FIXED INCOME ANALYSIS
LECTURE 8

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Fall 2019

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Agenda

- Credit Derivative and Methods
  - Credit Default Swap
  - Collateralized Debt Obligation
  - Copula

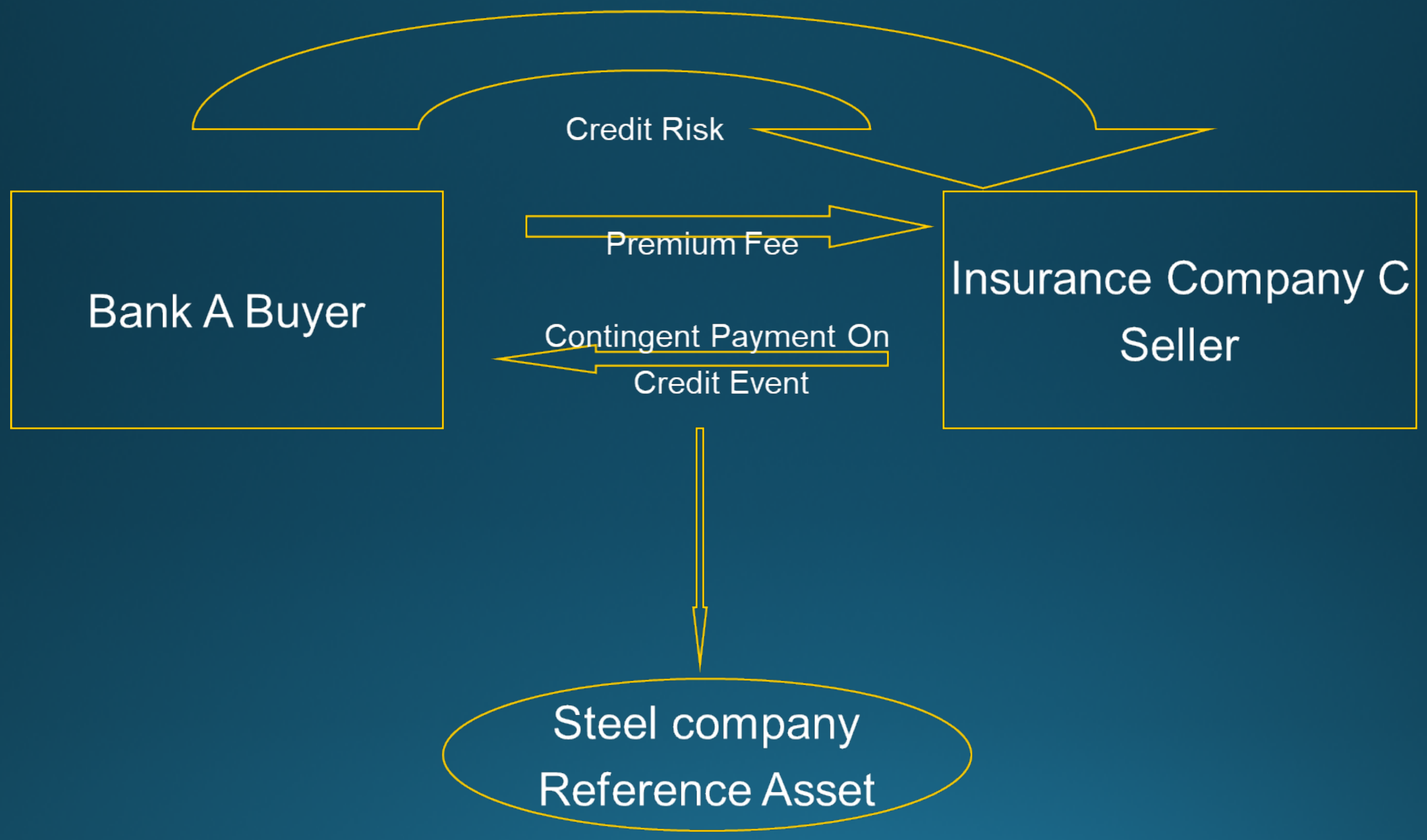
Credit Default Swap (CDS)

- Credit default swaps allow one party to "buy" protection from another party for losses that might be incurred as a result of default by a specified reference credit (or credits).
- The "buyer" of protection pays a premium for the protection, and the "seller" of protection agrees to make a payment to compensate the buyer for losses incurred upon the occurrence of any one of several specified "credit events."

Example

- Suppose Bank A buys a bond which issued by a Steel Company.
- To hedge the default of Steel Company:
  - Bank A buys a credit default swap from Insurance Company C.
  - Bank A pays a fixed periodic payments to C, in exchange for default protection.

Example



Credit Risk
Bank A Buyer
Premium Fee
Contingent Payment On
Credit Event
Insurance Company C
Seller
Steel company
Reference Asset

Attractions of the CDS

- Allows credit risks to be traded in the same way as market risks
- Can be used to transfer credit risks to a third party
- Can be used to diversify credit risks

Using a CDS to Hedge a Bond Position

- Portfolio consisting of a 5-year par yield corporate bond that provides a yield of 6% and a long position in a 5-year CDS costing 100 basis points per year is (approximately) a long position in a riskless instrument paying 5% per year

- This shows that bond yield spreads (measured relative to LIBOR) should be close to CDS spreads

CDS Valuation

- Hazard rate for reference entity is 2%.
- Assume payments are made annually in arrears, that defaults always happen half way through a year, and that the expected recovery rate is 40%, risk free rate 5%
- Suppose that the breakeven CDS rate is s per dollar of notional principal

Unconditional Default and Survival Probabilities

Time (years)	Survival Probability	Default Probability
1	0.9802	0.0198
2	0.9608	0.0194
3	0.9418	0.0190
4	0.9231	0.0186
5	0.9048	0.0183

Calculation of PV of Payments

Time (yrs)	Survival Prob	Expected Payment	Discount Factor	PV of Exp Pmt
1	0.9802	$0.9802s$	0.9512	$0.9324s$
2	0.9608	$0.9608s$	0.9048	$0.8694s$
3	0.9418	$0.9418s$	0.8607	$0.8106s$
4	0.9231	$0.9231s$	0.8187	$0.7558s$
5	0.9048	$0.9048s$	0.7788	$0.7047s$
Total				$4.0728s$

Present Value of Expected Payoff

Time (yrs)	Default Probab.	Rec. Rate	Expected Payoff	Discount Factor	PV of Exp. Payoff
0.5	0.0198	0.4	0.0119	0.9753	0.0116
1.5	0.0194	0.4	0.0116	0.9277	0.0108
2.5	0.0190	0.4	0.0114	0.8825	0.0101
3.5	0.0186	0.4	0.0112	0.8395	0.0094
4.5	0.0183	0.4	0.0110	0.7985	0.0088
Total					0.0506

Putting it all together

- PV of expected payments is $4.0728s$

- The breakeven CDS spread is given by
  $4.0728s = 0.0506$ or $s = 0.0124$ (124 bps)

Implying Default Probabilities from CDS spreads

- Suppose that the mid market spread for a 5 year newly issued CDS is 100bps per year
- We can reverse engineer our calculations to conclude that the hazard is 1.63% per year.
- Is this 1.63% risk neutral default rate or real world default rate?

Mini-Quiz #1

- Suppose risk free rate is 5% per year with continuous compounding. There is a 2-year credit default swap with 30% recovery rate and default probability each year conditional on no earlier default is 3%. Estimate the credit default swap spread. Assume payments are made annually and defaults can occur at the end of each year.

Break

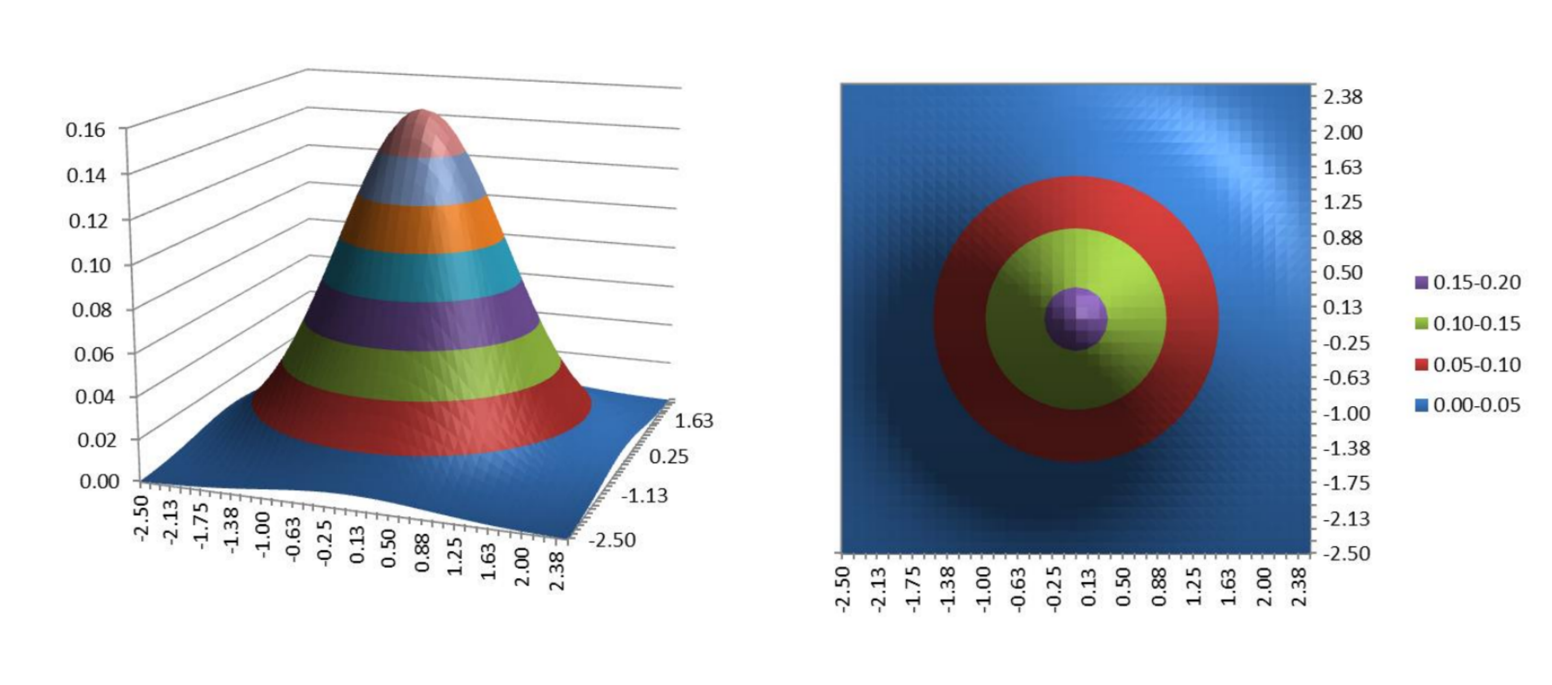
Asset Backed Securities

- Securities created from a portfolio of loans, bonds, credit card receivables, mortgages, auto loans, aircraft leases, music royalties, etc
- Usually the income from the assets is tranched
- A "waterfall" defines how income is first used to pay the promised return to the senior tranche, then to the next most senior tranche, and so on.

Collateralized Debt Obligations

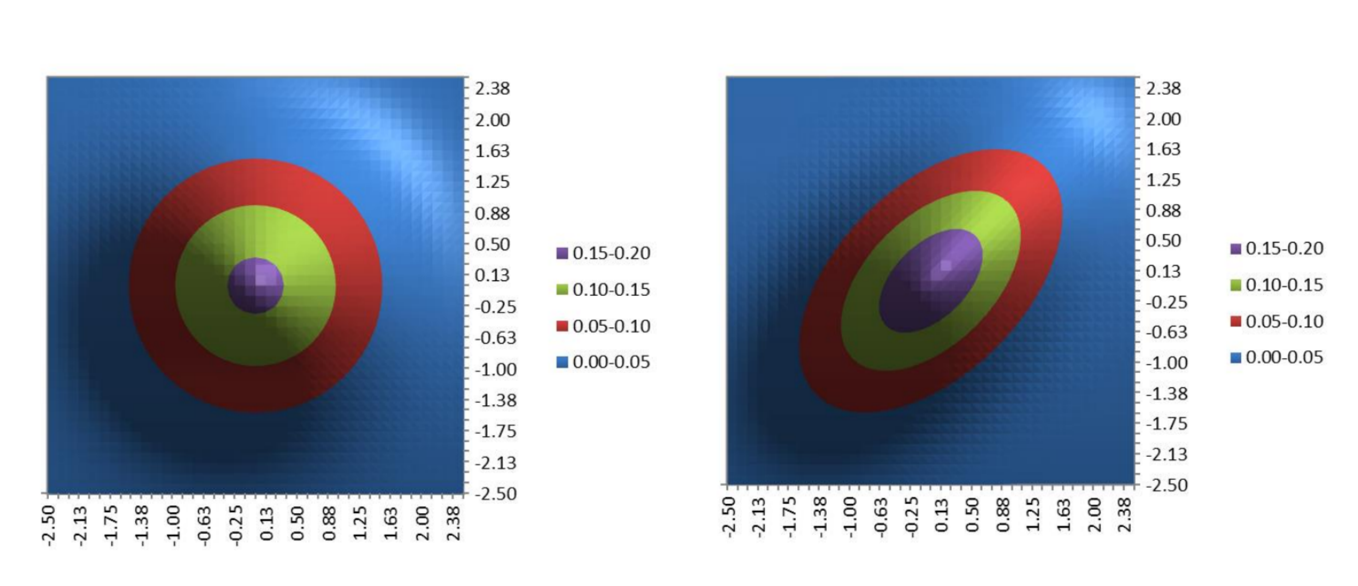
- A cash CDO is an ABS where the underlying assets are debt obligations
- A synthetic CDO involves forming a similar structure with short CDS contracts
- In a synthetic CDO most junior tranche bears losses first. After it has been wiped out, the second most junior tranche bears losses, and so on

Multivariate Distributions



Multivariate Distributions

- No Correlation vs. Correlation



Multivariate Distributions

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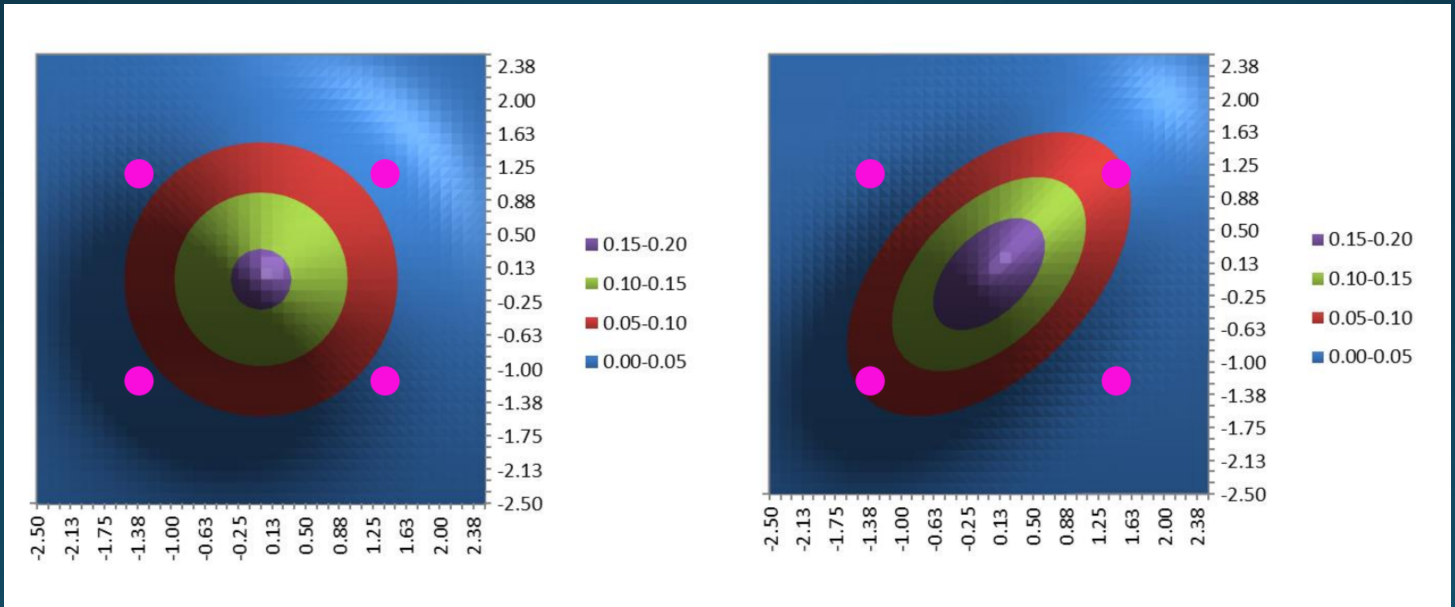
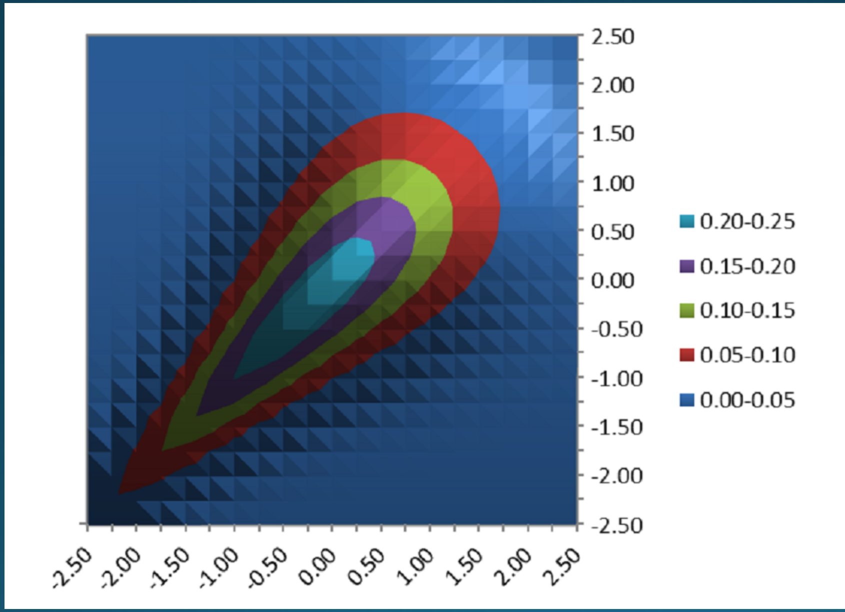


Mini-Quiz #2

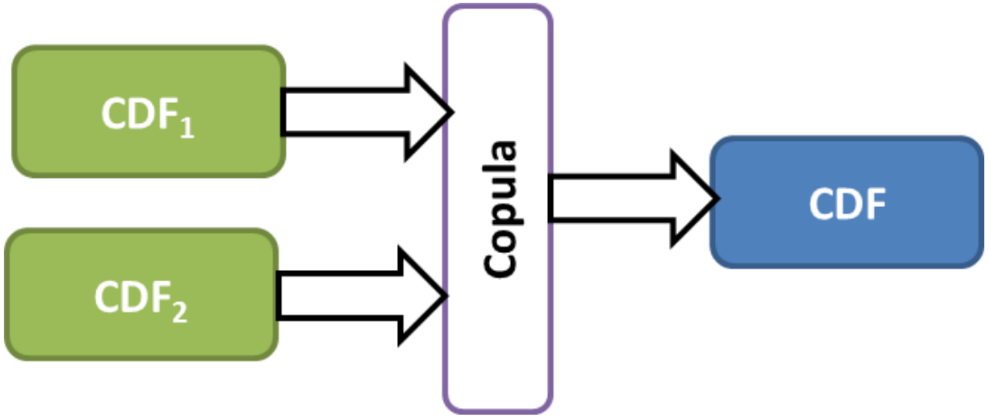
- What would the multivariate distribution look like for two variables with +100% correlation?
- -100%?

Copula

- Clayton Copula



Copula



Franks Copula: $C(u,v) = \frac{1}{\alpha} ln\left[1 + \frac{(e^{\alpha u} - 1)(e^{\alpha v} - 1)}{e^{\alpha} - 1}\right]$

Mini-Quiz #3

- Assume we have two standard uniform random variables, whose join distribution is defined by Frank's copula. What is the formula for the cumulative distribution of the two variables? Assume $\alpha = 1$.

Monte-Carlo Simulation in Copula

- Generate two independent random draws from a standard uniform distribution to determine u and C_1
  - $C_1 = \frac{\partial C}{\partial u}$
- Use the inverse CDF ($C_1 = \frac{\partial C}{\partial u}$) of the copula to determine v.
- Calculate values for x and y, using inverse cumulative distribution functions of the underlying distribution. For example if the underlying distributions are normal, use the inverse normal CDF to calculate x and y based on u and v.

Example

- Assume we are using Frank copula in a Monte Carlo simulation, with $\alpha = 3$. The underlying distributions are both standard normal. If our random number generator produces $u = 0.20$, $C_1 = 0.50$ what the values of our underlying random variables X and Y?
- $C_1 = \frac{\partial C}{\partial u} = \frac{(e^{-\alpha u}-1)(e^{-\alpha v}-1)-(e^{-\alpha v}-1)}{(e^{-\alpha u}-1)(e^{-\alpha v}-1)-(e^{-\alpha}-1)}$
- $v = -\frac{1}{\alpha}\ln\left[1 + \frac{C_1(e^{-\alpha}-1)}{1+(e^{-\alpha u}-1)(1-C_1)}\right] = -\frac{1}{3}\ln\left[1 + \frac{0.50\left(e^{-3}-1\right)}{1+(e^{-3*0.20}-1)(1-0.50)}\right] = 0.32$
- X = -0.84, Y = -0.48

Copula

- Pros
  - Very flexible
  - Used in Structured products
- Cons
  - Math is complicated
  - Not Intuitive

Homework

- Suppose risk free rate is 4% per year with continuous compounding. There is a 3-year credit default swap with 40% recovery rate and default probability each year conditional on no earlier default is 3%. Estimate the credit default swap spread. Assume payments are made annually and defaults can occur at the end of each year.
- Assume we are using Clayton copula in a Monte Carlo simulation, with $\alpha = 2$. The underlying distributions are both standard normal. If our random number generator produces $u = 0.10$, $C_1 = 0.30$ what the values of our underlying random variables X and Y?
  - Clayton copula: $C = (u^{-\alpha} + v^{-\alpha} - 1)^{-\frac{1}{\alpha}}$, $v = u\left(C_1^{-\frac{\alpha}{1+\alpha}} + u^{\alpha} - 1\right)^{-\frac{1}{\alpha}}$
- Monte Carlo problem: You are asked to calculate the probability of a portfolio returning less than −20%. The portfolio consists of 50% each of two securities. Each security has a standard deviation of 10%. Assume that the relationship between the securities can be described by a Clayton copula with α = 0.1. What would the probability have been if the joint distribution was elliptical with the
same correlation? Repeat for α = 0.3, 1, 3, and 10.