How to reduce the risk of executing VWAP orders?

- New approach to modeling intraday volume

Jedrzej Bialkowski*, Serge Darolles **, Gaëlle Le Fol***

November 27, 2006

Abstract

We propose an intraday dynamic VWAP (Volume Weighted Average Price) strategy with within the day adjustments. Our approach is based on factor decomposition and ARMA or SETAR models. It allows for a significant reduction of the execution risk in VWAP orders.

JEL classification: C53, G12, G29

Keywords: Intraday Volume; Factor models; Dynamic VWAP Strategies.

1. INTRODUCTION

We focus on an investor who is aiming to achieve the average price over a given time period (the trading day for example, which is the chosen time period for this article). This is typically the objective of:

- Any **institutional investor** reluctant to suffer the market impact and wanting his trades to move the market price as little as possible. The market impact measures the price change due to the execution of a trade. As a consequence, it is part of the total trading cost. By splitting orders over the course of the day at an average price, institutional investors are able to reduce the market impact of their trades and thus their trading costs.
- Any **broker-dealer** who is offering these types of services to clients. One possible average price is the volume weighted average price (VWAP), which has become a

* Department of Economics, Faculty of Business, Auckland University of Technology, Private Bag 92006, Auckland 1020, New Zealand, E-mail: jedrzej.bialkowski@aut.ac.nz.

**Société Générale Asset Management AI, Center for Research in Economics and Statistics (CREST), France, E-mail: serge.darolles@sgam.com.

*** *Corresponding author:* EPEE, University of Evry, Center for Research in Economics and Statistics (CREST), and Europlace Institute of Finance, France, E-mail: Gaelle.Le-Fol@ensae.fr. Tel : +331 41 17 35 90. Fax : +331 41 17 76 66.

benchmark. Brokers can guarantee the VWAP or just ensure that it is tracked at a lower rate of commission.

VWAP orders are used to balance time versus market impact and one way of doing this is to send a percentage of shares regularly over the course of an entire day. Another way of achieving this, whatever the price volatility over the chosen time period, is to analyse and use the daily volume allocation. To keep it simple, the idea is to split the day into smaller time periods and to calculate the proportion of the daily volume executed in each of these periods.

In fact, any trader willing to track the VWAP only needs to know the U-shape of volume for a particular day. With knowledge of the exact scope of any trading volume, any trader is able to calculate and time to perfection the proportions of his order to execute.

In practical terms, the investor needs to know the entire trading sequence of the day at the beginning of the trading day in order to be able to implement his "volume" strategies. The first step then is to predict the intraday volume sub-period by sub-period by taking the average daily volume allocation over the previous days, for example. He will then split his order according to the stock/market intraday volume scope predictions. These orders are then sent to an automated trading program to be executed over the course of an entire day.

2. CONTRIBUTIONS

Two problems arose as a result of the practice described above:

- Firstly, since the agent must predict the intradaily strategy, i.e. the intradaily splitting scheme, he is facing prediction errors. These errors will impact the trading price of the strategy and make it diverge from the target price. For that reason, brokers are charging commission fees for that "price guarantee" service. The commission fees must cover the maximum possible error. In Europe, the cost of VWAP type orders ranges from 10 to 20 basis points depending on the volume volatility.
- Secondly, our agent cannot correct his trading scheme during the day even if he notices that the true volumes traded in the first sub-periods of the day are far above or below his predictions. The reason for this comes from the prediction method of the intraday volume that is basically static as it is just a historical average.

Our main contribution is to propose a predictive method of volumes that allows for continuous updating of the volume allocation prediction for the rest of the day. This leads us to a dynamic VWAP execution strategy. In this sense, we do not follow the usual static way of improving VWAP trades (see Konishi (2002)). The main advantage of our approach is to remain simple (even in a dynamic framework) in comparison with the static improvement of VWAP proposed in the article above. Our empirical analysis shows that our approach allows a reduction in execution errors of such average price types of orders. The implication for the broker is straightforward: by reducing his margin for error, he can lower his commission fee in order to get a larger share of the business. This result contradicts Hobson who concludes that refinements to the volume profile do not yield significant benefits of VWAP execution strategies.

3. MODELLING VOLUME DYNAMICS

The intraday traded volume allocation shows some similarities across stocks motivating in turn a multivariate approach to predict volumes. We propose a new dynamic approach based on factor models. It assumes that the intraday volume, for stock *i* at time *t*, $x_{i,t}$, can be broken down into two parts, and for each, we consider a separate model.

The first, c_t, describes the movement of the whole market on which a particular stock is listed. In other words, the increased activity of investors across the whole market will result in changes in the first component of volume. For obvious reasons, we will call it the market component. This component is responsible for modelling the above-mentioned U-shape of volume. As this volume component is not stationary, we propose to predict it using a historical average.

The second component, $y_{i,t}$, depicts the intraday dynamics of the volume, which is different for each stock (specific component). Since this component is stationary, we predict it using an ARMA or a self-exciting threshold autoregressive (SETAR) model. The ARMA(1,1) with white noise is defined as:

$$y_{i,t} = \psi_1 y_{i,t-1} + \psi_2 + \varepsilon_{i,t}$$

The alternative model is SETAR defined as:

$$y_{i,t} = \begin{cases} \phi_{11} y_{i,t-1} + \phi_{12} + \varepsilon_{i,t} & y_{i,t-1} \le \tau, \\ \\ \phi_{21} y_{i,t-1} + \phi_{22} + \varepsilon_{i,t} & y_{i,t-1} > \tau. \end{cases}$$

The SETAR modelling of the specific component implies that the intraday dynamics of volume depends on its level – high ($y_{i,t} > \tau$) or low ($y_{i,t} \le \tau$). Parameters of the SETAR model are estimated by using sequential conditional least squares method. A description of the estimation technique can be found in Frances and van Dijk (2000). All statistical details can be found in Bialkowski, Darolles, Le Fol (2006).

4. THE TRADING ALGORITHM

Our unusual approach comes from the statistical treatment of the specific component that allows dynamic updates. At the beginning of the day, an agent willing to trade at the average price will:

1. Predict the common component. This component will remain unchanged throughout the day;
2. Make an initial prediction of the specific components for all sub-periods during the day. By summing up the two components over all the daily sub-periods, he will see the daily volume and the intraday volume allocation;
3. At the end of each sub-period, visibility of the true traded volume enables the trader to correct his predictions and thus update his trading mechanism for the remaining volume for the rest of the day. This update is obviously not possible in the basic case.

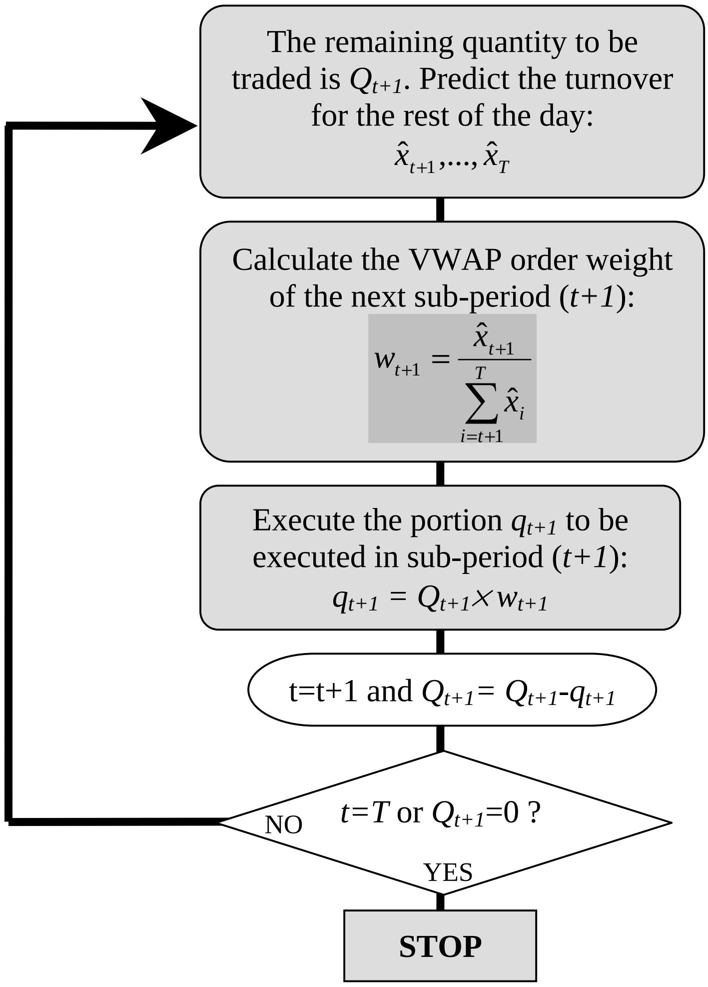

Figure 3: The dynamic VWAP order execution algorithm

Figure 3 presents the trading scheme. We break the day down into 25 sub-periods of 20 minutes. We start by predicting the turnover for the entire day using time series models applied to the specific part of the day, and the static (historical average) approach to the market component. As a result, we obtain 25 prediction points ($\hat{x}_1,\ldots,\hat{x}_{25}$). The proportion of the order to be executed at the beginning of the day – in the first sub-period – is equal to $\frac{\hat{x}_1}{\sum_{i=1}^{25}\hat{x}_i}$. At the end of the first period, we observe x_1 and use it to predict new ($\hat{x}_2,\ldots,\hat{x}_{25}$).

The proportion of the remaining volume to be executed in the second period is then equal to $\frac{\hat{x}_2}{\sum_{i=2}^{25} \hat{x}_i}$, and so on until the end of the day.

As a result, at the very beginning of the day, a trader using the strategy shown here trade without information. Then, as time passes, he/she can improve the predictions and can beat a trader who is predicting the whole U-shape at the beginning of the trading day.

5. RESULTS AND CONCLUSION

Our examination focuses on all stocks included within the CAC40 index at the beginning of September 2004. The analysis is based on a sample ranging from September 2003 to the end of August 2004. The intraday data is aggregated over 20 minute intervals. The 20-minute volume is defined as the sum of the traded volumes while a 20-minute price is the average over twenty minute periods. We have restricted the examination to continuous trading between 9:20am and 5:20pm. Finally, we take the turnover, as a proxy of volume, defined as the traded volume divided by the outstanding number of shares for a particular stock. Our selection is consistent with a few recent studies on volume; see, for example Lo & Wang (2000).

In order to examine the accuracy of our approach to reduce the risk of execution of VWAP orders, six of the largest French companies are analysed[1].

Table 2: Summary statistics for cost of execution of VWAP orders, September 3 to December 2, 2003.

	Basic approach			**PC-ARMA**			**PC-SETAR**		
Name of Company	**Mean**	**STD**	**Q95**	**Mean**	**STD**	**Q95**	**Mean**	**STD**	**Q95**
ALCATEL	16.12	17.67	51.95	8.870	9.130	31.13	7.870	9.991	21.12
FRANCE TELECOM	13.72	20.14	50.89	9.390	18.059	25.53	9.129	18.251	29.55
LAFARGE	13.97	17.24	70.02	8.315	8.213	26.00	7.019	7.823	20.65
LVMH	9.60	10.63	32.38	7.840	8.789	23.48	5.662	8.343	13.92
SUEZ	12.34	11.02	32.50	8.831	7.270	22.92	8.967	8.151	25.67
TOTAL	6.805	7.427	21.78	4.005	4.104	12.03	4.151	3.927	11.19
Overall	12.09	14.02	32.75	7.875	9.261	18.85	7.133	9.414	20.35

Note: The cost is expressed in basis points. The Basic approach is an historical average. PC-ARMA and PC-SETAR stand for Principal Component Auto-Regressive Moving Average and Principal Component Self-Exciting Auto-Regressive models respectively.

[1] The results for all companies from CAC40 are available upon request.

In Table 2, we report the average cost of VWAP order execution for the period between September 3 and December 2, 2003. Three models for intraday volume are assessed. Two of them are proposed by the authors (PC-ARMA and PC-SETAR). The third is based on a static approach to predicting daily dynamics of volume. It assumes that taking the historical average can approximate the volume during a particular time interval. Table 2 was prepared in the following way: each day the true daily VWAP is compared with that predicted by each of the models. The mean absolute percentage error (MAPE) is used as the error measurement.

The reported results confirm the effectiveness of the proposed factor decomposition models. The application of a PC-ARMA model allows for a reduction in the average cost of VWAP orders by more than 4 bp compared with the basic approach. In turn, selection of PC-SETAR enables a further 1 bp decrease compared with PC-ARMA. The superiority of PC models is also corroborated by the results of 95%-quantiles comparison. The higher the value, the less reliable the model, because it means that in 5% of all trading days, the cost of execution of VWAP orders is higher than the reported values. On average, the 95%-quantile is twice as low for PC models as using the basic approach. The best results are obtained for LAFARGE with reductions of 44 bp and 49 bp for PC-ARMA and PC-SETAR respectively.

In summary, by applying the proposed models, a trader is able to reduce the cost of VWAP orders and, moreover, his/her chances of losses measured by 95%-quantile are reduced. Therefore, VWAP trade orders can be offered at a lower rate of commission to investors.

If the PC-SETAR model globally outperforms the PC-ARMA model, this result clearly depends on the period, the market and even the stocks.

6. REFERENCES

Bialkowski, J., Darolles S. and G. Le Fol, 2006, "Improving VWAP Strategies: a Dynamical Volume Approach", working paper, Available at SSRN: http://ssrn.com/abstract=932699.

Frances, P., van Dijk, D., 2000, "Non-linear time series models in empirical finance", *Cambridge University Press*.

Hobson, D., 2006, "VWAP and Volume Profiles", *Journal of Trading, 1 (2)*, 38-42.

Konishi, H., 2002, "Optimal Slice of a VWAP Trade", *Journal of Financial Markets*, 5, 197-221.

Konishi, H. and N. Makimoto, 2000, "Optimal Slice of a Block Trade", *Journal of Risk, 3 (4)*, 33-51.

Lo, A. and J. Wang, 2000, "Trading Volume: Definition, Data Analysis, and Implication of Portfolio Theory", *Review of Financial Studies, 13*, 257-300.

Madhavan, A., 2002, "VWAP Strategies", *Investment Guides, Transaction Performance: The Changing Face of Trading, Spring 2002*, 32-38, edited by R. Bruce, Institutional Investor Journal, Inc., New York, NY.

Ting, C., 2006, “Which Daily Price is Less Noisy”, *Financial Management*, *35 (3)*, 81-95.