FM1 Notes, Spring 2020
Asset Pricing Test

I. The CAPM says that:

$$E(\tilde{r}_i) - r_f = \beta_i [E(\tilde{r}_m) - r_f]$$

1. Testable implications

(1) The relationship between expected return and beta is linear.

(2) Beta is a complete measure of risk of security *i* in the efficient portfolio *m*. No other measure of risk in security *i* should appear in the above equation.

(3) The equity risk premium is positive, i.e., $[E(\tilde{r}_m) - r_f] > 0$.

2. Problems with testing the CAPM

(1) Expected returns are not directly observed. By the assumption of rational expectations, we can use the *ex post* (realized) returns to proxy for the *ex ante* returns.

(2) Errors-in-variable problems: the beta is not observed. How to deal with that?

(3) Market portfolio is not observed: the Roll critique.

(4) Conditioning information.

II. How to test the unconditional CAPM

1. Time-series approach (Black, Jensen and Scholes approach)

(1) Estimate each stock's beta.
(2) Group stocks into *m* portfolios (10 in their case) based on beta. Get time series of returns for the portfolios.
(3) For each portfolio *i*, run *time-series* regression

$$r_{i,t} - r_{f,t} = \alpha_i + \beta_i (r_{m,t} - r_{f,t}) + \varepsilon_{i,t}$$

using observations from $t = 1, 2, \ldots, T$.

Then test $H_0 : \alpha_1 = \alpha_2 = \ldots = \alpha_m = 0$.

They concluded that H_0 cannot be rejected.

2. Cross-section approach (Fama and MacBeth two-stage procedure)

(1) Estimate beta for each stock.
(2) Sort stocks into 20 portfolios based on betas. Then estimate the portfolio betas.
(3) For each month *t*, run *cross-section* regression as follows

$$r_{i,t} - r_{f,t} = \gamma_{0,t} + \gamma_{1,t}\beta_i + \gamma_{2,t}\beta_i^2 + \gamma_{3,t}s_i + \eta_{i,t}$$

using observations from $i = 1, 2, \ldots, m$. ($m = 20$ in their case)

(4) Run steps (1)-(3) for each month *t*. Then test the following hypothesis:

(i) $\gamma_0 = 0$
(ii) $\gamma_1 = E(\tilde{r}_m - r_f)$
(iii) $\gamma_2 = 0$
(iv) $\gamma_3 = 0$

Construct 20 portfolios from all N stocks.

Portfolio formation period: 7 years.
Beta estimation period: 5 years.
Testing period: 4 years.

Do these for non-overlapping intervals.

Computer *t*-statistics based on the time series of the parameter estimates.

Results: Testing for CAPM

	estimate	t-stat	estimate	t-stat	estimate	t-stat	estimate	t-stat
γ_0	0.0048	**2.55**	0.0036	1.42	0.0041	**1.59**	0.0008	0.20
γ_1	0.0085	**2.57**	0.0105	**1.79**	0.0072	**2.20**	0.0114	**1.85**
γ_2			-0.0008	-0.029			-0.0026	-0.86
γ_3					0.0198	0.46	0.0516	1.11

Conclusions

Results support important implications of the two-parameter model.

(1) There is a trade off between return and risk.
(2) The linearity assumption cannot be rejected.
(3) Cannot reject the hypothesis that market portfolio is the only systematic risk for stock returns.
(4) The Sharpe-Lintner hypothesis that the coefficient is equal to the risk-free rate is rejected.

III. Challenge to the single-factor model

Fama and French use more recent data.

1. Methodology

(1) First estimate portfolio betas and assign a portfolio's beta to each stock in the portfolio. This allows them to use individual stocks in the Fama-MacBeth tests.

(2) For each month, run the cross-section regression of return on variables (beta, size, BE/ME, E/P, etc.).

(3) The time series means of the slope coefficients provide standard test statistics.

2. Main results

β	ln(ME)	ln(BE/ME)	ln(A/ME)	ln(A/BE)	E/P Dummy	E(+)/P
0.15 (0.46)						
	−0.15 (−2 58)					
−0.37 (−1.21)	−0.17 (−3.41)					
		0.50 (5.71)				
			0.50 (5 69)	−0.57 (−5.34)		
					0.57 (2.28)	4.72 (4.57)
	−0.11 (−1.99)	0.35 (4.44)				
	−0 11 (−2 06)		0.35 (4.32)	−0 50 (−4.56)		
	−0.16 (−3.06)				0.06 (0 38)	2.99 (3.04)
	−0.13 (−2.47)	0.33 (4.46)			−0.14 (−0.90)	0.87 (1.23)
	−0.13 (−2.47)		0.32 (4.28)	−0.46 (−4.45)	−0.08 (−0.56)	1.15 (1.57)

3. Summary

(1) Earlier research found a positive relationship between expected return and beta. Fama and French find weak relation in the more recent data.

(2) Univariate relations between expected return and size, leverage, E/P, and BE/ME are strong.

(3) In multivariate tests, size is robust to the inclusion of other variables.

(4) BE/ME has a stronger role than size.

(5) The combination of size and BE/ME seems to absorb the roles of leverage and E/P. The results seem to suggest that stock risks are multidimensional. One dimension of risk is proxied by size and another dimension is proxied by BE/ME. It is possible that the risk captured by BE/ME is the relative stress factor.