FIXED INCOME ANALYSIS LECTURE 4

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1. INTRODUCTION

- Any analysis of fixed-rate securities starts with an understanding of their risk and return characteristics.
- The yield-to-maturity, or internal rate of return on future cash flows, is of particular focus.
- The return on a fixed-rate bond is affected by many factors, such as credit risk (potential default on payments) and interest rate risk (varying coupon reinvestment rate and sale price).

2. SOURCES OF RETURN

A fixed-rate bond has three sources of return:

- Receipt of the promised coupon and principal payments on the scheduled dates
- Reinvestment of coupon payments
- Potential capital gains or losses on the sale of the bond prior to maturity

A horizon yield

 the internal rate of return between the total return for the investment horizon and the purchase price of the bond

A carrying value

 the purchase price plus (minus) the amortized amount of the discount (premium) if the bond is purchased at a price below (above) par value

SOURCES OF RETURN

Example: An investor purchases a 10-year, 8% annual coupon bond at \$85.503075 per \$100 of par value and holds it to maturity. The bond's yield to maturity is 10.40%. Show the sources of return:

Bondholder receives 1) Coupon payments 10 × \$8 = \$80; 2) Par value at maturity \$100; 3) Reinvestment income from coupons (at 10.40%).

$$[8\times(1.1040)^9]+[8\times(1.1040)^8]+[8\times(1.1040)^7]+[8\times(1.1040)^6]+[8\times(1.1040)^5]+[8\times(1.1040)^4]+[8\times(1.1040)^3]+[8\times(1.1040)^2]+[8\times(1.1040)^4]+[8\times(1.1040)^3]+[8\times(1.1040)^4]+[8\times(1.1040)^4]+[8\times(1.1040)^4]+[8\times(1.1040)^4]+[8\times(1.1040)^4]+[8\times(1.1040)^4]+[8\times(1.1040)^4]+[8\times(1.1040)^4]+[8\times(1.1040)^4]+[8\times(1.1040)^4]+[8\times(1.1040)^4]+[8\times(1.1040)^4]+[8\times(1.1040)^4]+[8\times(1.1040)^4]+[8\times(1.1040)^4]+[8\times(1.1040)^4]+[8\times(1.1040)^4]+[8\times(1.1040)^4]+[8\times(1.1040)^4]+[8\times(1.1040)^4]+[8\times(1.1040)^4]+[8\times(1.1040)^4]+[8\times(1.1040)^4]+[8\times(1.1040)^4]+[8\times(1.1040)^4]+[8\times(1.1040)^4]+[8\times(1.1040)^4]+[8\times(1.1040)^4]+[8\times(1.1040)^4]+[8\times(1.1040)^4]+[8\times(1.1040)^4]+[8\times(1.1040)^4]+[8\times(1.1040)^4]+[8\times(1.1040)^4]+[8\times(1.1040)^4]+[8\times(1.1040)^4]+[8\times(1.1040)^4]+[8\times(1.1040)^4]+[8\times(1.1040)^4]+[8\times(1.1040)^4]+[8\times(1.1040)^4]+[8\times(1.1040)^4]+[8\times(1.1040)^4]+[8\times(1.1040)^4]+[8\times(1.1040)^4]+[8\times(1.1040)^4]+[8\times(1.1040)^4]+[8\times(1.1040)^4]+[8\times(1.1040)^4]+[8\times(1.1040)^4]+[8\times(1.1040)^4]+[8\times(1.1040)^4]+[8\times(1.1040)^4]+[8\times(1.1040)^4]+[8\times(1.1040)^4]+[8\times(1.1040)^4]+[8\times(1.1040)^4]+[8\times(1.1040)^4]+[8\times(1.1040)^4]+[8\times(1.1040)^4]+[8\times(1.1040)^4]+[8\times(1.1040)^4]+[8\times(1.1040)^4]+[8\times(1.1040)^4]+[8\times(1.1040)^4]+[8\times(1.1040)^4]+[8\times(1.1040)^4]+[8\times(1.1040)^4]+[8\times(1.1040)^4]+[8\times(1.1040)^4]+[8\times(1.1040)^4]+[8\times(1.1040)^4]+[8\times(1.1040)^4]+[8\times(1.1040)^4]+[8\times(1.1040)^4]+[8\times(1.1040)^4]+[8\times(1.1040)^4]+[8\times(1.1040)^4]+[8\times(1.1040)^4]+[8\times(1.1040)^4]+[8\times(1.1040)^4]+[8\times(1.1040)^4]+[8\times(1.1040)^4]+[8\times(1.1040)^4]+[8\times(1.1040)^4]+[8\times(1.1040)^4]+[8\times(1.1040)^4]+[8\times(1.1040)^4]+[8\times(1.1040)^4]+[8\times(1.1040)^4]+[8\times(1.1040)^4]+[8\times(1.1040)^4]+[8\times(1.1040)^4]+[8\times(1.1040)^4]+[8\times(1.1040)^4]+[8\times(1.1040)^4]+[8\times(1.1040)^4]+[8\times(1.1040)^4]+[8\times(1.1040)^4]+[8\times(1.1040)^4]+[8\times(1.1040)^4]+[8\times(1.1040)^4]+[8\times(1.1040)^4]+[8\times(1.1040)^4]+[8\times(1.1040)^4]+[8\times(1.1040)^4]+[8\times(1.1040)^4]+[8\times(1.1040)^4]+[8\times(1.1040)^4]+[8\times(1.1040)^4]+[8\times(1.1040)^4]+[8\times(1.1040)^4]+[8\times(1.1040)^4]+[8\times(1.1040)^4]+[8\times(1.1040)^4]+[8\times(1.1040)^4]+[8\times(1.1040)^4]+[8\times(1.1040)^4]+[8\times(1.1040)^4]+[8\times(1.1040)^4]+[8\times(1.1040)^4]+[8\times(1.1040)^4]+[8\times(1.1040)^4]+[8\times(1.1040)^4]+[8\times(1.1040)^4]+[8\times(1.1040)^4]+[8\times($$

\$129.970678 = Future value of the coupons on the bond's maturity date

\$49.970678 = Interest on reinvested coupons (\$129.970678 – \$80)

\$229.970678 = Total return (\$129.970678 + \$100)

Realized rate of return: $r = \left(\frac{229.970678}{85.503075}\right)^{1/10} - 1 = \mathbf{0.1040} \text{ or } \mathbf{10.40}\%.$

SOURCES OF RETURN

Example: An investor purchases a 10-year, 8% annual coupon bond at \$85.503075 and sells it in four years. The bond's yield-to-maturity goes up from 10.40% to 11.40% straight after the purchase. Show the sources of return:

Bondholder receives 1) Coupon payments 4 × \$8 = \$32; 2) Sale price (at 11.40% YTM) \$85.780408; 3) Reinvestment income from coupons (at 11.40%). calculate the present value of cash flows

$$[8 \times (1.1140)^3] + [8 \times (1.1140)^2] + [8 \times (1.1140)^1] + 8 = \$37.899724$$

\$37.899724 = Future value of the reinvested coupons

\$5.899724 = Interest on reinvested coupons (\$37.899724 – \$32)

\$123.680132 = Total return (\$37.899724 + \$85.780408)

Realized rate of return:
$$r = \left(\frac{123.680132}{85.503075}\right)^{1/4} - 1 = 0.0967 \text{ or } 9.67\%.$$

INVESTMENT HORIZON AND INTEREST RATE RISK

A few important points about fixed-rate bonds:

• The *investment horizon* is at the heart of understanding interest rate risk and return.

There are two offsetting types of interest rate risk

Coupon reinvestment risk

Market price risk

COUPON REINVESTMENT RISK AND MARKET PRICE RISK

The future value of **reinvested coupon** payments (and in a portfolio, the principal on bonds that mature before the horizon date) *increases* when interest rates go up and *decreases* when rates go down.

The sale price on a bond that matures after the horizon date (and thus needs to be sold) *decreases* when interest rates go up and *increases* when rates go down.

Coupon reinvestment risk matters more when the investor has a long-term horizon relative to the time-to-maturity of the bond.

3. INTEREST RATE RISK ON FIXED-RATE BONDS

The **duration** of a bond measures the **sensitivity of the bond's full price** (including accrued interest) to changes in the bond's yield-to-maturity or, more generally, to changes in benchmark interest rates.

There are several types of bond duration. In general, these can be divided into **yield duration** and **curve duration**.

Yield duration is the sensitivity of the bond price with respect to the bond's own yield-to-maturity.

Curve duration is the sensitivity of the bond price (or more generally, the market value of a financial asset or liability) with respect to a benchmark yield curve.

YIELD DURATION STATISTICS

Yield duration statistics used in fixed-income

• Modified duration
• Money duration analysis include

- Macaulay duration

- Price value of a basis point (PVBP)

The Macaulay duration (D) formula (for the period)

$$D = \begin{bmatrix} \frac{(1-\frac{t}{T}) \times \text{PMT}}{(1+r)^{1-t/T}} + \frac{(2-\frac{t}{T}) \times \text{PMT}}{(1+r)^{2-t/T}} + \dots + \frac{(N-\frac{t}{T}) \times (\text{PMT} + \text{FV})}{(1+r)^{N-t/T}} \\ \frac{\text{PMT}}{(1+r)^{1-t/T}} + \frac{\text{PMT}}{(1+r)^{2-t/T}} + \dots + \frac{\text{PMT} + \text{FV}}{(1+r)^{N-t/T}} \end{bmatrix}$$

where t is the number of days from the last coupon payment to the settlement date; T is the number of days in the coupon period; PMT is the coupon payment per period; FV is par value; r is YTM/discount rate per period; and **N** is the number of coupon periods to maturity.

MACAULAY DURATION

Another way to calculate Macaulay duration is by using the following formula:

$$D = \left\{ \frac{1+r}{r} - \frac{1+r+[N \times (c-r)]}{c \times [(1+r)^N - 1] + r} \right\} - \left(\frac{t}{T}\right)$$

where **c** is the coupon rate per period.

 The Macaulay duration is usually expressed in annual terms. To convert it to an annual duration, divide the Macaulay duration by the number of coupon payment periods per year.

CALCULATING THE MACAULAY DURATION

Example: A 6% annual payment bond matures on 14 February 2022 and is purchased for settlement on 11 April 2014. The YTM is 4%. Calculate the bond's Macaulay duration (actual/actual convention):

Period	Time to Receipt	CF (cash flow)	PV of CF	Time-Weighted PV of CF
1	309/365 = 0.8466	6	6/(1 + 0.04)^0.8466 = 5.80	0.8466 × 5.80 = 4.91
2	1.8466	6	5.58	10.31
3	2.8466	6	5.37	15.28
4	3.8466	6	5.16	19.85
5	4.8466	6	4.96	24.05
6	5.8466	106	84.28	492.74
			111.15	567.13

D = 567.13/111.15 = 5.1 years

CALCULATING THE MACAULAY DURATION WITH THE ALTERNATIVE FORMULA

Using the alternative formula, the calculation is as follows:

$$D = \frac{1 + 0.04}{0.04} - \frac{1 + 0.04 + [6 \times (0.06 - 0.04)]}{0.06 \times [(1 + 0.04)^6 - 1] + 0.04} - \frac{56}{365}$$

= 5.10 years.

MODIFIED DURATION

Modified duration (MD) is a direct measure of the interest rate sensitivity of a bond. It assumes that yield changes do not change the expected cash flows.

$$MD = \frac{D}{1+r}$$

where *r* is the yield per period.

Modified duration provides a linear estimate of the percentage price change for a bond given a change in its yield-to-maturity.

$$\%\Delta PV^{Full} \approx -MD \times \Delta Yield(\%)$$

- Note: MD is expressed in annual terms.
- To get the % change in bond price, the % change must be multiplied by the original bond price.

APPROXIMATE MODIFIED DURATION

An alternative approach is to estimate the **approximate modified duration (AMD)** directly:

$$AMD = \frac{(PV_{-}) - (PV_{+})}{2 \times (\Delta Yield) \times (PV_{0})}$$

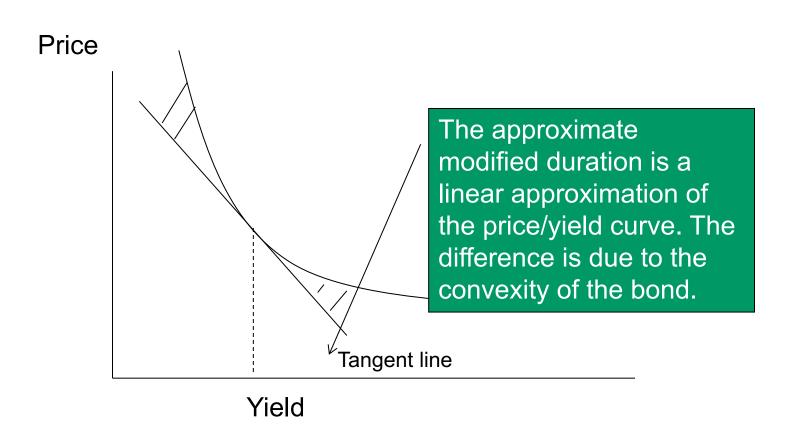
where PV_0 is the price of the bond at the current yield, PV_+ is the price of the bond if the yield increases (by Δ Yield), and PV_- is the price of the bond if the yield decreases (by Δ Yield).

Example: Consider a 6% semiannual coupon paying bond with 4 years to maturity currently priced at par (YTM = 6%).

If the YTM increases/decreases by annualized 20 bps, the price raises/decreases to 99.301 and 100.705, respectively:

AMD =
$$\frac{(100.705)-(99.301)}{2\times(0.002)\times(100)}$$
 = 3.51 years.

APPROXIMATE MODIFIED DURATION



APPROXIMATE MACAULAY DURATION

The approximate Macaulay duration (AD) is calculated from the approximate modified duration (AMD).

 $AD = AMD \times (1 + r)$

In the example from slide 15:

$$AD = 3.51 \times (1 + 0.03) = 3.615$$
 years

where

r = 3% = \$6 (annual coupon payment per \$100)/2.

GAME TIME

• You are playing a game with 3 doors. Behind one of the doors, there is a big prize (a nice car). The other two are empty. You pick one of the door. Now the show host looks at the remaining two doors, and he opens a door which is empty. Now there are two days. Do you switch your original choice?

EFFECTIVE DURATION

- Another approach to assess the interest rate risk of a bond is to estimate the percentage change in price given a change in a benchmark yield curve—for example, the government par curve.
- This estimate, which is very similar to the formula for approximate modified duration, is called the "effective duration":

EffDur =
$$\frac{(PV_{-}) - (PV_{+})}{2 \times (\Delta Curve) \times (PV_{0})}$$

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where Δ Curve is a parallel shift in the benchmark curve.

WHEN TO USE EFFECTIVE DURATION

essential to the measurement of the interest rate risk of a complex bond, such as a bond with an embedded option.

A callable/putable bond does not have a well-defined internal rate of return (yield-to-maturity). Therefore, yield duration statistics, such as modified and Macaulay durations, do not apply. Effective duration is the appropriate duration measure.

KEY RATE DURATION

A key rate duration (or partial duration) is a measure of a bond's sensitivity to a change in the benchmark yield curve at a specific maturity segment.

In contrast to effective duration, key rate durations help identify "shaping risk" for a bond—that is, a bond's sensitivity to changes in the shape of the benchmark yield curve (e.g., the yield curve becoming steeper or flatter).

PROPERTIES OF BOND DURATION

Bond duration is the basic measure of interest rate risk on a fixed-rate bond.

The duration for a fixed-rate bond is a function of these input variables.

- Coupon rate or payment per period
- Yield-to-maturity per period
- Time-to-maturity (as of the beginning of the period)
- Fraction of the period that has gone by
- Presence and nature of embedded options

COUPON RATE AND YIELD-TO-MATURITY RELATION TO MACAULAY DURATION

The coupon rate is inversely related to the Macaulay duration.

- A lower-coupon bond has a higher duration and more interest rate risk than a higher-coupon bond.
- The Macaulay duration of a zero-coupon bond is equal to its time-to-maturity.

The yield-to-maturity is inversely related to the Macaulay duration.

 A higher yield-to-maturity reduces the weighted average of the time to receipt of cash flow.

TIME-TO-MATURITY AND FRACTION OF THE PERIOD RELATION TO MACAULAY DURATION

Time-to-maturity is typically directly related to the Macaulay duration.

- This pattern always holds for bonds trading at par value or at a premium above par.
- The exception is deepdiscount bonds, where the relationship does not hold for a long time-to-maturity.

From the equation on slide 11, it is clear that the fraction of the period that has gone by (t/T) is inversely related to the Macaulay duration.

 Macaulay duration decreases smoothly as t goes from t = 0 to t = T and then jumps upward after the coupon is paid.

BONDS WITH EMBEDDED OPTIONS

 Bonds with embedded options (e.g., callable, putable) require the use of effective duration because Macaulay and modified yield duration statistics are not relevant.

The yield-to-maturity for callable and putable bonds is not well defined because future cash flows are uncertain.

- When benchmark yields are high (low), the effective durations of the callable (putable) and non-callable (nonputable) bonds are very similar. There is a large discrepancy in durations for callable (putable) and noncallable (non-putable) bonds when yields are low (high).
- In summary, the presence of an embedded option reduces the sensitivity of the bond price to changes in the benchmark yield curve (lower duration), assuming no change in credit risk.

CALCULATING THE DURATION OF A BOND PORTFOLIO

Bonds are typically held in a portfolio.

There are two ways to calculate the duration of a bond portfolio.

The weighted average of time to receipt of the aggregate cash flows

The weighted average of the individual bond durations that comprise the portfolio

This method is the theoretically correct approach, but it is difficult to use in practice.

This method is commonly used by fixed-income portfolio managers, but it has its own limitations.

MONEY DURATION

The **money duration** of a bond is a measure of the *price* change in units of the currency in which the bond is denominated.

- The money duration can be stated per 100 of par value or in terms of the actual position size of the bond in the portfolio.

Money duration (MoneyDur) is calculated as follows:

-

 $MoneyDur = AMD \times PV^{Full}$

The estimated change in the bond price in currency units is calculated by the following:

 $\Delta PV^{Full} \approx -MonD \times \Delta Yield$

MONEY DURATION CALCULATION

Example: Consider a 6% semiannual coupon bond with a current price of HKD100.940423 per HKD100 of par value and an annual modified duration of 6.1268 years. Suppose a life insurance company has a position in the bond of HKD100 million, and the market value of the investment is HKD100,940,423. Calculate the money duration and change in value of position as a result of a 100 bps decline in YTM.

- Money duration (MoneyDur) is calculated as $MoneyDur = 6.1268 \times HKD100,940,423 = HKD618,441,785.$
- The estimated change in the bond price in HKD is $\Delta PV^{Full} \approx -HKD618,441,785 \times 0.01 = -HKD6,184,418.$

PRICE VALUE OF A BASIS POINT (PVBP)

 The price value of a basis point (PVBP) is an estimate of the change in the full price given a 1 bp change in the yield-to-maturity.

The PVBP is calculated as follows:

$$PVBP = \frac{(PV_{-}) - (PV_{+})}{2}$$

Example: Assume a T-note is priced at 99.561006 and yields 0.723368%. An increase and decrease in 1 bp results in the price changing to 99.512707 and 99.609333, respectively. Calculate the PVBP:

$$PVBP = \frac{99.609333 - 99.512707}{2} = \mathbf{0.04831}.$$

CONVEXITY STATISTIC

 The true relationship between the bond price and the yieldto-maturity is the curved (convex) line, which shows the actual bond price given its market discount rate.

The linear approximation of estimated price change offered by duration is good for small yield-to-maturity changes. But for larger changes, the difference becomes significant.

• The **convexity statistic** for the bond is used to improve the estimate of the percentage price change provided by modified duration alone. There are various ways to estimate convexity:

$$\%\Delta PV^{Full} \approx (-AMD \times \Delta Yield) + \left[\frac{1}{2} \times Conv \times (\Delta Yield)^2\right]$$

CALCULATING A BOND'S CONVEXITY

Example: A 6% annual payment bond matures on 14 February 2022 and is purchased for settlement on 11 April 2014. The YTM is 4%. Calculate the bond's convexity (actual/actual convention):

Period	Time to Receipt	CF	PV of CF	<i>t</i> ^2+ <i>t</i>	$(t^2+t) \times PV$ of CF
1	0.8466	6	5.80	1.56	9.07
2	1.8466	6	5.58	5.26	29.34
3	2.8466	6	5.37	10.95	58.76
4	3.8466	6	5.16	18.64	96.19
5	4.8466	6	4.96	28.34	140.58
6	5.8466	106	84.28	40.03	3373.63
			111.15		3707.57

 $Conv = 1/(1 + 0.04)^2 \times 3707.57/111.15 = 30.84$

APPROXIMATE, MONEY, AND EFFECTIVE CONVEXITY

Like modified duration, convexity can be accurately approximated.

The approximate convexity is calculated by the following:

AConv =
$$\frac{(PV_{-}) + (PV_{+}) - [2 \times (PV_{0})]}{(\Delta Yield)^{2} \times (PV_{0})}$$

- •The **money convexity** of the bond is the annual convexity multiplied by the full price.
- •The **effective convexity** of a bond is a curve convexity statistic that measures the secondary effect of a change in a benchmark yield curve.

EffConv =
$$\frac{[(PV_{-})+(PV_{+})]-[2\times(PV_{0})]}{(\Delta Curve)^{2}\times(PV_{0})}$$

CALCULATING APPROXIMATE CONVEXITY

Example: Consider a 6% semiannual coupon paying bond with 4 years to maturity that is currently priced at par (YTM = 6%) and has an AMD of 3.51 years. If the YTM increases/ decreases by 20 bps, the price raises/decreases to 99.301 and 100.705, respectively. Calculate AConv and the effect of a 50 bps change in yield on the bond price:

AConv =
$$\frac{100.705 + 99.301 - (2 \times 100)}{(0.002)^2 \times 100}$$
 = **14.81**

 $\%\Delta PV^{Full} \approx -3.51 \times 0.005 + \frac{1}{2} \times 14.81 \times (0.005)^2 = 1.77\%$, including a 0.0185% convexity adjustment.

EFFECTS OF CONVEXITY ON BONDS

For the same decrease in yield-to-maturity, the more convex bond appreciates more in price. And for the same increase in yield-to-maturity, the more convex bond depreciates less in price.

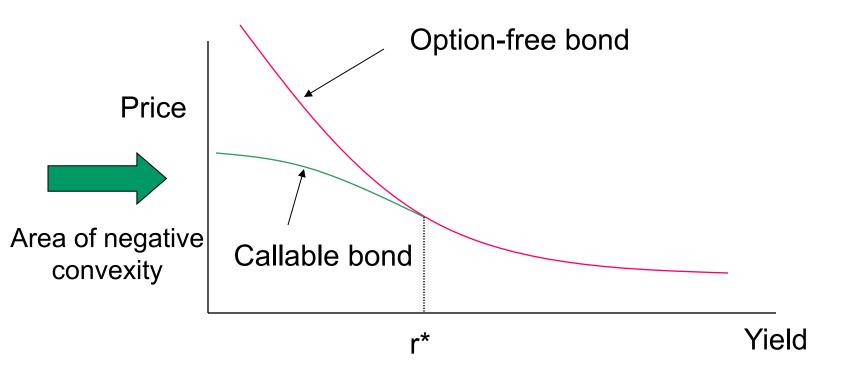


The conclusion is that the more convex bond outperforms the less convex bond in both bull (rising price) and bear (falling price) markets.

Option-free bonds always have positive convexity.

The negative convexity is present in callable bonds but not in putable bonds.

PRICE-YIELD RELATIONSHIP FOR A CALLABLE BOND



GAME TIME

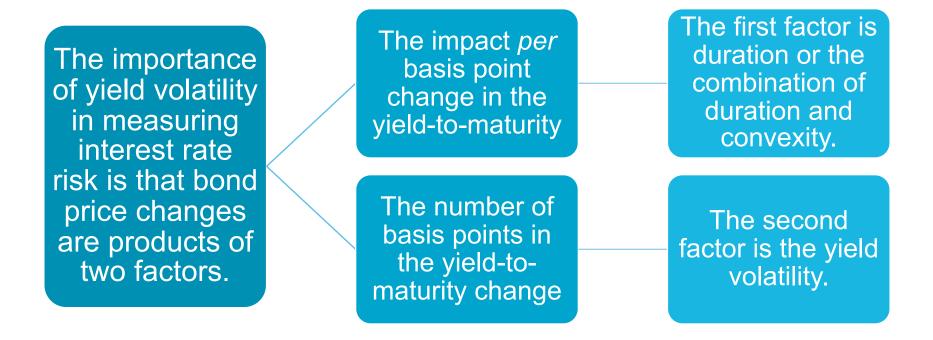
• 5 pirates looted a chest full of 100 gold coins. They agree on the following method to divide the root: The most senior pirate will propose a distribution of coins. All pirates vote. If at lease 50% of the pirates (3 in this case) agree, the gold is divided as proposed. If not, the most senior pirate will be killed and process goes to next senior pirate... until a plan is approved. Assume all pirates and perfectly rationale. They want to stay alive first, but also want to get as much gold as possible. Finally, being blood-thirsty, they want to have fewer pirates if given a choice between otherwise equal outcomes. How will the gold coins be divided in the end?

4. INTEREST RATE RISK AND THE INVESTMENT HORIZON

- An important aspect in understanding the interest rate risk and return characteristics of an investment in a fixed-rate bond is the time horizon.
- Bond duration is the primary measure of risk arising from a change in the yield-to-maturity; convexity is the secondary risk measure.
- The common assumption in interest rate risk analysis is a parallel shift in the yield curve. In reality, the shape of the yield curve changes based on factors affecting the supply and demand of shorter-term versus longer-term securities.

YIELD VOLATILITY

 The term structure of yield volatility is the relationship between the volatility of bond yields-to-maturity and timesto-maturity.



GENERAL RELATIONSHIPS AMONG INTEREST RATE RISK, THE MACAULAY DURATION, AND THE INVESTMENT HORIZON



When the investment horizon is greater than the Macaulay duration of a bond, coupon reinvestment risk dominates market price risk. The investor's risk is to lower interest rates.



When the investment horizon is equal to the Macaulay duration of a bond, coupon reinvestment risk offsets market price risk.



When the investment horizon is less than the Macaulay duration of a bond, market price risk dominates coupon reinvestment risk. The investor's risk is to higher interest rates.

• The difference between the Macaulay duration of a bond and the investment horizon is called the "duration gap."

5. CREDIT AND LIQUIDITY RISK

 The yield-to-maturity on a corporate bond is composed of a government benchmark yield and a spread over that benchmark. A change in the bond's yield-to-maturity can originate in either component or a combination of the two.

A change in the benchmark yield can arise from a change in either the expected inflation rate or the expected real rate of interest.

- The inflation duration would indicate the change in the bond price if expected inflation were to change by a certain amount.
- The real rate duration would indicate the bond price change if the real rate were to go up or down.

IMPACT OF CHANGES IN YIELD-TO-MATURITY

A change in the spread can arise from a change in the credit risk of the issuer or in the liquidity of the bond.

- For a bond with a given duration and convexity, the impact of changes in yield-to-maturity on the bond's price will be the same regardless of the source of the yield-tomaturity change.
- The problem for a fixed-income analyst is that it is rare for the changes in the components of the overall yield-tomaturity to occur in isolation.

6. SUMMARY

Sources of a bond's return

- The three sources of return on a fixed-rate bond purchased at par value are (1) receipt of the promised coupon and principal payments on the scheduled dates, (2) reinvestment of coupon payments, and (3) potential capital gains, as well as losses, on the sale of the bond prior to maturity.
- For a bond purchased at a discount or premium, the rate of return also includes the effect of the price being "pulled to par" as maturity nears, assuming no default.

Macaulay, modified, and effective durations

 Macaulay duration is the weighted average of the time to receipt of coupon interest and principal payments, in which the weights are the shares of the full price corresponding to each payment. This statistic is annualized by dividing by the periodicity.

Macaulay, modified, and effective durations (continued)

- Modified duration provides a linear estimate of the percentage price change for a bond given a change in its yield-to-maturity.
- Approximate modified duration approaches modified duration as the change in the yield-to-maturity approaches zero.
- Effective duration is a curve duration statistic that measures interest rate risk assuming a parallel shift in the benchmark yield curve.

Effective duration is the most appropriate measure of interest rate risk for bonds with embedded options

 Bonds with an embedded option do not have a meaningful internal rate of return because future cash flows are contingent on interest rates. Therefore, effective duration, not modified duration, is the appropriate interest rate risk measure.

Key rate duration

 Key rate duration is a measure of a bond's sensitivity to a change in the benchmark yield curve at specific maturity segments. Key rate durations can be used to measure a bond's sensitivity to changes in the shape of the yield curve.

A bond's maturity, coupon, embedded options, and yield level affect its interest rate risk

- Macaulay and modified durations are inversely related to the coupon rate and the yield-to-maturity.
- Time-to-maturity and durations are usually positively related. The exception is on long-term, low-coupon bonds, on which it is possible to have a lower duration than on an otherwise comparable shorter-term bond.

A bond's embedded options affect its interest rate risk

 The presence of an embedded call/put option reduces a bond's effective duration compared with that of an otherwise comparable non-callable/non-putable bond.

The duration of a portfolio and its limitations

- The duration of a bond portfolio can be calculated in two ways: (1) the weighted average of the time to receipt of aggregate cash flows and (2) the weighted average of the durations of individual bonds that compose the portfolio.
- The first method cannot be used for bonds with embedded options or for floating-rate notes.
- The second method is simpler to use and quite accurate when the yield curve is relatively flat. Its main limitation is that it assumes a parallel shift in the yield curve in that the yields on all bonds in the portfolio change by the same amount.

Money duration of a bond and price value of a basis point

- Money duration is a measure of the price change in terms of units of the currency in which the bond is denominated.
- The price value of a basis point (PVBP) is an estimate of the change in the full price of a bond given a 1 bp change in the yield-to-maturity.

Approximate convexity and effective convexity

- Convexity is the secondary, or second-order, effect on a bond's percentage price change given a change in the yieldto-maturity. It indicates the change in the modified duration as the yield-to-maturity changes.
- Effective convexity is the second-order effect on a bond price given a change in the benchmark yield curve.

Price change of a bond for a specified change in yield

• The change in a bond price is the product of (1) the impact per basis point change in the yield-to-maturity and (2) the number of basis points in the yield change.

Term structure of yield volatility and the interest rate risk of a bond

 For a particular assumption about yield volatility, the Macaulay duration indicates the investment horizon for which coupon reinvestment risk and market price risk offset each other. The assumption is a one-time parallel shift to the yield curve in which the yield-to-maturity and coupon reinvestment rates change by the same amount in the same direction.

Relationships among a bond's holding period return, duration, and investment horizon

- When the investment horizon is greater than the Macaulay duration of the bond, coupon reinvestment risk dominates price risk.
- When the investment horizon is equal to the Macaulay duration of the bond, coupon reinvestment risk offsets price risk.
- When the investment horizon is less than the Macaulay duration of the bond, price risk dominates coupon reinvestment risk.

Changes in credit spread and liquidity affect yield-tomaturity; duration and convexity can estimate the effects

 For a traditional (option-free) fixed-rate bond, the same duration and convexity statistics apply if a change occurs in the benchmark yield or a change occurs in the spread. The change in the spread can result from a change in credit risk or liquidity risk.

HOMEWORK

- 1. Read chapter 3 and 4
- 2. Read the material for Ethics
- 3. The (flat) price on a fixed-rate corporate bond falls one day from 92.25 to 91.25 per 100 of par value because of poor earnings and an unexpected ratings downgrade of the issuer. The (annual) modified duration for the bond is 7.24. What is the estimated change in the credit spread on the corporate bond, assuming benchmark yields are unchanged?

 increase 0.1497%
- 4. An investor buys a three-year bond with a 5% coupon rate paid annually. The bond, with a yield-to-maturity of 3%, is purchased at a price of 105.657223 per 100 of par value. Assuming a 5 bp change in yield-to-maturity, find the bond's approximate modified duration.
- 5. The interest rate risk of a fixed-rate bond with an embedded call option is *best* measured by:
 - A. effective duration.
 - B. modified duration.
 - C. Macaulay duration.
- 6. Which of the following is *most* appropriate for measuring a bond's sensitivity to shaping risk?
 - A. Key rate duration
 - B. Effective duration
 - C. Modified duration
- 7. A bond has an annual modified duration of 7.020 and annual convexity of 65.180. If the bond's yield-to-maturity decreases by 25 bps, the expected percentage price change is *closest* to:
 - A . 1.73%.
 - B . 1.76%.
 - **C** . 1.78%.