The University of Chicago, Booth School of Business
Business 41202, Spring Quarter 2016, Mr. Ruey S. Tsay

Solutions to Final Exam

Problem A: (40 points) Answer briefly the following questions.

1. Give two advantages of using the sub-sampling method over the method of optimal sampling interval in computing (daily) realized volatility of a stock.

   A: Mitigate the impact of market miscrostructure and (b) use more data.

2. Assume that x_t follows the stochastic diffusion equation $dx_t = \mu dt + \sigma dw_t$, where w_t is the Wiener process. Let $G(x_t) = \exp(x_t/2)$. Derive the stochastic diffusion equation for $G(x_t)$.

   A: $dG(x_t) = (\mu/2 + \sigma^2/8)G(x_t)dt + (\sigma/2)G(x_t)dw_t$ via Ito's lemma.

3. Give two methods that can be used to specify the order of an autoregressive time series.

   A: (a) Partial autocorrelation function (PACF) and (b) Information criteria.

4. Describe two methods discussed in class that can be used to predict the direction of a price movement (up or down).

   A: (a) Logistic regression and (b) Neural Network

5. Describe two methods that can be used to obtain time-varying correlations between two asset returns.

   A: (a) Bivariate volatility models and (b) use $\text{cov}(x, y) = [\text{var}(x + y) - \text{var}(x - y)]/4$.

6. Give two reasons that the observed log returns of an asset follow an MA(1) model.

   A: (a) Bid and Ask bounce and (b) data smoothing, i.e. average of two consecutive true log returns.

7. Consider the time series model $x_t = 0.1x_{t-1} + 0.72x_{t-2} + a_t$, where a_t are iid $N(0, \sigma^2)$. Is the model mean-reverting? If yes, what is the half-life?

   A: Yes, it is mean-reverting (solutions of its characteristic functions are 1.11 and -1.25). The half life is $\log(0.5)/\log(0.1 + 0.72) = 3.49$.

8. Give two special features of an ARCH(1) model that fare well with empirical features of asset returns.

   A: (a) Heavy tails and (b) volatility clustering

9. Correlations play an important role in statistical analysis and applications. Provide two weakness of the usual Pearson correlation in measuring relationship between variables.

   A: (a) Sensitive to outliers, (b) only measures linear dependence, or (c) the actual range may be much narrow than $[-1, 1]$.

10. Suppose that we are interested in the log returns of an asset for the next five (5) trading days, i.e. $r_t^{(5)} = r_{t+1} + r_{t+2} + \cdots + r_{t+5}$. State two conditions under which $\mathrm{Var}[r_t^{(5)}|F_{t-1}] = 5\mathrm{Var}[r_{t+1}|F_{t-1}]$, where F_{t-1} denotes the information available at time $t-1$.

    A: (a) the log returns are uncorrelated and (b) the volatility of the returns follows a IGARCH(1,1) model without a drift. [The condition of the log returns bing iid is not realistic, but acceptable.]

11. **(For Questions 11-15)**: Consider the adjusted daily stock prices of BHP and VALE used in Lecture Note 10. A linear model is used to find the linear relationship between the prices. Based on the fitted model, we define $w_t = \mathrm{bhp}_t - 2.718\mathrm{vale}_t$. An AR(4) model is selected. Write down the fitted model, including residual variance.

    A: Let $\tilde{w}_t = w_t - 5.30$. Then, the model is

    $$\tilde{w}_t = 0.838\tilde{w}_{t-1} + 0.054\tilde{w}_{t-2} + 0.010\tilde{w}_{t-3} + 0.069\tilde{w}_{t-4} + a_t,$$

    where $\sigma_a^2 = 0.110$.

12. The Box-Ljung statistics give $Q(10) = 6.00$ for the residuals of the fitted AR(4) model. Based on the test statistic, are there residual serial correlations in the first 10 lags? Why?

    A: No, there are no significant serial correlations in the first 10 lags. The p-value of the $Q(10)$ is 0.423 even after adjusting for degrees of freedom.

13. Based on the ADF unit-root test and using the 1% type-I error, is there any opportunity for pairs-trading using the two stocks? Why?

    A: Yes, there is opportunity because the unit-root hypothesis is rejected at the 5% level. Since I asked for 1%, there is no arbitrage opportunity at the 1% level.

14. Consider a bi-variate time series of the two stock prices, namely $\boldsymbol{x}_t = (\mathrm{bhp}_t, \mathrm{vale}_t)'$. If vector autoregressive models are entertained, what order is selected for the series? Why?

    A: A VAR(2) model is selected by AIC and HQ or A VAR(1) model is selected by BIC.

15. A VAR(2) model is entertained for $\boldsymbol{x}_t$ series. Write down the fitted AR(1) coefficient matrix. Interpret the meaning of the (1,2)-th element ϕ_{12} of the AR(1) coefficient.

    A:

    $$\hat{\boldsymbol{\phi}}_1 = \begin{bmatrix} 0.909 & 0.42 \\ 0.031 & 1.02 \end{bmatrix}.$$

    The (1,2)-th element measures the linear dependence of BHP daily price on the VALE price one day earlier. [More pricisely, it shows the aforementioned linear dependence in the presense of lag-1 value of BHP price.]

16. The Ljung-Box statistics are widely used in analyzing financial time series. Describe two specific applications of the statistics.

    A: (a) Model checking and (b) testing the existence of serial dependence in the observed data.

17. Give two characteristics of price changes in tick-by-tick transaction data.

    A: (a) heavy tails, (b) assumes discrete values, [or (c) concentration at zero, or (d) symmetric versus zero.]

18. R^2 is commonly used in linear regression analysis. Why is it not useful in time series analysis? State a condition under which R^2 can still be used in analyzing financial time series.

    A: The time series under study is weakly stationary.

19. Describe two cases under which nonlinear models can be useful in analyzing financial data.

    A: (a) Time-varying volatility (or beta) and (b) quantify the leverage effects

20. Give one advantage and one disadvantage of imposing price limits on stock trading.

    A: Advantage: prevent panic selling or reduce unwarranted volatility. Disadvantage: create abitrary opportunity or prolong volatility clustering.

Problem B. (16 points) Consider the daily log returns of Costco stock (COST) and the yield-to-maturity of U.S. 10-year treasure notes (TNX) for a period with sample size 2367. Suppose that the portfolio of interest consists of 10K of long position on each of the asset. Based on the attached output, answer the following questions. The variables used in R are "cost" and "tnx", respectively.

1. Are the daily log returns of COST serially correlated? Write down the null hypothesis and draw your conclusion.

   A: $H_0 : \rho_1 = \cdots = \rho_{10} = 0$ vs $H_a : \rho_i \neq 0$ for some $1 \leq i \leq 10$. Based on Q(10) = 36.87 with small p-value, the null hypothesis is rejected. There are serial correlations in the COST returns.

2. (3 points) A Gaussian ARMA-GARCH(1,1) model is entertained, but the estimated AR coefficients are relatively small and the normality test rejects the normality assumption. A GARCH(1,1) model with standardized Student-t distribution is fitted. Write down the fitted model, including innovations.

   A: The model is

$$\begin{aligned} r_t &= -5.71 \times 10^{-4} + a_t, \quad a_t = \sigma_t \epsilon_t, \quad \epsilon_t \sim t^*_{5.29} \\ \sigma_t^2 &= 1.12 \times 10^{-6} + 0.036 a_{t-1}^2 + 0.959 \sigma_{t-1}^2, \end{aligned}$$

   where t^*_v denotes standardized Student-t distribution with v degrees of freedom.

3. Predictions of the fitted GARCH(1,1) model are provided. Based on the predictions, compute VaR and Expected shortfall (ES) of holding the COST stock.

   A: VaR is \$334 and ES is \$439.

4. Compute the VaR and ES of the COST stock for the next 5 trading days.

   A: VaR is \$731 and ES is \$967.

5. If RiskMetrics is used, what are the VaR and ES for holding the COST stock?

   A: VaR is \$300 and ES is \$344.

6. (3 points) The sample correlations between the two log returns is 0.197. Compute the VaR of the portfolio.

   A: For TNX, VaR is \$506. So for the portfolio, the VaR is $\sqrt{300^2 + 506^2 + 2 * 0.197 * 300 * 506}$ $= \$637$.

7. If the long position of the treasury notes is changed to a short position, compute the VaR of the new portfolio.

   A: The VaR is $\sqrt{300^2 + 506^2 - 2 * .197 * 300 * 506} = \$535$.

Problem C. (19 points) Consider the daily log returns of Amazon (AMZN) stock and the ETF (exchange-traded fund) of VIX index (VXX) starting from January 30, 2009 for 1846 observations. In the R output, the variables of the daily log returns are "amzn" and "vxx", respectively.

$$\mathrm{yd}_t = \begin{cases} 1 & \text{if } \mathrm{amzn}_t \geq 0, \\ 0 & \text{if } \mathrm{amzn}_t < 0. \end{cases}$$

Similarly, we define $\mathrm{y1d}_t$ and $\mathrm{y2d}_t$ using amzn_{t-1} and amzn_{t-2}, respectively. Let the lag-2 value of amzn_t as "y2" in the R output.

1. A simple logistic regression is used to model yd_t using $\mathrm{y1d}_t$ as the predictor. Write down the fitted model. What is the meaning of the fitted coefficient -0.172 of $\mathrm{y1d}_t$.

   A: $P(\mathrm{yd}_t = 1) = \frac{\exp(0.153 - 0.172\mathrm{y1d}_t)}{1+\exp(0.153-0.172\mathrm{y1d}_t)}$. The coefficient -0.172 is the log(odd-ratio) for up market.

2. Another logistic regression is fitted to yd_t using y_{t-2} and $\mathrm{y1d}_t$ as regressors. Based on the AIC criterion, does y_{t-2} contribute significantly to modeling yd_t? Why?

   A: No, the coefficient is not statistically significant at the 5% level.

3. Suppose $y_{t-2} = 0.01$ and $\mathrm{y1d}_t = 0$. Use the fitted logistic model to compute the probability $P(\mathrm{yd}_t = 1)$.

   A: $P(\mathrm{yd}_t = 1) = \frac{\exp[0.160 - 3.72(0.01) - 0.176(0)]}{1+\exp[0.160-3.72(0.01)-0.176(0)} = 0.531$.

4. (5 points) Using y_{t-2} and $\mathrm{y1d}_t$ as input, a 2-4-1 neural network is fitted to yd_t. Write down the fitted model.

A: The model is

$$
\begin{aligned}
h_1 &= \frac{\exp[1.13 + 0.15 y_{t-2} - 0.43 \mathrm{y1d}_t]}{1 + \exp[1.13 + 0.15 y_{t-2} - 0.43 \mathrm{y1d}_t]} \\
h_2 &= \frac{\exp[0.66 + 0.50 y_{t-2} - 0.06 \mathrm{y1d}_t]}{1 + \exp[0.66 + 0.50 y_{t-2} - 0.06 \mathrm{y1d}_t]} \\
h_3 &= \frac{\exp[1.06 + 0.44 y_{t-2} - 0.95 \mathrm{y1d}_t]}{1 + \exp[1.06 + 0.44 y_{t-2} - 0.95 \mathrm{y1d}_t]} \\
h_4 &= \frac{\exp[1.03 + 0.42 y_{t-2} - 0.73 \mathrm{y1d}_t]}{1 + \exp[1.03 + 0.42 y_{t-2} - 0.73 \mathrm{y1d}_t]} \\
\mathrm{yd}_t &= \begin{cases} 1 & \text{if } o_t \geq 0 \\ 0 & \text{otherwise, where} \end{cases} \\
o_t &= -0.52 + 0.08 h_1 + 0.59 h_2 + 0.46 h_3 - 0.01 h_4 - 0.29 y_{t-2} + 0.05 \mathrm{y1d}_t
\end{aligned}
$$

5. Based on the fitted neural network model, compute $P(\mathrm{yd}_t = 1)$ given $y_{t-2} = 0.01$ and $\mathrm{y1d}_t = 0$.

   A: 0.565.

6. Define the directions of lag-1 and lag-2 VXX log returns in a similar way as those of $\mathrm{y1d}_t$ and $\mathrm{y2d}_t$ and denote the variables by $\mathrm{v1d}_t$ and $\mathrm{v2d}_t$. A logistic regression is entertained with regressors $y_{t-1}, y_{t-2}, \mathrm{y1d}_t, \mathrm{y2d}_t, \mathrm{v1d}_t$ and $\mathrm{v2d}_t$. Based on the fitted model, do lagged values of VXX log return contribute significantly in predicting the direction of AMZN price movement? Why?

   A: No, they do not. The estimated coefficients are insignificant at the 5% level.

7. Finally, let $\boldsymbol{r}_t = (\mathrm{amzn}_t, \mathrm{vxx}_t)'$ be a bi-variate log return series. A dynamic conditional correlation model is fitted. Write down the fitted coefficients $\hat{\theta}_1$ and $\hat{\theta}_2$? Are these estimates statistically significant at the 5% level? Why?

   A: $\hat{\theta}_1 = 0.877(0.0762)$ and $\hat{\theta}_2 = 0.0134(0.00736)$, where the number in parentheses denotes standard error. The $\hat{\theta}_1$ is significant, but $\hat{\theta}_2$ is not.

8. Based on the output, what is the time-varying correlation between the two asset returns at $t = 1843$?

   A: -0.870.

Problem D. (25 points) Consider the daily log returns of Costco stock (COST) for a recent period with 2368 observations. For risk assessment, we assume a long position of \$1 million on the stock. Based on the attached R output, answer the following questions.

1. Is the expected value of the log return zero? Why?

   A: Yes, because the null hypothesis of zero return cannot be rejected at the 5% level.

2. Is the distribution of the log returns skewed? Why? Perform a statistical test to support your answer.

   A: The test is $t = -0.0611/sqrt6/2368 = -1.21$, which is less than 1.96 in absolute. Thus, there is no evdience to reject the hypothesis that the log returns are not skewed.

3. Based on the fitted GARCH(1,1) model, does the stock have a leverage effect? Why?

   A: Yes, the leverage effect is significant because the t-ratio is -3.25.

4. Compute VaR and ES of the financial position using the fitted GARCH(1,1) model (denoted by **h1** in the output).

   A: VaR is \$33400 and ES is \$43400.

5. Compute the VaR and ES of the position for the next 5 trading days using the fitted GARCH(1,1) model.

   A: VaR is \$73100 and ES is \$95600.

6. Compared with Question 3 of Problem B, does the leverage effect affect the VaR calculation? Why?

   A: It does not affect the VaR, but affects ES. This is expected, because ES depends on the tail behavior.

7. If the extreme value theory with block maximum (size 21) is used, what is the VaR for the next trading day? What is the VaR for the next 10-trading days?

   A: VaR = \$32100 and for the next 10 trading days, VaR = \$63100.

8. If the theory of Peaks over Threshold with threshold 0.02 is used, write down the parameter estimates. Are these estimates significantly different from zero? Why?

   A: Yes, both estimates are significant at the 5% level; their t-ratios are 2.09 and 7.81, respectively.

9. If the theory of Peaks over Threshold with threshold 0.02 in employed, what are the VaR for the next trading day and the next 10 trading days?

   A: The VaR is \$38800 and for the next 1- trading days the VaR is \$62800.

10. If the theory of Peaks over Threshold with threshold 0.03 is used, what are the VaR and ES?

    A: VaR = \$37600 and ES = \$57000.

11. (1 point) Is VaR sensitive to the choice of thresholds in this particular case? Why?

    A: The two VaR values are close so that they are not sensitive to the choice of thresholds in this case.

12. The log returns of VIX index might be helpful in computing VaR for the financial position. To this end, a quantile regression (tau = 0.95) is used with four input variables (lag-1 and lag-2 of COST stock returns and lag-1 and lag-2 of VIX returns). Based on the fitted model, do the past values of VIX returns contribute significantly to model? Why?

    A: Yes, because the coefficient of "vixm1" is statistically significant at the 5% level.

13. Based on the quantile regression, what is the $\mathrm{VaR}_{0.95}$ if the input variables assume the value (0.08,0.01,0.03, 0.01)?

    A: VaR = $26100.