FM1 Notes, Spring 2020

Interest Rate Parity and Carry Trade

I. International arbitrage

1. Locational arbitrage

Example: Consider the BP

HSBC		Chase	
Bid	Ask	Bid	Ask
\$1.59	\$1.60	\$1.61	\$1.62

You can buy BP from HSBC at \$1.60, and sell BP to Chase at \$1.61
Make a profit =\$0.01/BP

Net investment =0
Risk=0
Profit >0

This is an arbitrage opportunity. Eventually, the price in HSBC will be driven up until its ask price is above Chase's bid price.

2. Triangular arbitrage

Let S_1=\$1.60/BP, S_2=\$1.30/€, then the implied S_3=1.60/1.30=€1.23/BP
Suppose that the current market cross rate is S_3 = €1.25/BP
Strategy: invest \$1.60 to buy BP1, sell for €1.25, use €1.25 to buy 1.25*1.30 =\$1.63
Profit = \$1.63 - \$1.60 = \$0.03

Arbitrage will drive up the BP price in terms of \$ until profit is zero, so that $S_3=S_1/S_2$.

3. Covered interest arbitrage

(1) Suppose that you have \$1 to invest. Should you invest in the dollar denominated security or in the pound denominated security? Let the time be 1 year for simplicity.

Return from investing in the dollar denominated security

$\$1\ [1 + r_t]$ This is for sure

Return from investing in the pound denominated security

$\$1/S_t [1 + r_t^*] S_{t+1}$ This is uncertain.

You can sell a 1-year forward contract to have a complete hedge, so that

$\$1/S_t [1 + r_t^*] F_{t,1}$ This is for sure.

If $(1+ r_t) > \$1/S_t [1+ r_t^*] F_{t,1}$ you invest in dollar denominated security
If $(1+ r_t) < \$1/S_t [1+ r_t^*] F_{t,1}$ you invest in pound denominated security

(2) Borrowing

Suppose that you borrow \$1 and have to pay back in 1 year. What is the cost?

If borrow in US: $\$1[1+ r_t]$
If borrow in UK: $\$1/S_t [1+ r_t^*] F_{t,1}$, where you buy a forward contract to hedge your position

If $(1+ r_t) > \$1/S_t [1+ r_t^*] F_{t,1}$ you borrow in UK
If $(1+ r_t) < \$1/S_t [1+ r_t^*] F_{t,1}$ you borrow in US

(3) Borrowing and investing for an arbitrage profit

If $(1+ r_t) > \$1/S_t [1+ r_t^*] F_{t,1}$ you borrow in UK and use the proceeds to invest in US to get

Profit = $(1+ r_t) - \$1/S_t [1+ r_t^*] F_{t,1} > 0$

and vice versa.

Arbitrage will continue until

$$(1+ r_t) = \$1/S_t [1+ r_t^*] F_{t,1}$$

Taking log, we have

$$i_t - i_t^* = \ln(F_t) - \ln(S_t) = f_t - s_t$$

where $i_t = \ln(1+ r_t)$, $i_t^* = \ln(1+ i_t^*)$, are continuously compounded interest rates, and $f_t = \ln(F_t)$, $s_t = \ln(S_t)$.

The above equation says that interest differential = forward exchange rate premium.

This is called the *covered interest rate parity* (CIP).

II. Uncovered interest rate parity

If investors are risk neutral, then we should have $F_{t,1} = E_t S_{t+1}$

Invest in US, sure return $[1+ r_t]$
Invest in UK, expected return $1/S_t [1+ r_t^*] E_t S_{t+1}$

If you are risk neutral, you do not care whether it is an expected return or a sure return. Hence, we should have

$$[1+ r_t] = 1/S_t [1+ r_t^*] E_t S_{t+1}$$

Using log approximation

$$i_t - i_t^* = E_t(\ln S_{t+1}) - \ln(S_t)$$

$$i_t - i_t^* = E_t(s_{t+1}) - s_t$$

This is the *uncovered interest rate parity* (UIP).

The UIP says interest rate differential should equal expected currency depreciation rate.

III. Real interest rate parity

Purchasing Power Parity (PPP) in expectations form

$$E_t(\ln P_{t+1}^{US} - \ln P_t^{US}) - E_t(\ln P_{t+1}^{UK} - \ln P_t^{UK}) = E_t(\ln S_{t+1} - \ln S_t)$$

The PPP says that expected inflation differential should equal expected currency depreciation rate.

Substituting into the UIP above, we obtain

$$E_t(\ln P_{t+1}^{US} - \ln P_t^{US}) - E_t(\ln P_{t+1}^{UK} - \ln P_t^{UK}) = i_t - i_t^*$$

Notice that the left hand side is the inflation differential between the US and the UK. Rearranging yields,

$$i_t - E_t(\ln P_{t+1}^{US} - \ln P_t^{US}) = i_t^* - E_t(\ln P_{t+1}^{UK} - \ln P_t^{UK})$$

$$i_t - E_t(\pi_{t+1}^{US}) = i_t^* - E_t(\pi_{t+1}^{UK})$$

or

$$\rho_t = \rho_t^*$$

where π -- inflation rate
ρ -- real interest rate

The above condition is called the *real interest rate parity*.