HW 10 1900013956

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a). We denote by r the mean $E[Z]$. by $-\eta$ the Value at Risk (to be found) and by v_j the shortfall below η in scenario j. The primal problem is:

$$\min_{r,\eta,v} \; -\frac{1}{3}r+\frac{2}{3}\{-\eta+\frac{1}{\alpha}\sum_{j=1}^{n}p_j v_j\}$$

$$r=\sum_{j=1}^{n}p_j z_j$$

$$-\eta+v_j \geq -z_j, \quad j=1,\cdots,n$$

$$v_j \geq 0, \quad j=1,\cdots,n$$

b) We assign multiplier u to the first constraint, and multipliers $\lambda_i \geq 0$ to the second group of constraints. The dual problem has the form:

$$\max_{u,\lambda} \{u\sum_{j=1}^{n}p_j z_j - \sum_{j=1}^{n}\lambda_j z_j\}$$

$$u=-\frac{1}{3},$$

$$\sum_{j=1}^{n}\lambda_j=\frac{2}{3},$$

$$0 \leq \lambda_j \leq \frac{2p_j}{3\alpha}, \quad j=1,\cdots,n$$

observe that the quantities $\xi_j=-up_j+\lambda_j$ are probabilities (they are nonnegative and total one).

The dual problem is to find $\max_{\xi\in\phi}\{-\sum_{j=1}^{n}\xi_j z_j\}$

with the set ϕ defined by the constraints of the dual problem.

c) $\lim_{\alpha\to 0}\rho(Z)=-\frac{1}{3}E[Z]-\min_{w}Z(w)$

The last term represents the worst case and is $+\infty$ if Z is not bounded from below.

$$\lim_{\alpha\to 1}\rho(Z)=-EZ$$