FE570 Financial Markets and Trading

Lecture 6. Volatility Models and Sequential Trade Models (Ref. Joel Hasbrouck - *Empirical Market Microstructure*)

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Outline

The Glosten-Harris Model

The Glosten-Harris model (1998) expands the Roll's model in that it treats the bid/ask spread as a dynamic variable and splits it into the transitory, C_t, and the adverse selection, Z_t, components

$$S_t = 2(C_t + Z_t) \tag{1}$$

Hence the observable transactional price equals

$$P_t = P_t^* + (C_t + Z_t)q_t \tag{2}$$

- This implies that the fundamental price is now affected by the adverse selection component

$$P_t^* = P_{t-1}^* + Z_t q_t + \epsilon_t \tag{3}$$

Glosten & Harris (1998) assume in the spirit of Kyle's model that both spread components are linear upon the trading size V_t.

The Glosten-Harris Model

The Glosten-Harris model (1998):

$$C_t = c_0 + c_1 V_t \ , \ Z_t = z_0 + z_1 V_t \tag{4}$$

It follows from (1)-(4) that the price change equals

$$\Delta P_t = c_0(q_t - q_{t-1}) + c_1(q_t V_t - q_{t-1} V_{t-1}) + z_0 q_t + z_1 q_t V_t + \epsilon_t \tag{5}$$

Let's calculate the price change for the round-trip transaction of a sale immediately following a purchase of the same asset size. Namely, let's put $q_t = 1$ and then $q_{t-1} = -1$ into (5). Then

$$\Delta P_t = 2C_t + Z_t + \epsilon_t \tag{6}$$

In fact, ΔP_t is a measure of the effective spread that is conditioned on the round-trip transaction.

Structural Models

- Both Roll's and Glosten-Harris models depart from the efficient market cannon in that the observable (transactional) prices are not treated as martingales anymore.
- One can argue that bid/ask bounces described in the Roll's model are small short-lived frictions in otherwise efficient market, and the mid-price equated with the asset fundamental value still follows the random walk.
- Glosten & Harris (1998) go further by offering a price discovery equation (3) in which trading volume of informed investors can result in a market impact.

Structural Models

- The information-based market impact may have lagged components related to quote revisions and past trades.
- Hasbrouck (1991) suggests that information-based price impact can be separated from the inventory-based price impact since the former remains persistent at intermediate time intervals while the latter is transient.
- Hasbrouck (1991) offers a structural model in which it is assumed that the expectation of the quoted mid-price conditioned on public information I_t available at time t.

Structural Models

- The true asset value P_T at some future time T:

$$E\left[(p_t^a + p_t^b)/2 - P_T | I_t\right] \to 0 \text{ as } t \to T \tag{7}$$

here p_t^a and p_t^b are ask and bid prices at time t, respectively. Let's introduce the revision of the mid-price:

$$r_t = \frac{1}{2}\left[(p_t^a + p_t^b)/2 - (p_{t-1}^a + p_{t-1}^b)/2\right] \tag{8}$$

- In the general case, r_t can be represented in the following form:

$$r_t = a_1 r_{t-1} + a_2 r_{t-2} + \ldots + b_0 x_t + b_1 x_{t-1} + \ldots + \epsilon_{1,t} \tag{9}$$

where a_i and b_i are the coefficients; $\epsilon_{1,t}$ is a disturbance caused by new public information; x_t is the signed order flow (positive for buy orders and negative for sell orders).

Structural Models

- In turn, we have the general form

$$x_t = c_1 r_{t-1} + c_2 r_{t-2} + ... + d_1 x_{t-1} + d_2 x_{t-2} + ... + \epsilon_{2,t} \quad (10)$$

- The specific of the Hasbrouck's model (1991) is that the equation for r_t (9) has the contemporaneous term, $b_0 x_t$, while (10) has only lagging terms. This implies that the quote revision follows trade impact immediately but the latter cannot instantly reflect the former.

- Hasbrouck assumes also that disturbances have zero means and are not serially correlated:

$$E(\epsilon_{1,t}) = E(\epsilon_{2,t}) = 0;$$
$$E(\epsilon_{1,t}, \epsilon_{1,s}) = E(\epsilon_{1,t}, \epsilon_{2,s}) = E(\epsilon_{2,t}, \epsilon_{2,s}) = 0 \text{ for } t \neq s \quad (11)$$

Structural Models

- The system (9) - (11) represents a bivariate *vector auto-regression* VAR.
- Assuming that $x_0 = \epsilon_{2,0}$ and $\epsilon_{1,t} = 0$, the long-term price impact can be estimated using the *cumulative impulse response*:

$$\alpha_m(\epsilon_{2,t}) = \sum_{t=0}^{m} E[r_t|\epsilon_{2,0}] \tag{12}$$

- Hasbrouk (1991) offers a structural model that is formulated in terms of the efficient price M_t, bid/ask mid-price p_t, and signed trading size x_t:

$$M_t = M_{t-1} + \epsilon_{1,t} + z\epsilon_{2,t} \tag{13}$$

Structural Models

- And the following

$$p_t = M_t + a(p_t - M_t) + bx_t \tag{14}$$

$$x_t = -c(p_{t-1} - M_{t-1}) + \epsilon_{2,t} \tag{15}$$

where x_t is positive (negative) if a trade is initiated by a buyer (seller).

The random shocks $\epsilon_{1,t}$ and $\epsilon_{2,t}$ satisfy (11) and are related to non trading and trading information, z is intensity of the latter, and a, b, and c are the model coefficients that satisfy the following conditions:

$$0 < a \leq 1, b > 0, c > 0 \tag{16}$$

where a is responsible for inventory control (which is imperfect when $a < 1$).

Structural Models

- While the efficient price is unobservable, the system (13) - (18) can be expressed in terms of observable variables x_t and $r_t = p_t - p_{t-1}$. The case with $a < 1$ yields an infinite VAR:

$$\begin{aligned} r_t = (a+b)x_t + [zbc - (1-a)b]x_{t-1} \\ +a[zbc - (1-a)b]x_{t-2} + \ldots + \epsilon_{1,t} \end{aligned} \tag{17}$$

$$x_t = -bcx_{t-1} - abcx_{t-2} - a^2bcx_{t-3} + \ldots + \epsilon_{2,t} \tag{18}$$

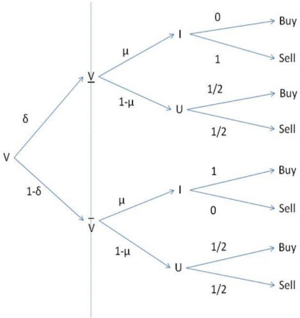
  where $a < 1$ the coefficients of higher terms decrease and regressions can be truncated at some lag with acceptable accuracy. Then the cumulative impulse response (12) has a rather fast convergence.

A Simple Sequential Trade Model - A special case of Glosten and Milgrom (1985)

- There is one security with a value [payoff] V that is either high or low, $\bar{V}$ or $\underline{V}$. The probability of the low outcome is δ. The value is revealed after the market closes - it is not affected by trading.
- The trading population comprises informed and uninformed traders. Informed traders (insiders) know the value outcome. The proportion of informed traders in the population is μ.
- A dealer posts bid and ask quotes, B and A. A trader is drawn at random from the population. If the trader is informed, she buys if $V = \bar{V}$ and sells if $V = \underline{V}$. If the trader is uninformed, he buys or sells randomly and with equal probability. The dealer does not know whether the trader is informed.
- I and U denote the arrivals of informed and uninformed traders.

Event Tree of the Sequential Trade Model

- A buy is a purchase by the customer at the dealer's ask price, A; a sell is a customer sale at the bid.



** The value attached to the arrow is the probability of the indicated transition. Total probabilities are obtained by multiplying along a path.

Event Tree Probabilities

- For example, the probability of a low realization for V, followed by the arrival of an uninformed trader who buys is $\delta(1-\mu)/2$.
- The sum of the total probabilities over terminal *Buy* nodes gives the unconditional probability of a buy:

$$Pr(\text{Buy}) = \frac{(1+\mu(1-2\delta))}{2}$$

- Similarly,

$$Pr(\text{Sell}) = \frac{(1-\mu(1-2\delta))}{2}$$

In the case where $\delta = 1/2$ (equal probabilities of good and bad outcomes), the buy and sell probabilities are also equal.

Dealer's Ask Quotes

- If the dealer is a monopolist, expected profits are maximized by setting the bid infinitely low and the ask infinitely high.
- In practice, the dealer's market power is constrained by competition and regulation, and we assume that competition among dealers drives expected profits to zero.
- The dealer's inference given that the first trade is a buy or sell can be summarized by her revised beliefs about the probability of a low outcome:

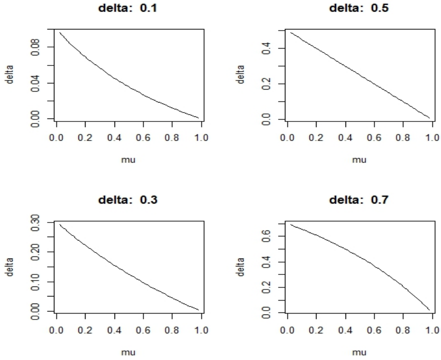
$$\delta_1(\text{Buy}) = Pr(\underline{V}|\text{Buy}) = \frac{Pr(\underline{V}, \text{Buy})}{Pr(\text{Buy})} = \frac{\delta(1-\mu)}{1+\mu(1-2\delta)}. \quad (19)$$

Because μ and δ are between zero and one, $\partial\delta_1(\text{Buy})/\partial\mu < 0$: The revision in beliefs is stronger when there are more informed traders in the population.

Dealer's Ask Quotes

- The dealer's beliefs:

$$\frac{\partial \delta_1(\text{Buy})}{\partial \mu} = -\frac{\delta\mu + \delta\mu^2(1-2\delta) + \delta(1-\mu)(1-2\delta)}{[1+\mu(1-2\delta)]^2}. \quad (20)$$



Dealer's Realized Profit:

- The dealer's realized profit on the transaction is $\pi = A - V$.
- Immediately after the trade, the dealer's expectation of this profit is:

$$E[\pi|\text{Buy}] = A - E[V|\text{Buy}],$$
$$E[V|\text{Buy}] = \delta_1(\text{Buy})\underline{V} + (1 - \delta_1(\text{Buy}))\bar{V}$$

- If competition drives this expected profit to zero, then

$$A = \frac{\underline{V}(1-\mu)\delta + \bar{V}(1-\delta)(1+\mu)}{1+\mu(1-2\delta)}. \tag{21}$$

A dealer's quote is essentially a proposal of terms of trade. When the bid is hit or the offer is lifted, this proposal has been accepted. In the present model, the ask is simply what the dealer believes the security to be worth.

Net Wealth Transfer:

- The ask quote strikes a balance between informed and uninformed traders. The conditional expectation of value can be decomposed as

$$E[V|\text{Buy}] = E[V|U, \text{Buy}]Pr(U|\text{Buy}) + E[V|I, \text{Buy}]Pr(I|\text{Buy}).$$

- Substitute this into the zero-expected profit condition $A = E[V|\text{Buy}]$ and rearrange, we have

$$\underbrace{(A - E[V|U, \text{Buy}])}_{\substack{\text{Gain from an} \\ \text{uninformed trader}}} \; Pr(U|\text{Buy}) = - \underbrace{(A - E[V|I, \text{Buy}])}_{\substack{\text{Loss to an} \\ \text{informed trader}}} \; Pr(I|\text{Buy}) \tag{22}$$

The expected gains from uninformed traders are balanced by the losses to informed traders. In this model, therefore, there is a net wealth transfer.

Dealer's Bid Quotes

- Following a sale to the dealer:

$$\delta_1(\text{Sell}) = Pr(\underline{V}|\text{Sell}) = \frac{Pr(\underline{V}, \text{Sell})}{Pr(\text{Sell})} = \frac{\delta(1+\mu)}{1-\mu(1-2\delta)}. \quad (23)$$

Because μ and δ are between zero and one, $\delta_1(\text{Sell}) > \delta_1(\text{Buy})$. $\underline{V}$ is less likely if the customer bought, reasons the dealer, because an informed customer who knew $V = \underline{V}$ would have sold. Further more $\partial\delta_1(\text{Sell})/\partial\mu > 0$

$$B = E[V|\text{Sell}] = \frac{\underline{V}(1+\mu)\delta + \bar{V}(1-\delta)(1-\mu)}{1-\mu(1-2\delta)}. \quad (24)$$

- The bid-ask spread is

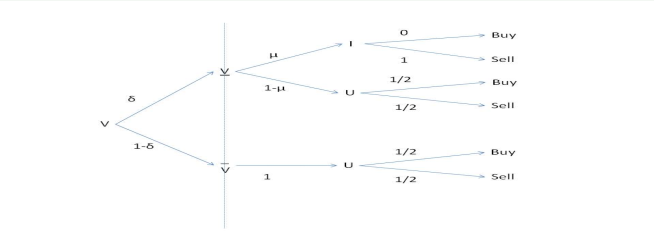
$$A - B = \frac{4(1-\delta)\delta\mu(\bar{V} - \underline{V})}{1-(1-2\delta)^2\mu^2}. \quad (25)$$

Example

Example: consider a variation of the model in which there is no informed trading in the low state (V=$\underline{V}$). Verify that

$$\delta_1(\text{Buy}) = \frac{\delta(1-\mu)}{1-\delta\mu}; \delta_1(\text{Sell}) = \frac{\delta(1+\mu)}{1+\delta\mu}$$

$$A = \frac{\underline{V}(1-\mu)\delta + \bar{V}(1-\delta)}{(1-\mu\delta)}; B = \frac{\underline{V}(1+\mu)\delta + \bar{V}(1-\delta)}{(1+\mu\delta)}.$$



Market Dynamics: Bid and Ask Quotes over Time

- Let δ_k denote the probability of a low outcome given δ_{k-1} and the direction of the kth trade, with the original (unconditional) probability being $\delta_0 \equiv \delta$. Then we have:

$$\delta_k(\mathrm{Buy}_k; \delta_{k-1}) = \frac{\delta_{k-1}(1-\mu)}{1+\mu(1-2\delta_{k-1})}. \tag{26}$$

$$\delta_k(\mathrm{Sell}_k; \delta_{k-1}) = \frac{\delta_{k-1}(1+\mu)}{1-\mu(1-2\delta_{k-1})}. \tag{27}$$

- Market dynamics have the following features:
  - The trade price series is a martingale. (A sequence conditioned on expanding information sets is a martingale)
  - The order flow is not symmetric.
  - Spread declines over time. (estimate more precisely.)
  - The orders are serially correlated (one subset always trade in the same direction).
  - There is a price impact of trades.