FE620 Pricing and Hedging

Lecture 9: Options on Stock Indices and Currencies

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Index Options (page 365-367)

- The most popular underlying indices in the U.S. are
  - The S&P 100 Index (OEX and XEO)
  - The S&P 500 Index (SPX)
  - The Dow Jones Index times 0.01 (DJX)
  - The Nasdaq 100 Index (NDX)

- Exchange-traded contracts are on 100 times index; they are settled in cash; OEX is American; the XEO and all others are European

Index Option Example

- Consider a call option on an index with a strike price of 880
- Suppose 1 contract is exercised when the index level is 900
- What is the payoff?

Using Index Options for Portfolio Insurance

- Suppose the value of the index is S_0 and the strike price is K
- If a portfolio has a β of 1.0, the portfolio insurance is obtained by buying 1 put option contract on the index for each $100S_0$ dollars held
- If the β is not 1.0, the portfolio manager buys β put options for each $100S_0$ dollars held
- In both cases, K is chosen to give the appropriate insurance level

Example 1

- Portfolio has a beta of 1.0
- It is currently worth $500,000
- The index currently stands at 1000
- What trade is necessary to provide insurance against the portfolio value falling below $450,000?

Example 2

- Portfolio has a beta of 2.0
- It is currently worth $500,000 and index stands at 1000
- The risk-free rate is 12% per annum
- The dividend yield on both the portfolio and the index is 4%
- How many put option contracts should be purchased for portfolio insurance?

Calculating Relation Between Index Level and Portfolio Value in 3 months

- If index rises to 1040, it provides a 40/1000 or 4% return in 3 months
- Total return (incl. dividends) = 5%
- Excess return over risk-free rate = 2%
- Excess return for portfolio = 4%
- Increase in Portfolio Value = 4+3−1=6%
- Portfolio value=$530,000

Determining the Strike Price (Table 17.2, page 367)

Value of Index in 3 months	Expected Portfolio Value in 3 months ($)
1,080	570,000
1,040	530,000
1,000	490,000
960	450,000
920	410,000

An option with a strike price of 960 will provide protection against a 10% decline in the portfolio value

European Options on Assets Providing a Known Yield

We get the same probability distribution for the asset price at time T in each of the following cases:

1. The asset starts at price S_0 and provides a yield = q
2. The asset starts at price $S_0 e^{-qT}$ and provides no income

European Options on Assets Providing Known Yield
continued

We can value European options by reducing the asset price to $S_0 e^{-qT}$ and then behaving as though there is no income

Extension of Chapter 11 Results

(Equations 17.1 to 17.3)

Lower Bound for calls:

$$c \geq \max\left(S_0 e^{-qT} - Ke^{-rT}, 0\right)$$

Lower Bound for puts

$$p \geq \max\left(Ke^{-rT} - S_0 e^{-qT}, 0\right)$$

Put Call Parity

$$c + Ke^{-rT} = p + S_0 e^{-qT} \qquad c + Ke^{-rT} = p + F_0 e^{-rT}$$

Extension of Chapter 15 Results (Equations 17.4 and 17.5)

$$c = S_0 e^{-qT} N(d_1) - K e^{-rT} N(d_2)$$

$$p = K e^{-rT} \; N(-d_2) - S_0 e^{-qT} N(-d_1)$$

$$\text{where} \quad d_1 = \frac{\ln(S_0 / K) + (r - q + \sigma^2 / 2)T}{\sigma\sqrt{T}}$$

$$d_2 = \frac{\ln(S_0 / K) + (r - q - \sigma^2 / 2)T}{\sigma\sqrt{T}}$$

Alternative Formulas (page 375)

$$c = e^{-rT}[F_0 N(d_1) - KN(d_2)]$$

$$p = e^{-rT}[KN(-d_2) - F_0 N(-d_1)]$$

$$d_1 = \frac{\ln(F_0 / K) + \sigma^2 T / 2}{\sigma\sqrt{T}}$$

$$d_2 = d_1 - \sigma\sqrt{T}$$

where

$$F_0 = S_0 e^{(r-q)T}$$

Valuing European Index Options

We can use these formulas for an option on an asset paying a dividend yield

Set S_0 = current index level

Set F_0= futures or forward index price for a contract maturing at the same time as the option

Set q = average dividend yield expected during the life of the option

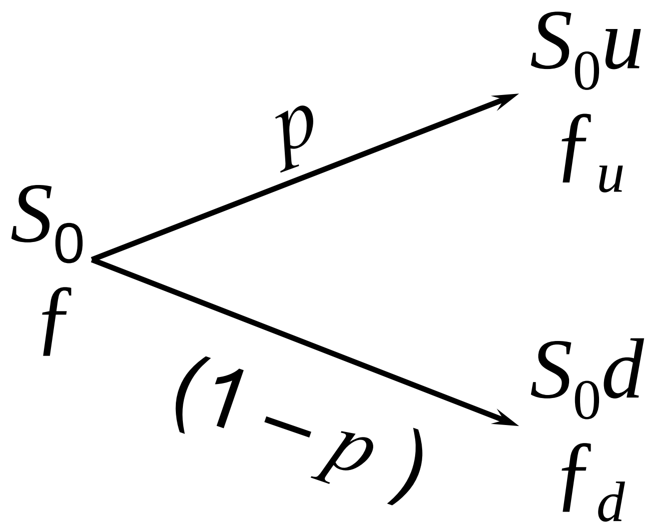
Implied Forward Prices and Dividend Yields

- From European calls and puts with the same strike price and time to maturity

$$F_0 = K + (c - p)e^{rT} \qquad\qquad q = -\frac{1}{T}\ln\frac{c - p + Ke^{-rT}}{S_0}$$

- These formulas allow term structures of forward prices and dividend yields to be estimated
- OTC European options are typically valued using the forward prices (Estimates of *q* are not then required)
- American options require the dividend yield term structure

The Binomial Model



$$f = e^{-rT}[pf_u + (1-p)f_d]$$

The Binomial Model continued

- In a risk-neutral world the asset price grows at $r-q$ rather than at r when there is a dividend yield at rate q
- The probability, p, of an up movement must therefore satisfy

$$pS_0u+(1-p)S_0d = S_0e^{(r-q)T}$$

so that

$$p=\frac{e^{(r-q)T}-d}{u-d}$$

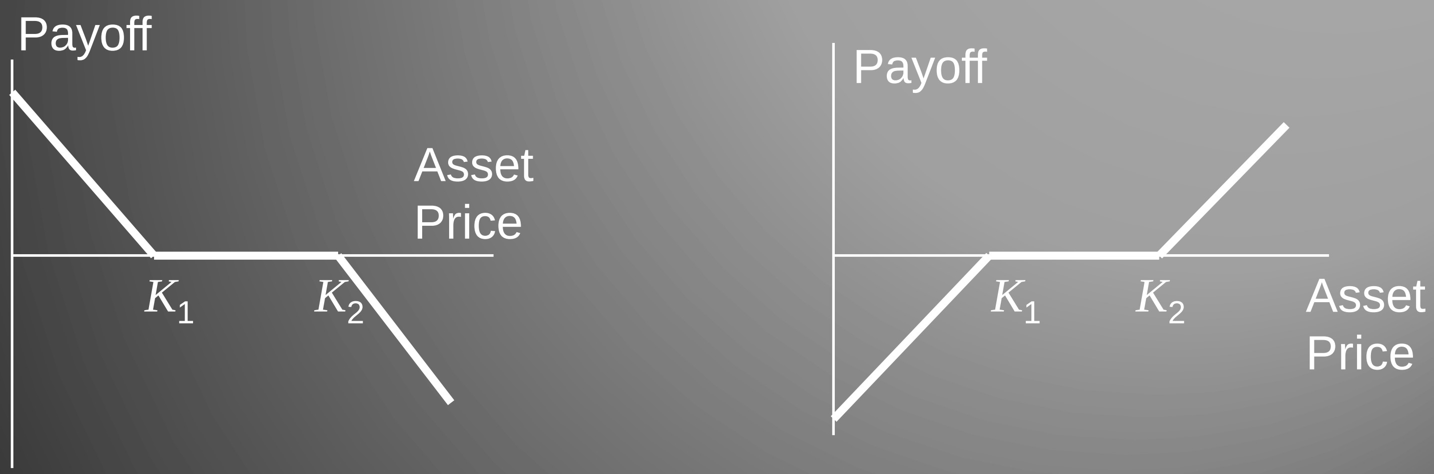
Currency Options

- Currency options trade on NASDAQ OMX
- There also exists a very active over-the-counter (OTC) market
- Currency options are used by corporations to buy insurance when they have an FX exposure

Range Forward Contracts

- Have the effect of ensuring that the exchange rate paid or received will lie within a certain range
- When currency is to be paid it involves selling a put with strike K_1 and buying a call with strike K_2 (with $K_2 > K_1$)
- When currency is to be received it involves buying a put with strike K_1 and selling a call with strike K_2
- Normally the price of the put equals the price of the call

Range Forward Contract continued
Figure 17.1, page 368



Short Position

Long Position

The Foreign Interest Rate

- We denote the foreign interest rate by r_f
- When a U.S. company buys one unit of the foreign currency it has an investment of S_0 dollars
- The return from investing at the foreign rate is $r_f S_0$ dollars
- This shows that the foreign currency provides a yield at rate r_f

Valuing European Currency Options

- A foreign currency is an asset that provides a yield equal to r_f
- We can use the formula for an option on a stock paying a dividend yield :
  - S_0 = current exchange rate
  - $q = r_f$

Formulas for European Currency Options (Equations 17.11 and 17.12, page 375)

$$c = S_0 e^{-r_f T} N(d_1) - K e^{-rT} N(d_2)$$

$$p = K e^{-rT} N(-d_2) - S_0 e^{-r_f T} N(-d_1)$$

$$\text{where} \quad d_1 = \frac{\ln(S_0 / K) + (r - r_f + \sigma^2 / 2)T}{\sigma\sqrt{T}}$$

$$d_2 = \frac{\ln(S_0 / K) + (r - r_f - \sigma^2 / 2)T}{\sigma\sqrt{T}}$$

Alternative Formulas (Equations 17.13 and 17.14)

Using $F_0 = S_0 e^{(r - r_f)T}$

$$c = e^{-rT}[F_0 N(d_1) - KN(d_2)]$$

$$p = e^{-rT}[KN(-d_2) - F_0 N(-d_1)]$$

$$d_1 = \frac{\ln(F_0 / K) + \sigma^2 T / 2}{\sigma\sqrt{T}}$$

$$d_2 = d_1 - \sigma\sqrt{T}$$