FE 610 Stochastic Calculus for Finance Midterm

March 3, 2017

Name:

Please circle your section:

FE610A-Thursday Afternoon
FE610B-Thursday Afternoon
FE610WS-Webcampus

- There are 4 problems, worth a total of 100 points.
- Before you start, make sure your exam is not missing any pages.
- Showcase your work: providing just the answer will result in a minimum of points.
- Closed book. No internet enabled devices. Only one hand written sheet of notes are allowed.

For instructor's use only

Problem	Points	Score
1	25	
2	25	
3	25	
4	25	
Total	100	

1. For $\Omega = \{a, b, c, d\}$, with $\mathbb{P}(a) = 4\mathbb{P}(b) = 2\mathbb{P}(c) = 3\mathbb{P}(d)$. Define the random variables:

$$X(\omega) = \begin{cases} 5 & \omega \in \{a, b\} \\ -2 & \omega \in \{c, d\} \end{cases}$$

$$Y(\omega) = \begin{cases} 3 & \omega \in \{a, c\} \\ 4 & \omega \in \{b, d\} \end{cases}$$

Express:

(a) $\mathbb{E}[X|Y]$

(b) $\mathbb{E}[Y|X]$

2. For a process

$$Z(t) = \log(\pi) + \int_0^t W^3(u)du + \int_0^t \cos(u)dW(u)$$

Find:

(a)

$$d(e^{Z(t)})$$

(b)

$$[Z, Z](t)$$

3. Simplify the following integral: (express in its simplest form i.e. non-stochastic integrals)

$$\int_0^t 4ts^2W^2(s)dW(s)$$

4. For a process $X(t)$ governed by the dynamic:

$$dX(t) = X(t)dt + \sqrt{X(t)}dW(t)$$

with $X(0) = 2$.

(a) Determine:

$$\mathbb{E}[X(t)]$$

(Hint: investigate the process $Y(t) = e^{-t}X(t)$)

(b) Is $X(t)$ a martingale?