NOTE 28/9/2022

Sia $\lambda \in \mathbb{R}$, poniamo:

$$e_\lambda(t) \overset{\text{def.}}{=} e^{2\pi i \lambda t} \qquad \left(= \cos 2\pi\lambda t + i \operatorname{sen}(2\pi\lambda t)\right)$$

allora

$$e_\lambda^n(t) = (e_\lambda(t))^n = \left(e^{2\pi i \lambda t}\right)^n = e^{2\pi i \lambda n t} = e_{n\lambda}(t)$$

① p. 17 Gasquet-Witomski:

$$RCv' + v = x \qquad (\text{eq. diff. circ. } RC)$$

se $x = e_\lambda$ allora $v = H(\lambda)\, e_\lambda$

$$v' = H(\lambda)\, e_\lambda'(t) = H(\lambda)\, e^{2\pi i \lambda t} \cdot 2\pi i \lambda$$

$$\left(e_\lambda(t) = e^{2\pi i \lambda t}\right)$$

sostituisco v, v', x nell'eq. diff:

$$RC\, H(\lambda)\, \cancel{e^{2\pi i \lambda t}}\, 2\pi i \lambda + H(\lambda)\, \cancel{e^{2\pi i \lambda t}} = \cancel{e^{2\pi i \lambda t}}$$

$$H(\lambda)\left(RC \cdot 2\pi i \lambda + 1\right) = 1$$

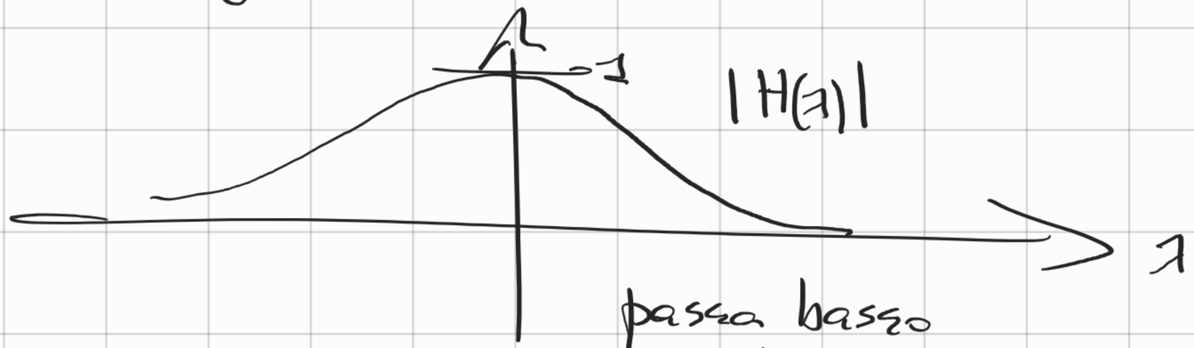
$$\boxed{H(\lambda) = \frac{1}{2\pi i \lambda RC + 1}}$$

funz. di trasferim. del filtro RC

$$|H(\nu)| = \left| \frac{1}{1+i\,2\pi\nu RC} \right| = \frac{1}{|1+i\,2\pi\nu RC|}$$

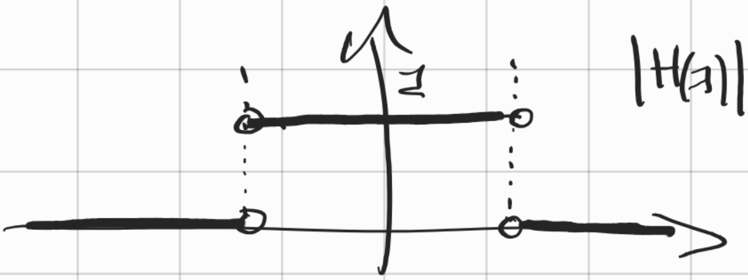
$|a+ib| = \sqrt{a^2+b^2}$

$$= \frac{1}{\sqrt{1+4\pi^2R^2C^2\nu^2}}$$



passa basso realizzabile

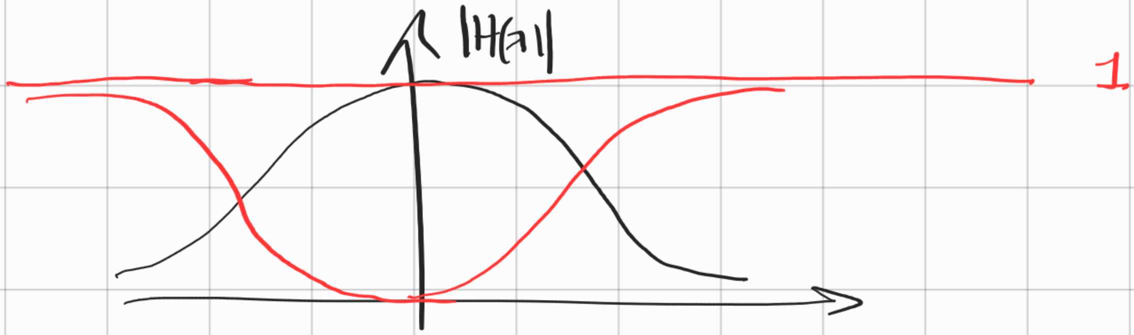
<u>Nota</u> un filtro passa basso ideale avrebbe funz. di trasf.:



Vedremo che è non realizzabile!

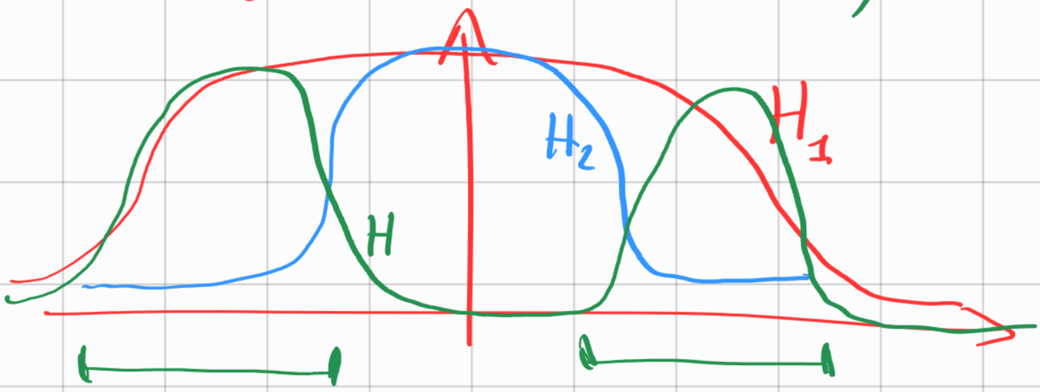
<u>Nota</u>

Sia B filtro con funz. di trasf. $1-H(\nu)$ allora B è filtro "passa alto"

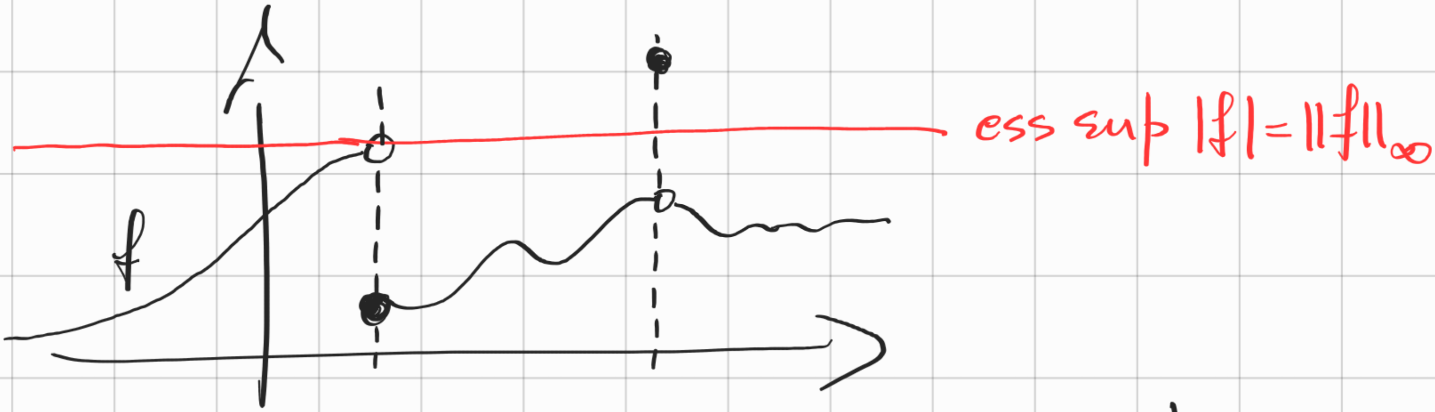


<u>Nota</u> funz. trasf. di un filtro "passa banda"

$$H_1(f) - H_2(f) = H(f)$$



Nota agli appunti: Trasf. di Fourier e Filtri

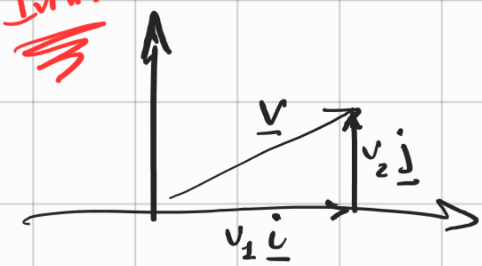


Nota

"Seno cardinale"

① p. 4

In $\mathbb{R}^2$



base ⊥ norm. $\underline{i}, \underline{j}$

coeff.

$$\boxed{\underline{v} = v_1 \underline{i} + v_2 \underline{j}} = \sum_{n=1}^{2} c_n e_n$$

$e_1 = \underline{i}$
$e_2 = \underline{j}$

$$= \underbrace{(\underline{v}, \underline{i})}_{\underline{v} \cdot \underline{i}} \underline{i} + \underbrace{(\underline{v}, \underline{j})}_{\underline{v} \cdot \underline{j}} \underline{j}$$

$c_n = (\underline{v}, e_n)$

In $L^2(I)$:

$$\boxed{f = \sum_{n=-\infty}^{+\infty} c_n \phi_n}$$

$c_n = (f, \phi_n)$
coeff. di Fourier

base ⊥ n.

Es $\quad f(t) = 3e_2(t) + 5e_3(t)$

$$= 3\,e^{2\pi i \cdot 2 \cdot t} + 5\,e^{2\pi i \cdot 3 \cdot t}$$

$c_2 = (f,\phi_2)$ $\quad$ ϕ_2 freq. = 2 $\quad$ $c_3 = (f,\phi_3)$ $\quad$ ϕ_3 freq. = 3

ϕ_n base ln di $L^2(I)$

$$\boxed{\|f\|_2^2 = \sum_{n=-\infty}^{+\infty} |c_n|^2}$$ uguaglianza di Parseval

dove $\quad f = \sum_n c_n \phi_n \qquad c_n = (f,\phi_n)$

$$\|c_n\phi_n\|_2^2 =$$

$$\int_I |c_n\phi_n(t)|^2\,dt =$$

$$\int_I |c_n|^2\,|\phi_n(t)|^2\,dt =$$

$$|c_n|^2 \int_I |\phi_n(t)|^2\,dt =$$

$$|c_n|^2 \underbrace{\|\phi_n\|_{L^2}^2}_{1} = |c_n|^2$$

In $\mathbb{R}^2$ o $\mathbb{R}^3$ Parseval corrisponde al:

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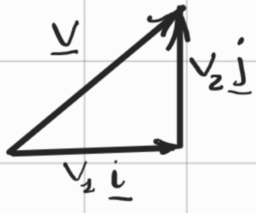
es in $\mathbb{R}^2$:

sia $\quad \underline{v} = v_1\underline{i} + v_2\underline{j}$



allora

$$\|\underline{v}\|^2 = \|v_1\underline{i}\|^2 + \|v_2\underline{j}\|^2$$

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