

# FE 610 Stochastic Calculus for Finance Midterm

March 8, 2018

**Name:**

- There are 4 problems, worth a total of 100 points.
- Simplify your answers as much as possible. If the result could be simplified further, you will potentially lose points.
- Showcase your work: providing just the answer will result in a minimum of points.
- Closed book (preferably). No internet enabled devices (except to take the test of course). Only one hand written sheet of notes should be used.

**For instructor's use only**

Problem	Points	Score
1	25	
2	25	
3	25	
4	25	
Total	100	

1. For a positive constant  $X(0)$  we have the process  $X(t)$  defined as:

$$X(t) = X(0) + \int_0^t W^2(u) dW(u) + \int_0^t W(u) du$$

- (a) Find  $[X, X](t)$   
 (b) Find  $\mathbb{E}[X, X](t)$

2. Given a three-dimensional Brownian Motion, and for processes defined by:

$$X(t) = \int_0^t W_1(u) \underline{W_2(u)} dW_3(u)$$

$$Y(t) = Y(0) + \int_0^t \delta(u) dW_1(u) + \underline{W_2(t)}$$

$$Z(t) = X(t)Y(t)$$

with  $Y(0)$  being a positive constant and  $\delta(t)$  an adapted process. Is  $Z(t)$  a martingale?

3. Let  $S(t)$  be the stock price governed by

$$S(t) = S(0) e^{\int_0^t (\alpha(u) - \frac{1}{2}\sigma^2(u)) du + \int_0^t \sigma(u) dW(u)}$$

with  $\alpha(t)$  an adapted process and  $\sigma(t)$  a non-random process. Determine the Quadratic Variation and Variance of  $S(t)$ .

4. For a process  $X(t)$  governed by the dynamic:

$$dX(t) = X(t) dW(t)$$

with  $X(0) = 2$ . Determine:

$$\mathbb{E}[e^{-t} X^2(t)]$$