Weekly Homework 2 570

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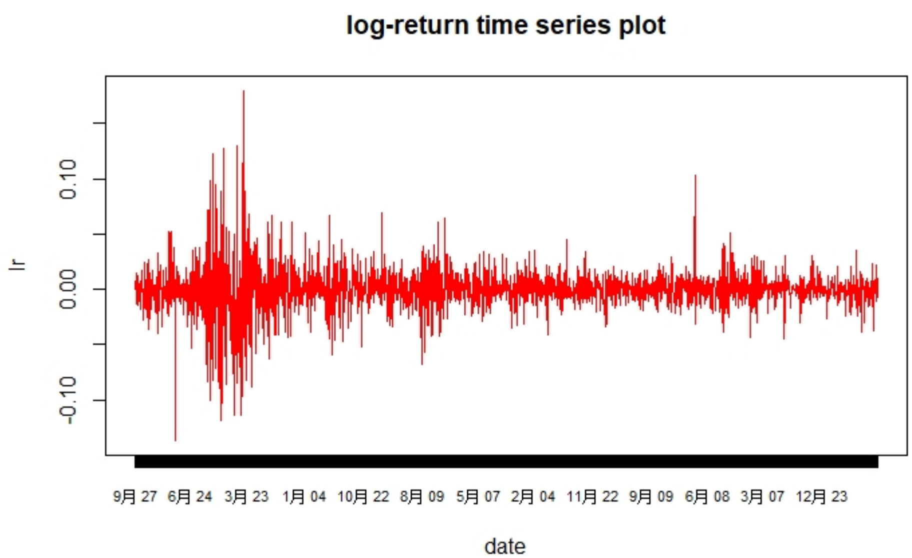
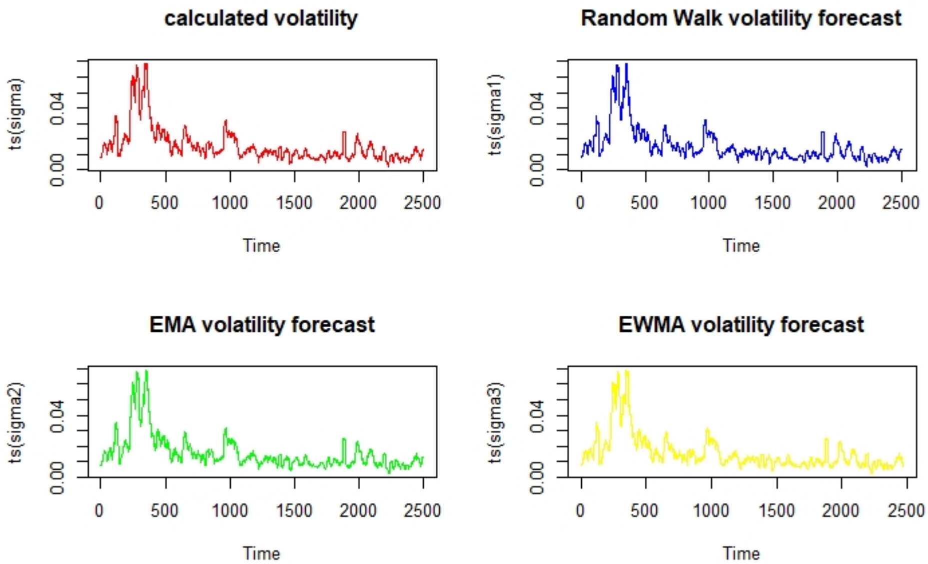
Assignment 1.

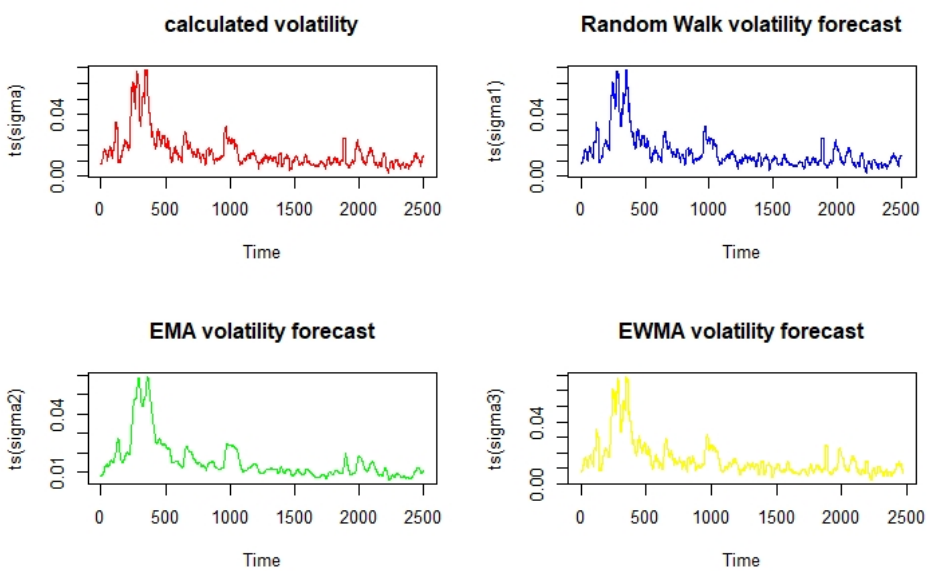
Solution.
The results were printed in the following pictures:



The compare of these three estimation methods($\beta = 2/(n+1)$):



The compare of these three estimation methods($\beta = 0.95$,bigger beta):



comments:

From the result, the compare of these three estimation methods under$\beta = 2/(n+1)$ seems very similar to each other. However, from intuition, EWMA must be the most accurate methods to estimate σ_t. In my opinion, β is too small here, so for EMA, $(1-\beta)\sigma_{t-1}$ is very close to σ_{t-1},$\beta\hat{\sigma}_{t-1}$ is close to zero, so the estimation will be very close to those by Random Walk. Therefore, if the β is very small, the estimation between Random Walk forecast and EMA will be very similar. So when I modify β to be bigger, there are more differemces, the EMA estimation curves seems smoother than others. But, i do not know how to explain why Random Walk forecast and EWMA forecast seems to similar. The fitted value evolves in a different way, but their time behaviour are smilar.

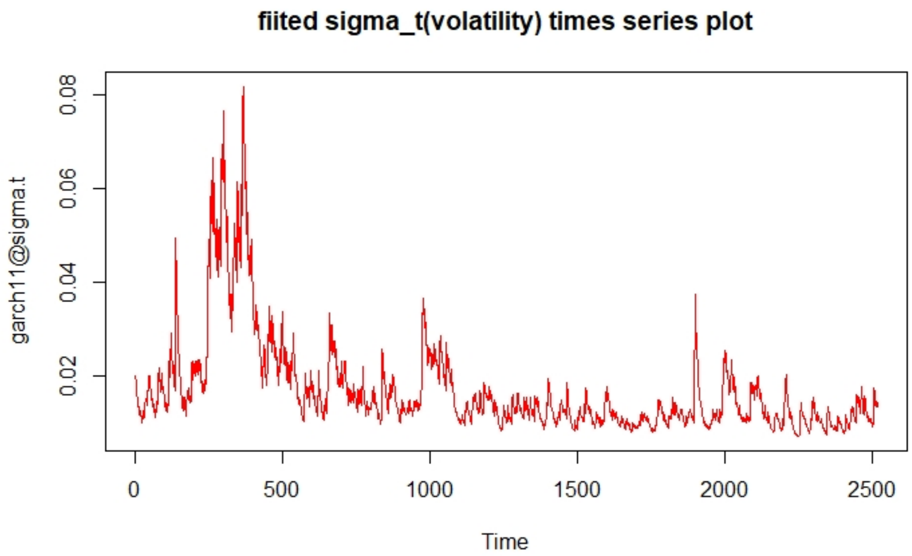
R code attached:

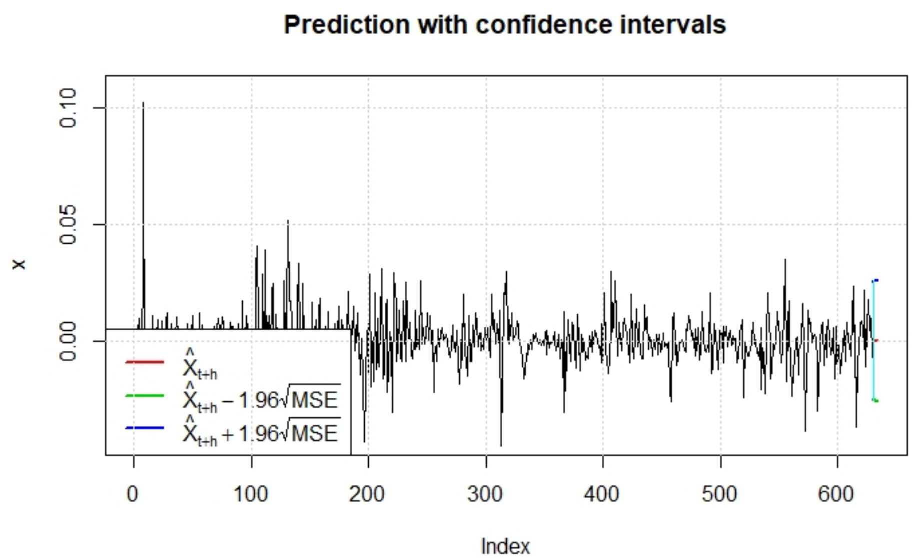
```
install.packages("fGarch")
library(fGarch)
#Problem 1
ge<-read.csv(file.choose())
names(ge)
close<-ge$Adj.Close
n1<-length(close)
date<-ge$Date
lr<-log(close[-1]/close[-n1])
n2<-length(lr)
date <- as.Date(date, "%m/%d/%Y")[-n1]
par(mfrow=c(1,1))
plot(lr ~ date,xaxt = "n", type = "l",col='red',
main="log-return time series plot")
axis(1, date, format(date, "%b %d"), cex.axis = .7)
n<-22

#calculate sigma
sigma<-c()
for(i in n:n2)
{
mean<-mean(lr[(i-n+1):i])
s_temp<-sqrt(sum((lr[(i-n+1):i]-mean)^2)/n)
sigma<-c(sigma,s_temp)
}

#The Random walk forecast
sigma1<-c(rep(0,n2))
sigma1[1]=sigma[1]
for(i in 2:n2)
{
  sigma1[i]=sigma[i-1]
}

#The EMA forecastast
sigma2<-c(rep(0,length(sigma)))
beta<-2/(n+1)
sigma2[1]<-sigma[1]
for(i in 2:length(sigma2))
{
sigma2[i]<-(1-beta)*sigma[i-1]+beta*sigma2[i-1]
}
```

```
#The EWMA forecast
sigma3<-c(rep(0,length(sigma)))
weight<-c(rep(0,n))
total<-sum(beta^c(1:22))

#calculate the weight for each historical information
for(i in 1:22)
{
  weight[i]<-(beta)^i/total
}
for(j in (n+1):length(sigma))
{
  sigma3[j]<-sum(sigma[(j-1):j-n]*weight)
}
sigma3<-sigma3[-(1:22)]

#Compare the differences of these three estimation methods
par(mfrow=c(2,2))
ts.plot(ts(sigma),col="red",
main="calculated volatility")
ts.plot(ts(sigma1),col="blue",
main="Random Walk volatility forecast")
ts.plot(ts(sigma2),col="green",
main="EMA volatility forecast")
ts.plot(ts(sigma3),col="yellow",
main="EWMA volatility forecast")
par(mfrow=c(1,1))
```

Assignment 2.

Solution.
The results were printed in the following pictures:






```
> predict(garch11,5,plot=TRUE)
  meanForecast  meanError standardDeviation lowerInterval upperInterval
1 0.0001659892 0.01286648        0.01286648   -0.02505184    0.02538382
2 0.0001659892 0.01299422        0.01299422   -0.02530222    0.02563420
3 0.0001659892 0.01311923        0.01311923   -0.02554723    0.02587921
4 0.0001659892 0.01324160        0.01324160   -0.02578707    0.02611905
5 0.0001659892 0.01336142        0.01336142   -0.02602191    0.02635388
> for1<-sum(sigma[length(sigma):(length(sigma)-21)]*weight)
> for1
[1] 0.01127855
```

commets:
From the volatility times series plot, we can tell that EWMA forecast is very similar to GARCH(1,1) forecast. The 1day ahead implied volatility forecast from GARCH(1,1) is 0.01286648, from EWMA is 0.01304082.They are very close.

R code attached:

```
#Problem 2 Garch(1,1)
garch11<-garchFit(formual~garch(1,1),data=lr)
summary(garch11)
garch11@sigma.t
par(mfrow=c(1,1))
ts.plot(garch11@sigma.t,col="red",
main="fiited sigma_t(volatility) times series plot")
predict(garch11,5,plot=TRUE)
for1<-sum(sigma[length(sigma):(length(sigma)-21)]*weight)
for1
```

Assignment 3.

Solution.
For $l \geq 2$:

$$\begin{aligned}\gamma_l \equiv Cov(\Delta p_{t-l}, \Delta p_t) &= E(\Delta p_{t-l}, \Delta p_t)\\ &= E((p_{t-l} - p_{t-l-1}) \times (p_t - p_{t-1}))\end{aligned}$$

Since $l \geq 2, p_{t-l} - p_{t-l-1}$is independent of $p_t - p_{t-1}$,So γ_l is zero for any l greater than 2.

According to the Roll model:

$$c = \sqrt{-\gamma_1} \tag{1}$$
$$\sigma_u^2 = \gamma_0 + 2\gamma_1 \tag{2}$$

Here,γ_0=0.0003917542,γ_1=-3.936707e-06

Therefore,

$$spread = 2c = 0.003968227, fundemental\ volatility = 0.0003838808$$

R code attached:

```
#Problem 3
#gamma_{l}is 0 for every l>=2
which(ge$Date=="9/26/17")
diff_p<-log(close[-1]/close[-n1])
gamma<-acf(diff_p,type="covariance",lag.max=1,plot=FALSE)
gamma
gamma0<-gamma$acf[1]
gamma1<-gamma$acf[2]
```

```
c<-sqrt(-gamma1)
sigma_u<-gamma0+2*gamma1
spread<-2*c
spread
sigma_u
```