

Q1~Q3:

HW4.

① Consider a portfolio that long 1 option with strike price  $K_1$  and  $K_3$ ,

short 2 option with strike price  $K_2$

long position	$S_T \leq K_1$	$K_1 \leq S_T \leq K_2$	$K_2 \leq S_T \leq K_3$	$S_T > K_3$
	$K_1 < K_2$	$S_T - K_1$	$S_T - K_1$	$2S_T - K_1 - K_3$
short position	0	0	$2K_2 - 2S_T$	$2K_2 - 2S_T$
sum	0	$S_T - K_1$	$2K_2 - K_1 - S_T$	0

$$P_{K_1} = (S_T - K_1)^+ - C_1, \quad P_{K_3} = (S_T - K_3)^+ - C_3, \quad P_{K_2} = 2C_2 - 2(S_T - K_2)^+$$

$$\text{Profit} = \begin{cases} 2C_2 - (C_1 + C_3) & S_T \leq K_1 \\ 2C_2 - (C_1 + C_3) + S_T - K_1 & K_1 \leq S_T \leq K_2 \\ 2C_2 - (C_1 + C_3) + 2K_2 - K_1 - S_T & K_2 \leq S_T \leq K_3 \\ 2C_2 - (C_1 + C_3) & S_T \geq K_3 \end{cases}$$

the minimum of Profit is  $2C_2 - (C_1 + C_3)$ . if  $C_2 \geq 0.5(C_1 + C_3)$ , there is an arbitrage to make profits which shouldn't exist.

Thus,  $C_2 \leq 0.5(C_1 + C_3)$

②  $1000 e^{-0.03 \times 5} \approx 860.71$ . we suppose the strike price of the call options are  $K_1$  and  $K_2$ .

Considering the situation of principal-protected notes. and bull spread.

$$P = \begin{cases} C_2 - C_1 & S_T \leq K_1 \\ S_T - K_1 + C_2 - C_1 & K_1 \leq S_T \leq K_2 \\ K_2 - K_1 + C_2 - C_1 & S_T \geq K_2 \end{cases}$$

the volatility of stock is 20%.

the risk-free rate is 3%

time to maturity is 5

Spot price is  $S_0$ .

Strike price (low) is  $K_1$ ,  $K_1 = S_0$ .

$$-(C_2 - C_1) = 139.29.$$

Judge from the BSM model.  $C, S_0, K$  can change as the same scalar. so we should

solve the function, suppose

$$\begin{cases} C(S_0, K_1, 6, r, t) = C_1 \\ C(S_0, K_2, 6, r, t) = C_2 \end{cases} \Rightarrow \frac{K_2}{K_1} = \frac{142.25}{100}.$$

$$C_1 - C_2 = 139.29$$

③ calculated by DerivaGem:

Call:

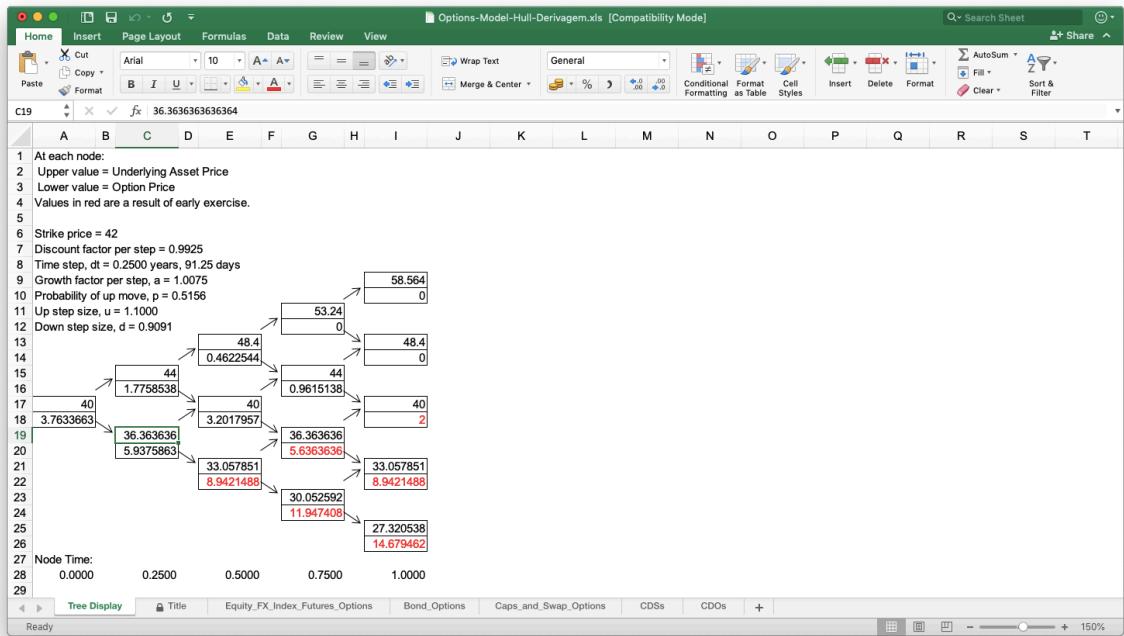
Strike price	call	Cost of call option
0.55	0.094	$= 0.094 + 0.02 - 0.023 \times 2 = 0.05$
0.66	0.023	
0.75	0.002	

put

strike price	put	Cost of put option
0.55	0.001	$= 0.001 + 0.102 - 0.027 = 0.049$
0.65	0.027	
0.75	0.002	

So. We ~~can~~ concluded that the cost are the same.

Q4:



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A1													
1	1.1000000000	42.0000000000											
2	0.9000000000												
3													
4													
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Use the different calculator,  
when we use the DerivaGem:

The price of European put option is 3.544602

The price of American put option is 3.763366

Use my own excel to calculate:

The price of European put option is 3.7519254557

The price of American put option is 3.9541420520

Q5:

G	H	I	J	K
strike price				
49.571				
			0	
		0.75955393		
	3.16400336			1.655
6.26439898		6.011		
9.571	9.971			10.367
		13.931		
13.571			17.495	
	17.171			
		20.411		
			23.327	
0.5376	0.4624			

Calculated by the excel, we can get the strike price is 49.571