Chapter_1.4_Inverses, Algebraic Properties of Matrices

THEOREM 1.4.5 *The matrix*

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

is invertible if and only if $ad - bc \neq 0$, in which case the inverse is given by the formula

$$A^{-1} = \frac{1}{ad - bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix} \qquad (2)$$

5. Compute the inverse of the matrix

$$\boldsymbol{A} = \begin{bmatrix} \mathbf{2} & \mathbf{-3} \\ \mathbf{4} & \mathbf{4} \end{bmatrix}$$

Solution: Given matrix is

$$A = \begin{bmatrix} 2 & -3 \\ 4 & 4 \end{bmatrix}$$

Here $ad - bc = 8 + 12 = 20 \neq 0$

$$\therefore A^{-1} = \frac{1}{20}\begin{bmatrix} 4 & 3 \\ -4 & 2 \end{bmatrix}$$

7. Compute the inverse of the matrix

$$\boldsymbol{A} = \begin{bmatrix} \mathbf{2} & \mathbf{0} \\ \mathbf{0} & \mathbf{3} \end{bmatrix}$$

Solution: Given matrix is

$$A = \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}$$

Here $ad - bc = 6 \neq 0$

$$\therefore A^{-1} = \frac{1}{6}\begin{bmatrix} 3 & 0 \\ 0 & 2 \end{bmatrix}$$

8. Compute the inverse of the matrix

$$\boldsymbol{A} = \begin{bmatrix} \mathbf{6} & \mathbf{4} \\ \mathbf{-2} & \mathbf{-1} \end{bmatrix}$$

Solution: Given matrix is

$$\boldsymbol{A} = \begin{bmatrix} \mathbf{6} & \mathbf{4} \\ \mathbf{-2} & \mathbf{-1} \end{bmatrix}$$

Here $ad - bc = -6 + 8 = 2 \neq 0$

$$\therefore A^{-1} = \frac{1}{2}\begin{bmatrix} -1 & -4 \\ 2 & 6 \end{bmatrix}$$

9. FInd the inverse of the matrix

$$\boldsymbol{A} = \begin{bmatrix} \frac{1}{2}(\boldsymbol{e^x + e^{-x}}) & \frac{1}{2}(\boldsymbol{e^x - e^{-x}}) \\ \frac{1}{2}(\boldsymbol{e^x - e^{-x}}) & \frac{1}{2}(\boldsymbol{e^x + e^{-x}}) \end{bmatrix}$$

Solution: Given matrix is

$$A = \begin{bmatrix} \frac{1}{2}(e^x + e^{-x}) & \frac{1}{2}(e^x - e^{-x}) \\ \frac{1}{2}(e^x - e^{-x}) & \frac{1}{2}(e^x + e^{-x}) \end{bmatrix}$$

Here $ad - bc = \frac{1}{2}(e^x + e^{-x}) \times \frac{1}{2}(e^x + e^{-x}) - \frac{1}{2}(e^x - e^{-x}) \times \frac{1}{2}(e^x - e^{-x})$

$$= \frac{1}{4}[(e^x + e^{-x})^2 - (e^x - e^{-x})^2]$$

$$= \frac{1}{4}[(e^x + e^{-x} + e^x - e^{-x})(e^x + e^{-x} - e^x + e^{-x})]$$

$$= \frac{1}{4} \times 2e^x \times 2e^{-x} = 1 \neq 0$$

$$\therefore A^{-1} = \frac{1}{1}\begin{bmatrix} \frac{1}{2}(e^x + e^{-x}) & -\frac{1}{2}(e^x - e^{-x}) \\ -\frac{1}{2}(e^x - e^{-x}) & \frac{1}{2}(e^x + e^{-x}) \end{bmatrix}$$

$$or, \quad A^{-1} = \begin{bmatrix} \frac{1}{2}(e^x + e^{-x}) & -\frac{1}{2}(e^x - e^{-x}) \\ -\frac{1}{2}(e^x - e^{-x}) & \frac{1}{2}(e^x + e^{-x}) \end{bmatrix}$$

10. Compute the inverse of the matrix

$$\boldsymbol{A} = \begin{bmatrix} \boldsymbol{cos\theta} & \boldsymbol{sin\theta} \\ \boldsymbol{-sin\theta} & \boldsymbol{cos\theta} \end{bmatrix}$$

Solution: Given matrix is

$$A = \begin{bmatrix} cos\theta & sin\theta \\ -sin\theta & cos\theta \end{bmatrix}$$

Here $ad - bc = cos^2\theta + sin^2\theta = 1 \neq 0$

$$\therefore A^{-1} = \frac{1}{1}\begin{bmatrix} cos\theta & sin\theta \\ -sin\theta & cos\theta \end{bmatrix}$$

$$or, A^{-1} = \begin{bmatrix} cos\theta & -sin\theta \\ sin\theta & cos\theta \end{bmatrix}$$

Solution: Given that,

$(5A^T)^{-1} = \begin{bmatrix} -3 & -1 \\ 5 & 2 \end{bmatrix}$

$\therefore \; 5A^T = \dfrac{1}{-1}\begin{bmatrix} 2 & 1 \\ -5 & -3 \end{bmatrix} \qquad [\because (A^{-1})^{-1} = A]$

$or, \qquad A^T = \dfrac{1}{5}\begin{bmatrix} -2 & -1 \\ 5 & 3 \end{bmatrix}$

$or, \qquad A = \dfrac{1}{5}\begin{bmatrix} -2 & 5 \\ -1 & 3 \end{bmatrix}$

$or, \qquad A = \begin{bmatrix} -\dfrac{2}{5} & 1 \\ -\dfrac{1}{5} & \dfrac{3}{5} \end{bmatrix}$

21. c) Given that $p(x) = x^3 - 2x + 1$. $A = \begin{bmatrix} 3 & 1 \\ 2 & 1 \end{bmatrix}$. Compute $p(A)$

Solution:

Given that. $A = \begin{bmatrix} 3 & 1 \\ 2 & 1 \end{bmatrix}$.

$p(x) = x^3 - 2x + 1$

$\therefore p(A) = A^3 - 2A + I$

$A = \begin{bmatrix} 3 & 1 \\ 2 & 1 \end{bmatrix}$.

$A^2 = \begin{bmatrix} 3 & 1 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} 3 & 1 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} 11 & 4 \\ 8 & 3 \end{bmatrix}$..

$A^3 = A^2 \times A = \begin{bmatrix} 11 & 4 \\ 8 & 3 \end{bmatrix} \times \begin{bmatrix} 3 & 1 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} 41 & 15 \\ 30 & 11 \end{bmatrix}$.

$\therefore p(A) = A^3 - 2A + I$

$$= \begin{bmatrix} 41 & 15 \\ 30 & 11 \end{bmatrix} - 2\begin{bmatrix} 41 & 15 \\ 30 & 11 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 41 - 82 + 1 & 15 - 30 \\ 30 - 60 & 11 - 22 + 1 \end{bmatrix}$$

$$= \begin{bmatrix} 40 & -15 \\ -30 & -10 \end{bmatrix}$$