Financial Econometrics (FIN620): Overview of the Course

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Welcome

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Financial Econometrics

- Modeling the statistical behavior of variables in financial markets
  - Prices
  - Returns
  - Trade volume
- Modeling: measurement, distribution, and prediction of financial variables

Major Applications of Financial Econometrics

- Pricing
- Risk management (and regulatory requirements)
- Theory testing
- Forecasting
- Portfolio selection
- Model calibration

Examples of financial time series

1. Daily log returns of Apple stock: 2004 to 2013 (10 years)
2. The VIX index
3. CDS spreads: Daily 3-year CDS spreads of JP Morgan from July 20, 2004 to September 19, 2014.
4. Quarterly earnings of Coca-Cola Company: 1983-2009

Examples of Financial Time Series

5. US monthly interest rates (3m & 6m Treasury bills)
   - Relations between the two series?
   - Term structure of interest rates
6. Exchange rate between US Dollar vs Euro Fixed income, hedging, carry trade
7. Size of insurance claims: values of fire insurance claims ($\times 1000$ Krone) that exceeded 500 from 1972 to 1992.
8. High-frequency financial data: tick-by-tick data of Caterpillars stock (on January 04, 2010)

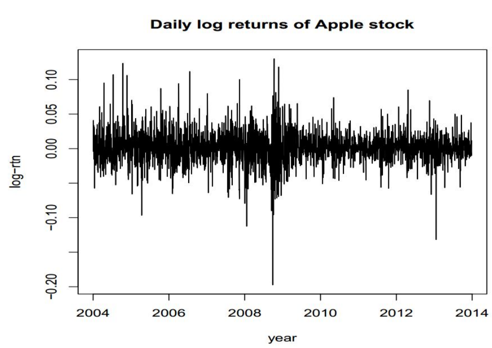


Figure: Daily log returns of Apple stock from 2004 to 2013

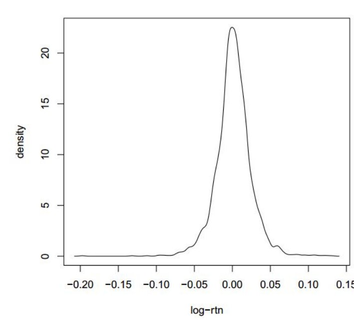


Figure: Density of daily log returns of Apple stock: 2004 to 2013

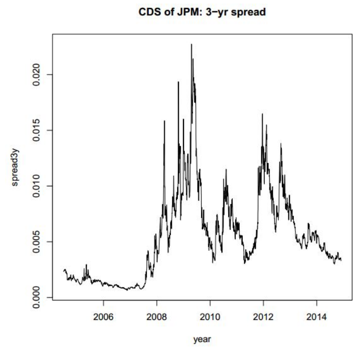


Figure: Time plot of daily 3-year CDS spreads of JPM: from July 20, 2004 to September 19, 2014.

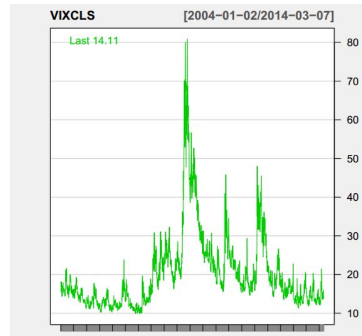


Figure: CBOE Vix index: January 2, 2004 to March 7, 2014.

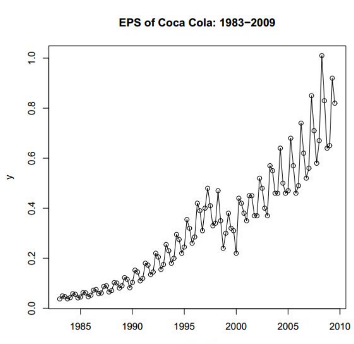


Figure: Quarterly earnings per share of Coca-Cola Company

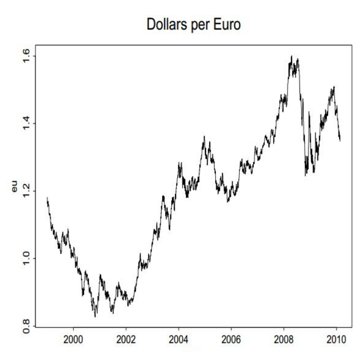


Figure: Daily Exchange Rate: Dollars per Euro

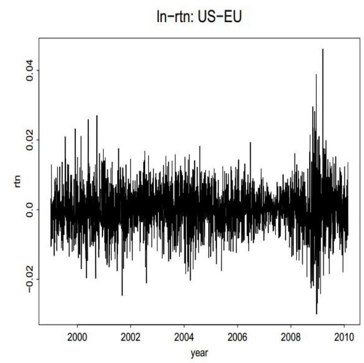


Figure: Daily log returns of FX (Dollar vs Euro)

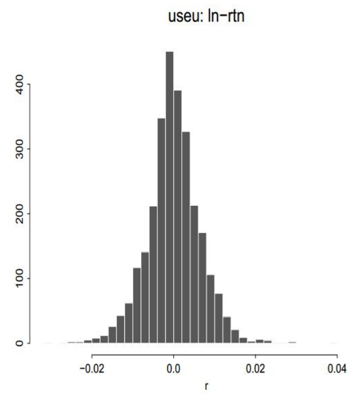


Figure: Histogram of daily log returns of FX (Dollar vs Euro)

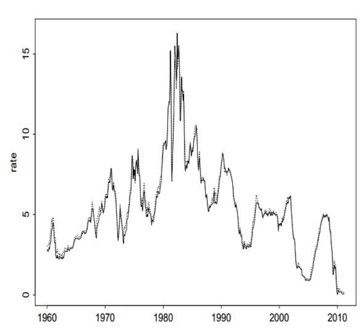


Figure: Monthly US interest rates: $3m$ & $6m$ TB

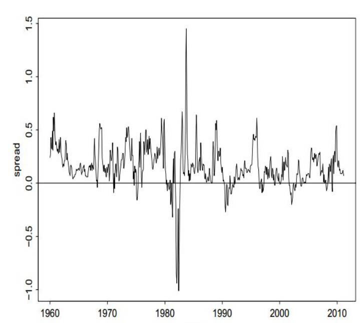


Figure: Spread of monthly US interest rates: $3m$ & $6m$ TB

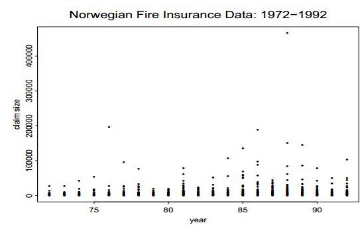


Figure: Claim sizes of the Norwegian fire insurance from 1972 to 1992, measured in 1000 Krone and exceeded 500.

Financial Econometrics versus Econometrics

- Objects of study: prices and returns
- The critical role of second moments
- Heavy use of time-series, stochastic processes, and simulation methods
- Knowledge of the finance theory

Some Properties of Financial Data

- Data problems
- Unit-root (random walk)
- Heteroskedasticity (volatility clustering)
- Fat tails and jumps
- Trend and seasonality
- High-frequency components

Objective of the course

- To learn ways to get financial information from web directly and to process the information.
- To provide some basic knowledge of financial time series data such as skewness, heavy tails, and measure of dependence between asset returns
- To introduce some statistical tools & econometric models useful for analyzing these series.
- To gain experience in analyzing FTS

Objective of the course

- To introduce recent developments in financial econometrics and their applications, e.g., high-frequency finance
- To study methods for assessing market risk, credit risk, and expected loss. The methods discussed include Value at Risk, expected shortfall, and tail dependence.
- To analyze high-dimensional asset returns, including co-movement

Outline of the course

- Returns & their characteristics: empirical analysis (summary statistics)
- Simple linear time series models & their applications
- Multi-variate time-series models (VAR, VECM)
- Univariate volatility models & their implications
- Nonlinearity in level and volatility
- High-frequency financial data and market micro-structure
- Value at Risk, extreme value theory and expected shortfall (also known as conditional VaR)
- Analysis of multiple asset returns: factor models, dynamic and cross dependence
- MCMC models

Components of the Course

- Textbook(s)
- Papers
- Projects
- Software

Main Textbook

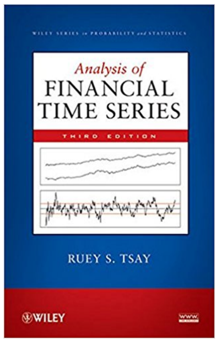
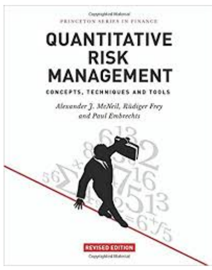


Figure: Analysis of Financial Time Series by Ruey S. Tsay, Wiley

Optional Textbook



Projects

- Pick an asset class: stock, fixed-income, commodity, alternative, etc
- Spot and futures prices with sufficiently long history
- Run all techniques learned in the course on that asset
- Produce analysis and business implications
- Reports and presentations

Paper Presentations

- Each week one paper on the application of financial econometrics
- Teams of 3-4 students
- Presentation time: 20-25 minutes
- Use Powerpoint or LaTeX
- Be very well prepared

Paper Presentations: Guidelines

- Motivation and research question(s) of the paper
- Contribution of the paper
- Methods and models (in details)
- Data sources and empirical preparation
- Results and discussions (in great details)
- Lessons to learn from this paper

Exams

- Mid-term exam
- Final exam
- Remarks
  - Rule 1: you can not pass the course if you receive less than 60% in both exams
    - Implication: one needs at least one grade > 60% in one of the exams to pass the course.
  - Rule 2: you can not get an A if you receive less than 90% in both exams.
    - Implication: one needs at least one A- in either mid-term or final exams to get a final A.

Software

- Eviews: time-series aspects
- R
  - Automatic data downloading
  - Plotting and visualization
  - Econometrics

Some Useful Journals

- Journal of Finance (JF)
- Review of Financial Studies (RFS)
- Journal of Finance Economics (JFE)
- The Journal of Financial and Quantitative Analysis (JFQA)
- Journal of Econometrics (JOE)
- Journal of Applied Econometrics (JAE)
- Journal of Financial Econometrics (JOEcs)
- Journal of Banking & Finance (JBF)
- Journal of Futures Markets (JFM)

A Quick Overview of Basic Concepts

Asset Returns

Let P_t be the price of an asset at time t, and assume no dividend.
One-period simple return: Gross return

$$1 + R_t = \frac{P_t}{P_{t-1}} \quad \text{on} \quad P_t = P_{t-1}(1 + R_t)$$

Simple return:

$$R_t = \frac{P_t}{P_{t-1}} - 1 = \frac{P_t - P_{t-1}}{P_{t-1}}.$$

Multiperiod simple return: Gross return

$$1 + R_t(k) = \frac{P_t}{P_{t-k}} = \frac{P_t}{P_{t-1}} \times \frac{P_{t-1}}{P_{t-2}} \times \cdots \times \frac{P_{t-k+1}}{P_{t-k}}.$$

The k-period simple net return is $R_t(k) = \frac{P_t}{P_{t-k}} - 1$.

Asset Returns

Example: Table below gives five daily closing prices of Apple stock in December 2011. The 1-day gross return of holding the stock from 12/8 to 12/9 $1 + R_t = 393.62/390.66 \approx 1.0076$ so that the daily simple return is 0.76%, which is $(393.62 - 390.66)/390.66$.

Date	12/02	12/05	12/06	12/07	12/08	12/09
Price($)	389.70	393.01	390.95	389.09	390.66	393.62

Time interval is important! Default is one year.

Annualized (average) return

$\text{Annualized}[R_t(k)] = \left[\prod_{j=0}^{k-1}(1 + R_{t-j})\right]^{1/k} - 1$. An approximation:$\text{Annualized}[R_t(k)] \approx \frac{1}{k}\sum_{j=0}^{k-1} R_{t-j}$.

Annualized (average) return

Continuously compounding: Illustration of the power of compounding (int. rate 10% per annum)

Type	#(payment)	Int.	Net
Annual	1	0.1	$1.10000
Semi-Annual	2	0.05	$1.10250
Quarterly	4	0.025	$1.10381
Monthly	12	0.0083	$1.10471
Weekly	52	$\frac{0.1}{52}$	$1.10506
Daily	365	$\frac{0.1}{365}$	$1.10516
Continuously	∞		$1.10517

$$A = C\exp[r \times n]$$

where r is the interest rate per annum, C is the initial capital, n is the number of years, and exp is the exponential function.
Present value:

$$C = A\exp[-r \times n]$$

Continuously compounded (or log) return

$$r_t = \ln(1 + R_t) = \ln \frac{P_t}{P_{t-1}} = p_t - p_{t-1},$$

where $p_t = \ln(P_t)$.

Multi period log return:

$$\begin{aligned} r_t(k) &= \ln[1 + R_t(k)] \\ &= \ln[(1 + R_t)(1 + R_{t-1}) \dots (1 + R_{t-k+1})] \\ &= \ln(1 + R_t) + \ln(1 + R_{t-1}) + \dots + \ln(1 + R_{t-k+1}) \\ &= r_t + r_{t-1} + \dots + r_{t-k+1}. \end{aligned}$$

Continuously compounded (or log) return

Example Consider again the Apple stock price.

1. What is the log return from 12/8 to 12/9:
   A:$r_t = \ln(393.62) - \ln(390.66) = 7.5\%$.
2. What is the log return from day 12/2 to 12/9?
   A:$r_t(4) = \ln(393.62) - \ln(389.7) = 1\%$.

Portfolio return: N assets

$$R_{p,t} = \sum_{i=1}^{N} \omega_i R_{it}$$

Example: An investor holds stocks of IBM, Microsoft and CitiGroup. Assume that her capital allocation is 30%, 30% and 40%. Use the monthly simple returns in Table 1.2 of the text. What is the mean simple return of her stock portfolio?
Answer: $E(R_t) = 0.3 \times 1.35 + 0.3 \times 2.62 + 0.4 \times 1.17 = 1.66$.

Continuously compounded (or log) return

Dividend payment:

$$R_t = \frac{P_t + D_t}{P_{t-1}} - 1, \quad r_t = \ln(P_t + D_t) - \ln(P_{t-1}).$$

Excess return: (adjusting for risk)

$$Z_t = R_t - R_{0t}, \quad z_t = r_t - r_{0t}$$

where r_{0t} denotes the log return of a reference asset (e.g. risk-free interest rate).
Relationship:

$$r_t = \ln(1 + R_t), R_t = e^{rt} - 1.$$

If the returns are in **percentage**, then

$$r_t = 100 \times \ln(1 + \frac{R_t}{100}), R_t = [\exp(r_t/100) - 1] \times 100.$$

Continuously compounded (or log) return

Temporal aggregation of the returns produces

$$\begin{aligned} 1 + R_t(k) &= (1 + R_t)(1 + R_{t-1}) \dots (1 + R_{t-k+1}), \\ r_t(k) &= r_t + r_{t-1} + \dots + r_{t-k+1}. \end{aligned}$$

These two relations are important in practice, e.g. obtain annual returns from monthly returns.

Example: If the monthly log returns of an asset are 4.46%, -7.34% and 10.77%, then what is the corresponding quarterly log return?
Answer: 4.46 - 7.34 + 10.77 = 7.89%.
Example: If the monthly simple returns of an asset are 4.46%, -7.34% and 10.77%, then what is the corresponding quarterly simple return?
Answer: R = (1+0.0446)(1-0.0734)(1+0.1077)-1 = 1.0721-1 = 0.0721 = 7.21%

Distributional properties of returns

Key: What is the distribution of $\{r_{it}; i=1,\dots,N; t=1,\dots,T\}$?

Some theoretical properties:

Moments of a random variable X with density $f(x)$: ℓ-th moment

$$m'_\ell = E(X^\ell) = \int_{-\infty}^{\infty} x^\ell f(x)dx$$

First moment: mean or expectation of X.

ℓ-th central moment

$$m_\ell = E[(X-\mu_x)^\ell] = \int_{-\infty}^{\infty} (x-\mu_x)^\ell f(x)dx,$$

2nd c.m.: Variance of X.

Skewness (symmetry) and kurtosis (fat-tails)

$$S(x) = E\left[\frac{(X-\mu_x)^3}{\sigma_x^3}\right], \quad K(x) = E\left[\frac{(X-\mu_x)^4}{\sigma_x^4}\right].$$

$K(x) - 3$: *Excess kurtosis*.

Distributional properties of returns

- Why are the mean and variance of returns important?
  - They are concerned with long-term return and risk, respectively.
- Why is return symmetry of interest in financial study?
  - Symmetry has important implications in holding short or long financial positions and in risk management.
- Why is kurtosis important?
  - Related to volatility forecasting, efficiency in estimation and tests, etc.
  - High kurtosis implies heavy (or long) tails in distribution.

Distributional properties of returns

Estimation:

Data:$\{x_1,\dots,x_T\}$

- sample mean:

$$\hat{\mu}_x = \frac{1}{T}\sum_{t=1}^{T} x_t,$$

- sample variance:

$$\hat{\sigma}_x^2 = \frac{1}{T-1}\sum_{t=1}^{T}(x_t-\hat{\mu}_x)^2,$$

- sample skewness:

$$\hat{S}(x) = \frac{1}{(T-1)\hat{\sigma}_x^3}\sum_{t=1}^{T}(x_t-\hat{\mu}_x)^3,$$

Distributional properties of returns

- sample kurtosis:

$$\hat{K}(x) = \frac{1}{(T-1)\hat{\sigma}_x^4}\sum_{t=1}^{T}(x_t-\hat{\mu}_x)^4.$$

Under normality assumption,

$$\hat{S}(x) \sim N(0, \frac{6}{T}), \quad \hat{K}(x) - 3 \sim N(0, \frac{24}{T}).$$

Some simple tests for normality (for large T).

1. Test for symmetry:

$$S^* = \frac{\hat{S}(x)}{\sqrt{6/T}} \sim N(0,1)$$

   if normality holds.

   Decision rule: Reject H_o of a symmetric distribution if $|S^*| > Z_{\alpha/2}$ or p-value is less than α.

Distributional properties of returns

2. Test for tail thickness:

$$K^* = \frac{\hat{K}(x) - 3}{\sqrt{24/T}} \sim N(0,1)$$

if normality holds.
Decision rule: Reject H_o of normal tails if $|K^*| > Z_{\alpha/2}$ or p-value is less than α.

3. A joint test (Jarque-Bera test):

$$JB = (K^*)^2 + (S^*)^2 \sim \chi^2_2$$

if normality holds, where χ^2_2 denotes a chi-squared distribution with 2 degrees of freedom.
Decision rule: Reject H_o of normality if $JB > \chi^2_2(\alpha)$ or p-value is less than α.

Empirical properties of returns

Data sources:

- Course web on Canvas
- CRSP: Center for Research in Security Prices (Wharton WRDS) http://wrds.wharton.upenn.edu/
- Various web sites, e.g. Federal Reserve Bank at St. Louis http://research.stlouisfed.org/fred2/
- Yahoo and Google Finance
- Data sets of the Textbook: http://faculty.chicagobooth.edu/ruey.tsay/teaching/fts3/

Empirical dist of asset returns tends to be skewed to the left with heavy tails and has a higher peak than normal dist. See Table 1.2 of the text.

Demonstration of Data Analysis

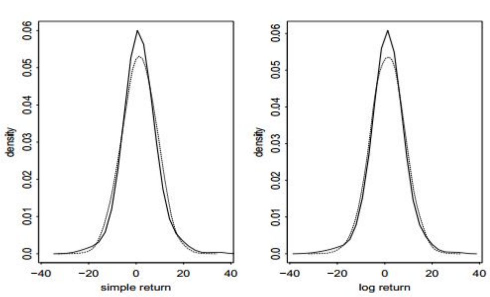


Figure: Comparison of empirical IBM return densities (solid) with Normal densities (dashed)

Normal and lognormal dists

Y is lognormal if $X = \ln(Y)$ is normal.
If $X \sim N(\mu, \sigma^2)$, then $Y = \exp(X)$ is lognormal with mean and variance

$$E(Y) = \exp(\mu + \frac{\sigma^2}{2}), \qquad V(Y) = \exp(2\mu + \sigma^2)[\exp(\sigma^2) - 1].$$

Conversely, if Y is lognormal with mean μ_y and variance σ^2_y,then $X = \ln(Y)$ is normal with mean and variance

$$E(X) = \ln\left[\frac{\mu_y}{\sqrt{1 + \frac{\sigma^2_y}{\mu^2_y}}}\right], \qquad V(X) = \ln\left[1 + \frac{\sigma^2_y}{\mu^2_y}\right].$$

Normal and lognormal dists

Application: If the log return of an asset is normally distributed with mean 0.0119 and standard deviation 0.0663, then what is the mean and standard deviation of its simple return?
Answer: Solve this problem in two steps.
Step 1: Based on the prior results, the mean and variance of $Y_t = \exp(r_t)$ are

$$E(Y) = \exp\left[0.0119 + \frac{0.0663^2}{2}\right] = 1.014$$

$$V(Y) = \exp(2 \times 0.0119 + 0.0663^2)[\exp(0.0663^2) - 1] = 0.0045$$

Step 2: Simple return is $R_t = \exp(r_t) - 1 = Y_t - 1$. Therefore,

$$E(R) = E(Y) - 1 = 0.014$$

$$V(R) = V(Y) = 0.0045, \quad \text{standard dev} = \sqrt{V(R)} = 0.067$$

Remark: See the monthly IBM stock returns in Table 1.2.