# FIXED INCOME ANALYSIS

Tony Zhang
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CFA Institute

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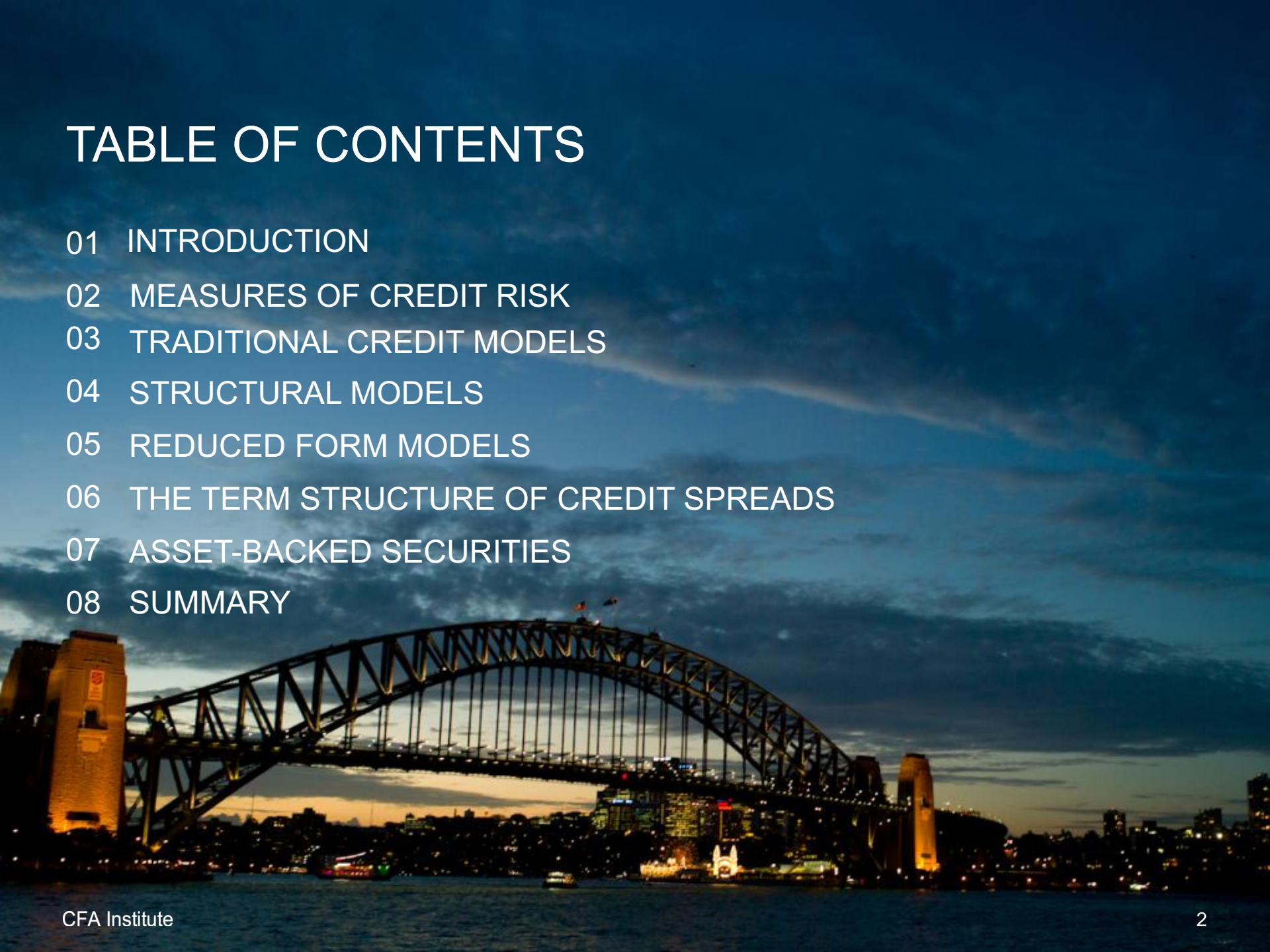
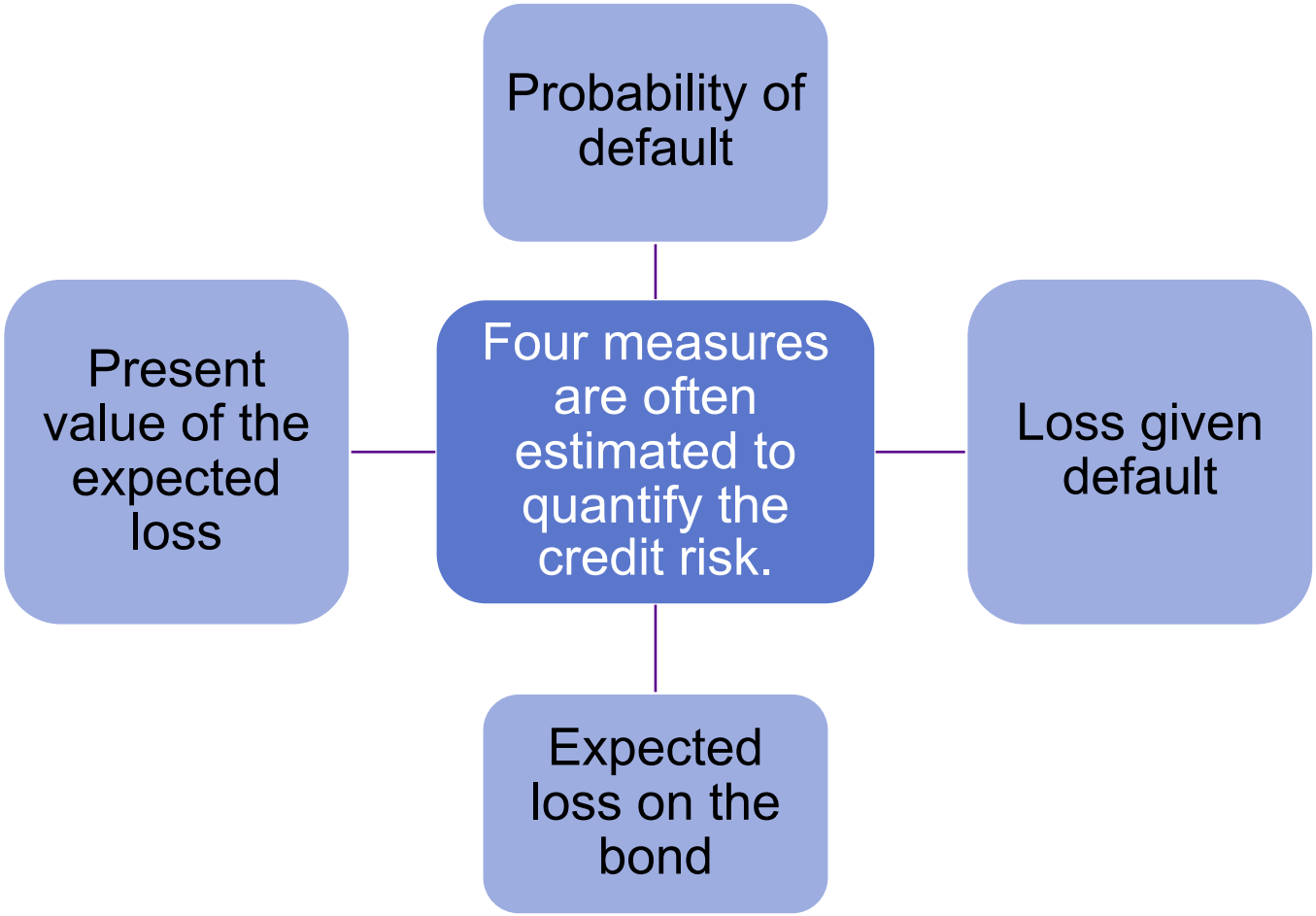


# 1. INTRODUCTION

- Since 1990, credit-related financial crises and consequent defaults by governments, companies, and individuals have spurred developments in credit risk analysis.
- Traditional credit ratings were only partially effective in capturing the changes in the default risks.
- As a result, additional tools to quantify and manage risks have been developed.

# 2. MEASURES OF CREDIT RISK



# PRESENT VALUE OF THE EXPECTED LOSS

- The present value of the expected loss is the most complex credit risk measure to calculate because it involves two modifications to the expected loss:

The first modification is to explicitly adjust the probabilities to account for the risk of the cash flows (the risk premium).

The second modification is to include the time value of money in the calculation—that is, the discounting of the future cash flows to the present.

- The present value of the expected loss is the most important credit risk measure, followed by expected loss and, finally, probability of default.

# MINI-QUIZ #1

1. The risk-neutral probabilities are not the same as the true real-world probabilities of the future states. True or False?
2. The rating agency's ratings are based on real-world default probabilities, rather than risk-neural probabilities. True or False?
3. Credit default swap (CDS) is priced based on real-world default probabilities, rather than risk-neural probabilities. True or False?

MBS, risk-neutral

# 3. TRADITIONAL CREDIT MODELS

- **Credit scoring** and **credit ratings**, two traditional approaches to credit risk analysis, apply to different types of borrowers.
- They are called "**ordinal ranking**" because they only "order" borrowers' riskiness from highest to lowest.

**Credit scoring**

- is used for small owner-operated businesses and individuals.

**Credit ratings**

- are used for companies, sovereigns, sub-sovereigns, and those entities' securities, as well as asset-backed securities.

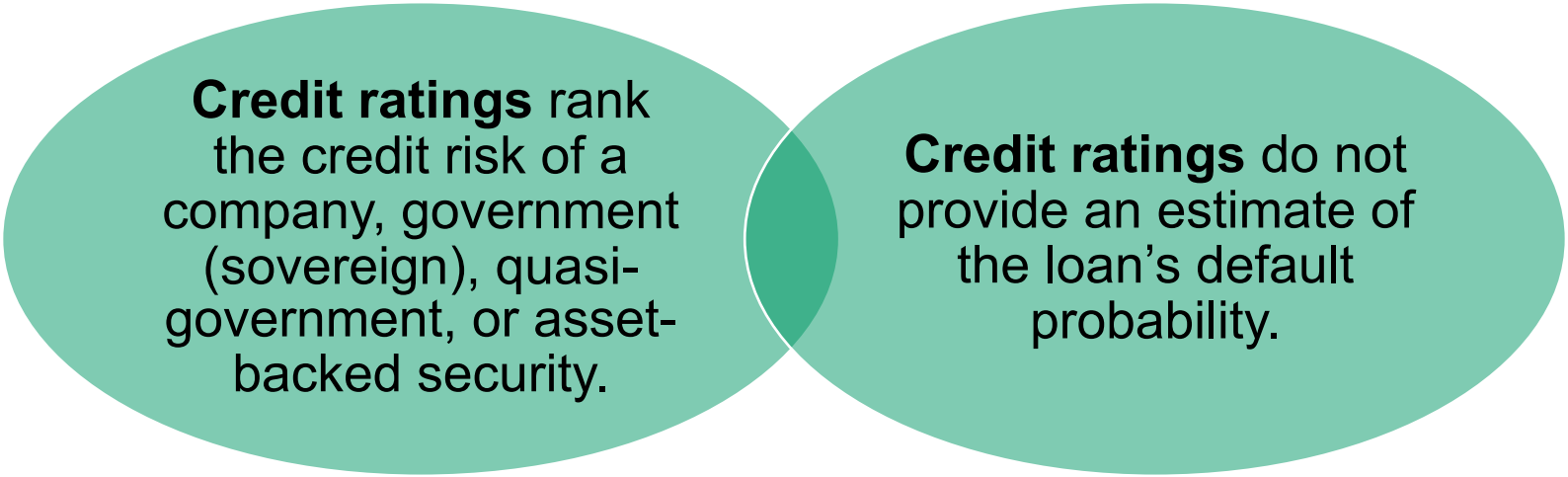
# CREDIT SCORING

**Credit scoring** ranks a borrower's credit riskiness. It does not provide an estimate of a borrower's default probability.

## The main features of credit scoring

- It does not explicitly depend on current economic conditions.
- It is not the percentile ranking of the borrower among a universe of borrowers.
- It has different implications for the probability of default depending on the borrower and the nature of the loan that has been extended.
- There is emphasis on credit scoring stability over time.

# CREDIT RATINGS




- The number of rating grades and their definitions vary among third-party rating agencies and among financial services firms, but their objective is the same:

  → Create an ordinal ranking of borrowers by riskiness as an aid to portfolio selection and risk management.

  - Rating agencies like Standard & Poor's and Moody's Investors Service use more than 20 rating grades, from **AAA** to **D**.

# STRENGTHS AND WEAKNESSES OF CREDIT RATINGS

Relative strengths:

- Simplicity
- Stability

Relative weaknesses:

- Inconsistent link with default probability
- No explicit link with the business cycle
- Compensation system with potential conflict of interest that may distort the accuracy of credit ratings

# MINI-QUIZ #2

1. A bank analyst is considering the loan applications of three individuals. Each is requesting a personal loan of $55,000. The bank can lend to only one of them. The bank's criteria emphasize the FICO score. Which individual is the bank analyst *most likely* to recommend lending to?

A. Individual A has a salary of $157,000, a net worth of $300,000, five credit cards, and a FICO score of 550.
B. Individual B has a salary of $97,000, a net worth of $105,000, two credit cards, and a FICO score of 700.

C. Individual C has a salary of $110,000, a net worth of $300,000, no credit cards, and a FICO score of 600.

# 4. STRUCTURAL MODELS

**Structural models** aim to understand the economics of a company's liabilities and build on the insights of option pricing theory.

- The link between option pricing theory and structural models comes from the *call option analogy* for equity.

  - The company's owners (equity holders) have limited liability.

  - If the equity holders default on the debt payment at time *T*, the debtholders' only recourse is to the company's assets. They have no additional claim on the equity holders' personal wealth.

# EXPECTED LOSS

Structural models can help estimate expected loss and the present value of expected loss. Expected loss is equal to the following:

$$E(\text{loss}) = \text{KN}(-e_2) - A_t e^{u(T-t)} N(-e_1)$$

where

$$e_1 = \frac{\ln\left(\frac{A_t}{K}\right) + u(T-t) + \frac{1}{2}\sigma^2(T-t)}{\sigma\sqrt{T-t}}$$

distance to default

$$e_2 = e_1 - \sigma\sqrt{T-t}$$

$A_t$ is the value of assets at time $t$; $K$ is the face value of debt; $N(.)$ is the cumulative standard normal distribution function with mean 0 and variance 1; $T - t$ is the debt's maturity of debt; $u$ and $\sigma$ are the annual expected return and volatility of the company's assets, respectively.

# PRESENT VALUE OF EXPECTED LOSS

Present value of expected loss is calculated as follows:

$$KP(t,T) - D(t,T) = Ke^{-r(T-t)}N(-d_2) - A_tN(-d_1)$$

where

$$d_1 = \frac{\ln\left(\frac{A_t}{K}\right) + r(T-t) + \frac{1}{2}\sigma^2(T-t)}{\sigma\sqrt{T-t}}$$

$$d_2 = d_1 - \sigma\sqrt{T-t}$$

$$P(t,T) = e^{-r(T-t)}$$

***r*** is the risk-free rate of interest.

# CALCULATING EXPECTED LOSS

**Example**. Assume a company has the following values: $A_t$ = \$1,000; $u_t$=0.03 per year; $r$ =0.01 per year; $K$ = \$700; time to maturity of debt, $T - t$ = 1 year; and $\sigma$ = 0.3 per year. Estimate the expected loss and the present value of the expected loss on this debt:

$$e_1 = \frac{\ln\left(\frac{1{,}000}{700}\right) + 0.03\times 1 + \frac{1}{2}(0.3)^2\times 1}{0.3\sqrt{1}} = \mathbf{1.43892}$$

$$e_2 = 1.43892 - 0.3\sqrt{1} = \mathbf{1.13892}$$

Using normal distribution table $N(-e_1) = 0.0751, N(-e_2) = 0.1274$

$$E(loss) = 700\times 0.1274 - 1{,}000e^{0.03\times 1}\times 0.0751 = \$\mathbf{11.78}$$

# CALCULATING PRESENT VALUE OF EXPECTED LOSS

**Example (continued)**. The present value of expected loss is calculated by:

$$d_1 = \frac{\ln\left(\frac{1{,}000}{700}\right) + 0.01\times 1 + \frac{1}{2}(0.3)^2\times 1}{0.3\sqrt{1}} = \mathbf{1.37225}$$

$$d_2 = 1.37225 - 0.3\sqrt{1} = \mathbf{1.07225}$$

Using a normal distribution table $N(-d_1) = 0.085, N(-d_2) = 0.1418$

$$KP(t,T) - D(t,T) = 700e^{-0.01\times 1}\times 0.1418 - 1{,}000\times 0.0850$$
$$= \$\mathbf{13.28}$$

- The $1.50 difference between expected loss and present value of expected loss includes both a discount for the time value of money and the risk premium required by the market to bear the risk of credit loss.

# STRENGTHS AND WEAKNESSES OF STRUCTURAL MODELS

Strengths:

- Optional analogy of a company's default probability and recovery rate
- Estimated using current market prices

Weaknesses:

- Balance sheets hard to model
- Can be estimated only using implied estimation
- Inherit errors in the model's formulation
- Business cycles are not taken into account

# 5. REDUCED FORM MODELS

- Reduced form models were originated to overcome a key weakness of the structural model—the assumption that the company's assets trade.
- Reduced form models replace this assumption with a more robust one—that some of the company's debt trades.

They are called "**reduced form models**" because they impose their assumptions on the outputs of a structural model—the probability of default and the loss given default—rather than on the balance sheet structure itself.

This change in perspective gives reduced form models tremendous flexibility in matching actual market conditions.

# REDUCED FORM MODEL ESTIMATIONS

Using the reduced form model, the following three credit risk measures can be estimated.

1. Probability of the debt defaulting over (t, *T*):

$$\text{prob}(\tau \leq T) = 1 - e^{-\lambda (T-t)}$$

2. The expected loss:

$$E(\text{loss}) = K\left[1 - e^{-\lambda\gamma(T-t)}\right]$$

3. The present value of the loss given default:

$$KP(t,T) - D(t,T) = KP(t,T)\left[1 - e^{-\lambda\gamma(T-t)}\right]$$

where λ is the default intensity and γ is loss given default.

# CALCULATING REDUCED FORM MODELS

**Example**. Assume a company has the following values for its debt issue: $K$ = \$700; time to maturity of debt, $T - t$ = 1 year; $\lambda = 0.01$; $\gamma = 0.4$; $P(t,T) = 0.96$. Using the reduced form model, estimate the credit risk measures:

- Probability of the debt defaulting over (0, $T$):

$$\text{prob}(\tau \leq T) = 1 - e^{-0.01\times 1} = \mathbf{0.00995}$$

- The expected loss:

$$E(\text{loss}) = 700[1 - e^{-0.4\times 0.01\times 1}] = \$\mathbf{2.79}$$

- The present value of the expected loss:

$$KP(t,T) - D(t,T) = 700\times 0.96[1 - e^{-0.4\times 0.01\times 1}] = \$\mathbf{2.68}$$

# ESTIMATION

## Two approaches to estimate a model's parameters

The **implicit approach** can be used for both structural and reduced form models. To use implicit estimation, one must completely specify the inputs to the model and the probability distributions for the macroeconomic state variables.

The **historical approach** can be used only for reduced form models because the economy's macroeconomic state variables and the company's debt prices are both observable. Estimating a reduced form model's parameters using historical estimation is an application of *hazard rate* estimation.

# STRENGTHS AND WEAKNESSES OF REDUCED FORM MODELS

Strengths:

- The model inputs are observable
- Business cycle is taken into account
- No need for specification of balance sheet structure

Weaknesses:

- Hard to properly formulate the model and back test it

# MINI-QUIZ #3

- You are estimating a simple regression:

$$Y = \alpha + \beta X + \epsilon$$

and find that Y = 0.05 + 1.91 X . $R^2 = 0.92$

Now you are asked to estimate the simple regression:

$$X = \alpha' + \beta' Y + \epsilon$$

with the same data. What will be $\beta'$?

# 6. THE TERM STRUCTURE OF CREDIT SPREADS

- The **term structure of credit spreads** corresponds to the spread between the yields on default-free and credit risky zero-coupon bonds.
- Using either the structural or reduced form model, under the frictionless market assumption, the credit spread is entirely due to credit risk.

**Credit spread** is equal to the difference between the average yields on the risky zero-coupon bond and the riskless zero-coupon bond.

In practical applications, the "true" credit spread will consist of both = Expected percentage loss (as in the structural and reduced form models) + Liquidity risk premium

# COMPUTING THE PRESENT VALUE OF EXPECTED LOSS USING THE TERM STRUCTURE OF CREDIT SPREADS

The present value of expected loss, PV *of E*(*l*oss), is calculated as follows:

$$\text{PV } of\ E(\text{loss}) = [P(t,T) - D(t,T)]X_T$$

- This represents the present value of the cash flow, if riskless, minus the present value of the cash flow considering credit risk.

**Example**. Consider Powder Corporation, which has promised to pay investors 25 euros on 30 September 2014. Today is 11 August 2011. The risk-free zero-coupon yield is 0.3718%. Credit spread for payment due on 30 September 2014 is 0.2739%. Calculate the PV of the expected loss implied by the credit spread.

# COMPUTING THE PRESENT VALUE OF EXPECTED LOSS USING THE TERM STRUCTURE OF CREDIT SPREADS

**Example (continued).**

**Solution:**

- Total yield = 0.3718% + 0.2739% = **0.6457%**
- Years to maturity = 3 years and 51 days = **3.1397 years**

$$\text{PV of CF} = 25 \times e^{-0.006457 \times 3.1397} = \$\mathbf{24.4983}$$

$$\text{PV of CF}_{rf} = 25 \times e^{-0.003718 \times 3.1397} = \$\mathbf{24.7099}$$

$$\text{PV of } E(\text{loss}) = 24.7099 - 24.4983 = \$\mathbf{0.2116}$$

*Note*: For a coupon bond, the sum of PV of the loss for each cash flow is calculated.

# 7. ASSET-BACKED SECURITIES

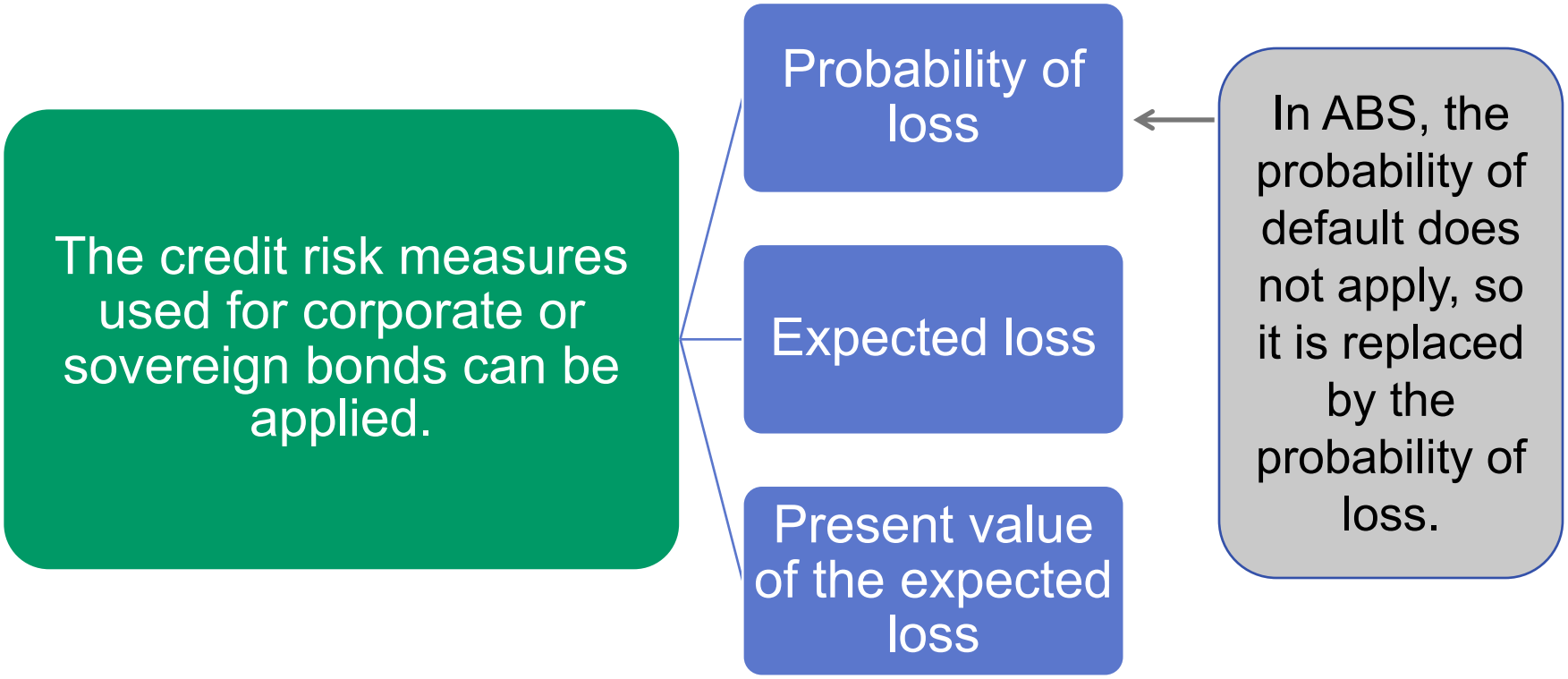
An **asset-backed security** (ABS) is a type of bond issued by a legal entity called a "**special purpose vehicle**" (SPV).

An SPV is formed to own a collection of assets, called its "**collateral pool**."

The collateral pool usually consists of a collection of loans of a particular type.

- Unlike corporate debt, an asset-backed security does not default when an interest payment is missed.

# MEASURES FOR ASSET-BACKED SECURITIES




To calculate these measures, a model analogous to those used for corporate and sovereign debt is used.

- However, the calculations are much more complex.

# 8. SUMMARY

## Credit measures of a bond

- There are four credit risk measures of a bond: the probability of default, the loss given default, the expected loss, and the present value of the expected loss.
- The present value of the expected loss is the most important because it incorporates an adjustment for risk and the time value of money.

## Credit scoring and credit ratings

- Credit scoring and credit ratings are traditional approaches to credit risk assessment and are used to rank retail borrowers versus companies, governments, and structured products.

# SUMMARY

## Structural models of corporate credit risk

- Structural models of credit risk assume a simple balance sheet for the company consisting of a single liability, a zero-coupon bond.
- In a structural model, the company's equity can be viewed as a European call option on the assets of the company, with a strike price equal to the debt's face value.

## Reduced form models of corporate credit risk

- Reduced form models of credit risk consider a company's traded liabilities.
- Using option pricing methodology, reduced form models provide insights into the debt's expected loss and the present value of the expected loss.

# SUMMARY

## Determinants of the term structure of credit spreads

- The term structure of credit spreads is the difference between yields on risky bonds and default-free zero-coupon bonds.
- These yields can be estimated from the market prices of traded coupon bonds of both types.

## Present value of the expected loss

- The present value of the expected loss on any bond can be estimated using the term structure of credit spreads.

# SUMMARY

## Credit analysis required for asset-backed securities

- Asset-backed securities (ABS) are liabilities issued by a special purpose vehicle (SPV). The SPV's assets, called a "collateral pool," consist of a collection of loans.
- ABS do not default, but they can lose value as the SPV's collateral pool incurs defaults. Modeling an ABS's credit risk—the probability of loss, the loss given default, the expected loss, and the present value of the loss—is a complex exercise.