FE 610 Stochastic Calculus for Finance Midterm

March 16^{th}, 2019

When you submit your file, please make sure that you have your name, CWID, and pledge on the document. I recommend the name you choose for your file include your last name and the word midterm so you can ensure that you upload the correct file (I've had students just upload the original document back up and of course received a 0).

- There are 3 problems, worth a total of 100 points.
- Anything you provide should be your work.
- Unless otherwise stated, you can assume that $W(t)$ is a Brownian Motion
- Showcase your work: providing just the answer will result in a minimum of points.

1. (50 pts) For this problem, you will be working with three stochastic processes:

$$X(t) = X(0) + \int_0^t \alpha X(u)du + \int_0^t \sigma X(u)dW(u)$$

$$Y(t) = e^{-rt+W(t)}$$

$$Z(t) = e^{-\int_0^t B(u)du}$$

where α, σ, and r are positive constants and $B(t)$ is a non-random process.

(a) Find $\mathbb{E}[X(t)]$, $\mathbb{E}[Y(t)]$, and $\mathbb{E}[Z(t)]$. (Hint: For $X(t)$ you might want to look at $d(\log(X(t)))$.

(b) Express the Ito Decomposition of the processes $Y(t)$ and $Z(t)$ ($X(t)$ is already expressed as an Ito process).

(c) For $A(t) = X(t)Y(t)Z(t)$, find:

$$d(A(t))$$

2. (20 pts) For a stochastic process given by:

$$F(t) = \begin{cases} 3W(t) & t < 2 \\ 3W(t) + 2 & t \in [2, 4) \\ 3W(t) - 1 & t \geq 4 \end{cases}$$

Find $[F, F](6)$

3. (30 pts) For a process defined as:

$$G(t) = e^{3W(t) - 4t}$$

(a) Is this process a martingale?

(b) Is this process Markov?