FE 610 Stochastic Calculus for Finance Final Section A

May 8, 2018

Name:

- There are 4 problems, worth a total of 100 points.
- Before you start, make sure your exam is not missing any pages.
- Showcase your work: providing just the answer will result in a minimum of points.
- Closed book. No internet enabled devices. Two hand written sheets of notes are allowed.
- For the duration of the exam, you should assume that $W(t)$ is Brownian Motion.

For instructor's use only

Problem	Points	Score
1	25	
2	25	
3	25	
4	25	
Total	100	

1. Assume that the stock $S(t)$ follows Geometric Brownian Motion and that there is a constant risk-free interest rate r. We will create a new type of forward, known as the "Cross Forward". This derivative security will have as payout at maturity the formula:

$$V(T) = S(T) - KS(T) + K$$

for some constant K. For what value of K do we have $V(0) = 0$?

2. (a) Prove or Disprove: The product of two martingales is a martingale.

(b) Prove or Disprove: All Markov Processes are Martingales.

3. We have an interest rate $R(t) = r + \sigma\widetilde{W}(t)$ for some positive constants r and σ. Determine the formula, at any time $t < T$ for a zero-coupon bond that pays one dollar at time T. You may assume that:

$$B(t,T) = e^{-R(t)C(t,T)-A(t,T)}$$

4. Simplify:

$$\int_0^t W(t)(W(u) - uW^2(u))dW(u)$$