Sample Final Exam (Fall 2019)

Problem 1

Consider the problem

$$\begin{aligned} &\min \sum_{j=1}^{n} \frac{a_j}{x_j} \\ &\text{subject to} \sum_{j=1}^{n} x_j \le b \\ &0 \le x_j \le u_j, \quad j = 1, \ldots, m, \end{aligned}$$

in which all coefficients a_j , u_j and b are positive.

Formulate the dual problem. Treat simple constraints directly.

Problem 2

A random variable Z has realizations $z_1, z_2, \ldots, z_n$, attained with probabilities $p_1, p_2, \ldots, p_n$. It represents profits. Formulate a linear programming problem to calculate the following measure of risk:

$$\rho(Z) = -\frac{1}{2}\mathbb{E}[Z] + \frac{1}{2}\mathrm{AVaR}_\alpha^-(Z),$$

where

$$\mathrm{AVaR}_\alpha^-(Z) = \frac{1}{\alpha}\int_0^\alpha \mathrm{VaR}_\beta(Z)\, d\beta = \min_\eta \Big\{ -\eta + \frac{1}{\alpha}\mathbb{E}\big[(\eta - Z)_+\big] \Big\}.$$

Formulate the dual problem.

Problem 3

The monthly return rates of assets 1 and 2 have joint distributions with the mean return rates $r_1 = 0.02$ and $r_2 = 0.01$ and with the covariance matrix

$$C = \begin{bmatrix} 0.002 & 0 \\ 0 & 0.001 \end{bmatrix}$$

Suppose shorting is allowed.

(a) Find the minimum variance portfolio.

(b) Find the best portfolio with the return rate 0.02.

(c) Describe the set of all efficient portfolios, with return rate μ. What is the range of μ?

(d) Suppose a riskless asset with return rate $r_0 = 0.005$ is available. Find the market portfolio.

Problem 4

Suppose the returns of assets in problem 3 have a joint normal distribution. Find the value at risk and the average value at risk at level $\alpha = 0.05$ of the portfolio with asset weights 1.5 and -0.5 and the total amount invested equal to $100,000. The 5% critical value for the standard normal distribution equals 1.65.