Sample Midterm Exam

Problem 1

Prove that the set of points $x \in \mathbb{R}^3$ defined by the inequalities:

$$\begin{aligned}
&(x_1)^2 + 3(x_2)^2 + (x_3)^2 + x_1x_2 - x_1x_3 + 2x_2x_3 \le 6\\
&3x_1 + (x_2)^2 - x_3 \le 4,\\
&x_1 \ge 0,\ x_2 \ge 0,\ x_3 \ge 0,
\end{aligned}$$

is convex.

Problem 2

You have \$20,000 to invest. Stock XYZ sells for \$20 per share. A European call option to buy 100 shares of XYZ at \$15 exactly six months from now sells for \$1,000. You can also raise additional funds by selling call options with the above characteristics. In addition, a 6-month riskless zero-coupon bond with \$100 face value sells for \$90. You have decided to limit the number of call options that you buy or sell to 50. The stock and bonds cannot be sold short.

You consider three scenarios for the price of stock XYZ six months from today: the price will be the same as today, the price will go to \$40, or drop to \$12. You estimate that each of these scenarios is equally likely.

(a) Formulate a linear programming problem to determine the portfolio of stocks, bonds and options to maximize the expected value of the profit.

Formulate the dual problem.

(b) Suppose you want a profit of at least \$2,000 in any of the three scenarios. Formulate a linear programming problem to determine the portfolio of stocks, bonds and options to maximize the expected value of the profit, under this condition.

Formulate the dual problem.

Problem 3

Solve the following nonlinear programming problem:

$$\begin{aligned}
\min\quad & (x_1)^2 + 2(x_2)^2 + 3(x_3)^2 - x_1x_2 - x_1x_3 + 2x_2x_3\\
\text{subject to}\quad & x_1 + x_2 + x_3 = 1,\\
& x_1 \ge 0, x_2 \ge 0, x_3 \ge 0.
\end{aligned}$$