Financial Econometrics
Lecture 3: Seasonal Time Series

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Introduction

Seasonal time series is time series with periodic patterns and useful in:

- predicting quarterly earnings
- pricing weather-related derivatives
- analysis of transactions data (high-frequency data), e.g., U-shaped pattern in intraday trading intensity, volatility, etc.

Examples of Seasonal Time-Series

Example 1. Monthly U.S. Housing Starts from January 1959 to February 2012. The data are in thousand units. See Figure 1 and compute the sample ACF of the series and its differenced data.

Example 2. Quarterly earnings of Johnson & Johnson See the time plot, Figures 2 and 3, and sample ACFs

Example 3. Quarterly earning per share of Coca Cola from 1983 to 2009.

Multiplicative model

Consider the housing-starts series:
Let y_t be the monthly data. Denoting 1959 as year 0, we can write the time index as t = year + month, e.g, $y_1 = y_{0,1}, y_2 = y_{0,2}$, and $y_{14} = y_{1,2}$, etc. The multiplicative model is based on the following consideration:

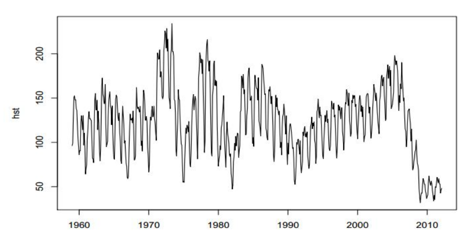


Figure 1: Time plot of monthly U.S. housing starts: 1959.1-2012.2. Data obtained from US Bureau of the Census.

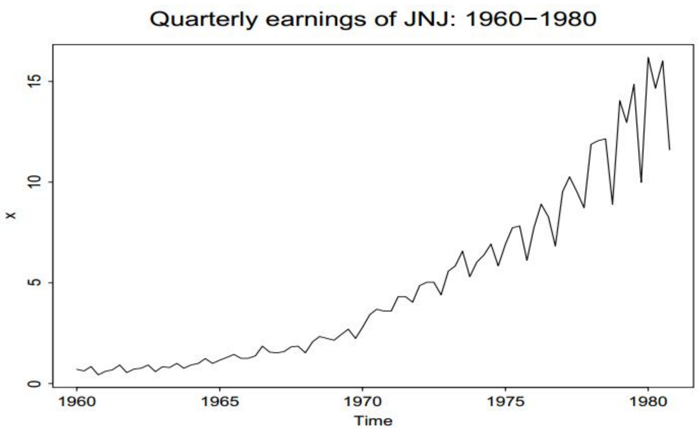


Figure 2: Time plot of quarterly earnings of Johnson and Johnson: 1960-1980

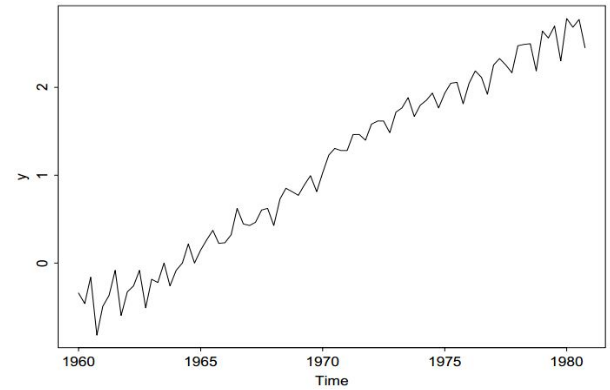


Figure 3: Time plot of quarterly logged earnings of Johnson and Johnson: 1960-1980

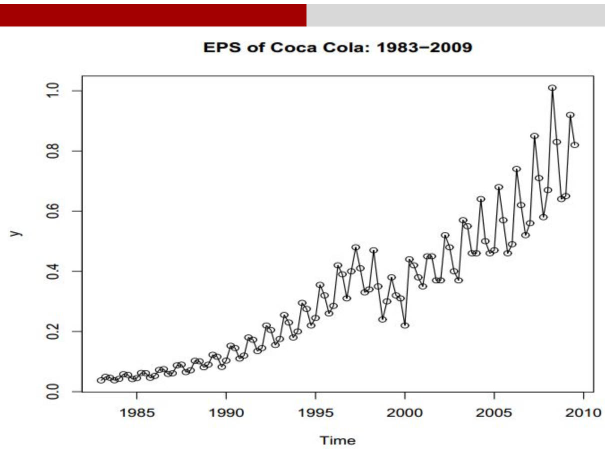


Figure 4: Time plot of quarterly earnings per share of KO (Coca Cola) from 1983 to 2009

Multiplicative model

	Month						
Year	Jan	Feb	Mar	$\dots$	Oct	Nov	Dec
1959	$y_{0,1}$	$y_{0,2}$	$y_{0,3}$	$\cdots$	$y_{0,10}$	$y_{0,11}$	$y_{0,12}$
1960	$y_{1,1}$	$y_{1,2}$	$y_{1,3}$	$\cdots$	$y_{1,10}$	$y_{1,11}$	$y_{1,12}$
1961	$y_{2,1}$	$y_{2,2}$	$y_{2,3}$	$\cdots$	$y_{2,10}$	$y_{2,11}$	$y_{2,12}$
1962	$y_{3,1}$	$y_{3,2}$	$y_{3,3}$	$\cdots$	$y_{3,10}$	$y_{3,11}$	$y_{3,12}$
$\vdots$	$\vdots$	$\vdots$	$\vdots$		$\vdots$	$\vdots$	$\vdots$

The column dependence is the usual lag-1, lag-2, ... dependence. That is, monthly dependence. We call them the regular dependence. The row dependence is the year-to-year dependence. We call them the seasonal dependence.
Multiplicative model says that the regular and seasonal dependence are *orthogonal* to each other.

Airline model for quarterly series

- Form:

$$r_t - r_{t-1} - r_{t-4} + r_{t-5} = a_t - \theta_1 a_{t-1} - \theta_4 a_{t-4} + \theta_1\theta_4 a_{t-5}$$

  or

$$(1-B)(1-B^4)r_t = (1-\theta_1 B)(1-\theta_4 B^4)a_t$$

- Define the differenced series w_t as

$$w_t = r_t - r_{t-1} - r_{t-4} + r_{t-5} = (r_t - r_{t-1}) - (r_{t-4} - r_{t-5}).$$

  It is called *regular* and *seasonal* differenced series.

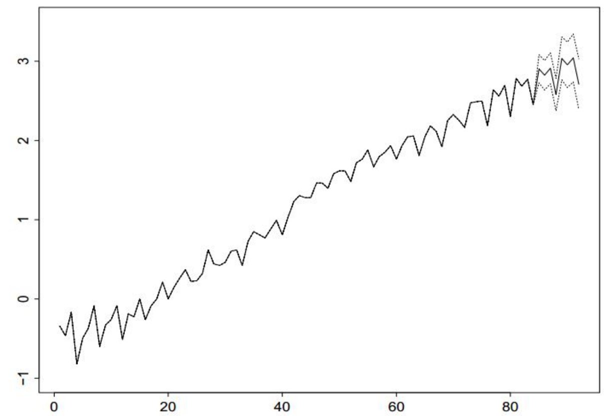


Figure 5: Forecast plot for the quarterly earnings of Johnson and Johnson. Data: 1960-1980, Forecasts: 1981-82.

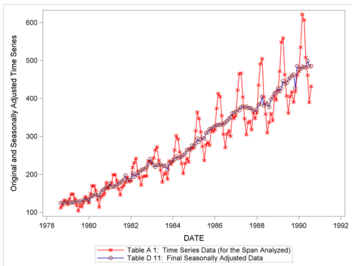
Airline model for quarterly series

- ACF of wt has a nice symmetric structure (see the text), i.e. $\rho_{s-1} = \rho_{s+1} = \rho_1\rho_s$. Also, $\rho_\ell = 0$ for $\ell > s+1$.
- This model is widely applicable to several seasonal time series.
- Multiplicative model means that the regular and seasonal dependences are roughly orthogonal to each other.
- Forecasts: exhibit the same pattern as the observed series. See Figure 5.
- Exponential Smoothing method

Seasonal Adjustment Using X11/X12 Procedure

- Receive a time-series and remove the seasonal component
- X11: Applies symmetric moving averages to a time series in order to estimate the trend, seasonal and irregular components.
- X12: Developed by U.S. Census Bureau
- Available for free on “Gretl” package

Example of X12 Output



Time-Series Before and After the X12 Procedure

- Source of Plot: SAS (`http://support.sas.com/documentation/cdl/en/etsug/63939/HTML/default/viewer.htm#etsug_x12_sect003.htm`)

Regression Models with Time Series Errors

Question: Why don't we use R-square in this course?

- Has many applications
- Impact of serial correlations in regression is often overlooked. It may introduce biases in estimates and in standard errors, resulting in unreliable t-ratios.
- Detecting residual serial correlation: Use Q-stat instead of DWstatistic, which is not sufficient!
- Joint estimation of all parameters is preferred
- Avoid the problem of spurious regression. **R-square** can be misleading!!!

Long-memory models

- Meaning? ACF decays to zero very slowly!
- Example: ACF of squared or absolute log returns ACFs are small, but decay very slowly.
- How to model long memory? Use "fractional" difference: namely, $(1-B)^d r_t$, where $-0.5 < d < 0.5$.
- Importance? In theory, Yes. In practice, yet to be determined.
- In R, the package **rugarch** may be used to estimate the fractionally integrated ARMA models. The package can also be used for GARCH modeling.

Summary of the chapter

- Sample ACF $\Rightarrow$ MA order
- Sample PACF $\Rightarrow$ AR order
- Some packages have "automatic" procedure to select a simple model for "conditional mean" of a FTS, e.g., R uses "ar" for AR models.
- Check a fitted model before forecasting, e.g. residual ACF and hetroscedasticity (chapter 3)

Summary of the chapter

- Interpretation of a model, e.g. constant term & For an AR(1) with coefficient ϕ_1, the speed of mean reverting as measured by half-life is
$$k = \frac{\ln(0.5)}{\ln(|\phi_1|)}$$
For an MA(q) model, forecasts revert to the mean in $q+1$ steps.

- Make proper use of regression models with time series errors, e.g. regression with AR(1) residuals.
  - Perform a joint estimation instead of using any two-step procedure, e.g. Cochrane-Orcutt (1949).

- Basic properties of a random-walk model

- Multiplicative seasonal models, especially the so-called airline model.