FE620 Pricing and Hedging

Lecture 4: Interest Rate Futures

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Day Count Convention

- Defines:
  - the period of time to which the interest rate applies
  - The period of time used to calculate accrued interest (relevant when the instrument is bought of sold

Day Count Conventions in the U.S. (Page 135-136)

Treasury Bonds:	Actual/Actual (in period)
Corporate Bonds:	30/360
Money Market Instruments:	Actual/360

Examples

- Bond: 8% Actual/ Actual in period.
  - 4% is earned between coupon payment dates. Accruals on an Actual basis. When coupons are paid on March 1 and Sept 1, how much interest is earned between March 1 and April 1?
- Bond: 8% 30/360
  - Assumes 30 days per month and 360 days per year. When coupons are paid on March 1 and Sept 1, how much interest is earned between March 1 and April 1?

Examples continued

- T-Bill: 8% Actual/360:
  - 8% is earned in 360 days. Accrual calculated by dividing the actual number of days in the period by 360. How much interest is earned between March 1 and April 1?

The February Effect (Business Snapshot 6.1)

- How many days of interest are earned between February 28, 2015 and March 1, 2015 when
  - day count is Actual/Actual in period?
  - day count is 30/360?

Treasury Bill Prices in the US

$$P = \frac{360}{n}(100 - Y)$$

Y is cash price per $100

P is quoted price

Treasury Bond Price Quotes in the U.S

Cash price = Quoted price +
Accrued Interest

Treasury Bond Futures
Pages 138–143

Cash price received by party with short position =

Most recent settlement price × Conversion factor + Accrued interest

Example

- Most recent settlement price = 90.00
- Conversion factor of bond delivered = 1.3800
- Accrued interest on bond =3.00
- Price received for bond is $1.3800 \times 90.00 + 3.00 =$ \$127.20 per \$100 of principal

Conversion Factor

The conversion factor for a bond is approximately equal to the value of the bond on the assumption that the yield curve is flat at 6% with semiannual compounding

CBOT T-Bonds & T-Notes

Factors that affect the futures price:

- Delivery can be made any time during the delivery month
- Any of a range of eligible bonds can be delivered
- The wild card play

Eurodollar Futures (Page 143-148)

- A Eurodollar is a dollar deposited in a bank outside the United States
- Eurodollar futures are futures on the 3-month LIBOR rate
- One contract is on the rate earned on $1 million
- A change of one basis point or 0.01 in a Eurodollar futures quote corresponds to a contract price change of $25

Eurodollar Futures continued

- A Eurodollar futures contract is settled in cash
- When it expires (on the third Wednesday of the delivery month) the final settlement price is 100 minus the actual three month LIBOR rate

Example

Date	Quote
Nov 1	97.12
Nov 2	97.23
Nov 3	96.98
…….	……
Dec 21	97.42

Example

- Suppose you buy (take a long position in) a contract on November 1
- The contract expires on December 21
- The prices are as shown
- How much do you gain or lose a) on the first day, b) on the second day, c) over the whole time until expiration?

Example continued

- If on Nov. 1 you know that you will have \$1 million to invest on for three months on Dec 21, the contract locks in a rate of

  $$100 - 97.12 = 2.88\%$$

- In the example you earn 100 – 97.42 = 2.58% on \$1 million for three months (=\$6,450) and make a gain day by day on the futures contract of 30×\$25 =\$750

Formula for Contract Value

(equation 6.2, page 144)

- If Q is the quoted price of a Eurodollar futures contract, the value of one contract is $10{,}000[100-0.25(100-Q)]$
- This corresponds to the $25 per basis point rule

Forward Rates and Eurodollar Futures (Page 143-145)

- Eurodollar futures contracts last as long as 10 years
- For Eurodollar futures lasting beyond two years we cannot assume that the forward rate equals the futures rate

There are Two Reasons

- Futures is settled daily whereas forward is settled once
- Futures is settled at the beginning of the underlying three-month period; FRA is settled at the end of the underlying three- month period

Forward Rates and Eurodollar Futures continued

- A "convexity adjustment" often made is

$$\text{Forward Rate} = \text{Futures Rate} - 0.5\sigma^2 T_1 T_2$$

  - T_1 is the start of period covered by the forward/futures rate
  - T_2 is the end of period covered by the forward/futures rate (90 days later that T_1)
- σ is the standard deviation of the change in the short rate per year

Convexity Adjustment when σ=0.012 (page 147)

Maturity of Futures (yrs)	Convexity Adjustment (bps)
2	3.2
4	12.2
6	27.0
8	47.5
10	73.8

Extending the LIBOR Zero Curve

- LIBOR deposit rates define the LIBOR zero curve out to one year
- Eurodollar futures can be used to determine forward rates and the forward rates can then be used to bootstrap the zero curve

Example (page 147–148)

so that $$F = \frac{R_2 T_2 - R_1 T_1}{T_2 - T_1}$$

$$R_2 = \frac{F(T_2 - T_1) + R_1 T_1}{T_2}$$

If the 400-day LIBOR zero rate has been calculated as 4.80% and the forward rate for the period between 400 and 491 days is 5.30 the 491 day rate is 4.893%

Duration Matching

- This involves hedging against interest rate risk by matching the durations of assets and liabilities
- It provides protection against small parallel shifts in the zero curve

Use of Eurodollar Futures

- One contract locks in an interest rate on $1 million for a future 3-month period
- How many contracts are necessary to lock in an interest rate on $1 million for a future six-month period?

Duration-Based Hedge Ratio

$$\frac{P D_P}{V_F D_F}$$

V_F Contract price for interest rate futures

D_F Duration of asset underlying futures at maturity

P Value of portfolio being hedged

D_P Duration of portfolio at hedge maturity

Example

- It is August. A fund manager has $10 million invested in a portfolio of government bonds with a duration of 6.80 years and wants to hedge against interest rate moves between August and December
- The manager decides to use December T-bond futures. The futures price is 93-02 or 93.0625 and the duration of the cheapest to deliver bond will be 9.2 years at the futures contract maturity
- The number of contracts that should be shorted is

$$\frac{10,000,000}{93,062.50} \times \frac{6.80}{9.20} = 79$$

Limitations of Duration-Based Hedging

- Assumes that only parallel shift in yield curve take place
- Assumes that yield curve changes are small
- When T-Bond futures is used assumes there will be no change in the cheapest-to-deliver bond

GAP Management (Business Snapshot 6.3)

This is a more sophisticated approach used by banks to hedge interest rate. It involves

- Bucketing the zero curve
- Hedging exposure to situation where rates corresponding to one bucket change and all other rates stay the same