FE570 Financial Markets and Trading

Lecture 4. Liquidity, Volatility, and Regulation

(Ref. Larry Harris - *Trading & Exchanges 19, 20, 12*)

Steve Yang

Stevens Institute of Technology

09/18/2018

Outline

The Random-Walk Model of Security Prices

- *1900 - Louis Bachelier:* The French mathematician Louis Bachelier first documented the idea and provided insights about stock market prices in his Ph.D. dissertation titled "The Theory of Speculation".
- *1953 - Maurice Kendal:l* A British statistician proposed the Random-Walk hypothesis in his paper titled "The Analytics of Economic Time Series, Part 1: Prices".
- *1964 - Paul Cootner:* MIT Sloan professor developed the same idea in his book titled "The Random Character of Stock Market Prices".
- *1965 - Eugene Fama:* Eugene Fama at University Chicago further developed the idea in a paper titled "Random Walks In Stock Market Prices", and eventually he infused the idea into the Efficient-Market hypothesis.
- **The Random-Walk Model** is no longer considered to be a complete and valid description of a short-term price dynamics, but it nevertheless retains an important role as a model for the fundamental security value. (Martin Weber, Andrew Lo, etc.)

The Random-Walk Model - General Model

- Let p_t denote the transaction price at time t, where t indexes regular points of real ("calendar" or "wall-clock") time, for example, end-of-day, end-of-hour, end-of-minute, end-of-second, etc. Because it is unlikely that trades occur exactly at these times, we will approximate these observations by using the prices of the last (most recent) trade, for example, the day's closing price.

- **The Random-Walk model** (with drift) is defined as:

$$\underline{p_t = p_{t-1} + \mu + u_t,}$$

  - where u_t, $t = 0, 1, \ldots$ are independently and identically distributed random variables. Intuitively, they arise from new information that bears on the security value.
  - μ is the expected price change (the drift).
  - The units of p_t are either levels (e.g. dollars per share) or logarithms. The log form is sometimes more convenient because price changes can be interpreted as continuously compounded returns.

The Random-Walk Model - Martingale

- The drift can be dropped in this model in most of the microstructure analysis. When $\mu = 0$, p_t cannot be forecast beyond its most recent value:

  $E[p_{t+1}|p_t, p_{t-1}, ...] = p_t,$

  A process with this property is generally described as a *martingale*.

  One definition of a martingale is a discrete stochastic process x_t where $E|x_t| < \infty$ for all t, and $E(x_{t+1}|x_t, x_{t-1}, ...) = x_t$
- Note that expectation in this formulation is conditioned on lagged p_t or x_t that is the history of the process.
- A more general definition of a martingale process involves conditioning on broader information set. The process x_t is a martingale with respect to another (possibly multidimensional) process z_t if $E|x_t| < \infty$ for all t and $E(x_{t+1}|z_t, z_{t-1}, ...) = x_t$
- When the conditioning information is "all public information", the conditional expectation is sometimes called the fundamental value or the efficient price of the security.

The Random-Walk Model - Observations

- To define a Random-Walk formally, take independent random variables $Z_1, Z_2, \ldots$, where each variable is either 1 or -1, with a 50% probability for either value, and $S_0 = 0$ and $S_n = \sum_{j=1}^{n} Z_j$. The series S_n is called the **simple random walk on $\mathbb{Z}$**
  - This series (the sum of the sequence of -1s and 1s) gives the distance walked, if each part of the walk is of length one. The expectation $E(S_n)$ of S_n is zero.
  - This follows by the finite additivity property of expectation: $E(S_n) = \sum_{j=1}^{n} E(Z_j) = 0$.
- A Random-Walk is a process constructed as the sum of independently and identically distributed (i.i.d) zero-mean random variables - a special case of martingale.
- In microstructure analysis, transaction prices are usually not martingales, but by imposing economic and statistical structure it is often possible to identify a martingale component of the price.

Liquidity

- **Liquidity** is the ability to trade large size quickly, at low cost, when you want to trade. It is the most important characteristic of well-functioning markets.

  Everyone in the markets has some affect on liquidity. Impatient traders take liquidity. Dealers, limit order traders, and some speculators offer liquidity. Brokers and exchanges organize liquidity.

  Given its importance, one would expect that the term *liquidity* would be well defined and universally understood. The confusion is due to the many dimensions of liquidity.

Liquidity Dimensions

Liquidity is the object of bilateral search. The primary input of this process is the time spent, and the main outputs are good prices and adequate sizes.

CHANGE	HOLD CONSTANT	IMPLICATION
Spend more time searching	Size of trade Price you are willing to pay or receive	Expect to find a better average price Expect to find more size
Increase size of desired trade	Time spent searching Price	Expect to find worst average price Expect to spend more time searching
Offer a better price	Size of trade Time spent searching	Expect to spend less time searching Expect to find more size

Asset Returns and Distributional Properties of Returns

Asset Returns:

Let P_t be the price of an asset at time t, and assume no dividend.

One-period simple return: Gross return

$$1 + R_t = \frac{P_t}{P_{t-1}} or P_t = P_{t-1}(1 + R_t) \tag{1}$$

Simple return:

$$R_t = \frac{P_t}{P_{t-1}} - 1 = \frac{P_t - P_{t-1}}{P_{t-1}} \tag{2}$$

Multiperiod simple return: Gross return

$$\begin{aligned} 1 + R_t(k) &= \frac{P_t}{P_{t-k}} \\ &= \frac{P_t}{P_{t-1}} \times \frac{P_{t-1}}{P_{t-2}} \times \ldots \times \frac{P_{t-k+1}}{P_{t-k}} \\ &= (1 + R_t)(1 + R_{t-1})\ldots(1 + R_{t-k+1}). \end{aligned}$$

Example

Asset Returns Example: Suppose the closing prices of an asset are

Period	1	2	3	4	5
Price	37.84	38.49	37.12	37.60	36.30

- What is the simple return from period 1 to period 2?
  Ans: $R_2 = \frac{38.49-37.84}{37.84} = 0.017$.
- What is the simple return from period 1 to period 5?
  Ans: $R_5(4) = \frac{36.30-37.84}{37.84} = -0.041$.
- Verify that $1 + R_5(4) = (1 + R_2)(1 + R_3)...(1 + R_5)$.

- **Continuously Compounded (or Log) Returns:**
  - **Log return:**

$$r_t = ln(1 + R_t) = ln\frac{P_t}{P_{t-1}} = p_t - p_{t-1} \tag{3}$$

where $p_t = ln(P_t)$.

  - **Multiperiod log return**

$$\begin{aligned} r_t(k) &= ln[1 + R_t(k)] \\ &= ln[(1 + R_t)(1 + R_{t-1})...(1 + R_{t-k+1})] \\ &= ln(1 + R_t) + ln(1 + R_{t-1}) + ... + ln(1 + R_{t-k+1}) \\ &= r_t + r_{t-1} + ... + r_{t-k+1}. \end{aligned}$$

Example

Log Returns Example: Suppose the closing prices of an asset are

Period	1	2	3	4	5
Price	37.84	38.49	37.12	37.60	36.30

- What is the log return from period 1 to period 2?

  Ans: $r_2 = ln(38.49) - ln(37.84) = 0.017.$

- What is the log return from period 1 to period 5?

  Ans: $r_5(4) = ln(36.30) - ln(37.84) = -0.042.$

- Is it easy to verify that $r_5(4) = r_2 + r_3 ... r_5$?

Distributional Properties of Returns:

- *Moments of a random variable* X with density $f(x) : \ell - th$ moment

$$m_\ell^{'} = E(X^\ell) = \int_{-\infty}^{\infty} x^\ell f(x) dx \tag{4}$$

- $\ell - th$ **central moment**

$$m_\ell = E[(X - \mu_x)^\ell] = \int_{-\infty}^{\infty} (x - \mu_x)^\ell f(x) dx \tag{5}$$

- **First moment: mean or expectation of** X
- **Second moment: variance of** X
- **Skewness (symmetry) and Excess Kurtosis (fat-tails)**

$$S(x) = E[\frac{(X - \mu_x)^3}{\sigma_x^3}] \tag{6}$$

$$K(x) = E[\frac{(X - \mu_x)^4}{\sigma_x^4}] \tag{7}$$

Example

Properties of Returns:

- Why are mean and variance of returns important?

  Ans: They are concerned with long-term return and risk, respectively.

- Why is symmetry of interest in financial study?

  Ans: Symmetry has important implications in holding short or long financial positions and in risk management.

- Why is kurtosis important?

  Ans: Related to volatility forecasting, efficiency in estimation and tests, etc. High kurtosis implies heavy (or long) tails in distribution.

Estimation for a given sample data $\{x_1, ..., x_T\}$:

- **Sample mean:**

$$\hat{\mu_x} = \frac{1}{T}\sum_{t=1}^{T} x_t, \tag{8}$$

- **Sample variance:**

$$\hat{\sigma}_x^2 = \frac{1}{T-1}\sum_{t=1}^{T}(x_t - \hat{\mu}_x)^2, \tag{9}$$

- **Sample skewness:**

$$\hat{S}(x) = \frac{1}{(T-1)\hat{\sigma}_x^3}\sum_{t=1}^{T}(x_t - \hat{\mu}_x)^3, \tag{10}$$

- **Sample kurtosis:**

$$\hat{K}(x) = \frac{1}{(T-1)\hat{\sigma}_x^4}\sum_{t=1}^{T}(x_t - \hat{\mu}_x)^4, \tag{11}$$

Normality Test: Under the normality assumption, we have:

$$\hat{S}(x) \sim N(0, \frac{6}{T}), \hat{K}(x) - 3 \sim N(0, \frac{24}{T}) \tag{12}$$

- **Test for symmetry:**

$$S^* = \frac{\hat{S}(x)}{\sqrt{6/T}} \sim N(0, 1), \tag{13}$$

  Description rule: Reject H_0 of a symmetric distribution if $|S^*| > Z_{\alpha/2}$ or p-value is less than α.

- **Test for tail thickness:**

$$K^* = \frac{\hat{K}(x) - 3}{\sqrt{24/T}} \sim N(0, 1), \tag{14}$$

  Description rule: Reject H_0 of a symmetric distribution if $|K^*| > Z_{\alpha/2}$ or p-value is less than α.

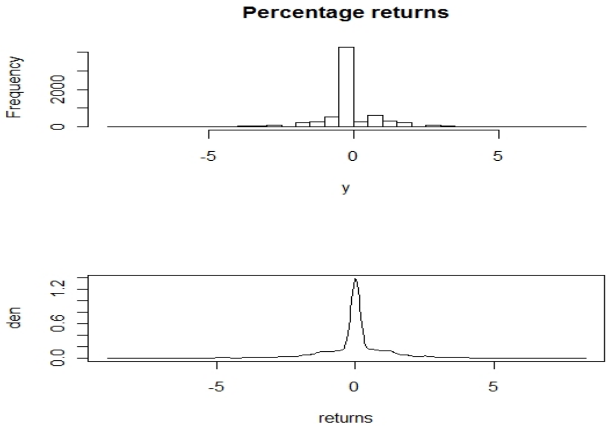


Figure: Simulated E-mini Market Return Distribution.

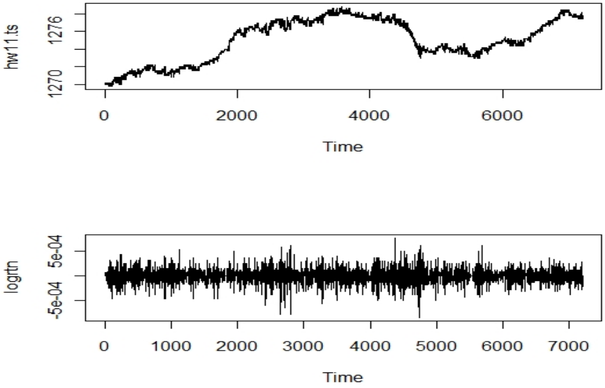
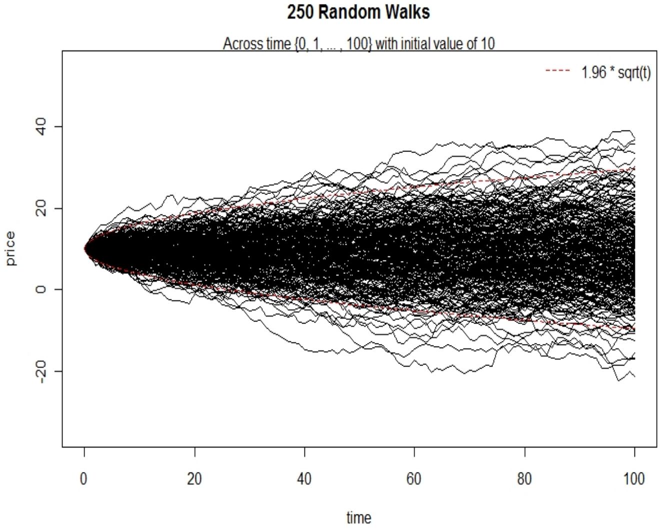


Figure: Simulated E-mini Market Prices and Returns.

Suppose we have a sample $\{p_1, p_2, ..., p_T\}$ generated from a random walk process.



2D Random Walk(for fun!!!).

Statistical Analysis of Price Series - Empirical Evidence

- By the Random-Walk model (with drift):
  $p_t = p_{t-1} + \mu + u_t$,
  Price changes $\Delta p_t = p_t - p_{t-1}$ should be i.i.d with mean $E(u_t) = \mu$ and variance $Var(u_t) = \sigma_u^2$, for which we can calculate the usual estimates. However the empirical evidence shows the following features:
  - **Near-Zero Mean Returns.** In microstructure data samples μ is usually small relative to the estimation error of its usual estimate, the arithmetic mean. Zero is, of course, a biased estimate of μ, but its estimation error will generally be lower than that of the arithmetic mean.
  - **Extreme Dispersion.** The convenient assumption that price changes are normally distributed is routinely violated. Statistical analysis of price changes at all horizons generally encounter sample distribution with fat tails.
  - **Dependence of Successive Observations.** The increments (changes) in random walk should be uncorrelated. In actual samples, the first order autocorrelations of short-run price changes are usually negative.

Spread

- **Spread:** The size of the bid/ask spread is an important object of the microstructure theory.
  - *The quoted spread* between ask A_t and bid B_t that is averaged over T periods equals:
    $S^Q = \frac{1}{T}\sum_{t=1}^{T}(A_t - B_t)$
  - *The average spread* in terms of the asset fundamental price P_t^* is defined as:
    $S = \frac{1}{T}\sum_{t=1}^{T} 2q_t(P_t - P_t^*)$
    where q_t is 1 for buy orders and -1 for sell orders. Since the value of P_t^* is not observable, the *effective spread* in terms of mid-price $M_t = \frac{1}{2}(A_t + B_t)$ is usually used:
    $S^E = \frac{1}{T}\sum_{t=1}^{T} 2q_t(P_t - M_t)$
  - *The realized spread* is applied in post-trade analysis:
    $S^R = \frac{1}{T}\sum_{t=1}^{T} 2q_t(P_t - M_{t+1})$

- **Spread Components:** The bid/ask spread is the price of immediacy of trading.
  - The spread incorporates the dealers' operational costs, such as trading system development and maintenance, clearing and settlement, etc. If dealers are not compensated for their expenses, there is no rationale for them to stay in the business.
  - Dealers' inventory costs contribute to the bid/ask spread, because they must recover their potential losses by widening the spread. Since deals must satisfy order flows on both sides of the market, they maintain inventories of risky instruments (and sometimes undesirable).
  - Spread covers the dealers' risk of trading with counterparts who have superior information about true security value. Informed traders trade at one side of the market and may profit from trading with dealers. This component of the bid/ask spread is called the *adverse-selection* component since dealers confront one-sided selection of their order flow.

- **Liquidity:** is a notion that is widely used in finance, yet it has no strict definition and in fact may have different meanings. Generally, the term *liquid asset* implies that it can be quickly and cheaply sold for cash. A popular notion defines liquidity as the market's *breadth, depth, and resiliency*.
  - It means that buying price and selling price of a liquid instrument are close, that is, the bid/ask spread is small. In a deep market, there are many orders from multiple market makers, so that order cancellations and transactions do not affect notably the total order inventory available for trading.
  - Market resiliency means that if some liquidity loss does occur, it is quickly replenished by market makers. In other words, market impact has only temporary effect.
  - Sometimes inverse liquidity - *illiquidity* based on the price impact caused by trading volume, is used: $ILLIQ = \frac{1}{N}\sum_{k=1}^{N} |r_k|/V_k$, where r_k and V_k are the return and trading volume at time k.