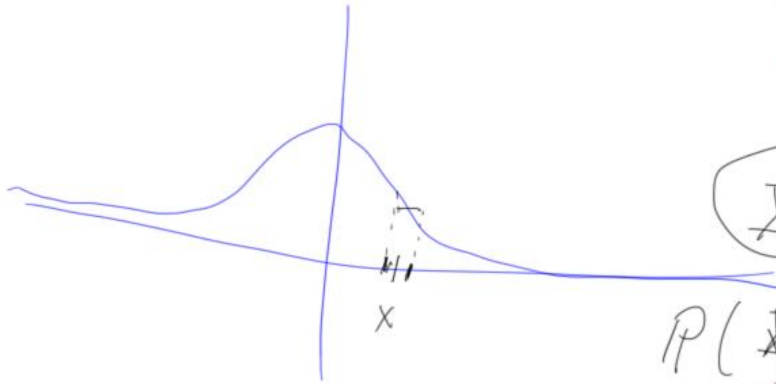
$$B(x,\varepsilon) = \left(x - \frac{\varepsilon}{2}, x + \frac{\varepsilon}{2}\right)$$

$$B(b,\varepsilon) = \left(b - \frac{\varepsilon}{2}, b + \frac{\varepsilon}{2}\right)$$

$\varepsilon > 0$

ε is "small"



$X \sim N(0,1)$

$\mathbb{P}(X \in B(x,\varepsilon))$

$\mathbb{P}(X \in B(b,\varepsilon))$

$\Rightarrow \mathbb{P}\left(b - \frac{\varepsilon}{2} \le X \le b + \frac{\varepsilon}{2}\right)$

$$\rightarrow \mathbb{P}\left(x - \frac{\varepsilon}{2} \le X \le x + \frac{\varepsilon}{2}\right)$$

$$= \int_{x-\frac{\varepsilon}{2}}^{x+\frac{\varepsilon}{2}} f_X(y)\,dy$$

$$= \int_{x-\frac{\varepsilon}{2}}^{x+\frac{\varepsilon}{2}} \frac{1}{\sqrt{2\pi}} e^{-y^2/2}\,dy$$

$$\approx \frac{1}{\sqrt{2\pi}} e^{-x^2/2}\left(x + \frac{\varepsilon}{2} - \left(x - \frac{\varepsilon}{2}\right)\right)$$

$$= \varepsilon \frac{1}{\sqrt{2\pi}} e^{-x^2/2}$$

$X \sim N(0,1) \qquad Y = XZ$

$$Z = \begin{cases} 1, & \omega = H \\ -1, & \omega = T \end{cases} \qquad \mathbb{P}(\omega = H) = 1/2$$

$E[e^{uX + vY}]$ for u & v d.v.'s

$$= E[e^{uX + vXZ}]$$

$$= E\left[\frac{1}{2} e^{uX + vX} + \frac{1}{2} e^{uX - vX}\right]$$

$$= \frac{1}{2} E[e^{X(u+v)}] + \frac{1}{2} E[e^{X(u-v)}]$$

$W \sim N(\mu, \sigma^2)$

$$\varphi_W(b) = E[e^{bW}] = e^{b\mu + \frac{1}{2}\sigma^2 b^2}$$

$\mu = 0$
$\sigma^2 = 1$
$b = u + v$

$$= \frac{1}{2} e^{\frac{1}{2}(u+v)^2} + \frac{1}{2} e^{\frac{1}{2}(u-v)^2}$$

Quadratic Variation

Brownian Motion

Moment GF's

Prob. Meas.

Martingale