher ϕ_i means the firm must spend more to achieve the same effort $\tilde{e}_i$.

Marginal Cost Reduction: Efforts translate perfectly into cost reductions for the firm and its neighbors:

$$c_i(\tilde{\mathbf{e}}, \mathbf{G}) = \bar{c} - \tilde{e}_i - \underbrace{\sum_{j \in \mathcal{N}_i(\mathbf{G})} \tilde{e}_j}_{\text{collaboration benefit}}$$



Stage 3: Market Competition

Firms compete in quantities facing linear inverse demand:

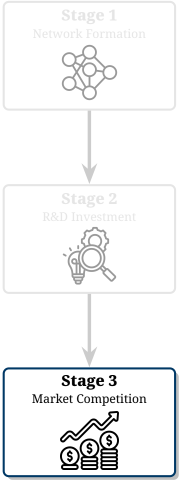
$$p(\mathbf{q}) = \alpha - \sum_{i \in \mathcal{N}} q_i$$

Profit Maximization:

$$\pi_i(\mathbf{q}, \tilde{\mathbf{e}}, \mathbf{G}) = \underbrace{q_i \cdot p(\mathbf{q})}_{\text{Total Revenue}} - \underbrace{q_i \cdot c_i(\tilde{\mathbf{e}}, \mathbf{G})}_{\text{Production Cost}} - \underbrace{\phi_i \cdot \tilde{e}_i^2}_{\text{R\&D Cost}}$$

Equilibrium Output given the production technologies (costs):

$$q_i^*(\tilde{\mathbf{e}}, \mathbf{G}) = \frac{1}{n+1}\left(\alpha - c_i(\tilde{\mathbf{e}}, \mathbf{G}) + \sum_{k \in \mathcal{N}} \left[c_k(\tilde{\mathbf{e}}, \mathbf{G}) - c_i(\tilde{\mathbf{e}}, \mathbf{G})\right]\right)$$



Normalization

By defining $\phi := \min\{\phi_1, \ldots, \phi_n\}$, and $\theta_i = \sqrt{\phi/\phi_i}$, we rewrite:

$$\left.\begin{aligned} c_i\left(\tilde{\mathbf{e}}, \mathbf{G}\right) &= \bar{c} - \tilde{e}_i - \sum_{j \in \mathcal{N}_i(\mathbf{G})} \tilde{e}_j \\ \pi_i\left(\mathbf{q}, \tilde{\mathbf{e}}, \mathbf{G}\right) &= q_i \cdot \left[p\left(\mathbf{q}\right) - c_i\left(\tilde{\mathbf{e}}, \mathbf{G}\right)\right] - \phi_i \cdot {\tilde{e}_i}^2 \end{aligned}\right\} \rightarrow \left\{\begin{aligned} c_i\left(\mathbf{e}, \mathbf{G}\right) &= \bar{c} - \theta_i\, e_i - \sum_{j \in \mathcal{N}_i(\mathbf{G})} \theta_j\, e_j \\ \pi_i\left(\mathbf{q}, \mathbf{e}, \mathbf{G}\right) &= q_i \cdot \left[p\left(\mathbf{q}\right) - c_i\left(\mathbf{e}, \mathbf{G}\right)\right] - \phi \cdot {e_i}^2 \end{aligned}\right.$$

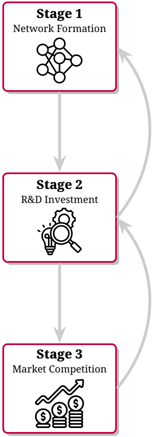
where

- θ_i: relative productivity of firm i,
- e_i: rescaled R&D effort $e_i = \tilde{e}_i/\theta_i$.

Backward Induction

We solve for the sub-game perfect equilibrium.

1. Solve the **optimal production quantities**, given the collaboration network, and R&D efforts (and costs) as given.
2. Solve for the **optimal R&D effort**, knowing the production behavior, and taking the network as given.
3. Solve for the **equilibrium collaboration structure**, knowing the R&D effort, and production behavior.



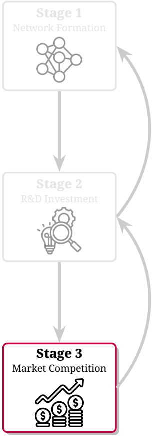
Equilibrium Outputs

The equilibrium outputs, given the collaboration network **G**, and effort profile **e** is:

$$q_i^*(\mathbf{e}, \mathbf{G}) = \underbrace{\frac{\alpha - \bar{c}}{n+1}}_{\text{baseline}} + \underbrace{\eta_i(\mathbf{G})\,\theta_i\,e_i}_{\text{own effort effect}} + \underbrace{\sum_{j \in \mathcal{N}_i(\mathbf{G})} \eta_j(\mathbf{G})\,\theta_j\,e_j}_{\text{neighbors' effort effect } (+)} - \underbrace{\sum_{k \in \mathcal{N}_{-i}(\mathbf{G})} [1 - \eta_k(\mathbf{G})]\,\theta_k\,e_k}_{\text{non-neighbors' effort effect } (-)},$$

where

- $\eta_i(\mathbf{G}) := \frac{n - d_i(\mathbf{G})}{n+1} \in \left[\frac{1}{n+1}, \frac{n}{n+1}\right]$ is the sparsity coefficient,
- $d_i(\mathbf{G}) := |\mathcal{N}_i(\mathbf{G})|$ is the degree of firm i in network $\mathbf{G}$.



Equilibrium Efforts I

Given the collaboration network $\mathbf{G}$, the best response effort of firm i given the effort of other firms $\mathbf{e}_{-i}$ is:

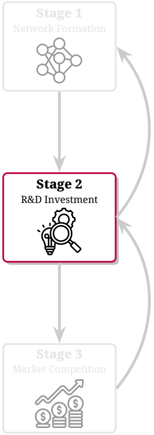
$$e_i^*\left(\mathbf{G},\mathbf{e}_{-i}\right) = \underbrace{\frac{\theta_i\,\eta_i(\mathbf{G})}{\phi-\theta_i^2\,\eta_i^2(\mathbf{G})}}_{\text{sparsity}} \cdot \left[\underbrace{\frac{\alpha-\bar{c}}{n+1}}_{\text{baseline}} + \underbrace{\sum_{j\in\mathcal{N}_i(\mathbf{G})}\eta_j(\mathbf{G})\,\theta_j\,e_j}_{\text{neighbor effect}} - \underbrace{\sum_{k\in\mathcal{N}_{-i}(\mathbf{G})}\left[1-\eta_k(\mathbf{G})\right]\theta_k\,e_k}_{\text{non-neighbor effect}}\right]$$

The solution is generally intractable as: $\mathbf{e}^*(\mathbf{G}) = (\alpha-\bar{c})\,\mathbf{A}(\mathbf{G})^{-1}\,\mathbf{1}_n$.

Proposition 1

There exists a **unique, interior equilibrium effort profile** $e_i^*(\mathbf{G}) > 0,\ \forall i \in \mathcal{N}$ for any network $\mathbf{G} \in \mathcal{G}^n$, provided the cost of R&D is sufficiently high:

$$\phi > \underline{\phi} := \frac{n^3-n^2-n+2}{n^2+2n+1}.$$



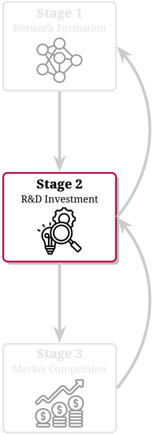
Equilibrium Efforts II

Proposition 2 (efforts)

Let firms $i, j \in \mathcal{N}$ have a symmetric position in a given network $\mathbf{G}$. That is $\forall k \in \mathcal{N} : G_{ik} = G_{jk}$. Then, the ratio of their equilibrium efforts are:

$$\frac{e_i^*(\mathbf{G})}{e_j^*(\mathbf{G})} = \frac{\theta_i}{\theta_j} \frac{\phi - \theta_j^2\, \eta_j(\mathbf{G})\,(1 - G_{ij})}{\phi - \theta_i^2\, \eta_i(\mathbf{G})\,(1 - G_{ji})}.$$

- The firm with higher productivity exerts higher efforts ($\theta_i \geq \theta_j \Rightarrow e_i^* \geq e_j^*$),
- Firms i and j have the same effective effort if $G_{ij} = 1$ ($e_i^*/\theta_i = e_j^*/\theta_j$).



Equilibrium Efforts III

Proposition 2 (profits)

Let firms $i, j \in \mathcal{N}$ have a symmetric position in a given network $\mathbf{G}$. That is $\forall k \in \mathcal{N} : G_{ik} = G_{jk}$. Then, the ratio of their equilibrium profits are:

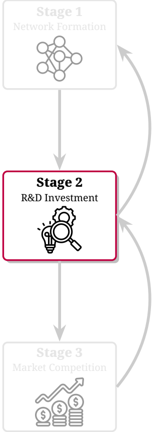
$$\frac{\pi_i^*(\mathbf{G})}{\pi_j^*(\mathbf{G})} = \frac{\phi - \theta_i^2\, \eta_i^2(\mathbf{G})}{\phi - \theta_j^2\, \eta_j^2(\mathbf{G})} \left[\frac{\phi - \theta_j^2\, \eta_j(\mathbf{G})\,(1 - G_{ij})}{\phi - \theta_i^2\, \eta_i(\mathbf{G})\,(1 - G_{ji})} \right]^2 .$$

- Comparing two firms with different productivities $\theta_i \geq \theta_j$

$$\pi_i^*(\mathbf{G}_{+ij}) \leq \pi_j^*(\mathbf{G}_{+ij}), \qquad \text{and} \qquad \pi_i^*(\mathbf{G}_{-ij}) \geq \pi_j^*(\mathbf{G}_{-ij}).$$

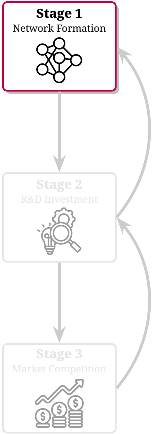
- If the more productive firm benefits from the connection, the less productive firm also benefits

$$\pi_i^*(\mathbf{G}_{-ij}) < \pi_i^*(\mathbf{G}_{+ij}) \quad \Rightarrow \quad \pi_j^*(\mathbf{G}_{-ij}) < \pi_j^*(\mathbf{G}_{+ij}).$$



Equilibrium Network

- We adopt Pairwise Stability to identify equilibrium networks.
- Characterizing the full set of stable networks is intractable. The same challenge is present even in the homogeneous framework of Goyal and Moraga-Gonzalez (2001).
- Goyal and Moraga-Gonzalez (2001) show that the **Complete Network** is stable.



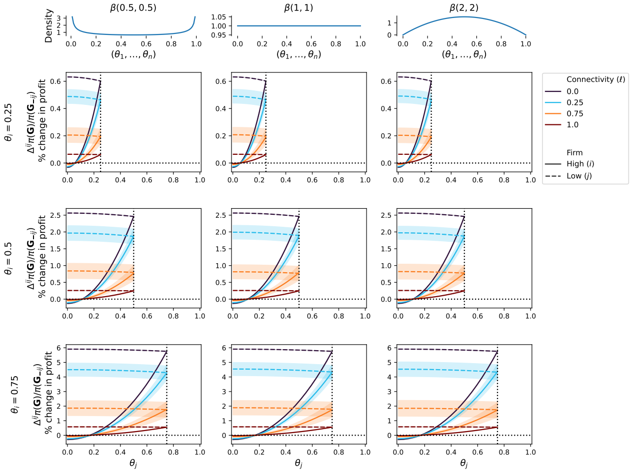
Proposition 3

There exists a threshold $\theta^* \in (0, 1)$ such that if there is a firm j with productivity $\theta_j < \theta^*$, then the complete network $\mathbf{G}^C$ is **not pairwise stable**.

Single-link Deviation

We use simulation to identify the creation of a link between two firms i and j, where $\theta_i \geq \theta_j$.

- The profit gain for a more productive firm ($\Delta\pi_i$) is strictly increasing in the partner's productivity θ_j.
- For any network **G**, there exists a cutoff $\theta_i^*(\mathbf{G})$ such that firm i **rejects** any partner with $\theta_j < \theta_i^*(\mathbf{G})$.



New Environment: The Two-Type Setting

To fully characterize the set of stable networks, we introduce a simplified environment.

Two Types of Firms:

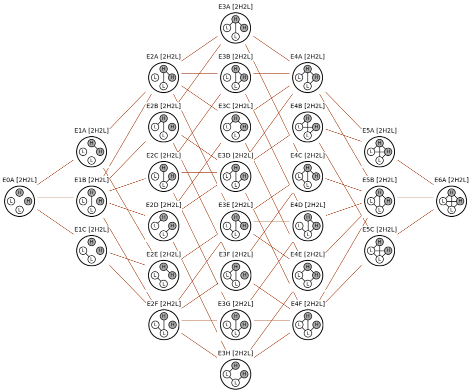
- **High-productive (H):** Normalized productivity $\theta_H = \theta_{\texttt{max}} = 1$.
- **Low-productive (L):** Productivity $\theta := \theta_L \in (0, 1)$.

Population Structure (ρ): Let n_H be the number of high-productive firms. We define the ratio:

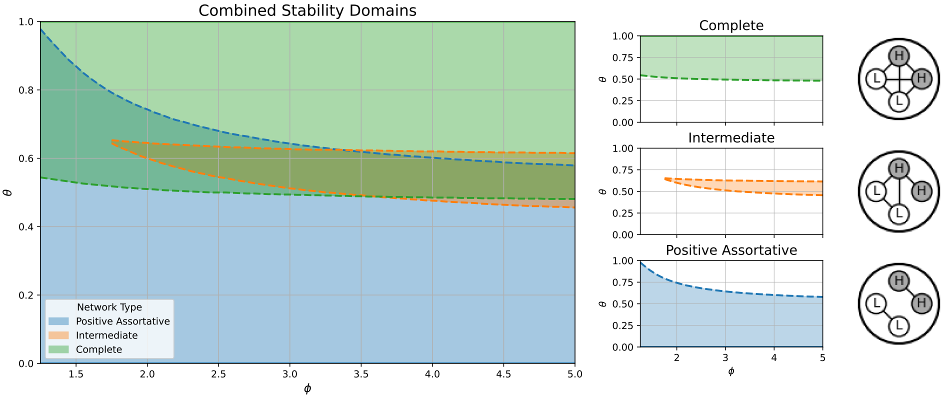
$$\rho = \frac{n_H}{n} \in [0, 1].$$

We initially set $\rho = 1/2$, and experiment with it later.

Transition Graph for 4 Nodes with 2 H-Type Nodes (with Images)



Full Characterization for $n = 4$



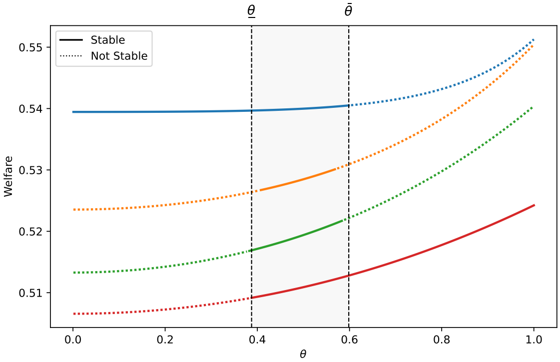
- Complete is stable when $\theta \geq \underline{\theta}$.
- Positive Assortative (PA) is stable when $\theta \leq \overline{\theta}$.

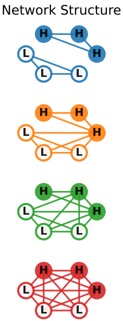
Welfare Analysis

$$W(\mathbf{G}) = \underbrace{\sum_{i \in \mathcal{N}} \pi_i^*(\mathbf{G})}_{\text{Producer Surplus}} + \underbrace{\frac{1}{2}\Big[\sum_{i \in \mathcal{N}} q_i^*(\mathbf{G})\Big]^2}_{\text{Consumer Surplus}}$$

We define Welfare $W(\mathbf{G})$ as the sum of Producer Surplus and Consumer Surplus.

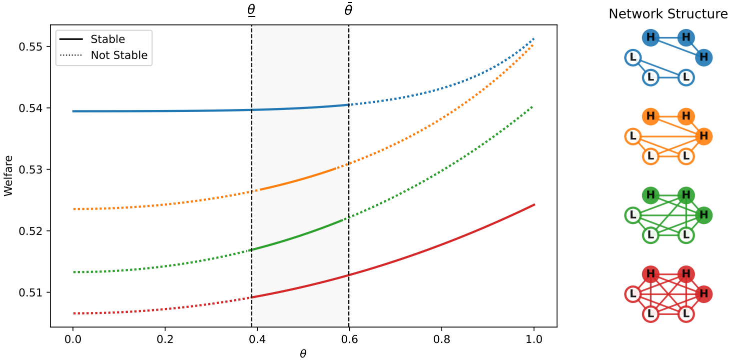
- Welfare is ranked by the level of connectivity.
- When θ is sufficiently low/high, the PA/complete network are uniquely stable.





Welfare Paradox: The Stability Cliff

- Welfare increases with θ for any given stable $\mathbf{G}$.
- The level of θ determines which structure is stable.



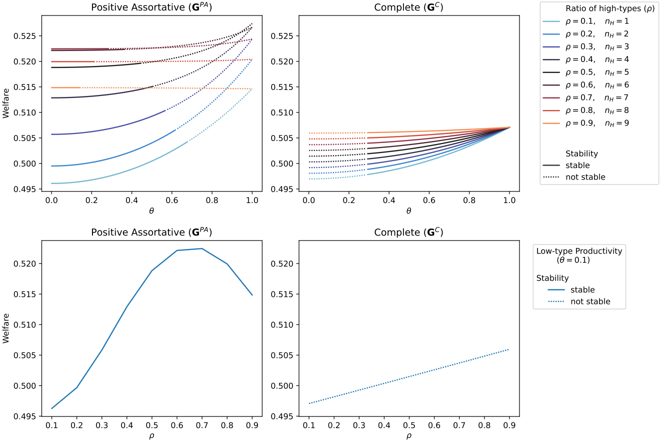
Government and/or firm-level R&D policies can have negative welfare effects even if they increase average productivity in the economy.

Welfare Paradox: Crowding-Out

- We initially set $\rho = 1/2$.
- Here we set $n = 10$ and experiment with ρ.

Results:

- $W(\mathbf{G}^C)$ increases with ρ.
- $W(\mathbf{G}^{PA})$ shows an inverted U-shape relation with ρ.



Conclusion (1/2): What Heterogeneity Does

Contribution

- Heterogeneous productivities reshape collaboration incentives.
- Complete Network becomes unstable when productivity gaps grow.
- Positive Assortative (PA) Network emerges as the stable structure.

Key Finding: The "Welfare Paradox"

- Raising average productivity can reduce aggregate welfare.
- Stability Cliff: PA $\rightarrow$ Complete jump lowers welfare.
- Crowding-Out: welfare is inverted-U in the share of high types.

Conclusion (2/2): Policy Implications

Core Insight

Policies that **raise average R&D productivity** (e.g., increasing θ or changing the share of high types ρ) can push the economy into a **lower-welfare stable network equilibrium**.

Why?

- Network formation is endogenous, not fixed.
- Small policy changes can trigger equilibrium switches.
- Welfare depends on network structure more that the overall productivity.

Effective R&D policy must anticipate equilibrium network responses.

References I

Aghion, P. and Howitt, P. (1992). A Model of Growth Through Creative Destruction. *Econometrica*, 60(2):323–351.

Billand, P., Bravard, C., Durieu, J., and Sarangi, S. (2019). Firm heterogeneity and the pattern of R&D collaborations. *Economic Inquiry*, 57(4):1896–1914.

Bischi, G. I. and Lamantia, F. (2012). A dynamic model of oligopoly with R&D externalities along networks. part i. *Mathematics and Computers in Simulation*, 84:51–65.

Calero, C., van Leeuwen, T., and Tijssen, R. (2007). Research cooperation within the bio-pharmaceutical industry: Network analyses of co-publications within and between firms. *Scientometrics*, 71(1):87–99.

Goyal, S. and Joshi, S. (2003). Networks of collaboration in oligopoly. *Games and Economic behavior*, 43(1):57–85.

Goyal, S. and Moraga-Gonzalez, J. L. (2001). R&D networks. *Rand Journal of Economics*, pages 686–707.

Hsieh, C.-S., König, M. D., and Liu, X. (2024). Endogenous technology spillovers in dynamic R&D networks. *The RAND Journal of Economics*.

Jackson, M. and Wolinsky, A. (1996). A strategic model of economic and social networks. *Journal of Economic Theory*, 71(1):44–74.

Jones, C. I. (1995). R&D-based models of economic growth. *Journal of Political Economy*, 103(4):759–784.

König, M. D., Liu, X., and Zenou, Y. (2019). R&D networks: Theory, empirics, and policy implications. *Review of Economics and Statistics*, 101(3):476–491.

Lentz, R. and Mortensen, D. T. (2008). An empirical model of growth through product innovation. *Econometrica*, 76(6):1317–1373.

Romer, P. M. (1990). Endogenous technological change. *Journal of Political Economy*, 98(5, Part 2):S71–S102.

Zirulia, L. (2012). The role of spillovers in R&D network formation. *Economics of Innovation and New Technology*, 21(1):83–105.