Deterministic regime based Black-Scholes model in short-term equity option pricing

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Abstract

In this paper, we propose a deterministic time regime based Black Scholes model for pricing short term options in high frequency markets. To implement our model, we use high frequency data to estimate parameters in two different ways; one using simple returns and one using activity weighted returns. To establish the efficacy of our approach we compare it to the classic Black Scholes model, chosen for its comparable calculation times.

Key words: Short-term option pricing, High-frequency data, Black-Scholes model

1 Introduction

The trading volume of short-term options has increased sharply during recent years. In 2010, the short-term S&P 500 options contributed less than 5% in average daily trading volume. This number increased to more than 30% in 2013 [13], and then in 2017, the average daily volume of S&P 500 weekly option exceeded 520,000 contracts [14]. As short-term equity options attract more interest from investors, there is an associated increase in interest in short-term option pricing. The goal of this paper is to provide an effective, accurate methodology to price these options, even at high frequency.

With the increased trading volume in high-frequency trading, market micro-structure is changing [12]. Martens and Zein [11] show that historical returns from high-frequency data has better performance in predicting financial volatility. Garcia et al. [5] use high-frequency spot prices to estimate parameters in stochastic volatility model to have better performance. The work from Andersen et al. [1] also show that high-frequency data is more robust in pricing tail risk. Therefore, we apply tick-by-tick data from the equity market and estimate realized volatility for the short-term option pricing.

The Black-Scholes model [2] is widely used in option pricing. When estimating the option price, we can use realized volatility which is calculated based on the historical equity data. We can also use implied volatility, which describes the current dynamic of trading activity. However, the volatility in Black-Scholes model is a constant value. According to empirical study from Ye and Florescu [15], the trading activity is more frequent when the equity markets open and close. For most equities, the intraday return is higher in the head and tail parts during the trading day. Therefore, it is not adequate to use a single volatility to describe the change of price for intraday trading.

Using multiple regimes in option pricing could reduce the error. This is not a new idea as it has been investigated in a variety of papers, all of which showed higher accuracy. For example, Bollen [3] introduced a two-regime Black-Scholes model. He then used Monte-Carlo simulation to show that this model improved pricing accuracy. Ishijima and Kihara [8] used a similar idea but their work can be expanded into multi-dimensions. Guo [6] used a two-state model in option pricing, letting the sentiment analysis determine which state the model was in. Fuh et al. [4] implemented a regime switching model with the regimes determined by a Markov switching model. They then examined the numerical results using

Monte-Carlo simulations. However, despite the increase in accuracy, they found that their regime-switching model is slower in option pricing compared to the classic Black-Scholes model.

To handle this particular issue, we propose a deterministic regime based model for high-frequency option pricing, which will have comparable calculation time to the classic BS. In our results, by effectively determining our parameters, we can still take advantage of the increased accuracy without sacrificing our computation time. The deterministic time regimes are estimated based on historical high-frequency equity data. This allows us to obtain the time regimes before market opening. Thus, we can price the option in a faster way when real-time data comes in. Two types of realized volatility are used in the short-term option pricing. One is the classic realized volatility, which is calculated based on the spot prices at a fixed frequency. The other is activity-weighted volatility, which is estimated based on the equity trading activity. In Section 2.1, we explain how to estimate the deterministic regimes estimation. The deterministic regime Black-Scholes model is shown in Section 2.2. Our result from each model is presented in 3. The t-test is designed in Section 2.3 and the result is shown in Section 4. Additionally, we show the interaction among all factors using analysis of variance (ANOVA) in Section 5.

2 Model description and evaluation

2.1 Deterministic time regimes

Two types of returns are used in this research to obtain the regimes: the classic simple return and the activity-weighted return. The classic simple returns are calculated based on the last price in each minute. The activity-weighted return is calculated based on the mean value of trade price within each minute:

$$AP_t = hiding \tag{1}$$

$$AWR_t = hiding \tag{2}$$

The AP_t is the ... within each minute and AWR is the activity-weighted return. TP_t is the observed trade price within each minute.

Potentially, the sampling frequency can be increase to second level or even smaller. However, one shall consider the influence from the dark pool. By using raw data from the exchange, which is commonly recorded at nanosecond frequency. It is possible to receive the delayed trade price from dark pools, which may not present the current trade price.

We let $r_{i,j}$ denote the return observed on day i and time index j where $i \in \{1, \dots, d\}$ and $j \in \{0, \dots, J\}$ where J is 389 in this paper (or more generally the number of partitions in a given day). For each equity, we group the data from the same time index j, such as $\{r_{i,j=1}\}_{i \in 1 \dots d}, \{r_{i,j=2}\}_{i \in 1 \dots d}, \dots$. Using these data sets, we split a trading day into multiple regimes by using the following steps:

1. Set the first regime of the day based on the first data set corresponding to $\{r_{i,j=1}\}_{i \in 1 \dots d}$.
2. For the remaining return groups $\{r_{i,j}\}_{i \in 1 \dots d, j \in \{2, \dots, N\}}$:
   (a) We perform the t-test on $\{r_{i,j=1}\}$ and $\{r_{i,j=2}\}$ to check whether they have the same mean value. In this t-test, we have the null hypothesis and alternative hypothesis as:

   $$H_0 : \text{The mean value from two data sets are same.}$$
   $$H_\alpha : \text{The mean value from two data sets are different.}$$

   (b) If the P-value from the t-test is not significant, we think the data from $j = 1$ and $j = 2$ belong to the same time regime. Thus, we merge the later group into the first regime and repeat step (a).

(c) If the P-value from the t-test is significant, we think a new time regime starts from $j = 2$. Thus, we set the second regime of the day based on the $\{r_{i,j=2}\}_{i \in 1 \ldots d}$, and repeat step (a).

Based on the deterministic time regimes, we calculate the realized standard deviation using the following expression[10][7][9]. In this paper, we define the realized standard deviation as regime-based realized volatility:

$$\sigma_n = \sqrt{\frac{1}{d * (t(n+1) - t(n))} \sum_{i=1}^{d} \sum_{j=t(n)}^{T(n)} r_{i,j}^2} \tag{3}$$

Here the realized volatility σ_n is calculated based on minute frequency return. The $t(n)$ is the index of the first time-stamp included in regime n, while $T(n)$ is the last time-stamp. Thus, $t(n+1) - t(n)$ is the duration for the corresponding time regimes.

In our experience, we often obtained different regime divisions when using the classic simple return as opposed to activity-weighted returns. As an example, Table 1 shows the deterministic regimes when using the activity-weighted return, and Table 2 shows different regimes when sampling on the simple return. As can be seen, these not only disagree on the number of regimes, but also on the parameters for each regime. In Fig. 1, we graph the corresponding volatility for each regime from these tables to better show its dynamic nature throughout the day.

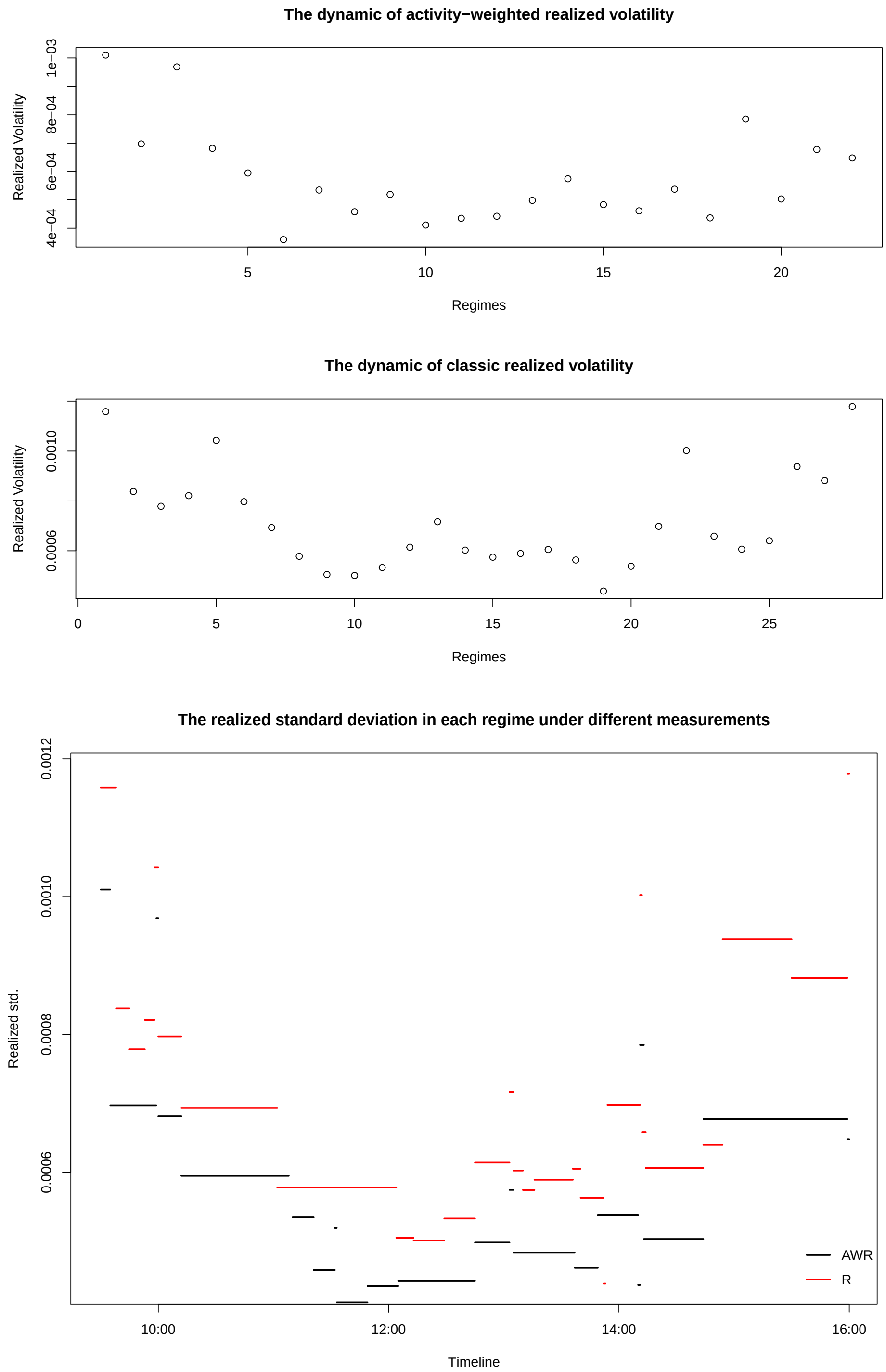
Table 1: An example of SPY when using the activity-weighted return to estimate the deterministic time regime on March 6th, 2018

Regime	Realized std.	Regime duration	Regime	Realized std.	Regime duration
1	0.00101028	5	12	0.00044217	40
2	0.00069715	24	13	0.00049807	18
3	0.00096870	1	14	0.00057447	2
4	0.00068143	12	15	0.00048320	32
5	0.00059480	56	16	0.00046123	12
6	0.00035984	2	17	0.00053753	21
7	0.00053462	11	18	0.00043654	1
8	0.00045797	11	19	0.00078475	2
9	0.00051905	1	20	0.00050311	31
10	0.00041127	16	21	0.00067752	75
11	0.00043499	16	22	0.00064766	1

Table 2: An example of SPY when using the simple return to estimate the deterministic time regime on March 6th, 2018

Regime	Realized vol	Regime duration	Regime	Realized vol	Regime duration
1	0.00115839	8	15	0.00057428	6
2	0.00083775	7	16	0.00058914	20
3	0.00077852	8	17	0.00060506	4
4	0.00082103	5	18	0.00056310	12
5	0.00104262	2	19	0.00043842	1
6	0.00079702	12	20	0.00053791	1
7	0.00069331	50	21	0.00069792	17
8	0.00057783	62	22	0.00100232	1
9	0.00050496	9	23	0.00065836	2
10	0.00050110	16	24	0.00060620	30
11	0.00053292	16	25	0.00064020	10
12	0.00061400	18	26	0.00093795	36
13	0.00071667	2	27	0.00088185	29
14	0.00060245	5	28	0.00117858	1

Figure 1: An example of the realized volatility in each time regime



2.2 Deterministic regime based Black-Scholes model

The deterministic regime based BS model (RBS) can be considered as a combination of BS model, which only estimate the option price for a specific time period. In order to combine them together, the remaining regime time RT is used as the weight factor, which can be obtained through following expression:

$$RT_n = hiding \tag{4}$$

and the weight factor w_n is calculated as:

$$w_n = hiding \tag{5}$$

In Equ 4, we use $frac()$ to express the fractional part of ..., where the time to maturity τ is expressed in minutes. When τ is less than one day (...), we have .... The remaining time RT_n indicates the "life time" for corresponding time regime. When this value goes to 0, the corresponding time regime is no longer used when pricing the short-term option.

The deterministic regime based BS model can be written as:

$$C(S_t, \tau) = \sum_{n=1}^{N} w_n C_n(S_t, \tau) \tag{6}$$

$$P(S_t, \tau) = \sum_{n=1}^{N} w_n P_n(S_t, \tau) \tag{7}$$

where

$$C_n(S_t, \tau) = N(d_{1,n})S_t - N(d_{2,n})Ke^{-r\tau} \tag{8}$$

$$P_n(S_t, \tau) = N(-d_{2,n})Ke^{-r\tau} - N(-d_{1,n})S_t \tag{9}$$

$$d_{1,n} = \frac{1}{\sigma_n\sqrt{\tau}}[ln(\frac{S_t}{K}) + (r + \frac{\sigma_n^2}{2})\tau] \tag{10}$$

$$d_{2,n} = d_{1,n} - \sigma_n\sqrt{\tau} \tag{11}$$

Compared with the classic BS model, the RBS model support multiple volatility in short-term option pricing. Since we price an option at minute frequency, this results in the risk-free rate r extremely small. Therefore, to simply the calculations, we treat r as effectively 0 throughout this study.

2.3 Evaluation for option pricing result

We use the root mean square error (RMSE) to summarize the intraday option pricing results at the end of each trading day. When using the historical data from Thomson Reuters, it is possible to observe missing points at 1 minute frequency. This indicates there was no option transaction during the specific time interval. Therefore, we use the mid price of the best bid/ask for each option as the benchmark during calculation.

As our basis of comparison, we use four different models in this study:

- The Black-Scholes model using a constant volatility. The realized volatility is calculated based on historical simple return. We use *BS_R* to notate this model throughout this paper. This model is our benchmark model.

- The Black-Scholes model using a constant volatility. The realized volatility is calculated based on historical activity-weighted return. We use *BS_AWR* to notate this model throughout this paper.

- The deterministic time regime based Black-Scholes model, which uses the historical simple return to obtain the time regimes. We use *RBS_R* to notate this model throughout this paper.

- The deterministic time regime based Black-Scholes model, which uses the historical activity-weighted return to obtain the time regimes. We use *RBS_AWR* to notate this model throughout this paper.

In this study, we introduce two new concept in option pricing. One is the deterministic time regime, the other one is activity-weighted volatility. In order to figure out which factor improves the accuracy in short-term option pricing, we will perform two tests on the RMSE values in the end.

First, we will perform two t-test on following groups: BS_R with BS_AWR, and BS_R with RBS_R. In these t-tests, we use all available option contracts. Result will be shown in Section 4. The null hypothesis and alternative hypothesis are:

$$H_0 : \text{The mean value of RMSE are indifferent for the two samples}$$
$$H_\alpha : \text{The mean value of RMSE are different for the two samples}$$

Second, we will perform analysis of variance (ANOVA) on all four models. This time, we will use the options which are at the money. Additional information can be found in Section 5.

3 Data description and results

3.1 Data description

We select 10 equity options in this study, their ticker names are recorded in Table 3. All selected tickers are highly traded in both equity markets and option markets. Even though

SPY has three maturities in each week (Monday, Wednesday and Friday), for consistency, we only analyze the SPY options which expire on Friday.

Table 3: 10 equity options we selected in analyzing short-term option pricing results

Ticker	Equity name	Ticker	Equity name
AAPL	Apple Inc.	BAC	Bank of America Corporation
C	Citigroup Inc.	FB	Facebook, Inc.
NVDA	NVIDIA Corporation	QCOM	QUALCOMM Incorporated
TWTR	Twitter, Inc.	SPY	SPDR S&P 500 ETF
XLE	Energy Select Sector SPDR ETF	XLF	Financial Select Sector SPDR ETF

We use one month of training data when estimating the deterministic time regimes for each equity. When calculating the activity-weighted return, we use tick data at nanosecond resolution from Thomson Reuters DataScope Select. Meanwhile, we use the Intraday Summarize Database from Thompson Reuters to obtain the equity spot price and calculate the simple returns. We also use this database to download historical option data from Feb 1st, 2018 to May 30th, 2018 at 1 minute frequency to check our accuracy. As we mentioned before, we focus our analysis on options which have maturities on Friday.

3.2 Option pricing result

We will evaluate the performance of our models for all of these options in later sections. But first, we provide an example using the BAC call option and put option with strike price $30 in Fig. 2. In this example, the option expires on April 20th and we stand on April 19th. We show the price as calculated by each of our four approaches, in addition to providing the bid-ask spread as a shaded grey region. As can be seen in this plot, pricing results from RBS_AWR model has the best performance for these options. The pricing line is within the best bid/ask interval for most of the time. Conversely, when using BS_R model, the option is over priced throughout the whole trading day.

Figure 2: An example for the option pricing result from four models. The gray area is the price range between the best bid price and best ask price. We use different color to distinguish the result from each model

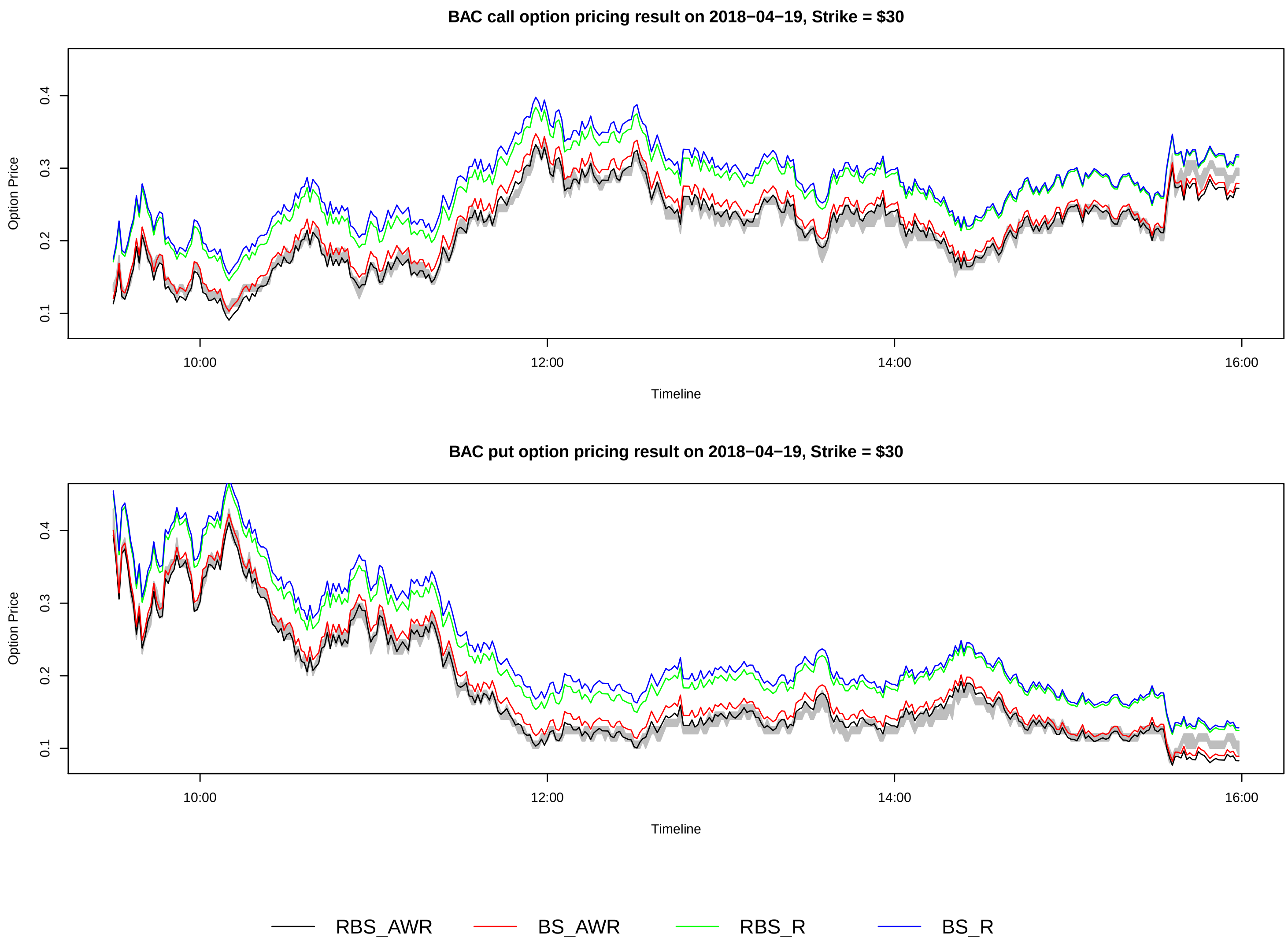
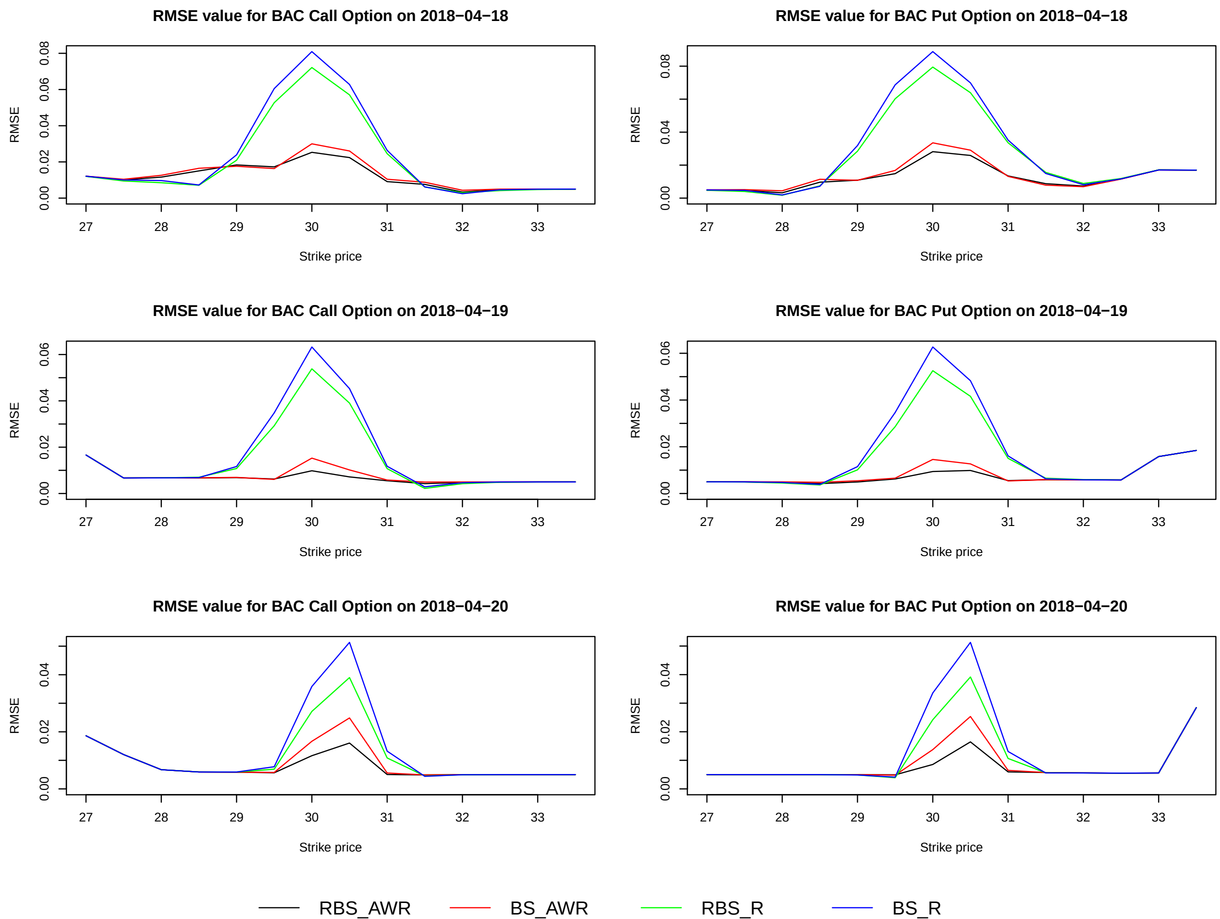


Figure 3: The RMSE value for BAC options. The X-axis records the available strike prices and the Y-axis records the RMSE values. The gray area is the price range based on the equity price movements



To further highlight the accuracy of this approach, Fig. 3 show an example of the RMSE values for BAC options in three consecutive trading days for multiple strikes each day. In this example, the maturity for all options are April 20th, 2018. Based on the equity price movements, we highlight the price range between daily low and daily high using gray color. Based on the plot, the RMSE value from each model presents a "Λ" shape in each trading day. Compared to the other three models, the RMSE value from RBS_AWR maintains a relatively impressive performance. No matter whether the strike price is at the money or not, the RMSE value from this model stays at a low level for each of these different times until maturity. When looking at the BS_R model, it is clear to see that when the strike price is at the money, the pricing result has huge differences as compared to the mid price.

4 T-test on model performance using all contracts

In the previous section, the RBS_AWR model generated the most accurate pricing results in Fig. 2 and Fig. 3. Compared with the benchmark model, this model contains two new factors in option pricing. In this section, we concentrate on just single factor improvements and so will perform t-tests on these two groups: BS_R with BS_AWR, and BS_R with RBS_R. Thus, we shall determine whether a single factor can improve model accuracy when setting the significant level at 5% for all available contracts.

4.1 Factor: model structure

The first experiment will test the RMSE from BS_R model and RBS_R model. More specifically, we want to test whether the deterministic model improves the accuracy when pricing an option. Based on the data sets, we have 1760 t-test results in total.

Surprisingly, the P-values from all t-tests are not significant, which implies that improvement from RBS_R is not significant at 5% level when checking the overall performance. Therefore, we simply calculate the mean value of RMSE for equity options which are grouped based on option type, ticker and maturity. If the RBS_R has lower mean value for a specific group, we count it as a success. Otherwise, we count is as fail. By summarizing this information, we obtain the percentage of success and record it in Table 4. Based on this result, the RBS_R model always outperform BS_R.

Based on the result, although the difference of RMSE value from two models is not significant, the RBS_R still have better accuracy compared with the benchmark model. For most equity options, more than 70% of the time we observe better pricing result. Furthermore, the ratio between call option and put option are similar, which indicates the deterministic

Table 4: The percentage of success time for each model. When calculating this number, we separate the result from call options and put options.

Ticker	Option type	BS_R	RBS_R
AAPL	Call	0.28	0.72
	Put	0.28	0.72
BAC	Call	0.25	0.75
	Put	0.22	0.78
C	Call	0.27	0.73
	Put	0.3	0.7
FB	Call	0.22	0.78
	Put	0.17	0.83
NVDA	Call	0.25	0.75
	Put	0.23	0.77

Ticker	Option type	BS_R	RBS_R
QCOM	Call	0.41	0.59
	Put	0.42	0.58
SPY	Call	0.23	0.77
	Put	0.22	0.78
TWTR	Call	0.31	0.69
	Put	0.3	0.7
XLE	Call	0.47	0.53
	Put	0.44	0.56
XLF	Call	0.22	0.78
	Put	0.23	0.77

time regimes structure works for both type.

4.2 Factor: activity-weighted volatility

The activity-weighted realized volatility is another factor we introduce in the short-term option pricing. In this test, we compare the RMSE value from between BS_R and BS_AWR. When the P-value is not significant, we think both models presents the same accuracy. If the P-value is significant, we check the mean RMSE value for the corresponding group and denote it as success. The result in Table 5 indicates that both models generate indifferent result for the most of time. Especially for AAPL, BAC, QCOM and XLF. Therefore, we use the same method as we have done for the first experiments. We look into the detailed information record the result in Table 6. For AAPL, BAC and XLF, although the t-test result shows that the mean value of RMSE are indifferent for two models, the BS_AWR still has better performance. For QCOM and XLE, BS_R outperform the RBS_R for the most of time.

Based on the information from Table 5 and Table 6, we notice that the activity-weighted realized volatility may not work for all the equity options, such as QCOM and XLE. For the remaining options, the BS_AWR model may be a better selection.

Table 5: We compare the RMSE value from BS_R and BS_AWR, and calculate the percentage of success time for each model. When both model has indifferent result, we denote it as same performance.

Ticker	Option	Same	BS_R	BS_AWR	Ticker	Option	Same	BS_R	BS_AWR
AAPL	Call	1	0	0	QCOM	Call	1	0	0
	Put	1	0	0		Put	1	0	0
BAC	Call	1	0	0	SPY	Call	0.66	0.06	0.28
	Put	1	0	0		Put	0.7	0.03	0.27
C	Call	0.92	0	0.08	TWTR	Call	0.97	0.02	0.02
	Put	0.97	0	0.03		Put	0.97	0	0.03
FB	Call	0.95	0.03	0.02	XLE	Call	0.92	0.02	0.06
	Put	0.94	0.05	0.02		Put	0.92	0.05	0.03
NVDA	Call	0.67	0.09	0.23	XLF	Call	1	0	0
	Put	0.62	0.11	0.27		Put	1	0	0

Table 6: The overall performance of BS_R and BS_AWR, higher percentage indicates the corresponding method has better performance

Ticker	Option type	BS_R	BS_AWR	Ticker	Option type	BS_R	BS_AWR
AAPL	Call	0.47	0.53	QCOM	Call	0.72	0.28
	Put	0.48	0.52		Put	0.75	0.25
BAC	Call	0.31	0.69	SPY	Call	0.45	0.55
	Put	0.28	0.72		Put	0.47	0.53
C	Call	0.36	0.64	TWTR	Call	0.42	0.58
	Put	0.38	0.62		Put	0.39	0.61
FB	Call	0.31	0.69	XLE	Call	0.66	0.34
	Put	0.3	0.7		Put	0.69	0.31
NVDA	Call	0.42	0.58	XLF	Call	0.48	0.52
	Put	0.41	0.59		Put	0.44	0.56

5 Analysis of variance using contracts which are at the money

When using the t-test, we can compare the influence from an individual factor. However, we may neglect the interaction between factors. Nevertheless, we use all available contract for each option, which potentially may smooth out the improvements for contracts which are at the money.

In this section, we perform the ANOVA on the RMSE values from all four models. Instead of using all contracts, we only select the contracts which are at money. To select contracts which are at the money, we use equity open price as the benchmark. If the strike price is within 7% boundary of the open price, we think this contract is at the money. When using the ANOVA method, we not only consider the model structure and type of volatility as factors, but also the maturity date and ticker name. For the convenient of readers, we list the factors in Table 7. In ANOVA analysis, we have the null hypothesis and alternative hypothesis expressed as:

$$H_0 : \text{The mean value from each groups are equal.}$$

$$H_\alpha : \text{At least one groups is not equal to the remaining groups.}$$

Table 7: List of factors in the ANOVA test.

Factor	Number of elements
Model structure	BS model and RBS model
Volatility type	R and AWR
Maturity	13 maturity dates
Ticker	10 equity options

When using ANOVA, we can focus our analysis based on the interaction among factors. Based on the result from Table 8, we can see that the interaction among all four factors is not significant. However, we do find that the interaction among volatility, model structure and ticker is significant. According to this information, we analyze the model accuracy based on these three factors. If we can determine which model has the best performance, we can use it at any time in the days leading up to expiration. Therefore, we summarize the mean RMSE value for these three factors in Table 9. When looking into the details, we notice that the RBS_AWR model has the best performance for most of the tickers. QCOM, XLE and XLF has RBS_R as the best model for option pricing. This shows that our deterministic regime switching model will out-perform the standard Black-Scholes approach, and that the

Table 8: The result of Four-way ANOVA, the bold P-values show significant factors or the interaction between factors are significant

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
Volatility	1	2.87	2.87	41.79	**0.0000**
Model S.	1	0.40	0.40	5.87	**0.0154**
Maturity	12	470.89	39.24	570.44	**0.0000**
Ticker	9	2208.32	245.37	3566.94	**0.0000**
Volatility:Model S.	1	0.31	0.31	4.53	**0.0333**
Volatility:Maturity	12	91.33	7.61	110.64	**0.0000**
Model S.:Maturity	12	1.27	0.11	1.54	0.1022
Volatility:Ticker	9	12.96	1.44	20.93	**0.0000**
Model S.:Ticker	9	1.42	0.16	2.29	**0.0143**
Maturity:Ticker	108	4834.79	44.77	650.77	**0.0000**
Volatility:Model S.:Maturity	12	0.09	0.01	0.11	0.9999
Volatility:Model S.:Ticker	9	1.25	0.14	2.02	**0.0330**
Volatility:Maturity:Ticker	108	140.27	1.30	18.88	**0.0000**
Model S.:Maturity:Ticker	108	6.72	0.06	0.90	0.7512
Volatility:Model S.:Maturity:Ticker	108	0.90	0.01	0.12	1.0000
Residuals	87136	5994.06	0.07		

introduction of the activity weighted returns will further improve our accuracy in many situations.

When we narrow down the option range to at the money contract, we see significant improvement from the activity-weighted realized volatility and the deterministic time regime structure. However, the former factor has more credit in the improvements. This conclusion is consistent to our observations in Section 4.

6 Conclusion

In this study, we propose the deterministic regime based Black-Scholes model (RBS) in short-term equity option pricing. Meanwhile, we compare the model efficiency by using the activity-weighted realized volatility (AWR) and the classic realized volatility (R). Based on the empirical result, the RBS_AWR works best for majority of the equity options.

As we mentioned, QCOM, XLE, and XLF prefer to use RBS_R model when pricing short-term option. At the current stage, we are not able to carry out a reliable conclusion

Table 9: Mean RMSE values based on each model. The lowest value indicates the corresponding model is prefer for this equity.

Ticker	RBS_AWR	BS_AWR	RBS_R	BS_R	Best Model
AAPL	0.1923	0.1942	0.1930	0.1992	RBS_AWR
BAC	0.0250	0.0257	0.0310	0.0333	RBS_AWR
C	0.0914	0.0917	0.0994	0.1023	RBS_AWR
FB	0.2884	0.2922	0.3079	0.3236	RBS_AWR
NVDA	0.6412	0.6425	0.6974	0.7565	RBS_AWR
QCOM	0.1729	0.1693	0.1510	0.1519	RBS_R
SPY	0.1506	0.1517	0.1618	0.1639	RBS_AWR
TWTR	0.1132	0.1133	0.1141	0.1285	RBS_AWR
XLE	0.0764	0.0760	0.0681	0.0693	RBS_R
XLF	0.0292	0.0294	0.0288	0.0296	RBS_R

about why this scenario happens. However, we guess the trading frequency for the option may plays an important role in this part. Based on information from CBOE, the option of QCOM has the lowest average daily trading volume among all equity options. In future research, we will group the equity option based on trading volume and see whether they share the same property.

The data we are using in this study is from February to May, 2018. During that time period, there is no major events influence the equity markets. In our model, we haven't consider the news sentiments as a factor in option pricing. In later study, we think this factor potentially can be used as risk factor in the short-term option pricing.

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