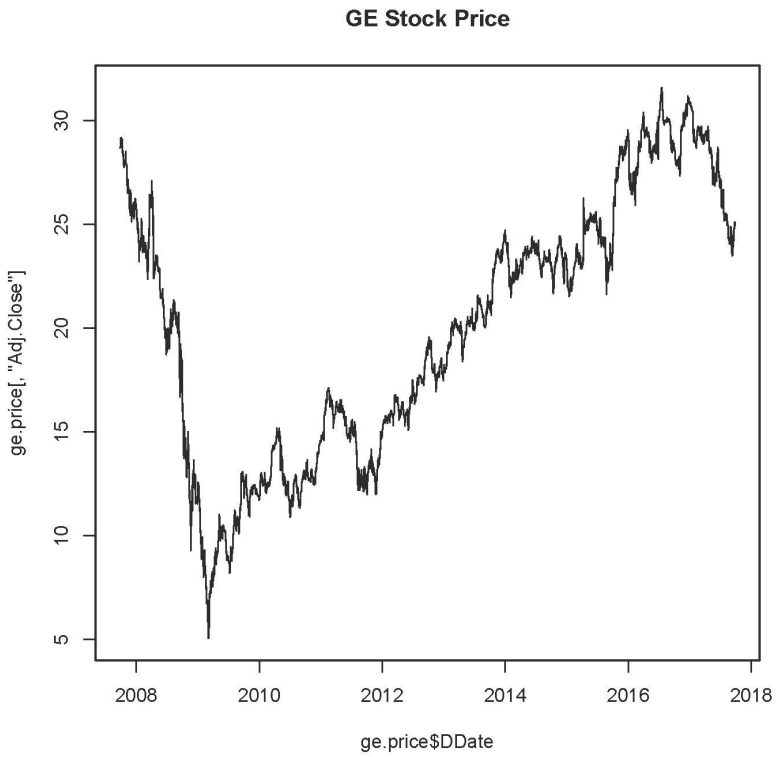
FE570 Homework Assignment #3

Due Date: In class on October 09 (Tuesday).

Problem 1. Realized volatility is normally calculated based on n data points using $\sigma_t = \left(\frac{1}{n}\sum_{i=t-n+1}^{t}(r_i - \bar{r})^2\right)^{1/2}$. Consider the daily stock price for General Electric (GE) from September 26, 2007 to September 26, 2017 in the file GE_2007−2017.csv.



Use the following volatility forecasting method to calculate "expected" volatility using the given GE stock prices. Please use n = 22 to calculate σ_t and $\beta = 2/(n+1)$ for the following volatility forecasting methods. Note: please use logarithmic return.

(a) The *Random Walk* forecast: $\hat{\sigma}_t = \sigma_{t-1}$

(b) *Exponential smoothing average* (often called exponential moving average or EMA):

$$\hat{\sigma}_t = (1-\beta)\sigma_{t-1} + \beta\hat{\sigma}_{t-1}, 0 < \beta < 1 \tag{1}$$

(c) *Exponentially weighted moving average* (EWMA), which is a truncated version of EMA:

$$\hat{\sigma}_t = \sum_{i=1}^{n} \beta^i \sigma_{t-i} / \sum_{i=1}^{n} \beta^i, 0 < \beta < 1 \tag{2}$$

(d) Compare and contrast the differences of these three estimation methods. Write your analysis and recommendation.

Problem 2. For implied volatility, the most frequently used model is the generalized autoregressive conditional heteroskedasticity - GARCH (1,1) model. It can be expressed in the following form:

$$\sigma_t^2 = \omega + a\epsilon_{t-1}^2 + b\sigma_{t-1}^2 \tag{3}$$

The advantage of the stationary GARCH(1,1) model is that it can be easily used for forecasting. Namely, the conditional expectation of volatility at time $(t+k)$ equals (please use logarithmic return):

$$E[\sigma_{t+k}^2] = (a+b)^k[\sigma_t^2 - \omega/(1-a-b)] + \omega/(1-a-b) \tag{4}$$

(a) Please use the same dataset (GE_2007−2017.csv) from the last problem and estimate a GARCH(1,1) model, and then use Equation (4) to forecast the implied volatility for GE stock for the next 1, 2, ..., 5 days. (Note: You may use fGarch R package to build the model and perform forecasting.)

(b) Compare the EWMA realized volatility forecast and the 1 day ahead implied volatility forecast from the GARCH(1,1) model, and discuss your observations.

Problem 3. In the Roll model, we define efficient price by m_t, and we assume $m_t = m_{t-1} + u_t$, where u_t are i.i.d. zero-mean random variables ($u_t \sim N(0, \sigma_u^2)$). The bid-ask spread is $a_t - b_t = 2c$, a constant. At time t, there is a trade at transaction price p_t, which may be expressed as: $p_t = m_t + q_t c$ where q_t is a trade direction indicator set to +1 if the customer is buying and -1 if the customer is selling. We also assume that buys and sells are equally likely, serially independent (a buy this period does not change the probability of a buy next period), and that agents buy or sell independently of u_t (a customer buy or sell is unrelated to the evolution of m_t). By definition we have $\Delta p_t = (q_t - q_{t-1})c + u_t$, and we can write the variance and autocovariance as:

$$\gamma_0 \equiv Var(\Delta p_t) = 2c^2 + \sigma_u^2.$$
$$\gamma_1 \equiv Cov(\Delta p_{t-1}, \Delta p_t) = -c^2.$$

For this assignment, you need to do the following (please use logarithmic return):

(a) Please derive the value $\gamma_\ell, \ell \geq 2$?

(b) Use the Roll model to estimate the bid-ask spread and fundamental volatility of GE stock during the period from September 26, 2007 to September 26, 2017. Please use the same dataset for the last two problems (GE_2007−2017.csv).

Homework Honor Policy: You are allowed to discuss the problems between yourselves, but once you begin writing up your solution, you must do

so independently, and cannot show one another any parts of your written solutions. The HW is to be pledged (that it adheres to this).