Yifu_He_Financial_Modeling_HW1

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1 Yifu_He_190003956_Homework_1

In [1]:
```
# import necessary packages
import numpy as np
import pandas as pd
from datetime import datetime
import matplotlib.pyplot as plt
%matplotlib inline
import seaborn as sns
```

In [2]:
```
# define the function to calculate statistics
def mean_(arr):
    """
    Parameters
    ----------
    arr: np.array
        array of data

    Returns
    ----------
    res: float
        mean - first moment
    """
    res = sum(arr)/len(arr)
    return res

def std_deviation_(arr):
    """
    Parameters
    ----------
    arr: np.array
        array of data

    Returns
    ----------
    res: float
         standard deviation - second moment
```

```
    """
    x_head = mean_(arr)
    res = np.sqrt(sum( pow(arr-x_head,2))/ (len(arr) - 1))
    return res

def skewness_(arr):
    """
    Parameters
    ----------
    arr: np.array
        array of data

    Returns
    ----------
    res: float
          skewness - third moment
    """
    x_head = mean_(arr)
    sigma = std_deviation_(arr)
    mu_3 = sum( pow(arr - x_head,3)) / (len(arr)-1)
    res = mu_3/pow(sigma,3)
    return res

def kurtosis_(arr):
    """
    Parameters
    ----------
    arr: np.array
        array of data

    Returns
    ----------
    res: float
          kurtosis - fourth moment
    """
    x_head = mean_(arr)
    sigma = std_deviation_(arr)
    mu_4 = sum( pow(arr - x_head,4)) / (len(arr)-1)
    res = mu_4/pow(sigma,4)
    return res

def JB_test_(arr):
    """
    Parameters
    ----------
    arr: np.array
        array of data
```

```
    Returns
    ----------
    res: float
          Jarque-Bera statistic
    """
    T = len(arr)
    res = pow(skewness_(arr),2) * T / 6 + pow(kurtosis_(arr)-3,2) * T / 24
    return res
def Cov_(arr1,arr2):
    """
    Parameters
    ----------
    arr1: np.array
        array of data
    arr2: np.array
        array of data

    Returns
    ----------
    res: float
          Covariance
    """
    x_head = mean_(arr1)
    y_head = mean_(arr2)
    res = 0
    for i in range(len(arr1)):
        res +=(arr1[i] - x_head)(arr2[i] - y_head)
    return res/(len(arr1)-1)
```

In [3]:
```
def convert_type(x):
    """
    Parameters
    ----------
    x: np.int64
        date in type of int64

    Returns
    ----------
    res: datetime.datetime
          date in type of datetime.datetime
    """
    res = str(x)[:7]
    year = int(res[:4])
    month = int(res[4:6])+1
    if month > 12:
        month = 1
        year +=1
    res = datetime(year,month,1,0,0)
```

```
    return res

# set begin date and end date
begin_date = datetime(1998,12,1,0,0)
end_date = datetime(2018,12,1,0,0)

# import and preprocessing oil, gold and house data
oil = pd.read_excel("/Users/yifuhe/Learning/RBS/fin_model/homework1/OilPrice_Jan2020.xl
gold = pd.read_excel("/Users/yifuhe/Learning/RBS/fin_model/homework1/GoldPriceData_Jan2
house = pd.read_excel("/Users/yifuhe/Learning/RBS/fin_model/homework1/HousingData_Jan20
CRSP = pd.read_csv("/Users/yifuhe/Learning/RBS/fin_model/homework1/Marker_value_weighte
Tbill_cpi = pd.read_csv("/Users/yifuhe/Learning/RBS/fin_model/homework1/tbill_cpi.csv")


oil = oil.iloc[10:,:]
oil.columns = ["date","oil_p"]
index = oil[(oil.date == begin_date) | (oil.date == end_date)].index.tolist()
oil = oil.loc[index[0]:index[1]].reset_index().drop("index",axis=1)

gold = gold.iloc[10:,:]
gold.columns = ["date","gold_p"]
index = gold[(gold.date == begin_date) | (gold.date == end_date)].index.tolist()
gold = gold.loc[index[0]:index[1]].reset_index().drop("index",axis=1)

house = house.iloc[10:,:]
house.columns = ["date","house_p"]
index = house[(house.date == begin_date) | (house.date == end_date)].index.tolist()
house = house.loc[index[0]:index[1]].reset_index().drop("index",axis=1)

CRSP["date"] = CRSP["caldt"].apply(convert_type)
CRSP = CRSP.loc[:,["date","vwretd","vwindd"]]
index = CRSP[(CRSP.date == begin_date) | (CRSP.date == end_date)].index.tolist()
CRSP = CRSP.loc[index[0]:index[1]].reset_index().drop(["index","date"],axis=1)

Tbill_cpi["date"] = Tbill_cpi["caldt"].apply(convert_type)
Tbill_cpi = Tbill_cpi.loc[:,["date","t90ret","cpiret","t90ind","cpiind"]]
index = Tbill_cpi[(Tbill_cpi.date == begin_date) | (Tbill_cpi.date == end_date)].index
Tbill_cpi = Tbill_cpi.loc[index[0]:index[1]].reset_index().drop(["index","date"],axis=

# generate dataset
data = pd.merge(oil,gold,on = ["date"])
data = data.merge(house)
data = pd.concat([data,CRSP],axis =1)
data = pd.concat([data,Tbill_cpi],axis =1)
# calculate returns based on Rt = (Pt - Pt-1)/Pt-1
data["oil_r"] = data["oil_p"].diff().shift(-1)/data["oil_p"]
data["gold_r"] = data["gold_p"].diff().shift(-1)/data["gold_p"]
data["house_r"] = data["house_p"].diff().shift(-1)/data["house_p"]
```

```
#data["vwretd"] = data["vwretd"].diff()/data["vwretd"]


# drop "NA"
data = data.dropna().reset_index().drop("index",axis =1)
data = data.drop(["oil_p","gold_p","house_p"],axis=1)

data.head()
```

Out[3]:

```
                 date    vwretd    vwindd    t90ret    cpiret    t90ind  \
0 1998-12-01 00:00:00  0.062023  2271.938  0.003565  0.000000  681.4067
1 1999-01-01 00:00:00  0.063051  2415.186  0.003951 -0.000610  684.0990
2 1999-02-01 00:00:00  0.038456  2508.064  0.003572  0.002441  686.5426
3 1999-03-01 00:00:00 -0.038085  2412.545  0.003042  0.001217  688.6310
4 1999-04-01 00:00:00  0.037932  2504.058  0.004331  0.003040  691.6135

   cpiind      oil_r       gold_r     house_r
0   385.9   0.103084  -0.00833912  0.00543502
1   385.6 -0.0407348   0.00578136  0.00495517
2   386.6   0.222315   -0.0264762  0.00582723
3   387.1   0.179155     0.025586  0.00603752
4   388.2  0.0236857   -0.0628053  0.00589582
```

1.1 Question 1

In [4]:

```
Assets = [data.columns[i] for i in [1,3,4,7,8,9]]
Statistics = ["mean","std","skewness","kurtosis"]

q1_value = []
for i in Assets:
    L = []
    L.append(mean_(data[i]))
    L.append(std_deviation_(data[i]))
    L.append(skewness_(data[i]))

    L.append(kurtosis_(data[i]))
    q1_value.append(L)
q1_result = pd.DataFrame(q1_value,index = Assets,columns=Statistics)
q1_result
```

Out[4]:

	mean	std	skewness	kurtosis
vwretd	0.006634	0.043623	-0.650342	4.193085
t90ret	0.001600	0.001728	0.890505	2.492211
cpiret	0.001813	0.003719	-0.878723	7.020951
oil_r	0.009978	0.086367	-0.447242	3.767702
gold_r	0.007435	0.049216	0.036966	3.973547
house_r	0.003352	0.005938	-0.966303	3.498207

1.2 question 2

1.From the line plot we can find that the lines of discrete return and continuous return almost overlapped, which means they are almose the same. 2.Also, I calculated the average return of discrete return and continuous return below.

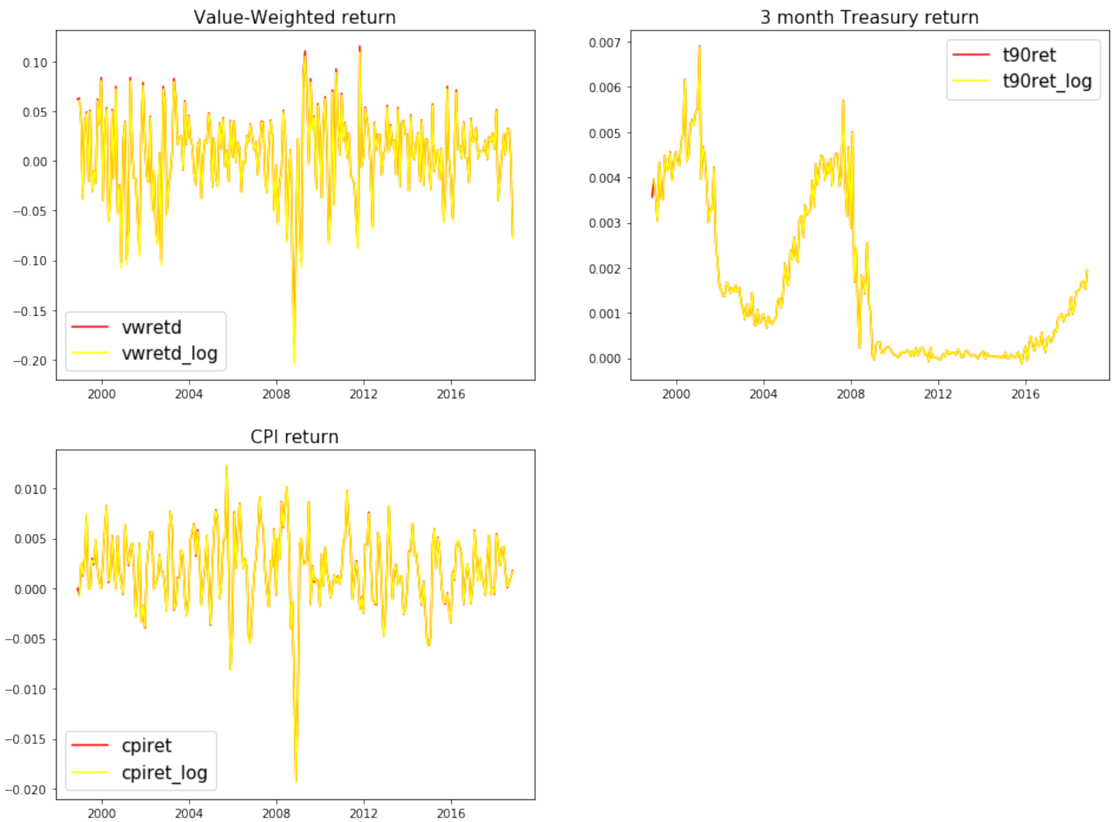
```
In [5]: q2 = data.loc[:,["date","vwretd","vwindd","t90ret","cpiret","t90ind","cpiind"]]
        temp = np.log(data.loc[:,["vwindd","t90ind","cpiind"]])
        q2["vwretd_log"] = temp["vwindd"].diff()
        q2["t90ret_log"] = temp["t90ind"].diff()
        q2["cpiret_log"] = temp["cpiind"].diff()
        q2 =  q2.loc[:,["date","vwretd_log","vwretd","t90ret_log","t90ret","cpiret_log","cpire
        #sns.relplot(x="timepoint", y="signal", kind="line",hue='event',style='event',data=q2)
```

```
In [6]: Assets = [q2.columns[i] for i in range(1,7)]
        Statistics = ["mean"]

        L = q2.iloc[1:,1:].apply(mean_)

        q2_result = pd.DataFrame(L,index = Assets, columns=Statistics)
        q2_result
        print(q2_result)

        fig = plt.figure(figsize=(16,12))
        ax1 = fig.add_subplot(2,2,1)
        ax1.plot(q2["date"],q2["vwretd"],color = "red")
        ax1.plot(q2["date"],q2["vwretd_log"], color = "yellow")
        ax1.legend(fontsize = 15, loc="best")
        ax1.set_title("Value-Weighted return", fontsize = 15)

        ax2 = fig.add_subplot(2,2,2)
        ax2.plot(q2["date"],q2["t90ret"],color = "red")
        ax2.plot(q2["date"],q2["t90ret_log"],color = "yellow")
        ax2.legend(fontsize = 15, loc="best")
        ax2.set_title("3 month Treasury return", fontsize = 15)

        ax3 = fig.add_subplot(2,2,3)
        ax3.plot(q2["date"],q2["cpiret"],color = "red")
        ax3.plot(q2["date"],q2["cpiret_log"],color = "yellow")
        ax3.legend(fontsize = 15, loc="best")
        ax3.set_title("CPI return", fontsize = 15)


        plt.show()
        plt.close(fig)
```

```
                mean
vwretd_log  0.005427
vwretd      0.006402
```

```
t90ret_log  0.001589
t90ret      0.001592
cpiret_log  0.001812
cpiret      0.001821
```

```
/Users/yifuhe/anaconda3/lib/python3.7/site-packages/pandas/plotting/_converter.py:129: FutureWa

To register the converters:
        >>> from pandas.plotting import register_matplotlib_converters
        >>> register_matplotlib_converters()
  warnings.warn(msg, FutureWarning)
```

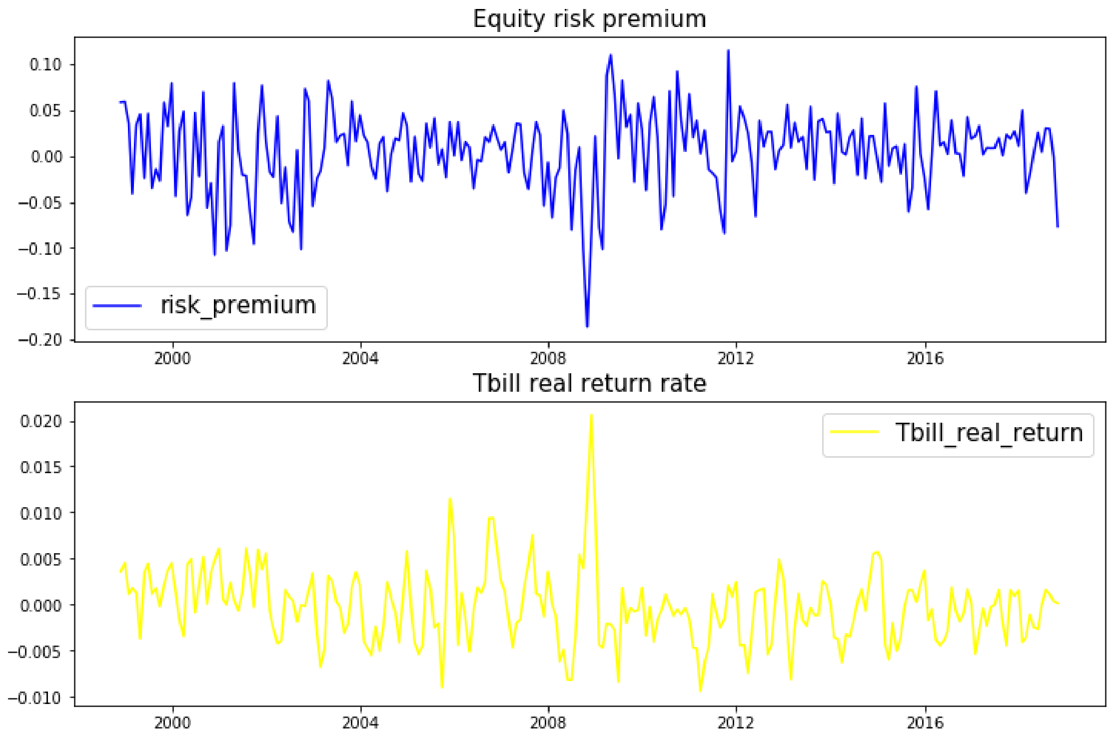


1.3 question 3

1.T-distribution critical level at infinite df with 99% confidence level: 2.345 student - t statistics: 1.780072515139707 T-statistics is less than the t-distribution critical level, so we cannot reject the null hypothesis. Thus equity risk premium is not significantly positive 2.Also, I calculate the average real return rate for the T-bill using the CPI index.

```
In [7]: # data processing
        data["risk_premium"] = data["vwretd"] - data["t90ret"]
```

```
data["Tbill_real_return"] = (data["t90ret"]+1) / (data["cpiret"]+1) - 1
fig = plt.figure(figsize=(12,8))
ax = fig.add_subplot(2,1,1)
ax.plot(data["date"],data["risk_premium"],color = "blue")
ax.legend(fontsize = 15, loc="best")
ax.set_title("Equity risk premium", fontsize = 15)

ax2 = fig.add_subplot(2,1,2)
ax2.plot(data["date"],data["Tbill_real_return"],color = "yellow")
ax2.legend(fontsize = 15, loc="best")
ax2.set_title("Tbill real return rate", fontsize = 15)
plt.show()
plt.close(fig)

print("u = 0")
print(f"degree of freedom = {len(data['risk_premium'])}")
print(f"t-distribution critical level at infinite df with 99% confidence level: 2.345")
print(f'student - t statistics: \
      {mean_(data["risk_premium"]- 0)/std_deviation_(data["risk_premium"])*np.sqrt(len
      end="\n\n")
print(f"t-statistics is less than the t-distribution critical level, so we cannot reje
print(f'average Tbill real return rate: {mean_(data["Tbill_real_return"])}')
```




```
u = 0
```

```
degree of freedom = 240
t-distribution critical level at infinite df with 99% confidence level: 2.345
student - t statistics:       1.780072515139707

t-statistics is less than the t-distribution critical level, so we cannot reject the null hypo

average Tbill real return rate: -0.00019919958704201767
```

1.4 question 4

1.From the chart below, we can find that actually oil has the highest average return, thus oil is a better investment than real estate market in long run. 2.Real estate market has the lowest volatility, so real estate market is safer than gold market. 3.Oil market has lower sharpe ration than stock market, thus stock market performed better in the last 20 years

```
In [8]: q4 = data.loc[:,["house_r","oil_r","gold_r","vwretd"]]
        values = ["avrg_return","volatility","sharp ratio"]
        q4_value = []
        for i in q4.columns:
            L = []
            L.append(mean_(q4[i]))
            L.append(std_deviation_(q4[i]))
            L.append(mean_(q4[i]-data['t90ret']) / std_deviation_(q4[i]))
            q4_value.append(L)
        q4_result = pd.DataFrame(q4_value,columns=values,index = q4.columns)
        q4_result
```

Out[8]:

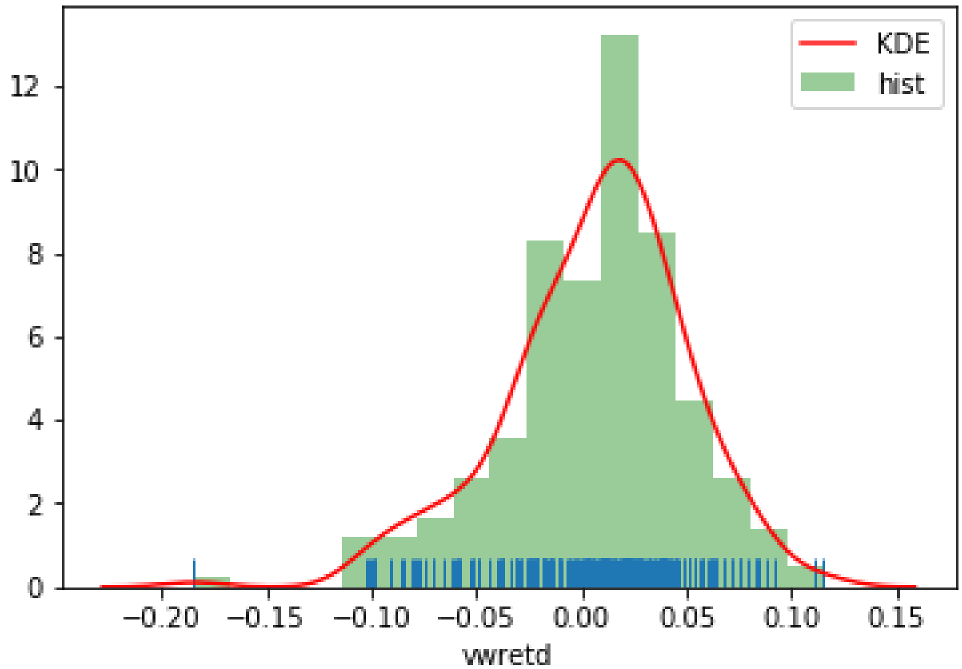
	avrg_return	volatility	sharp ratio
house_r	0.003352	0.005938	0.294993
oil_r	0.009978	0.086367	0.096995
gold_r	0.007435	0.049216	0.118551
vwretd	0.006634	0.043623	0.115387

1.5 Question 5

1. I generate the skewness and kurtosis from question 1. Then I ploted the distribution of stock market returns.
2. Jarque-Bera test statistic: 31.152305228111018 It is greater than 4.61, the chi-square distribution critical value at 99% confidence level. Thus, we rejected the null hypothesis. The return of stock market doesn't follow normal distribution.

```
In [9]: print(q1_result.loc[["vwretd"],["skewness","kurtosis"]])
        # visualization
        sns.distplot(data["vwretd"],
                     rug=True,
                     hist_kws={'color':'green','label':'hist'},
                     kde_kws={'color':'red','label':'KDE'})
        print(f'Jarque-Bera test statistic: {JB_test_(data["vwretd"])}')
```

```
        skewness  kurtosis
vwretd -0.650342  4.193085
Jarque-Bera test statistic: 31.152305228111018
```



1.6 Question 6

Based on the covariance matrix, we cannot find any value which is significantly different from zero. Based on the t(12) distribution critical value with 12 degree of freedom on 99% confidence level, the value is greater than t-statistics, which means we cannot reject the null hypothesis. We can conclude that the value is not significantly different from zero.

```
In [12]: def Cov_(arr1,arr2):
             """
             Parameters
             ----------
             arr1: np.array
                 array of data
             arr2: np.array
                 array of data

             Returns
             ----------
             res: float
                  Covariance
```

```
    """
    x_head = mean_(arr1)
    y_head = mean_(arr2)
    res = 0
    for i in range(len(arr1)):
        res +=(arr1[i] - x_head)(arr2[i] - y_head)
    return res/(len(arr1)-1)

Stock_p = data["vwretd"]
q6_value = [Stock_p.shift(i) for i in range(13)]
q6_data = pd.DataFrame(q6_value).T.iloc[12:,]
q6_data = q6_data.reset_index().drop("index",axis =1)
q6_data.columns = ["lag_"+str(i) for i  in range(13)]
CovMatrix = np.cov(q6_data,rowvar= False)
CovMatrix = pd.DataFrame(CovMatrix,index = q6_data.columns,columns=q6_data.columns)
CovMatrix
```

Out[12]:

```
               lag_0     lag_1     lag_2     lag_3     lag_4         lag_5  \
lag_0   1.913318e-03  0.000231 -0.000092  0.000122  0.000184  1.753828e-07
lag_1   2.309853e-04  0.001898  0.000222 -0.000087  0.000122  1.952100e-04
lag_2  -9.218982e-05  0.000222  0.001902  0.000224 -0.000082  1.161478e-04
lag_3   1.217280e-04 -0.000087  0.000224  0.001900  0.000224 -8.498212e-05
lag_4   1.842119e-04  0.000122 -0.000082  0.000224  0.001903  2.169190e-04
lag_5   1.753828e-07  0.000195  0.000116 -0.000085  0.000217  1.912019e-03
lag_6  -1.431460e-04  0.000001  0.000199  0.000116 -0.000083  2.116440e-04
lag_7  -2.043343e-05 -0.000133 -0.000004  0.000196  0.000109 -7.455606e-05
lag_8   1.644182e-04 -0.000022 -0.000138 -0.000004  0.000194  1.150671e-04
lag_9  -1.754027e-04  0.000137 -0.000017 -0.000129  0.000009  1.852129e-04
lag_10 -1.334627e-05 -0.000151  0.000134 -0.000024 -0.000140  1.495800e-05
lag_11  8.918766e-05  0.000003 -0.000159  0.000129 -0.000034 -1.285902e-04
lag_12  2.163949e-05  0.000110 -0.000004 -0.000165  0.000118 -2.343260e-05

           lag_6     lag_7     lag_8     lag_9    lag_10    lag_11    lag_12
lag_0  -0.000143 -0.000020  0.000164 -0.000175 -0.000013  0.000089  0.000022
lag_1   0.000001 -0.000133 -0.000022  0.000137 -0.000151  0.000003  0.000110
lag_2   0.000199 -0.000004 -0.000138 -0.000017  0.000134 -0.000159 -0.000004
lag_3   0.000116  0.000196 -0.000004 -0.000129 -0.000024  0.000129 -0.000165
lag_4  -0.000083  0.000109  0.000194  0.000009 -0.000140 -0.000034  0.000118
lag_5   0.000212 -0.000075  0.000115  0.000185  0.000015 -0.000129 -0.000023
lag_6   0.001913  0.000207 -0.000076  0.000125  0.000177  0.000008 -0.000137
lag_7   0.000207  0.001921  0.000213 -0.000085  0.000131  0.000188  0.000018
lag_8  -0.000076  0.000213  0.001923  0.000201 -0.000075  0.000139  0.000198
lag_9   0.000125 -0.000085  0.000201  0.001922  0.000204 -0.000085  0.000133
lag_10  0.000177  0.000131 -0.000075  0.000204  0.001918  0.000211 -0.000081
lag_11  0.000008  0.000188  0.000139 -0.000085  0.000211  0.001932  0.000224
lag_12 -0.000137  0.000018  0.000198  0.000133 -0.000081  0.000224  0.001943
```

In [18]:

```
temp = CovMatrix.iloc[0,1:]
```

```
t_12 = temp.dot(temp)*240
t_12
```

```
Out[18]: 4.7499244464231004e-05
```

1.7 Question 7

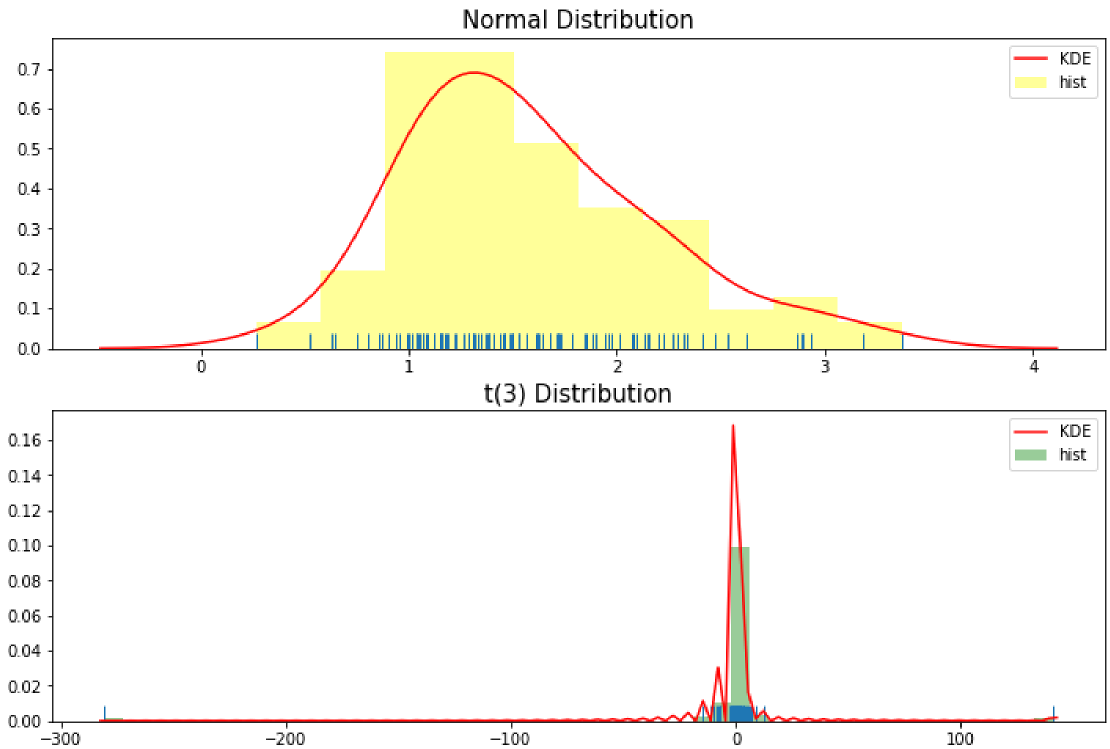
Based on the result, we can find the values in 100 simulations. 1.There are only 18 times of loss under normal distribution assumption. There are 29 times when stock market was worse than Tbill under normal distribution assumption. 2.There are 68 times of loss under t(3) distribution assumption. There are 69 times when stock market was worse than Tbill under t(3) distribution assumption. 3.It depends on the assumption and distribution of returns. If it is normal distribution, the advice makes sense. If it is t(3) distribution, it doesn't make sense.

```
In [317]: Rf = 0.04
          Tbill = (1+Rf)**5
          print(f"Tbill: {Tbill}")
          mu, sigma = 0.1, 0.2
          df = 3
          normal_stock = 0
          normal_tbill = 0
          t_dist_stock = 0
          t_dist_tbill = 0
          L1 , L2 = [],[]
          for i in range(100):
              n1 = (1+np.random.normal(mu,sigma,size = 5)).prod()
              n2 = (1+np.random.standard_t(df,size = 5)).prod()
              L1.append(n1)
              L2.append(n2)
              if n1 < 1:
                  normal_stock += 1
              if n1 < Tbill:
                  normal_tbill += 1
              if n2 < 1:
                  t_dist_stock += 1
              if n2 < Tbill:
                  t_dist_tbill += 1
          print(normal_stock,normal_tbill,t_dist_stock,t_dist_tbill)
```

```
Tbill: 1.2166529024000001
12 31 73 75
```

```
In [366]: fig = plt.figure(figsize=(12,8))
          ax = fig.add_subplot(2,1,1)
          sns.distplot(L1,rug=True,hist_kws={'color':'yellow','label':'hist'},kde_kws={'color'
          #ax.plot(Tbill,np.linspace(0, 1, 5),color = "blue",sizes=1000)
          ax.legend(fontsize = 10, loc="best")
          ax.set_title("Normal Distribution", fontsize = 15)
```

```
ax2 = fig.add_subplot(2,1,2)
sns.distplot(L2,rug=True,hist_kws={'color':'green','label':'hist'},kde_kws={'color':
ax2.legend(fontsize = 10, loc="best")
ax2.set_title("t(3) Distribution", fontsize = 15)

plt.show()
plt.close(fig)
```



1.8 Question 8

Based on the bootstrap method, we can the scenario is quite similar to it under normal distribution. There are only 16 times of loss. There are 3 times when stock market was worse than Tbill.

```
In [368]: res1 = 0
          res2 = 0
          L3 = []
          for i in range(100):
              n = (data["vwretd"].sample(n=60, replace =False)+1).prod()
              L3.append(n)
              if n < 1:
                  res1 +=1
```

```
            if n < Tbill:
                res2 +=1
        print(f"Tbill: {Tbill}")
        print(res1,res2)
```

```
Tbill: 1.2166529024000001
8 31
```

```
In [371]: fig = plt.figure(figsize=(12,4))
          ax = fig.add_subplot(111)
          sns.distplot(L3,rug=True,hist_kws={'color':'yellow','label':'hist'},kde_kws={'color'
          #ax.plot(Tbill,np.linspace(0, 1, 5),color = "blue",sizes=1000)
          ax.legend(fontsize = 10, loc="best")
          ax.set_title("Bootstrap method", fontsize = 15)
          plt.show()
          plt.close(fig)
```