Booth School of Business, University of Chicago
Business 41202, Spring Quarter 2016, Mr. Ruey S. Tsay

Midterm

ChicagoBooth Honor Code:
I pledge my honor that I have not violated the Honor Code during this examination.

Signature: **Name**: **ID**:

Notes:

- Open notes and books. Exam time: 180 minutes.
- You may use a calculator or a PC. However, turn off Internet connection and cell phones. **Internet access and phone communication are strictly prohibited during the exam.**
- The exam has 8 pages and the R output has 12 pages. Please **check** that you have all 20 pages.
- For each question, write your answer in the blank space provided.
- **Manage** your time carefully and answer as many questions as you can.
- For simplicity, if not specifically given, use 5% Type-I error in hypothesis testings.
- Round your answer to 3 significant digits.

Problem A: (30 pts) Answer briefly the following questions. Each question has two points.

1. Give two reasons by which the return series of an asset tend to contain outliers.

2. Describe two differences between an AR(1) model and an MA(1) model of a time series.

3. Give two characteristics of the return r_t if it follows the model $r_t = 0.05 + a_t$, $a_t = \sigma_t \epsilon_t$, where ϵ_t are iid $N(0, 1)$ and $\sigma_t^2 = 0.02 + 0.4a_{t-1}^2$.

4. **(Questions 4 to 6)**: Suppose that the asset return r_t follows the model

$$
\begin{aligned}
r_t &= a_t \\
a_t &= \sigma_t \epsilon_t, \quad \epsilon_t \sim \text{iid } t_6^* \\
\sigma_t^2 &= 0.09 + 0.145a_{t-1}^2 + 0.855\sigma_{t-1}^2.
\end{aligned}
$$

Does the unconditional variance of r_t exist? Why?

5. Suppose that $r_{100} = -0.05$ and $\sigma_{100} = 0.3$. Compute 1-step and 2-step ahead volatility forecasts at the forecast origin $t = 100$. (Note that it is volatility, not σ^2.)

6. Compute the 22-step ahead mean and volatility forecasts (one month ahead).

7. Give an advantage of Spearman's ρ over the Pearson correlation.

8. Give a feature that GARCH-M models have, but the GARCH models do not.

9. Suppose that r_t follows the model

$$r_t = r_{t-1} + a_t - 0.9a_{t-1},$$

and we have $r_{1001} = 1.2$ and $r_{1000}(1) = 1.0$, where $r_t(1)$ denotes the 1-step ahead prediction of r_{t+1} at the forecast origin t. Compute $r_{1001}(1)$.

10. Why is the usual R^2 measure not proper in time series analysis?

11. Give two real applications of seasonal time series models in finance.

12. **(Questions 12-13)** Suppose that the daily simple returns of an asset in week 1 were -0.5%, 1.2%, 2.5%, -1.0%, and 0.6%. What are the corresponding daily log returns?

13. What is the weekly simple return of the asset?

14. **(Questions 14-15)**: The summary statistics of daily simple returns of an asset are given below:

```
> basicStats(rtn)
                    rtn
nobs        2515.000000
Mean           0.000410
SE Mean        ????????
LCL Mean      -0.000257
UCL Mean       0.001077
Stdev          0.017060
Skewness       0.517184
Kurtosis       6.661044
```

What is the standard deviation of the mean? Is the expected return of the asset significantly different from zero? Why?

15. Based on the summary statistics, are the returns normally distributed? Perform a statistical test to justify your conclusion.

Problem B. (23 points) Consider the monthly U.S. unemployment rates from January 1947 to March 2016. Due to strong serial dependence, we analyze the differenced series $x_t = r_t - r_{t-1}$, where r_t is the seasonally adjusted unemployment rate. Answer the following questions, using the attached R output. Note: A fitted ARIMA model should include **residual variance**.

1. (2 points) The `auto.arima` command specifies an ARIMA(2,0,2) model for x_t. The fitted model is referred to as **m1** in the output. Write down the fitted model.

2. (3 points) Model checking shows two large outliers. An ARIMA(2,0,2) model with two outliers are then specified, **m3**. Write down the fitted model.

3. (3 points) Model checking shows some serial correlations at lags 12 and 24. A seasonal model is then employed and called **m4**. Write down the fitted model.

4. (3 points) The outliers remain in the seasonal model. Therefore, a refined model is used and called `m5`. Write down the fitted model.

5. (2 points) Based on the model checking statistics provided, are there serial correlations in the residuals of model **m5**? Why?

6. (2 points) Among models **m1,m3, m4** and **m5**, which model is preferred under the in-sample fit? Why?

7. (2 points) If root mean squares of forecast errors are used in out-of-sample prediction, which model is preferred? Why?

8. (2 points) If mean absolute forecast errors are used in out-of-sample comparison, which model is selected?

9. (2 points) Consider models **m1** and **m3**. State the impact of outliers on in-sample fitting.

10. (2 points) Again, consider models **m1** and **m3**. State the impact of outliers on out-of-sample predictions.

Problem C. (27 points) Consider the daily log returns of Amazon (AMZN) stock obtained via `quantmod`. Statistical analysis is included in the attached R output. Answer the following questions. Note, a model should include both mean and volatility equations and the innovation distribution used.

1. (2 points) Are there serial correlations in the daily log returns? Why? Write down the proper null hypothesis for testing.

2. (3 points) A standard GARCH(1,1) model is fitted. Write down the fitted model.

3. (3 points) Model checking shows the normality is rejected. A skew standardized Student-t distribution is used. Write down the fitted model. Model **m3**.

4. (2 points) Based on the fitted model **m3**. Does the model support that the innovation is skewed? Perform a test to support your conclusion.

5. (2 points) Compute the 95% interval forecasts for 1-step and 2-step ahead predictions using model **m3**.

6. (2 points) An IGARCH(1,1) model is also entertained. Write down the fitted model. Model **m4**.

7. (2 points) Why are the 1-step to 5-step ahead volatility forecasts of the IGARCH(1,1) model not constant?

8. (2 points) An EGARCH model is also entertained. Write down the fitted model? Model **m5**.

9. (2 points) Based on the fitted EGARCH model, is the leverage effect significant? Why?

10. (3 points) The lag-1 VIX index is used as an explanatory variable for volatility. Write down the fitted model. Model **m6**

11. (2 points) Based on the fitted model, does the lag-1 VIX index affect significantly the AMZN volatility? Why?

12. (2 points) Among all volatility models entertained, which model provides best in-sample fit? Why?

Problem D. (10 points) Consider the monthly log returns of Procter and Gamble stock from January 1960 to March 2015. Use the R output to answer the following questions.

1. (2 points) An IGARCH(1,1) model is entertained. Write down the fitted model.

2. (2 points) Based on the statistics provided, is the model adequate? Why?

3. (4 points) Based on the fitted IGARCH(1,1) model, compute the 1-step and 2-step ahead forecasts for mean and volatility of the log returns.

4. (2 points) A GARCM-M model is entertained. Based on the fitted model, is the risk premium statistically significant? Perform a test to justify your answer.

Problem E. (10 points) Consider the monthly log returns of value-weighted index and the S&P composite index from January 1960 to March 2015. Our goal is to study the relationship between the volatility of the two market indexes. Based on the output provided, answer the following questions:

1. (1 points) A GARCH(1,1) model with skew standardized Student-t innovations is employed for the S&P index returns. Does the fitted model support the use of skew innovations? Why?

2. (2 points) A similar GARCH(1,1) model is also employed for the value-weighted index returns. Let the resulting volatility be volvw_t. Let volsp_t be the corresponding volatility of the S&P index return. Write down the fitted simple linear regression model for the dependent variable volsp_t. Is this simple linear regression model adequate? Why?

3. (2 points) A refined model is employed. Write down the fitted linear regression model with time series errors.

4. (3 points) Alternatively, one can use volvw_t as an explanatory variable in volatility modeling of the S&P index return. Write down the fitted volatility model.

5. (2 point) Does volvw_t significantly contribute to the volatility modeling of the S&P index returns? Why?

R output: edited to shorten the output

```
### Problem B #########
> rate <- as.numeric(UNRATE[,1])
> xt <- diff(rate)  ### Differenced series
> require(forecast)
> auto.arima(xt)
Series: xt
ARIMA(2,0,2) with zero mean
> m1 <- arima(xt,order=c(2,0,2),include.mean=F)
> m1
Call:arima(x=xt,order=c(2,0,2),include.mean=F)
Coefficients:
         ar1      ar2      ma1     ma2
      1.6546  -0.7753  -1.6288  0.8440
s.e.  0.0427   0.0468   0.0420  0.0477

sigma^2 estimated as 0.03838:  log likelihood = 172.36,  aic = -334.71
> which.min(m1$residuals)
[1] 22
> i22[22]=1;  i22 <- rep(0,818)
> m2 <- arima(xt,order=c(2,0,2),xreg=i22,include.mean=F)
> m2
         ar1      ar2      ma1     ma2      i22
      1.6953  -0.7965  -1.6286  0.8164  -1.5038
s.e.  0.0454   0.0477   0.0484  0.0509   0.1837

sigma^2 estimated as 0.03545:  log likelihood = 204.92,  aic = -397.84
> which.max(m2$residuals)
[1] 21
> i21 <- rep(0,818)
> i21[21]=1
> out <- cbind(i22,i21)
> m3 <- arima(xt,order=c(2,0,2),xreg=out,include.mean=F)
> m3
Call: arima(x = xt, order = c(2, 0, 2), xreg = out, include.mean = F)
Coefficients:
         ar1      ar2      ma1     ma2      i22     i21
      1.6901  -0.7909  -1.6128  0.8014  -1.5302  1.1472
s.e.  0.0466   0.0504   0.0534  0.0592   0.1755  0.1757

sigma^2 estimated as 0.03368: log likelihood = 225.86, aic = -437.72
> Box.test(m3$residuals,lag=12,type='Ljung')
        Box-Ljung test
data:  m3$residuals
X-squared = 31.83, df = 12, p-value = 0.00147

> m4 <- arima(xt,order=c(2,0,2),seasonal=list(order=c(1,0,1),period=12),
```

```
  include.mean=F)
> m4
Call:arima(x = xt,order=c(2,0,2),seasonal=list(order=c(1,0,1),period=12),
    include.mean = F)
Coefficients:
         ar1      ar2      ma1     ma2    sar1     sma1
      1.2357  -0.3608  -1.2354  0.5151  0.5542  -0.8220
s.e.  0.2413   0.2221   0.2241  0.1702  0.0662   0.0473

sigma^2 estimated as 0.03538:  log likelihood = 204.21, aic = -394.43

> m5 <- arima(xt,order=c(2,0,2),seasonal=list(order=c(1,0,1),period=12),
  include.mean=F,xreg=out)
> m5
Call:arima(x=xt,order=c(2,0,2),seasonal=list(order=c(1,0,1),period=12),
    xreg = out, include.mean = F)

Coefficients:
         ar1      ar2      ma1     ma2    sar1     sma1      i22     i21
      1.5743  -0.6591  -1.4869  0.6720  0.5488  -0.8208  -1.4762  1.1441
s.e.  0.1159   0.1110   0.1111  0.0913  0.0659   0.0448   0.1620  0.1616

sigma^2 estimated as 0.03062:  log likelihood = 263.2,  aic = -508.4
> Box.test(m5$residuals,lag=24,type='Ljung')
        Box-Ljung test
data:  m5$residuals
X-squared = 27.826, df = 24, p-value = 0.2674

> source("backtest.R")
> backtest(m1,xt,750,include.mean=F)
[1] "RMSE of out-of-sample forecasts"
[1] 0.1621524
[1] "Mean absolute error of out-of-sample forecasts"
[1] 0.1242145
> backtest(m3,xt,750,include.mean=F,xre=out)
[1] "RMSE of out-of-sample forecasts"
[1] 0.1625846
[1] "Mean absolute error of out-of-sample forecasts"
[1] 0.1236525
> backtest(m4,xt,750,include.mean=F)
[1] "RMSE of out-of-sample forecasts"
[1] 0.1499355
[1] "Mean absolute error of out-of-sample forecasts"
[1] 0.1164277
> backtest(m5,xt,750,include.mean=F,xre=out)
[1] "RMSE of out-of-sample forecasts"
[1] 0.1492887
```

```
[1] "Mean absolute error of out-of-sample forecasts"
[1] 0.1162959

##### Problem C
> getSymbols("AMZN")
[1] "AMZN"
> getSymbols("^VIX")  ## to be used later.
[1] "VIX"
> vix <- as.numeric(VIX[,6])
> vixm1 <- vix[-1]
> amzn <- diff(log(as.numeric(AMZN[,6])))
> Box.test(amzn,lag=10,type='Ljung')
        Box-Ljung test
data:  amzn
X-squared = 14.015, df = 10, p-value = 0.1723

> require(rugarch)
> spec1 <- ugarchspec(variance.model=list(model="sGARCH"),
     mean.model=list(armaOrder=c(0,0)))
> m1 <- ugarchfit(data=amzn,spec=spec1)
> m1
*---------------------------------*
*          GARCH Model Fit        *
*---------------------------------*
Conditional Variance Dynamics
-----------------------------------
GARCH Model     : sGARCH(1,1)
Mean Model      : ARFIMA(0,0,0)
Distribution    : norm

Optimal Parameters
------------------------------------
        Estimate  Std. Error   t value Pr(>|t|)
mu      0.001364    0.000496    2.7513 0.005936
omega   0.000002    0.000001    2.9754 0.002927
alpha1  0.008162    0.000591   13.8111 0.000000
beta1   0.988780    0.000329 3004.2634 0.000000

Information Criteria
------------------------------------
Akaike       -4.5338
Bayes        -4.5240
Shibata      -4.5338
Hannan-Quinn -4.5302

Weighted Ljung-Box Test on Standardized Residuals
------------------------------------
```

```
                        statistic p-value
Lag[1]                     0.5403  0.4623
Lag[4*(p+q)+(p+q)-1][5]    5.3300  0.1288
d.o.f=0
H0 : No serial correlation

Weighted Ljung-Box Test on Standardized Squared Residuals
------------------------------------
                        statistic  p-value
Lag[1]                      8.774 0.003056
Lag[4*(p+q)+(p+q)-1][9]     9.439 0.065919
d.o.f=2

> spec2 <- ugarchspec(variance.model=list(model="sGARCH"),
mean.model=list(armaOrder=c(0,0)),distribution.model="std")
> m2 <- ugarchfit(data=amzn,spec=spec2)
> m2
Conditional Variance Dynamics
-----------------------------------
GARCH Model     : sGARCH(1,1)
Mean Model      : ARFIMA(0,0,0)
Distribution    : std

Optimal Parameters
------------------------------------
        Estimate  Std. Error  t value Pr(>|t|)
mu      0.000865    0.000384   2.2551 0.024124
omega   0.000004    0.000003   1.3177 0.187617
alpha1  0.021701    0.003228   6.7223 0.000000
beta1   0.972435    0.006398 151.9797 0.000000
shape   3.601122    0.252180  14.2800 0.000000

Information Criteria
------------------------------------
Akaike       -4.8059
Bayes        -4.7937
Shibata      -4.8059
Hannan-Quinn -4.8015

> spec3 <- ugarchspec(variance.model=list(model="sGARCH"),mean.model=
list(armaOrder=c(0,0)),distribution.model="sstd")
> m3 <- ugarchfit(data=amzn,spec=spec3)
> m3
Conditional Variance Dynamics
-----------------------------------
GARCH Model     : sGARCH(1,1)
Mean Model      : ARFIMA(0,0,0)
```

```
Distribution    : sstd

Optimal Parameters
------------------------------------
        Estimate  Std. Error  t value Pr(>|t|)
mu      0.001461    0.000453   3.2223 0.001272
omega   0.000004    0.000003   1.5644 0.117724
alpha1  0.022732    0.003547   6.4097 0.000000
beta1   0.971489    0.004124 235.5422 0.000000
skew    1.076577    0.031540  34.1333 0.000000
shape   3.573507    0.275739  12.9597 0.000000

Information Criteria
------------------------------------
Akaike       -4.8078
Bayes        -4.7931
Shibata      -4.8078
Hannan-Quinn -4.8024

Weighted Ljung-Box Test on Standardized Residuals
------------------------------------
                        statistic p-value
Lag[1]                     0.7015  0.4023
Lag[4*(p+q)+(p+q)-1][5]    4.4511  0.2029
d.o.f=0
H0 : No serial correlation

Weighted Ljung-Box Test on Standardized Squared Residuals
------------------------------------
                        statistic p-value
Lag[1]                      1.694  0.1930
Lag[4*(p+q)+(p+q)-1][9]     2.709  0.8059
d.o.f=2

> ugarchforecast(m3,n.ahead=5)
*------------------------------------*
*       GARCH Model Forecast         *
*------------------------------------*
Model: sGARCH
Horizon: 5
0-roll forecast [T0=1976-06-05 19:00:00]:
      Series   Sigma
T+1 0.001461 0.02494
T+2 0.001461 0.02495
T+3 0.001461 0.02496
T+4 0.001461 0.02497
T+5 0.001461 0.02498
```

```
> spec4 <- ugarchspec(variance.model=list(model="iGARCH"),
mean.model=list(armaOrder=c(0,0)),distribution.model="sstd")
> m4 <- ugarchfit(data=amzn,spec=spec4)
> m4
Conditional Variance Dynamics
-----------------------------------
GARCH Model     : iGARCH(1,1)
Mean Model      : ARFIMA(0,0,0)
Distribution    : sstd

Optimal Parameters
------------------------------------
        Estimate  Std. Error  t value Pr(>|t|)
mu      0.001528    0.000462   3.3095 0.000935
omega   0.000002    0.000002   1.5149 0.129803
alpha1  0.024331    0.004152   5.8598 0.000000
beta1   0.975669          NA       NA       NA
skew    1.079980    0.031788  33.9747 0.000000
shape   3.280276    0.137902  23.7871 0.000000

Information Criteria
------------------------------------
Akaike       -4.8073
Bayes        -4.7950
Shibata      -4.8073
Hannan-Quinn -4.8028

Weighted Ljung-Box Test on Standardized Residuals
------------------------------------
                        statistic p-value
Lag[1]                     0.6145  0.4331
Lag[4*(p+q)+(p+q)-1][5]    4.2644  0.2229
d.o.f=0
H0 : No serial correlation

Weighted Ljung-Box Test on Standardized Squared Residuals
------------------------------------
                        statistic p-value
Lag[1]                      1.864  0.1721
Lag[4*(p+q)+(p+q)-1][9]     2.849  0.7837
d.o.f=2
> ugarchforecast(m4,n.ahead=5)
*------------------------------------*
*       GARCH Model Forecast         *
*------------------------------------*
Model: iGARCH
```

```
Horizon: 5

0-roll forecast [T0=1976-06-05 19:00:00]:
      Series   Sigma
T+1 0.001528 0.02624
T+2 0.001528 0.02628
T+3 0.001528 0.02633
T+4 0.001528 0.02638
T+5 0.001528 0.02642

> spec5 <- ugarchspec(variance.model=list(model="eGARCH"),
mean.model=list(armaOrder=c(0,0)))
> m5 <- ugarchfit(data=amzn,spec=spec5)
> m5
Conditional Variance Dynamics
-----------------------------------
GARCH Model     : eGARCH(1,1)
Mean Model      : ARFIMA(0,0,0)
Distribution    : norm

Optimal Parameters
------------------------------------
        Estimate  Std. Error  t value Pr(>|t|)
mu      0.001807    0.000386   4.6873 0.000003
omega  -0.456107    0.021264 -21.4494 0.000000
alpha1 -0.049813    0.013608  -3.6605 0.000252
beta1   0.936172    0.003462 270.4202 0.000000
gamma1  0.127728    0.022494   5.6783 0.000000

Information Criteria
------------------------------------
Akaike       -4.5318
Bayes        -4.5195
Shibata      -4.5318
Hannan-Quinn -4.5273

Weighted Ljung-Box Test on Standardized Residuals
------------------------------------
                        statistic p-value
Lag[1]                      0.264  0.6074
Lag[4*(p+q)+(p+q)-1][5]     4.364  0.2121
d.o.f=0
H0 : No serial correlation

Weighted Ljung-Box Test on Standardized Squared Residuals
------------------------------------
                        statistic p-value
```

```
Lag[1]                        3.630 0.05673
Lag[4*(p+q)+(p+q)-1][9]      5.067 0.41958
d.o.f=2

> spec6 <- ugarchspec(variance.model=list(model="sGARCH",external.regressors=
as.matrix(vixm1)),mean.model=list(armaOrder=c(0,0)),distribution.model="sstd")
> m6 <- ugarchfit(data=amzn,spec=spec6)
> m6
Conditional Variance Dynamics
-----------------------------------
GARCH Model     : sGARCH(1,1)
Mean Model      : ARFIMA(0,0,0)
Distribution    : sstd

Optimal Parameters
------------------------------------
        Estimate  Std. Error  t value Pr(>|t|)
mu      0.001905    0.000455   4.1847 0.000029
omega   0.000000    0.000010   0.0000 1.000000
alpha1  0.089867    0.031530   2.8502 0.004369
beta1   0.000173    0.133352   0.0013 0.998963
vxreg1  0.000026    0.000004   6.0994 0.000000
skew    1.118515    0.033620  33.2692 0.000000
shape   3.958441    0.320506  12.3506 0.000000

Information Criteria
------------------------------------
Akaike       -4.8371
Bayes        -4.8200
Shibata      -4.8372
Hannan-Quinn -4.8309

Weighted Ljung-Box Test on Standardized Residuals
------------------------------------
                        statistic p-value
Lag[1]                      1.457 0.22734
Lag[2*(p+q)+(p+q)-1][2]     5.013 0.04035
Lag[4*(p+q)+(p+q)-1][5]     7.953 0.03027
d.o.f=0
H0 : No serial correlation

Weighted Ljung-Box Test on Standardized Squared Residuals
------------------------------------
                        statistic p-value
Lag[1]                  3.139e-05  0.9955
Lag[4*(p+q)+(p+q)-1][9] 1.149e+00  0.9792
d.o.f=2
```

```
#### Problem D ##
> da=read.table("m-pg3dx-6015.txt",header=T)
> head(da)
  PERMNO     date       RET    vwretd    ewretd    sprtrn
1  18163 19600129 -0.081667 -0.066244 -0.039202 -0.071464
> pg <- log(da[,3]+1)
> source(‘‘Igarch.R’’)
> m3 <- Igarch(pg)
Estimates:  0.9164492
Maximized log-likehood:  -967.2926

Coefficient(s):
      Estimate  Std. Error  t value   Pr(>|t|)
beta 0.9164492   0.0163915  55.9099 < 2.22e-16 ***
---
> names(m3)
[1] "par"        "volatility"
> r3 <- pg/m3$volatility
> Box.test(r3,lag=12,type='Ljung')
        Box-Ljung test
data:  r3
X-squared = 10.684, df = 12, p-value = 0.5562

> Box.test(r3^2,lag=12,type='Ljung')
        Box-Ljung test
data:  r3^2
X-squared = 5.7124, df = 12, p-value = 0.9299

> length(pg)
[1] 663
> pg[663]
[1] -0.03819212
> m3$volatility[663]
[1] 0.03961325

> source("garchM.R")
> m4 <- garchM(pg,type=1)
Maximized log-likehood:  991.3017

Coefficient(s):
         Estimate  Std. Error  t value   Pr(>|t|)
mu    0.007111978 0.004706627  1.51106 0.13077411
gamma 0.707355559 1.574949256  0.44913 0.65333852
omega 0.000416370 0.000223498  1.86297 0.06246653 .
alpha 0.165418629 0.046037973  3.59309 0.00032678 ***
beta  0.709835860 0.101923405  6.96440  3.298e-12 ***
```

```
#### Problem E
> da=read.table("m-pg3dx-6015.txt",header=T)
> head(da)
  PERMNO     date       RET    vwretd    ewretd    sprtrn
1  18163 19600129 -0.081667 -0.066244 -0.039202 -0.071464
> sp <- log(da[,6]+1)
> vw <- log(da[,4]+1)

> m1 <- garchFit(~garch(1,1),data=sp,trace=F,cond.dist="sstd")
> summary(m1)
Title: GARCH Modelling
Call:
 garchFit(formula = ~garch(1, 1), data = sp, cond.dist = "sstd", trace = F)

Mean and Variance Equation:
 data ~ garch(1, 1)[data = sp]

Conditional Distribution: sstd

Error Analysis:
        Estimate  Std. Error  t value Pr(>|t|)
mu     6.092e-03   1.444e-03    4.218 2.46e-05 ***
omega  9.338e-05   4.132e-05    2.260 0.023822 *
alpha1 1.304e-01   3.236e-02    4.031 5.56e-05 ***
beta1  8.244e-01   3.764e-02   21.899  < 2e-16 ***
skew   7.699e-01   4.459e-02   17.267  < 2e-16 ***
shape  7.901e+00   2.212e+00    3.571 0.000355 ***
---
Standardised Residuals Tests:
                                Statistic p-Value
 Ljung-Box Test     R    Q(10)  7.547206  0.6729704
 Ljung-Box Test     R    Q(20)  12.2038   0.9088825
 Ljung-Box Test     R^2  Q(10)  5.408172  0.8622992
 Ljung-Box Test     R^2  Q(20)  8.436177  0.9885659

> volsp <- volatility(m1)

> n1 <- garchFit(~garch(1,1),data=vw,trace=F,cond.dist="sstd")
> summary(n1)
Title: GARCH Modelling
Call:garchFit(formula = ~garch(1, 1), data = vw, cond.dist = "sstd",
    trace = F)

Mean and Variance Equation:
 data ~ garch(1, 1) [data = vw]
```

```
Conditional Distribution: sstd

Error Analysis:
        Estimate  Std. Error  t value Pr(>|t|)
mu     8.787e-03   1.485e-03    5.918 3.27e-09 ***
omega  9.953e-05   4.376e-05    2.274 0.022940 *
alpha1 1.271e-01   3.203e-02    3.967 7.27e-05 ***
beta1  8.274e-01   3.841e-02   21.542  < 2e-16 ***
skew   7.383e-01   4.391e-02   16.811  < 2e-16 ***
shape  7.428e+00   1.919e+00    3.871 0.000108 ***
---
> volvw <- volatility(n1)
> k1 <- lm(volsp~volvw)
> summary(k1)
Call: lm(formula = volsp ~ volvw)

Coefficients:
             Estimate Std. Error t value Pr(>|t|)
(Intercept) 0.0007918  0.0002818    2.81   0.0051 **
volvw       0.9487851  0.0062352  152.17   <2e-16 ***
---
Residual standard error: 0.001891 on 661 degrees of freedom
Multiple R-squared:  0.9722,    Adjusted R-squared:  0.9722
F-statistic: 2.315e+04 on 1 and 661 DF,  p-value: < 2.2e-16

> k2 <- ar(k1$residuals)
> k2$order
[1] 1
> k3 <- arima(volsp,order=c(1,0,0),xreg=volvw)
> k3
Call:arima(x = volsp, order = c(1, 0, 0), xreg = volvw)
Coefficients:
         ar1  intercept   volvw
      0.8861     0.0012  0.9392
s.e.  0.0178     0.0004  0.0073

sigma^2 estimated as 7.603e-07:  log likelihood = 3729.18,  aic = -7450.36
> tsdiag(k3)
> Box.test(k3$residuals,lag=12,type='Ljung')
        Box-Ljung test
data:  k3$residuals
X-squared = 11.216, df = 12, p-value = 0.5105

> spec1 <- ugarchspec(variance.model=list(model="sGARCH",external.regressors=
as.matrix(volvw)),mean.model=list(armaOrder=c(0,0)),distribution.model="sstd")
> n4 <- ugarchfit(data=sp,spec=spec1)
> n4
```

```
Conditional Variance Dynamics
-----------------------------------
GARCH Model     : sGARCH(1,1)
Mean Model      : ARFIMA(0,0,0)
Distribution    : sstd

Optimal Parameters
------------------------------------
        Estimate  Std. Error   t value Pr(>|t|)
mu      0.005915    0.001475  4.009230 0.000061
omega   0.000000    0.000027  0.000003 0.999998
alpha1  0.115915    0.034151  3.394183 0.000688
beta1   0.771342    0.067762 11.383034 0.000000
vxreg1  0.004918    0.002628  1.871405 0.061289
skew    0.769959    0.044977 17.118825 0.000000
shape   7.983262    2.245018  3.555990 0.000377

Weighted Ljung-Box Test on Standardized Residuals
------------------------------------
                        statistic p-value
Lag[1]                     0.4769  0.4898
Lag[4*(p+q)+(p+q)-1][5]    2.1466  0.5842
d.o.f=0
H0 : No serial correlation
```