If futures price above spot price:
- sell futures
- buy spot
- make delivery

If future price below spot price:
- buy future
- sell asset

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Daily Gains/Losses = $200 \times (1250 - 1241) = -\$1,800$

Assume today is January 15.
A manufacturer will require 100,000 pounds of copper on May 15. At this time:
- Spot price = 340 ¢ / pound
- future with May delivery = 320 ¢ / pound

Manufacturer requires long position in 4 futures contracts.

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Suppose

1) Spot price on May 15 = 325¢ / pound.

Because May is the delivery month

Spot price ≈ futures price

$\Rightarrow$ $100{,}000 \times (3.25 - 3.20) = \$5000$ on futures contracts

and pays : $100{,}000 \times 3.25 = \$325{,}000$

Net cost : $\$325{,}000 - \$5000 = \$320{,}000$

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2) Spot price is 305¢ / pound on May 15

Manufacturer : $100{,}000\,(3.20 - 3.05) = \$15{,}000$ On futures contract

Pays . $100{,}000 \times 3.05 = \$305{,}000$

Net cost = $320,000

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Shot Futres Hedge

Assume today is May 15
- Oil producer negotiated a contract to sell $1 mill barrels of oil.
- it was agreed that the price applied is market price on Aug 15

→ will gain $10,000 for each 1 cent increase in the price of oil.

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Suppose on May 15 spot price $60/barrel
futures with Aug delivery is 59$/barrel
Company can hedge by shorting 1000 future contracts.
→ Assume that on Aug 15
1) spot $55/barrel

Total amount = $55 million + $4 mill =
= $ 59 mil

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2) Spot = \$65 / barrel

Total amount = \$65 - \$6 = \$59 mill.

$$B = S - F$$

Ex Consider $S_1 = 2.50$ | $S_2 = 2.00$

$F_1 = 2.20$ | $F_2 = 1.90$

$b_1 = S_1 - F_1 = 0.30$

$b_2 = S_2 - F_2 = 0.10$

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Consider situation of a hedger who knows that it will buy at t_2 and he can initiate a longhedg at t_1

$$S_2 - (F_2 - F_1) = F_1 + b_2 = 2.20 + 0.3 = \$2.30$$

Consider the situation of a hedger who knows that the asset will be sold t_2. Take short future position

$$S_2 + (F_1 - F_2) = F_1 + b_2 = 220 + 0.10 = \$2.30$$

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Optimal Hedge Ratio

Suppose we expect to sell N_A units of an asset at time t_2. and choose to hedge at time t_1 by shorting futures contracts on N_F units of a similar asset.

Hedge ratio $h = \frac{N_F}{N_A}$

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Total amount realized for the asset:

$$Y = S_2 \cdot N_A - (F_2 - F_1) N_F$$

$$Y = S_1 N_A + (S_2 - S_1) N_A - (F_2 - F_1) N_F$$

$$\boxed{Y = S_1 \cdot N_A + N_A(\Delta S - h \Delta F)}$$

$\Delta S = S_2 - S_1$

$\Delta F = F_2 - F_1$

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S_1 & N_A known

$\Rightarrow$ variance of Y minimized when $\Delta S - h\Delta F$ is minimized

Variance of $\Delta S - h\Delta F$:

$$v = \sigma_S^2 + h^2\sigma_F^2 - 2\rho\sigma_S\sigma_F h$$

$$\frac{dv}{dh} = 2h\sigma_F^2 - 2\rho\sigma_S\sigma_F = 0 \Rightarrow h^* = \rho\frac{\sigma_S}{\sigma_F}$$

$$\frac{d^2v}{dh^2} > 0$$

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Def P = current value of portfolio

F = Current value of futures contract.

If portfolio mirror the index, $h^* = 1$

$$N^* = \frac{P}{F}$$

When portfolio doesn't mirror the index

β = slope of best fit line obtained regressing excess return of portfolio over risk free rate / market return over risk free rate

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$$N^* = \beta \cdot \frac{P}{F}$$

Ex $$N^* = 1.5 \times \frac{5{,}000{,}000}{250 \times 1{,}000} = 30$$

In general changing beta form β to β^*, $\beta > \beta^*$

- short position $(\beta - \beta^*) \frac{P}{F}$

whe $\beta < \beta^*$

- long positio $(\beta^* - \beta) \cdot \frac{P}{F}$

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