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Emerging markets and heavy tails

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ABSTRACT

Emerging countries are held to be subject to more frequent and more pronounced external and internal shocks than their developed counter-parts. This suggests that key variables pertaining to their markets, including their exchange rates, will be marked by greater likelihood of extreme observations and large fluctuations. We focus on the hypothesis that compared to developed country exchange rates, emerging country exchange rates will be more pronouncedly heavy-tailed. We find support for the hypothesis using recently proposed robust tail index estimation methods which, in particular, perform well under heavy-tailed dependent GARCH processes that are often used for modeling exchange rates. According to the estimation results reported in the paper, variances may be infinite for several emerging country exchange rates. Tail index values $\zeta = p \in (2.6, 2.8)$ appear to be at the dividing boundary between the two sets of countries: while the moments of order $p \in (2.6, 2.8)$ are finite for most of the developed country exchange rates, they may be (or are) infinite for most of the emerging country exchange rates. We also study the impact of the on-going financial and economic crisis, and find that heavy-tailedness properties of most exchange rates did not change significantly with the onset of the crisis. At the same time, some foreign exchange markets have experienced structural changes in their heavy-tailedness properties during the crisis.

1. Introduction

1.1. Heavy tails in economics and finance

Foreign exchange markets are arguably the world's largest markets, operating continuously, and bringing together a wide variety of buyers and sellers, within and across national borders. In recent years these markets have been characterized by turbulence and volatility, with extreme variations marking some exchange rates. As the literature on the determination of exchange rates points out, there are many processes capable of generating extreme exchange rate variations. These include economic crises, speculative attacks, bailouts, stabilization efforts, regime reforms and regulatory changes, among others. Recent theoretical literature contains useful models that explain extreme changes in financial returns, in terms of trading actions of large market participants (see, for instance, Gabaix et al., 2006), and in terms of government interventions in the case of foreign exchange markets.

Large fluctuations in exchange rates carry significant real consequences for international trade, foreign investment, asset prices, and a wide range of other economic and financial outcome variables. The on-going financial and economic crisis has raised the need for accurate estimates of probabilities associated with large changes in financial returns and exchange rates. Emerging and developing countries are generally held to be subject to more frequent and more pronounced external and internal shocks than their developed counter-parts, and in that context it is ever more important to identify currencies that are relatively more prone to large fluctuations.

Numerous contributions in economics, finance, risk management and insurance have concluded that distributions of many variables of interest deviate from the Normal distribution paradigm, particularly in terms of heavy tails. For example, it is virtually impossible for Gaussian distributions to generate variations of the order of magnitude observed on Black Monday in 1987, according to the striking illustrations in Ch. 2 in Stock and Watson (2006). Detailed discussion and reviews are available in Embrechts et al. (1997), Cont (2001), Rachev et al. (2005), Gabaix (2009), Ibragimov and Walden (2007), Ibragimov (2009a), and references

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therein. This stream of literature goes back to Mandelbrot (1963) who pioneered the study of heavy-tailed distributions in economics and finance (see also Fama, 1965, and the papers in Mandelbrot (1997)).

The potential for large changes in random variables is characterized by the probability mass in the tails of their distributions. In models involving a heavy-tailed risk, return or foreign exchange rate, the variable of interest r is usually assumed to have a distribution with power tails, such that:

$$P(r > x) \sim \frac{C_1}{x^{\zeta_1}}, \tag{1}$$

$$P(r < -x) \sim \frac{C_2}{x^{\zeta_2}}, \tag{2}$$

$\zeta_1, \zeta_2 > 0$, $C_1, C_2 > 0$, as $x \to +\infty$, that implies, with $\zeta = \min(\zeta_1, \zeta_2)$,

$$P(|r| > x) \sim \frac{C}{x^{\zeta}}, \tag{3}$$

$C > 0$, as $x \to +\infty$ (here and throughout the paper, $f(x) \sim g(x)$ as $x \to +\infty$ means that $\lim_{x \to +\infty} \frac{f(x)}{g(x)} = 1$). The parameters ζ, ζ_1 and ζ_2 in (3) and (1), (2) are referred to, respectively, as the tail index (or the tail exponent), the right tail index and the left tail index of the distribution of r. These indices characterize the heaviness (the rate of decay) of the tails of power law distributions (1)–(3). The greater the probability mass in the tails, the smaller the tail index parameters, and vice versa. Thus smaller values of ζ correspond to larger likelihood of outliers in realizations of the variable.

The models (1)–(3) allow one to determine the quantiles of variable r that correspond to small tail probability levels, and the corresponding loss exceedance probabilities and risk measures including the value at risk (VaR) and the expected shortfall. Differences between markets in their tail index estimates point to differences in risk that they carry. The heavy-tailedness property is of key interest to risk managers, financial regulators, financial stability analysts and policy makers concerned with the likelihood of extreme values in financial and exchange rate returns.

Tail indices ζ characterize the maximal order of finite moments of the variable r considered, as discussed in Appendix. In particular, the fourth moments and thus the kurtosis of the variable are finite if and only if $\zeta > 4$; the variances of r are finite if and only if $\zeta > 2$; and the means of r are finite if and only if $\zeta > 1$. The tail index can be regarded as being infinite for normal distributions where the decay to zero of distributional tails in (1)–(3) is faster than exponential, and thus the moments of an arbitrary order are finite. Many recent studies have found that in developed market economies, tail indices in the heavy-tailed model (3) for financial returns and exchange rates typically lie in the interval $\zeta \in (2, 5)$ (see, among others, Loretan and Phillips, 1994; Gabaix et al., 2006; Ibragimov and Walden, 2007; Gabaix, 2009; Ibragimov, 2009a, and references therein). The estimates $\zeta \in (2, 4)$ imply that the return variables have finite variances and finite first moments; however, their fourth moments are infinite.

Finiteness of first moments is of key importance for the optimality of diversification in the value at risk framework and for the robustness of a number of economic models for the variable of interest (see Ibragimov and Walden, 2007; Ibragimov, 2009a; Ibragimov et al., 2009). The finiteness of variances is crucial for the applicability of classical statistical and econometric approaches, including regression and least squares methods, to economic and financial variables of interest. The problem of potentially infinite fourth moments of economic and financial time series needs to be taken into account when analysing them using autocorrelation-based methods and related inference procedures (see, among others, the discussion in Granger and Orr (1972), and in a number of more recent studies, e.g., Ch. 7 in Embrechts et al. (1997), Cont (2001), and references therein).

1.2. Robust inference for heavy-tailed exchange rates: emerging vs. developed countries

Our principal goal in this paper is the robust analysis of heavy-tailedness properties of exchange rates of emerging and developing countries, in comparison with developed countries. This comparative examination is motivated by the generally held view that the former set of countries are more subject to severe external and internal shocks, and therefore suffer greater potential for extreme changes in financial returns and exchange rates. We use recently proposed robust tail index estimation methods, based on log–log rank-size regressions with optimal shifts in ranks, and correct standard errors (see Gabaix and Ibragimov, 2011, and Appendix), applying them to large data sets on daily exchange rates for a number of countries. This is in contrast to earlier studies of exchange rates of emerging countries which have tended to use model-specific parametric maximum likelihood procedures or (semiparametric) Hill's estimators, with a number of contributions using relatively small data sets, with potentially non-robust conclusions (see the discussion in Appendix).[3,4] We also report results of an analysis of asymmetry in extreme upward, as against downward changes in the exchange rates considered.

A further dimension to our analysis is the on-going economic crisis. We assess whether the crisis led to significant changes in the likelihood of large variations in exchange rates. We also draw conclusions on the applicability of standard economic and econometric models, including regression methods, and models explaining heavy tails in financial markets.

We find that the tail indices for exchange rates of emerging countries are indeed considerably smaller than those of developed countries. Our estimates imply that, in contrast to developed countries, the value of the tail index $\zeta = 2$ is not rejected at commonly used statistical significance levels for the exchange rates of several emerging countries (Section 3), implying that their variances may be infinite. Tail index values $\zeta = p \in (2.6, 2.8)$ appear to be at the dividing boundary between developed country exchange rates on the one hand, and emerging country exchange rates on the other: while the moments of order $p \in (2.6, 2.8)$ are finite for most of the developed country exchange rates, they may be (or are) infinite for most of the emerging country exchange rates.

With respect to the on-going financial and economic crisis, we find that while the heavy-tailedness properties of most exchange rates did not change significantly, a few foreign exchange markets did see structural changes. There was significant increase in the degree of heavy-tailedness of the Swiss franc and pound sterling, and surprisingly, a decrease in the degree of heavy-tailedness of the Russian rouble.

These results have a number of implications. They underscore the need for robust econometric and statistical methods in the analysis of emerging country financial markets (see the discussion in Sections 1.1 and 4). They highlight aspects of macroeconomic management and policy in emerging countries. In a structural

[3] Robustness of the tail index estimation approaches based on log–log rank-size regressions is illustrated by their favorable performance under deviations from power laws (1)–(3) in the form of slowly varying factors and dependent GARCH processes that are often used for modeling financial returns, exchange rates and other economics and financial time series (see Gabaix and Ibragimov, 2011, and the discussion in Appendix).

[4] For illustration, we compare the tail index estimates obtained using the log–log rank-size regression approach with those obtained using Hill's estimation procedure used in previous works in the literature (see Section 3). The comparisons typically lead to similar conclusions for both estimation approaches.

model that explains the determination of the tail indices of exchange rates, the tail indices of trading volumes as well as of sizes of market participants have bearing (Gabaix et al., 2006). These reflect the extent of official intervention in the currency market (see the discussion in Section 4). Further, estimates for emerging exchange rates may be used to forecast patterns in their future development and convergence to distributions with $\zeta \in (2,4)$ as in the case of developed countries.

The paper is organized as follows. Section 2 discusses the data on exchange rates of developed and emerging countries used in the analysis. Section 3 presents our tail index estimates for the exchange rates considered. In Section 4, we discuss the implications of our results for some economic, financial and econometric models, as well as for economic policy and forecasting. Section 5 concludes with suggestions for further research. The appendix reviews the main properties of heavy-tailed power laws and discusses the tail index estimation approaches available in the literature. In particular, it describes the main tail index inference methods used in the paper – robust, bias-corrected log–log rank-size regressions with optimal shifts in ranks and correct standard errors presented in Gabaix and Ibragimov (2011). It also discusses Hill's tail index estimation procedures used in a number of works in the previous literature.

2. Data

We analyse daily exchange rates to US dollar (USD) for the currencies listed below, for the period from 1 January 1999 to 22 June 2012.[5] The developed country currencies we analyse in this paper are: Australian dollar (AUD), Canadian dollar (CAD), Swiss franc (CHF), Danish krone (DKK), Euro (EUR), pound sterling (GBP), Japanese yen (JPY), Norwegian kroner (NOK) and Swedish krona (SEK). The currencies of emerging countries in our analysis are: Chinese renminbi (CNY), Hong Kong dollar (HKD), Indian rupee (INR), South Korean won (KRW), Malaysian ringgit (MYR), Russian rouble (RUB), Singapore dollar (SGD), Taiwan dollar (TWD) and Thai baht (THB). For an overview of exchange rate regimes in the emerging countries, see (Patnaik et al., 2011) (Table 1 therein) and IMF AREAER, various issues.[6,7] Fig. 1 illustrates and contrasts the dynamics of exchange rates of developed and emerging country currencies. The exchange rates of currencies with respect to CNY were calculated from their exchange rates with respect to USD. We also present the results of estimation of the tail indices of GBP, EUR and the base currency USD, using Gold and Silver Fix prices in these currencies per Troy ounce, using data from the London Bullion Market Association.[8]

3. Estimation results

We present (in Tables 1–4), the estimation results for the tail indices in power law models (3) for all the exchange rates discussed in Section 2. The results in Tables 1 and 2, and in this section, concern the exchange rates with respect to USD. We also present results for the exchange rates with respect to CNY (in Tables 3 and 4), for illustration and comparison. Table 7 presents the results for exchange rates of some of the developed countries in terms of Gold and Silver prices.

A stylized statistical fact common to many financial returns, in addition to the heavy-tailedness property discussed in Section 1.1, is the so-called gain/loss asymmetry (see, among others, Cont, 2001). According to this, in many return variables one typically observes large drawdowns but not equally large upward movements. This property has not been found to characterize exchange rates. Motivated by these empirical findings, we present estimates of exchange rate tail indices for the right and left tails in power law models (1) and (2) in Tables 5 and 6.

Due to sensitivity of the commonly used Hill's tail index inference approach to dependence and sample sizes used in estimation, and the robustness of the log–log rank-size regression tail index estimation methods discussed in Section 1.2 and Appendix (including favorable performance of the methods under deviations from power laws and heavy-tailed dependent GARCH processes that are often used for exchange rate modeling), we mainly focus on applications of the log–log rank-size regression approaches to inference on the foreign exchange rate tail indices. For illustration, we compare a number of the conclusions to those obtained using Hill's estimation approach (see Tables 1 and 2). The comparisons typically indicate similar conclusions for both the log–log rank-size regression and Hill's tail index estimation approaches.

Tables 1–7 report the tail index estimates $\hat{\zeta}_{RS}$ obtained from log–log rank-size regressions (8, Appendix), with the optimal shift $\gamma = 1/2$ and the correct standard errors $\sqrt{\frac{2}{n}}\hat{\zeta}_{RS}$. The estimates pertaining to up/down movements in the exchange rates (Tables 5 and 6) are based on the analogues of regression (8) with $\gamma = 1/2$ for positive and negative values of their growth rates (see Appendix). Tables 1–7 also provide the correct 95% confidence intervals (9) for the true tail index values ζ. For comparison, the last three columns in Tables 1 and 2 provide Hill's approach estimates (5), their standard errors ($s.e._{Hill} = \frac{1}{\sqrt{n}}\hat{\zeta}_{Hill}$) and the corresponding 95%-confidence intervals (6) for ζ. The estimates relate to the 5% and 10% truncation levels for extreme observations as in (4): $n \approx 0.05N$ and $n \approx 0.1N$, where N is the total sample size for the time series.

Through the rest of this section, for brevity, we refer to the extreme observations used for estimation, defined with respect to the 5% and 10% truncation levels in (4), by $AUD_{5\%}$, $AUD_{10\%}$, $AUD(+)_{5\%}$, $AUD(+)_{10\%}$, $AUD(-)_{5\%}$, $AUD(-)_{10\%}$, etc. Failure to reject the null hypothesis H_a:$\zeta = \zeta_0$ refers to the 5% significance level and the two-sided alternative H_a: $\zeta \neq \zeta_0$. Rejection of H_0 refers to the 2.5% significance level and the one-sided alternatives H_a: $\zeta < \zeta_0$ or H_a: $\zeta > \zeta_0$. Similar to the results in Tables 1–7, the confidence intervals discussed below are 95% confidence intervals.

The results in Tables 1 and 2 point to remarkable differences in heavy-tailedness properties of the exchange rates of developed and emerging countries. The point estimates $\hat{\zeta}_{RS}$ for developed country exchange rates in Table 1 lie between 2.7 and 4.6. This range of values is preserved by estimates $\hat{\zeta}_{Hill}$, which lie between 2.8 and 4.3. These results are in line with the results for developed country financial markets, which report ζ estimates in the interval (2,5) for returns on stocks and stock indices.

The null hypothesis $\zeta = 2$ is rejected in favor of $\zeta > 2$ for all developed country exchange rates by both the log–log rank-size regression and Hill's estimation procedures (Table 1). The null hypothesis $\zeta = 3$ is not rejected for $AUD_{5\%}$, $AUD_{10\%}$, $CAD_{5\%}$, $CAD_{10\%}$,$GBP_{5\%}$, $JPY_{5\%}$ and $JPY_{10\%}$ using the log–log rank-size regression approach, and for $AUD_{5\%}$, $AUD_{10\%}$, $CAD_{5\%}$, $CAD_{10\%}$, $JPY_{10\%}$, $NOK_{5\%}$, $NOK_{10\%}$ and $SEK_{10\%}$ using Hill's estimation. Both approaches reject the null

[5] The exchange rate of the Russian Ruble is available from the Central Bank of Russia, http://www.cbr.ru/eng/currency_base/default.aspx. The data source for all other exchange rates considered in the paper is the Board of Governors of the Federal Reserve System, http://www.federalreserve.gov/datadownload

[6] The classification of the countries considered as emerging follows the *Economist*; this list includes Hong Kong, Singapore and Saudi Arabia and the following economies in the Morgan Stanley Emerging Markets Index: Brazil, Chile, China (mainland), Colombia, Czech Republic, Egypt, Hungary, India, Indonesia, Iran, Israel, Jordan, Malaysia, Mexico, Morocco, Pakistan, Peru, Philippines, Poland, Russia, South Africa, South Korea, Taiwan, Thailand, Tunisia, Turkey and Vietnam (Morgan Stanley Capital International classifies the economies of Hong Kong and Singapore as developed countries).

[7] Two of these exchange rates, CNY and MYR, were pegged to the US dollar till 2005, and HKD regime is the related linked exchange (see Table 1 in Patnaik et al. (2011)). Existence of peg periods for currencies, however, does not affect tail index estimates for their exchange rates since the estimates are based on largest absolute values of exchange rates (see Appendix).

[8] Estimation uses the Gold a.m. Fix prices.

Table 1
Tail index estimates for exchange rates of developed countries (base currency: USD).

Currency	Truncation (%)	$\hat{\zeta}_{RS}$	$s.e._{RS}=\sqrt{\frac{2}{n}}\hat{\zeta}_{RS}$	$95\%CI_{RS}$ (Eq. (9))	$\hat{\zeta}_{Hill}$	$s.e._{Hill}=\sqrt{\frac{1}{n}}\hat{\zeta}_{Hill}$	$95\%CI_{Hill}$ (Eq. (6))
AUD	10	2.80	0.22	(2.38,3.23)	2.85	0.15	(2.54,3.15)
	5	2.71	0.29	(2.14,3.29)	2.88	0.22	(2.45,3.32)
CAD	10	3.26	0.25	(2.77,3.76)	2.99	0.16	(2.67,3.30)
	5	3.46	0.37	(2.72,4.19)	3.31	0.25	(2.81,3.81)
CHF	10	3.92	0.30	(3.33,4.51)	3.70	0.20	(3.31,4.10)
	5	3.90	0.42	(3.07,4.73)	4.14	0.32	(3.51,4.76)
DKK	10	3.86	0.30	(3.28,4.44)	3.64	0.20	(3.26,4.03)
	5	3.98	0.43	(3.13,4.83)	4.01	0.31	(3.40,4.61)
EUR	10	4.23	0.33	(3.60,4.87)	3.77	0.20	(3.37,4.18)
	5	4.59	0.50	(3.62,5.57)	4.28	0.33	(3.64,4.92)
GBP	10	3.54	0.27	(3.01,4.08)	3.65	0.20	(3.26,4.04)
	5	3.34	0.36	(2.63,4.05)	3.70	0.28	(3.14,4.26)
JPY	10	3.39	0.26	(2.88,3.90)	3.09	0.17	(2.76,3.42)
	5	3.72	0.40	(2.93,4.50)	3.54	0.27	(3.00,4.07)
NOK	10	3.56	0.27	(3.03,4.10)	3.24	0.18	(2.90,3.59)
	5	3.90	0.42	(3.07,4.73)	3.52	0.27	(2.99,4.05)
SEK	10	3.64	0.28	(3.09,4.19)	3.22	0.18	(2.88,3.57)
	5	3.98	0.43	(3.13,4.82)	3.67	0.28	(3.12,4.22)

Note. 1 January 1999–22 June 2012: N = 3390, 10%N = 339, 5%N = 170.

hypothesis $\zeta = 3$ in favor of $\zeta > 3$ for $CHF_{5\%}$, $CHF_{10\%}$, $DKK_{5\%}$, $DKK_{10\%}$, $EUR_{5\%}$, $EUR_{10\%}$, $GBP_{10\%}$ and $SEK_{5\%}$. The hypothesis is also rejected for $GBP_{5\%}$, and at the margin for $JPY_{5\%}$ using Hill's estimation, and for $NOK_{5\%}$, $NOK_{10\%}$ and $SEK_{10\%}$ using log–log rank-size regression. Both approaches do not reject the null hypothesis $\zeta = 4$ for $CHF_{5\%}$, $CHF_{10\%}$, $DKK_{5\%}$, $DKK_{10\%}$, $EUR_{5\%}$, $EUR_{10\%}$, $GBP_{5\%}$, $GBP_{10\%}$, $JPY_{5\%}$, $NOK_{5\%}$ and $SEK_{5\%}$; this hypothesis is also not rejected for $CAD_{5\%}$, $NOK_{10\%}$ and $SEK_{10\%}$ by the confidence intervals based on log–log rank-size regressions. Both methods reject the null hypothesis $\zeta = 4$ in favor of $\zeta < 4$ for $AUD_{5\%}$, $AUD_{10\%}$, $CAD_{10\%}$ and $JPY_{10\%}$. This hypothesis is also rejected in favor of $\zeta < 4$ for $CAD_{5\%}$, $NOK_{10\%}$ and $SEK_{10\%}$ using Hill's estimation.

The conclusions that can be drawn from Table 1 on the existence of the second, third and fourth moments for the exchange rates of developed countries, are similar for both estimation approaches. All developed country exchange rates have finite variances. In addition, CHF, DKK, EUR, and, apparently, GBP and SEK have finite third moments; however, the fourth moments of these exchange rates may be infinite. In contrast, according to both approaches, AUD and, apparently, CAD have infinite fourth moments, and both may have infinite third moments. The third and fourth moments may be infinite for JPY and NOK. All in all, the results in Table 1 show that CHF, DKK and EUR exchange rates are less heavy-tailed than AUD, CAD, GBP, JPY, NOK and SEK exchange rates, with AUD and CAD being the most heavy-tailed. The latter set of exchange rates would appear to be subject to more extreme shocks.

The results in Table 2 for emerging countries exchange rates can be summarized as follows. The point estimates $\hat{\zeta}_{RS}$ lie between 2.1 and 4.1, and the point estimates $\hat{\zeta}_{Hill}$ lie between 1.9 and 3.5. In particular, for CNY, HKD, KRW, RUB, THB and TWD, the point estimates of both $\hat{\zeta}_{RS}$ and $\hat{\zeta}_{Hill}$ are less than 2.9. The null hypothesis $\zeta = 1$ is rejected in favor of $\zeta > 1$ for all emerging country exchange rates. The null hypothesis H_0: $\zeta = 2$ is not rejected for $CNY_{5\%}$, $CNY_{10\%}$, $HKD_{10\%}$, $KRW_{5\%}$ and $KRW_{10\%}$ using the log–log rank-size regression approach, and for $CNY_{10\%}$, $HKD_{5\%}$, $HKD_{10\%}$, $KRW_{5\%}$, $KRW_{10\%}$ and $RUB_{10\%}$ using Hill's estimation. Both approaches reject the null hypothesis H_0: $\zeta = 2$ in favor of H_a: $\zeta > 2$ for $INR_{5\%}$, $INR_{10\%}$, $MYR_{5\%}$, $MYR_{10\%}$, $RUB_{5\%}$, $SGD_{5\%}$, $SGD_{10\%}$, $THB_{5\%}$, $THB_{10\%}$, $TWD_{5\%}$ and $TWD_{10\%}$. The hypothesis is also rejected in favor of H_a: $\zeta > 2$ for $HKD_{5\%}$ and $RUB_{10\%}$ using the log–log rank-size regression approach and for $CNY_{5\%}$ using Hill's method. The log–log rank-size regression approach does not reject the hypothesis H_0: $\zeta = 3$ for $HKD_{5\%}$, $INR_{5\%}$, $INR_{10\%}$, $MYR_{10\%}$, $RUB_{5\%}$, $SGD_{5\%}$, $SGD_{10\%}$, $THB_{5\%}$, $THB_{10\%}$ and $TWD_{5\%}$, while the hypothesis H_0:$\zeta = 3$ is rejected in favor of H_a: $\zeta < 3$ for all other exchange rates in Table 2, except $MYR_{5\%}$, for which the hypothesis is rejected in favor of H_a: $\zeta > 3$. The hypothesis H_0:$\zeta = 3$ is also not rejected for $INR_{5\%}$, $MYR_{5\%}$, $SGD_{5\%}$, $SGD_{10\%}$ and $TWD_{5\%}$ using Hill's estimation, which also leads to the rejection of the hypothesis H_0: $\zeta = 3$ in favor of H_a: $\zeta < 3$ for all other exchange rates in Table 2. The null hypothesis H_0: $\zeta = 4$ is not rejected only for $SGD_{5\%}$ using the log–log rank-size regression approach, and for $MYR_{5\%}$ using both the estimation methods; it is rejected in favor of H_0: $\zeta < 4$ for all the other exchange rates.

The results in Table 2 thus imply that the first moments are finite for all emerging country exchange rates. The variance is finite for INR, MYR, SGD, THB, TWD and, apparently, for RUB. The variances may be infinite for CNY, HKD and KRW. CNY and KRW have infinite third moments. The same conclusion holds, apparently, for HKD and RUB. The third moments may also be infinite for INR, MYR, SGD, THB and TWD. The fourth moments may be infinite for MYR and are, apparently, infinite for SGD; they are infinite for all the remaining exchange rates in Table 2.

Summarizing the results in Tables 1 and 2, typically the tail indices of exchange rates in emerging countries are considerably smaller than those of developed economies. The heavy-tailedness properties of exchange rates of emerging countries are indeed more pronounced than those of their developed counter-parts. A key difference is that the exchange rates of developed countries appear to have finite variances, in contrast to the exchange rates of several emerging countries. Similarly, the third moments are infinite, or may be infinite, for most of the emerging country exchange rates, while they are finite for most developed country exchange rates.

These difference can be described in terms of the maximal order p of the finite moments as follows (see Appendix): For $p = 2.7$, while the null hypothesis $\zeta = p$ is rejected in favor of $\zeta > p$ for all developed country exchange rates in Table 1 except AUD, it is not rejected or is rejected in favor of $\zeta < p$ for all the emerging country exchange rates in Table 2 except $MYR_{5\%}$. Tail index values $\zeta = p \in (2.6, 2.8)$ are in some sense at the dividing boundary between those characteristic of developed and emerging countries:

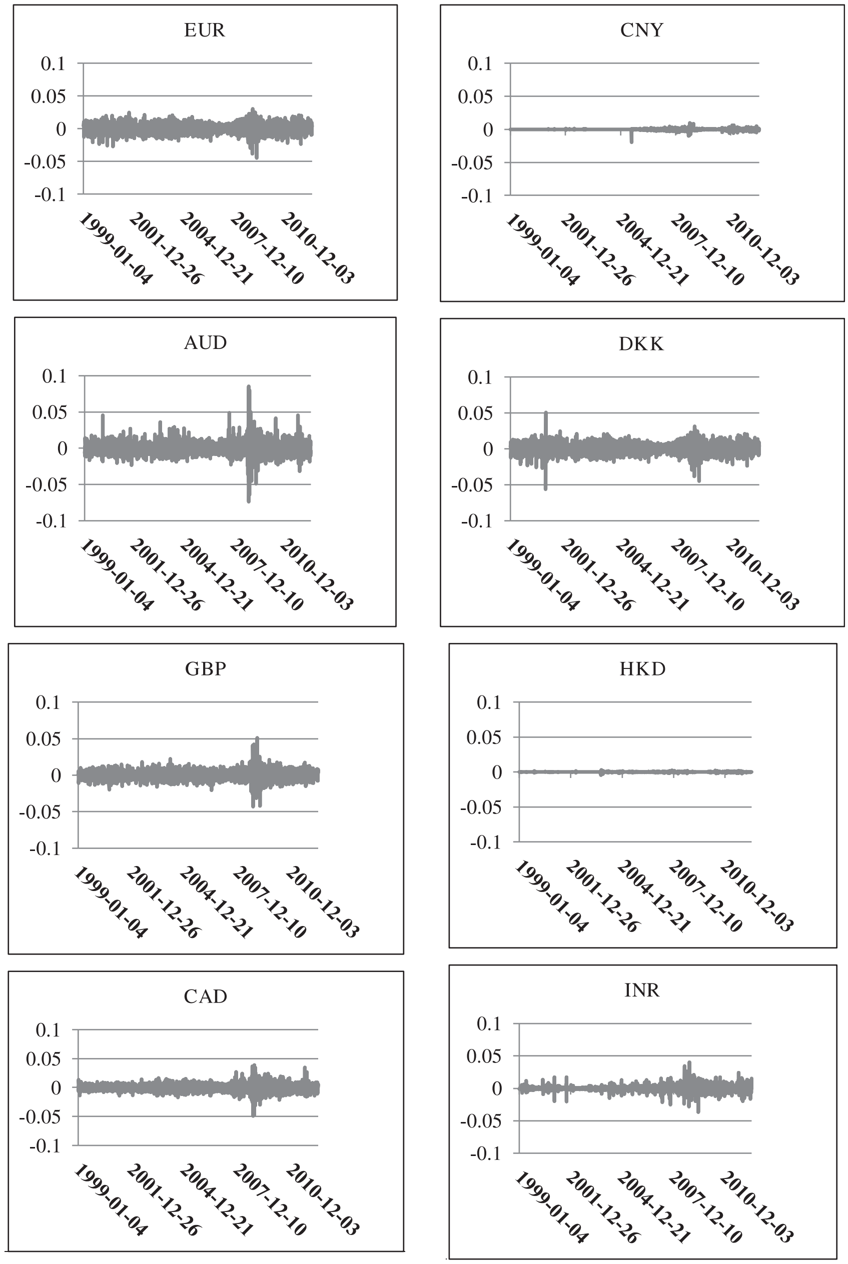

Fig. 1. The dynamics of the rate of growth of developed and emerging country exchange rates.

while the moments of order $p \in (2.6, 2.8)$ are finite for most of the developed country exchange rates, they may be infinite, or are infinite, for most of the emerging country exchange rates.

The conclusions for the developed and emerging country exchange rates with respect to CNY (Tables 3 and 4) are mostly similar to those for the exchange rates with respect to USD. Developed country exchange rates tend to be less heavy-tailed than emerging country exchange rates. While moments of order $p \in (2.6, 2.8)$ are finite for most of the developed countries, they may be, or are infinite for most of the emerging countries.

The tail indexes for signed returns in models (1) and (2) for developed and emerging countries are presented in Tables 5 and 6. The point estimates and confidence intervals are quite similar between models (1) and (2) for developed countries. The confidence intervals for upward and for downward moves intersect (Table 4), implying that these tail indices are statistically indistinguishable. These results support symmetry in heavy-tailedness between upward and downward moves, consistent with symmetry in volatility of foreign exchange markets reported in the literature (see the beginning of this section). It may however be noted that the downward moves of DKK, EUR, JPY, NOK and SEK appear to be somewhat more heavy-tailed than their upward moves (Table 5). There is some indication that heavy-tailedness is somewhat more pronounced in the upward moves of AUD, CHF and GBP exchange rates.

Symmetry between upward and downward moves in exchange rates is preserved among all the emerging countries considered (Table 6). Again, the confidence intervals for upward and downward moves intersect, and the tail indices are statistically indistinguishable. Exchange rates of HKD, INR and MYR are seen to have somewhat more pronounced heavy-tailedness in downward moves than in upward movements. Upward moves appear to be a little more heavy-tailed than downward moves for CNY and RUB. This latter type of asymmetry is the opposite of the gain/loss

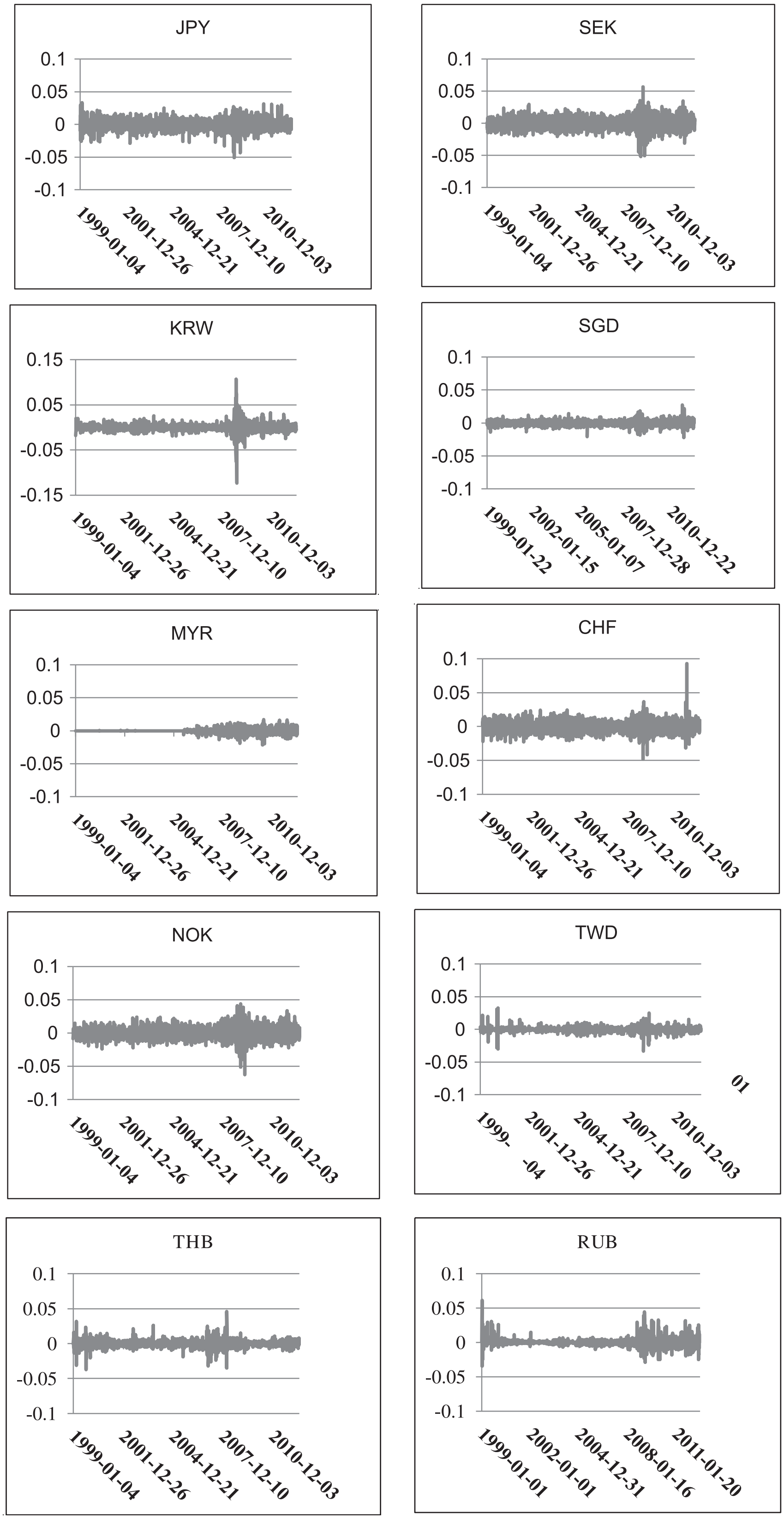

Fig. 1. (*continued*)

asymmetry found in financial markets, and may indicate regulatory interventions during large drawdowns in these currency markets.

Table 7 presents tail index estimates for USD, GBP and EUR calculated using the London Gold and Silver Fix prices in these currencies. The results are qualitatively similar to those in Table 1 for

Table 2
Tail index estimates for exchange rates of emerging countries (base currency: USD).

Currency	Truncation (%)	$\hat{\zeta}_{RS}$	$s.e._{RS} = \sqrt{\frac{2}{n}}\hat{\zeta}_{RS}$	$95\%CI_{RS}$ (Eq. (9))	$\hat{\zeta}_{Hill}$	$s.e._{Hill} = \sqrt{\frac{1}{n}}\hat{\zeta}_{Hill}$	$95\%CI_{Hill}$ (Eq. (6))
CNY	10	2.18	0.17	(1.85,2.51)	1.90	0.10	(1.70,2.10)
	5	2.40	0.26	(1.89,2.92)	2.36	0.18	(2.01,2.72)
HKD	10	2.25	0.17	(1.91,2.59)	1.91	0.10	(1.70,2.11)
	5	2.57	0.28	(2.02,3.11)	2.30	0.18	(1.95,2.64)
INR	10	2.86	0.22	(2.43,3.29)	2.52	0.14	(2.25,2.78)
	5	3.16	0.34	(2.49,3.83)	2.81	0.22	(2.39,3.23)
KRW	10	2.32	0.18	(1.97,2.66)	2.20	0.12	(1.97,2.44)
	5	2.36	0.26	(1.86,2.86)	2.23	0.17	(1.90,2.57)
MYR	10	3.25	0.25	(2.76,3.74)	2.56	0.14	(2.28,2.83)
	5	4.08	0.44	(3.21,4.94)	3.51	0.27	(2.98,4.03)
RUB	10	2.41	0.18	(2.04,2.77)	1.96	0.11	(1.75,2.17)
	5	2.81	0.31	(2.22,3.41)	2.57	0.20	(2.18,2.95)
SGD	10	3.12	0.24	(2.65,3.60)	2.84	0.15	(2.54,3.14)
	5	3.36	0.36	(2.65,4.07)	3.07	0.24	(2.61,3.53)
THB	10	2.66	0.20	(2.26,3.06)	2.37	0.13	(2.11,2.62)
	5	2.85	0.31	(2.25,3.46)	2.47	0.19	(2.10,2.85)
TWD	10	2.53	0.19	(2.15,2.91)	2.35	0.13	(2.10,2.60)
	5	2.54	0.28	(2.00,3.08)	2.77	0.21	(2.36,3.19)

Note. 1 January 1999–22 June 2012: $N = 3390$, $10\%N = 339$, $5\%N = 170$.

Table 3
Tail index estimates for exchange rates of developed countries (base currency: CNY).

Currency	Truncation (%)	$\hat{\zeta}_{RS}$	$s.e._{RS} = \sqrt{\frac{2}{n}}\hat{\zeta}_{RS}$	$95\%CI_{RS}$
AUD	10	2.83	0.22	(2.41,3.26)
	5	2.89	0.31	(2.28,3.50)
CAD	10	3.25	0.25	(2.76,3.74)
	5	3.50	0.38	(2.75,4.24)
CHF	10	3.90	0.30	(3.31,4.48)
	5	3.89	0.42	(3.07,4.72)
DKK	10	3.85	0.30	(3.27,4.43)
	5	3.99	0.43	(3.15,4.84)
EUR	10	4.21	0.32	(3.58,4.84)
	5	4.57	0.50	(3.60,5.55)
GBP	10	3.53	0.27	(3.00,4.07)
	5	3.38	0.37	(2.66,4.10)
JPY	10	3.32	0.25	(2.82,3.82)
	5	3.70	0.40	(2.91,4.48)
NOK	10	3.55	0.27	(3.02,4.09)
	5	3.91	0.42	(3.08,4.75)
SEK	10	3.62	0.28	(3.07,4.16)
	5	3.93	0.43	(3.09,4.76)

Note. 1 January 1999–22 June 2012: $N = 3390$, $10\%N = 339$, $5\%N = 170$.

developed countries, with the null hypothesis $\zeta = 2$ rejected in favor of $\zeta > 2$ (finite variances) for USD, GBP and EUR prices of Gold and Silver, both at the 5% and 10% truncation levels. However, in contrast to the estimates in Table 1, the 95% confidence intervals in Table 7 for the tail indices of GBP and EUR prices of Gold and Silver contain $\zeta = 3$, and so $\zeta = 3$ is not rejected. The null hypothesis $\zeta = 4$ is rejected in favor of $\zeta < 4$ for all the exchange rate time series considered in Table 7, except for the estimates calculated using Gold prices and 5% truncation levels. This suggests that the estimates in terms of Silver prices are more robust due to lower volatility. The estimates obtained for longer exchange rate time series confirm the robustness of the above conclusions.

Tables 8 and 9 present estimation results pertaining to the effects of the on-going economic and financial crisis on the heavy-tailedness of exchange rates. Developed country exchange rates appear to have become more pronouncedly heavy-tailed since the beginning of the crisis (Table 8). The crisis-period confidence intervals for the tail indices of $AUD_{10\%}$, $CHF_{5\%}$, $CHF_{10\%}$, $GBP_{5\%}$ and $GBP_{10\%}$ lie to the left of their pre-crisis confidence intervals. This points to structural breaks and statistically significant decreases in the tail indices of these exchange rates after the beginning of the crisis in 2008 that correspond to the increase in the degree of their heavy-tailedness and the likelihood of large fluctuations. The pre- and post-crisis confidence intervals in Table 8 for other currencies intersect implying that the tail indices of these currencies before and after the beginning of the crisis are statistically indistinguishable from each other. This is also true of emerging country currencies in Table 9 with two notable exceptions: for *RUB*, and for $MYR_{10\%}$, the post-crisis confidence intervals lie to right of the pre-crisis confidence intervals. Heavy-tailedness properties

Table 4
Tail index estimates for exchange rates of emerging countries (base currency: CNY)

Currency	Truncation (%)	$\hat{\zeta}_{RS}$	$s.e._{RS} = \sqrt{\frac{2}{n}}\hat{\zeta}_{RS}$	$95\%CI_{RS}$
HKD	10	2.30	0.18	(1.96,2.65)
	5	2.45	0.27	(1.93,2.97)
INR	10	2.83	0.22	(2.40,3.25)
	5	3.24	0.35	(2.55,3.93)
KRW	10	2.27	0.17	(1.93,2.62)
	5	2.33	0.25	(1.84,2.83)
MYR	10	3.23	0.25	(2.75,3.72)
	5	3.99	0.43	(3.15,4.84)
SGD	10	3.03	0.23	(2.57,3.48)
	5	3.25	0.35	(2.56,3.94)
THB	10	2.64	0.20	(2.24,3.03)
	5	2.89	0.31	(2.28,3.51)
TWD	10	2.49	0.19	(2.12,2.87)
	5	2.49	0.27	(1.96,3.02)

Note. 1 January 1999–22 June 2012: $N = 3390$, $10\%N = 339$, $5\%N = 170$.

Table 5
Tail index estimates for upward (+) and downward (−) movements in exchange rates of developed countries (base currency: USD).

Currency	N	Truncation (%)	n	$\hat{\zeta}_{RS}$	$s.e._{RS} = \sqrt{\frac{2}{n}}\hat{\zeta}_{RS}$	$95\%CI_{RS}$
AUD (+)	1573	10	157	2.64	0.30	(2.06,3.23)
		5	79	2.59	0.41	(1.78,3.39)
AUD (−)	1777	10	178	3.04	0.32	(2.41,3.68)
		5	89	2.93	0.44	(2.07,3.78)
CAD (+)	1642	10	164	3.20	0.35	(2.51,3.90)
		5	82	3.50	0.55	(2.43,4.57)
CAD (−)	1713	10	171	3.28	0.35	(2.58,3.97)
		5	86	3.34	0.51	(2.34,4.34)
CHF (+)	1687	10	169	3.63	0.40	(2.86,4.41)
		5	84	3.50	0.54	(2.44,4.56)
CHF (−)	1684	10	168	4.17	0.45	(3.28,5.06)
		5	84	4.23	0.65	(2.95,5.51)
DKK (+)	1681	10	168	4.32	0.47	(3.40,5.25)
		5	84	4.68	0.72	(3.26,6.09)
DKK (−)	1693	10	169	3.47	0.38	(2.73,4.21)
		5	85	3.43	0.53	(2.40,4.46)
EUR (+)	1687	10	169	4.70	0.51	(3.70,5.71)
		5	84	5.58	0.86	(3.89,7.27)
EUR (−)	1662	10	166	3.83	0.42	(3.01,4.66)
		5	83	3.85	0.60	(2.68,5.02)
GBP (+)	1646	10	165	3.29	0.36	(2.58,4.01)
		5	82	3.20	0.50	(2.22,4.17)
GBP (−)	1689	10	169	3.88	0.42	(3.05,4.71)
		5	84	3.53	0.55	(2.47,4.60)
JPY (+)	1654	10	165	3.51	0.39	(2.75,4.27)
		5	83	4.07	0.63	(2.83,5.31)
JPY (−)	1713	10	171	3.24	0.35	(2.55,3.93)
		5	86	3.37	0.51	(2.37,4.38)
NOK (+)	1659	10	166	3.59	0.39	(2.81,4.36)
		5	83	4.33	0.67	(3.01,5.65)
NOK (−)	1710	10	171	3.57	0.39	(2.81,4.32)
		5	86	3.51	0.54	(2.46,4.56)
SEK (+)	1672	10	167	3.81	0.42	(3.00,4.63)
		5	84	4.76	0.73	(3.32,6.20)
SEK (−)	1707	10	171	3.43	0.37	(2.70,4.16)
		5	85	3.34	0.51	(2.34,4.34)

Note. 1 January 1999–22 June 2012: N and $n = 10\%N$, $5\%N$ are provided in the table.

have become less pronounced for these currencies, suggesting a corresponding decrease in the likelihood of large fluctuations in their exchange rates.

In order to illustrate the appropriateness of the tail truncation levels (5% and 10%) used in this section, we follow the analysis and suggestions in Embrechts et al. (1997) and Mikosch and Stărică

Table 6
Tail index estimates for upward (+) and downward (−) movements in exchange rates of emerging countries (base currency: USD).

Currency	N	Truncation (%)	n	$\hat{\zeta}_{RS}$	$s.e._{RS} = \sqrt{\frac{2}{n}}\hat{\zeta}_{RS}$	$95\%CI_{RS}$
CNY (+)	1259	10	126	2.02	0.25	(1.52,2.51)
		5	63	2.26	0.40	(1.47,3.05)
CNY (−)	1462	10	146	2.39	0.28	(1.84,2.94)
		5	73	2.38	0.39	(1.60,3.15)
HKD (+)	1616	10	162	2.52	0.28	(1.97,3.06)
		5	81	2.80	0.44	(1.94,3.66)
HKD (−)	1404	10	140	2.19	0.26	(1.67,2.70)
		5	70	2.53	0.43	(1.69,3.36)
INR (+)	1495	10	150	3.00	0.35	(2.32,3.68)
		5	75	3.31	0.54	(2.25,4.36)
INR (−)	1533	10	153	2.77	0.32	(2.15,3.39)
		5	77	3.04	0.49	(2.08,4.00)
KRW (+)	1518	10	152	2.30	0.26	(1.78,2.81)
		5	76	2.43	0.39	(1.66,3.20)
KRW (−)	1717	10	172	2.32	0.25	(1.83,2.81)
		5	86	2.28	0.35	(1.60,2.96)
MYR (+)	802	10	80	4.60	0.73	(3.18,6.03)
		5	40	5.42	1.21	(3.04,7.79)
MYR (−)	850	10	85	3.63	0.56	(2.54,4.72)
RUB (+)	1494	10	149	2.42	0.28	(1.87,2.98)
		5	75	2.96	0.48	(2.01,3.91)
RUB (−)	1672	10	167	2.81	0.31	(2.21,3.42)
		5	84	3.21	0.50	(2.24,4.18)
SGD (+)	1622	10	162	3.12	0.35	(2.44,3.80)
		5	81	3.35	0.53	(2.32,4.39)
SGD (−)	1720	10	172	3.10	0.33	(2.45,3.76)
		5	86	3.30	0.50	(2.31,4.28)
THB (+)	1559	10	156	2.66	0.30	(2.07,3.25)
		5	78	3.03	0.48	(2.08,3.98)
THB (−)	1659	10	166	2.65	0.29	(2.08,3.22)
		5	83	2.70	0.42	(1.88,3.52)
TWD (+)	1449	10	145	2.55	0.30	(1.96,3.13)
		5	72	2.59	0.43	(1.74,3.43)
TWD (−)	1478	10	148	2.53	0.29	(1.96,3.11)
		5	74	2.37	0.39	(1.61,3.14)

Note. 1 January 1999–22 June 2012: N and n = 10%N, 5%N are provided in the table.

Table 7
Tail index estimates for USD, GBP and EUR in terms of Gold and Silver prices.

Currency	Truncation (%)	$\hat{\zeta}_{RS}$	$s.e._{RS} = \sqrt{\frac{2}{n}}\hat{\zeta}_{RS}$	$95\%CI_{RS}$
USD/Gold	10	3.23	0.25	(2.74,3.72)
	5	3.56	0.39	(2.80,4.32)
USD/Silver	10	2.85	0.22	(2.42,3.28)
	5	3.16	0.34	(2.49,3.83)
GBP/Gold	10	3.09	0.24	(2.62,3.55)
	5	3.32	0.36	(2.62,4.03)
GBP/Silver	10	2.87	0.22	(2.44,3.30)
	5	3.04	0.33	(2.39,3.68)
EUR/Gold	10	3.15	0.24	(2.68,3.63)
	5	3.49	0.38	(2.75,4.23)
EUR/Silver	10	2.84	0.22	(2.41,3.26)
	5	2.94	0.32	(2.31,3.56)

Note. 1 January 1999–22 June 2012: N = 3390, 10%N = 339, 5%N = 170.

(2000), and present the analogues of Hill's plots for the log–log rank-size regression tail index estimates for EUR, GBP and RUB (Figs. 2–4). These are graphs of the log–log rank-size regression point estimates $\hat{\zeta}_{RS}$ of the tail indices for the currencies' exchange rates, for different values of the truncation levels n for extreme observations, together with the corresponding 95%-confidence

Table 8
Tail index estimates for exchange rates of developed countries before and after the beginning of the on-going crisis in 2008 (base currency: USD).

Currency	Truncation (%)	4 January 1999 –15 September 2008			15 September 2008–22 June 2012		
		$\hat{\zeta}_{RS}$	$s.e._{RS} = \sqrt{\frac{2}{n}}\hat{\zeta}_{RS}$	$95\%CI_{RS}$	$\hat{\zeta}_{RS}$	$s.e._{RS} = \sqrt{\frac{2}{n}}\hat{\zeta}_{RS}$	$95\%CI_{RS}$
AUD	10	3.91	0.35	(3.22,4.61)	2.42	0.35	(1.73,3.10)
	5	3.99	0.51	(2.99,4.99)	2.54	0.52	(1.51,3.57)
CAD	10	4.48	0.41	(3.69,5.28)	3.19	0.46	(2.28,4.10)
	5	5.02	0.64	(3.76,6.28)	3.36	0.69	(2.00,4.73)
CHF	10	4.89	0.44	(4.02,5.75)	3.04	0.44	(2.18,3.91)
	5	5.84	0.75	(4.37,7.30)	2.78	0.57	(1.66,3.90)
DKK	10	4.07	0.37	(3.34,4.79)	3.84	0.56	(2.75,4.93)
	5	3.84	0.56	(2.75,4.93)	4.19	0.86	(2.49,5.88)
EUR	10	4.97	0.45	(4.09,5.85)	3.87	0.56	(2.77,4.98)
	5	5.67	0.73	(4.25,7.10)	4.24	0.87	(2.52,5.95)
GBP	10	5.43	0.49	(4.47,6.40)	2.86	0.42	(2.05,3.67)
	5	6.28	0.80	(4.71,7.86)	2.89	0.60	(1.72,4.06)
JPY	10	3.80	0.34	(3.12,4.47)	2.90	0.42	(2.07,3.72)
	5	4.26	0.55	(3.19,5.33)	3.37	0.70	(2.01,4.73)
NOK	10	4.65	0.42	(3.83,5.48)	3.54	0.51	(2.53,4.54)
	5	5.49	0.70	(4.11,6.86)	4.15	0.86	(2.47,5.83)
SEK	10	4.76	0.43	(3.92,5.61)	3.80	0.55	(2.72,4.88)
	5	6.23	0.80	(4.67,7.79)	3.75	0.77	(2.23,5.26)

Note. 1 January 1999–15 September 2008: $N = 2444$, $10\%N = 244$, $5\%N = 122$; 15 September 2008–22 June 2012: $N = 947$, $10\%N = 95$, $5\%N = 47$.

Table 9
Tail index estimates for exchange rates of emerging countries before and after the beginning of the on-going crisis in 2008 (base currency: USD).

Currency	Truncation (%)	4 January 1999 –15 September 2008			15 September 2008–22 June 2012		
		$\hat{\zeta}_{RS}$	$s.e._{RS} = \sqrt{\frac{2}{n}}\hat{\zeta}_{RS}$	$95\%CI_{RS}$	$\hat{\zeta}_{RS}$	$s.e._{RS} = \sqrt{\frac{2}{n}}\hat{\zeta}_{RS}$	$95\%CI_{RS}$
CNY	10	1.99	0.18	(1.64,2.35)	2.46	0.36	(1.76,3.15)
	5	2.20	0.28	(1.65,2.76)	2.60	0.54	(1.55,3.65)
HKD	10	2.05	0.19	(1.69,2.42)	2.69	0.39	(1.93,3.46)
	5	2.31	0.30	(1.73,2.89)	2.89	0.60	(1.72,4.06)
INR	10	2.45	0.22	(2.02,2.89)	3.41	0.49	(2.44,4.38)
	5	2.72	0.35	(2.04,3.40)	3.83	0.79	(2.28,5.37)
KRW	10	3.26	0.30	(2.68,3.84)	2.26	0.33	(1.62,2.90)
	5	3.93	0.50	(2.94,4.92)	2.44	0.50	(1.45,3.42)
MYR	10	2.28	0.21	(1.88,2.69)	4.01	0.58	(2.87,5.15)
	5	2.99	0.38	(2.24,3.74)	4.82	0.99	(2.87,6.77)
SGD	10	3.92	0.36	(3.23,4.62)	3.14	0.46	(2.25,4.03)
	5	4.10	0.52	(3.07,5.12)	3.62	0.75	(2.15,5.08)
THB	10	2.62	0.24	(2.15,3.08)	4.07	0.59	(2.91,5.22)
	5	2.95	0.38	(2.21,3.69)	5.09	1.05	(3.03,7.15)
RUB	10	1.87	0.17	(1.54,2.20)	3.32	0.48	(2.38,4.27)
	5	1.84	0.24	(1.38,2.31)	4.27	0.88	(2.54,6.00)
TWD	10	2.50	0.23	(2.06,2.95)	2.46	0.36	(1.76,3.16)
	5	2.61	0.33	(1.95,3.26)	2.30	0.47	(1.37,3.22)

Note. 1 January 1999–15 September 2008: $N = 2444$, $10\%N = 244$, $5\%N = 122$; 15 September 2008–22 June 2012: $N = 947$, $10\%N = 95$, $5\%N = 47$.

intervals $95\%CI_{RS}$ in (9) for the true tail index values computed using log–log rank-size regressions. The figures highlight the relative stability of the point estimates across truncation levels. In particular, we note the $95\%CI_{RS}$ confidence intervals constructed for different tail truncation levels intersect. This shows that the tail indices in power law approximations of the tails of distributions of the exchange rates are statistical indistinguishable for different tail truncation levels. In addition, according to confidence intervals in Figs. 2–4, the qualitative conclusions in this section on (in) finiteness of second, third and fourth moments for the exchange rates remain unchanged regardless of the choice of truncation levels for extreme observations used in estimation.

4. Implications for economic models and policy decisions

Heavy-tailedness has crucial implications for the robustness of many economic models, leading, in a number of settings, to reversals of conclusions drawn from them. The finding that the exchange rates of all countries considered in this paper have tail indices greater than one is reassuring. Ibragimov (2009a,b) show that the stylized facts on optimality of diversification are robust to heavy-tailedness of risks or returns in value at risk models as long as the distributions of these risks or returns are moderately heavy-tailed with tail indices $\zeta > 1$ and have finite means. However, the stylized fact that portfolio diversification is preferable is reversed when tail indices are less than one and first moments are infinite. A similar conclusion holds for the efficiency property of the sample mean, as shown in Ibragimov (2007) (see also the discussion in Ibragimov (2009a): the sample mean is the best linear unbiased estimator of the population mean in the sense of peakedness properties for moderately heavy-tailed populations with tail indices $\zeta > 1$. In addition, for such populations, the sample mean exhibits the property of monotone consistency, and thus an increase in the sample size always improves its performance. However the efficiency of the sample mean in the sense of its peakedness in estimating the population center decreases with sample size under extreme heavy-tailedness, when tail indices $\zeta < 1$.

Our estimates indicate that the tail indices may be less than two for exchange rates of several emerging countries. This presents a challenge to the applicability of standard statistical and econometric methods. In particular, as pointed out by Granger and Orr (1972) and a number of recent studies (see, among others, Ch. 7 in Embrechts et al. (1997), and references therein), many classical approaches to inference based on variances and (auto) correlations, such as regression and spectral analysis, least squares methods and autoregressive models, may not apply directly in the case of heavy-tailed observations with infinite second or higher moments. Our results imply that traditional econometric and statistical methods should be applied with care in studies of exchange rates for many emerging and developing countries. This is especially important in cases where tail indices are close to critical robustness boundary ($\zeta = 1$), or close to the threshold value for applicability of standard inference methods ($\zeta = 2$).

The results obtained are pertinent to economic policy making and macroeconomic forecasting. The finding that exchange rates of emerging and developing countries are typically more heavy-tailed than those of their developed counter-parts may reflect their susceptibility to more frequent and extreme external and internal shocks. Secondly, estimated tail indices for the exchange rates of emerging and developing countries can be used to forecast the patterns in their future development, as they converge to distributions with tail indices $\zeta \in (2, 4)$ implied by empirical results and theoretical models in the literature for financial returns and exchange rates of developed country currencies (see Gabaix et al. (2006)), Section 1.1 and references therein). Third, our results suggest modification of structural models of heavy tailedness to suit emerging and developing countries exchange rates. For example, Gabaix et al. (2006) proposes a model where the tail index value $\zeta \approx 3$ for financial returns in developed economies is generated by the trading actions of market participants who have a size distribution with tail indices $\zeta = 1$ (Zipf's law). Due to government intervention and regulation, it seems likely that the tail indices may be less than $\zeta = 1$ for the participants in emerging and developing foreign exchange markets (implying deviations from the Zipf's law), with the states being some of the largest key traders. It is also likely that price impacts of trading volume differ between emerging and developed country currency markets. It will be interesting and important to provide estimates of these characteristics for different countries.[9]

[9] In most developing and emerging countries, the respective central banks tend to be large market participants who hold, relative to the scale of global trading in their currencies, substantial international reserves. They are fairly placed to intervene credibly and effectively (in terms of volume traded) to prevent appreciation or depreciation as policy directs. Indeed devaluing currencies artificially has become a global strategy. Bernanke (2011) identified Taiwan, Singapore and Thailand as aggressively trying to hold their currencies down, while India, Chile and Turkey were not seen to be engaging in "currency manipulation". Typically, central banks intervene more readily to prevent appreciation of the domestic currency. Hence, in addition to increases in heavy-tailedness, interventionist policy could also generate asymmetry in exchange rate returns tail behavior.

Finally, our tail index estimates can be used, together with estimates of the constant in log–log rank-size regressions and the implied estimates of the scaling constants in heavy-tailed models (3), for evaluation of commonly used risk measures such as the value at risk and expected shortfall relatively far in the tails of exchange rate distributions (see Appendix).

5. Conclusion and suggestions for further research

The results of our robust, comparative, assessment of heavy-tailedness properties of exchange rates indicate that the exchange rates of emerging and developing countries are typically more heavy-tailed than those of developed countries. The implication that several emerging countries may have infinite variance exchange rate distributions highlight the need for care in applying economic and financial models, diversification analysis, and econometric and statistical inference methods.

Extending the list of currencies covered by our analysis to those of Latin American, Middle East and African countries will be of much interest. It should be revealing to examine tail index estimates for a larger number of cross exchange rates, relative to countries' largest trading partners, and relative to neighbors in the same geographical region, e.g., Asia, Latin America. It is likely that heavy-tailedness may be less marked, due to similarity of underlying economic fundamentals and due to trade flow concentration.

Developing structural models that explain the presence of heavy tails in emerging country exchange rates and the factors (including macroeconomic and institutional variables) that affect them would be an important direction for further research (see Gabaix et al., 2006, and the discussion in Section 4). This will require estimates of tail indices for size distributions of market participants, and of trading volume distributions. Cross-country analysis of relationships between tail index estimates and macro economic variables that are proximate to government intervention in foreign exchange markets should be interesting. Similar estimates for export and import volumes and their concentration across industries and trade partners are also important.

Application of other estimation and inference approaches, including the methods for adaptive selection of the tail sample fraction used in inference on heavy-tailedness (see Section 4.7 in Beirlant et al. (2004)), small sample analogues of Hill's estimates developed in Huisman et al. (2001) and other robust econometric and statistical procedures under heavy tails such as the *t*-statistic based robust inference methods proposed in Ibragimov and Müller (2010) to this research issue would be of interest. Further analysis of structural breaks in the heavy-tailedness of exchange rates, using for example, tests developed in Quintos et al. (2001) will be useful. Going beyond the on-going economic and financial crisis, it would be of interest to study the impacts of changes in currency regimes, and of other shocks (such as the 1998 Russian financial crisis).

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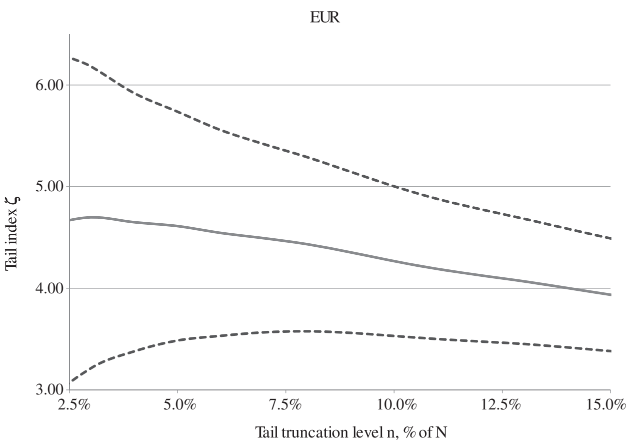

Fig. 2. Log–log rank-size estimates and 95% confidence intervals for EUR tail index with different tail truncation levels.

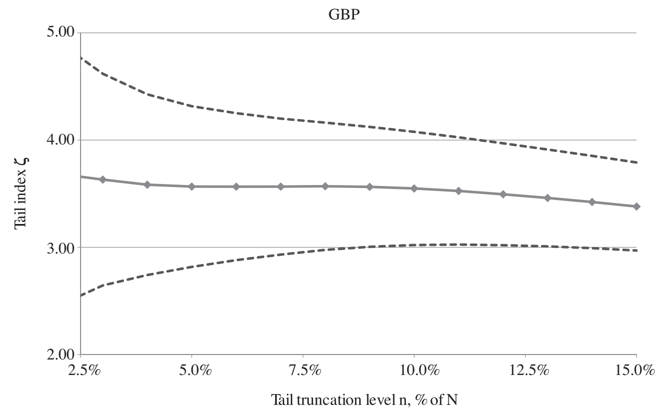

Fig. 3. Log–log rank-size estimates and 95% confidence intervals for GBP tail index with different tail truncation levels.

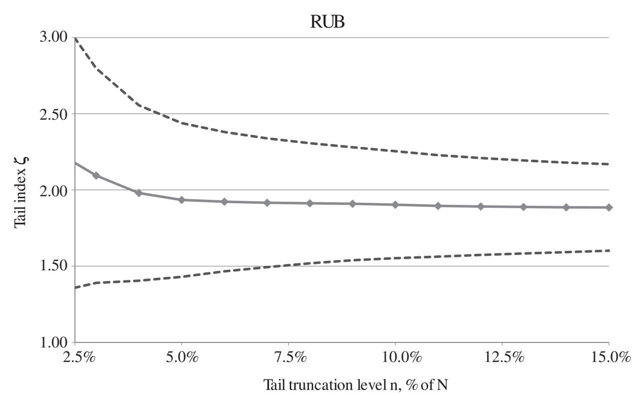

Fig. 4. Log–log rank-size estimates and 95% confidence intervals for RUB tail index with different tail truncation levels.

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Appendix A. Heavy-tailed power law distributions: Main properties and inference

An important property of risks or returns r satisfying heavy-tailed power laws (1)–(3) is that the absolute moments of r are finite if and only if their order is less than $\zeta = \min(\zeta_1, \zeta_2)$: $E|r|^p < \infty$ if $p < \zeta$ and $E|r|^p = \infty$ if $p \geqslant \zeta$. Thus returns r that follow (1)–(3) have finite second moments $Er^2 < \infty$ (and therefore, well-defined finite variances) if and only if $\zeta > 2$. The first absolute moment of r in (1)–(3) is finite ($E|r| < \infty$) if and only if $\zeta > 1$.

Examples of power laws (1) are given by stable and Student-t distributions and also by Singh–Maddala families and Pareto distributions with equality in (1) for all $x > x_0$ (widely used in income distribution modeling, see the discussion and references in Cowell and Flachaire (2007), and Davidson and Flachaire (2007), and Ibragimov (2009a). Important classes of heavy-tailed time series with power law distributions are provided by GARCH and stochastic volatility processes, that are used in models of a number of economic and financial variables, including financial returns and exchange rates (see, among others, Ch. 12 in Campbell et al. (1997), Cont (2001) and references therein).[10,11]

The empirical literature indicates that, in addition to financial returns and exchange rates, heavy-tailed power law behavior is also exhibited by variables such as income and wealth (with $\zeta \in (1.5, 3)$ and $\zeta \approx 1.5$, respectively; see, among others, (Gabaix, 2009), and references therein); city sizes and firm sizes ($\zeta \approx 1$, see Gabaix, 1999; and Axtell, 2001); financial returns from technological innovations, losses from operational risks and those from earthquakes and other natural disasters (with tail indices that can be considerably less than one, see Nešlehova et al., 2006; Silverberg and Verspagen, 2007, and Ibragimov et al., 2009).

To our knowledge there are very few studies on heavy-tailedness properties in emerging and developing economies. Akgiray et al. (1988) focus on maximum likelihood estimation of parametric stable and Generalized Pareto power law families fitted to monthly observations for a number of Latin American exchange rates. The confidence intervals obtained using Hill's estimator suggest that the variances and even first moments of these time series may be infinite (see also Fofack and Nolan, 2001, for maximum likelihood estimates for infinite variance stable distributions fitted to different exchange rates). Koedijk et al. (1992) estimate tail indices for Latin American exchange rates and find evidence for different tail behavior in exchange rate returns under different exchange rate regimes. The analysis in Akgiray et al. (1988) and Koedijk et al. (1992) are based on relatively small samples of monthly observations and therefore have wide confidence intervals. Quintos et al. (2001) develop tests for structural breaks in the tail index, and motivated by the Asian financial crisis, apply these tests to emerging Asian stock prices. Using extensions of tests in Quintos et al. (2001) to allow for multiple tail index breaks, Candelon and Straetmans (2006) focus on changes in the tail indices of six emerging Asian currencies (Indonesian Rupiah, Malaysian Ringgit, Thai Baht, Philippine Peso, South Korean Won and Pakistan Rupee) and five developed currencies (Japanese Yen, British Pound, Swiss Frank, Canadian Dollar and German Mark) over the period from the beginning of 1994 to the middle of 2003.[12] The empirical results in Candelon and Straetmans

[10] It is important to emphasize that, as discussed in Gabaix and Ibragimov (2011), the inference methods employed in this paper perform well under GARCH data-generating processes.

[11] Concerning the latter time series classes, it is important to note that while the concepts of volatility and heavy-tailedness are related, they refer to different properties of random variables and time series. Namely, in view of definition (3), heavy-tailedness (tail index ζ) of the variable r (e.g., risk, financial return or an exchange rate) governs the likelihood of observing extreme fluctuations in the variable, for instance, during crisis periods. The smaller values of the tail index ζ correspond to a higher degree of heavy-tailedness in r and, thus, to a larger likelihood of observing outliers and extreme fluctuations in the realizations of this variable. Heavy-tailedness properties thus may be intuitively thought of as high volatility in extreme observations of the variable dealt with. Such processes as GARCH model the important stylized fact of volatility clustering (see, among others, Cont, 2001) in economic and financial time series (e.g., financial returns or exchange rates) and, in addition, their stationary solutions exhibit the stylized fact of heavy tails.

[12] See also (Payaslioglu, 2009), for applications of tests in Quintos et al. (2001) in the analysis of structural breaks in the tail index of the exchange rate in Turkey over periods with different foreign exchange regimes.

(2006) point to statistically significant changes in the tail indices of exchange rates of most of the above emerging currencies over the 1997 Asian crisis period (with tail index drops corresponding to increases in the degrees of heavy-tailedness). In a number of cases the tail index breaks can be linked to changes in monetary and exchange rate policies. In contrast, statistically significant breakpoints are not observed in the tail indices of exchange rates of the developed countries. The estimates for (relatively small) samples of quarterly data on exchange rates in Asian, Latin American and European economies in Pozo and Amuedo-Dorantes (2003) produce confidence intervals that indicate that the variances of the time series considered may be infinite.

Several approaches to inference about the tail index ζ of heavy-tailed distributions are available in the literature (see, among others, the reviews in Embrechts et al. (1997), and Beirlant et al. (2004). The two most commonly used, are Hill's estimator and the OLS approach using the log–log rank-size regression.

Let $r_1, r_2, \ldots, r_N$ be a sample from a population satisfying power law (3). Further, let, for $n < N$,

$$|r|_{(1)} \geqslant |r|_{(2)} \geqslant \cdots \geqslant |r|_{(n)} \tag{4}$$

be decreasingly ordered largest absolute values of observations in the sample. Hill's estimator $\hat{\zeta}_{Hill}$ of the tail index ζ is given by

$$\hat{\zeta}_{Hill} = \frac{n}{\sum_{t=1}^{n}(\log|r|_{(t)} - \log|r|_{(n+1)})}. \tag{5}$$

The standard error of the estimator is $s.e._{Hill} = \frac{1}{\sqrt{n}}\hat{\zeta}_{Hill}$. The corresponding 95%-confidence interval for the true tail index ζ is thus given by

$$\left(\hat{\zeta}_{Hill} - \frac{1.96}{\sqrt{n}}\hat{\zeta}_{Hill}, \hat{\zeta}_{Hill} + \frac{1.96}{\sqrt{n}}\hat{\zeta}_{Hill}\right). \tag{6}$$

A number of studies have concluded that inference on the tail index using Hill's estimator suffers from several problems, including sensitivity to dependence and small sample sizes (see, among others, Embrechts et al., 1997 Chapter 6). Motivated by this, several studies have focused on alternative approaches to tail index estimation. For instance, Huisman et al. (2001) propose a weighted analogue of Hill's estimator that was reported to correct its small sample bias for sample sizes less than 1000. Embrechts et al. (1997), among others, advocate sophisticated non-linear procedures for tail index estimation.

Despite the availability of more sophisticated methods, a popular way to estimate the tail index ζ is still to run the following OLS log–log rank-size regression with $\gamma = 0$:

$$\log(t-\gamma) = a - b\log|r|_{(t)}, \tag{7}$$

$t = 1, \ldots, n$, or (calling t the rank of an observation, and $|r|_{(t)}$ its size): log (Rank − γ) = $a - b$ log (Size) (here and throughout the paper, log $(\cdot)$ stands for the natural logarithm). The estimate of the tail index is given by b. Similar log–log rank-size regressions applied to positive and negative observations r_t in the sample are employed to estimate the tail indices ζ_1 and ζ_2 in (1) and (2). The reason for the popularity of the OLS approaches to tail index estimation is arguably the simplicity and the robustness of these methods. The log–log rank-size regressions of form (7) in the case $\gamma = 0$ and closely related procedures have been employed in various frameworks, in particular, in Levy (2003), Levy and Levy, 2003, Helpman et al. (2004)), and many other works (see also the review and references in Gabaix and Ibragimov, 2011).

Unfortunately, tail index estimation procedures based on OLS log–log rank-size regressions (7) with $\gamma = 0$ are strongly biased in small samples. The recent study by Gabaix and Ibragimov (2011) provides a simple practical remedy for this bias, and argues that if one wants to use an OLS regression, one should use the Rank − 1/2, and run log (Rank − 1/2) = $a - b$log (Size), that is,

$$\log(t-1/2) = a - b\log|r|_{(t)}, \tag{8}$$

$t = 1, \ldots, n$. In (8), one takes the OLS estimate $\hat{b}$ as the log–log rank-size regression estimate $\hat{\zeta}_{RS}$ of the tail index ζ. The shift of 1/2 is optimal, and reduces the bias to a leading order. The standard error of the estimator $\hat{\zeta}_{RS}$ is $s.e._{RS} = \sqrt{\frac{2}{n}}\hat{\zeta}_{RS}$ (the standard error is thus different from the OLS standard error).[13] The corresponding 95% confidence interval for ζ (denoted by 95%CI_{RS} in estimation results in the paper) is

$$\left(\hat{\zeta}_{RS} - 1.96 \times \sqrt{\frac{2}{n}}\hat{\zeta}_{RS}, \hat{\zeta}_{RS} + 1.96 \times \sqrt{\frac{2}{n}}\hat{\zeta}_{RS}\right). \tag{9}$$

Numerical results in Gabaix and Ibragimov (2011) demonstrate the advantage of the proposed approach over the standard OLS estimation procedures (7) with $\gamma = 0$ and indicate that it performs well under deviations from power laws and heavy-tailed dependent GARCH processes that are often used for modeling exchange rates and other economic and financial variables. The modifications of the OLS log–log rank-size regressions with the optimal shift and the correct standard errors provided by Gabaix and Ibragimov (2011) have subsequently been used in Hinloopen and Marrewijk (2006), Bosker et al. (2007, 2008), Gabaix and Landier (2008), Ioannides et al. (2008), Le Gallo and Chasco (2008), Zhang et al. (2009) and several other works. Due to inherent heterogeneity and dependence properties and data availability constraints, exchange rates in emerging and developing countries provide natural areas for applications of robust inference methods as discussed in Gabaix and Ibragimov (2011). These methods provide the main tools used in the empirical analysis reported in this paper.

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[13] Similar to the analysis in Gabaix and Ibragimov (2011), one can also obtain results on the standard error of the estimate a of the constant term in the above log–log rank-size regression (evidently, a is an estimate of the logarithm of the scaling constant C in heavy-tailed population model (3)). As discussed in Section 1.1, together with estimates and standard errors on the tail index ζ, these results can be used in calculating loss exceedance probabilities and in assessing commonly used risk measures such as the value at risk and expected shortfall relatively far in the tails of the heavy-tailed distributions considered.

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