Assignment 10 - Solution

(a) We denote by r the mean $\mathbb{E}[Z]$, by $-\eta$ the Value at Risk (to be found) and by v_j the shortfall below η in scenario j. The primal problem is:

$$\begin{aligned}
\min_{r,\eta,v} \quad & -\frac{1}{3}r + \frac{2}{3}\Big\{ -\eta + \frac{1}{\alpha}\sum_{j=1}^{n} p_j v_j \Big\} \\
& r = \sum_{j=1}^{n} p_j z_j \\
& -\eta + v_j \geq -z_j, \quad j = 1,\dots,n \\
& v_j \geq 0, \quad j = 1,\dots,n.
\end{aligned}$$

(b) We assign multiplier μ to the first constraint, and multipliers $\lambda_j \geq 0$ to the second group of constraints. The dual problem has the form:

$$\begin{aligned}
\max_{\mu,\lambda} \quad & \Big\{ \mu \sum_{j=1}^{n} p_j z_j - \sum_{j=1}^{n} \lambda_j z_j \Big\} \\
& \mu = -\frac{1}{3}, \\
& \sum_{j=1}^{n} \lambda_j = \frac{2}{3}, \\
& 0 \leq \lambda_j \leq \frac{2p_j}{3\alpha}, \quad j = 1,\dots,n.
\end{aligned}$$

Observe that the quantities $\xi_j = -\mu p_j + \lambda_j$ are probabilities (they are nonnegative and total one). The dual problem is to find

$$\max_{\xi \in \mathscr{A}} \Big\{ -\sum_{j=1}^{n} \xi_j z_j \Big\}$$

with the set $\mathscr{A}$ defined by the constraints of the dual problem.

(c)

$$\lim_{\alpha \to 0} \rho(Z) = -\frac{1}{3}\mathbb{E}[Z] - \min_{\omega} Z(\omega).$$

The last term represents the worst case and is $+\infty$ if Z is not bounded from below.

$$\lim_{\alpha \to 1} \rho(Z) = -\mathbb{E}[Z]$$