PART A

1. $f(-1) = 2(-1)^3 + 3(-1)^2 + 1$
$= -2 + 3 + 1$
$= \boxed{2}$ ⇐ Remainder

2. $y = k(x-1)(x-2)(x+3)$ ⇐ substitute $(0,24)$
$24 = k(0-1)(0-2)(0+3)$
$24 = 6k$
$\boxed{k=4}$

3. $f(1) = 0$
Check $f(-1) \stackrel{?}{=} 0$

$(-1)^3 - (-1)^2 - (-1) + 1$
$= -1 - 1 + 1 + 1$
$= 0$

∴ The other factor is $\boxed{x+1}$ because $a = \pm 1$, $b = \pm 1$ so the only 2 possible factors are $(x-1)$ and $(x+1)$

4.

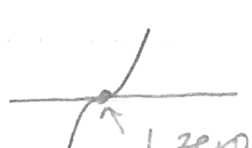
x	$f(x)$			
0	0			
1	2	2		
2	10	8	6	
3	30	20	12	6
4	68	38	18	6

∴ It is a cubic function because the 3rd difference is the same
$\boxed{\text{Degree} = 3}$

5. a) Even degree function Degree = 4

b) $y = a(x+2)^2(x-0)(x-3)$

6. 1



PART A

7. $y = \frac{1}{x+2}$ as $x \to +\infty$ $y \to 0$
$x \to -\infty$ $y \to 0$

8. $y = \frac{2x^2-1}{x^2+3}$ H.A. $y = 2$

9a) $y = \frac{1}{(x+3)(x-2)}$; $x = -3$ (VA), $x = 2$ (VA)

b) y-int. $= -\frac{1}{6}$

★ 10. $\{x \in R \mid 0 < x \leq 1\}$ and $\{x \in R \mid x \leq -1\}$

? 11. $\log 3^2 - \log 7 - \log 11 \Rightarrow \log\left[\frac{9}{7(11)}\right] \Rightarrow \log\left(\frac{9}{77}\right)$

12. $2^3 = x+1$
$8 = x+1$
$\boxed{x = 7}$

13. $a = \log_c d^b$
$c^a = d^b$

14. $x = \frac{\log 10}{\log 3}$

$x = 2.096$

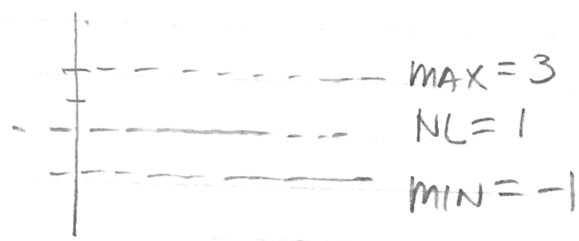
PART A

15. $P = \frac{2\pi}{k}$

$P = \frac{2\pi}{3}$

phase shift : $\frac{\pi}{4}$ (rad) to the left

16.



$\{y \in \mathbb{R} \mid -1 \leq y \leq 3\}$

17. $y = \sin x$

18. $1600 \times \frac{\pi}{180}$

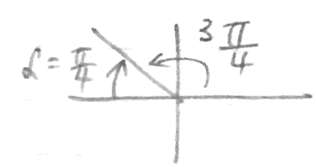
$= \frac{80\pi}{9}$ rad.

(17) The other # 17

$\cos \frac{3\pi}{4} \Rightarrow$ Quadrant 2

$= -\cos\left(\frac{\pi}{4}\right)$

$= -\frac{\sqrt{2}}{2}$



19. $\sin 2\theta = 2\sin\theta\cos\theta$

$= 2\left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{\sqrt{2}}\right)$

$= 1$

20. $k = \frac{2\pi}{p}$

$= \frac{2\pi}{2}$

$k = \pi$

PART A

21. $f(t) = 5\cos 200t$; $k = 200$. $P = \frac{2\pi}{200}$

$50\left(\frac{\pi}{100}\right) = \frac{\pi}{2}$ seconds $P = \frac{\pi}{100}$ ← length of 1 cycle

22. $\sqrt{2}\cos x - 1 = 0$ (Quadrant 1)

$\cos x = \frac{1}{\sqrt{2}}$

$x = \frac{\pi}{4}$ rad

23. C

24 $f(g(1))$ $g(1) = 2$

$f(2) = 4$ $\therefore f(g(1)) = 4$

Page 2 21. $(g-f)(x)$

$= g(x) - f(x)$

$= x^3 + 5x^2 - 2 - (x^3 - x^2 + 5x)$

$= x^3 + 5x^2 - 2 - x^3 + x^2 - 5x$

$= 6x^2 - 5x - 2$

22. $f(g(x)) = 3(2x+1)^2$

$= 3(4x^2 + 4x + 1)$

$= 12x^2 + 12x + 3$

23. $c(d(10)) = ?$ $d(10) = 80(10)$ $c(800) = 0.09(800)$

$= 800$ $= 72$

PART B

1. $2x^3 - dx^2 + (1-d^2)x + 5$

$f(d) = 0$

$$2d^3 - d(d^2) + (1-d^2)(d) + 5 = 0$$
$$2d^3 - d^3 + d - d^3 + 5 = 0$$
$$\boxed{d = -5}$$

2. $f(x) = x^3 - 5x^2 + 2x + 8$

a) $a = \pm 1$
$b = \pm 1, \pm 2, \pm 4, \pm 8$

$$f(-1) = (-1)^3 - 5(-1)^2 + 2(-1) + 8$$
$$= -1 - 5 - 2 + 8$$
$$f(-1) = 0$$

$\therefore$ $(x+1)$ is a factor

$$\begin{array}{r|l} & x^2 - 6x + 8 \\ \hline x+1 & x^3 - 5x^2 + 2x + 8 \\ & x^3 + x^2 \\ \hline & -6x^2 + 2x \\ & -6x^2 - 6x \\ \hline & 8x + 8 \\ & 8x + 8 \\ \hline \end{array}$$

$\rightarrow x^2 - 6x + 8$

$(x-2)(x-4)$ are factors

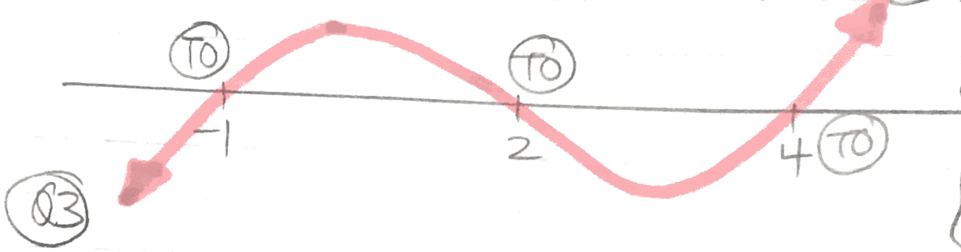
$\therefore$ The zeros are:

$$x + 1 = 0 \Rightarrow x = -1$$
$$x - 2 = 0 \Rightarrow x = 2$$
$$x - 4 = 0 \Rightarrow x = 4$$

☆

PART B

2b) $$f(x) = (x+1)^{\overset{\boxed{TO}}{1}}(x-2)^{\overset{\boxed{TO}}{1}}(x-4)^{\overset{\boxed{TO}}{1}}$$ Q1



☆ ODD DEGREE FUNCTION (QUADRANT 3 TO 1)
☆ Positive leading coefficient $+1x^3$

c) As $x \to -\infty$ $y \to -\infty$
$x \to +\infty$ $y \to +\infty$

3. $$\frac{x^3-8}{x^2+x-6} \geq 0$$

Recall
$$x^3-y^3 = (x-y)(x^2+xy+y^2)$$

$$\frac{\cancel{(x-2)}(x^2+2x+4)}{(x+3)\cancel{(x-2)}} \geq 0$$

$x \neq -3$
$x \neq 2$

$$\frac{x^2+2x+4}{x+3} \geq 0$$

$$x^2+2x+4=0$$
$$x = \frac{-2 \pm \sqrt{2^2-4(1)(4)}}{2}$$
$$= \frac{-2 \pm \sqrt{4-16}}{2}$$
No real roots

Oblique asymptote (Ignore)

3) Solve: $\dfrac{x^2+2x-3}{x^2-9x+20} \geq 0$

$$\frac{(x+3)(x-1)}{(x-4)(x-5)} \geq 0$$

VA = $x=4$, $x=5$

Zeros ⇒ $x=-3$, $x=1$

	1 $x=-4$	2 $x=0$	3 $x=2$	4 $x=4.5$	5 $x=6$
$(x+3)(x-1)$	$(-)(-)=+$	$(+)(-)=-$	$(+)(+)=+$	$(+)(+)=+$	$(+)(+)$ $+$
$(x-4)(x-5)$	$(-)(-)=+$	$(-)(-)=+$	$(-)(-)=+$	$(+)(-)=-$	$(+)(+)=+$
Sign	$+$	$-$	$+$	$-$	$+$
Satisfies ≥ 0	✓	×	✓	×	✓

$\{x \in \mathbb{R} \mid x \leq -3\}$ $\{x \in \mathbb{R} \mid +1 \leq x < 4\}$ and

$\{x \in \mathbb{R} \mid x > 5\}$

4. $f(x) = \dfrac{x^2-1}{2x^2+x-3}$

$\boxed{f(x) = \dfrac{x+1}{2x+3}}$ $= \dfrac{\cancel{(x-1)}(x+1)}{(2x+3)\cancel{(x-1)}}$ $x \neq 1$ → Hole! (x-co-ord) $x \neq -\frac{3}{2}$ → VA

$2x^2+x-3$
$= 2x^2+3x-2x-3$
$= x(2x+3)-1(2x+3)$
$= (2x+3)(x-1)$

a) x-intercept(s) (set $y=0$) $x+1=0$ $\boxed{x=-1}$

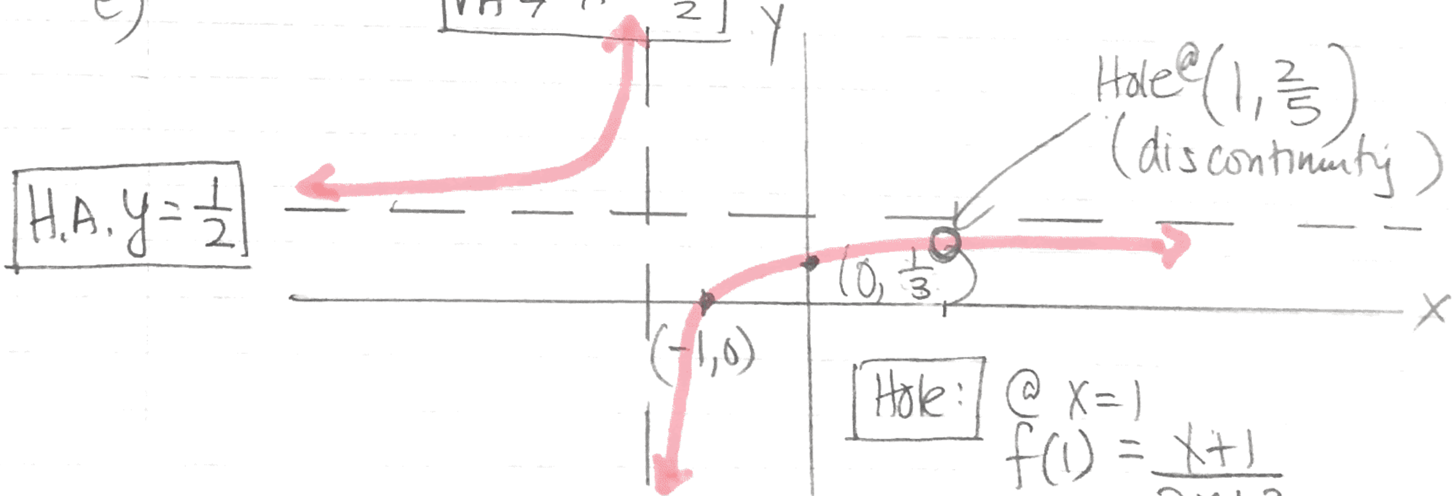
b) y-intercept (set $x=0$) $f(0) = \frac{1}{3}$ $\boxed{y=\frac{1}{3}}$

c) $\cancel{x=1}$, $x = -\frac{3}{2}$

d) As $x \to +\infty$, $y \to \frac{1}{2}$
$x \to -\infty$, $y \to \frac{1}{2}$

Because ....
H.A. $= \dfrac{1x^2}{2x^2}$
$\boxed{\text{H.A } y = \frac{1}{2}}$ ☆

e) $\boxed{\text{VA} \Rightarrow x = -\frac{3}{2}}$



$\boxed{\text{Hole:}}$ @ $x=1$

$f(1) = \dfrac{x+1}{2x+3}$

$f(1) = \dfrac{2}{5}$ $\boxed{(1, \frac{2}{5})}$

PART B

5. - Graph from quadrant 3 to 4 (opens down)
- Even degree function
- Leading coefficient must be negative
- X - intercepts

$x = -4$ (TO)
$x = -1$ (BE)
$x = 3$ (TO)

Quartic Degree = 4

$f(x) = a(x+4)(x+1)^2(x-3)$ y-int = 3 or (0,3)

Substitute a known point (0,3) to find "a")

$3 = a(0+4)(0+1)^2(0-3)$
$3 = a(4)(1)(-3)$
$3 = -12a$
$-\frac{1}{4} = a$

$$\boxed{\therefore\ y = -\frac{1}{4}(x+4)(x+1)^2(x-3)}$$