

FE570 Financial Markets and Trading

Lecture 7. Information-based Models, and Inventory Models (Ref. Anatoly Schmidt - *Market Microstructure: Information-Based Models*)

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Outline

- 1 Information-based Models
- 2 Inventory Models
- 3 Agent-based Modeling of Financial Markets

One-Period Model (Kyle's Model)

Kyle (1985) considers a one-period model with a risk-neutral informed trader (insider) and several liquidity traders who trade a single risky security with a risk-neutral market maker (dealer).

- The terminal security value is $v \sim N(p_0, \Sigma_0)$.
- There is one informed trader who knows v and enters a demand x .
- Liquidity ("noise") traders submit a net order flow $u \sim N(0, \sigma_u^2)$, independent of v .
- The market maker (MM) observes the total demand $y = x + u$ and then sets a price p .
- All of the trades are cleared at p .
- If there is an imbalance between buyers and sellers, the MM makes up the difference.

The Informed Trader's Problem

We first consider the informed trader's problem (given a conjectured MM price function) and then show that the conjectured price function is consistent with the informed trader's optimal strategy.

- The informed trader conjectures that the MM uses a linear price adjustment rule:

$p = \lambda y + \mu$ where y is the total order flow: $y = u + x$. λ is an inverse measure of liquidity.

- The informed trader's profits are: $\pi = (v - p)x$.
- Substitute in for the price conjecture and y yields:

$$\pi = x[v - \lambda(u + x) - \mu]$$

- The expected profits are:

$$E\pi = x(v - \lambda x - \mu).$$

In the sequential trade models, an informed trader always makes money, but this is not true anymore.

- If the informed trader is buying ($x > 0$), it is possible that a large surge of uninformed trader is buying ($u \gg 0$) drives $\lambda(u + x) + \mu > v$ - loss
- The informed trader chooses x to maximize $E\pi$, yielding $x = \frac{v - \mu}{2\lambda}$; the second-order condition is $\lambda > 0$.

The Market Maker's Problem

- The MM conjectures that the informed trader's demand is linear in v : $x = \alpha + \beta v$.
- Knowing the optimization process that the informed trader followed, the MM can solve for α and β

$$\frac{v - \mu}{2\lambda} = \alpha + \beta v \text{ for all } v. \quad (1)$$

which implies

$$\alpha = -\mu/2\lambda \text{ for all } \beta = 1/2\lambda. \quad (2)$$

Bivariate Normal Projection

- The inverse relation between β and λ is particularly important. As liquidity drops (i.e. as λ rises) the informed agent trades less. Now the MM must figure out $E[v|y]$.
- Suppose that X and Y are bivariate normal random variables with means μ_X and μ_Y , variances σ_X^2 and σ_Y^2 , and covariance σ_{XY} .
- The conditional expectation of Y given X is

$$E[Y|X = x] = \mu_Y + \frac{(x - \mu_X)\sigma_{XY}}{\sigma_X^2} \quad (3)$$

Because this is linear in X , conditional expectation is equivalent to projection. The Variance of the projection error is

$$\text{Var}[Y|X = x] = \sigma_Y^2 - \frac{\sigma_{XY}^2}{\sigma_X^2} \quad (4)$$

- Given the MM's conjecture, $y = u + \alpha + \beta v$, we have:

$$Ey = \alpha + \beta Ev = \alpha + \beta p_0,$$

$$\text{Var}(y) = \sigma_u^2 + \beta^2 \Sigma_0, \text{ and}$$

$$\text{Cov}(y, v) = \beta \Sigma_0.$$

- Using these in the projection results gives

$$E[v|y] = p_0 + \frac{\beta(y - \alpha - \beta p_0)\Sigma_0}{\sigma_u^2 + \beta^2 \Sigma_0} \text{ and}$$

$$\text{Var}[v|y] = \frac{\sigma_0^2 \Sigma_0}{\sigma_u^2 + \beta^2 \Sigma_0}.$$

- This must equal to $p = \lambda y + \mu$ for all values of y , so

$$\mu = \frac{-\alpha\beta\Sigma_0 + \sigma_u^2 p_0}{\sigma_u^2 + \beta^2 \Sigma_0} \text{ and } \lambda = \frac{\beta\Sigma_0}{\sigma_u^2 + \beta^2 \Sigma_0}.$$

Solution

- Solving the mentioned equations for α , β , μ , and λ , we have

$$\alpha = p_0 \sqrt{\frac{\sigma_u^2}{\Sigma_0}}; \mu = p_0; \lambda = \frac{1}{2} \sqrt{\frac{\Sigma_0}{\sigma_u^2}}; \beta = \sqrt{\frac{\sigma_u^2}{\Sigma_0}}.$$

Discussion

- Both the liquidity parameter λ and the informed trader's offer coefficient β depend only on the value uncertainty Σ_0 relative to the intensity of noise trading σ_u^2 .
- The informed trader's expected profit is:

$$E[\pi] = \frac{(v - p_0)^2}{2} \sqrt{\frac{\sigma_u^2}{\Sigma_0}} \quad (5)$$

One-Period Model

- The expected profit $E[\pi]$ is increasing in the divergence between the true value and the unconditional mean.
- The expected profit $E[\pi]$ is also increasing in the variance of noise trading. We can think of the noise trading as providing camouflage for the informed trader.
- All else equal, an agent trading on inside information will be able to make more money in a widely held and frequently traded stock.

Extension

- The essential properties of the Kyle model that make it tractable arise from the multivariate normality (which gives linear conditional expectations) and a quadratic objective function (which has a linear first-order condition).
- The multivariate normality can accommodate a range of modifications, such as, partially informed traders, signal, etc.

Multiple Rounds of Trading

We consider the case of N auctions that are equally spaced over a unit interval time. The time between auction is $\Delta t = 1/N$. The auctions are indexed by $n = 1, \dots, N$. The noise order flow arriving at the n th auction is defined as Δu_n : $\Delta u_n \sim N(0, \sigma_u^2 \Delta t)$ where σ_u^2 has the units of variance per unit time. The use of the difference notation facilitates the passage to continuous time. α_n and δ_n define a quadratic profit function. We have the following properties:

- The informed trader's demand is $\Delta x_n = \beta_n(v - p_n)\Delta t_n$, where β_n is trading intensity per unit time.
- The price change at the n th auction is $p_n = p_{n-1} + \lambda_n(\Delta x_n + \Delta u_n)$.
- Market efficiency requires $p_n = E[v|y_n]$ where y_n is the cumulative order flow over time.
- In the end of n auctions, the dealer's expected profits: $E(\pi_n | p_1, \dots, p_{n-1}, v) = \alpha_{n-1}(v - p_{n-1})^2 + \delta_{n-1}$.

Multiple Rounds of Trading - Equilibrium

- In equilibrium, the coefficients satisfy the following relations:

$$\lambda_n = \beta_n \sigma_n^2 / \sigma_u^2 \quad (6)$$

$$\beta_n \Delta t = \frac{1 - 2\alpha_n \lambda_n}{2\lambda_n(1 - \alpha_n \lambda_n)} \quad (7)$$

$$\alpha_{n-1} = \frac{1}{4\lambda_n(1 - \alpha_n \lambda_n)} \quad (8)$$

$$\delta_{n-1} = \delta_n + \alpha_n \lambda_n^2 \sigma_u^2 \Delta t \quad (9)$$

- In contrast to the single-period model, the variance is now time-dependent:

$$\sigma_n^2 = (1 - \beta_n \lambda_n \Delta t) \sigma_{n-1}^2 \quad (10)$$

****** Need an iterative procedure to solve the non-linear equations.

Multiple Rounds of Trading

- The conditional expectation of the incremental order flow is:

$$E[\Delta y_n | y_{n-1}] = E[\Delta u_n + \Delta x_n | y_{n-1}]$$

$$E[\Delta u_n + \Delta x_n | y_{n-1}] = E[\beta_n(v - p_{n-1})\Delta t | y_{n-1}]$$

$$E[\beta_n(v - p_{n-1})\Delta t | y_{n-1}] = 0.$$

- The first equality is definitional.
- The second holds because the noise order flow has zero expectation.
- The third equality reflects the market efficiency condition that $E[v | y_{n-1}] = p_{n-1}$.

****** From a strategic viewpoint, the informed trader hides behind the uninformed order flow (means she trades so that the MM can't predict what she will do next- on the basis of the net order flow).

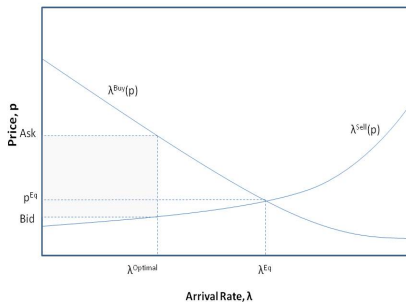
Inventory Models - Garman (1976)

Garman (1976) suggests that a dealer is needed because buyers and sellers do not arrive synchronously. In this model, buyers and sellers arrive randomly in continuous time, as Poisson arrival process.

- The arrival intensity for buyers is $\lambda^{Buy}(p)$, a function of the price they pay.
- The arrival intensity of sellers is $\lambda^{Sell}(p)$, a function of price they receive.
- Market clearing in this context means that buyers and sellers arrive at the same average rate, that is, which have the same intensity.
- If the dealer were to quote the same price to buyers and sellers, market clearing would occur where the intensity functions cross.

Garman Model

- Market clearing when $\lambda^{Buy}(p^{Eq}) = \lambda^{Sell}(p^{Eq}) = \lambda^{Eq}$.
As long as the intensities are the same, the dealer is on average buying and selling at the same rate.



** Arrival rates of buyers and sellers.

Garman Model

- If the dealer is buying and selling at the same price, of course, there is no profit. If the dealer quotes an ask price to the buyers and a bid quote to the sellers, he will make the spread on every unit turned over (the "turn").
- The dealer's average profit (trading revenue) per unit time is:

$$\begin{aligned}\pi(Bid, Ask) &= (Ask - Bid)\lambda^{Buy}(Ask) \\ &= (Ask - Bid)\lambda^{Sell}(Bid),\end{aligned}$$

where we have maintained the condition that is $\lambda^{Buy}(Ask) = \lambda^{Sell}(Bid)$, that is, supply and demand balance (on average). Setting a wide spread increases the profit on each trade but depress the rate of arrivals.

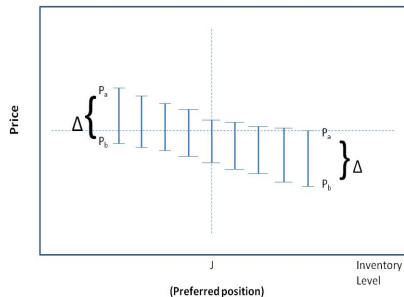
- The profits are defined by the shaded rectangle in the figure. The dealer sets the bid and ask to maximize this area.

Garman Model - Discussion

- To accommodate the asynchronous buying and selling, the dealer needs to maintain buffer stocks of the security and cash. The key constrain is that the dealer's inventories of the security and cash cannot drop below given levels (maybe zero).
- If $\lambda^{Buy}(Ask) = \lambda^{Sell}(Bid)$, holdings of stock follow a zero-drift random walk. Cash holdings follow a positive-drift random walk (due to the turn).
 - * Garman points out that in this case, the dealer is eventually ruined with probability one. (A zero-drift random walk will eventually hit any finite barrier with probability one.)
- The inventory control principle: the essential mechanism is that dealers change their bid and ask quotes to elicit an expected imbalance of buy and sell orders to push their inventories in the direction of their preferred long-run position.

Amihud and Mendelson Model (1980)

- The market maker's inventory of the security is constrained to lie between given upper and lower bounds, which may be thought of as arising from credit constraints.



****** The dependence of bid and ask prices on inventory position.

Amihud and Mendelson Model - Discussion

- The market maker maximizes expected profits per unit time.
- The dealer has a preferred inventory level lying between the two extremes. This arises from the opportunity cost of being at an extreme point.
- The relation between the inventory position and bid/ask prices:
 - Bid and ask are monotone decreasing functions of the inventory level.
 - There is a positive spread.
 - The spread is increasing in distance from preferred position.
 - The quotes are not set symmetrically about the "true" value.
- Another important prediction: Because bids and asks are dependent only on the inventory level, and the inventory level is mean reverting toward the preferred position, the price effects of inventory imbalances are transient.

Agent-based Modeling of Financial Markets

- Agent-based modeling has become very popular in economics and finance (Hommes 2006, LeBaron 2006, and Chiarella et al. 2009, Paddrick et al. 2012, Bookstaber and Paddrick 2014).
- The notion of agents has been used in many market microstructure models discussed in former chapters. But the two fields differ in their assumptions and methodology.
- The current market microstructure theory is focused primarily on the properties of the order book in equilibrium. It is usually assumed that all investors are rational and returns follow the random walk in the spirit of EMH.
- The agent-based modeling is to describe how the asset price dynamics are affected by traders' adaptive behaviors as well as the complex interactions among the heterogeneous agents.

Agent-based Modeling of Financial Markets

- Current agent-based models of financial markets differ in various ways. There are two major groups of models depending on how price discovery is formulated:
- In the first group, *the adaptive equilibrium models*, agents adapt their heterogeneous beliefs using past returns. Price in these models is derived from the supply-demand equilibrium (Brock and Hommes 1998, Chiarella and He 2001)
- The second group, *non-equilibrium price models*, price is a dynamic variable determined via its empirical relation to excess demand (Lux 1998, Farmer and Joshi 2002).

Note: if you are interested in this topic you may choose FE622 - Simulation Methods in Computational Finance and Economics.