NORTHWESTERN UNIVERSITY | SCHOOL OF PROFESSIONAL STUDIES

Final Exam

Points possible: 100

Description: The final exam will cover topics from sessions 1-9.

Resources: The exam is completely open book. You may use course textbooks, materials provided on Canvas, or basic graphing calculators (such as TI 83 or 84). Any more advanced calculators, Excel Solver, Web calculators, Web-graphic calculators, or simplex method calculators are not allowed. Programming languages other than Python are also not permitted.

For questions that require calculations, all calculations should be shown, not just the final answer. This will allow for partial credit for those answers that might be set up correctly but have calculation errors. For questions that specifically require Python, the code and output should be included with your answer. For questions that require graphs, only use Python.

Restrictions: All answers are to be your work only. You are not to receive assistance from any other person.

To complete the exam:

1. Answer all questions on the exam thoroughly. Create a Microsoft Word document, including the question number, the question, your typed answer, and graphs if required. You may use Word's equation editor to complete your answers.

2. Once you have completed your exam, return to the exam item where you downloaded the exam PDF, click View/Complete Assignment, and submit your document.

1. A local copy center needs to buy white paper and yellow paper. They can buy from three suppliers. Supplier 1 sells a package of 20 reams of white and 10 reams of yellow for $60. Supplier 2 sells a package of 10 reams of white and 10 reams of yellow for $40. Supplier 3 sells a package of 10 reams of white and 20 reams of yellow for $50. The copy center needs 350 reams of white and 400 reams of yellow. **Using Python**, determine (1) how many packages they should buy from each supplier in order to minimize cost and (2) the minimum cost.

2. A new test has been developed to detect a particular type of cancer. A medical researcher selects a random sample of 1,000 adults and finds (by other means) that 4% have this type of cancer. Each of the 1,000 adults is given the new test and it is found that the test indicates cancer in 99% of those who have it and in 1% of those who do not. Based on these results, what is the probability of a randomly chosen person having cancer given that the test indicates cancer? What is the probability of a person having cancer given that the test does not indicate cancer?

3. If a tank holds 5000 gallons of water, which drains from the bottom of the tank in 40 minutes, then the volume of the water remaining in the tank after t minutes is given by $V(t) = 5000\left(1 - \frac{t}{40}\right)^2, 0 \leq t \leq 40$. **Using Python**, determine the rate at which water is draining from the tank.

4. A rectangular container with a volume of 475 ft^3 is to be constructed with a square base and top. The cost per square foot for the bottom is \$0.20, for the top is \$0.10, and for the sides is \$0.015. Find the dimensions of the container that minimize the amount of material used.

5. Assume the total revenue from the sale of x items is given by $R(x) = 27\ln(6x + 1)$ while the total cost to produce x items is $C(x) = x/7$. Find the approximate number of items that should be manufactured so that profit is maximized.

6. For the following function, determine the domain, critical points, intervals where the function is increasing or decreasing, inflection points, intervals of concavity, intercepts, and asymptotes where applicable. Use this information and **Python to graph** the function.

$$f(x) = -\frac{1}{(x+2)^2} + 4$$

7. The rate of growth of the profit (in millions) from an invention is approximated by the function $P'(x) = xe^{-x^2}$ where x represents time measured in years. The total profit in year 2 that the invention is in operation is \$25,000. Find the total profit function. Round to three decimals where appropriate.

8. For a certain drug, the rate of reaction in appropriate units is given by

$$R'(t) = \frac{6}{t+1} + \frac{1}{\sqrt{t+1}}$$

where t is time (in hours) after the drug is administered. Find the total reaction to the drug from 1 to 8 hours after it is administered.

9. Show that the following function is a probability density function on $[0, \infty)$.

$$f(x) = \begin{cases} {}^{x^3}/_{8}, if\ 0 \le x \le 2 \\ {}^{4}/_{x^3}, if\ x > 2 \end{cases}$$

Determine $P(1 \le x \le 5)$.

10. Researchers have shown that the number of successive dry days that occur after a rainstorm for a particular region is a random variable that is distributed exponentially with a mean of 9 days. **Using Python**, determine the (separate) probabilities that 13 or more successive dry days occur after a rainstorm, and fewer than 2 dry days occur after a rainstorm.