

Solutions of Final Exam

Problem A: (40 points) Answer briefly the following questions.

1. Describe two econometric methods discussed in the lectures that can be used to aid in making decision of a loan application.

Answer: (1) Neural network and (2) logistic linear regression

2. Assume that x_t follows the stochastic diffusion equation $dx_t = \mu dt + \sigma dw_t$, where w_t is the Wiener process. Let $G(x_t) = x_t^2$. Derive the stochastic diffusion equation for $G(x_t)$.

Answer: $\frac{\partial G(x_t)}{\partial x_t} = 2x_t$, $\frac{\partial G(x_t)}{\partial t} = 0$, and $\frac{\partial^2 G(x_t)}{\partial x_t^2} = 2$. Applying the Ito's Lemma, we have

$$dG(x_t) = 2x_t dx_t + \frac{1}{2} 2\sigma^2 dt = (2x_t\mu + \sigma^2)dt + 2x_t\sigma dw_t.$$

3. Consider a call option contingent on Stock A, which pays no dividend. Suppose that we have $c_t = \$1.12$, $P_t = 30$, $K = 29$, $r = 3\%$ per annum, and the time to expiration is $T - t = 0.25$. Is there any arbitrage opportunity? Why?

Answer: Yes, there is arbitrage opportunity, because the lower bound of call price is $P_t - K \exp[-r(T - t)] = \$ [30 - 29\exp(-0.03 \cdot 0.25)] = \1.22 , which is higher than the call price $c_t = \$1.12$.

4. Give two applications of seasonal time series models in finance.

Answer: Any two of (a) quarterly earnings forecast, (b) modeling series in energy sector, and (c) handling the diurnal pattern in high-frequency data analysis.

5. What are the two critical assumptions used by RiskMetrics to justify the square root of time rule?

Answer: (a) Return is distributed as $N(0, \sigma_t^2)$ and (b) the volatility follows an IGARCH(1,1) model without drift, i.e. $\sigma_t^2 = (1 - \beta)r_t^2 + \beta\sigma_{t-1}^2$.

6. Describe two approaches to VaR calculation that use the extreme value theory in statistics.

Answer: (a) Block approach and (b) peaks over threshold.

7. Describe two methods that can be used to mitigate the impact of market micro-structure noises in computing realized volatility of a stock.

Answer: (a) Optimal sampling interval and (b) sub-sampling method.

8. State two financial applications that require use of high-frequency transactions data.

Answer: Any two of (a) price discovery, (b) impact of daily price limits, (c) liquidity, (d) design of trading system, and (e) realized volatility.

9. The univariate GARCH(1,1) model is commonly used in volatility modeling, but it has its shares of weakness. Describe two weaknesses of model.

Answer: Any two of (a) no leverage effect (assumes symmetric responses to past returns), (b) parameter restrictions, and (c) slow response to large shocks.

10. **(For Questions 10-16):** Consider the daily log return of Netflix (NFLX) stock, starting from January 3, 2007. A GARCH(1,1) model with skew Student- t innovations was entertained. Write down the fitted model.

Answer: The fitted model is

$$\begin{aligned} r_t &= 0.00182 + a_t, \quad a_t = \sigma_t \epsilon_t, \quad \epsilon_t \sim t_{2.92, 1.06}^* \\ \sigma_t^2 &= 1.21 \times 10^{-5} + .0195a_{t-1}^2 + 0.974\sigma_{t-1}^2, \end{aligned}$$

where 2.92 and 1.06 are the degrees of freedom and skewness parameter, respectively, of the Student- t innovations.

11. Based on the fitted model, perform the hypothesis $H_0 : v = 5$ vs $H_a : v \neq 5$, where v denotes the degrees of freedom of the Student- t innovations. Draw your conclusion.

Answer: $t = \frac{2.92-5}{1.98} = -1.051$, which is less in 1.96 in absolute value. Therefore, the null hypothesis cannot be rejected at the 5% level.

12. Based on the fitted model, is the distribution of the returns indeed skew? Perform test to justify your answer.

Answer: $t = \frac{1.062-1}{.031} = 2$, which is greater than 1.96. Therefore, the symmetric distribution is rejected at the 5% level, i.e. the distributed is skewed.

13. To check the risk premium of investing in the stock, a GARCH-M model is entertained. Write down the fitted model.

Answer: The fitted model is

$$\begin{aligned} r_t &= 0.00314 + 0.0139\sigma_t^2 + a_t, \quad a_t = \sigma_t \epsilon_t, \quad \epsilon_t \sim N(0, 1) \\ \sigma_t^2 &= 0.148 + 0.0118a_{t-1}^2 + 0.977\sigma_{t-1}^2. \end{aligned}$$

14. Based on the fitted model, is the risk premium statistically significant? Perform test to justify your answer.

Answer: No, risk premium is not significant at the 5% level, because its t -ratio is 0.572 with p -value 0.567.

15. The volatility obtained from model in Question 10 and the VIX index are used to better estimate the risk of shorting the stock. Specifically, we employ quantile regression with explanatory variables (a) the lag-1 return r_{t-1} (rtm1), (b) the lag-1 VIX index (vixm1), and (c) the lag-1 volatility (volm1). Write down the parameter estimates when the 95% quantiles are used. Are these estimates significantly different from zero? Why?

Answer: The parameters (standard errors) are 0.00849(0.0116), 0.0888(0.0448), 0.0679(0.0235), and 0.758(0.325). Except for the intercept term, all other three estimates are significant at the 5% level. In other words, these three parameters are significantly different from zero at the 5% level.

16. Given $(\text{rtm1}, \text{vixm1}, \text{volm1}) = (-0.00185, 0.1213, 0.03204)$, what is the VaR of the position with tail probability 0.05?

Answer: Plugging the values, we have 0.0408.

17. Let p_{1t} and p_{2t} be the daily log prices of two stocks. Assume that both p_{it} series are unit-root nonstationary. Let $w_t = p_{1t} - \gamma p_{2t}$ be a linear combination of the two log price series, where γ is a constant. It is also known that w_t follows the model

$$w_t = 0.787w_{t-1} + 0.213w_{t-2} + a_t, \quad a_t \sim_{iid} N(0, 2.4 \times 10^{-4}).$$

Can the two stocks be used in pairs trading? Why?

Answer: No, because w_t contains a unit root with $(1 - 0.787x - 0.213x^2) = (1 - x)(1 - 0.213x)$.

18. Describe two models that can be used to model price changes in intraday stock trading.

Answer: The ADS decomposition and ordered the Probit model.

19. Give two applications of the regression model with time series errors in finance.

Answer: (a) Handling outliers and (b) forecasting turn points.

20. Why is expected shortfall a preferred risk measure over value at risk in finance? Under what condition are the two risk measures coherent?

Answer: Because ES is a coherent risk measures. VaR and ES are coherent if the underlying returns are normally distributed.

Problem B. (12 points) Consider the monthly returns r_t of the Proctor & Gamble stock. Let

$$M_t = \begin{cases} 1 & \text{if } r_t > 0, \\ 0 & \text{if } r_t \leq 0. \end{cases}$$

Similarly, we define the directions of the S&P composite return and the value-weighted CRSP index return as follows:

$$S_t = \begin{cases} 1 & \text{if } sp_t > 0, \\ 0 & \text{if } sp_t \leq 0. \end{cases} \quad \text{and} \quad VW_t = \begin{cases} 1 & \text{if } vw_t > 0, \\ 0 & \text{if } vw_t \leq 0. \end{cases}$$

We model M_t using lagged values of M_t , S_t and VW_t as explanatory variables.

1. A logistic linear regression is employed using S_{t-1} as the explanatory variable. Write down the fitted model. Explain the meaning of the estimated coefficient of S_{t-1} .

Answer: The model is $P(M_t = 1) = \frac{\exp(0.415 - 0.313S_{t-1})}{1 + \exp(0.415 - 0.313S_{t-1})}$. The coefficient -0.313 is the log of odd ratio.

2. Based on the fitted model, compute $P(M_t = 1|S_{t-1} = 1)$ and $P(M_t = 1|S_{t-1} = 0)$.

Answer: $P(M_t = 1|S_{t-1} = 1) = \frac{\exp(0.102)}{1 + \exp(0.102)} = 0.525$. $P(M_t = 1|S_{t-1} = 0) = \frac{\exp(0.415)}{1 + \exp(0.415)} = 0.602$.

3. Another logistic linear regression is employed using VW_{t-1} as the explanatory variable. Compared with the model in part (1), which model do you prefer? Why?

Answer: The model in Part (1) is preferred as it has lower AIC value.

4. A third logistic linear regression using M_{t-1} , S_{t-1} and M_{t-2} as explanatory variables is employed. Write down the fitted model. Is the model helpful in predicting $P(M_t = 1)$ given the past information? Why?

Answer: The fitted model is

$$P(M_t = 1) = \frac{\exp(0.444 + 0.238M_{t-1} - 0.381S_{t-1} - 0.216M_{t-2})}{1 + \exp(0.444 + 0.238M_{t-1} - 0.381S_{t-1} - 0.216M_{t-2})}.$$

Yes, the model is helpful because the coefficient of S_{t-1} is significant at the 5% level.

5. A 3-3-1 neural network with skip layer is entertained for M_t using the input variables M_{t-1} , S_{t-1} and M_{t-2} . White down the models for the two hidden nodes h_1 and h_3 .

Answer: The models are

$$\begin{aligned} h_{1t} &= \frac{\exp(-1.07 - 6.46M_{t-1} - 7.70S_{t-1} - 1.04M_{t-2})}{1 + \exp(-1.07 - 6.46M_{t-1} - 7.70S_{t-1} - 1.04M_{t-2})} \\ h_{3t} &= \frac{\exp(-1.79 - 5.52M_{t-1} - 0.07S_{t-1} - 4.73M_{t-2})}{1 + \exp(-1.79 - 5.52M_{t-1} - 0.07S_{t-1} - 4.73M_{t-2})}. \end{aligned}$$

6. Write down the model for the output node.

The model is $h(y_1) = 1$ if $y_t > 0$ and $h(y_t) = 0$, otherwise, where

$$y_t = 1.46 - 4.81h_{1t} - 1.46h_{2t} + 2.87h_{3t} + 0.00M_{t-1} - 0.40S_{t-1} - 0.56M_{t-2}.$$

Problem C. (30 points) Consider a portfolio consisting of two stocks: (a) Exxon-Mobil (XOM) and (b) Qualcomm (QCOM) each with \$1 million. To estimate the risk of the portfolio, we use daily log returns of the stocks, starting from January 3, 2003.

1. (2 points) Focus on the XOM stock. Is the expected return different from zero? Why?

Answer: No, because the 95% confidence interval of the mean contains zero.

2. (2 points) A Gaussian AR(1)+GARCH(1,1) model is entertained for the XOM stock. Write down the fitted model.

Answer: The fitted model is

$$\begin{aligned} r_t &= 5.86 \times 10^{-4} - 0.0615r_{t-1} + a_t, & a_t &= \sigma_t \epsilon_t, & \epsilon_t &\sim N(0, 1) \\ \sigma_t^2 &= 3.07 \times 10^{-6} + 0.0683a_{t-1}^2 + 0.914\sigma_{t-1}^2. \end{aligned}$$

3. (2 points) Based on the fitted model in part (2). What are the VaR and ES for the XOM position?

Answer: VaR = \$20,815. and ES = \$24,055.

4. (4 points) Focus on the QCOM stock. If the GARCH(1,1) model with Gaussian innovations is used, what are the VaR and Expected shortfall? Also, what are the VaR and ES for the next 10-trading days?

Answer: VaR = \$30,569 and ES = \$35,143. For the next 10-trading days, VaR = \$96,988 and ES = \$112,326.

5. (3 points) Focus again on the QCOM stock. If the GARCH(1,1) model with Student- t innovations and leverage effect is used, what are the VaR and ES for the position? Is the leverage effect significant? Why?

Answer: VaR = \$30,746 and ES = \$40,826. The leverage effect is significant as the estimate has a p -value 0.000635.

6. (4 points) Using the RiskMetrics method, what are the VaR for the portfolio for the next trading day and for the next ten (10) trading days? You may use the sample correlation 0.45.

Answer: One day VaR = \$ $\sqrt{20442^2 + 33835^2 + 2 * 0.45 * 20442 * 33835}$ = \$46,746. Ten days VaR = \$ $\sqrt{10} \times 46746$ = \$147,823.

7. (4 points) Focus on the QCOM stock and apply the generalized extreme value theory with block size 42. Write down the parameter estimates. What is the corresponding VaR?

Answer: The estimates (standard errors) are 0.170(0.089), 0.0159(0.00157), 0.0321(0.00207), respectively, for ξ , σ , and μ . The VaR is \$46,878.

8. (5 points) Focus on the QCOM stock and apply the generalized Pareto distribution with threshold 1%. Write down the parameter estimates? Are the estimates statistically significant at the 5% level? What are the VaR and ES implied by the model? What is the VaR for the next 10 trading days?

Answer: The parameter estimates (standard errors) are 0.139(0.0397) and 0.0108(0.000564), respectively, for ξ and β . These estimates are significant at the 5% level because their t -ratios are greater than 2. The VaR is \$53,540 and ES is \$73,090. The VaR for the next 10 trading days is VaR = \$ $53540 * 10^{0.1387}$ = \$73,685.

9. (2 points) For the QCOM stock, we also considered threshold 0.02 and 0.03 in applying generalized Pareto distribution. Are the VaR and ES sensitive to the choice of the threshold when generalized Pareto distribution is used? Why?

Answer: For threshold 0.02, the VaR and ES are \$53,134 and \$73,120, respectively. For threshold 0.03, the VaR and ES are \$52,637 and \$75,135. They are close. Therefore, VaR and ES are not sensitive to the choices of threshold in this particular case.

10. (2 points) Revisit the RiskMetrics method, but consider a new portfolio that holds a short position on XOM with \$1 million and a long position on QCOM with \$2 million. What is the VaR for the portfolio if the sample correlation is used?

Answer: For XOM, VaR = \$20,442. For QCOM, the VaR is \$67,670. The VaR for the portfolio is \$ $\sqrt{20442^2 + 67670^2 - 2 * 0.45 * 20442 * 67670}$ = \$61,255.

Problem D. (18 points) Consider the daily log returns of Exxon-Mobil (XOM) and Qualcomm (QCOM) stocks used before. For numerical stability, we use percentage log returns. That is, $\mathbf{x}_t = 100(\text{XOM}_t, \text{QCOM}_t)$. Some analysis of this bivariate series is given in the attached R output. Answer the following questions.

1. (2 points) Are there serial correlations in the \mathbf{x}_t series? Answer the question using the multivariate $Q(m)$ statistics with $m = 5$.

Answer: $Q(5) = 126.1$ with p -value close to 0 so that there are serial correlations in the bivariate series.

2. (4 points) A VAR(2) model is selected via the BIC and HQ criteria. In addition, we refined the fitted model by removing any estimate with t -ratio less than 1.5. What are the AR(1) and AR(2) coefficient matrices of the refined model?

Answer: The coefficient matrices are

$$\phi_1 = \begin{bmatrix} -0.158 & 0 \\ -0.151 & 0 \end{bmatrix}, \quad \phi_2 = \begin{bmatrix} -0.1125 & 0 \\ -0.0462 & -0.0382 \end{bmatrix}.$$

3. (2 points) Does the refined model imply any Granger causality between the two daily log return series? Why?

Answer: Yes, because both ϕ_1 and ϕ_2 have the (1,2)th element being zero.

4. (2 points) Are there any ARCH effects in the residuals of the fitted VAR(2) model? Why?

Answer: Yes, there are ARCH effects, because all four test statistics have p -values close to 0.

5. (2 points) The exponentially weighted moving average method is used to estimate the volatility matrices of \mathbf{a}_t , the residual series of the VAR(2) model. Write down the fitted model.

Answer: The model is

$$\Sigma_t = (0.023)\mathbf{a}_{t-1}\mathbf{a}_{t-1}' + 0.977\Sigma_{t-1}.$$

6. (2 points) Write down the range and mean of the resulting time-varying correlations.

Answer: The mean is 0.377 and the range is $(-0.0613, 0.835)$.

7. (2 points) A DCC model with bivariate Student- t innovations is applied to the residual series \mathbf{a}_t . Write down the estimates of θ_1 , θ_2 and degrees of freedom of Student- t innovations of the DCC model.

Answer: The estimates are $\hat{\theta}_1 = 0.965$ and $\hat{\theta}_2 = 0.0199$. The degrees of freedom are 5.65.

8. (2 points) Write down the range and mean of the time-varying correlations based on the fitted DCC model.

Answer: The mean is 0.326 and the range is $(0.0635, 0.594)$.