FE570 Financial Markets and Trading

Lecture 5. Volatility and the Roll Model of Trade Prices
(Ref. Joel Hasbrouck - *Empirical Market Microstructure*)

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Outline

Volatility and Its Components

- **Volatility** is the tendency for prices to change unexpectedly. Prices change in response to new information about values and in response to the demands of important traders for liquidity.
  - Volatility, risk, and profit are closely related. Every drop in prices creates losses for traders who have long positions and profits for traders with short positions. Likewise, every price rise causes losses for traders with short positions and profits for traders with long positions. Traders therefore are very interested in volatility because it can have a significant impact on their wealth.
  - We define two main sources of volatility and distinguish them as *fundamental volatility* and *transitory volatility*. *Fundamental volatility* is due to unanticipated changes in instrument values, and *transitory volatility* is due to trading activities by uninformed traders.

Fundamental Volatility

- Values change when the fundamental factors that determine them change, which contributes to fundamental volatility.

- When new information about changes in fundamental values is common knowledge, prices may change without any trading. When only a few people know new information about changes in fundamental values, prices generally will change on high trading volume. The pressure of their trades will cause prices to change to reflect the new fundamental values.

- Uncertain knowledge about fundamental factors often causes substantial fundamental volatility. The stocks of companies involved in technological research and companies with high price to earnings (P/E) ratios tend to be highly volatile.

Transitory Volatility

- Transitory volatility is due to the demands of impatient uninformed traders that cause prices to diverge from fundamental values.
- The simplest form of transitory volatility is bid/ask bounce. Bid/ask bounce occurs when market order traders buy at the ask and sell at the bid. Their trades cause prices to bounce from bid to ask.
- Large orders and cumulative order imbalances created by uninformed traders also cause prices to move from their fundamental values. The price changes reverse when value traders or arbitrageurs recognize that prices differ from fundamental values.

Measure Volatility and Its Components

- People generally measure total volatility by using variances, standard deviations, or mean absolute deviation of price changes. The statistical models are necessary to identify and estimate the two components of the total volatility.

- These models exploit the primary distinguishing characteristics of the two types of volatility: fundamental volatility consists of seemingly random price changes that do not revert while transitory volatility consists of price changes that revert.

- The presence of negative serial correlation in price series is therefore a strong indicator of transitory volatility. Transaction-induced negative serial correlation in price changes may appear over various horizons.

Measure Volatility and Its Components

- Volatility is a genetic notion for measuring price variability. This concept is very important in risk measurement and is widely used in defining trading strategies.

- Standard deviation of returns is usually used for quantifying volatility. For a data sample with N returns r_i at $i = 1, 2, \ldots, N$ and the average value $\bar{r}$, or *realized volatility* (also called historical volatility),

$$\sigma = \left[\frac{1}{N}\sum_{i=1}^{N}(r_i - \bar{r})^2\right]^{1/2}$$

Measure Volatility and Its Components

- Usually, returns are calculated on a homogeneous time grid with spacing Δt.
- For financial reporting, volatility is often calculated as the annualized percentage, that is $\sigma(T/\Delta t)^{1/2}100$ percent, where T is the annual period in units of Δt.
- Usage of all available data points for calculating the historic volatility may not be a good idea. Normally, we use only last n data points:

$$\sigma_t = \left(\frac{1}{n} \sum_{i=t-n+1}^{t} (r_i - \bar{r})^2 \right)^{1/2}$$

Measure Volatility and Its Components

- Poon & Granger (2003) describe several practical ways for forecasting volatility. The simplest forecast in the spirit of the martingale hypothesis is referred to as the *random walk forecast*: $\hat{\sigma}_t = \sigma_{t-1}$. Other popular forecasting methods are:
- *Simple moving average* (SMA):

$$\hat{\sigma}_t = (\sigma_t + \sigma_{t-1} + \ldots + \sigma_{t-T})/T \tag{1}$$

- *Exponential smoothing average* (often called exponential moving average or EMA):

$$\hat{\sigma}_t = (1-\beta)\sigma_{t-1} + \beta\hat{\sigma}_{t-1}, 0 < \beta < 1 \tag{2}$$

Here, the value of $\hat{\sigma}_1$ is not defined. Often it is assumed that $\hat{\sigma}_1 = \sigma_1$; sometimes SMA for a few initial values of σ_t is used instead.

Measure Volatility and Its Components

- *Exponentially weighted moving average* (EWMA), which is a truncated version of EMA:

$$\hat{\sigma}_t = \sum_{i=1}^{n} \beta^i \sigma_{t-i} / \sum_{i=1}^{n} \beta^i, 0 < \beta < 1 \tag{3}$$

  It can be shown by comparing EMA and EWMA in the limit of high n that the smoothing parameter equals

$$\beta = 2/(n+1) \tag{4}$$

  EMA with relation (4) is widely used in technical analysis.
- The notion of *implied volatility* is based on the Black-Scholes theory of option price. In this theory, volatility is one of the parameters that determines the price of an option.

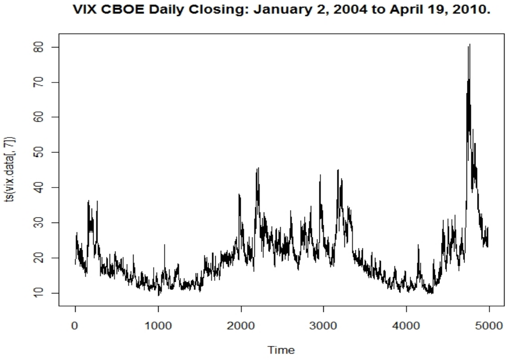
Application of Volatility

- Volatility has many financial applications:
  - Option (derivative) pricing, e.g., Black-Scholes formula.
  - Risk management, e.g. value at risk (VaR).
  - Asset allocation, e.g., mean-variance portfolio.
- Use high-frequency data: French, Schwert & Stambaugh (1987);
  - Realized volatility of daily returns in recent literature.
  - Use daily high, low, and closing (log) prices, e.g. range.
- Implied volatility of options data, e.g, VIX of CBOE.

  The volatility index of a market has become a financial instrument.

  The VIX volatility index complied by the Chicago Board of Option Exchange (CBOE) started to trade in futures on March 26, 2004.

Characteristics of Volatility



*A special feature of stock volatility is that it is not directly observable.

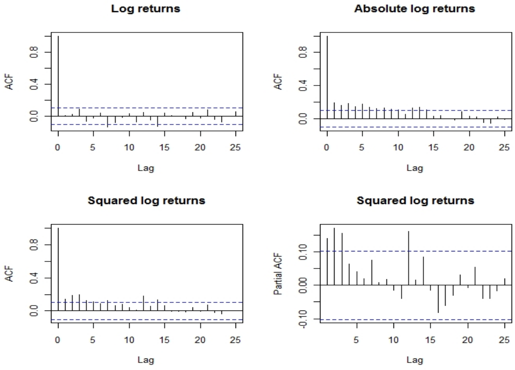
Characteristics of Volatility

- There exist volatility clusters (i.e., volatility may be high for certain time periods and low for other periods).
- Volatility evolves over time in a continuous manner - that is, volatility jumps are rare.
- Volatility does not diverge to infinity - that is, volatility varies with some fixed range. (this means that volatility is often stationary.)
- Volatility seems to react differently to a big price increase or a big price drop, referred to as the *leverage* effect.

** Important in the development of the volatility models.

Structure of a Model

- Let r_t be the return of an asset at time t. The basic idea behind volatility study is that the series $\{r_t\}$ is either serially uncorrelated or with minor lower order serial correlations, but it is a dependent series.



Structure of a Model

- To put the volatility models in a proper perspective, it is informative to consider the conditional mean and variance of r_t given F_{t-1}; that is

$$\mu_t = E(r_t|F_{t-1}), \sigma_t^2 = Var(r_t|F_{t-1}) = E[(r_t - \mu_t)^2|F_{t-1}], \quad (5)$$

where F_{t-1} denote the information set, and typically F_{t-1} consists of all linear functions of the past returns.

- We can model r_t as a stationary $ARMA(p, q)$ model with some explanatory variables.

$$r_t = \mu_t + a_t, \mu_t = \phi_0 + \sum_{i=1}^{k} \beta_i x_{it} + \sum_{i=1}^{p} \phi_i r_{t-i} - \sum_{i=1}^{q} \theta_i a_{t-i}, \quad (6)$$

for r_t, where k, p and q are non-negative integers, and x_{it} are explanatory variables.

Structure of a Model

- Combine the last two equations, we have

$$\sigma_t^2 = Var(r_t|F_{t-1}) = Var[(a_t|F_{t-1}],$$

- The conditional heteroscedastic models are concerned with the evolution of σ_t^2. The manner under which σ_t^2 evolves over time distinguishes one volatility model from another. They can be classified into two general categories:

  1. Those use an exact function to govern the evolution of σ_t^2;
  2. Those use a stochastic equation to describe σ_t^2.

- Autoregressive Conditional Heteroscedastic (ARCH) model of Engle (1982), and the Generalized Autoregressive Conditional Heteroscedastic (GARCH) model of Bollerslev (1986) belong to the first category whereas the stochastic volatility model is in the second category.

Volatility Model Building

1. Specify a mean equation by testing for serial dependence in the data and, if necessary, building an econometric model (e.g. an ARMA model) for the return series to remove any linear dependence.
2. Use the residuals of the mean equation to test for ARCH effects.

$$a_t = \sigma_t \epsilon_t, \sigma_t^2 = \alpha_0 + \alpha_1 a_{t-1}^2 + ... + \alpha_m a_{t-m}^2.$$

   where $\{\epsilon\}$ is a sequence of independent and identically distributed (iid) random variables with mean zero and variance 1, $\alpha_0 > 0$ and $\alpha_i \geq 0$ for $i > 0$.

3. Specify a volatility model if ARCH effects are statistically significant and perform a joint estimation of the mean and volatility equations.
4. Check the fitted model carefully and refine it if necessary.

Volatility GARCH Model

Volatility ARCH(m,n) process combines the ARCH(m) process with the AR(n) process for lagged variance:

$$\sigma_t^2 = \omega + a_1\epsilon_{t-1}^2 + a_2\epsilon_{t-2}^2 + \ldots + a_m\epsilon_{t-m}^2 + \\ + b_1\sigma_{t-1}^2 + b_2\sigma_{t-2}^2 + \ldots + b_n\sigma_{t-n}^2 +$$

The simple GARCH(1,1) model is widely used in financial applications:

$$\sigma_t^2 = \omega + a\epsilon_{t-1}^2 + b\sigma_{t-1}^2 \tag{7}$$

It can be transformed into

$$\sigma_t^2 = \omega + (a+b)\sigma_{t-1}^2 + a[\epsilon_t^2 - \sigma_{t-1}^2] \tag{8}$$

Volatility GARCH Model

- The last term in (18) conditioned on information available at time $t-1$ has zero mean and can be treated as a shock to volatility.

  Therefore, the unconditional expectation of volatility for the GARCH(1,1) model equals

$$E[\sigma_t^2] = \omega/(1-a-b)$$

- This implies that the GARCH(1,1) process is weakly stationary when $a+b<1$. The advantage of the stationary GARCH(1,1) model is that it can be easily used for forecasting. Namely, the conditional expectation of volatility at time $(t+k)$ equals:

$$E[\sigma_{t+k}^2] = (a+b)^k[\sigma_t^2 - \omega/(1-a-b)] + \omega/(1-a-b) \quad (9)$$

Volatility GARCH Model

- The GARCH(1, 1) model (8) can be rewritten as

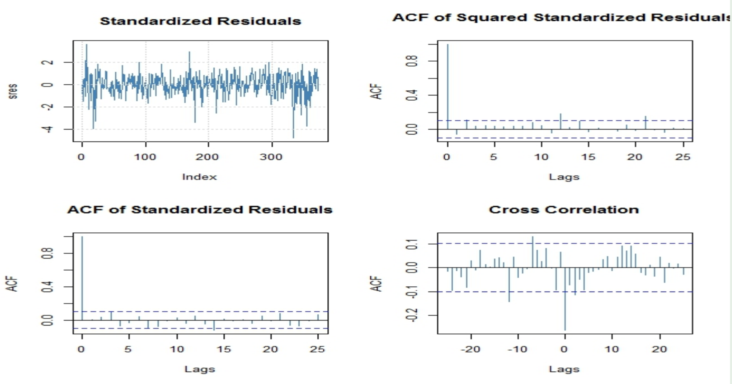
$$\sigma_t^2 = \omega/(1 - b) + a(\epsilon_{t-1}^2 + b\epsilon_{t-2}^2 + b^2\epsilon_{t-3}^2 + \ldots) \qquad (10)$$

- This implies that the GARCH(1,1) model is equivalent to the infinite ARCH model with exponentially weighted coefficients. This explains why the GARCH models are more efficient that the ARCH models.

- Several other GARCH models have been derived fro addressing specifics of various economic and financial time series. One popular GARCH(1,1) model, in which $a + b = 1$, is called *integrated* GARCH (IGARCH).

Example

Volatility ARCH(1) Example



- Fit an ARCH (3) model first based on PACF
- Examine the fitness of the model and further reduce the model to ARCH(1), and residual past Ljung-Box Q(m) test.

The Roll Model of Bid, Ask, and Transaction Prices

- Keep the random-walk assumption, and apply it to the efficient price instead of the actual transaction price.
- Denote efficient price by m_t, and we assume $m_t = m_{t-1} + u_t$, where u_t are i.i.d. zero-mean random variables.
- All trades are conducted through dealers - the best dealer quotes are bid and ask prices, b_t and a_t, i.e. if a customer wants to buy, he/she must pay the dealer's ask price (therefore lifting the ask); if a customer wants to sell, he/she receives the dealer's bid price (hitting the bid).
- Dealer incurs a cost of c per trade. This charge reflects costs like clearing fees and per trade allocations of fixed costs.
- If dealers compete to the point where the costs are just covered, the bid and ask are $m_t - c$ and $m_t + c$, respectively.

The Roll Model - Two Parameters

- The bid-ask spread is $a_t - b_t = 2c$, a constant.
- At time t, there is a trade at transaction price p_t, which may be expressed as: $p_t = m_t + q_t c$

  where q_t is a trade direction indicator set to $+1$ if the customer is buying and -1 if the customer is selling.
- We also assume that buys and sells are equally likely, serially independent (a buy this period does not change the probability of a buy next period), and that agents buy or sell independently of u_t (a customer buy or sell is unrelated to the evolution of m_t).
- The Roll model has two parameters, c and σ_u^2. These are most conveniently estimated from the variance and first-order autocovariance of the price changes, Δp_t:

  The variance $\gamma_0 \equiv \mathcal{F}(c, \sigma_u^2)$

  The first order autocovariance $\gamma_1 \equiv \mathcal{G}(c, \sigma_u^2)$

The Roll Model - Variance and Auto-covariance

Define variance and auto-covariances as follows:

$$\begin{aligned}\gamma_0 \equiv Var(\Delta p_t) &= E[(\Delta p_t)^2] - (E[\Delta p_t])^2 \\ &= E[q_{t-1}^2 c^2 + q_t^2 c^2 - 2q_{t-1}q_t c^2 \\ -2q_{t-1}u_t c + 2q_t u_t c + u_t^2] &= 2c^2 + \sigma_u^2.\end{aligned}$$

In the expectation of this equation all of the cross-products vanish except for those involving q_t^2, q_{t-1}^2, and u_t^2.

- The first-order auto-covariance:

$$\begin{aligned}\gamma_1 \equiv Cov(\Delta p_{t-1}, \Delta p_t)^2 &= E(\Delta p_{t-1}\Delta p_t) \\ = E[c^2(q_{t-2}q_{t-1} - q_{t-1}^2 - q_{t-2}q_t + q_{t-1}q_t) \\ +c(q_t u_{t-1} - q_{t-1}u_{t-1} + u_t q_{t-1} - u_t q_{t-2})] &= -c^2.\end{aligned}$$

- It can be easily verified that all autocovariances of order 2 or higher are zero.

The Roll Model - Estimate of Model Parameters

- From the equations obtained earlier, we can compute the model parameters as:

$$c = \sqrt{-\gamma_1} \tag{11}$$

$$\sigma_u^2 = \gamma_0 + 2\gamma_1 \tag{12}$$

- Given a sample data, it is sensible to estimate γ_0 and γ_1 and apply these transformations to obtain estimate of the model parameters.
- Example: Based on all trades for PCO in October 2003, the estimated first-order autocovariance of the price changes is $\hat{\gamma}_1 = -0.0000294$. This implies $c = \$0.017$ and a spread of $2c = \$0.034$.

  Note: The Roll model is often used in situations where we don't possess bid and ask data. In this example, we do know that the time-weighted average NYSE spread in the sample is \$0.032, so the Roll estimate is fairly close.