Week 1

Reference 5 → Bible

WRDS → mqf2020 / Mqf_2020

Housing → Case-Shiller Housing Index

Ass1 5. real return → we discrete $\frac{1+r}{1+inf} = 1+real$

$\sqrt{T}\,\rho_1 \sim N(0,1)$

autocorrelation: $\rho_1 = \frac{Cov(r_t, r_{t-1})}{Var(r_t)}$

Randomization with replacement

Week 2

Slope of Indiff Curve = Marginal Rate of Substitution

MRS

$\frac{\partial c_1}{\partial c_0}$ | utility is held constant

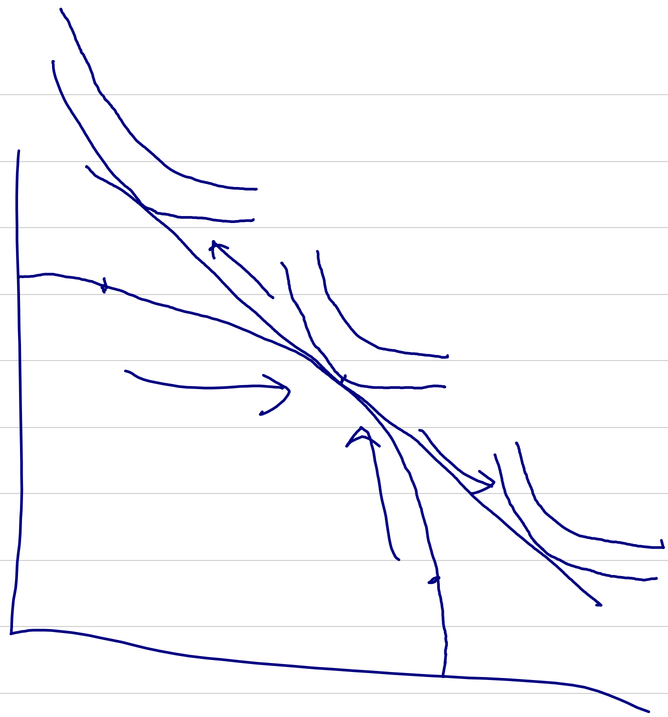
Slope of PPF = marginal rate of techical Substitution.

RTS.

$\frac{\partial c_1}{\partial c_0}$ given resources and techology

(PPF?)

Trade on the production line and trade beyond time and get the market interest line tangent to the utility function.



Final MRS = HR
= RTS

Fisher - Seperation Theorem..

Assume n risky stock. r_j $j = 1 \cdots n$
risk free asset r_f.
W_0: initial wealth
$E[u(\hat{w})]$ expected utility

$$= \begin{cases} \sum P_i \, u(w_i) & \text{discrete} \\ \int f(w) u(w) \, dw \end{cases}$$

$u'(w) > 0$
$u''(w) < 0$

$$\max_{\{a_j\}} E[u(\hat{w})]$$

budget constraint : $\hat{w} = a_j (1 + \tilde{r}_j) = \sum_{j=1}^{n} a_j (1 + \tilde{r}_j) + (W_0 - \sum_{j=1}^{n} a_j)(1 + r_f)$

F.O.C $\frac{\partial L}{\partial a_j} = 0$

$$E\left[u'(\tilde{w})\frac{\partial \tilde{w}}{\partial a_j}\right] = E\left[u'(\tilde{w})\left[(1+\tilde{r}_j) - (1+r_f)\right]\right]$$

$$= E\left[u'(\tilde{w})(\tilde{r}_j - r_f)\right] = 0$$

margin utility　　risk premium

stochastic discount rate

pricing kernel

$$E\left[u'(\tilde{w})\tilde{r}_j\right] = E\left[u'(\tilde{w})r_f\right]$$

$\Rightarrow$ when we invest one more dollar at j,

They have the same utility

$$E\left[u'(w)\tilde{r}_i\right] = E\left[u'(\tilde{w})\tilde{r}_j\right] \quad \forall i,j$$

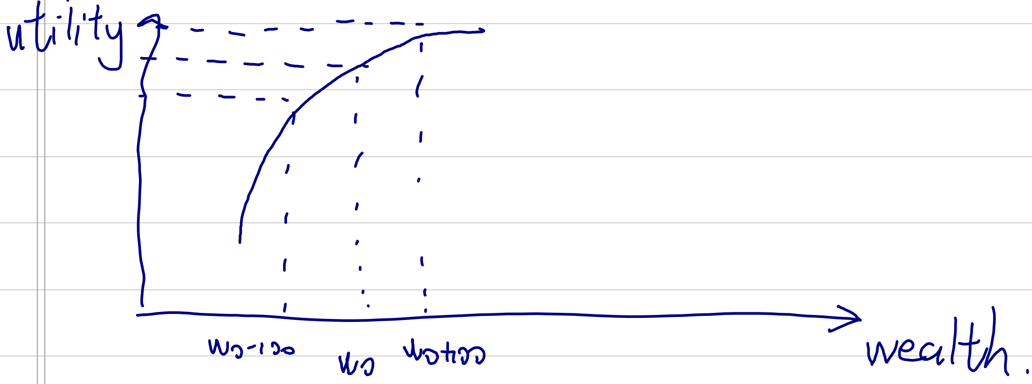
also equal among stocks

$r_t \sim (N_t, \sigma_t^2)$　　assume distribution not changing

Exam Bove 220
1:00 pm.

Week 3.

F.O.C.



$E[u'(\tilde{w})(\tilde{r}_j - r_f)] = 0$

$u' > 0$, $u'' < 0$.

If $u'' = 0$, $u' = c$ $\quad u = cW + D$

$\Rightarrow cE[\tilde{r}_j - r_f] = 0$

$E[\tilde{r}_j] = E[r_f] \quad \forall j$

In order for investors to invest in stocks, at least one stock must have a positive risk premium

Proof: By contradiction

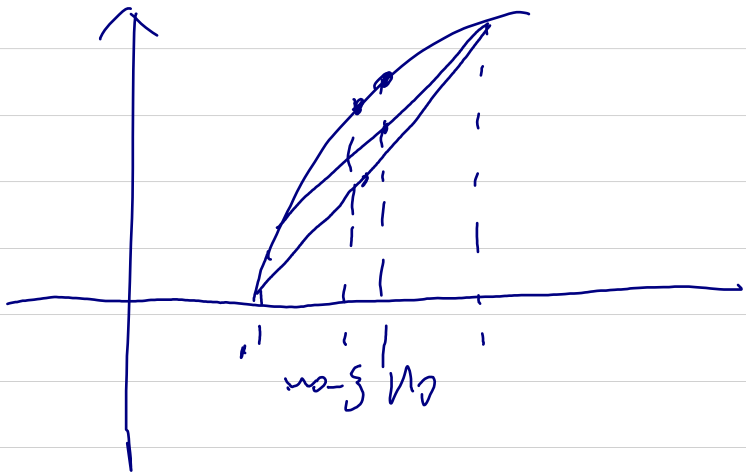
For investor to have \$0 investment in stocks or even short sell stocks, the following must held, $E[u'(W_0(1+r_f))(\tilde{r}_j - r_f)] \leq 0 \quad \forall j$

$u'(W_0(1+r_f)) E[r_j^2 - r_f] \leq 0$

negative premium $\rightarrow$ insurance

to make things certain

$\rightarrow$ push utility higher.



$E[\tilde{r}_m]$ (market portfolio) is higher than r_f,
but not for all $\tilde{r}_j$ in r_m

diminishing returns.

minimum Risk premium

$E[u'(w_0(1+\tilde{r}))(\tilde{r}-r_f)] \geq 0$

Do the taylor expansion of u' around r_f

$$u'(w_0(1+r_f)) + u''(w_0(1+r_f)) \cdot w_0(\tilde{r}-r_f) + O(\tilde{r}-r_f)$$

multiply by $(\tilde{r}-r_f)$ and take E

$$u'(w_0(1+r_f))E[\tilde{r}-r_f] + u''(w_0(1+r_f))w_0 E[\tilde{r}-r_f]^2$$

$$\Rightarrow E[\tilde{r}-r_f] = \left[(-)\frac{u''(w_0(1+r_f))w_0}{u'(w_0(1+r_f))}\right] E[\tilde{r}-r_f]^2$$

like volatility

investor's risk attitude

How much percentage of risk adverse the investor would like to take at wealth w_0

Relative Risk Aversion

$R_r = (-)\frac{u''}{u'} W$

Absolute Risk Aversion

$R_A = (-)\frac{u''}{u'}$

$R_A(z)$, $R_R(z)$ $z \to$ another measure of wealth

$R_A(z) = (-)\frac{u''(z)}{u'(z)}$

$R_A'(z) > 0$ increasing absolute risk Aversion

< 0 decreasing ...

$= 0$ constant

Initial Allocation

Stock = 800000,

T-bill = 200000

When wealth increase.

$\Delta W = 200000$

1. If $R_A'(z) > 0$ $\frac{da}{dw} < 0$

Stock = 800000 − 100000 = 700000

T-bill = 200000 + 300,000 = 500000

2. If $R_A'(z) < 0$ $\frac{da}{dw} > 0$

Stock = 800000 + 50000 = 850000

T-bill = 200000 + 150000 = 350000

3. If $R_A'(z) = 0$ $\frac{da}{dw} = 0$

Stock = 800000

T-bill = 200000 + 200000 = 400000

$\frac{da}{dw_0}$ take the second

Start with the F.O.C

$E[u'(\tilde{w})(\tilde{r}-r_f)]=0$

$\tilde{w}=a(1+\tilde{r})+(w_0-a)(1+r_f)$

$=w_0(1+r_f)+a(\tilde{r}-r_f)$

using the implicit function theorem

$$\frac{da}{dw_0}=(-)\frac{\partial[\ ]/\partial w_0}{\partial[\ ]/\partial a}=(-)\frac{E[u''(\tilde{w})(1+r_f)(\tilde{r}-r_f)]}{E[u''(\tilde{w})(\tilde{r}-r_f)^2]}$$

Sign of $\frac{da}{dw_0}$ $\Leftrightarrow$ sign $E[u''(\tilde{w})(\tilde{r}-r_f)]$

combination

If $\tilde{r}\geq r_f$ we will have $\tilde{w}\geq w_0(1+r_f)$

$\Rightarrow R_A(\tilde{w})\leq R_A(w_0(1+r_f))$

If $\tilde{r}<r_f$ we will have $\tilde{w}<w_0(1+r_f)$

$\Rightarrow R_A(\tilde{w})>R_A(w_0(1+r_f))$

multiply both sides by $(-)u'(\tilde{w})(\tilde{r}-r_f)$

If $\tilde{r}\geq r_f$ $u''(\tilde{w})(\tilde{r}-r_f)\geq R_A(w_0(1+r_f))u'(\tilde{w})(\tilde{r}-r_f)$

If $\tilde{r}<r_f$ $u''(\tilde{w})(\tilde{r}-r_f)>R_A(w_0(1+r_f))u'(\tilde{w})(\tilde{r}-r_f)$

Combining:

$$E[u''(\tilde{w})(\tilde{r}-r_f)]>R_A(w_0(1+r_f))E[u'(\tilde{w})(\tilde{r}-r_f)]$$

$\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad$ finite $\quad\quad\quad\quad$ 0

$E[u''(\tilde{w})(\tilde{r}-r_f)]>0$

If $R_R'(z) > 0$ increasing relative risk aversion.

< 0 decreasing

$= 0$ constant

$\Rightarrow$ the rescaling by wealth $\Rightarrow$ percentage

① Stock 800000 +160000

T-bill 200000 +40000 $R_R'(z) = 0$

$\Delta W = 200000$

② $\Delta S = 120000$

$\Delta T = 80000$ $R_R'(z) > 0$

③ $\Delta T = 180000$

$\Delta T = 20000$ $R_R'(z) < 0$

Spirit of capitalizim.

For most people: $R_R'(z) = 0$. $R_A'(z) < 0$

Power utility function:

$$u(w) = \begin{cases} \dfrac{w^{1-\gamma} - 1}{1-\gamma} & \gamma \neq 1 \\ \ln(w) & \gamma = 1 \end{cases}$$

$R_R = (-)\frac{u''}{u'} \cdot W =$

$u' = W^{-\gamma} > 0$

$u'' = (-\gamma) w^{-\gamma-1} < 0 \quad \text{if } \gamma > 0$

$u' = \frac{1}{W} > 0$

$u'' = \frac{-1}{W^2} < 0$

$R_R = (-)\frac{u''}{u'} w$

$= \frac{(-)(-\gamma) w^{-\gamma-1} w}{w^{-\gamma}}$ $\Rightarrow$ constant

$= \gamma$

$R_R = (-)\frac{u''}{u'} w = \frac{(-)\frac{1}{w}}{\frac{-1}{w^2}} w = 1$

bigger γ, bigger risk reverse

Very risk averse $\Rightarrow$ $\gamma > 10$

Negative exponential utility function

$u(w) = -\exp(-\delta w^2), \ \delta > 0$

$u' = \delta(\exp)\exp(-\delta\tilde{w})$

$u'' = -\delta^2 \exp(-\delta\tilde{w}) < 0$

$R_A = (-1)\frac{u''}{u'} = \delta$

Week 4

$R_A = (-)\frac{u''}{u'}$. R_R.

Suppose we have two investors i and k.

$R_A^i(z) \geq R_A^k(z) \quad \forall z$

Then investor i will require a higher risk premium to fully invest money in stock than investor k.

Start with the optimal choice of k, for a given premium $E[\tilde{r} - r_f]$. He puts all money in stock.

$$E[u_k'(W_0(1+\tilde{r}))(\tilde{r} - r_f)] = 0$$

Need to show for i

$E[u_i'(W_0(1+\tilde{r}))(\tilde{r} - r_f)] \leq 0$

Lemma: The following are equivalent:

(1) $R_A^i(z) \geq R_A^k(z) \quad \forall z$.

(2) There exists an increasing and concave function G. Such that $u_i = G(u_k)$

$u_i'(W_0(1+\tilde{r})) = G'(u_k(W_0(1+\tilde{r}))) \cdot u_k'(W_0(1+\tilde{r}))$

If $\tilde{r} - r_f \geq 0$, $W_0(1+\tilde{r}) \geq W_0(1+r_f)$

If $\tilde{r} - r_f \geq 0$, $W_0(1+\tilde{r}) \geq W_0(1+r_f)$

$G'(u_k(W_0(1+\tilde{r}))u_k'(W_0(1+\tilde{r})(\tilde{r} - r_f) \leq$

$G'(u_k(W_0(1+r_f))u_k'(W_0(1+\tilde{r}))(\tilde{r} - r_f)$

If $\tilde{r} - r_f < 0$, $W_0(1+\tilde{r}) < W_0(1+r_f)$

The same equality hold.

Then $E[G'(u_k(W_0(1+\tilde{r})) \cdot u_k'(W_0(1+\tilde{r}))(\tilde{r}-r_f)]$
$\leq G'(u_k(W_0(1+r_f))E[u_k'(W_0(1+\tilde{r}))(\tilde{r}-r_f)]$

Stochastic Dominance

we say $A \underset{FSD}{\geq} B$

if all nonsatiated investors will prefer A to B. Equivalent to $E[u(A)] \geq E[u(B)]$ $\forall u$ increasing

we can show the following are equivalent

(i) $A \geq_{FSD} B$

(ii) $F_A(z) \leq F_B(z)$ $\forall z \in [0,1]$

$P\{\tilde{r}_A < z\} \leq P\{\tilde{r}_B < z\}$ $\forall z$

$P\{\tilde{r}_A \geq z\} \geq P\{\tilde{r}_B \geq z\}$ $\forall z$

(iii) $\tilde{r}_A \stackrel{d}{=} \tilde{r}_B + \tilde{\alpha}$, $\tilde{\alpha} \geq 0$

iii $\Rightarrow$ $E(\tilde{r}_A) = E(\tilde{r}_B) + E[\tilde{\alpha}] \geq E(\tilde{r}_B)$

necessary condition

We say $A \underset{SSD}{\geq} B$ if all risk-averse investors will prefer A to B. Equivalent to $E[u(A)]$ $\geq E[u(B)]$ $\forall u$ concave

we can show the following are equivalent

(i) $A \succeq_{SSD} B$

(ii) $\int_0^y F_A(z)\,dz \le \int_0^y F_B(z)\,dz \quad \forall y \in [0,1]$

(iii) $\tilde{r}_B \overset{d}{=} \tilde{r}_A + \tilde{\varepsilon} \quad E[\tilde{\varepsilon} \mid \tilde{r}_A] = 0$

fair game property

$\tilde{r}$ and $\tilde{\varepsilon}$ are independent

$\not\Uparrow$

$\Rightarrow E(\varepsilon^2 \mid r_A^2) = 0$

$\not\Uparrow$

$\Rightarrow Cov(\varepsilon^2, r_A^2) = 0$

$E(\tilde{\varepsilon}) = E(E[\tilde{\varepsilon} \mid \tilde{r}_A])$

law of Interative expectation

$E[\tilde{r}_B] = E[\tilde{r}_A] + E[\tilde{\varepsilon}] = E[\tilde{r}_A]$

$Var[\tilde{r}_B] = Var(\tilde{r}_A) + Var(\tilde{\varepsilon}) \geq Var(\tilde{r}_A)$

Week 5

(1) $A \succsim_{SSD} B$

(2) $\int F_A(z) \leq \int F_B(z) \quad \forall z \in [0, N]$

(3) $\tilde{r}_B = \tilde{r}_A + \tilde{\varepsilon} \quad E[\tilde{\varepsilon} | \tilde{r}_A] = 0$

For a concave utility function

$E[u(1+\tilde{r}_B)] \leq E[u(1+\tilde{r}_A)]$ to show

$E[f(x)] \leq f(E[x]) \rightarrow$ Jensen. for f concave

$E[u(1+\tilde{r}_B) | \tilde{r}_A] \leq u(E(1+\tilde{r}_B | \tilde{r}_A)) = u(E(1+\tilde{r}_A + \tilde{\varepsilon} | \tilde{r}_A))$
$= u(1+\tilde{r}_A)$

Take expectation on both sides.

$E[u(1+\tilde{r}_B)] = E[u(1+\tilde{r}_A)]$

$u' > 0, \quad u'' > 0, \quad u''' > 0$ decreasing absolute risk aversion.

$A \succsim_{TSD} B$ skew preference

$\iint F_A(z) \leq \iint F_B(z) \quad \forall z \in [0, N]$

4th Degree.

Time Varying volatility: $\varepsilon_t \sim N(0, \sigma_t^2)$

Stochastic Volatility $\Rightarrow$ variance of variance

$\sigma_t^2 = \sigma_0^2 + V_t$

Taylor expand around $E(\tilde{w})$

$$u(\tilde{w}) = u(E(\tilde{w})) + u'(E(\tilde{w}))(\tilde{w} - E(\tilde{w})) + \frac{1}{2}u''(E(\tilde{w}))[\tilde{w} - E(\tilde{w})]^2 + R_3$$

$$E[u(\tilde{w})] = u(E(\tilde{w})) + \frac{1}{2}u''(E(\tilde{w}))\sigma_w^2 + E(R_3)$$

☆ Sufficient Conditions for mean-variance analysis to be consistent with utility maximization.

(1) Quadratic utility → higher derivative disapper

$u(w) = A + BW + CW^2$

$u'(w) = B + CW > 0$ $\quad w < -\frac{B}{2C}$ $\quad B > 0$

$u''(w) = 2C < 0$ $\quad C < 0$

(2) Normally distributed

N Risky stocks. →higher moment disappear

$\tilde{r} = [\,]$. $E[\tilde{r}] = \sim$

$$\min \frac{1}{2} w^T V w$$

$$\text{s.t. } w^T 1 = 1$$

$$w^T e = E(\tilde{r}_p)$$

$$\mathcal{L} = \frac{1}{2} w^T V w + \lambda(w^T 1 - 1) + \gamma(w^T e - E(r_p))$$

$$\frac{\partial L}{\partial w} = 0 \quad Vw - \lambda e - \gamma 1 = 0$$

$$\frac{\partial L}{\partial \lambda} = 0 \quad E[\tilde{r}_p] - w^T e = 0$$

$$\frac{\partial L}{\partial \gamma} = 0 \quad 1 - w^T 1 = 0$$

$$W_p = g + h E(\tilde{r}_p)$$

efficient frontier

Two fund theorem.

choose λ : $W_p = \lambda W_1 + (1-\lambda) W_2$

$$E[\tilde{r}_q] = \lambda E[\tilde{r}_1] + (1-\lambda) E(\tilde{r}_2)$$

$$W_p = \lambda W_1 + (1-\lambda) W_2$$

$$= g + h[\lambda E(\tilde{r}_1) + (1-\lambda) E(\tilde{r}_2)]$$

$$= g + h E[r_q]$$

Any two distinct frontier portfolios can generate the entire portfolio frontier. Let q be an arbitrary frontier portfolio.

The covariance of any portfolio and the mvp is equal to the variance of MVP.

Let q be any portfolio

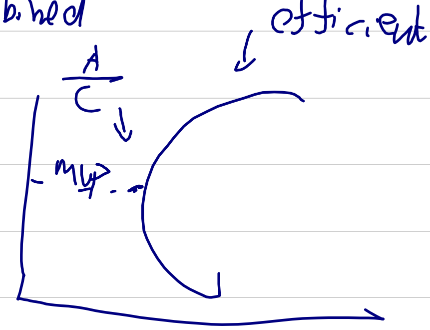
$$\tilde{r}_p = \lambda \tilde{r}_q + (1-\lambda) \tilde{r}_{mvp}$$

$$\sigma_p^2 = \lambda^2 \sigma_q^2 + (1-\lambda^2) \sigma_{mvp}^2 + 2\lambda(1-\lambda) \sigma_{q,mvp}$$

$$\frac{\partial \sigma_p^2}{d\lambda} = 0 \quad \text{F.O.C.}$$

Also we know that $\lambda = 0$. will gives us the optimal weight. → combined

$$\sigma_{q,mvp} = \sigma_{mvp}^2 = \frac{1}{c}$$



Any combinaton of frontier portfolios is a frointer portfolio

$$w_p = \sum_{i=1}^{K} \lambda_i w_i \qquad \sum_{i=1}^{K} \lambda_i = 1$$

including $\lambda_i < 0$

$$= \sum_{i=1}^{K} \lambda_i \left(g + h E(\hat{r}_i)\right)$$

$$= g + h \left(\sum_{i=1}^{K} \lambda_i E(\hat{r}_i)\right)$$

$$= g + h E(\hat{p})$$

Any convex combination of the Efficient frointor is efficient portfolio.

$$0 \leq \lambda_i \leq 1 \qquad \sum \lambda_i = 1$$

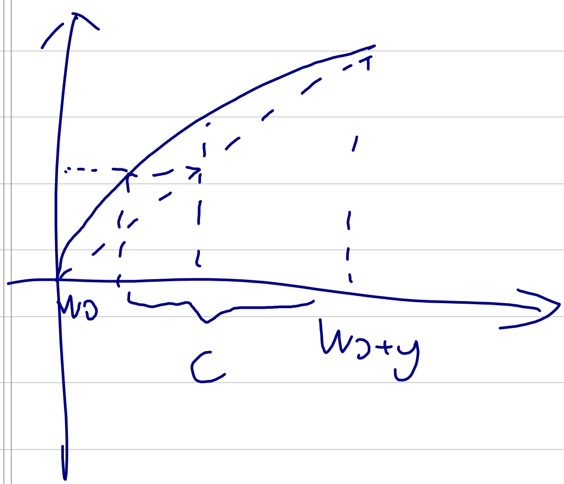
Need to $E(\hat{r}_p) > \frac{A}{C}$

$$= \sum_{r=1}^{K} \lambda_i E(r_i) > \frac{A}{C}$$

Week 6

1.

$E(u)$



w_0

$\begin{cases} y & p \\ 0 & 1-p \end{cases}$

$u[w_0+y-c] = E[u]$

$= p\,u[w_0+y] + (1-p)u(w_0)$

$u(w) = \dfrac{w^{1-\gamma}-1}{1-\gamma}$ $\quad \gamma \neq 1$

$= \ln w$ $\quad \gamma = 1$

when $\gamma = 2$

$u(w) = -\dfrac{1}{w}$

$p\left(\dfrac{-1}{w_0+y}\right) + (1-p)\dfrac{-1}{w_0} = \dfrac{-1}{w_0+y+c}$

$w_0 = \dfrac{(y-c)(1-p)y}{c-(1-p)y}$

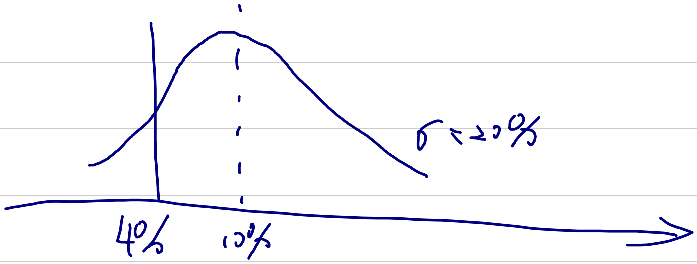
Since $u(w)$ is constant R_R,

Then consider the margain utility

Decreasing R_A

The risk premium exceed

[2]



Consider. $u(z) = -e^{(-z)}$

$$\tilde{r}_A = \begin{cases} 1, & p = \frac{1}{10} \\ 0 & 1-p = \frac{9}{10} \end{cases}$$

$E(\tilde{r}_A) = \frac{1}{10} = 0.1$

$\tilde{r}_B = 0.09 \qquad E(\tilde{r}_A) > E[\tilde{r}_B)$

$E[u(\tilde{r}_A)] = -e^{-0.1} \times \frac{1}{10} + \frac{9}{10} \times -e^{-0}$

$= -0.94$

$E[u(\tilde{r}_B)] = -e^{-0.09} = -0.91$

What if $\tilde{r}_B = 0.04 \Rightarrow$ make $u(z) = -e^{(-10z)}$

[3]

3rd moment of normal distribution $= 0$

Consider

$$\tilde{r}_A = \begin{cases} 2 & p = \frac{1}{5} \\ 12 & (1-p) = \frac{4}{5} \end{cases} \qquad \tilde{r}_B = \begin{cases} 22 & p = \frac{1}{5} \\ 7 & 1-p = \frac{4}{5} \end{cases}$$

$E(\tilde{r}_A) = E(\tilde{r}_B) = 10 \qquad Var(\tilde{r}_A) = 16 \qquad Var(\tilde{r}_B) = 36$

$E[u(\tilde{r}_A)] = \frac{1}{5}\ln(2) + \frac{4}{5}\ln(12) = 2.13 \qquad E[u(\tilde{r}_B)] > E[u(\tilde{r}_A)]$

$E[u(\tilde{r}_B)] = \frac{1}{5}\ln(22) + \frac{4}{5}\ln(7) = 2.17$

Lottery Type tail $\rightarrow$ preference of skewness

$\boxed{4}$ EWP $\geq^{SSD}$ other portfolio

We can show that the EWP is the minimum variance portfolio

To show sufficient condition, need to show for any portfolio P,

$$\tilde{r}_p \overset{d}{=} \tilde{r}_{EWP} + \tilde{\varepsilon}, \quad E(\tilde{\varepsilon} \mid \tilde{r}_{EWP}) = 0$$

WLOG, assume $n=2$

For any portfolio p

$$\tilde{r}_p = \lambda \tilde{r}_1 + (1-\lambda)\tilde{r}_2$$

$$\tilde{r}_{EWP} = \frac{1}{2}\tilde{r}_1 + \frac{1}{2}\tilde{r}_2$$

Let ε be distributed as $\tilde{\varepsilon} \overset{d}{=} \tilde{r}_p - \tilde{r}_{EWP}$

By the iid assumption, we have

$$E[\tilde{r}_1 \mid \tilde{r}_{EWP}] = E[\tilde{r}_2 \mid \tilde{r}_{EWP}]$$

$$E[\tilde{r}_1 \mid \tilde{r}_{EWP}] + E[\tilde{r}_2 \mid \tilde{r}_{EWP}] = E[\tilde{r}_1 + \tilde{r}_2 \mid \tilde{r}_{EWP}] = \tilde{r}_1 + \tilde{r}_2$$

$$E[\tilde{r}_1 \mid \tilde{r}_{EWP}] = E(\tilde{r}_2 \mid \tilde{r}_{EWP}) = \frac{\tilde{r}_1 + \tilde{r}_2}{2}$$

$$\begin{aligned}
E[\tilde{\varepsilon} \mid \tilde{r}_{EWP}] &= E[(\lambda \tilde{r}_1 + (1-\lambda)\tilde{r}_2) - \tilde{r}_{EWP}) \mid \tilde{r}_{EWP}] \\
&= \lambda \frac{\tilde{r}_1 + \tilde{r}_2}{2} + (1-\lambda)\frac{\tilde{r}_1 + \tilde{r}_2}{2} - \tilde{r}_{EWP} \\
&= \frac{\tilde{r}_1 + \tilde{r}_2}{2} - \tilde{r}_{EWP} \\
&= 0
\end{aligned}$$

iid $\rightarrow$ key

$\boxed{4}$ will still hold if we have $N(\mu, \sigma_i^2)$. Then we can have $mvp \overset{SSD}{\succcurlyeq} \forall p$

$\boxed{5}$ $R_A^i(z) \geq R_A^k(z)$

$E(\tilde{r}) - r_f > 0$

Given that investor i is willing to invest all in stock, i.e. $E[u_i'(w_0(1+\tilde{r}))(\tilde{r} - r_f)] = 0$

We need to show

$$E[u_k'(w_0(1+\tilde{r}))(\tilde{r} - r_f)] \geq 0$$

We can find a convex and increasing function such that $u_k(z) = G(u_i(z))$

$$u_k'(w_0(1+\tilde{r}))(\tilde{r} - r_f) = G'(u_i(w_0(1+\tilde{r})))u_i'(w_0(1+\tilde{r}))(\tilde{r} - r_f)$$

When $\tilde{r} - r_f \geq 0$, $w_0(1+\tilde{r}) \geq w_0(1+r_f)$

higher increasing rate $G'(u_i(w_0(1+\tilde{r})))u_i'(w_0(1+\tilde{r}))(\tilde{r} - r_f) \geq$
$G'(u_i(w_0(1+r_f)))u_i'(w_0(1+\tilde{r}))(\tilde{r} - r_f)$

When $\tilde{r} - r_f \leq 0$, the same inequality hold.

$\Rightarrow$ then take E

$$E[u_k'(w_0(1+\tilde{r}))(\tilde{r} - r_f)] \geq 0$$

$\boxed{6}$ (1) favorable of higher return

(2) hard to say, maybe depend on risk attitude

limited → take risk

unlimited → more cautious.