FE 515 Final exam

December 2018

This is an open-book final, you can use all materials that are available to you. However, any kind of communication tools is not allowed. Please notice that this final can be solved in multiple approaches. Regardless of the method you would use, you are required to answer all questions.

For this final, **10 points** is reserved for the answers' formatting. For this part you are required to use the Google's R style as discussed in class. Your submission should be easy to understand and clear to read.

Question 1

In this question, the goal is using Monte-Carlo simulation and Euler Method to estimate equity price movement. At the current stage, we don't know how the price moves. However, we guess it may apply to one of the following approaches:

- $dS_t = \mu S_t dt + \sigma S_t dW_t$
- $dS_t = (0.5 + \mu S_t)dt + \sigma S_t^{0.9} dW_t$

In order to figure out which approach is better, you need to download 3 year-length of data in total. The first two years is your training data set and the last year is testing data set.

Part 1: Questions related to training data set

Before you perform the Monte-Carlo simulation, you need to obtain all necessary parameters.

1. (20 points) Assume we have a bond, which pays \$5 coupon every half year. The maturity of this bond is two year with face value \$100. If the current price of this coupon is \$109.28, can you calculate the interest rate μ?

2. (10 points) Using quantmod to download daily data for one equity and calculate the realized variance. You can select any equity you want.

Part 2: Questions related to testing data set

Starting from here, you need to use the parameters from the previous questions, and perform Monte-Carlo simulation on two approaches.

1. (30 points) For the same equity, use the the first close price from the training data set as the initial price S_0 and perform Month-Carlo simulation with following detailed requirements
    - Time to maturity $T = 1$ year
    - Number of path $p = 5000$
    - Number of steps $n =$ this factor depends on the length of testing data set.
2. (10 points) During the simulation, you need to record all paths. In the end, you shall calculate the mean value for every single step.
3. (10 points) Now you should have two vectors represents the equity price movement from two approaches. Compare your simulation result to the testing data using root mean square errors. Which approach is more accurate?
4. (10 points) Collect all S_T from the best approach you selected. Assuming you buy 100 shares of this equity at the beginning of this year, what is the probability to gain profit from this investment after one year?

Bonus part 1 (20 points)

1. (10 points) If you use Rmarkdown for this submission, you will get bonus points. However, a sufficient rmd file and the report are required.
2. (10 points) Make a ggplot which presents the averaged simulated path and the real price movements. In this plot, you should contain following information
    - Proper X-label, Y-label and title
    - Your X input has to be a time line

Bonus part 2 (30 points)

In this bonus question, you need to create a simulation function for a lottery game. **For this self-defined function, the input is the round of games I want to play.**

Here is the probability to win each prize:

Prize	Probability
A	3%
B	50%
C	47%

Additionally:

- If I fail to get a level A prize from the lottery 5 times in a row, the probability to win an A prize will go up 0.5% in the next play. Correspondingly, the probability to obtain B prize and C prize will go down 0.25% respectively.
- If I obtain a level A prize, the probability will be reset to initial values.

Now, please answer following questions:

1. (20 points) For this function, your output should be a table, which record following information:
    - The index of game play sequence. (For example: 1, 2, 3 ...)
    - The result for current play. (For example: A, B, C)
    - The probability for next play. (For example: If I fail to get A prize 5 time in a row, I should receive the probability for the next play as A - 3.5%, B - 49.75%, C - 46.75%)
2. (10 points) If I want to obtain a level A prize (a success simulation), how many times I need to play in average? Answer this question based on 1000 success simulations.