COMPRESSIVE SENSING AND ROBOTIC MAPPING IN RESOURCE CONSTRAINED ROBOTS

Presenter: Zili Wang

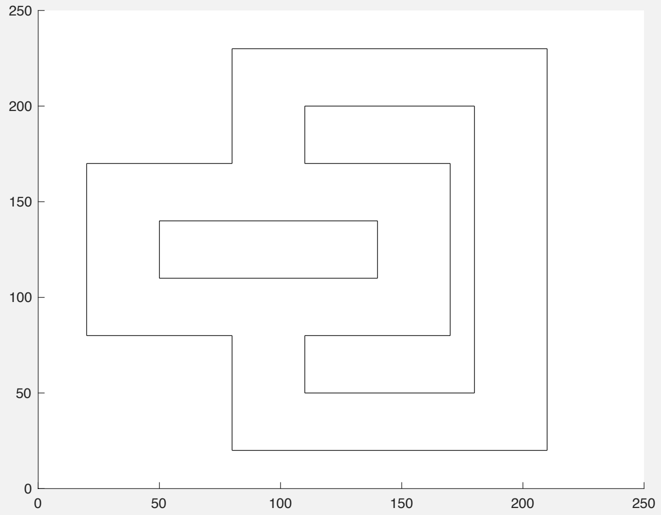
Date: July 22nd, 2020

PROBLEM DEFINITION

- Given resource-constrained robot, which has limited on-board power or payload to carry a 3D lidar/ high resolution camera, we want it to navigate and map an environment.

  Question: How to use the sparse and incomplete data (single beam laser, low resolution laser range-finder, single pixel camera etc) to reconstruct the environment efficiently?

- Question of simplicity: How to reconstruct a 2D/3D indoor environment with collections of sparse data?



TWO EXISTING SOLUTIONS

Observation Reconstruction

Spatial: Given incomplete information of a single measurement, to reconstruct the depth profile.

before SLAM

Map Reconstruction

Temporal and Spatial: Given geometric descriptors, to estimate the map

In SLAM

OBSERVATION RECONSTRUCTION

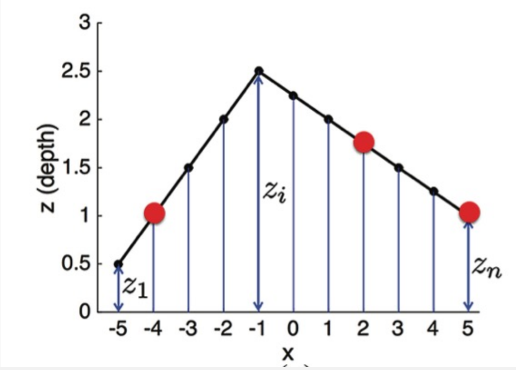
Sparse Sensing [8] to reconstruct 2D depth profile $z \in R^n$

- Assumption: z is sufficiently regular and contains few corner indicators
- Measurement model

$$y = Az + \eta$$

  where the sparse sampling matrix $A \in R^{m \times n}$ is with $m \ll n$, $A = I_M$ with M the measured entries of depth profile, and η is the measurement noise

- Fact: the corners are from change of slopes



- Curvature at i: $z_{i-1} - 2z_i + z_{i+1}$

i. Zero: collinear

ii. Positive: locally convex

iii. Negative: locally concave

OBSERVATION RECONSTRUCTION

Sparse Sensing [8] to reconstruct 2D depth profile $z \in R^n$

- Number of corners: $||Dz||_0$ where $D = \begin{bmatrix} 1 & -2 & 1 & 0 & \cdots & 0 \\ 0 & 1 & -2 & 1 & \cdots & 0 \\ \vdots & 0 & \ddots & \ddots & \ddots & \vdots \\ 0 & \cdots & 0 & 1 & -2 & 1 \end{bmatrix} \in R^{(n-2)\times n}$
- Problem $\min_{z} ||Dz||_0 \ \ subject\ to\ \ Az = y$ is NP-hard
- Relaxing the problem to

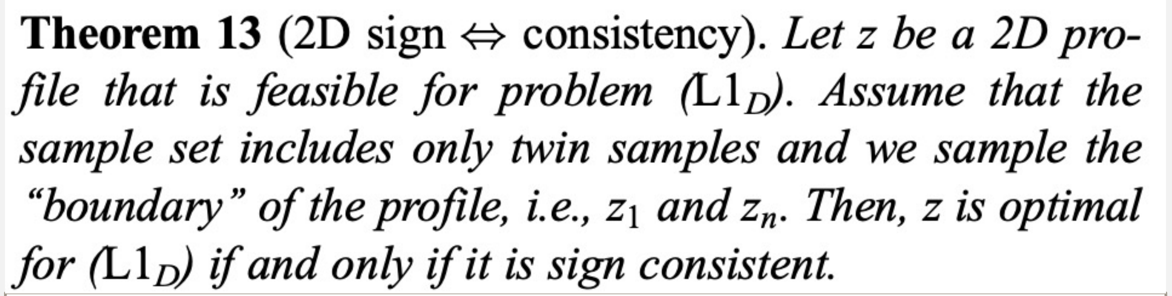
$$\min ||Dz||_1 \ \ subject\ to\ \ Az = y$$

- With bounded measurement noise ε, it becomes

$$\min ||Dz||_1 \ \ subject\ to\ \ ||Az - y||_\infty \le \varepsilon$$

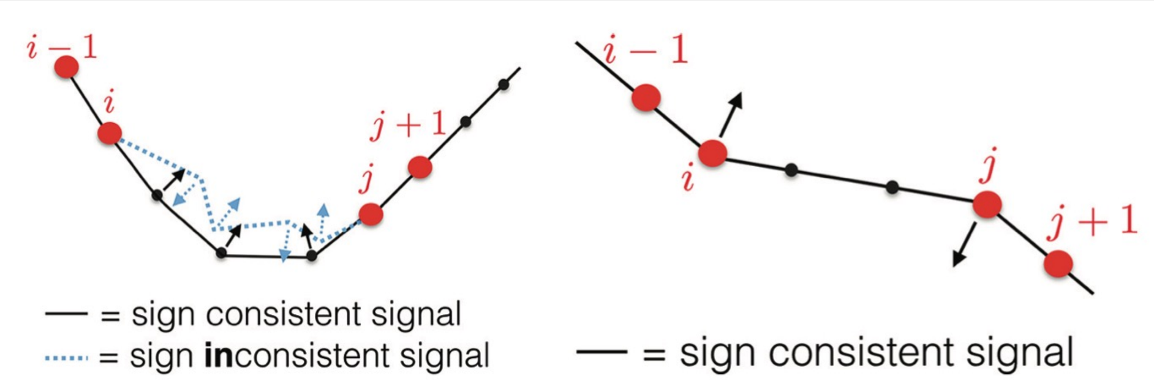
OBSERVATION RECONSTRUCTION

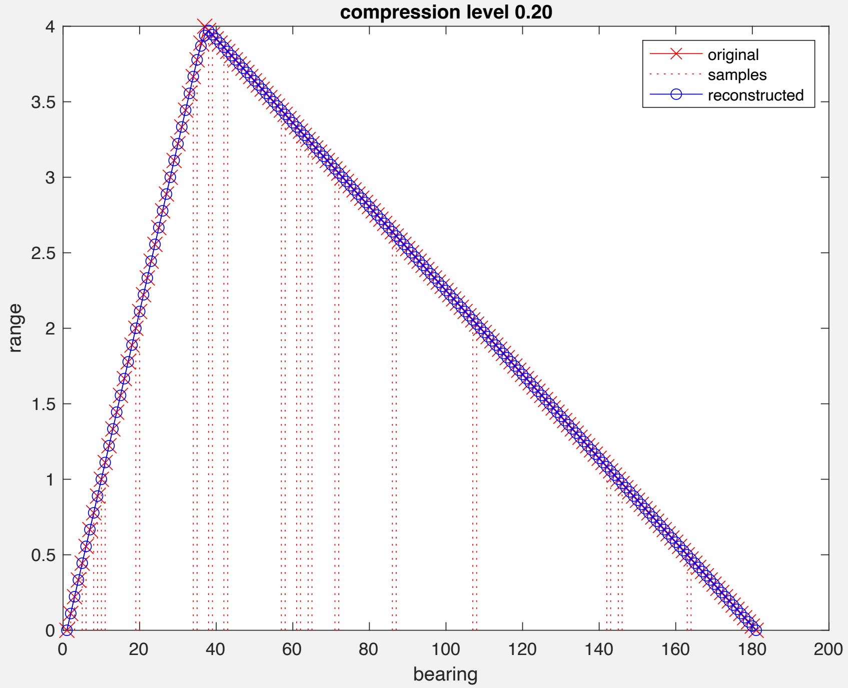
- Sparse Sensing [8] to reconstruct 2D depth profile $z \in R^n$



Theorem 13 (2D sign $\Leftrightarrow$ consistency). *Let z be a 2D profile that is feasible for problem* ($\mathrm{L1}_D$). *Assume that the sample set includes only twin samples and we sample the "boundary" of the profile, i.e., z_1 and z_n. Then, z is optimal for* ($\mathrm{L1}_D$) *if and only if it is sign consistent.*

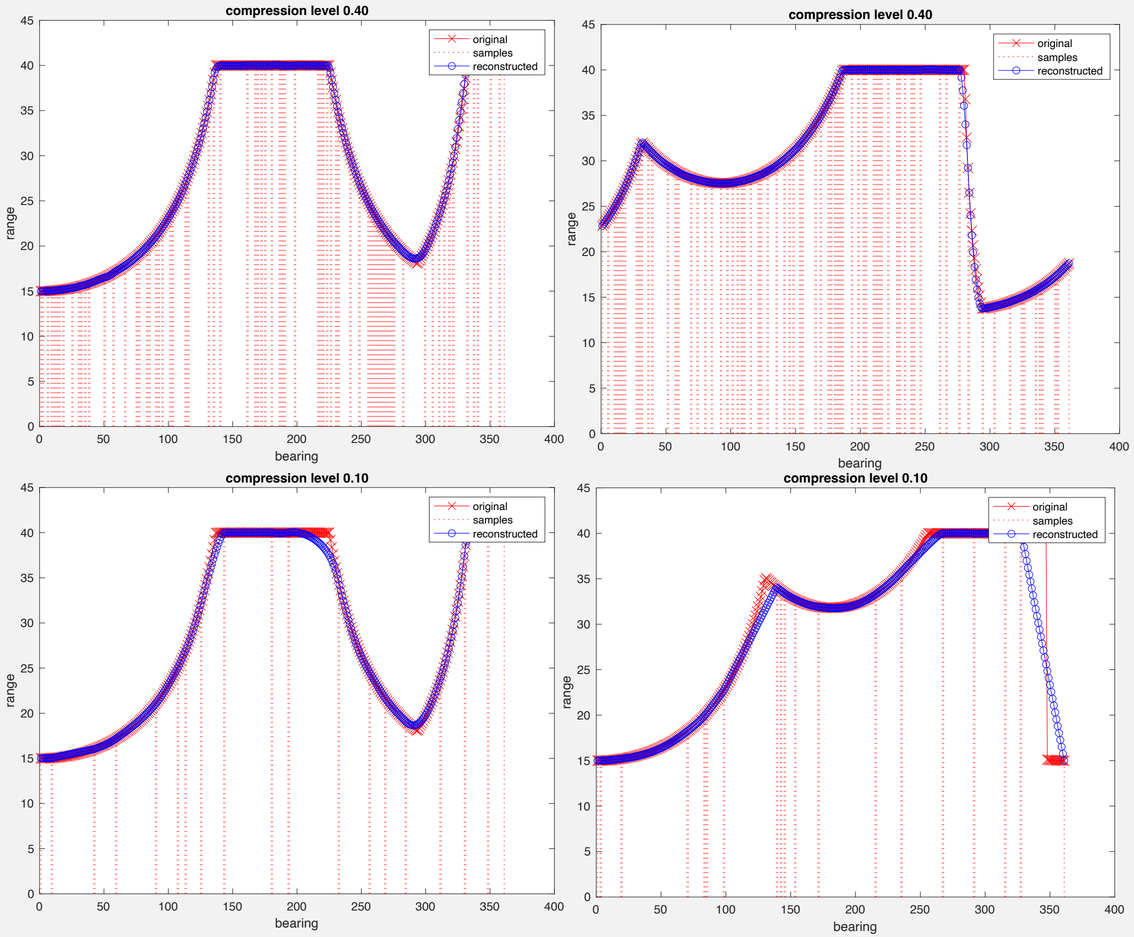
Where the sign consistency is determined by curvature within the interval of consecutive samples

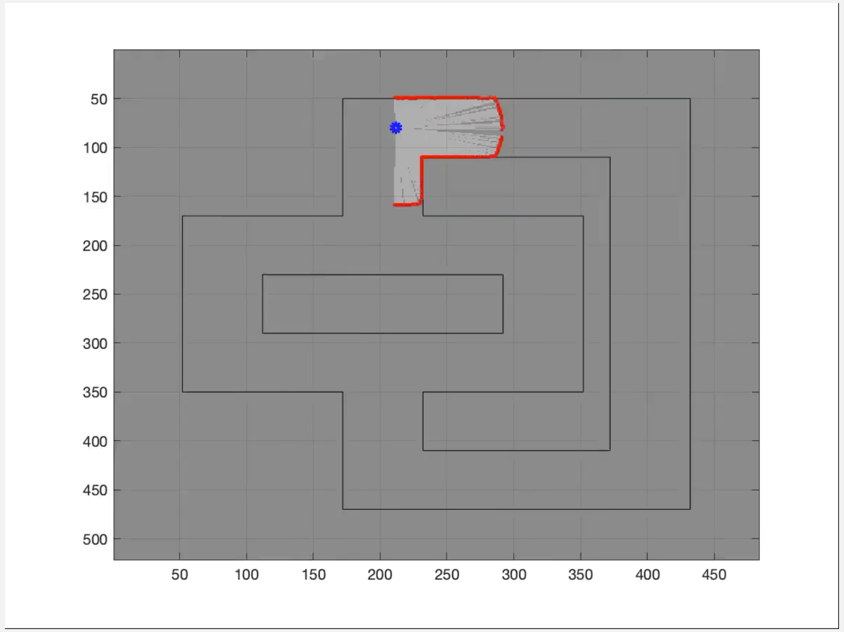

$i-1$
i
$j+1$
j
— = sign consistent signal
= sign **in**consistent signal
$i-1$
i
j
$j+1$
— = sign consistent signal

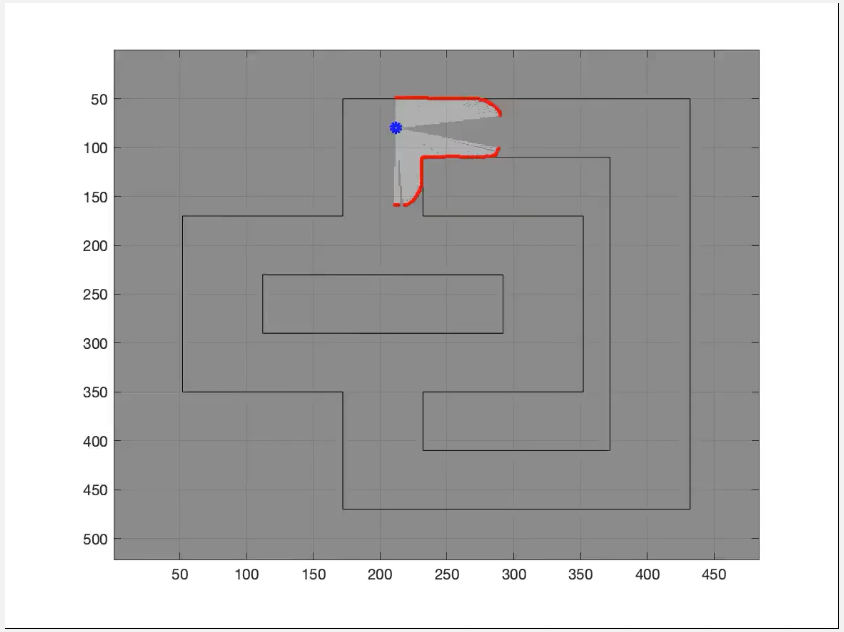


OBSERVATION RECONSTRUCTION

- $resolution\ 0.5^{\circ} /\ fielf\ of\ view 180^{\circ} / \max range\ 40m$







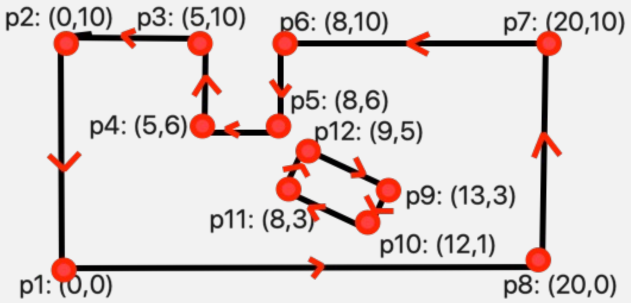
All videos: https://drive.google.com/drive/folders/1b-0oIgQ1xv7jFb74fyEu3p3ti0SeDLEN?usp=sharing

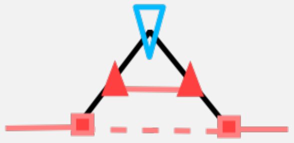
MAP RECONSTRUCTION

Wireframes mapping given robot trajectory[9]

- Goal: directly represent the metric geometry of the environment in a memory efficient way → build a map that is easy to store, communicate with others, and used for navigation
- Assumption: sensor layer extracts environment corners from raw measurements → measurement scan gives noisy environment corner locations relative to robot
- Wireframe map representation

- $W = \left\{ \underbrace{V_W}_{vertices}, \underbrace{E_W}_{edges}, \underbrace{\emptyset}_{\substack{function \\ V_W \to R^2}}, \underbrace{\lambda}_{\substack{function \\ V \to \mathcal{L}}} \right\}$
- $\mathcal{L} = \{nominal, occlusion, frontier\}$

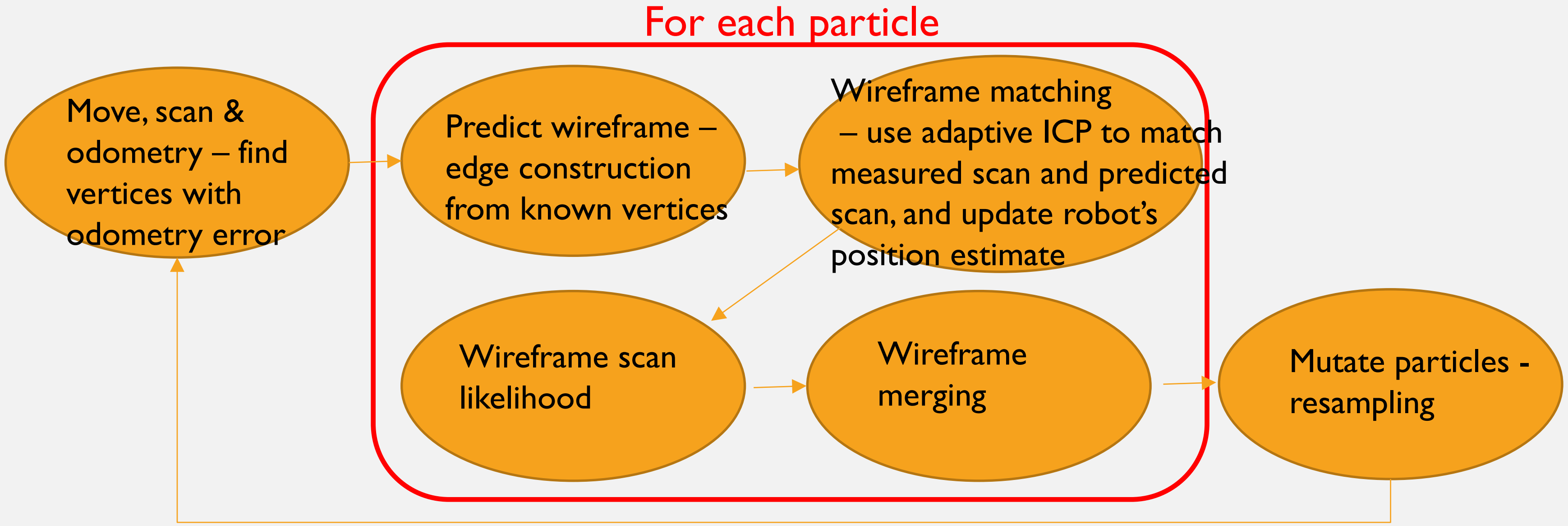




MAP RECONSTRUCTION

Wireframes mapping given robot trajectory[9]

- Particle filter to deal with the multiple hypotheses behind the uncertainty of binary decision in the existence of edges in the wireframe (unlike RBPF slam where each particle is a candidate robot trajectory, here the particles are candidate map geometry)



For each particle

Move, scan & odometry – find vertices with odometry error

Predict wireframe – edge construction from known vertices

Wireframe matching – use adaptive ICP to match measured scan and predicted scan, and update robot's position estimate

Wireframe scan likelihood

Wireframe merging

Mutate particles - resampling

$$p(S|W_i) = \prod_i \underbrace{g_i(p_i, scan)}_{\substack{Gaussian\ for \\ vertice\ measurement}} \prod_i \underbrace{b\ (e_{scan}(i,j), e_{predict}(i,j))}_{\substack{Bernoulli\ for \\ the\ existence\ of\ edge}}$$

INTRODUCTORY TUTORIAL OF SLAM

Presenter: Zili Wang

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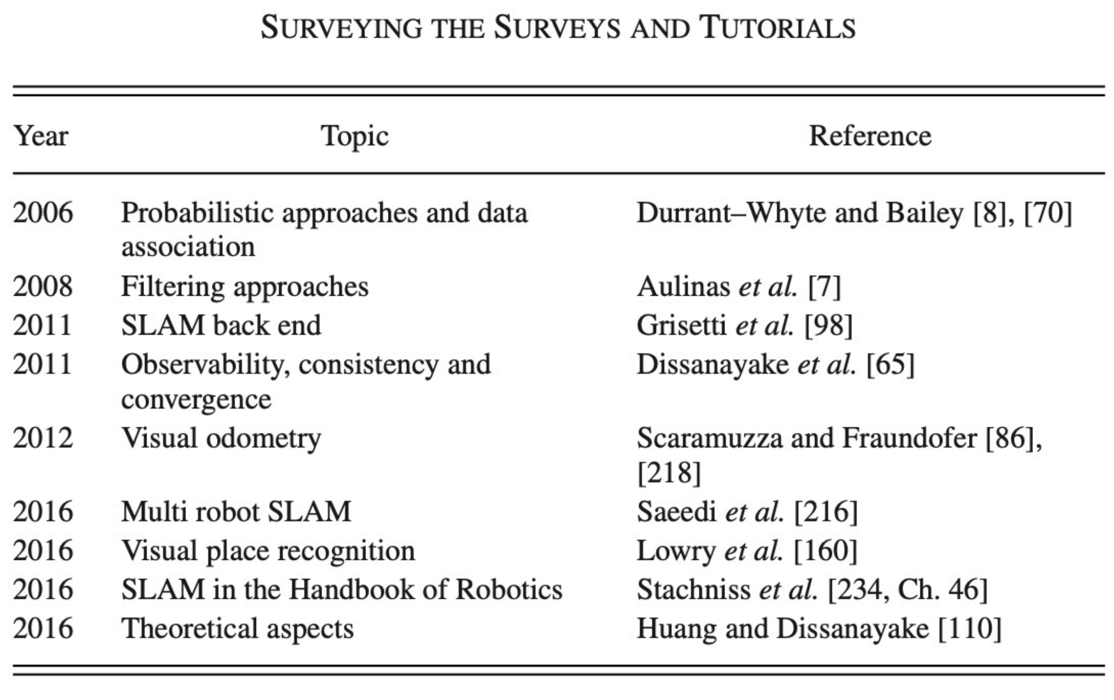
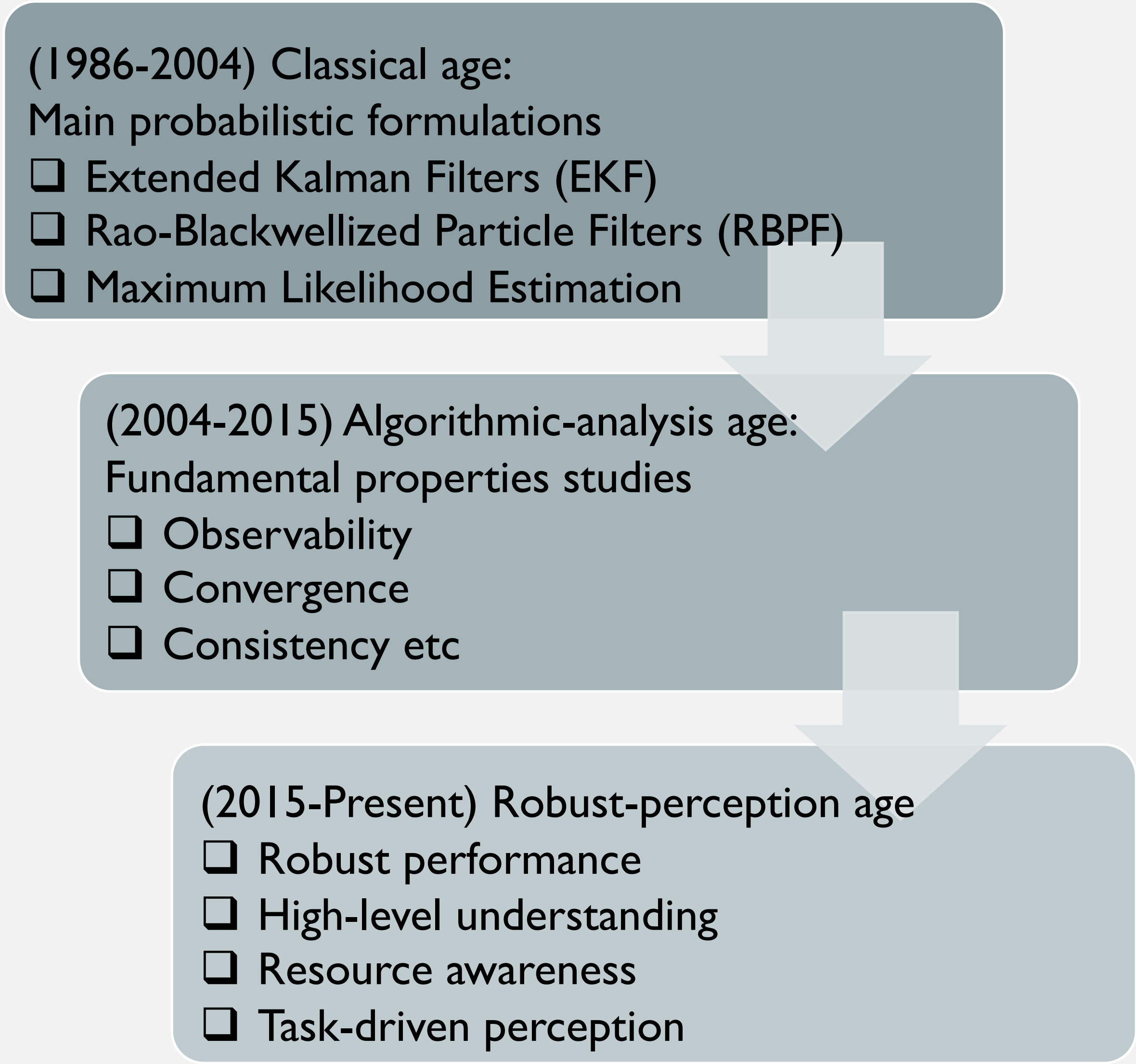
SIMULTANEOUS LOCALIZATION AND MAPPING



SURVEYING THE SURVEYS AND TUTORIALS

Year	Topic	Reference
2006	Probabilistic approaches and data association	Durrant–Whyte and Bailey [8], [70]
2008	Filtering approaches	Aulinas *et al.* [7]
2011	SLAM back end	Grisetti *et al.* [98]
2011	Observability, consistency and convergence	Dissanayake *et al.* [65]
2012	Visual odometry	Scaramuzza and Fraundofer [86], [218]
2016	Multi robot SLAM	Saeedi *et al.* [216]
2016	Visual place recognition	Lowry *et al.* [160]
2016	SLAM in the Handbook of Robotics	Stachniss *et al.* [234, Ch. 46]
2016	Theoretical aspects	Huang and Dissanayake [110]

Surveys and Tutorials in SLAM [1]



(1986-2004) Classical age:
Main probabilistic formulations

- □ Extended Kalman Filters (EKF)
- □ Rao-Blackwellized Particle Filters (RBPF)
- □ Maximum Likelihood Estimation

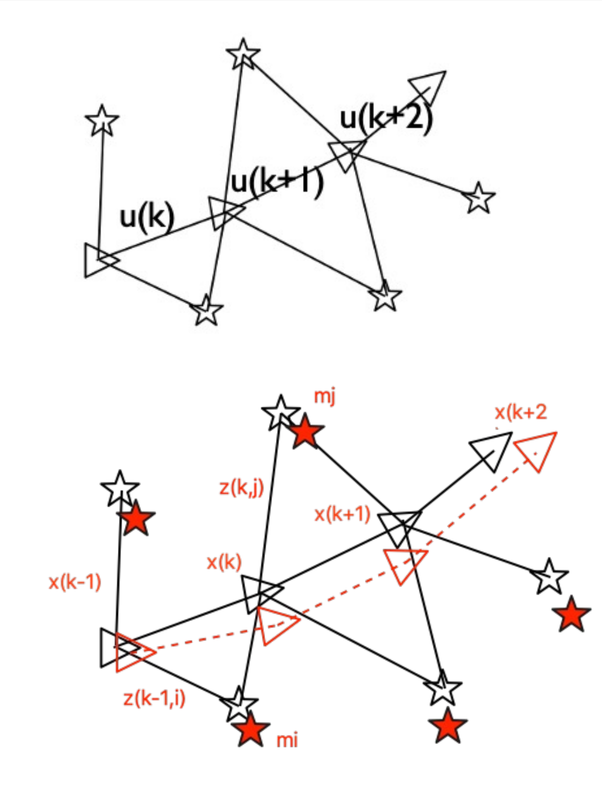
(2004-2015) Algorithmic-analysis age:
Fundamental properties studies

- □ Observability
- □ Convergence
- □ Consistency etc

(2015-Present) Robust-perception age

- □ Robust performance
- □ High-level understanding
- □ Resource awareness
- □ Task-driven perception

SLAM PROBLEM DEFINITION



Given

- Robot controls $u_{1:T} = \{u_1, u_2, u_3, \dots, u_T\}$
- Observations $z_{1:T} = \{z_1, z_2, z_3, \dots, z_T\}$

Want

- Map of the environment m (landmarks, occupancy grids, surface maps, point clouds)
- Robot path $\mathrm{x}_{1:\mathrm{T}} = \{x_0, x_1, x_2, \dots, x_T\}$

Full SLAM estimates the entire path

$$p\left(x_{0:T}, m \middle| z_{1:T,}, u_{1:T}\right)$$

Online SLAM estimated the most recent pose

$$p\left(x_t, m \middle| z_{1:t,}, u_{1:t}\right)$$

FOUNDATIONS IN SLAM

Kalman Filter based SLAM

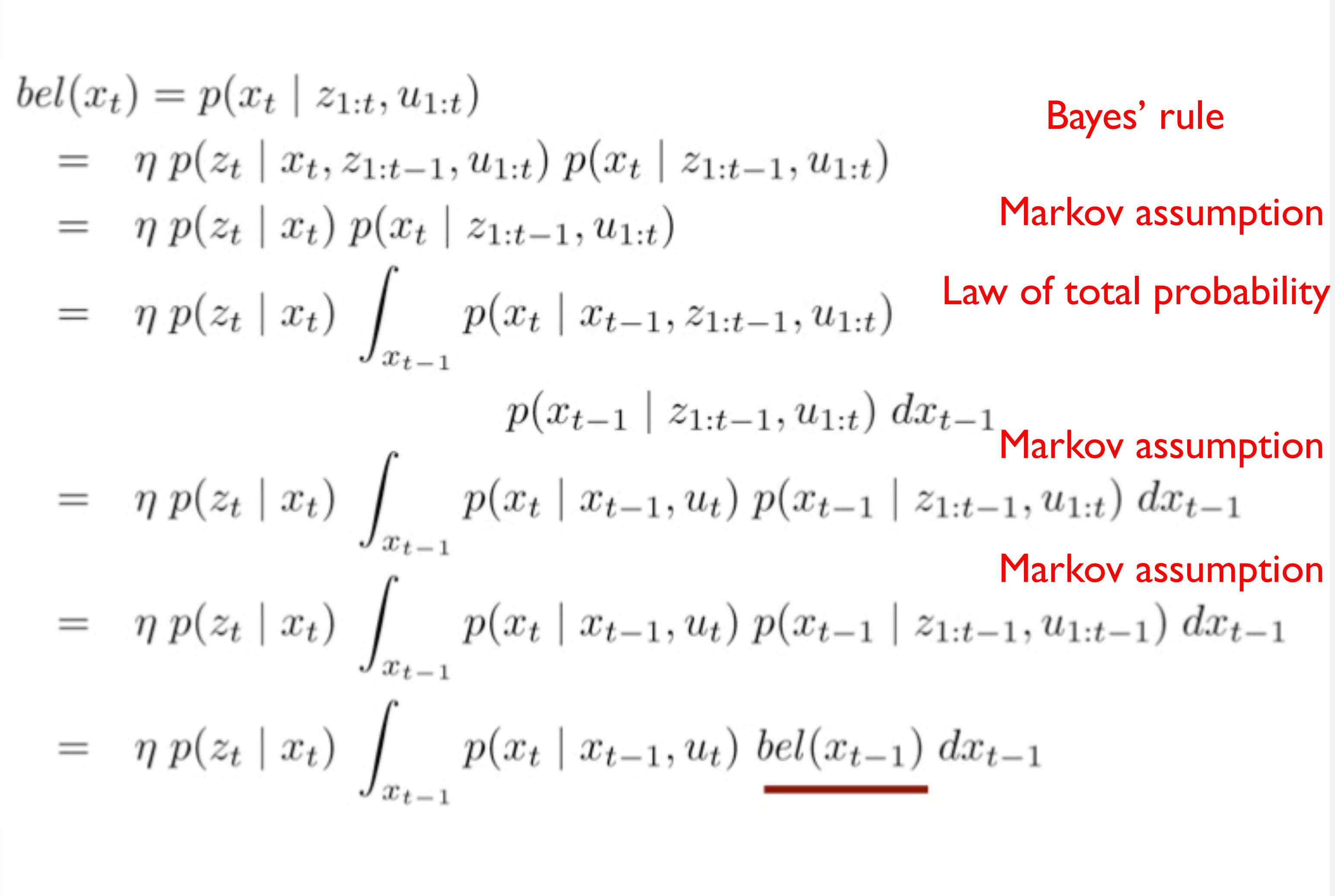
Particle Filter based SLAM

Graph based SLAM

FOUNDATIONS IN SLAM

- Kalman Filter based SLAM
- Particle Filter based SLAM
- Graph based SLAM

BAYES FILTER



$$bel(x_t) = p(x_t \mid z_{1:t}, u_{1:t})$$

Bayes' rule

$$= \eta \, p(z_t \mid x_t, z_{1:t-1}, u_{1:t}) \, p(x_t \mid z_{1:t-1}, u_{1:t})$$

Markov assumption

$$= \eta \, p(z_t \mid x_t) \, p(x_t \mid z_{1:t-1}, u_{1:t})$$

Law of total probability

$$= \eta \, p(z_t \mid x_t) \int_{x_{t-1}} p(x_t \mid x_{t-1}, z_{1:t-1}, u_{1:t}) \, p(x_{t-1} \mid z_{1:t-1}, u_{1:t}) \, dx_{t-1}$$

Markov assumption

$$= \eta \, p(z_t \mid x_t) \int_{x_{t-1}} p(x_t \mid x_{t-1}, u_t) \, p(x_{t-1} \mid z_{1:t-1}, u_{1:t}) \, dx_{t-1}$$

Markov assumption

$$= \eta \, p(z_t \mid x_t) \int_{x_{t-1}} p(x_t \mid x_{t-1}, u_t) \, p(x_{t-1} \mid z_{1:t-1}, u_{1:t-1}) \, dx_{t-1}$$

$$= \eta \, p(z_t \mid x_t) \int_{x_{t-1}} p(x_t \mid x_{t-1}, u_t) \, \underline{bel(x_{t-1})} \, dx_{t-1}$$

BAYES FILTER

$$bel(x_t) = \eta\, p(z_t|x_t) \int_{x_{t-1}} p(x_t|x_{t-1}, u_t)\, bel(x_{t-1})\, dx_{t-1}$$

- Assumption

I. Observations are conditionally independent given the map and current robot pose

II. State transition is a Markov process

- Prediction step

$$\overline{bel}(x_t) = \int \underbrace{p(x_t|x_{t-1}, u_t)}_{motion\ model} bel(x_{t-1})\, dx_{t-1}$$

- Correction step

$$bel(x_t) = \eta \underbrace{p(z_t|x_t)}_{sensor\ model} \overline{bel}(x_t)$$

EKF-SLAM

- Assume $\boldsymbol{w}_t$ and $\boldsymbol{v}_t$ are additive, zero mean uncorrelated Gaussian noise with covariance $\boldsymbol{Q}_t$ and $\boldsymbol{R}_t$ (linear Gaussian)
- Motion model: $\mathrm{p}(x_t|x_{t-1},u_t) \Leftrightarrow \boldsymbol{x}_t = \boldsymbol{f}(\boldsymbol{x}_{t-1},\boldsymbol{u}_t) + \boldsymbol{w}_t$
- Observation model: $\mathrm{p}(z_t|x_t) \Leftrightarrow \boldsymbol{z}(t) = \boldsymbol{h}(\boldsymbol{x}_t, m) + \boldsymbol{v}_t$
- Algorithm ($\widehat{\boldsymbol{x}}_{t|t}, \widehat{\boldsymbol{m}}_t$, and $\boldsymbol{P}_{t|t}$: mean and covariance of joint posterior distribution)

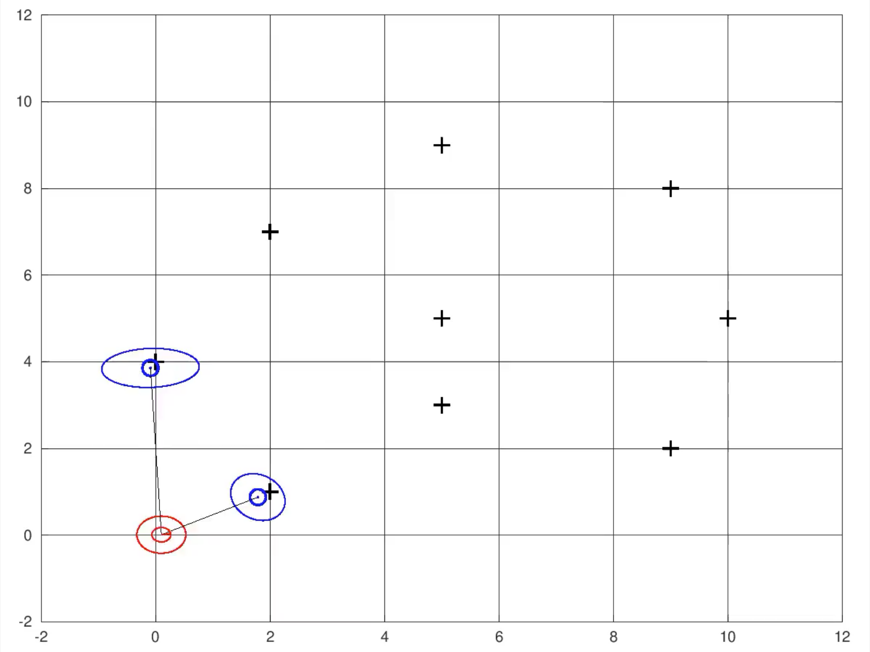
i. Prediction update

$$\widehat{\boldsymbol{x}}_{t|t-1} = \boldsymbol{f}(\hat{x}_{t-1|t-1}, \boldsymbol{u}_k)$$
$$\boldsymbol{P}_{xx,t|t-1} = \nabla\boldsymbol{f}\, \boldsymbol{P}_{xx,t-1|t-1}\, \nabla\boldsymbol{f}^T + \boldsymbol{Q}_t$$

i. Correction update

$$\boldsymbol{K}_t = \boldsymbol{P}_{t|t-1}\, \nabla\boldsymbol{h}^T \left(\nabla\boldsymbol{h}\, \boldsymbol{P}_{t|t-1} \nabla\boldsymbol{h}^T + \boldsymbol{R}_t\right)^{-1}$$
$$\begin{bmatrix} \widehat{\boldsymbol{x}}_{t|t} \\ \widehat{\boldsymbol{m}}_t \end{bmatrix} = \begin{bmatrix} \widehat{\boldsymbol{x}}_{t|t-1} \\ \widehat{\boldsymbol{m}}_{t-1} \end{bmatrix} + \boldsymbol{K}_t\, (\boldsymbol{z}(t) - \boldsymbol{h}(\widehat{\boldsymbol{x}}_{t|t-1}, \widehat{\boldsymbol{m}}_{t-1}))$$
$$\boldsymbol{P}_{t|t} = (I - \boldsymbol{K}_t \nabla\boldsymbol{h})\boldsymbol{P}_{,t|t-1}$$

EKF SLAM



Properties

- Convergence::

  the standard deviation of a landmark decreases monotonically towards a lower bound
- Computational effort:

  Computation grows quadratically with the number of landmarks
- Data association:

  Fragile to incorrect association of observations to landmarks
- Non-linearity:

  Local linearization may deviate from consistency and convergence

Improvements [3]

FOUNDATIONS IN SLAM

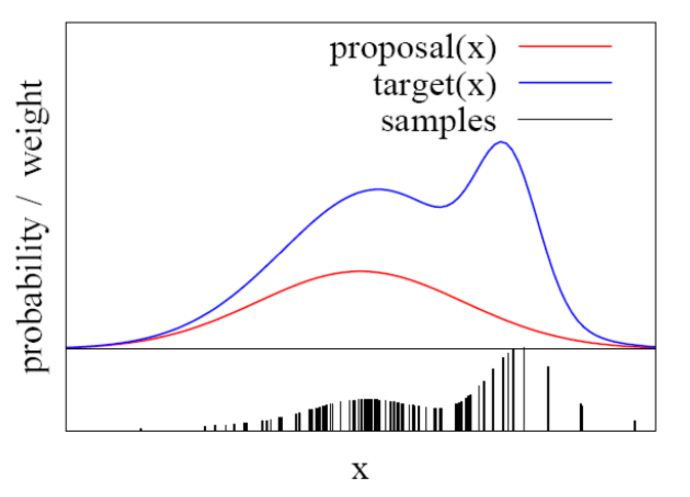
Kalman Filter based SLAM

Particle Filter based SLAM

Graph based SLAM

RBPF SLAM

- Based on Monte Carlo sampling/ particle filtering to directly represent non-linear process model and non-Gaussian pose distribution
- Particle Filter and Importance sampling principle



- Particle Filter algorithm

For N particles:

i. Sample the particles using the proposal distribution $x_t^{[j]} = \pi(x_t)$

ii. Compute the importance weights $w_t^{[j]} = \frac{target\left(x_t^{[j]}\right)}{proposal\ \left(x_t^{[j]}\right)}$

Resampling: draw samples i with probability $w_t^{[j]}$ and repeat N times

RBPF SLAM

- Rao-Blackwellisation: joint state factorization

$$p(x_1, x_2) = \underbrace{p(x_2|x_1)}_{\substack{can\ be\ represented \\ analytically}} \underbrace{p(x_1)}_{\substack{need\ be \\ sampled}\ x_1^{(i)} \sim p(x_1)}$$

$$\Leftrightarrow \{x_1^{(i)}, p\left(x_2\middle|x_1^{(i)}\right)\}_i^N$$

- Rao-Blackwellisation: SLAM

$$p(x_{1:t}, m|z_{1:t}, u_{1:t}) = \underbrace{p(x_{1:t}|z_{1:t}, u_{1:t})}_{\substack{path\ posterior \\ particle\ filter}} \underbrace{p(m|x_{1:t}, z_{1:t})}_{\substack{map\ posterior \\ ananytic}}$$

- Importance weights

i. Target distribution: $p(x_{1:t}|z_{1:t}, u_{1:t})$

ii. Proposal distribution: $\pi(x_{1:t}|z_{1:t-1}, u_{1:t}) = \pi(x_t|x_{t-1}, u_t)\pi(x_{1:t-1}|z_{1:t-1}, u_{1:t-1})$ gives FastSLAM 1.0

$\pi(x_{1:t}|z_{1:t}, u_{1:t}) = \pi(x_t|x_{t-1}, z_{1:t}, u_t)\pi(x_{1:t-1}|z_{1:t-1}, u_{1:t-1})$ gives FastSLAM 2.0

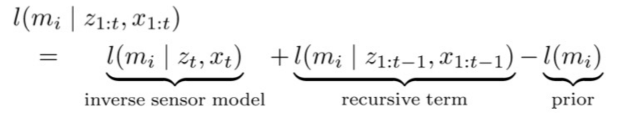
RBPF SLAM

i. Sampling based on proposal distribution π
ii. Importance weighting
iii. Resampling such that all particles have same weight
iv. Map estimation based on robot trajectory and observations $\underbrace{(p(m|x_{1:t}, z_{1:t}))}$

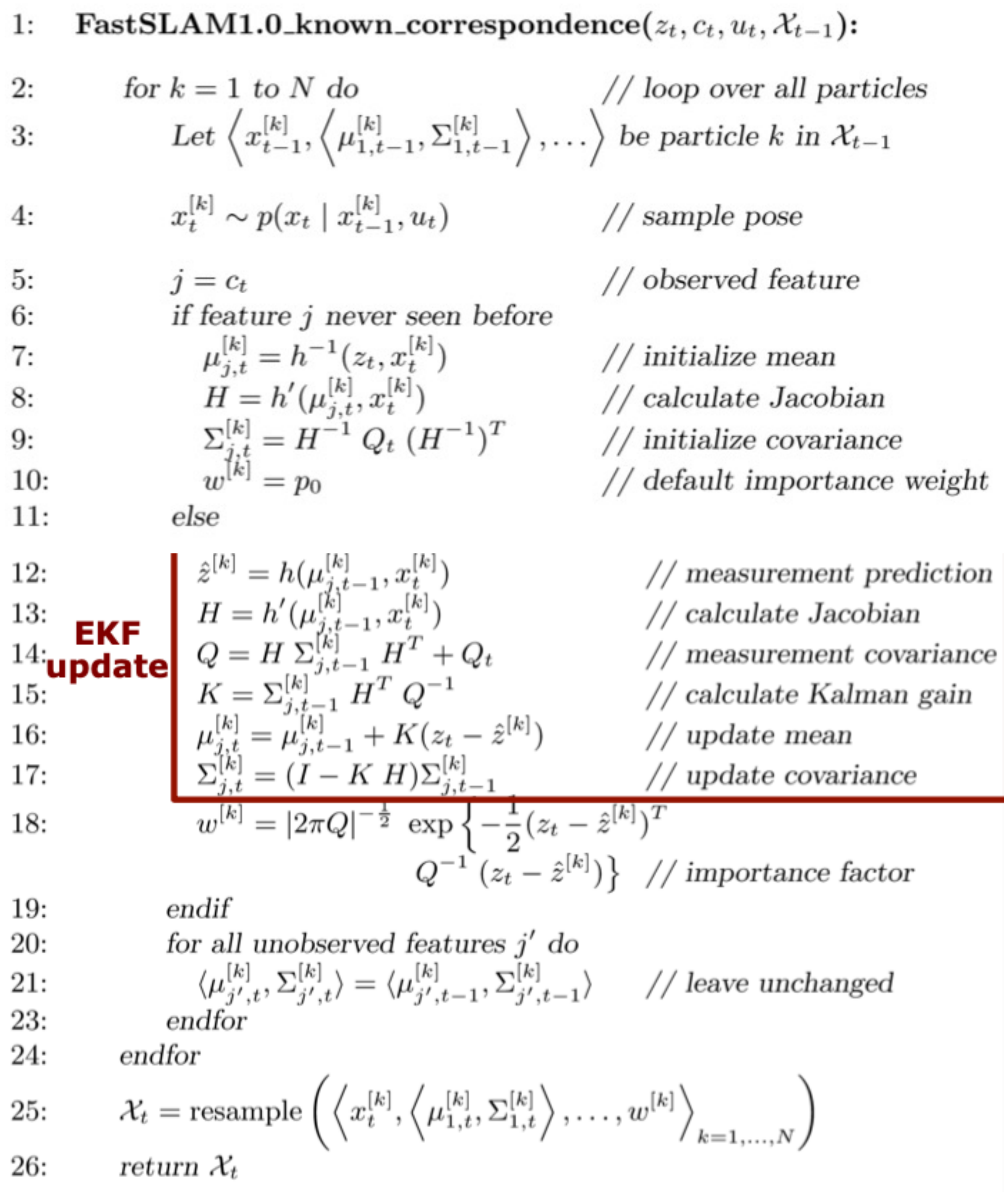
- Feature based

$$\underbrace{\prod_{i=1}^{M} p(m_i|x_{0:t}, z_{1:t})}_{\text{EKFs (conditioned on the whole trajcotyr,landmakrs are independent Gaussians)}}$$

- Grid based (GMapping)



$$l(m_i \mid z_{1:t}, x_{1:t}) = \underbrace{l(m_i \mid z_t, x_t)}_{\text{inverse sensor model}} + \underbrace{l(m_i \mid z_{1:t-1}, x_{1:t-1})}_{\text{recursive term}} - \underbrace{l(m_i)}_{\text{prior}}$$



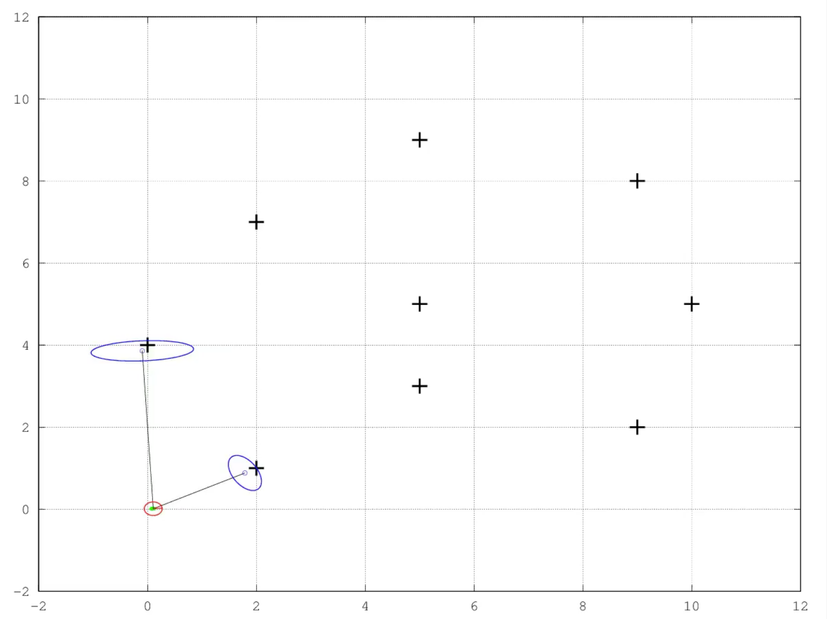
EKF update

```
FastSLAM1.0_known_correspondence($z_t, c_t, u_t, \mathcal{X}_{t-1}$):

    for $k = 1$ to $N$ do                                   // loop over all particles
        Let $\left\langle x_{t-1}^{[k]}, \left\langle \mu_{1,t-1}^{[k]}, \Sigma_{1,t-1}^{[k]} \right\rangle, \ldots \right\rangle$ be particle $k$ in $\mathcal{X}_{t-1}$

        $x_t^{[k]} \sim p(x_t \mid x_{t-1}^{[k]}, u_t)$                     // sample pose

        $j = c_t$                                           // observed feature
        if feature $j$ never seen before
            $\mu_{j,t}^{[k]} = h^{-1}(z_t, x_t^{[k]})$                    // initialize mean
            $H = h'(\mu_{j,t}^{[k]}, x_t^{[k]})$                      // calculate Jacobian
            $\Sigma_{j,t}^{[k]} = H^{-1}\, Q_t\, (H^{-1})^T$               // initialize covariance
            $w^{[k]} = p_0$                                   // default importance weight
        else
            $\hat{z}^{[k]} = h(\mu_{j,t-1}^{[k]}, x_t^{[k]})$                   // measurement prediction
            $H = h'(\mu_{j,t-1}^{[k]}, x_t^{[k]})$                      // calculate Jacobian
            $Q = H\, \Sigma_{j,t-1}^{[k]}\, H^T + Q_t$                    // measurement covariance
            $K = \Sigma_{j,t-1}^{[k]}\, H^T\, Q^{-1}$                      // calculate Kalman gain
            $\mu_{j,t}^{[k]} = \mu_{j,t-1}^{[k]} + K(z_t - \hat{z}^{[k]})$            // update mean
            $\Sigma_{j,t}^{[k]} = (I - K\, H)\Sigma_{j,t-1}^{[k]}$                 // update covariance
            $w^{[k]} = |2\pi Q|^{-\frac{1}{2}}\, \exp\left\{-\frac{1}{2}(z_t - \hat{z}^{[k]})^T\right.$
                          $\left. Q^{-1}\, (z_t - \hat{z}^{[k]})\right\}$    // importance factor
        endif
        for all unobserved features $j'$ do
            $\langle \mu_{j',t}^{[k]}, \Sigma_{j',t}^{[k]} \rangle = \langle \mu_{j',t-1}^{[k]}, \Sigma_{j',t-1}^{[k]} \rangle$      // leave unchanged
        endfor
    endfor
    $\mathcal{X}_t$ = resample $\left( \left\langle x_t^{[k]}, \left\langle \mu_{1,t}^{[k]}, \Sigma_{1,t}^{[k]} \right\rangle, \ldots, w^{[k]} \right\rangle_{k=1,\ldots,N} \right)$
    return $\mathcal{X}_t$
```

RBPF SLAM

- FastSLAM 1.0: motion model as proposal distribution (suboptimal, esp. when sensor information is much more precise than motion estimate from odometry) , importance weights are based on observation model.

  FastSLAM 2.0 integrate most recent sensor observation into the proposal (optimal, gives a more peaked proposal distribution), less particles are required, more robust and accurate
- Improved techniques for Grid Mapping with RBPFs (improved proposal distribution, scan matching, and adaptive sampling) – details refer to [4]



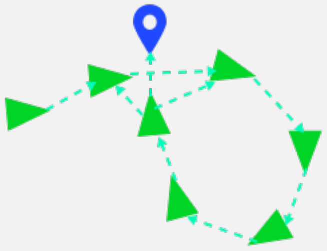
FOUNDATIONS IN SLAM

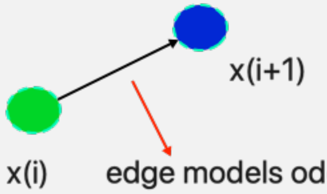
Kalman Filter based SLAM

Particle Filter based SLAM

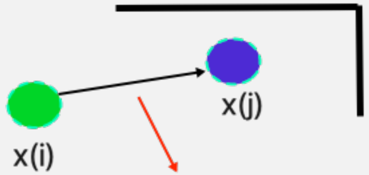
Graph based SLAM

GRAPH





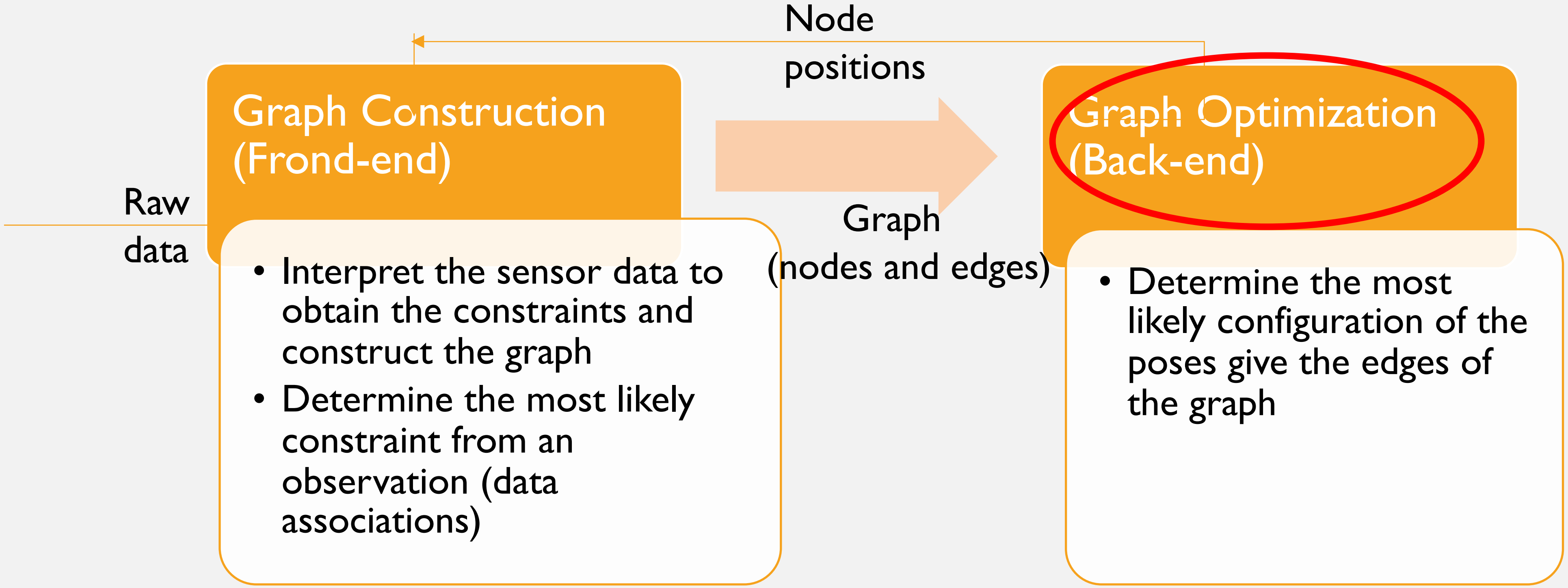
edge models odemetry measurements



edge aries from multiple observations

- Node: a robot position and a measurement acquired at that position
- Edge: spatial constraint relating the corresponding two robot poses, a constraint consists in a probability distribution over the relative transformation between two poses ($< z_{ij}, \Omega_{ij} >$, where Ω_{ij} is the information matrix, larger value make the edge matter more)
- Graph SLAM: Build the graph and find a node configuration which minimizes the error introduced by constraints

GRAPH SLAM



Node positions

Raw data

Graph Construction (Frond-end)

- Interpret the sensor data to obtain the constraints and construct the graph
- Determine the most likely constraint from an observation (data associations)

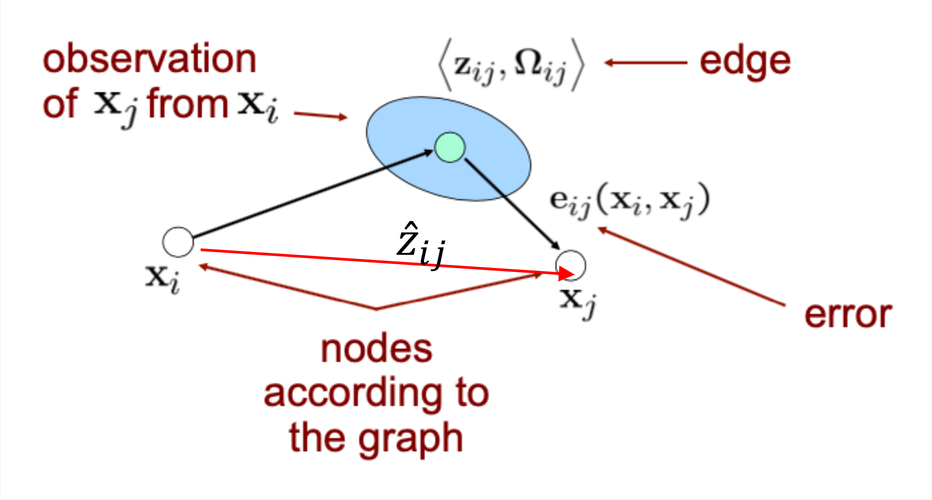
Graph (nodes and edges)

Graph Optimization (Back-end)

- Determine the most likely configuration of the poses give the edges of the graph

For correct data association, the frond-end requires consistent estimate of $p(x_{1:T}|z_{1:T}, u_{1:T})$, which is fed in from back-end

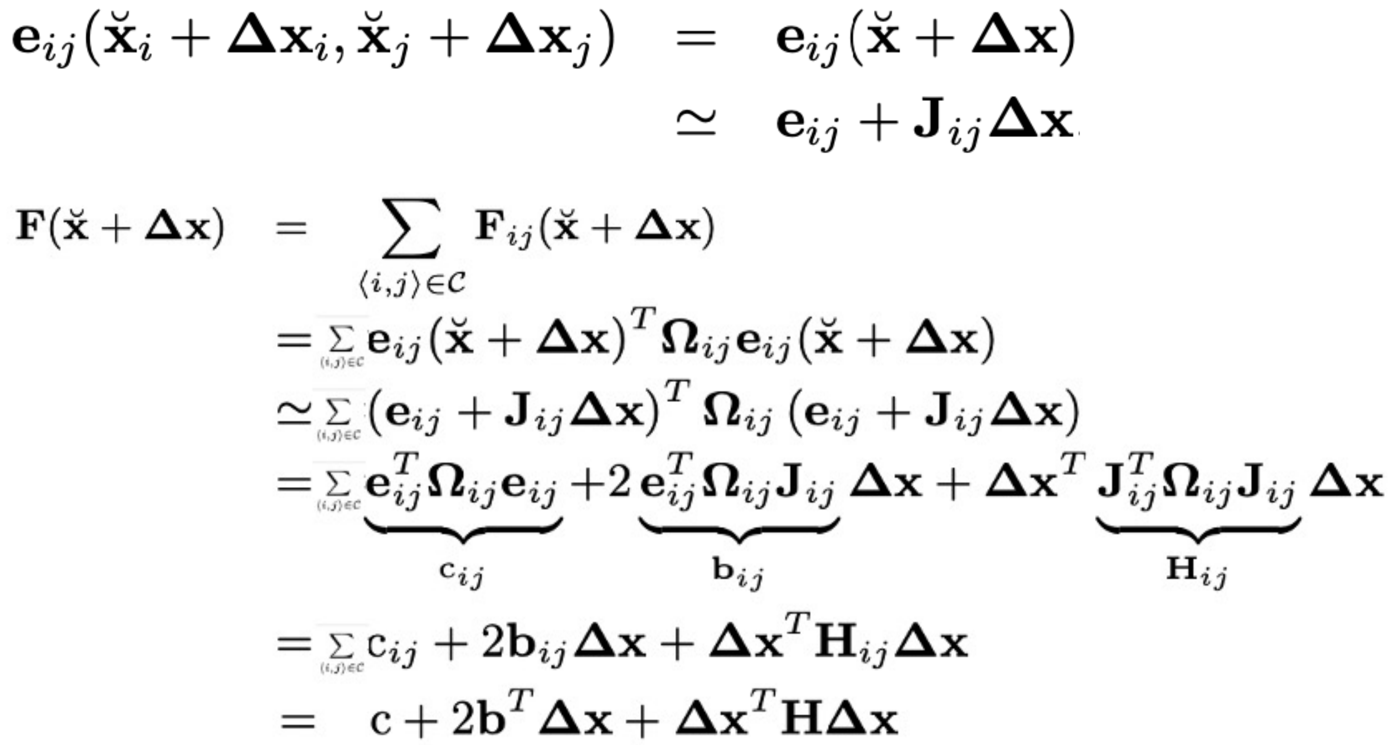
PROBLEM DEFINITION


observation of $\mathbf{x}_j$ from $\mathbf{x}_i$
$\langle \mathbf{z}_{ij}, \mathbf{\Omega}_{ij} \rangle$
edge
$\mathbf{e}_{ij}(\mathbf{x}_i, \mathbf{x}_j)$
$\hat{z}_{ij}$
$\mathbf{x}_i$
$\mathbf{x}_j$
error
nodes according to the graph

- z_{ij} : mean of virtual measurement
- $\hat{z}_{ij}$: prediction of virtual measurement
- $e_{ij}\left(x_i, x_j\right) = z_{ij} - \hat{z}_{ij}$
- Given gaussian noise, the log-likelihood of a measurements is proportional to $e_{ij}^T \, \Omega_{ij} \, e_{ij}$
- Maximum likelihood goal is to find x^* to minimize the negative log likelihood of all the observations

$$x^* = \underset{x}{argmin}\, F(x) = \underset{x}{argmin} \sum_{(i,j)} e_{ij}^T \, \Omega_{ij} \, e_{ij}$$

GAUSS-NEWTON ERROR MINIMIZATION



$$\begin{aligned}\mathbf{e}_{ij}(\breve{\mathbf{x}}_i + \mathbf{\Delta x}_i, \breve{\mathbf{x}}_j + \mathbf{\Delta x}_j) &= \mathbf{e}_{ij}(\breve{\mathbf{x}} + \mathbf{\Delta x}) \\ &\simeq \mathbf{e}_{ij} + \mathbf{J}_{ij}\mathbf{\Delta x}.\end{aligned}$$

$$\begin{aligned}\mathbf{F}(\breve{\mathbf{x}} + \mathbf{\Delta x}) &= \sum_{\langle i,j\rangle \in \mathcal{C}} \mathbf{F}_{ij}(\breve{\mathbf{x}} + \mathbf{\Delta x}) \\ &= \sum_{\langle i,j\rangle \in \mathcal{C}} \mathbf{e}_{ij}(\breve{\mathbf{x}} + \mathbf{\Delta x})^T \mathbf{\Omega}_{ij} \mathbf{e}_{ij}(\breve{\mathbf{x}} + \mathbf{\Delta x}) \\ &\simeq \sum_{\langle i,j\rangle \in \mathcal{C}} (\mathbf{e}_{ij} + \mathbf{J}_{ij}\mathbf{\Delta x})^T\, \mathbf{\Omega}_{ij}\, (\mathbf{e}_{ij} + \mathbf{J}_{ij}\mathbf{\Delta x}) \\ &= \sum_{\langle i,j\rangle \in \mathcal{C}} \underbrace{\mathbf{e}_{ij}^T \mathbf{\Omega}_{ij} \mathbf{e}_{ij}}_{\mathbf{c}_{ij}} + 2\, \underbrace{\mathbf{e}_{ij}^T \mathbf{\Omega}_{ij} \mathbf{J}_{ij}}_{\mathbf{b}_{ij}}\, \mathbf{\Delta x} + \mathbf{\Delta x}^T\, \underbrace{\mathbf{J}_{ij}^T \mathbf{\Omega}_{ij} \mathbf{J}_{ij}}_{\mathbf{H}_{ij}}\, \mathbf{\Delta x} \\ &= \sum_{\langle i,j\rangle \in \mathcal{C}} \mathrm{c}_{ij} + 2\mathbf{b}_{ij}\mathbf{\Delta x} + \mathbf{\Delta x}^T \mathbf{H}_{ij} \mathbf{\Delta x} \\ &= \mathrm{c} + 2\mathbf{b}^T \mathbf{\Delta x} + \mathbf{\Delta x}^T \mathbf{H} \mathbf{\Delta x}\end{aligned}$$

Locally minimizing solution: $\mathrm{H}\ \Delta x^* = -b, \qquad \mathrm{x}^* = \breve{x} + \Delta x^*$

GAUSS-NEWTON ERROR MINIMIZATION

- Due to the sparsity of the Jacobian

$$J_{ij} = \begin{pmatrix} 0 & \dots 0 & \underbrace{A_{ij}}_{\substack{derivative \\ w.r.t \\ node\ i}} & 0 & \dots 0 & \underbrace{B_{ij}}_{\substack{derivative \\ w.r.t \\ node\ j}} & 0 & \dots 0 \end{pmatrix}$$

H_{ij} and b_{ij} are also sparse

- At each iteration, constraint-based update of H_{ij} and b_{ij} is considered due to the sparsity, referring [5] for details in 2D laser-based mapping
- Most of the visual SLAM algorithms are in this category, referring [1] to see the extensions of Graph-based SLAM

REFERENCE

[1]A complete survey of SLAM

[2][3]Tutorial of filtering-based approach

[4]GMapping

[5]Tutorial of graph-based approach

[6]Course in robot mapping, search professor for newer release

[7]Book: old but provides all the fundamental details, errata

[8][9]Project reference, both have extensions in newer release

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