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Problem 1

1)

When using the OIS zero rates to discount, we need to suppose that the value of the swap(gain fixed rate 3.2% per annual and give floated rate LIBOR) should be 0 on the beginning. In order to calculate the swap's value, we can sum up the value of a series of forward contracts and assume that the forward rates will come true. So we can give this formula:

$$0 = \frac{3.2 - 3}{(1 + 2.5\%)} + \frac{3.2 - 3.2}{(1 + 2.7\%)^2} + \frac{3.2 - 100F}{(1 + 2.9\%)^3}$$

Then F=3.4126%.

2)

If in such situation, we can calculate the value of this swap by using the forward rate we have in 1). In the same way we draw the formula of swap value:

$$V = 100million \times \left[\frac{4\% - 3\%}{(1 + 2.5\%)} + \frac{4\% - 3.2\%}{(1 + 2.7\%)^2} + \frac{4\% - F}{(1 + 2.9\%)^3}\right]$$

Use the conclusion of 1):

$$V = 100million \times \left[\frac{0.8\%}{(1 + 2.5\%)} + \frac{0.8\%}{(1 + 2.7\%)^2} + \frac{0.8\%}{(1 + 2.9\%)^3} + 0\right]$$

Then V= $2273226.33

Problem 2

We can calculate the value of this swap by summing up a series of forward contracts' cash flows(FRA pricing) and discounting them by OIS rate.

For the fixed cash flows, we can have 3.6%*0.25*100 million=0.9 million.

For the floated cash flows, first of all, the first cash flow has been determined one month ago. The first cash flow is 3.2%*0.25*100 million=0.8 million. Then for the rest of cash flows, we need to use the forward LIBOR rate and assume it will come true in the future. This forward rate is 4% per annum for all maturities and compounded quarterly. So the floated cash flows is some how "fixed" and they are 4%*0.25*100 million = 1 million for all maturities.

For the discount rate, we use the OIS rate which is currently 3.8% for all maturities with continuous compounding.

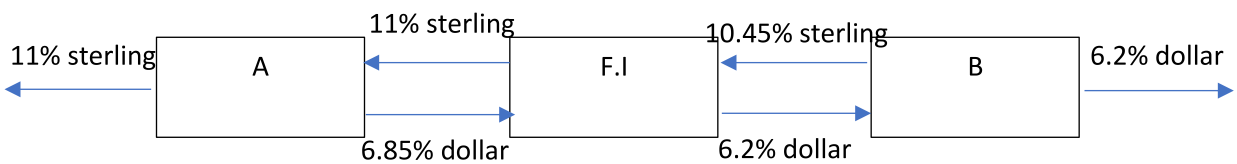
$$e^{-3.8\%\times\frac{1}{6}}, e^{-3.8\%\times\frac{5}{12}}, e^{-3.8\%\times\frac{8}{12}}, e^{-3.8\%\times\frac{11}{12}}, e^{-3.8\%\times\frac{14}{12}}$$

This is the table of the cash flows:

time	Fixed cash flows(million dollar)	Floated cash flows(million dollar)	Net cash flows(million dollar)	Discount factors	Net discounted cash flows(million dollar)
1/6	-0.9	0.8	-0.1	0.994	-0.09937
5/12	-0.9	1	0.1	0.984	0.09843
8/12	-0.9	1	0.1	0.975	0.09750
11/12	-0.9	1	0.1	0.966	0.096577
14/12	-0.9	1	0.1	0.957	0.0956635
total					0.2888

So the value of this swap is 288799.09 dollars. A Python code file is attached with this file.

Problem 3



There are (7%-6.2%)-(11%-10.6%)-0.4% discrepancy that we can use in this case. So the company A can borrow the dollar with the rate that is 0.15% smaller than it could with no swap involved. And same to the company B, while the financial institution can make a profit of 10% of the underlying.

Problem 4

a)

Because the trader write 5 naked put option contracts, the margin requirement is the greater of these two number:

$$500 \times (10 + 20\% \times 58) = 10800$$

$$500 \times (10 + 10\% \times 64) = 8200$$

So the margin requirement is 10800 dollars in all.

b)

For the index options, the parameter 20% should be replaced by 15% because the index have less risk than a single stock's option. Then the margin requirement is the greater of this two:

$$500 \times (10 + 15\% \times 58) = 9350$$

$$500 \times (10 + 10\% \times 64) = 8200$$

So the margin requirement is 9350 dollars in all.

c)

Similar to a) but this time the options have the virtual value which is 6 dollars for each stocks. Then the margin requirement is the greater of this two numbers:

$$500 \times (10 + 20\% \times 70 - 6) = 9000$$

$$500 \times (10 + 10\% \times 64) = 8200$$

So the margin requirement is 9000 dollars in all.

d)

Trader don't need to pay a margin when buying option contracts.

Problem 5

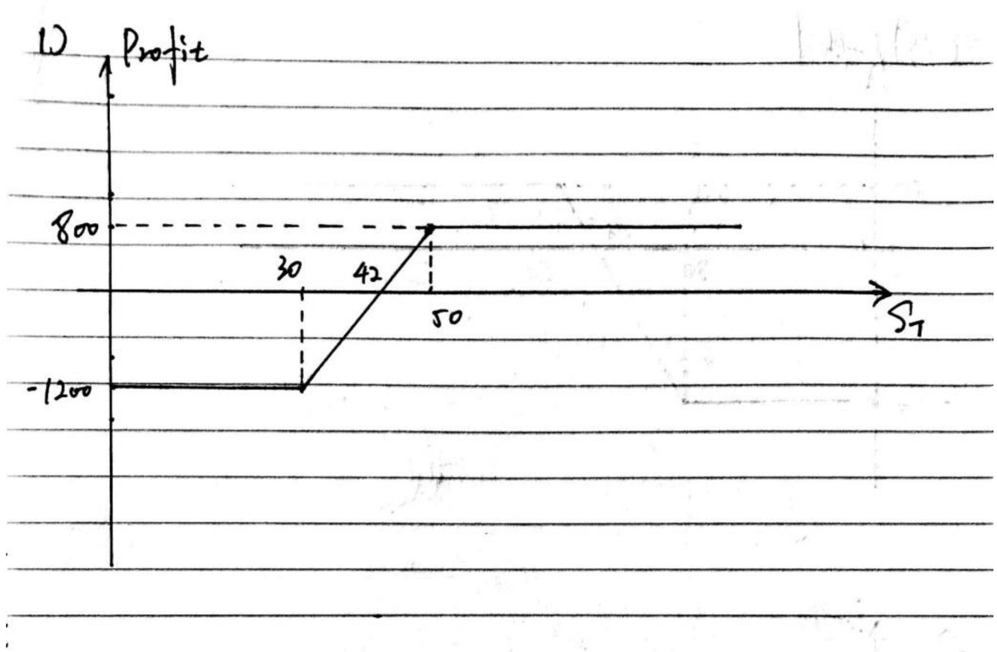
1)

We can write the payoff function of the next year and simplify it:

$$Profit = 100[(S_T - 40) - (max(S_T - 50, 0) - 5) + (max(30 - S_T, 0) - 7)]$$

$$Profit = 100\left[\frac{1}{2}(|30 - S_T| - |S_T - 50|) - 2\right]$$

$$Profit = \begin{cases} -1200, & S_T < 30 \\ 100(S_T - 42), & 30 \le S_T < 50 \\ 800, & 50 \le S_T \end{cases}$$



2)

We can also write the payoff function and simplify it:

$$Profit = 100[(S_T - 40) - 2(max(S_T - 50, 0) - 5) + 2(max(30 - S_T, 0) - 7)]$$

$$Profit = 100[36 - S_T + |30 - S_T| - |S_T - 50|]$$

$$Profit = \begin{cases} 100(16 - S_T), & S_T < 30 \\ 100(S_T - 44), & 30 \le S_T < 50 \\ 100(56 - S_T), & 50 \le S_T \end{cases}$$