FE-620 – Assignment 4

Problem 1.

Suppose that c_1, c_2, *and* c_3 *are the prices of European call options with strike prices* K_1, K_2, *and* K_3, *respectively, where* $K_3 > K_2 > K_1$ *and* $K_3 - K_2 = K_2 - K_1$. *All options have the same maturity. Show that*

$$c_2 \leq 0.5(c_1 + c_3)$$

(Hint: Consider a portfolio that is long one option with strike price K_1, *long one option with strike price* K_3, *and short two options with strike price* K_2.*)*

Problem 2.

A bank decides to create a five-year principal-protected note on a non-dividend-paying stock by offering investors a zero-coupon bond plus a bull spread created from calls. The risk-free rate is 3% and the stock price volatility is 20%. The low strike price option in the bull spread is at the money. What is the maximum ratio of the higher strike price to the lower strike price in the bull spread?

Problem 3.

A foreign currency is currently worth $0.64. A one-year butterfly spread is set up using European call options with strike prices of $0.55, $0.65, and $0.75. The risk-free interest rates in the United States and the foreign country are 4% and 3% respectively, and the volatility of the exchange rate is 10%. Calculate the cost of setting up the butterfly spread position. Show that the cost is the same if European put options are used instead of European call options.

Problem 4.

A stock price is currently $40. Over each of the next four three-month periods it is expected to go up by 10% or down by 10%. The risk-free interest rate is 3% per annum with continuous compounding.

a. *What is the value of a 1 year European put option with a strike price of $42?*
b. *What is the value of a 1 year American put option with a strike price of $42?*

Problem 5.

Using a "trial-and-error" approach, estimate how high the strike price has to be in Problem 4. such that it would be optimal to exercise the option immediately.