FE570 Financial Markets and Trading

Lecture 9. Arbitrage Trading Strategies

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Outline

Market Risk Measurement: there are several possible causes of financial losses (see Jorion 2000).

- *Market risk* is resulted from unexpected changes in the market prices, interest rates, or foreign exchange rates.
- Other types of financial risk include *liquidity risk*, *credit risk*, and *operational risk*.
  - *Liquidity risk* is determined by a finite number of assets available at a given price, and another form of liquidity risk refers to the inability to pay off debt on time.
  - *Credit risk* arises when one of the counterparties involved in a financial transaction does not fulfill its obligation.
  - *Operational risk* is a generic notion for unforeseen human and technical problems, such as fraud, accidents, and so on.
- We will focus exclusively on market risk here. Historical volatility σ calculated using the daily returns is the most straightforward measure of market risk. Other measures (e.g. Beta, Sharpe-ratio, VaR and ETL) are also widely used.

Beta β: In equities, the parameter Beta is defined as:

$$\beta_i = Cov(R_i, R_p)/Var(R_p) \tag{1}$$

where $R_i = r_i - r_f$ and $R_p = r_p - r_f$ are the excess returns of a stock i and the entire market portfolio, respectively; r_i is the return of the stock i, r_p is the portfolio return, and r_f is the return of a risk-free asset.

- The definition of Beta stems from the classical *Capital Asset Pricing Model* (CAPM) (e.g., Bodie & Merton 1998)

$$E[r_i] = r_f + \beta_i(E[r_p] - r_f) \tag{2}$$

- Usually, the S&P 500 index and three-month U.S. Treasury bills are used as proxies to the market portfolio and the risk-free assets, respectively.
- Beta defines sensitivity: namely, $\beta_i > 1$ means that the asset is more volatile than the entire market, while β_i implies that the asset has lower sensitivity to the market movements. CAPM is valid only if $\beta_i \geq 0$ (not for a bear market).

Sharpe Ratio: A ratio developed by Nobel laureate William F. Sharpe to measure risk-adjusted performance. The Sharpe ratio formula is:

$$\text{Sharpe ratio} = \frac{\bar{r}_p - r_f}{\sigma_p} \tag{3}$$

where $\bar{r}_p$ is expected portfolio return; r_f is the return of a risk-free asset, and σ_p is portfolio standard deviation.

- The Sharpe ratio tells us whether a portfolio's returns are due to smart investment decisions or a result of excess risk. This measurement is very useful because although one portfolio or fund can reap higher returns than its peers, it is only a good investment if those higher returns do not come with too much additional risk. The greater a portfolio's Sharpe ratio, the better its risk-adjusted performance has been. A negative Sharpe ratio indicates that a risk-less asset would perform better than the security being analyzed.

VaR: A statistical technique used to measure and quantify the level of financial risk within a firm or investment portfolio over a specific timeframe. Value at risk is used by risk managers in order to measure and control the level of risk which the firm undertakes. The risk manager's job is to ensure that risks are not taken beyond the level at which the firm can absorb the losses of a probable worst outcome.

- VaR refers to the maximum amount of an asset that is likely to be lost over a given period at a specific confidence level α. It is often assumed that the probability density function of *profit* and *loss* follows a normal distribution, and hence

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$$\mathrm{VaR}(\alpha) = -\sigma z_\alpha - \mu \tag{4}$$

where μ is the mean return and σ is the standard deviation.

- The value of z_σ can be determined from the cumulative distribution function:

The standard normal distribution:

$$\text{VaR}(\alpha) = -\sigma z_\alpha - \mu$$

$$\Pr(Z \leq z_\alpha) = -\int_{-\infty}^{z_\alpha} \frac{1}{\sqrt{2\pi}} exp[-z^2/2]dz = 1 - \alpha$$

Since $z_\alpha < 0$ at $\alpha > 50\%$, it implies that positive values of VaR point to losses.

- In general, $VaR(\alpha)$ grows with the confidence level α. 置信水平 Sufficiently high values of mean move $VaR(\alpha)$ into the negative region, which implies profits for a given α rather than losses.
- The advantage of VaR is that it is a simple and universal measure that can be used for determining the risks of different financial assets and entire portfolios.
- VaR may discourage investment diversification, because adding volatile assets to a portfolio may move VaR above the chosen risk threshold.

Expected Tail Loss (ETL): While VaR is an estimate of loss within a given confidence level, ETL is an estimate of loss within the remaining tail (i.e. an average worst-case scenario).

$$\text{ETL} = E[L|L > VaR]$$

- For a given probability distribution of P/L and a given α, ETL is always higher than VaR.
- As a simple example of calculating VaR and ETL, consider a sample of 100 P/L values. Say the chosen confidence level is 95%. Then, VaR is the sixth smallest number in the sample, while ETL is the average of the five smallest numbers within the sample.
- After all, VaR and ETL are connected in the sense that from the VaR surface of the tail ETL figures can easily be calculated.

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Arbitrage Trading Strategies: According to the Law of One Price, equivalent assets (i.e., assets with the same payoff) must have the same price (see, e.g. Bodie & Merton 1998). In competitive markets, the price of an asset must be the same worldwide providing that it is expressed in the same currency, and the transportation and transaction costs can be neglected. Violation of the Law of One Price leads to *arbitrage*, which is risk-free profiteering by buying an asset in a cheaper market and immediately selling it in a more expensive market.

- Arbitrage can continue only for a limited time until mispricing is eliminated due to increased demand and finite supply of a cheaper asset. Such a clear-cut opportunity, which guarantees a risk-less gain, is sometimes called *pure arbitrage* (Bondarenko 2003). Deterministic arbitrage should be discerned from *statistical arbitrage*, which is based on statistical deviation of asset prices from their expected values and may incur losses (Jarrow et al. 2005).

Arbitrage Trading Strategies:

- Many arbitrage trading strategies are based on hedging the risk of financial losses by combining long and short positions in the same portfolio. An ultimate long/short hedging yields a market-neutral portfolio, in which risks from having long and short positions compensate each other. Market-neutral strategy can be described in terms of the CAPM.
- Since market-neutral strategy is supposed to completely eliminate market risk, the parameter β_i for the market-neutral portfolio equals zero.
- Not all arbitrage trading strategies are market-neutral: the extent of long/short hedging is often determined by the asset fund manager's discretion. The hedge fund industry has been using a taxonomy that includes not only strategy type but also instruments and geographic investment zones (Stefanini 2006; Khandani & Lo 2007).

Example

- *A simple example of market-neutral strategy:* consider two companies within the same industry, A and B, one of which, say A, often yields higher returns.
- A simple investing approach might be buying shares A and neglecting shares B. A more sophisticated strategy named *pair trading* may involve simultaneously buying shares A and short-selling shares B. Obviously, if the entire sector rises, this strategy does not bring as much money as simply buying shares A. However, if the entire market falls, it is expected that shares B will have higher losses than share A. Then, profit from short selling shares B would compensate for the loss resulting from buy shares A.
- Generally, pair trading is based on the idea of *mean reversion*. Namely, it is expected that the divergence in returns of similar companies A and B is a temporary effect due to market inefficiency, which is eliminated over some time.

Hedging Strategies:

- **Relative Value Arbitrage**
  - **Equity Hedging** One of two positions (e.g. the long one) is a stock index future while the other (short) one consists of all stocks that constitute this index (so-called *index arbitrage*). Pair trading is its special case, and American Depository Receipts (ADRs) fit in this category. ADR is a security that represent shares of non-U.S. companies traded in the U.S. markets.
  - **Equity market-neutral strategy and statistical arbitrage** This strategy creates a hedge against market factors. "Highly technical short-term mean-reversion strategies involving large numbers of securities, very short holding periods (measured in days to seconds), and substantial computational, trading, and IT infrastructure". .
  - **Convertible arbitrage**. Convertible bonds often decline less in a falling market than shares of the same company does. Hence, the idea of convertible arbitrage may be buying convertible bonds and short selling the underlying stocks.

Relative Value Arbitrage:

- **Fixed-income arbitrage** This strategy implies taking long and short positions in different fixed-income securities. Using analysis of correlations between different securities, one can buy those securities that seem to become underpriced and sell short those that look overpriced.

  One example is issuance-driven arbitrage, for example, on-the-run versus off-the-run U.S. Treasury bonds. Newly issued (on-the-run) Treasuries usually have yields lower than older off-the-run Treasuries but both yields are expected to converge with time. A more generic yield curve arbitrage is based on anomalies in dependence of bound yield on maturity. Other opportunities may appear in comparison of yields for Treasuries, corporate bonds, and municipal bonds.

- **Mortgage-backed securities (MBS) arbitrage**. MBS is actually a form of fixed income with a prepayment option. Namely, mortgage borrowers can prepay their loans fully or partially prior to the mortgage terms, which increases the uncertainty of the MBS value.

Event-driven Arbitrage

- A typical example here is *merger arbitrage* (also called risk-arbitrage). This form of arbitrage involves buying shares of the acquirer. The rationale behind this strategy is that business are usually acquired at a premium, which sends down the stock prices of the acquiring companies.
- Another event-driven arbitrage strategy focuses on financially distressed companies. Their securities are sometimes sold below their fair values as a result of market overreaction to the news of distress.

Multi-strategy Arbitrage

- Multi-strategy hedge funds take a synthetic approach that utilizes several hedging strategies and different securities. Looking for arbitrage opportunities across the board is technically more challenging yet potentially rewarding.

Pair Trading: - a strategy of matching a long position with a short position in two stocks of the same sector. This creates a hedge against the sector and the overall market that the two stocks are in. The hedge created is essentially a bet that you are placing on the two stocks; the stock you are long in versus the stock you are short in.

- *Arbitrage Pricing Theory* (APT) provides some theoretical background to this trading strategy (see, e.g. Grinold & Kahan 2000). The CAPM equation implies that return on a risky asset is determined by a single non-diversifiable risk factor. APT offers a generic extension of CAPM into the multi-factor paradigm. Namely, APT states that the return for an asset i at every time period is a weighted sum of the risk factor contributions $f_j(t)(j = 1, ..., K)$ plus an asset-specific random shock $\epsilon_i(t)$:

$$R_i(t) = a_i + \beta_{i1} f_1 + \beta_{i2} f_2 + ... + \beta_{iK} f_K + \epsilon_i(t)$$

Pair Trading:

- Assume that the expectations of all factor values and for the asset-specific innovations are zeros:

$$E[f_1(t)] = ... = E[f_K(t)] = E[\epsilon_i(t)] = 0$$

Also, the risk factors of the asset-specific innovations are independent and uncorrelated:

$$\underline{Cov[f_j(t), f_j(t')] = Cov[\epsilon_i(t), \epsilon_i(t')] = 0}, t \neq t';$$

$$\underline{Cov[f_j(t), \epsilon_i(t)] = 0}$$

APT states that there exist such $K + 1$ constants that

$$\underline{E[R_i(t)] = \lambda_0 + \beta_{i1}\lambda_1 + ... + \beta_{iK}\lambda_K}$$

where λ_0 has the sense of the risk-free asset return, and λ_j is called the risk premium for the jth risk factor.

Pair Trading:

- APT implies that similar companies have the same risk premiums and any deviation of returns is a mispricing, which yields arbitrage opportunities.

  For example, shares of an underpriced company should be bought while shares of an overpriced company should be shorted. Alas, APT does not contain a recipe for defining risk factors. These factors can be chosen among multiple fundamental and technical parameters.

- APT turns out to be more accurate for portfolios rather than for individual stocks.
- APT offers a valuable rationale for pursuing long/short trading strategies. Yet, it should be noted that buying underpriced and selling overpriced (in respect to the APT benchmark) securities in equal cash amounts does not guarantee market neutrality of the resulting portfolio.

协整 Cointegration and Causality

- Cointegration is an effective statistical technique developed by Engle and Granger for an analysis of common trends in multivariate time series. The correlation analysis can only be applied to stationary variables. Correlation is a measure of a short-term relationship that may be affected by volatility. Therefore trading strategies based on the correlation analysis need frequent rebalancing. On the other hand, cointegration describes long-term trends that may be stable even when correlations are broken (Alexander 1999).
- Two non-stationary time series are cointegrated if their linear combination is stationary. While the difference between two arbitrary prices series (the spread) may vary unpredictably, it is stationary for cointegrated series. If the spread deviates from its stationary value, it is expected that mean reversion will bring it back, or mispricing will be eliminated.

Cointegration and Causality

- Formally, two time series $x, y \sim I(1)$ - means integrated with order one - are cointegrated if there is such a constant α that

$$z = x - \alpha y \sim I(0)$$

- The standard technique offered by Engle and Granger for finding if two time series are cointegrated is derivation of the linear regression

$$x_t = \alpha y_t + c + \epsilon_t$$

  The residuals ϵ_t are then tested for stationarity.

- Usually, the Augmented Dickey-Fuller (ADF) test is used for unit roots. If stationarity is not rejected, then the cointegration condition holds. In the context of cointegrated portfolio, the residuals are sometimes called *tracking errors*.

Cointegration and Causality

- The cointegration property is closely related to Granger causality, which implies that turning points in the direction of one series precede turning points of another one. Consider the *error correction model* (ECM)

$$\Delta x_t = \delta_1 + \sum_{i=1}^{M_1} \beta_{1i} \Delta x_{t-i} + \sum_{i=1}^{M_2} \beta_{2i} \Delta y_{t-i} + \gamma_1 z_{t-1} + \epsilon_{1t}$$

$$\Delta y_t = \delta_2 + \sum_{i=1}^{M_3} \beta_{3i} \Delta x_{t-i} + \sum_{i=1}^{M_4} \beta_{4i} \Delta y_{t-i} + \gamma_2 z_{t-1} + \epsilon_{2t}$$

  where Δ is the difference operator, and the coefficients δ_i, β_{ij}, and γ_i are estimated from linear regression.

- The *Granger representation theorem* states that cointegration and ECM are equivalent. It says that x "Grangerly causes" y, when x and y are cointegrated.

Pair Selection

- Vidyamurthry (2004) describes the following pair trading strategy: Buy a portfolio consisting of long shares A with log price x_t and short shares B with log price y_t when

$$x_t - \alpha y_t = c - \Delta$$

and sell the portfolio when

$$x_t - \alpha y_t = c + \Delta$$

The challenge is the selection of stocks A, B and Δ.

- Within the APT framework, it is natural to pick up stocks from the same industry as their prices are expected to be determined with the same risk factors. However, the search for cointegrated pairs can be extended into entire industrial sectors or even beyond them.

Pair Selection

什么时间用 return 什么时间用 price

- Several problems arise with the cointegration technique. First, there is arbitrariness in choosing an independent variable. In other words, two options exist:

$$p_t^A = \alpha^A p_t^B + c^A + \epsilon_t^A \text{ with } \alpha^A = Cov(\epsilon_t^A, \epsilon_t^B)/Var(\epsilon_t^A)$$

and

$$p_t^B = \alpha^B p_t^A + c^B + \epsilon_t^B \text{ with } \alpha^B = Cov(\epsilon_t^A, \epsilon_t^B)/Var(\epsilon_t^B)$$

Vidyamurthry (2004) suggests choosing a variable with lower volatility as the independent one.

- Another problem is that the Augmented Dickey-Fuller (ADF) being a probabilistic test does not always offer an unambiguous conclusion about the stationarity of residuals. Schmidt (2008) suggested using the Johansen test.

Pair Selection

- In general choosing Δ in the strategy should be based on an analysis of residuals ϵ_t. Practitioners sometimes suggest entering the market when prices diverge by more than two standard deviations.

类似作业题，2个标准差

- Vidyamurthry (2004) advises to calculate Δ by maximizing the trading profits. If the cumulative distribution function of the spread is $Pr(\Delta)$, then the probability that profits exceed Δ equals $(1 - Pr(\Delta))$ and the profits W depend on Δ as

$$W \sim \Delta(1 - Pr(\Delta))$$

Minimization of the right-hand side of the above equation yields an optimal Δ. In particular, if the spread follows the Normal distribution, the profits have a maximum when Δ is 0.75σ.

Pair Selection

- Rather than employing the Engle-Granger cointegration framework, Gatev et al. (2006) used the concept of weakly dependent stocks. Namely, they have chosen such counterparts in trading pairs that minimize the sum of squared deviations between the two normalized price series (the distance measure).
- Another practical criterion in choosing trading pairs is that a successful outcome must be realized within acceptable time frame. Since residuals are expected to fluctuate around zero value due to mean reversion, even visual inspection of the chart ϵ_t may give an idea about possible round-trip execution time for a given pair.

  The zero-crossing rate of a mean-reverting process can be used as a quantitative measure of the execution time. For a stationary ARMA process, see Abrahams 1986.

Arbitrage Risks

Arbitrage traders face multiple risks:

- An event-driven arbitrage may go wrong simply because an event does not happen (a merger may be annulled). It seems that sometime investors have unlimited patience and resources for living through times of widening spread until mean reversion brings it to zero.
- For the relative-value arbitrage, past mean reversion may be broken in the future. In real life, market-neutral portfolio is not self-funded as brokers who lend shares for short selling require collateral. If the spread keeps widening after the long/short portfolio was set up, an investor may receive a margin call from the broker.
- Systemic risks may cause liquidation of significant portfolio volumes. Initial liquidation might happen even outside the pure long/short group of hedge funds.