Booth School of Business, University of Chicago
Business 41202, Spring Quarter 2015, Mr. Ruey S. Tsay

Midterm

ChicagoBooth Honor Code:
I pledge my honor that I have not violated the Honor Code during this examination.

Signature: **Name**: **ID**:

Notes:

- Open notes and books. Exam time: 120 minutes.
- You may use a calculator or a PC. However, turn off Internet connection and cell phones. **Internet access and phone communication are strictly prohibited during the exam.**
- The exam has 8 pages and the R output has 11 pages. Please **check** that you have all 19 pages.
- For each question, write your answer in the blank space provided.
- **Manage** your time carefully and answer as many questions as you can.
- For simplicity, if not specifically given, use 5% Type-I error in hypothesis testings.
- Round your answer to 3 significant digits.

Problem A: (30 pts) Answer briefly the following questions. Each question has two points.

1. Give two situations under which returns of an assert follow an MA(1) model.

2. Describe two ways by which a GARCH(1,1) model can introduce heavy tails.

3. **(Questions 3 to 6)**: Suppose that the asset return r_t follows the model

$$
\begin{aligned}
r_t &= 0.005 + a_t \\
a_t &= \sigma_t \epsilon_t, \quad \epsilon_t \sim \text{iid } t_6^* \\
\sigma_t^2 &= 0.09 + 0.10 a_{t-1}^2 + 0.855 \sigma_{t-1}^2.
\end{aligned}
$$

Compute the mean and variance of r_t, i.e., $E(r_t)$ and $\text{Var}(r_t)$.

4. Express the volatility model in an ARMA(1,1) formation using $\eta_t = a_t^2 - \sigma_t^2$.

5. Describe two nice characteristics of the volatility model for r_t.

6. Suppose further that $a_{100} = -0.02$ and $\sigma_{100}^2 = 0.16$. Compute the 1-step ahead prediction (both mean and volatility) of the return at the forecast origin $t = 100$.

7. Give two nice features of the exponential GARCH model that the standard GARCH model does not have.

8. Give two empirical characteristics of daily asset returns.

9. **(Questions 9 - 11)**: Let p_t be the log price of an asset at time t. Assume that the log price follows the model

$$p_t = 0.001 + p_{t-1} + a_t, \quad a_t \sim \text{iid } N(0, 0.16),$$

where $N(\mu, \sigma^2)$ denotes normal distribution with mean μ and variance σ^2. Assume further that $p_{200} = 4.551$. Compute the 95% interval forecast for p_{201} at the forecast origin $t = 200$.

10. Compute the 2-step ahead point forecast and its standard error for p_{202} at the forecast origin $t = 200$.

11. What is the 100-step ahead forecast for p_{300} at the forecast origin $t = 200$?

12. Describe two methods for comparing two different models for a time series z_t.

13. **(Questions 13-15)**: Suppose that the quarterly growth rates r_t of an economy follows the model

$$r_t = 0.006 + 0.168 r_{t-1} + 0.338 r_{t-2} - 0.189 r_{t-3} + a_t, \quad a_t \sim \text{iid } N(0, .0016).$$

What is the expected growth rate of r_t?

14. Does the model imply existence of business cycles? Why?

15. What is the average length of business cycles of the economy, if any?

Problem B. (45 points) Consider the daily log returns r_t, in percentages, of the NASDAQ index for a certain period of time with 1841 observations. Answer the following questions, using the attached R output.

1. (2 points) Let μ be the expected value of r_t. Test $H_0 : \mu = 0$ versus $H_a : \mu \neq 0$. Obtain the test statistic and draw your conclusion.

2. (2 points) Is the distribution of r_t skew? Why?

3. (2 points) Does the distribution of r_t have heavy tails? Why?

4. (2 points) Let ρ_1 be the lag-1 ACF of r_t. Test $H_0 : \rho_1 = 0$ versus $H_a : \rho_1 \neq 0$. The sample lag-1 ACF is -0.086. Obtain the test statistic and draw your conclusion.

5. (2 points) An MA(1) model is fitted. Write down the fitted model, including σ^2 of the residuals.

6. (2 points) Is there any ARCH effect in the residuals of the fitted MA(1) model? Why?

7. (2 points) An ARMA(0,1)+GARCH(1,1) model with Gaussian innovations was entertained for r_t. See **m2**. Write down the fitted model.

8. (2 points) Is the fitted ARMA(0,1)+GARCH(1,1) model adequate? Why?

9. (3 points) A GARCH(1,1) model with standardized Student-t innovations was fitted. See **m3**. Write down the fitted model.

10. (3 points) To further improve the model, a GARCH model with skew standardized Student-t innovations was considered. See **m4**. Write down the fitted model.

11. (2 points) Based on the fitted model **m4**. Does the return r_t follow a skew distribution? Why? Perform proper test and draw the conclusion.

12. (2 points) Based on the model **m4**. Obtain 1-step and 2-step 95% interval forecasts for r_t at the forecast origin $t = 1841$.

13. (2 points) An IGARCH model is fitted to r_t. See **m5**. Write down the fitted model, including mean equation.

14. (2 points) Based on the output provided, is the IGARCH model adequate? Why?

15. (2 points) From the IGARCH model, we have $r_{1841} = -0.0955$ and $\sigma_{1841} = 0.745$. Compute the 1-step ahead forecast of r_t and its volatility at the forecast origin $t = 1841$?

16. (3 points) A GARCH-M model is entertained for the r_t series. See **m6**. Write down the fitted model.

17. (2 points) Based on the fitted model **m6**, is the risk premium statistically significant? Why?

18. (4 points) A Threshold GARCH model with standard Student-t innovations is considered. See **m7**. Write down the fitted model.

19. (2 points) Based on the fitted TGARCH model **m7**, is the leverage effect statistically significant? Why?

20. (2 points) Among models { **m7, m4, m2** }, which one is preferred? Why?

Problem C. (12 points) The consumption of natural gas in northern American cities depends heavily on temperature and daily activities. Let y be the daily sendout of natural gas and DHD be the degrees of heating days defined as DHD = $65^o F$ minus daily average temperature. Also, let x_1 = DHD, x_2 = lag-1 DHD, x_3 = windspeed, and x_4 be the indicator variable for weekend. The data were collected for 63 days. Statistical analysis is included in the attached R output. Answer the following questions.

1. (2 points) A multiple linear regression is applied. Write down the fitted model, including the R^2.

2. (3 points) Model checking shows that the residuals have serial correlations, and the AIC selects an AR(8) model. After removing insignificant parameters, we have the model **n5**. Write down the fitted model. Is the model adequate? Why?

3. (3 points) Let ϕ_i be the lag-i AR coefficient. It is seen that $\phi_1 \times \phi_7 \approx 0.2$, which is not far away from $-\phi_8$, especially in view of its standard error. This is indicative of a multiplicative model. Therefore, we fit a seasonal model. Denoted by **n6**. Write down the fitted seasonal model. Is the model adequate? Why?

4. (2 points) Compare models **n5** and **n6**. Which one is preferred? Why?

5. (2 points) The weekend effect is rather significant in the multiple linear regression model of Question 1, but it is not so in the seasonal model. Why?

Problem D. (13 points) Consider the monthly log return of Decile 10 portfolio of CRSP from 1961,1 to 2014.12 with $T = 648$. The returns include dividends. Let r_t denote the monthly log return. Answer the following questions based on the attached R output.

1. (2 points) The ACF of r_t shows $\hat{\rho}_1 = 0.203$ and $\hat{\rho}_{12} = 0.127$. Test $H_0 : \rho_{12} = 0$ versus $H_a : \rho_{12} \neq 0$. Compute the test statistic and draw the conclusion.

2. (2 points) The $\hat{\rho}_{12}$ is likely due to the *January effect*. To remove ρ_{12}, one can use a simple linear regression with January dummy variable. The fitted model is $r_t = 0.0064 + 0.06926\text{Jan}_t + \epsilon_t$. Let $\tilde{r}_t = r_t - 0.06928\text{Jan}_t$ be the adjusted log returns of Decile 10 portfolio. Several models were entertained for $\tilde{r}_t$; see model **g1, g2, g3** and **g4**. Which model is preferred? Why?

3. (2 points) Write down the fitted model **g4**.

4. (2 points) Based on the fitted model **g4**. Is the leverage effect significant? Why?

5. (3 points) The average of fitted volatility is 0.0614 and the 1% quantile of the residuals of model **g4** is -0.171. Compute the ratio $\frac{\sigma_t^2(-0.171)}{\sigma_t^2(0.171)}$ of model **g4**, where $\sigma_t^2(a_t)$ denotes the conditional variance of the series when innovation is a_t.

6. (2 points) Consider the forecasts of model **g4**. Why is the 1-step ahead forecast of $\tilde{r}_t$ different from multi-step ahead forecasts?

R output: edited

```
### Problem B #########
> getSymbols("^IXIC",from="XXXX",to=''XXXX'')
[1] "IXIC"
> rtn=diff(log(as.numeric(IXIC[,6]))) * 100
> require(fBasics)
> basicStats(rtn)
                    rtn
nobs        1841.000000
Mean           0.036063
Median         0.099857
Sum           66.391926
SE Mean        0.035029
LCL Mean      -0.032637
UCL Mean       0.104763
Variance       2.258907
Stdev          1.502966
Skewness      -0.245339
Kurtosis       6.831855

> m0=acf(rtn)
> m0$acf[2]
[1] -0.08602153
> m1=arima(rtn,order=c(0,0,1),include.mean=F)
> m1
Call: arima(x = rtn, order = c(0, 0, 1), include.mean = F)
Coefficients:
          ma1
      -0.0948
s.e.   0.0243

sigma^2 estimated as 2.241:  log likelihood = -3354.9,  aic = 6713.79
> Box.test(m1$residuals,lag=10,type='Ljung')
        Box-Ljung test
data:  m1$residuals
X-squared = 12.9361, df = 10, p-value = 0.2273
> Box.test(m1$residuals^2,lag=10,type='Ljung')
        Box-Ljung test
data:  m1$residuals^2
X-squared = 1507.765, df = 10, p-value < 2.2e-16

> require(fGarch)
> m2=garchFit(~arma(0,1)+garch(1,1),data=rtn,trace=F)
> summary(m2)
Title:  GARCH Modelling
Call:
 garchFit(formula = ~arma(0, 1) + garch(1, 1), data = rtn, trace = F)
```

```
Mean and Variance Equation:
 data ~ arma(0, 1) + garch(1, 1)
 [data = rtn]
Conditional Distribution: norm

Error Analysis:
        Estimate  Std. Error  t value Pr(>|t|)
mu      0.091456    0.022416    4.080 4.50e-05 ***
ma1    -0.036450    0.025636   -1.422    0.155
omega   0.028193    0.007026    4.013 6.00e-05 ***
alpha1  0.104304    0.013844    7.534 4.93e-14 ***
beta1   0.880167    0.014468   60.835  < 2e-16 ***
---
Log Likelihood:
 -2911.304    normalized:  -1.581371

Standardised Residuals Tests:
                                Statistic p-Value
 Jarque-Bera Test   R    Chi^2  93.64382  0
 Shapiro-Wilk Test  R    W      0.9860011 2.046596e-12
 Ljung-Box Test     R    Q(10)  7.810015  0.6473883
 Ljung-Box Test     R    Q(20)  15.9162   0.7218094
 Ljung-Box Test     R^2  Q(10)  17.04423  0.0733911
 Ljung-Box Test     R^2  Q(20)  27.94681  0.1106644
 LM Arch Test       R    TR^2   16.9636   0.1509718

Information Criterion Statistics:
     AIC      BIC      SIC     HQIC
3.168174 3.183161 3.168160 3.173700

> m3=garchFit(~garch(1,1),data=rtn,trace=F,cond.dist="std")
> summary(m3)
Title: GARCH Modelling
Call:
garchFit(formula = ~garch(1,1), data = rtn, cond.dist="std", trace = F)

Mean and Variance Equation:
 data ~ garch(1, 1)
 [data = rtn]
Conditional Distribution: std

Error Analysis:
        Estimate  Std. Error  t value Pr(>|t|)
mu      0.119076    0.022341    5.330 9.82e-08 ***
omega   0.025339    0.008176    3.099  0.00194 **
alpha1  0.106679    0.017710    6.024 1.70e-09 ***
```

```
beta1   0.882429    0.017503   50.417  < 2e-16 ***
shape   7.030196    1.254542    5.604 2.10e-08 ***
---
Log Likelihood:
 -2894.18    normalized:  -1.572069

Standardised Residuals Tests:
                                Statistic p-Value
 Ljung-Box Test     R    Q(10)  11.29792  0.334783
 Ljung-Box Test     R    Q(20)  19.92898  0.4623801
 Ljung-Box Test     R^2  Q(10)  15.97748  0.100279
 Ljung-Box Test     R^2  Q(20)  28.08922  0.1073041
 LM Arch Test       R    TR^2   16.01562  0.1905215

Information Criterion Statistics:
     AIC      BIC      SIC     HQIC
3.149571 3.164557 3.149556 3.155097

> m4=garchFit(~garch(1,1),data=rtn,trace=F,cond.dist="sstd")
> summary(m4)
Title: GARCH Modelling
Call:
 garchFit(formula=~garch(1,1),data=rtn,cond.dist="sstd",trace=F)

Mean and Variance Equation:
 data ~ garch(1, 1)
 [data = rtn]

Conditional Distribution:  sstd

Error Analysis:
        Estimate  Std. Error  t value Pr(>|t|)
mu      0.090509    0.022901    3.952 7.75e-05 ***
omega   0.023284    0.007639    3.048   0.0023 **
alpha1  0.103543    0.016509    6.272 3.57e-10 ***
beta1   0.884482    0.016813   52.607  < 2e-16 ***
skew    0.873450    0.027496   31.767  < 2e-16 ***
shape   8.123793    1.660601    4.892 9.98e-07 ***
---
Log Likelihood:
 -2884.789    normalized:  -1.566968

Standardised Residuals Tests:
                               Statistic p-Value
 Ljung-Box Test     R    Q(10)  11.36986  0.3294374
 Ljung-Box Test     R    Q(20)  19.82094  0.4691798
 Ljung-Box Test     R^2  Q(10)  15.70359  0.1084377
```

```
 Ljung-Box Test     R^2  Q(20)  27.38012  0.1249042

Information Criterion Statistics:
     AIC      BIC      SIC     HQIC
3.140455 3.158439 3.140434 3.147086

> predict(m4,5)
  meanForecast meanError standardDeviation
1   0.09050898 0.7804844         0.7804844
2   0.09050898 0.7906612         0.7906612
3   0.09050898 0.8005892         0.8005892
4   0.09050898 0.8102788         0.8102788
5   0.09050898 0.8197398         0.8197398

> source("Igarch.R")
> m5=Igarch(rtn)
Estimates:  0.9224943
Maximized log-likehood:  2939.672

Coefficient(s):
       Estimate  Std. Error  t value   Pr(>|t|)
beta 0.92249434  0.00841949  109.567 < 2.22e-16 ***
---
> names(m5)
[1] "par"        "volatility"
> vol5=m5$volatility
> rtn[1841]
[1] -0.09549699
> vol5[1841]
[1] 0.744785
> resi=rtn/vol5
> Box.test(resi,lag=10,type="Ljung")
        Box-Ljung test
data:  resi
X-squared = 13.1735, df = 10, p-value = 0.2141
> Box.test(resi^2,lag=10,type="Ljung")
        Box-Ljung test
data:  resi^2
X-squared = 21.4537, df = 10, p-value = 0.01814
>
> source("garchM.R")
> m6=garchM(rtn,type=1)
Maximized log-likehood:  -2915.346

Coefficient(s):
        Estimate  Std. Error  t value   Pr(>|t|)
mu    0.07601900  0.03238316  2.34749   0.018901 *
```

```
gamma 0.01408378  0.02140652  0.65792   0.510589
omega 0.02845490  0.00715017  3.97961 6.9027e-05 ***
alpha 0.10499661  0.01401392  7.49231 6.7724e-14 ***
beta  0.87913601  0.01471637 59.73864 < 2.22e-16 ***
---
> m7=garchFit(~garch(1,1),data=rtn,trace=F,cond.dist="std",leverage=T)
> summary(m7)
Title: GARCH Modelling
Call:
 garchFit(formula = ~garch(1, 1), data = rtn, cond.dist = "std",
  leverage = T, trace = F)

Mean and Variance Equation:
 data ~ garch(1, 1)
 [data = rtn]
Conditional Distribution: std

Error Analysis:
        Estimate  Std. Error  t value Pr(>|t|)
mu      0.078450    0.022038    3.560 0.000371 ***
omega   0.032456    0.007679    4.226 2.37e-05 ***
alpha1  0.052368    0.016678    3.140 0.001690 **
gamma1  0.992492    0.280199    3.542 0.000397 ***
beta1   0.874213    0.016132   54.192  < 2e-16 ***
shape   7.960959    1.584444    5.024 5.05e-07 ***
---
Log Likelihood:
 -2861.373    normalized:  -1.554249

Standardised Residuals Tests:
                                Statistic p-Value
 Ljung-Box Test     R    Q(10)  10.42453  0.40407
 Ljung-Box Test     R    Q(20)  18.25502  0.5706129
 Ljung-Box Test     R^2  Q(10)  18.89127  0.04166563
 Ljung-Box Test     R^2  Q(20)  30.74011  0.05871629
 LM Arch Test       R    TR^2   20.56976  0.05704693

Information Criterion Statistics:
     AIC      BIC      SIC     HQIC
3.115017 3.133000 3.114995 3.121648

##### Problem C ##########
> da=read.table("JW74.DAT",header=T)
> head(da)
  Sendout DHD DHDm1 Windspeed Weekend
1     227  32    30        12       1
2     236  31    32         8       1
```

```
3     228  30    31          8       0
4     252  34    30          8       0
5     238  28    34         12       0
6     195  24    28          8       0
> dim(da)
[1] 63  5
> n1=lm(Sendout~DHD+DHDm1+Windspeed+Weekend,data=da)
> n1
Call:
lm(formula = Sendout ~ DHD + DHDm1 + Windspeed + Weekend, data = da)
> summary(n1)
Call:
lm(formula = Sendout ~ DHD + DHDm1 + Windspeed + Weekend, data = da)

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)   1.8581    11.5561   0.161  0.87282
DHD           5.8742     0.2905  20.219  < 2e-16 ***
DHDm1         1.4052     0.2928   4.799 1.16e-05 ***
Windspeed     1.3154     0.5787   2.273  0.02675 *
Weekend     -15.8571     5.3344  -2.973  0.00429 **
---
Residual standard error: 18.32 on 58 degrees of freedom
Multiple R-squared:  0.9521,    Adjusted R-squared:  0.9488
F-statistic: 288.1 on 4 and 58 DF,  p-value: < 2.2e-16

> Box.test(n1$residuals,lag=10,type='Ljung')
        Box-Ljung test
data:  n1$residuals
X-squared = 48.1743, df = 10, p-value = 5.768e-07
> acf(n1$residuals)
> y=da[,1]
> X=da[,-1]
> pacf(n1$residuals)
> n2=ar(n1$residuals,method="mle")
> n2$order
[1] 8
> n3=arima(y,order=c(8,0,0),xreg=X)
> n3
Call:arima(x = y, order = c(8, 0, 0), xreg = X)

Coefficients:
        ar1     ar2      ar3     ar4      ar5     ar6     ar7      ar8
      0.5758  0.0043  -0.0511  0.2908  -0.3086  0.2350  0.2673  -0.3363
s.e.  0.1323  0.1401   0.1465  0.1352   0.1268  0.1565  0.1367   0.1287
      intercept     DHD   DHDm1  Windspeed   Weekend
        17.0627  5.6777  1.2451     1.2691  -14.8311
```

```
s.e.    14.9832  0.2226  0.2180     0.3755    8.1959

sigma^2 estimated as 161.3:  log likelihood = -250.66,  aic = 529.31
> c1=c(NA,0,0,NA,NA,0,NA,NA,0,NA,NA,NA,NA)
> n4=arima(y,order=c(8,0,0),xreg=X,fixed=c1)
> n4
Call: arima(x = y, order = c(8, 0, 0), xreg = X, fixed = c1)

Coefficients:
         ar1  ar2  ar3     ar4      ar5  ar6     ar7      ar8  intercept
      0.5229    0    0  0.2252  -0.1715    0  0.3746  -0.2835          0
s.e.  0.1069    0    0  0.1231   0.1152    0  0.1121   0.1184          0
         DHD   DHDm1  Windspeed   Weekend
      5.7673  1.4870     1.2958  -15.3126
s.e.  0.1945  0.1923     0.3565    5.9855

sigma^2 estimated as 177.1:  log likelihood = -253.33,  aic = 526.67
> c1=c(NA,0,0,0,0,0,NA,NA,0,NA,NA,NA,NA)
> n5=arima(y,order=c(8,0,0),xreg=X,fixed=c1)
> n5
Call:arima(x = y, order = c(8, 0, 0), xreg = X, fixed = c1)

Coefficients:
         ar1  ar2  ar3  ar4  ar5  ar6     ar7      ar8  intercept     DHD
      0.5362    0    0    0    0    0  0.3677  -0.2613          0  5.7669
s.e.  0.1064    0    0    0    0    0  0.1165   0.1210          0  0.1946
       DHDm1  Windspeed   Weekend
      1.4751     1.3192  -10.1304
s.e.  0.1897     0.3630    6.1649

sigma^2 estimated as 188.1:  log likelihood = -255.07,  aic = 526.13
> Box.test(n5$residuals,lag=10,type='Ljung')
        Box-Ljung test
data:  n5$residuals
X-squared = 7.2835, df = 10, p-value = 0.6984

> n6=arima(y,order=c(1,0,0),seasonal=list(order=c(1,0,0),period=7),xreg=X,
   include.mean=F)
> n6
Call:
arima(x=y,order=c(1,0,0),seasonal=list(order=c(1,0,0),period=7),
    xreg = X, include.mean = F)

Coefficients:
         ar1    sar1     DHD   DHDm1  Windspeed  Weekend
      0.5359  0.3677  5.7651  1.4732     1.2819  -9.5947
s.e.  0.1065  0.1171  0.1999  0.1953     0.3637   6.0260
```

```
sigma^2 estimated as 189.5:  log likelihood = -255.28,  aic = 524.56
> Box.test(n6$residuals,lag=10,type='Ljung')
        Box-Ljung test
data:  n6$residuals
X-squared = 6.5785, df = 10, p-value = 0.7645
>
#### Problem D ####
> da=read.table("m-dec12910-6114.txt",header=T)
> head(da)
      date      dec1      dec2      dec9     dec10
1 19610131  0.058011  0.068040  0.096754  0.087303
2 19610228  0.029241  0.042879  0.056564  0.060040
3 19610330  0.025896  0.025270  0.060563  0.073311
4 19610428  0.005667  0.000877  0.011911  0.025753
5 19610531  0.019208  0.037392  0.046248  0.052023
6 19610630 -0.024670 -0.025332 -0.050651 -0.052041
> dec10=da$dec10
> jan=rep(c(1,rep(0,11)),54)
> m0=acf(dec10)
> m0$acf[c(2,13)]
[1] 0.2031250 0.1271504
> m1=lm(dec10~jan)
> m1
Call:lm(formula = dec10 ~ jan)

Coefficients:
(Intercept)          jan
   0.006396     0.069263

> adj10=dec10-0.069263*jan
> acf(adj10)
> g1=garchFit(~arma(0,1)+garch(1,1),data=adj10,trace=F)
> summary(g1)
Title: GARCH Modelling
Call:
 garchFit(formula = ~arma(0, 1) + garch(1, 1), data = adj10, trace = F)

Mean and Variance Equation:
 data ~ arma(0, 1) + garch(1, 1)
 [data = adj10]

Conditional Distribution: norm

Error Analysis:
        Estimate  Std. Error  t value Pr(>|t|)
mu     0.0070817   0.0027669    2.559  0.01048 *
```

```
ma1    0.2340229   0.0429797    5.445 5.18e-08 ***
omega  0.0002646   0.0001126    2.350  0.01879 *
alpha1 0.0856456   0.0272147    3.147  0.00165 **
beta1  0.8484757   0.0468131   18.125  < 2e-16 ***
---
Log Likelihood:
 900.7496    normalized:  1.390046

Standardised Residuals Tests:
                                Statistic p-Value
 Jarque-Bera Test   R    Chi^2  309.9221  0
 Shapiro-Wilk Test  R    W      0.9706054 4.095619e-10
 Ljung-Box Test     R    Q(10)  2.717944  0.9873051
 Ljung-Box Test     R    Q(20)  14.40794  0.8092247
 Ljung-Box Test     R^2  Q(10)  1.275305  0.9994815
 Ljung-Box Test     R^2  Q(20)  4.427128  0.999894

Information Criterion Statistics:
      AIC       BIC       SIC      HQIC
-2.764659 -2.730139 -2.764777 -2.751268

> g2=garchFit(~arma(0,1)+garch(1,1),data=adj10,trace=F,cond.dist="std")
> summary(g2)
Title: GARCH Modelling
Call:
 garchFit(formula =~arma(0,1)+garch(1,1),data=adj10,cond.dist ="std",
    trace = F)

Mean and Variance Equation:
 data ~ arma(0, 1) + garch(1, 1)
 [data = adj10]

Conditional Distribution: std

Error Analysis:
        Estimate  Std. Error  t value Pr(>|t|)
mu     0.0071115   0.0024374    2.918  0.00353 **
ma1    0.2148526   0.0395801    5.428 5.69e-08 ***
omega  0.0002084   0.0001204    1.730  0.08363 .
alpha1 0.1182122   0.0434399    2.721  0.00650 **
beta1  0.8366444   0.0583986   14.326  < 2e-16 ***
shape  5.4767582   1.1049796    4.956 7.18e-07 ***
---
Standardised Residuals Tests:
                                Statistic p-Value
 Ljung-Box Test     R    Q(10)  3.092116  0.9791723
 Ljung-Box Test     R    Q(20)  14.15442  0.8225807
```

```
 Ljung-Box Test     R^2  Q(10)  2.437512  0.9917554
 Ljung-Box Test     R^2  Q(20)  4.907722  0.9997601
 LM Arch Test       R    TR^2   3.07941   0.9949569

Information Criterion Statistics:
      AIC       BIC       SIC      HQIC
-2.844666 -2.803241 -2.844835 -2.828596

> g3=garchFit(~arma(0,1)+garch(1,1),data=adj10,trace=F,cond.dist="std",
   leverage=T)
> summary(g3)
Title: GARCH Modelling
Call:
 garchFit(formula =~arma(0, 1)+garch(1,1),data = adj10, cond.dist = "std",
    leverage = T, trace = F)

Mean and Variance Equation:
 data ~ arma(0, 1) + garch(1, 1)
 [data = adj10]

Conditional Distribution:  std

Error Analysis:
        Estimate  Std. Error  t value Pr(>|t|)
mu     0.0060302   0.0024882    2.423   0.0154 *
ma1    0.2224592   0.0400198    5.559 2.72e-08 ***
omega  0.0003067   0.0001572    1.950   0.0511 .
alpha1 0.1294974   0.0495784    2.612   0.0090 **
gamma1 0.3189026   0.1376621    2.317   0.0205 *
beta1  0.7898273   0.0729151   10.832  < 2e-16 ***
shape  5.7288196   1.1917451    4.807 1.53e-06 ***
---
Standardised Residuals Tests:
                                Statistic p-Value
 Ljung-Box Test     R    Q(10)  3.126521  0.9782877
 Ljung-Box Test     R    Q(20)  16.21446  0.7032337
 Ljung-Box Test     R^2  Q(10)  3.290507  0.9737376
 Ljung-Box Test     R^2  Q(20)  5.845771  0.9990903

Information Criterion Statistics:
      AIC       BIC       SIC      HQIC
-2.852305 -2.803976 -2.852535 -2.833556

> g4=garchFit(~arma(0,1)+aparch(1,1),data=adj10,trace=F,cond.dist="std",
   delta=2,include.delta=F)
> summary(g4)
Title: GARCH Modelling
```

```
Call:
 garchFit(formula = ~arma(0, 1) + aparch(1, 1), data = adj10,
    delta = 2, cond.dist = "std", include.delta = F, trace = F)

Mean and Variance Equation:
 data ~ arma(0, 1) + aparch(1, 1)
 [data = adj10]

Conditional Distribution: std

Error Analysis:
        Estimate  Std. Error  t value Pr(>|t|)
mu     0.0060302   0.0024882    2.423   0.0154 *
ma1    0.2224592   0.0400198    5.559 2.72e-08 ***
omega  0.0003067   0.0001572    1.950   0.0511 .
alpha1 0.1294974   0.0495784    2.612   0.0090 **
gamma1 0.3189026   0.1376621    2.317   0.0205 *
beta1  0.7898273   0.0729151   10.832  < 2e-16 ***
shape  5.7288196   1.1917451    4.807 1.53e-06 ***
---
Standardised Residuals Tests:
                                Statistic p-Value
 Ljung-Box Test     R    Q(10)  3.126521  0.9782877
 Ljung-Box Test     R    Q(20)  16.21446  0.7032337
 Ljung-Box Test     R^2  Q(10)  3.290507  0.9737376
 Ljung-Box Test     R^2  Q(20)  5.845771  0.9990903
 LM Arch Test       R    TR^2   4.740345  0.9660941

Information Criterion Statistics:
      AIC       BIC       SIC      HQIC
-2.852305 -2.803976 -2.852535 -2.833556
> predict(g4,4)
  meanForecast  meanError standardDeviation
1  0.013232245 0.05388801        0.05388801
2  0.006030204 0.05619847        0.05490500
3  0.006030204 0.05715694        0.05583665
4  0.006030204 0.05803645        0.05669161

> v4=volatility(g4)
> r4=residuals(g4)
> mean(v4)
[1] 0.06144349
> quantile(r4,prob=c(0.01,0.99))
        1%        99%
-0.1712219  0.1462078
```