Rutgers Business School--Newark & New Brunswick
MQF 22:839:571, Financial Modeling I, Spring 2020
Assignment II
Assigned: 2/12/20, not to be graded

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1. An expected utility maximizing individual has utility of end-of-period wealth given by

$$u(W)=\begin{cases}\dfrac{W^{1-\gamma}-1}{1-\gamma}, \text{ if } \gamma \neq 1\\ \ln(W), \text{ if } \gamma = 1\end{cases}$$

(1) Show that this individual has constant relative risk aversion and decreasing absolute risk aversion.

(2) Consider the special case where $\gamma = 2$. Suppose that this individual is endowed with an initial wealth W_0 but his end-of-period wealth is subject to a random income shock given as follows:

$$\begin{cases}\$y, \text{ with probability p}\\ \$0, \text{ with probability 1-p}\end{cases}$$

where 0<p<1.

He can purchase an insurance at a cost of \$c to remove the risk of receiving no income. At what level of initial wealth will he be indifferent between taking on the risk of getting no income and buying the insurance that removes this risk?

2. Show that if A first degree stochastically dominates B, i.e., $A \geq^{FSD} B$, then $E(\tilde{r}_A) \geq E(\tilde{r}_B)$. However, the converse is not necessarily true.

3. Show that if Stock A second degree stochastically dominates Stock B, i.e., $A \geq^{SSD} B$, then $\text{var}(\tilde{r}_B) \geq \text{var}(\tilde{r}_A)$. However, the converse is not necessarily true.

4. Suppose that there are n risky stocks with rates of return, $\tilde{r}_1, \tilde{r}_2, ..., \tilde{r}_n$, which are independent and identically distributed, but not necessarily normally distributed. Show that any risk-averse investor will always choose the equally weighted portfolio.

5. Assume that there are two risk-averse individuals, i and k, with $R_A^i(z) \geq R_A^k(z), \forall\ z$. There is a risky asset and a risk-free asset. Assume that at the risk premium, $rp = E(\tilde{r}) - r_f > 0$, individual *i* invests all his wealth in the risky asset. Show that at this risk premium, individual *k* may either invest all his wealth in the risky asset or even take a leveraged position, i.e., he may invest more than his wealth at the risky asset by borrowing at the risk-free rate.

6. Consider the following two risky projects for a firm:

	project 1	
probability		cash flow
0.2		4,000
0.6		5,000
0.2		6,000

	project 2	
probability		cash flow
0.4		0
0.2		5,000
0.4		10,000

(1) The firm has a fixed debt payment of $6,000 and limited liability. Which project is more favorable to shareholders and why?

(2) Suppose now that the firm has a fixed debt payment of $6,000 and unlimited liability. Which project is more favorable to shareholders and why?

7. Assume that returns of individual stocks are multivariate normally distributed and there are two stock portfolios, A and B. In each of the following cases, compare portfolios A and B using both first degree stochastic dominance and second degree stochastic dominance:

Case 1	*Case* 2	*Case* 3
$\sigma_A > \sigma_B$	$\sigma_A = \sigma_B$	$\sigma_A > \sigma_B$
$\mu_A = \mu_B$	$\mu_A > \mu_B$	$\mu_A > \mu_B$

Suppose that stock returns are not normally distributed. Do your answers above still hold?