

FE 610 Stochastic Calculus for Finance Final

December 7, 2018

- There are 4 problems, worth a total of 100 points.
- Showcase your work: providing just the answer will result in a minimum of points.
- Unless otherwise stated, you may assume that $W(t)$ refers to a Brownian Motion.

For instructor's use only

Problem	Points	Score
1	25	
2	25	
3	25	
4	25	
Total	100	

1. Assume that your stock follows Geometric Brownian Motion with a constant risk-free interest rate r . Price the European option

$$V(T) = (S^2(T) - K)_+$$

2. For a process given by

$$X(t) = e^{\sin(t^2 W(t))}$$

find $[X, X](t)$.

3. (a) Prove or Disprove: The product of two independent continuous martingales is a martingale.
(b) Evaluate for $s < t$

$$\mathbb{E}[W^2(s)(W(t) - W(s))^2 | \mathcal{F}(s)]$$

From this, what conclusions can be drawn about $W(s)(W(t) - W(s))$ and $\mathcal{F}(s)$?

4. Let $W_1(t)$ and $W_2(t)$ be Brownian motions with $[W_1, W_2](t) = \rho t$ where $\rho \in [-1, 1]$. Compute $d(W_1^2(t)W_2^2(t))$.