FE620 Pricing and Hedging

Lecture 9: Futures Options and Black's Model

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Options on Futures

- Referred to by the maturity month of the underlying futures
- The option is American and usually expires on or a few days before the earliest delivery date of the underlying futures contract

Mechanics of Call Futures Options

- When a call futures option is exercised the holder acquires
  - A long position in the futures
  - A cash amount equal to the excess of the futures price at the time of the most recent settlement over the strike price

Mechanics of Put Futures Option

- When a put futures option is exercised the holder acquires
  - A short position in the futures
  - A cash amount equal to the excess of the strike price over the futures price at the time of the most recent settlement

Example 18.1 (page 382)

- Sept. call option contract on copper futures has a strike of 320 cents per pound. It is exercised when futures price is 331 cents and most recent settlement is 330. One contract is on 25,000 pounds
- Trader receives
  - Long Sept. futures contract on copper
  - 25,000 times 10 cents or $2,500 in cash

Example 18.2 (page 382)

- Dec put option contract on corn futures has a strike price of 600 cents per bushel. It is exercised when the futures price is 580 cents per bushel and the most recent settlement price is 579 cents per bushel. One contract is on 5000 bushels
- Trader receives
  - Short Dec futures contract on corn
  - $1,050 in cash

The Payoffs

If the futures position is closed out immediately:

Payoff from call = $F - K$

Payoff from put = $K - F$

where F is futures price at time of exercise

Interest Rate Futures Options

- Options on T-Bond futures (quoted as percentage of face value to the nearest 1/64 of 1%)
- Options on Eurodollar futures. Each one basis point in the quote represents $25
- If you think interest rates will go up should you buy call or put options?

Potential Advantages of Futures Options over Spot Options

- Futures contracts may be easier to trade and more liquid than the underlying asset
- Exercise of option does not lead to delivery of underlying asset
- Futures options and futures usually trade on same exchange
- Futures options may entail lower transactions costs

European Futures Options

- European futures options and European spot options are equivalent when futures contract matures at the same time as the option
- It is common to regard European spot options as European futures options when they are valued

Put-Call Parity for Futures Options
(Equation 18.1, page 385)

Consider the following two portfolios:

1. European call plus Ke^{-rT} of cash
2. European put plus long futures plus cash equal to F_0e^{-rT}

They must be worth the same at time T so that

$$c + Ke^{-rT} = p + F_0e^{-rT}$$

Other Relations

$$F_0 e^{-rT} - K < C - P < F_0 - Ke^{-rT}$$

$$c > (F_0 - K)e^{-rT}$$

$$p > (F_0 - K)e^{-rT}$$

Growth Rates For Futures Prices

- A futures contract requires no initial investment
- In a risk-neutral world the expected return should be zero
- The expected growth rate of the futures price is therefore zero
- The futures price can therefore be treated like a stock paying a dividend yield of r

Valuing European Futures Options

- We can use the formula for an option on a stock paying a dividend yield
  - S_0 = current futures price, F_0
  - q = domestic risk-free rate, r
- Setting $q = r$ ensures that the expected growth of F in a risk-neutral world is zero
- The result is referred to as Black's model because it was first suggested in a paper by Fischer Black in 1976

Black's Model (Equations 18.9 and 18.10, page 388)

$$c = e^{-rT}\left[F_0\ N(d_1) - K\ N(d_2)\right]$$

$$p = e^{-rT}\left[K\ N(-d_2) - F_0\ N(-d_1)\right]$$

where $d_1 = \dfrac{\ln(F_0 / K) + \sigma^2 T / 2}{\sigma\sqrt{T}}$

$$d_2 = \frac{\ln(F_0 / K) - \sigma^2 T / 2}{\sigma\sqrt{T}} = d_1 - \sigma\sqrt{T}$$

How Black's Model is Used in Practice

- Black's model is frequently used to value European options on the spot price of an asset
- This avoids the need to estimate income on the asset

Using Black's Model Instead of Black-Scholes-Merton (Example 18.7, page 389)

- Consider a 6-month European call option on spot gold
- 6-month futures price is 1,240, 6-month risk-free rate is 5%, strike price is 1,200, and volatility of futures price is 20%
- Value of option is given by Black's model with $F_0 = 1{,}240$, $K=1{,}200$, $r = 0.05$, $T=0.5$, and $\sigma = 0.2$
- It is 88.37

Binomial Tree Example

A 1-month call option on futures has a strike price of 29.

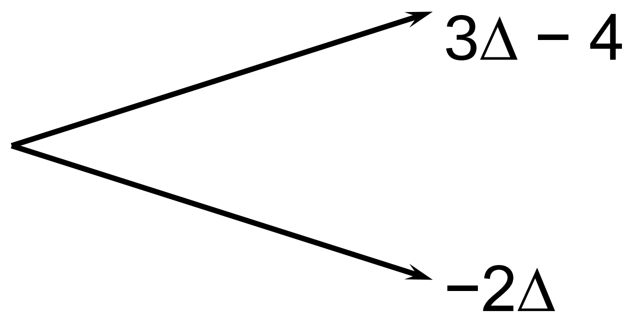
Futures Price = $33
Option Price = $4

Futures Price = $30
Option Price = ?

Futures Price = $28
Option Price = $0

Setting Up a Riskless Portfolio

- Consider the Portfolio: long Δ futures
  short 1 call option



- Portfolio is riskless when $3\Delta - 4 = -2\Delta$ or $\Delta = 0.8$

Valuing the Portfolio
(Risk-Free Rate is 6%)

- The riskless portfolio is:

  long 0.8 futures

  short 1 call option
- The value of the portfolio in 1 month is

  $$-1.6$$
- The value of the portfolio today is

  $$-1.6e^{-0.06/12} = -1.592$$

Valuing the Option

- The portfolio that is

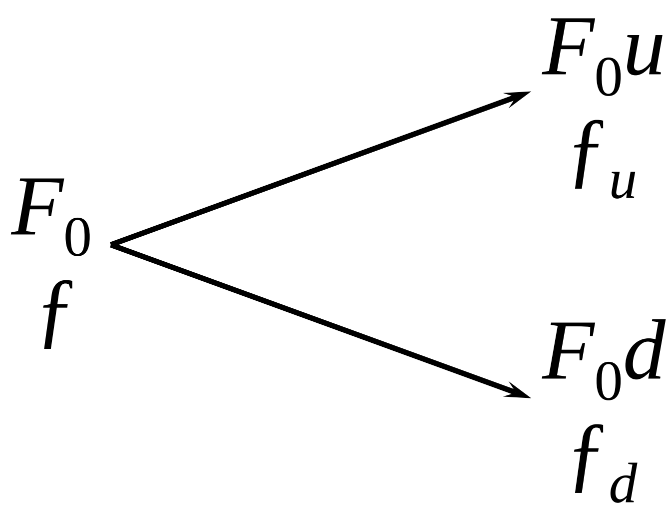
  long 0.8 futures

  short 1 option

  is worth −1.592
- The value of the futures is zero
- The value of the option must therefore be 1.592

Generalization of Binomial Tree Example (Figure 18.2, page 391)

- A derivative lasts for time T and is dependent on a futures price



Generalization (continued)

- Consider the portfolio that is long Δ futures and short 1 derivative

$$F_0 u\,\Delta - F_0\,\Delta - f_u$$

$$F_0 d\,\Delta - F_0 \Delta - f_d$$

- The portfolio is riskless when

$$\Delta = \frac{f_u - f_d}{F_0 u - F_0 d}$$

Generalization (continued)

- Value of the portfolio at time T is
  - $F_0 u\,\Delta - F_0\Delta - f_u$
- Value of portfolio today is $-f$
- Hence

$$f = -\,[F_0 u\Delta - F_0\Delta - f_u]e^{-rT}$$

Generalization
(continued)

- Substituting for Δ we obtain

$$f = [\, p\, f_u + (1-p)f_d \,]e^{-rT}$$

where

$$p = \frac{1-d}{u-d}$$

Futures Option Price vs Spot Option Price

- If futures prices are higher than spot prices (normal market), an American call on futures is worth more than a similar American call on spot. An American put on futures is worth less than a similar American put on spot.
- When futures prices are lower than spot prices (inverted market) the reverse is true.

Futures Style Options (page 393)

- A futures-style option is a futures contract on the option payoff
- Some exchanges trade these in preference to regular futures options
- The futures price for a call futures-style option is

$$F_0 N(d_1) - K N(d_2)$$

- The futures price for a put futures-style option is

$$K N(-d_2) - F_0 N(-d_1)$$

Summary: Put–Call Parity Results

Non-dividend-paying stock

$$c + Ke^{-rT} = p + S$$

Indices:

$$c + Ke^{-rT} = p + S_0 e^{-qT}$$

Foreign exchange:

$$c + Ke^{-rT} = p + S_0 e^{-r_f T}$$

Futures:

$$c + Ke^{-rT} = p + F_0 e^{-rT}$$

Futures style:

$$c + K = p + F_0$$

Summary of Key Results from Chapters 17 and 18

- We can treat stock indices, currencies, and futures like a stock paying a dividend yield of q
  - For stock indices, q is average dividend yield on the index over the option life
  - For currencies, $q = r_f$
  - For futures, $q = r$