ASSIGNMENT-2

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Answer to the Question No-1

An experiment consists of tossing a die and then flipping a coin once if the number on the die is even. If the number is odd, the coin is flipped twice. If A is the event that a number less than 4 occurs on the die and B is the event that two heads occur.

(i) Construct a tree diagram to show the 18 elements of the sample spaces.

Ans:

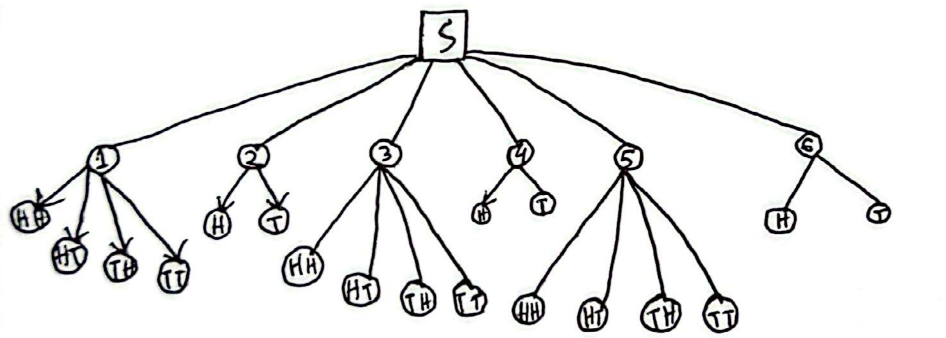
From rolling a die, outcomes: 1,2,3,4,5,6

Die is even: 2,4,6

Flip a coin once outcomes: H,T

Die is odd: 1,3,5

Flip a coin twice outcomes: HH,HT,TH,TT



So, the sample space will be,

$S = \{(1,HH),(1,HT),(1,TH),(1,TT),(2,H),(2,T),(3,HH),(3,HT),(3,TH),(3,TT),$
$(4,H),(4,T),(5,HH),(5,HT),(5,TH),(5,TT),(6,H),(6,T)\}$

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(ii) List the elements corresponding to the event $A' \cap B$.

Ans:

A: A number less than 4 occurs on the die (1, 2, 3)

$A = \{(1, HH), (1, HT), (1, TH), (1, TT), (2, H), (2, T), (3, HH), (3, TH), (3, HT), (3, TT)\}$

B: Two heads occur (HH)

$B = \{(1, HH), (3, HH), (5, HH)\}$

$A' = \{(4, H), (4, T), (5, HH), (5, HT), (5, TH), (5, TT), (6, H), (6, T)\}$

$\therefore A' \cap B = \{(5, HH)\}$ (Ans)

(iii) List the elements corresponding to the event $A \cup B$

Ans:

$A \cup B = \{(1, HH), (1, HT), (1, TH), (1, TT), (2, H), (2, T), (3, HH), (3, HT), (3, TH), (3, TT), (5, HH)\}$ (Ans)

Answer to the question No-2

In a certain assembly plant, three machine M1, M2, M3 make 35%, 40% and 30% respectively of the products. It is know from the past experience that 3%, 2%, 3% of the products made of each machine, respectively and defective. Now, suppose that a finished product is randomly selected.

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a) What is the probability that it is defective?

Ans:

$P(M_1) = 35\% = 0.35$

$P(M_2) = 40\% = 0.4$

$P(M_3) = 30\% = 0.3$

$P(D \backslash M_1) = 3\% = 0.03$

$P(D \backslash M_2) = 2\% = 0.02$

$P(D \backslash M_3) = 3\% = 0.03$

we know from the law of total probability,

$$P(D) = P(D \backslash M_1) \cdot P(M_1) + P(D \backslash M_2) \cdot P(M_2) + P(D \backslash M_3) \cdot P(M_3)$$

$$= 0.03 \times 0.35 + 0.02 \times 0.4 + 0.03 \times 0.3$$

$$= 0.0275 \text{ (Ans)}$$

b) If a product was chosen randomly and found to be defective, what is the probability that it was made by machine M_2?

Ans: we know,

$$P(M_2 \backslash D) = \frac{P(D \backslash M_2) \cdot P(M_2)}{P(D)}$$

$$= \frac{0.02 \times 0.4}{0.0275}$$

$$= 0.290 \text{ (Ans)}$$

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Answer to the question No-3

The probability that a husband enjoys cooking is 0.4, and the probability that his wife enjoys cooking is 0.7. The probability that a wife enjoys cooking, given that her husband does, is 0.6. Find the probability that:

a) Both husband and wife enjoy cooking

Ans:

$P(H) = 0.4$
$P(W) = 0.7$
$P(W \mid H) = 0.6$

$\therefore P(H \cap W) = P(W \mid H) \times P(H)$
$= 0.6 \times 0.4$
$= 0.24$ (Ans)

b) The husband enjoys cooking, given that his wife does.

Ans:
$$P(H \mid W) = \frac{P(H \cap W)}{P(W)}$$
$$= \frac{0.24}{0.7}$$
$= 0.3429$
$= 34.29\%$ (Ans)

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c) At least one of the enjoys cooking

Ans:

$$P(H \cup W) = P(H) + P(W) - P(H \cap W)$$
$$= 0.4 + 0.7 - 0.24$$
$$= 0.86$$
$$= 86\%$$

Answer to the question No-4

Four cards are drawn in succession, without replacement, from an ordinary deck of playing cards. Find the probability that the event $C_1 \cap C_2 \cap C_3 \cap C_4$ occurs where

a) C_1 is the event that the first card is a red ace.

Ans: We know,

$$P(C_1) = \frac{\text{Number of red aces}}{\text{total cards}}$$
$$= \frac{2}{52}$$
$$= \frac{1}{26} \text{ (Ans)}$$

b) C_2 is the event that the second card is a 10 or a king

Ans:
We know, ~~P(C2/C1)~~ $P(C_2 \mid C_1) = \frac{\text{Number of 10s or kings}}{\text{total remaing cards}}$

$$= \frac{8}{51} \text{ (Ans)}$$

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c) C_3 is the event that the third card is greater than 2 but less than 5.

Ans:
Greater than 2 but less than 5 is 3 or 4.

∴ we know,

$$P(C_3|C_1 \cap C_2) = \frac{\text{Number of 3s and 4s}}{\text{total remaining cards}}$$

$$= \frac{8}{50}$$

$$= \frac{4}{25} \text{ (Ans)}$$

d) C_4 is the event that the fourth card is greater than 7 but less than 9.

Ans: we know,

$$P(C_4|C_1 \cap C_2 \cap C_3) = \frac{\text{Number of 8s}}{\text{total remaining cards}}$$

$$= \frac{4}{49} \text{ (Ans)}$$

∴ Probability of the event,

$$P(C_1 \cap C_2 \cap C_3 \cap C_4) = \frac{2}{52} \times \frac{8}{51} \times \frac{8}{50} \times \frac{4}{49}$$

$$= 7.88 \times 10^{-5} \text{ (Ans)}$$

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Answer to the question No-5

A fair die is tossed and up face is observed. If the face is even, you will win, otherwise lose. What is the probability that you will win.

Ans:

A fair die has 6 faces that are: 1, 2, 3, 4, 5, 6

even number on a die are: 2, 4, 6

we know,

$$P(W) = \frac{3}{6} = \frac{1}{2} \text{ (Ans)}$$

Answer to the question No-6

a) state Bayes theorem;

Ans:

$$P(A|B) = \frac{P(B|A) \cdot P(A)}{P(B)}$$

where, $P(A)$ = prior probability of A

$P(B|A)$ = Likelihood of B given A

$P(B)$ = total probability of B

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b) suppose that there are two websites, A and B for renting books. Site A receives 60% of all oreders. Among the orders placed on A, 95% arrive on time. Among the orders placed on site B, 90% arrive on time. What is the probability that an order arrives on time. find the probability that it was placed on website B.

Ans:

$P(A) = 60\% = 0.6$

$P(B) = (100-60)\% = 40\% = 0.4$

$P(T \backslash A) = 95\% = 0.95$

$P(T \backslash B) = 90\% = 0.9$

We know,

$$P(T) = P(T \backslash A) \cdot P(A) + P(T \backslash B) \cdot P(B)$$
$$= 0.95 \times 0.6 + 0.9 \times 0.4$$
$$= 0.81$$

$$P(B \backslash T) = \frac{P(T \backslash B) \cdot P(B)}{P(T)}$$
$$= \frac{0.9 \times 0.4}{0.81}$$
$$= 0.44 \text{ (Ans)}$$

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Answer to the question No-7

x	-1	0	1
$f(x)$	0.2	0.3	0.5

compute $E(x), E(2x), E(3x+1)$

Ans: $E(x) = \sum_{i=1}^{n} x f(x)$

$= \sum_{i=1}^{3} (-1)(0.2) + (0)(0.3) + (1)(0.5)$

$= 0.3$ (Ans)

$E(2x) = 2E(x)$

$= 2 \times 0.3$

$= 0.6$ (Ans)

$E(3x+1) = 3E(x) + E(1)$

$= (3 \times 0.3) + 1$

$= 1.9$ (Ans)

Answer to the question No-8

The probability that a certain kind of component will survive a shock test is 3/4. Find the probability that exactly 2 of the next 4 components tested survive.

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Ans: $p = \frac{3}{4}$, $n = 2$

$$b(2; 4, \tfrac{3}{4}) = \binom{4}{2}\left(\frac{3}{4}\right)^2\left(\frac{1}{4}\right)^2$$

$$= \frac{4!}{2!\,2!} \cdot \frac{9}{16} \cdot \frac{1}{16}$$

$$= \frac{27}{128}$$

The probability that a patient recovers from a rare blood disease is 0.4. If 15 people are known to have contracted this disease, what is the probability that (a) at least 10 survive, (b) from 3 to 8 survive, (c) exactly 5 survive?

Ans:

(a) $P(x \geq 10) = 1 - P(x < 10) = 1 - \sum_{x=0}^{9} b(x; 15, 0.4)$

$= 1 - 0.9662$

$= 0.0338$ (Ans)

(b) $P(3 \leq x \leq 8) = \sum_{x=3}^{8} b(x; 15, 0.4)$

$= \sum_{x=0}^{8} b(x; 15, 0.4) - \sum_{x=0}^{2} b(x; 15, 0.4)$

$= 0.9050 - 0.0271$

$= 0.8779$ (Ans)

(c) $P(x = 5) = b(5; 15, 0.4)$

$= \sum_{x=0}^{5} b(x; 15, 0.4) - \sum_{x=0}^{4} b(x; 15, 0.4)$

$= 0.4032 - 0.2173$

$= 0.1859$ (Ans)

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Answer to the Question No-9

A large chain retailer purchase a certain kind of electronic device from a manufacturer. The manufacturer indicates that the defective rate of the device is 3%.

a) The inspector randomly picks 20 items from a shipment. What is the probability that there will be at least one defective item among these 20?

Ans:

We know,

$$P(x \geq 1) = 1 - P(x=0)$$
$$= 1 - b(0; 20, 0.03)$$
$$= 1 - (0.03)^0 (1-0.03)^{20-0}$$
$$= 0.4562 \text{ (Ans)}$$

b) Suppose that the retailer receives 10 shipments in a month and the inspector randomly tests 20 devices per shipment. What is the probability that there will be exactly 3 shipments each containing atleast one defective device among the 20 that are selected tested from the shipment?

Ans: from (a) we get,

$$p = 0.4562$$

Let Y another binominal distribution $b(y; 10, 0.4562)$

$$\therefore P(Y=3) = \binom{10}{3} 0.4562^3 (1-0.4562)^7$$
$$= 0.1602 \text{ (Ans)}$$

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Answer to the question No-10

It is conjectured that an impurity exists in 30% of all drinking wells in a certain rural community. In order to gain some insight into the true extent of the problem, it is determined that some testing is necessary. It is too expensive to test all of the wells in the area. So 10 are randomly selected for testing.

a) Using the binomial distribution, what is the probability that exactly 3 wells have the impurity, assuming that the conjecture is correct?

Ans: we know,

$$b(3; 10, 0.3) = \sum_{x=0}^{3} b(x; 10, 0.3) - \sum_{x=0}^{2} b(x; 10, 0.3)$$

$$= 0.6496 - 0.3828$$

$$= 0.2668 \text{ (Ans)}$$

b) What is the probability that more than 3 wells are impure?

Ans: we know,

$$P(x > 3) = 1 - 0.6496$$

$$= 0.3504 \text{ (Ans)}$$

c) The notion that 30% of the wells are impure is merely a conjecture put forth by the area water board. Suppose 10 wells are randomly selected and 6 are found to contain the impurity. What does this imply about the conjecture? Use a probability statement.

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Ans: We know,

$$P(x \geq 6) = \sum_{x=0}^{10} b(x; 10, 0.3) - \sum_{x=0}^{5} b(x; 10, 0.3)$$
$$= 1 - 0.9527$$
$$= 0.0473$$

As a result, it is very unlikely (4.7% chance) that 6 or more wells would be found impure if only 30% of all are impure.

Answer to the Question No. 11

During a laboratory experiment, the average number of radioactive particles passing through a counter in 1 millisecond is 4. What is the probability that 6 particles enter the counter in a given millisecond?

Ans:
we know,

$$P(6; 4) = \frac{e^{-4} 4^6}{6!}$$
$$= \sum_{x=0}^{6} p(x; 4)$$
$$= 0.8893 - 0.7851$$
$$= 0.1042 \text{ (Ans)}$$

Answer to the question No-12

Ten is the average number of oil tankers arriving each day at a certain point. The facilities at the port can handle at most 15 tankers per day. What is the probability that on a given day tankers have to be turned away?

Ans: we know,

$$P(x > 15) = 1 - P(x \leq 15)$$
$$= 1 - \sum_{x=0}^{15} p(x; 10)$$
$$= 1 - 0.9513$$
$$= 0.0487 \text{ (Ans)}$$

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Answer to the question No-13

In a certain industrial facility, accidents occur infrequently. It is known that the probability of an accident on any given day is 0.005 and accidents are independent of each other.

a) What is the probability that in any given period of 400 days there will be an accident on one day?

Ans: We know,

$$P(x=1) = e^{-2} 2^1$$
$$= 0.271$$

Here,
$n = 400$
$p = 0.005$
$np = 2$

b) What is the probability that there are at most three days with an accident?

Ans: we know,

$$P(x \leq 3) = \sum_{x=0}^{3} e^{-2} 2^x / x!$$
$$= 0.857 \text{ (Ans)}$$

Answer to the question No-14

A certain type of storage battery lasts on average 3.0 years. with a standard deviation of 0.5 years. Assuming that battery life is normally distributed, find the probability that a given battery will last less than 2.3 years.

Ans: we know,

$$z = \frac{2.3-3}{0.5}$$
$$= -1.4$$

$$P(x<2.3) = P(z<-1.4)$$
$$= 0.0808 \text{ (Ans)}$$

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Answer to the question No-15

An electrical firm manufacturers light bulbs that have a life, before burn out, that is normally distributed with mean equal to 800 hours and a standard deviation of 40 hours. Find the probability that a bulb burns between 778 and ~~84~~ 834 hours.

Ans: we know,

$$z_1 = \frac{778-800}{40} = -0.55$$

$$z_2 = \frac{834-800}{40} = 0.85$$

$$P(778 < x < 834) = P(-0.55 < z < 0.85)$$
$$= P(z < 0.85) - P(z < -0.55)$$
$$= 0.8023 - 0.2912$$
$$= 0.5111 \text{ (Ans)}$$

Answer to the question No-16

In an industrial process, the diameter of a ball bearing is an important measure ment. The buyer sets specifications for the diameter to be 3.0±0.01 cm. The implication is that no part falling outside these specification will be accepted. It is known that in the process the diameter of a ball bearing has a normal distribution with mean $\mu=3.0$ and standard deviation $\sigma=0.005$. On average, how many manufactured ball bearings will be ~~accepted~~ scrapped?

Ans: $x_1 = 2.99$

$x_2 = 3.01$

$$z_1 = \frac{2.99-3.0}{0.005}$$

$$= -2.0$$

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$$z_2 = \frac{3.01 - 3.0}{0.005}$$

$= +2.0$

$P(2.99 < x < 3.01) = P(-2.0 < z < 2.0)$

$= P(z < -2.0) + P(z > 2.0)$

$= 2(0.0228)$

$= 0.0456$ (Ans)

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