Yifu He
Assignment 5

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HW5.

17.Q1.

a). According to BSM.

$$C = S_0 N(d_1) - N(d_2) K e^{-r(T-t)}$$

$$d_1 = \frac{1}{\sigma\sqrt{T-t}}\left[\ln\frac{S_t}{K} + (r+\frac{\sigma^2}{2})(T-t)\right], \quad d_2 = d_1 - \sigma\sqrt{T-t}.$$

Calculated by Python. $S_0 = 30$, $K=29$, $t=\frac{1}{3}$, $r=0.03$.

$C = \$2.410559$.

b). American option without dividends will never be executed before the delivery date. So the American call option has the same price as the European call option which is $\$2.410559$.

c). $p = N(-d_2) K e^{-r(T-t)} - N(-d_1) S_t$

$P = \$1.122004$

d). call-put parity is

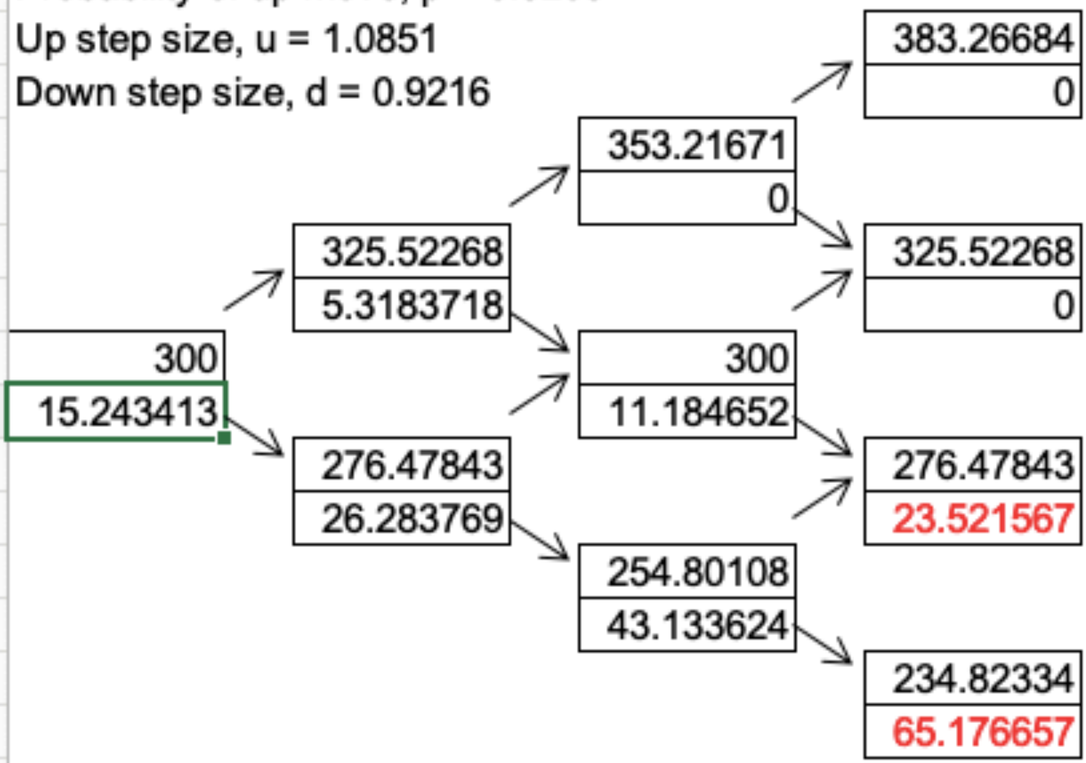
$$p + S_0 = c + K e^{-rT}$$

$p + S_0 = 30 + 1.12204$ $\quad$ $Ke^{-rT} + c \approx 2.410559 + 29e^{-0.03/3}$

$= 31.12204$ $\quad$ $\approx 31.12204.$

At each node:
Upper value = Underlying Asset Price
Lower value = Option Price
Values in red are a result of early exercise.

Strike price = 300
Discount factor per step = 0.9917
Time step, dt = 0.1667 years, 60.83 days
Growth factor per step, a = 1.0067
Probability of up move, p = 0.5205
Up step size, u = 1.0851
Down step size, d = 0.9216



Node Time:

0.0000	0.1667	0.3333	0.5000

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			383.26684
			0
		353.21671	
		0	
	325.52268		325.52268
	5.3183718		0
300		300	
15.71039		11.184652	
	276.47843		276.47843
	27.265831		23.521567
		254.80108	
		45.198921	
			234.82334
			65.176657
Node Time:			
0.0000	0.1667	0.3333	0.5000

27.Q2.

a) Use the Derivagem software, we can calculate the price of this European option, the price is \$ 15.24341.

b) Use the Derivagem, we have the price of American put option, the price is \$ 15.71039

contain the picture of binomial tree in the attached file.

Q3

37. According the ~~the~~ formula of put-call parity,

$$C+Ke^{-rT}=p+F_0e^{-rT}$$

$$C+80e^{-0.02\cdot0.5}=6.5+78e^{-0.02*0.5}$$

$$C=4.5200.$$

calculated by Python.

American option:

$$F_0e^{-rT}-K<C-P<F_0-Ke^{-rT}$$

$$78e^{-0.02*0.5}-80<C-6.5<78-80\cdot e^{-0.02*0.5}$$

$$3.7239<C<5.2960$$

Q4.

a). calculate the delta:

$$-1000\times 0.5 - 500\times 0.8 - 2000\times(-0.40) - 500\cdot 0.70 = -450$$

Calculate the gamma:

$$-1000\times 2.2 - 500\times 0.6 - 2000\times 1.3 - 500\times 1.8 = -6000$$

calculate the vega:

$$-1000\times 1.8 - 500\times 0.2 - 2000\times 0.7 - 500\times 1.4 = -4000$$

In order to neutralize this gamma and delta, we need w1 amount of traded options and w2 amout of sterling.

Thus. $\begin{cases} w_1 * 1.5 - 6000 = 0; \\ w_1 \times 0.6 - 450 + w_2 = 0. \end{cases}$

$\Rightarrow \begin{cases} w_1 = 4000 \\ w_2 = 1950 \end{cases}$. Long 4000 traded options
short 1950 position of sterling.

b). Similar to the question a, we assume that we need w1 amount of traded options and w2 amount of sterling. So $w1*0.8 - 4000 = 0$ and $w1*0.6 - 450 + w2 = 0$.

As a result, $\begin{cases} w_1 = 5000 \\ w_2 = 2550. \end{cases}$

5000 long position in options.
2550 short position in sterling.

Q5. see the attached file.

76. Q6.

Let X be the Gold and Y be the silver. The standard deviation of each asset is:

$\sigma_X = 300000 \cdot 1.8\% = 5400$

$\sigma_Y = 500000 \cdot 1.2\% = 6000$

$\sigma_{X+Y} = \sqrt{5400^2 + 6000^2 + 0.6 \times 2 \times 5400 \times 6000} = 10200$

We assume that the mean is equal to 0, $N^{-1}(1-0.975) = -1.95996$.
So 1-day 97.5% VaR is \$19991.63 and 10-day 97.5% VaR is \$63219.09.

10-day VaR for gold

$300000 \times 0.018 \times \sqrt{10} \times 1.96 = 33469.5$

silver.

$500000 \times 0.012 \times \sqrt{10} \times 1.96 = 37188.4$

the risk diffused is 7437.9.