PART B

6a) $\dfrac{s(4)-s(2)}{4-2}$

$s(4) = -16(4)^2 + 48(4) + 160$
$= 96$
$s(2) = -16(2)^2 + 48(2) + 160$
$= 192$

$= \dfrac{96-192}{4-2}$

$= \boxed{-48 \text{ m/s}}$ ← Average rate of change.

b) $\dfrac{s(4.001)-s(4)}{4.001-4}$

$s(4.001) = 95.92$

$= \dfrac{95.92-96}{0.001}$

$= \boxed{-80 \text{ m/s}}$ ← Instanteous tate of change.

? 7.

# Bounce (b)	Height (H)
0	2
1	$2(0.25)$
2	$2(0.25)^2$
3	$2(0.25)^3$
4	$2(0.25)^4$
5	$2(0.25)^5$

$H = 2(0.25)^b$

$0.25 = 2(0.25)^b$
$0.125 = 0.25^b$
$\log 0.125 = b \log 0.25$
$\boxed{1.5 = b}$ ← check

8. $7.9 = \log\left(\dfrac{250}{T}\right) + 5.7$

$2.2 = \log\left(\dfrac{250}{T}\right)$

$2.2 = \log 250 - \log T$
$\log T = \log 250 - 2.2$
$\log T = 0.1979$

$10^{0.1979} = T$
$\boxed{T = 1.58 \text{ seconds}}$

PART B

9. $\log_9(x+3) + \log_9(x-5) = 1$

$\log_9[(x+3)(x-5)] = 1$

$\log_9(x^2-2x-15) = 1$

$9^1 = x^2-2x-15$

$0 = x^2-2x-24$

$0 = (x-6)(x+4)$

$x-6=0$ $\boxed{x=6}$

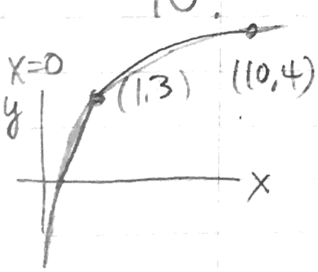
$x+4=0$ $\boxed{x=-4}$ × reject! extraneous root

10. $f(x) = \log(1000x)$

$= \log 1000 + \log x$

$= \log_{10} 10^3 + \log x$

$f(x) = 3 + \log x$



VA ⇒ $x = 0$

HA ⇒ none

Domain $\{x \in R \mid x > 0\}$

Range $\{y \in R\}$

x-int ⇒ $0 = 3 + \log x$

$\log x = -3$

$10^{-3} = x$

$\boxed{0.001 = x}$

y-int ⇒ none

11. $\frac{H(99) - H(9)}{99-9}$ ⇒ $\frac{40-20}{90}$

$= \frac{2}{9}$ or $\boxed{0.22 \text{ m/s}}$

$H(99) = 20\log 100$

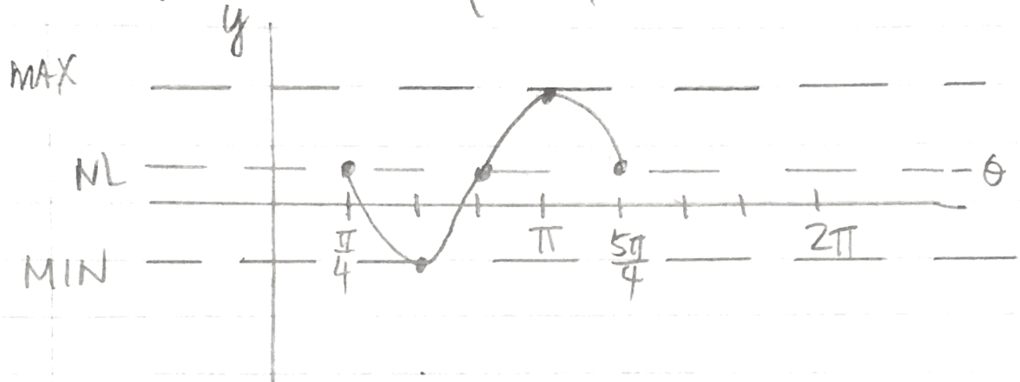
$= 40$ m

$H(9) = 20 \log 10$

$= 20$m

PART B

12. $y = -3\sin 2\left(\theta - \frac{\pi}{4}\right) + 1$ $P = \frac{2\pi}{2}$ or π



OR **Mapping Rule**

$(x, y) \rightarrow \left(\frac{x}{2} + \frac{\pi}{4}, -3y + 1\right) \rightarrow$ New Co-ord.

$(0, 0) \rightarrow \left(\frac{0}{2} + \frac{\pi}{4}, -3(0) + 1\right) \rightarrow \left(\frac{\pi}{4}, 1\right)$

$\left(\frac{\pi}{2}, 1\right) \rightarrow \left(\frac{\pi}{2} \div 2 + \frac{\pi}{4}, -3(1) + 1\right) \rightarrow \left(\frac{\pi}{2}, -2\right)$

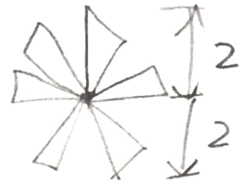
$(\pi, 0) \rightarrow \left(\frac{\pi}{2} + \frac{\pi}{4}, -3(0) + 1\right) \rightarrow \left(\frac{3\pi}{4}, 1\right)$

$\left(\frac{3\pi}{2}, -1\right) \rightarrow \left(\frac{3\pi}{2} \div 2 + \frac{\pi}{4}, -3(-1) + 1\right) \rightarrow (\pi, 4)$

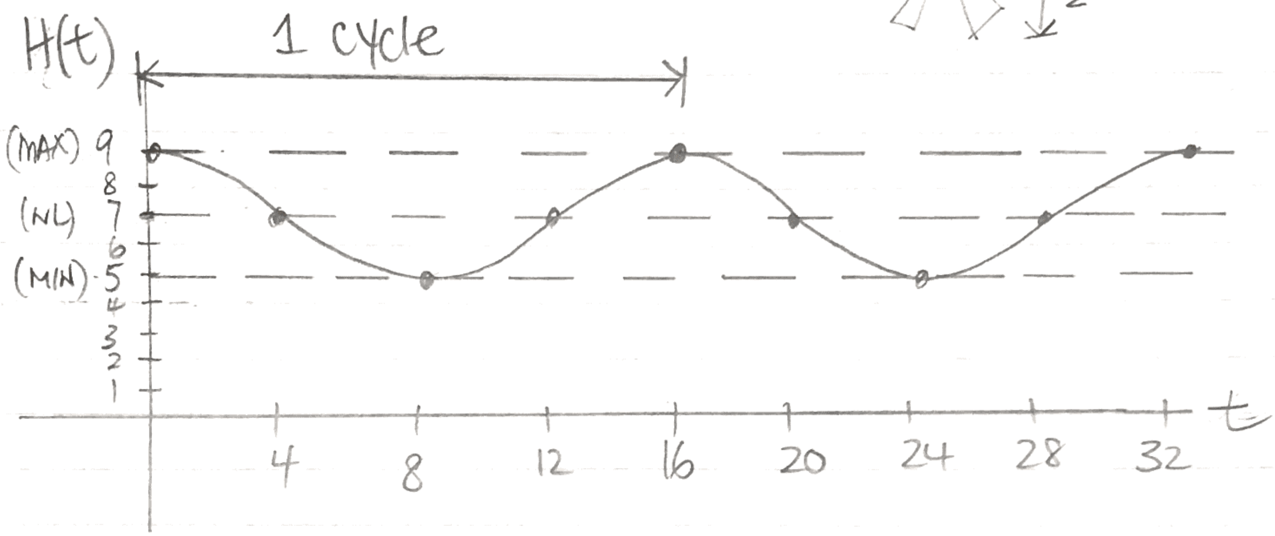
$(2\pi, 0) \rightarrow \left(\frac{2\pi}{2} + \frac{\pi}{4}, -3(0) + 1\right) \rightarrow \left(\frac{5\pi}{4}, 1\right)$

13

PART B



13 a)



b) $P = 16$ $\quad a = 2$

$k = \frac{2\pi}{P}$ $\quad d = 7$ $\quad H(t) = 2\cos\left(\frac{\pi}{8}t\right) + 7$

$\boxed{k = \frac{\pi}{8}}$

14a) $\frac{1}{1-\cos\theta} - \frac{1}{1+\cos\theta} = 2\csc\theta\cot\theta$

LS

$\frac{(1+\cos\theta) - (1-\cos\theta)}{(1-\cos\theta)(1+\cos\theta)}$

$= \frac{2\cos\theta}{1-\cos^2\theta}$

$= \frac{2\cos\theta}{\sin^2\theta}$ ✓

$RS = 2\left(\frac{1}{\sin\theta}\right)\left(\frac{\cos\theta}{\sin\theta}\right)$

$= \frac{2\cos\theta}{\sin^2\theta}$ ✓

LS = RS

PART B

14b) $\dfrac{1-\cos 2x}{\sin 2x}$ | $\tan x$

$\underline{\underline{LS}} = \dfrac{1-(\cos^2 x - \sin^2 x)}{2\sin x \cos x}$

$= \dfrac{1-\cos^2 x + \sin^2 x}{2\sin x \cos x}$

$\left.\begin{matrix}\sin^2 x + \cos^2 x = 1 \\ \sin^2 x = 1-\cos^2 x\end{matrix}\right\} = \dfrac{\sin^2 x + \sin^2 x}{2\sin x \cos x}$

$= \dfrac{2\sin^2 x}{2\sin x \cos x}$

$= \dfrac{\sin x}{\cos x}$

$= \tan x$ $\therefore LS = RS$

15a) $2\sin\left(x - \frac{\pi}{6}\right) = 1$ $\sin\theta = \frac{1}{2}$, $\sin 30° = \frac{1}{2}$

$\sin\left(x-\frac{\pi}{6}\right) = \frac{1}{2}$ ... but $\sin\frac{\pi}{6} = \frac{1}{2}$ (special angles)

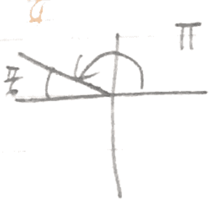
$x - \frac{\pi}{6} = \frac{\pi}{6}$

$x = \frac{\pi}{6} + \frac{\pi}{6}$

$x = \frac{2\pi}{6}$ or $\frac{\pi}{3}$ ⇐ Quadrant 1

But $\sin\theta = \frac{1}{2}$ is also positive in Quadrant 2 ! → check Quadrant 2

PART B



15a) Quadrant 2 : $\sin \frac{5\pi}{6} = \frac{1}{2}$

and we know $\sin\left(x - \frac{\pi}{6}\right) = \frac{1}{2}$

$\therefore \quad x - \frac{\pi}{6} = \frac{5\pi}{6}$

$$\boxed{x = \pi}$$

15b) $\cos^2 x - \cos x - \sin^2 x = 0$

we know ...

$\therefore \sin^2 x + \cos^2 x = 1$

$\therefore \sin^2 x = 1 - \cos^2 x$

$\cos^2 x - \cos x - (1 - \cos^2 x) = 0$

$\cos^2 x - \cos x - 1 + \cos^2 x = 0$

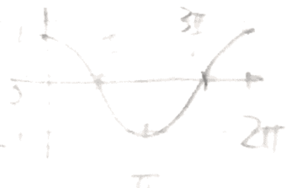
$2y^2 - y - 1 \qquad 2\cos^2 x - \cos x - 1 = 0$

$(2y+1)(y-1) \qquad (2\cos x + 1)(\cos x - 1) = 0$

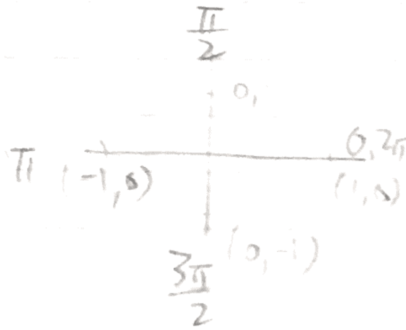
$2\cos x + 1 = 0 \qquad \cos x = 1$

$2\cos x = -1 \qquad \boxed{x_3 = 0 \quad x_4 = 2\pi}$

$\cos x = -\frac{1}{2}$ 4 Quadrant 2 & 3



$x = \frac{\pi}{3}$

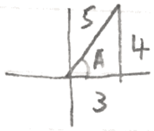


Quad 2 $\quad x_1 = \pi - \frac{\pi}{3}$ $\qquad$ Quad 3 $\quad x_2 = \pi + \frac{\pi}{3}$

$$\boxed{x_1 = \frac{2\pi}{3}} \qquad \boxed{x_2 = \frac{4\pi}{3}}$$

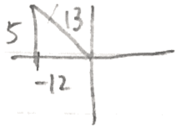
PART B

5a) $\sin A = \frac{4}{5}$



$\cos A = \frac{3}{5}$ (Quad 1)

$\cos B = \frac{-12}{13}$



$\sin B = \frac{5}{13}$ (Quad 2)

$$\begin{aligned}
\sin(A+B) &= \sin A \cos B + \cos A \sin B \\
&= \left(\frac{4}{5}\right)\left(\frac{-12}{13}\right) + \left(\frac{3}{5}\right)\left(\frac{5}{13}\right) \\
&= \frac{-48}{65} + \frac{15}{65} \\
&= \frac{-33}{65}
\end{aligned}$$

b) $\tan(B-A)$ → ignore.