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With no loss of generality, Problem 1.

$$\frac{u_1c_1}{a_1} \geq \frac{u_2c_2}{a_2} \geq \dots \geq \frac{u_nc_n}{a_n}$$

Lagrangian:

$$L(x,u) = \sum_{j=1}^{n} c_j(x_j)^2 + u(1-\sum_{j=1}^{n} a_j x_j)$$

$$= \sum_{j=1}^{n} (c_j(x_j)^2 - u a_j x_j) + u.$$

minimizing $x_j \in [0, u_j]$

$$x_j = \begin{cases} \frac{u a_j}{2c_j}, & 0 \leq \frac{u a_j}{2c_j} \leq u_j \\ u_j, & \frac{u a_j}{2c_j} > u_j \\ 0, & u < 0. \end{cases}$$

$u > 0$, there exist k that $x_j = \frac{u a_j}{2c_j}$ for $j = 1, \dots, k$ and $x_j = u_j$ for $j = k+1, \dots, n$

$$\sum_{j=1}^{n} x_j = \frac{u}{2} \sum_{j=1}^{k} \frac{a_j}{c_j} + \sum_{j=k+1}^{n} u_j = 1.$$

$$\Rightarrow \frac{u}{2} = \frac{1 - \sum_{j=k+1}^{n} u_j}{\sum_{j=1}^{k} \frac{a_j}{c_j}}$$

thus. $$\frac{u_k c_k}{a_k} \geq \frac{1 - \sum_{j=k+1}^{n} u_j}{\sum_{j=1}^{k} \frac{a_j}{c_j}} \geq \frac{u_{k+1} c_{k+1}}{a_{k+1}}$$

when we fix the value of k.

we can calculate u and x. according to. k.

find k. that minimize $\sum c_i x_i^2 = \left(\sum_{j=1}^{k} c_j \left(\frac{u a_j}{2c_j}\right)^2 + \sum_{j=k+1}^{n} c_j \cdot u_j^2\right)$