Bearing-Based Formation Control with Optimal Motion Trajectory

Zili Wang[1], Sean B. Andersson[1,2], Roberto Tron[1,2]

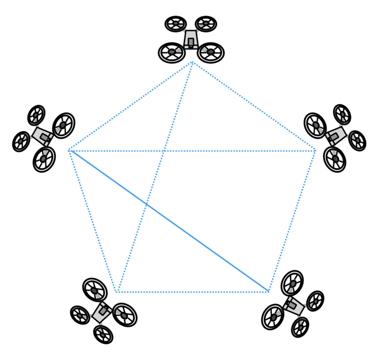
[1] Division of Systems Engineering, [2] Department of Mechanical Engineering

Boston University, MA 02155

Introduction

Problem Formulation

Given multiple mobile robots with single integrator dynamics, design a bearing-based controller that can steer the system of robots from random positions to a specific shape while minimizing the trajectory length.



Previous Work

- Distance-based formation control:
  non-reliable measurements when agents are far away.
- Bearing-based formation control:
  require distance, special graph topology, or a specific Lyapunov function.

Our Contribution

- Simplified sufficient conditions on global convergence of a bearing-based formation controller.
- Formulated an optimization problem to automatically tune the controller to minimize trajectory lengths.
- Generalizes well to new initial conditions and topologies.

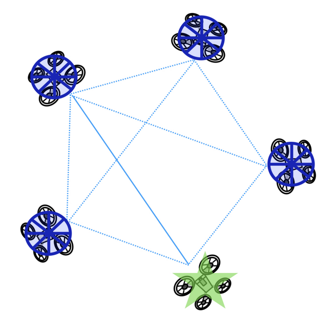
Existing Controller (Tron, CDC 16)

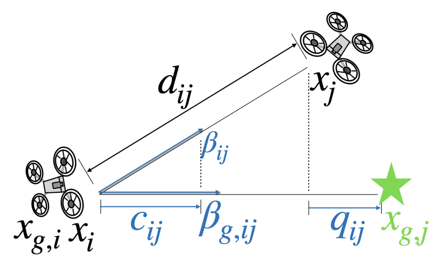
Gradient-based control law (bearing with optional range measurements).

\+ Global convergence.

\+ Allow flexible choice up to sufficient conditions, but choice was ad-hoc.

– Might give sub-optimal trajectories.





$$\varphi = \sum_{(i,j)\in E} \varphi_{ij}(c_{ij}, d_{ij})$$

$$\dot{\mathbf{x}} = -\mathrm{grad}_x \varphi$$

$$\varphi^b_{ij} = d_{ij} f_b(c_{ij}) \quad \text{bearing similarity}$$

$$\varphi^d_{ij} = f_d(q_{ij}) \quad \text{range similarity}$$

$$u_i = -\sum_{j:(i,j)\in E} \mathrm{grad}_{x_i} \varphi_{ij}(c_{ij}, d_{ij})$$

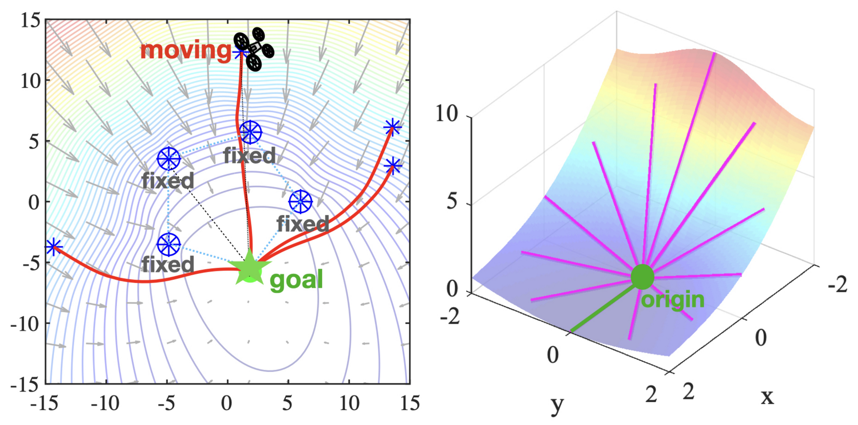
Convergence Conditions

C1. $f_b(1) = 0$,

C2. $f_b'(c_{ij}) = \begin{cases} \le 0, & c_{ij} = 1, \\ < 0, & \text{otherwise.} \end{cases}$

C3. $f_b(c_{ij}) + (1 - c_{ij}) f_b'(c_{ij}) \le 0$

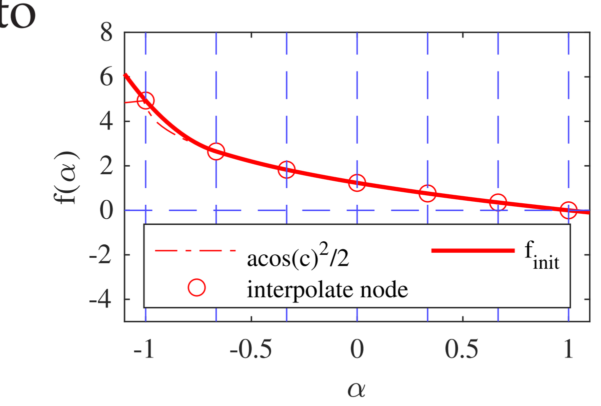
Agent Cost　　Edge Cost



Function Interpolator

We use a piecewise polynomial interpolator to parameterize f_b (f_d) in the controller.

$$\begin{aligned} f(\chi, \alpha) =& a_k^0 + \sum_{i=1}^{k-1} (a_i^1 h + a_i^2 h^2) + a_k^1 (\chi - \chi_k) \\ &+ a_k^2 (\chi - \chi_k)^2, \quad \chi \in [\chi_k, \chi_{k+1}] \\ h =& \chi_{k+1} - \chi_k, \\ 1 =& \chi_1 > \chi_2 > \cdots > \chi_K = -1 \end{aligned}$$

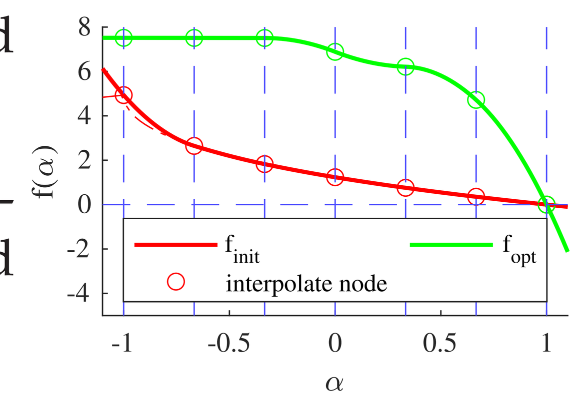


α: minimal parameterization of $\{a_k^r\}_{k\in\{1,\dots,K\}}^{r\in\{0,1,2\}}$.

Simplified Convergence Conditions

We simplify the sufficient conditions to C1-C2.

- Condition: control cost increases monotonically along radial lines, except in the desired bearing direction.
- Converge asymptotically to a scaled (congruent if with distance) version of the desired formation.
- Expand the class of cost functions, and give linear constraints on function parameters.



Nonlinear Optimization

We formulate a nonlinear optimization program, to minimize the path length and terminal cost.

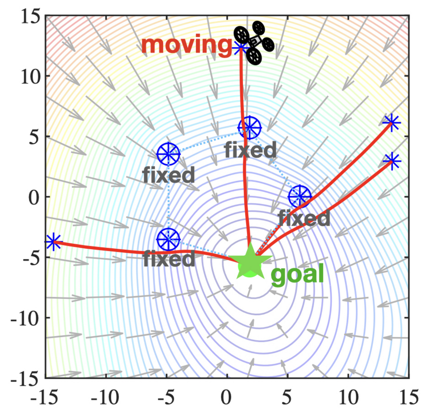
$$L(\alpha, \mathbf{x}_0) = \sum_{i \in V} \int_0^T ||\dot{\mathbf{x}}_i(\mathbf{x}(t), \alpha)|| \mathrm{d}t + \omega \varphi(\mathbf{x}(T)) \Big|_{\mathbf{x}(0)=\mathbf{x}_0}$$

$$\min_{\alpha} \quad \sum_{\mathbf{x}_0 \in X_0} L(\alpha, \mathbf{x}_0)$$

subj. to gradient control law, constraints on f_b (and f_d)

- Optimize over a small training set.
- Provide analytical derivative of the objective function using sensitivity function.
- Solved using Sequential Quadratic Program.
- Gives rounder level set of agent control cost and straighter trajectories.

Agent Cost



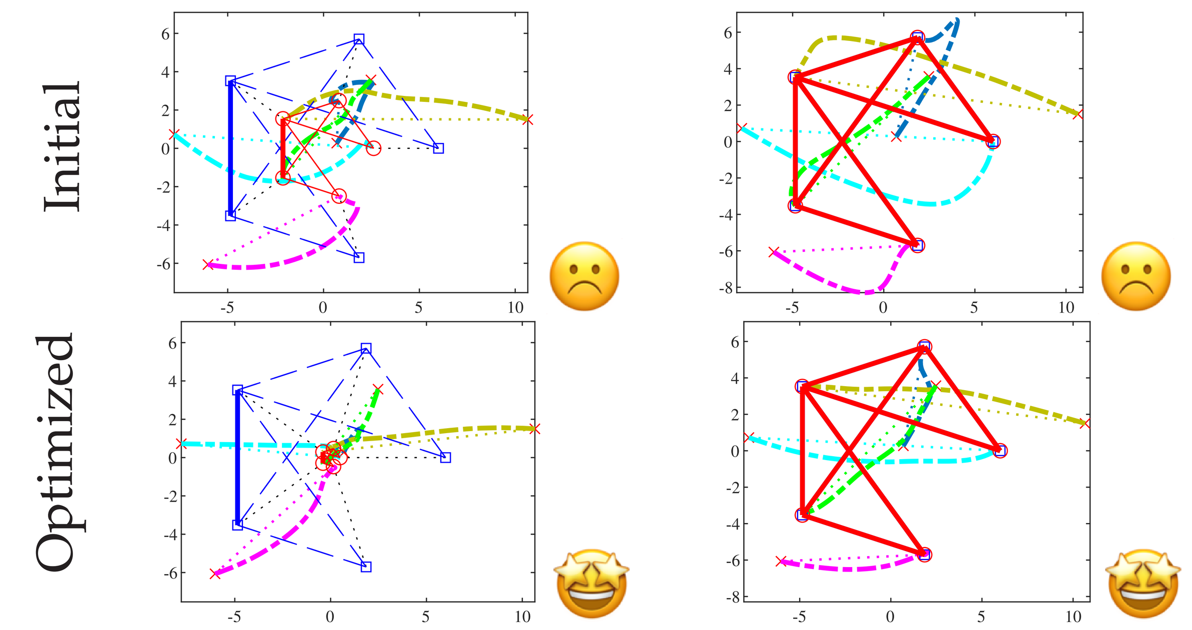
Results

We solve the optimization program on 1-7 initial conditions (ICs), then test the optimized controller on 200 random ICs.

Case	Model		Training		Test		
			$\delta path$ (%)	$\delta diff.$ (%)	$\delta path$(%)	$\delta diff.$(%)	+%
Train on 5 Test on 5	NoEd	7 ICs	7.26	22.04	7.87 (7.45)	**15.7** (19.76)	**90**
	OneEd	7 ICs	3.75	23.03	4.09 (4.32)	22.58 (24.05)	90
	SomeEd	7 ICs	11.02	53.67	9.14 (9)	49.42 (52.83)	97.5
	FullEd	7 ICs	**11.05**	**61.15**	**11.26** (11.73)	**65.64** (67.78)	**99.5**

Ed: edge for distance measurements　δ_{path}: change in path length

+%: % of improved samples　δ_{diff}: diff. of path length from a straight line

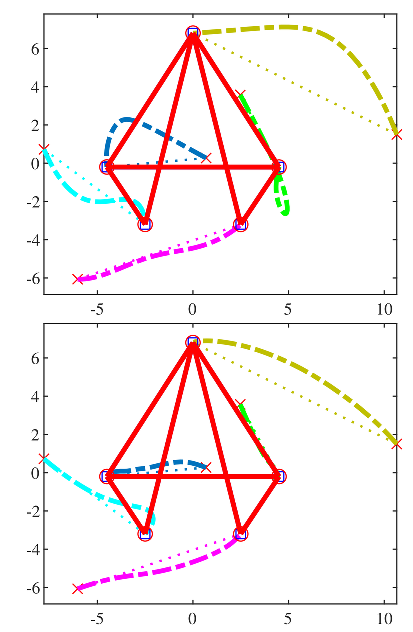


- Result from small training set improves large test set.
- Increasing training ICs gives straighter trajectories in average.

Generalizability

We test the optimized controllers on different formation shape and number of agents.

Case	Model		Test		
			$\delta path$(%)	$\delta diff.$(%)	+%
Train on 5 Test on 5 Alt.shape	NoEd	7 ICs	8.8 (8.19)	22.7 (21.9)	95.5
	OneEd	7 ICs	4.02 (4.25)	21.56 (22.11)	89
	SomeEd	7 ICs	7.91 (8.28)	39.12 (39.99)	94
	FullEd	7 ICs	**9.16** (9.52)	**56.55** (59.62)	**99**
Train on 5 Test on 3	NoEd	7 ICs	8.76 (7.91)	29.38 (37.79)	93
	OneEd	7 ICs	5.97 (6.25)	40.5 (47.31)	92.5
	SomeEd	7 ICs	6.44 (6.75)	40.54 (52.07)	91
	FullEd	7 ICs	**12.66** (14.14)	**87.53** (92.07)	**99.5**



- Yield similar trajectory improvement, and generalizes well to different topologies

Future Work

- Apply to different dynamic models
- Propose a collision avoidance solution

paper　example