Assignment 5

Due: Nov. 26th, 2018 at 1:59 pm

Question 1: Option pricing and Newton's Method (50 Points)

The Black-Scholes (BS) model is widely used when pricing the option. In this question, you are required to solve a set of questions. Before solving these question, here is the information you may need:

- When pricing a call option, the BS model can be written as:

  $$C(S_t, t) = N(d_1)S_t - N(d_2)Ke^{-r(T-t)}$$
  $$d_1 = \frac{1}{\sigma\sqrt{T-t}}[ln(\frac{St}{K}) + (r + \frac{\sigma^2}{2})(T-t)]$$
  $$d_2 = d_1 - \sigma\sqrt{T-t}$$

  Correspondingly, you can have:

  $$\frac{\partial C}{\partial \sigma} = Ke^{-r(T-t)}N'(d_2)\sqrt{T-t}$$

  where $N'()$ is the density function.

- To solve this question, you may need to use quantmod() package in R. If you want to download the option data, you need to do it between 10:00 am and 4:00 pm during a trading day. Otherwise, you may obtain after-market data.

- Set the yearly risk-free rate $r = 2.25\%$

Here is the list of questions you need to solve:

1. Data preparation: For this step, you don't need to do anything special, only thing you need to do is recording necessary parameters. Meanwhile, figure out which equity and its option you want to analyze. The maturity of the option has to be at least 3 month length. For example: I want to analyze equity XYZ on Nov 16th, 2018. Thus, the option I want to analyze should expire later than Feb 16th, 2018. The necessary parameter include: the equity price S_t, the option strike price K, the maturity date T, and the current date t. Additionally, record market buy/sell price for this option.

2. Download 1-year-length daily data for this equity and calculate daily log return. Then, calculate the realized volatility $\sigma_{realized}$.

3. Use the parameters you obtained from step 1 and step 2, price this call option. Compared your value to the market buy/sell, do you think your estimation is good?

4. Based on your market buy/sell price, calculate the mean value and set this parameter as call option price C_t. Now, calculate implied volatility $\sigma_{implied}$. (Hint: $\sigma_{realized}$ is your initial value)

Question 2: Geometric Brownian Motion and Monte-Carlo simulation (30 Points)

In the previous question, you collect enough parameters to do the following Monte-Carlo simulation. We assume the stock price movement applies to the GBM:

$$dS_t = rS_t dt + \sigma S_t dW_t$$

In this question, you need to use Monte-Carlo simulation to estimate the equity price after 1 year. Here is the additional parameter you may need.

$$Step = 252, \quad \sigma = \sigma_{implied}, \quad Path = 2000$$

In the end, you should answer following questions:

1. Based on your simulation, what is your average stock price $\bar{S_T}$ at maturity?
2. Perform normality test on your S_T data sets. Is your data set normal distributed? How about $ln(S_T)$?

Question 3: Gradient descent and linear regression (20 Points)

Recall the linear regression example we used in Lecture 7 (line 24 to line 77). From the Lecture code, you know the best model is:

$$\hat{y} = \beta x$$

where $\beta = 0.041956$
In this question, you are required to use gradient descent method to estimate the value of β. In order to estimate β, you need to minimize following function:

$$f(\beta) = \frac{1}{2}\sum(y - \hat{y})^2 = \frac{1}{2}\sum(y - \beta x)^2$$

1. Calculate $f'(\beta)$
2. Make a plot for $f(\beta)$, the X-axis input should be your estimated β range.
3. Set the step factor $\alpha = 0.01$, converge condition $\epsilon = 0.0001$, and initial value $\beta_0 = 0.06$. Use gradient descent to calculate the β value.

(Hint: the y values and x values are known numbers in $f(\beta)$)