Midterm Exam FE621 Section A

Saturday, March 30, 2019

Name:

- In this exam you may use any material you want. Please be aware the submission window closes 4 hours after it has been opened. So you need 4 continuous hours of work.
- Be very specific with your definitions and derivations. Showcase your work.
- Please submit a well written report containing all the answers to the exam questions. Submitting commented code is not appropriate and will earn 0 in this exam.
- Scanned handwritten pages are appropriate for certain problems requiring mathematical derivations.
- Communication with other students either physical or virtual is strictly forbidden.

For instructor's use only

Problem	Points	Score
1	20	
2	30	
3	30	
4	20	
Total	100	

Problem 1. Numerical integration

(a) Numerically compute the integral $\int_0^2 e^{x^2}\ dx$ using the trapezoid method with 100 steps

(b) Compute numerically the integral $\int_0^2 e^{x^2}\ dx$, using Simpson's quadrature rule, with 100 steps

(c) Please compare the two results obtained. Comment.

Problem 2. Option pricing using a trinomial tree Construct a trinomial tree to price an American put option. To this end start with the following given parameters: $S_0 = 100$, $K = 120$, maturity $T = 8$ months, $r = 0$, $\delta = 0$, volatility $\sigma = 30\%$, time steps $N = 200$.

(a) What is the best choice for Δx to obtain the best order of convergence? Calculate Δx

(b) Calculate the American Put option price using the tree.

(c) Estimate Gamma of the American Put at time $t = 0$.

Problem 3. Option price range Assume a Stock is at \$23.35. You look at the market to a European Call option with strike \$22.5 and Maturity 8 weeks. Assume $r = 0.01$. The listed best bid is \$3.2 and best ask is \$3.8. Use the code you turned in the assignments to answer the following questions.

(a) Calculate an interval of possible values for the European Put.

(b) Calculate an interval of possible values for the American Call.

Problem 4. We know that an option price under a certain stochastic model satisfies the following PDE:

$$\frac{\partial V}{\partial t} + 2\cos(S)\frac{\partial V}{\partial S} + 0.2S^{\frac{3}{2}}\frac{\partial^2 V}{\partial S^2} - rV = 0.$$

Assume you have an equidistant grid with points of the form $(i, j) = (i\Delta t, j\Delta x)$, where $i \in \{1, 2, \ldots, N\}$ and $j \in \{-N_S, N_S\}$. Let $V_{i,j} = (i\Delta t, j\Delta x)$. Discretize the derivatives and give the finite difference equation for an Explicit scheme. Use the notation introduced above.