COMPILER DESIGN LAB: CS47L

Implementation of a bottom up parser for a given grammar

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1 Introduction to Bottom-up Parsing

A bottom-up parse corresponds to the construction of a parse tree for an input string beginning at the leaves (the bottom) and working up towards the root (the top). They are also known as LR parsers or shift-reduce parsers. The order in which they parse a string is equivalent to the right most derivation of the string in the reverse order. An example of bottom up parsing for the expression id * id is shown below:

A bottom-up parser parses the string by applying two operations: Reduction and Handle pruning

REDUCTION A reduction is the reverse of a step in a derivation (recall that in a derivation, a non terminal in a sentential form is replaced by the body of one of its productions). The goal of bottom-up parsing is therefore to construct a derivation in reverse. We can think of bottom-up parsing as the process of reducing" a string w to the start symbol of the grammar. At each reduction step, a specific substring matching the body of a production is replaced by the non terminal at the head of that production.

The key decisions during bottom-up parsing are about when to reduce and about what production to apply, as the parse proceeds

HANDLE PRUNING Bottom-up parsing during a left-to-right scan of the input constructs a rightmost derivation in reverse. Informally, a handle is a substring that matches the body of a production, and whose reduction represents one step along the reverse of a rightmost derivation.

An example is shown below:

Right sententinal form	Handle	Reducing production
$id_1 * id_2$	id_1	F -> id
$\mathbf{F} * \mathbf{id}_2$	F	T -> F
$\mathbf{T} * \mathbf{id}_2$	id_2	F -> id
T * F	T * F	T -> T * F
T	T	E -> T

2 SHIFT REDUCE PARSER

Shift-reduce parsing is a form of bottom-up parsing in which a stack holds grammar symbols and an input buffer holds the rest of the string to be parsed. A shift-reduce parser can, at any moment, perform 4 operations:

- Shift: Shift the next input symbol onto the top of the stack.
- Reduce: The right end of the string to be reduced must be at the top of the stack. Locate the left end of the string within the stack and decide with what non terminal to replace the string
- Accept: Announce successful completion of parsing
- Error: Discover a syntax error and call an error recovery routine

We use two types of shift-reduce parsers, canonical-LR and lookahead-LR parser, to solve our problem. Even though, the algorithm for the parser is the same for both types of parsers, the way in which the parse table is created is different.

3 Constructing LR(1) set of items

FIRST **FIRST**(*X*) for a grammar symbol X is the set of terminals that begin the strings derivable from X.

PROCEDURE The method of construction of LR(1) set of items is essentially the same as that of construction of LR(0) items, only this time, we keep track of the symbols that could appear after the application of a particular production. We present a function to find the **CLOSURE** of a set of states and a function to generate the **GOTO** of a set of states for some grammar G as follows:

Now to generate set of LR(1) items, we use the following procedure:

```
SetOfLR1Items items(G : Grammar) {
    C = {CLOSURE({[S' -> .S, $$]})}
    repeat {
        for (each set of items I in C)
            for (each grammar symbol X in G)
                  if (GOTO(I, X) is not empty and doesn't exist in C)
                  add GOTO(I, X) to C;
    } until (no new set of items are added to C);
    return C;
}
```

4 LR Parsing Algorithm

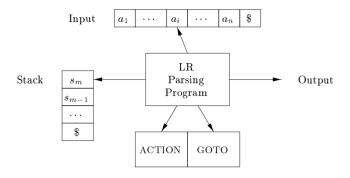


Figure 4.1: Schematic diagram of LR Parser

An LR Parser consists of an input buffer, an output, a stack, a driver program, and a parsing table that has two parts (ACTION and GOTO). The driver program is the same for all LR parsers; only the parsing table changes from one parser to another. The parsing program reads characters from an input buffer one at a time. Where a shift-reduce parser would shift a symbol, an LR parser shifts a state. Each state summarizes the information contained in the stack below it.

4.1 STRUCTURE OF LR PARSE TABLE

The parsing table consists of two parts: a parsing-action function **ACTION** and a goto function **GOTO**.

- The **ACTION** function takes as arguments a state i and a terminal a (or \$, the input end-marker). The value of **ACTION**[i, a] can have one of four forms:
 - Shift j, where j is a state. The action taken by the parser effectively shifts input a to the stack, but uses state j to represent a.
 - Reduce $A \rightarrow b$. The action of the parser effectively reduces b on the top of the stack to head A.
 - Accept. The parser accepts the input and finishes parsing.
 - Error. The parser discovers an error in its input and takes some corrective action.
- We extend the **GOTO** function, defined on sets of items, to states: if **GOTO**[I_i , A] = I_j , then **GOTO** also maps a state i and a non-terminal A to state j.

4.2 THE ALGORITHM

INPUT: An input string w and an LR-parsing table with functions **ACTION** and **GOTO** for a grammar G. **OUTPUT**: If w is in L(G), the reduction steps of a bottom-up parse for w; otherwise, an error indication. **METHOD**: Initially, the parser has s_0 on its stack, where s_0 is the initial state, and w\$ in the input buffer.

```
let a be the first symbol of w$;
while (1) {
    let s be the state on top of the stack;
    if (ACTION[s, a] == shift t) {
        push t onto the stack;
        let a be the next input symbol;
    } else if (ACTION[s, a] == reduce A -> b) {
        pop |b| symbols off the stack;
        let state t now be on top of the stack;
        push GOTO[t, A] onto the stack;
        output the production A -> b
    } else if (ACTION[s, a] == accept) break; // successful parse else call error-recovery routine;
}
```

5 OUR IMPLEMENTATION

5.1 Data-structures used

- For the parsing algorithm, we use a **stack** as our data-structure to keep track of the state of the parser. We use the **std**:: **stack** class that comes with the C++ standard library.
- To hold the **ACTION** and **GOTO** map, we use a dictionary. This could be implemented using a hash-map or a binary search tree. We chose to use C++ standard library's **std**:: **map**.
- We also use arrays through out the program to hold list of items as well as sets.

5.2 Code

```
1 #include <iostream>
 2 #include <iomanip>
 3 #include <string>
 4 #include <stack>
 5 #include <map>
 6 #include <set>
 7 #include <utility >
 8 #include <vector>
 9
    #include <sstream>
10 #include <algorithm>
11
    #include <functional>
12
13
    using namespace std;
14
    // FOR TREE DRAWING
15
    class TextBox {
16
17
    public:
18
        struct xy {
19
            TextBox& tb;
20
            int x, y;
21
22
            xy operator << (const string& s) const {
23
                 return \{tb, x + tb.puts(x, y, s), y\};
24
25
26
            xy operator<< (const int i) const {
27
                 string s = to_string(i);
28
                 return \{tb, x + tb.puts(x, y, s), y\};
29
            }
30
31
            xy operator << (const char c) const {
32
                 tb.putc(x, y, c);
33
                 return \{tb, x + 1, y\};
34
35
36
            xy operator<< (const TextBox& _tb) const {
37
                 tb.puttb(x, y, _tb);
38
                 return \{tb, x + \_tb.width(), y\};
39
            }
40
        };
41
42
        string str() const {
43
            stringstream ss;
44
            for (auto& s : m_buffer) {
45
                 //ss \ll s \ll ' n';
46
                 for (char c : s) {
47
                     if (c > 0b1111)
48
                         ss << c;
49
                     else {
50
                         switch (static_cast<int>(c) & 0b1111) {
51
                             case 0b01:
52
                                  ss << '-';
53
                                  break;
54
                             case 0b10:
                                  ss << '|';
55
56
                                  break;
```

```
case 0b11:
                             ss << '.';
                             break;
                        default:
                             ss \ll c;
                }
            }
            ss << '\n';
        return ss.str();
    TextBox::xy operator() (int x, int y) const {
        return {const_cast<TextBox&>(*this), x, y};
    int width() const {
        int w = -1;
        for (auto& s : m_buffer)
            w = max(w, static\_cast < int > (s.size()));
        return w;
    int height() const {
        return m_buffer.size();
    void hline(int x, int y, int width) {
        if (y >= m_buffer.size())
            m_buffer.resize(y+1);
        if (x+width >= m_buffer[y].size())
            m_buffer[y].resize(x+width, '');
        for (int i = 0; i < width; i++) {
            if (m_buffer[y][x+i] > 0b1111)
                m_buffer[y][x+i] = 0;
            m_buffer[y][x+i] = 0b1;
        }
    }
    void vline(int x, int y, int height) {
        if (y+height >= m_buffer.size())
            m_buffer.resize(y+height);
        for (int i = 0; i < height; i++) {
            if (x \ge m_buffer[y+i].size())
                m\_buffer[y+i].\,resize(x+1,\ '\ ');
            if (m_buffer[y+i][x] > 0b1111)
                m_buffer[y+i][x] = 0;
            m_buffer[y+i][x] = 0b10;
        }
private:
    void putc(int x, int y, char c) {
        if (y >= m_buffer.size())
            m_buffer.resize(y+1);
        if (x >= m_buffer[y].size())
            m_buffer[y].resize(x+1, '');
        m_buffer[y][x] = c;
    int puts(int x, int y, const string& s) {
        if (s.empty()) return x;
        for (int i = 0; i < s.size(); i++)
            putc(x+i, y, s[i]);
        return x + s.size();
    }
```

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 $\frac{117}{118}$

119 120 121

122

123

124

125

126

```
128
         void puttb(int x, int y, const TextBox& tb) {
129
             for (int i = 0; i < tb.height(); i++) {
130
                  puts(x, y+i, tb.m_buffer[i]);
131
132
         }
133
134
     private:
135
         vector<string> m_buffer;
136
137
138
139
     struct Tree {
140
         struct TreeNode {
             string data;
             vector<TreeNode> children;
142
143
144
             TreeNode(string _data) : data(_data) {}
145
             void add_child(string c) {
146
                  children.emplace_back(c);
147
148
149
         };
150
             bool rightmost_add(string& lhs, string& rhs) {
152
                      bool added = false;
                      _rightmost_add(root, lhs, rhs, added);
153
154
                      return added;
155
             }
156
             void _rightmost_add(TreeNode& n, string& lhs, string& rhs, bool& added) {
157
                      if (added) return;
158
159
160
                      if (n.children.size() == 0) {
161
                               if (n.data == lhs) {
                                       if (rhs != "id") {
162
                                               for (char c : rhs) n.children.emplace_back(string(1, c));
163
164
                                       } else {
                                               n.children.emplace_back("id");
165
166
167
                                       added = true;
168
                              }
169
                      }
170
                      for (auto i = n.children.rbegin(); i < n.children.rend(); i++) {
171
                               _rightmost_add(*i, lhs, rhs, added);
172
173
                      }
             }
174
175
176
         TreeNode root;
177
178
         Tree(string _root) : root(_root) {}
179
180
         friend ostream& operator<< (ostream& os, Tree& tree) {</pre>
181
             TextBox tb = create_tree_textbox(tree.root);
182
             os << tb.str();
183
             return os;
184
185
     private:
186
         static TextBox create_tree_textbox(const TreeNode& node) {
187
             TextBox tb;
188
             constexpr int padding = 2;
             tb(0, 0) \ll node.data;
189
190
              if (node.children.empty())
191
                  return tb;
192
             vector<TextBox> child_tbs;
193
             for (auto child : node.children)
194
195
                  child_tbs.push_back(create_tree_textbox(child));
196
             tb.vline(0, 1, 1);
197
198
             int i = 0;
```

```
199
              for (auto child : child_tbs) {
200
                  tb.vline(i, 2, 2);
201
                  tb(i, 4) \ll child;
202
                  i += child.width() + padding;
203
              tb.hline(0,\ 2,\ i\ -\ child\_tbs[child\_tbs.size()\ -\ 1].width()\ -\ padding\ +\ 1);
204
205
206
              return tb;
207
208
     };
209
210
     // TREE DRAWING END
211
212
213
     #define TOKEN_CASE(chr, tkn) \
214
              case chr: \
                      cur = lookahead + 1; \
215
216
                      return tkn:
217
     template <class T, class Container = deque<T>>>
218
     class printable_stack : public stack<T, Container> {
219
220
              friend ostream& operator << (ostream& os, const printable_stack < T, Container >& stk) {
221
                      stringstream ss;
222
                      ss << "[";
223
                      for (auto i : stk.c)
                               ss << " " << i;
                      ss << " ]";
226
                      os << ss.str();
227
                      return os:
228
              }
229
    };
230
231
    enum class Token {
232
              ID, PLUS, MULT, BRACKET_OPEN, BRACKET_CLOSE, EOI, ERR
233
234
     string token_to_str(const Token& token) {
235
236
              switch(token) {
237
                      case Token::ID:
238
                               return "<ID>";
239
                      case Token::PLUS:
240
                               return "<PLUS>";
241
                      case Token::MULT:
242
                               return "<MULT>";
243
                      case Token::BRACKET_OPEN:
                               return "<BRACKET_OPEN>";
244
                      {\bf case} \ \ {\bf Token::BRACKET\_CLOSE:}
245
                               return "<BRACKET_CLOSE>";
246
247
                      case Token::EOI:
248
                               return "<$>";
249
                      case Token::ERR:
250
                               return "<ERROR>";
251
                      default:
252
                               return "unrecogonized token";
253
              }
254
     }
255
     class Lexer {
256
     public:
257
258
              Lexer(const string input) {
259
                      input_buffer = input;
260
              }
261
262
              Token next() {
263
                      if (cur >= input_buffer.size()) {
264
                               // No more input to read, return end of input token
265
                               return Token::EOI;
266
                      }
267
268
                      // Ignore whitespace
                      while (input_buffer[cur] == ' ' || input_buffer[cur] == '\t') cur++;
269
```

```
270
271
                       int lookahead = cur;
272
                       switch(input_buffer[lookahead]) {
                               TOKEN_CASE('+', Token::PLUS)
TOKEN_CASE('*', Token::MULT)
TOKEN_CASE('(', Token::BRACKET_OPEN)
273
274
275
                                TOKEN_CASE(')', Token::BRACKET_CLOSE)
276
                                TOKEN_CASE('\0', Token::EOI)
277
                                case 'i':
278
279
                                         if (input_buffer[++lookahead] == 'd') {
280
                                                 cur = lookahead + 1;
281
                                                 return Token::ID;
282
                                        }
283
                                         error();
284
                                        return Token::ERR;
285
                                default:
286
                                         error();
287
                                         return Token::ERR;
288
                       }
289
290
291
              friend ostream& operator << (ostream& os, const Lexer& lex) {
292
                       stringstream ss;
                       for (int i = (lex.input_buffer[lex.cur] == ' ')? lex.cur + 1 : lex.cur; i < lex.input_buffer.size(); i+-
293
                       ss << " $";
294
295
                       os << ss.str();
296
                       return os:
297
              }
298
299
              void display_current_state(const string& msg) {
                       cout << input_buffer << endl;</pre>
300
                       for(int i = 0; i < cur; i++) cout << '';
301
                       cout << \ \text{"$\wedge$} \ \text{"} << msg << endl;
302
303
304
305
     private:
              void error() {
306
307
                       cout << "Lex error: " << endl << input_buffer << endl;</pre>
308
                       for(int i = 0; i < cur; i++) cout << '';
309
                       cout << "^ error occured while trying to lex." << endl;
310
              }
311
312
              int cur = 0;
313
              string input_buffer;
314
315
     class Action {
316
317
     public:
318
              enum ActionType {
319
                       Shift,
320
                       Reduce,
321
                       Accept,
322
                       Error
323
              };
324
325
              ActionType type;
326
     };
327
328
     class ShiftAction : public Action {
329
     public:
330
              ShiftAction(int shiftState)
331
                       : shift_state(shiftState) {
332
                       type = Action::Shift;
333
334
              int shift_state;
335
336
337
     class ReduceAction : public Action {
     public:
338
339
              ReduceAction(const string& productionLhs, const string& productionRhs, int i)
                       :production_lhs(productionLhs), production_rhs(productionRhs), pop_amt(productionRhs == "id" ? 1 : production
340
```

```
341
                                      type = Action::Reduce;
342
343
                       int production_id;
344
                       int pop_amt;
345
                       string production_lhs;
346
                       string production_rhs;
347
        }:
348
349
        class Parser {
350
        public:
351
                       Parser(map<pair<int, Token>, Action*> actionMap, map<pair<int, string>, int> gotoMap)
352
                                      :action_map(actionMap), goto_map(gotoMap) {
353
                                      parse_stack.push(0);
354
                       }
355
356
                       bool parse(const string& input) {
357
                                      Lexer lex(input);
                                      cout << left << setw(25) << "Stack"</pre>
                                                                                                              << setw(25) << "Current Token" << setw(25) << "Input" << setw(25)
358
                                                                                                                                                                       << setw(25) << lex
                                      cout << left << setw(25) << parse_stack << setw(25) << "- "
359
                                                                                                                                                                                                                 << setw(25
360
                                      Token a = lex.next();
361
                                      while (true) {
                                                     if (a == Token::ERR) return false;
362
363
                                                    int s = parse_stack.top();
364
                                                     if (action_map[{s, a}]->type == Action::Shift) {
365
                                                                   ShiftAction* sa = reinterpret_cast<ShiftAction*>(action_map[{s, a}]);
366
                                                                   parse_stack.push(sa->shift_state);
367
                                                                   cout << left << setw(25) << parse_stack << setw(25) << token_to_str(a) << setw(25) << le
368
                                                                   a = lex.next();
369
                                                    } else if (action_map[{s, a}]->type == Action::Reduce) {
370
                                                                   ReduceAction* ra = reinterpret_cast<ReduceAction*>(action_map[{s, a}]);
371
                                                                   for (int i = 0; i < ra \rightarrow pop_amt; i++) parse_stack.pop();
372
                                                                   int t = parse_stack.top();
373
                                                                   if (goto_map[\{t, ra->production_lhs\}] == -1) {
374
                                                                                  error(a, lex);
375
                                                                                  return false;
376
377
                                                                   parse_stack.push(goto_map[{t, ra->production_lhs}]);
378
                                                                   cout << left << setw(25) << parse_stack << setw(25) << token_to_str(a) << setw(25) << le
                                                                    //cout << "Using production" << ra->production\_lhs << " -> " << ra->production\_rhs << e <= color="block" formula | color="bl
379
380
                                                                   production_stack.push({ra->production_lhs, ra->production_rhs});
381
                                                    } else if (action_map[{s, a}]->type == Action::Accept) {
382
                                                                   cout << left << setw(25) << parse_stack << setw(25) << token_to_str(a) << setw(25) << le
383
                                                                   Tree pt = create_parse_tree();
                                                                   cout << "\nThe parse tree for the string is : \n" << pt << "\n";
384
385
                                                                   return true;
386
                                                    } else {
                                                                   error(a, lex);
387
388
                                                                   return false;
389
                                                    }
390
                                      }
391
        private:
392
393
                       printable_stack<int> parse_stack;
394
                       map<pair<int, Token>, Action*> action_map;
395
                       map<pair<int, string>, int> goto_map;
396
                       stack<pair<string, string>> production_stack;
397
398
                       void error(Token cur_token, Lexer& lex) {
399
                                      cout << "Encountered error while parsing : Unexpected token " << token_to_str(cur_token) << endl;</pre>
400
                                      lex.display_current_state("Parse error");
401
                       }
402
403
                       void print_state() {
404
                                      cout << "This is test string lol this string is supposed to be very big lol lets see";
                                      cout << "lol " << endl;
405
406
407
                       Tree create_parse_tree() {
408
                                      Tree parse_tree("E");
409
410
                                      while (!production_stack.empty()) {
```

```
411
                              auto prod = production_stack.top(); production_stack.pop();
412
                               parse_tree.rightmost_add(prod.first, prod.second);
413
414
                      return parse_tree;
415
             }
416
     };
417
418
     #define ERR_ACTN new Action {.type = Action::Error}
419
     #define ACC_ACTN new Action {.type = Action::Accept}
420
     #define SHFT_ACTN(i) new ShiftAction(i)
421
     #define REDC_ACTN(i) new ReduceAction(str_productions[i][0], str_productions[i][1], i)
422
423
     typedef pair<string, string> production;
424
425
     struct Item {
426
             Item(string 1, string r, string la = "$") : lhs(1), rhs(r), dot_idx(0), lookahead(la) \{ \} \}
427
             Item(string 1, string r, int di, string la = "$") : lhs(l), rhs(r), dot_idx(di), lookahead(la) {}
428
         Item(production \ p, \ string \ la = "$") : lhs(p.first), \ rhs(p.second), \ dot\_idx(0), \ lookahead(la) \ \{\} \}
429
         Item(production p, int di, string la = "$") : lhs(p.first), rhs(p.second), dot_idx(di), lookahead(la) {}
430
431
         bool operator==(const Item& other) const {
432
              return (lhs == other.lhs) && (rhs == other.rhs) && (dot_idx == other.dot_idx) && (lookahead == other.lookahead);
433
434
435
         bool operator < (const Item& other) const {
             return (lhs < other.lhs) && (rhs < other.rhs) && (dot_idx < other.dot_idx) && (lookahead < other.lookahead);
436
437
438
439
         friend ostream& operator << (ostream& os, const Item& i) {
440
             int idx = i.dot_idx;
              os << "[" << i.lhs << " -> " << i.rhs.substr(0, idx) << "." << i.rhs.substr(idx) << ", " << i.lookahead << "]";
441
442
443
444
445
              string lhs;
446
              string rhs;
         string lookahead = "$";
447
             int dot_idx;
448
449
     };
450
451
     string str_productions[7][2] = {
                                    ', "E"},
452
                               {"E'"
                                    , "E+T"},
                               { "E"
453
                                      "T"},
                               { "E"
454
                                      "T*F"},
                               { "T" }
455
                               { "T" }
                                    , "F"},
456
                                    , "(E)"},
                               {"F"
457
                               {"F" , "id"}
458
459
                      };
460
461
     struct Grammar {
462
         Grammar(vector<production> p) : productions(p) {
463
464
              for (auto& i : p)
465
                  non_terminals.insert(i.first);
466
              generate_lr1_items();
467
         }
468
469
         set<string> first(string s) {
              set < string > first_set;
470
471
              for (char c : s) {
                  if (non_terminals.find(string(1, c)) == non_terminals.end()) {
472
                      first_set.insert(c != 'i' ? string(1,c) : "id");
473
                      break;
474
475
                  } else {
476
                      string nt = string(1, c);
                      for (auto p: productions) if (p.first == nt && p.second[0] != nt[0]) {
477
478
                          auto tmp = first(p.second);
479
                           first_set.insert(tmp.begin(), tmp.end());
480
                      }
481
                  }
```

```
482
483
                                  return first_set;
484
485
486
                        vector<Item> closure(vector<Item> I) {
                                  bool done = false;
487
488
                                  while (!done) {
489
                                             done = true;
490
                                             for (int i = 0; i < I.size(); i++) {
                                                       Item item = I[i];
491
492
                                                        if (non_terminals.find(item.rhs.substr(item.dot_idx, 1)) != non_terminals.end()) {
493
                                                                  string nt = item.rhs.substr(item.dot_idx, 1);
494
                                                                  string remaining = item.rhs.substr(item.dot_idx+1) + item.lookahead;
495
                                                                  for (auto p : productions) if (p.first == nt) {
496
                                                                             auto s = first(remaining);
                                                                              \  \  \, \text{for} \  \  \, (\text{auto} \ k \ : \ s) \  \  \{
                                                                                       auto u = Item(p, k);
498
499
                                                                                       bool exists = false;
500
                                                                                       for (auto\& x : I) {
501
                                                                                                  if (x.dot_idx == u.dot_idx && x.lhs == u.lhs && x.rhs == u.rhs && x.lookahead == u.looka
502
503
                                                                                                            break;
504
505
                                                                                       if (!exists) {
506
                                                                                                  I.push_back(u);
508
                                                                                                 done = false;
509
510
                                                                            }
511
                                                                  }
512
                                                       }
513
                                            }
514
515
                                  return I;
516
517
                        vector<Item> Goto(vector<Item> I, string X) {
518
                                  vector<Item> J;
519
520
                                  for (auto i : I) {
521
                                              \textbf{if} \quad (i.rhs[i.dot_idx] == X[0]) \quad J.push_back(Item(i.lhs, i.rhs, X == "id" ? i.dot_idx + 2 : i.dot_idx + 1, i.look ) \\ \\ \textbf{i.dot}_idx + \textbf{i.dot}_idx +
522
523
                                  return closure(J);
524
                        }
525
                       bool is_in_item_set(vector<Item> set) {
526
527
                                  for (auto soi : item_set) {
528
                                             if (set == soi) return true;
529
530
                                  return false;
531
532
                        void generate_lr1_items() {
533
534
                                  item_set.push_back(closure({Item(productions[0])}));
                                  vector < string > \ grammar\_symbols \ = \ \{"E", "T", "F", "(", "id", ")", "+", "*", \};
535
536
                                  bool done = false;
537
                                  while (!done) {
538
                                             done = true;
539
                                             for (int i = 0; i < item_set.size(); i++) {</pre>
540
                                                       for (auto symb : grammar_symbols) {
                                                                  auto g = Goto(item_set[i], symb);
541
542
                                                                  if (g.size() != 0 && !is_in_item_set(g)) {
                                                                             goto_history.push_back({i, symb});
543
544
                                                                             item_set.push_back(g);
545
                                                                            done = false;
546
                                                                  } else if (g.size() != 0) {
                                                                                                                                            int k;
                                                                                                                                            for (k = 0; k < item_set.size() \&\& item_set[k] != g; k++);
548
549
                                                                                                                                            existing_goto_history.push_back({{i, symb}, k});
550
                                                                                                                       }
                                                       }
552
                                             }
```

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```
clean_item_set();
void clean_item_set() {
    for (auto\& x : item\_set) {
                    vector<Item> cleaned;
                    for (auto i : x)  {
                            bool exists = false;
                            for (auto k : cleaned) {
                                     if (k.lhs == i.lhs && k.rhs == i.rhs && k.dot_idx == i.dot_idx) {
                                             exists = true;
                                             break;
                                     }
                            if (exists) continue;
                            Item ni(i.lhs, i.rhs, i.dot_idx, "");
                            set<string> lookaheads;
                            for (auto tmp : x) {
                                     if (tmp.lhs == ni.lhs && tmp.rhs == ni.rhs && tmp.dot_idx == ni.dot_idx) {
                                             lookaheads.insert(tmp.lookahead);
                            ni.lookahead = *(lookaheads.begin());
                            for (auto i : lookaheads)
                                     if (i != *lookaheads.begin()) ni.lookahead = ni.lookahead + "/" + i;
                            cleaned.push_back(ni);
                    x = cleaned;
            }
    map<pair<int, Token>, Action*> action_map() {
            map<pair<int, Token>, Action*> action_map;
            // init action map
            for (auto token : {Token::PLUS, Token::MULT, Token::BRACKET_OPEN, Token::BRACKET_CLOSE, Token::ID, Token
                    for (int i = 0; i < item_set.size(); i++)
                            action_map[{i, token}] = nullptr;
            // add shift actions
            for (int i = 0; i < existing_goto_history.size(); i++) {
                    if (non_terminals.find(existing_goto_history[i].first.second) == non_terminals.end()) {
                            if (action_map[{existing_goto_history[i].first.first, token_map[existing_goto_history[i]
                                    action_map[{existing_goto_history[i].first.first, token_map[existing_goto_history
                    }
            // add reduce actions
            map<string, int> reduction_map = {
                    {"E", 0}, {"E+T", 1}, {"T", 2}, {"T*F", 3}, {"F", 4}, {"(E)", 5}, {"id", 6}
            };
            for (int i = 2; i < item_set.size(); i++) {
                    for (auto item : item_set[i]) {
                            if (item.dot_idx == item.rhs.size()) {
                                     for (char c : item.lookahead) if (c != '/') {
                                             string lah(1, c);
                                             action_map[{i, token_map[lah]}] = REDC_ACTN(reduction_map[item.rhs]);
                                     }
                            }
                    }
            // add accept action
            action_map[{1, Token::EOI}] = ACC_ACTN;
            // add err actions
            for (auto token : {Token::PLUS, Token::MULT, Token::BRACKET_OPEN, Token::BRACKET_CLOSE, Token::ID, Token
                    for (int i = 0; i < item_set.size(); i++) if (action_map[{i, token}] == nullptr) {</pre>
```

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622

```
action_map[{i, token}] = ERR_ACTN;
                                    }
                  return action_map;
}
map<pair<int, string>, int> goto_map() {
                  map<pair<int, string>, int> goto_map;
                   // initialize goto_map
                  for (int i = 0; i < productions.size(); i++)
                                    for (int j = 0; j \le item_set.size(); j++)
                                                      goto_map[{j, str_productions[i][0]}] = -1;
                  for (int i = 0; i < existing_goto_history.size(); i++) {</pre>
                                     if (non_terminals.find(existing_goto_history[i].first.second) != non_terminals.end()) {
                                                       if (goto_map[{existing_goto_history[i].first.first, existing_goto_history[i].first.secon
                                                                        goto_map[{existing_goto_history[i].first.first, existing_goto_history[i].first.s
                                    }
                  return goto_map;
}
bool has_same_core(vector<Item>& i1, vector<Item>& i2) {
                   if (i1.size() != i2.size()) return false;
                  for (auto p : i1) {
                                    bool exists = false;
                                    for (auto op: i2) if (p.lhs == op.lhs && p.rhs == op.rhs && p.dot_idx == op.dot_idx) {
                                                       exists = true:
                                                      break;
                                    if (exists == false) return false;
                  return true;
vector<pair<int, vector<int>>> lalr_grouping() {
                   int idx = 0;
                  set < int > used_states;
                   vector<pair<int, vector<int>>> ans;
                   for (int i = 0; i < item_set.size(); i++) {
                                     if (used_states.find(i) != used_states.end()) continue;
                                    vector<int> grouping {i};
                                     used_states.insert(i);
                                    for (int j = i + 1; j < item_set.size(); j++) {
                                                       if (used_states.find(j) != used_states.end()) continue;
                                                       if (has_same_core(item_set[i], item_set[j])) {
                                                                         grouping.push_back(j);
                                                                         used_states.insert(j);
                                    ans.push_back({idx++, grouping});
                  return ans;
map<pair<int, Token>, Action*> lalr_action_map(map<pair<int, Token>, Action*>& clr_action_map) {
                  auto grouping = lalr_grouping();
                  map<int, int> old_to_new;
                  map<pair<int, Token>, Action*> new_action_map;
                  for (auto token : {Token::PLUS, Token::MULT, Token::BRACKET_OPEN, Token::BRACKET_CLOSE, Token::ID, Token
                                     for(int i = 0; i < item_set.size(); i++) {
                                                       if (new_action_map.find({old_to_new[i], token}) == new_action_map.end() || new_action_ma
                                                                         if (clr_action_map[{i, token}]->type == Action::Shift) {
                                                                                           ShiftAction* a = reinterpret_cast<ShiftAction*>(clr_action_map[{i, token}
                                                                                           new\_action\_map\left[\left\{old\_to\_new\left[\,i\,\right]\,,\;\;token\left.\right\}\right] \;=\; SHFT\_ACTN(old\_to\_new\left[\left(\,a->s\,hift\_action\_new\left[\,a->s\,hift\_action\_new\left[\,a->s\,hift\_action\_new\left[\,a->s\,hift\_action\_new\left[\,a->s\,hift\_action\_new\left[\,a->s\,hift\_action\_new\left[\,a->s\,hift\_action\_new\left[\,a->s\,hift\_action\_new\left[\,a->s\,hift\_action\_new\left[\,a->s\,hift\_action\_new\left[\,a->s\,hift\_action\_new\left[\,a->s\,hift\_action\_new\left[\,a->s\,hift\_action\_new\left[\,a->s\,hift\_action\_new\left[\,a->s\,hift\_action\_new\left[\,a->s\,hift\_action\_new\left[\,a->s\,hift\_action\_new\left[\,a->s\,hift\_action\_new\left[\,a->s\,hift\_action\_new\left[\,a->s\,hift\_action\_new\left[\,a->s\,hift\_action\_new\left[\,a->s\,hift\_action\_new\left[\,a->s\,hift\_action\_new\left[\,a->s\,hift\_action\_new\left[\,a->s\,hift\_action\_new\left[\,a->s\,hift\_action\_new\left[\,a->s\,hift\_action\_new\left[\,a->s\,hift\_action\_new\left[\,a->s\,hift\_action\_new\left[\,a->s\,hift\_action\_new\left[\,a->s\,hift\_action\_new\left[\,a->s\,hift\_action\_new\left[\,a->s\,hift\_action\_new\left[\,a->s\,hift\_action\_new\left[\,a->s\,hift\_action\_new\left[\,a->s\,hift\_action\_new\left[\,a->s\,hift\_action\_new\left[\,a->s\,hift\_action\_new\left[\,a->s\,hift\_action\_new\left[\,a->s\,hift\_action\_new\left[\,a->s\,hift\_action\_new\left[\,a->s\,hift\_action\_new\left[\,a->s\,hift\_action\_new\left[\,a->s\,hift\_action\_new\left[\,a->s\,hift\_action\_new\left[\,a->s\,hift\_action\_new\left[\,a->s\,hift\_action\_new\left[\,a->s\,hift\_action\_new\left[\,a->s\,hift\_action\_new\left[\,a->s\,hift\_action\_new\left[\,a->s\,hift\_action\_new\left[\,a->s\,hift\_action\_new\left[\,a->s\,hift\_action\_new\left[\,a->s\,hift\_action\_new\left[\,a->s\,hift\_action\_new\left[\,a->s\,hift\_action\_new\left[\,a->s\,hift\_action\_new\left[\,a->s\,hift\_action\_new\left[\,a->s\,hift\_action\_new\left[\,a->s\,hift\_action\_new\left[\,a->s\,hift\_action\_new\left[\,a->s\,hift\_action\_new\left[\,a->s\,hift\_action\_new\left[\,a->s\,hift\_action\_new\left[\,a->s\,hift\_action\_new\left[\,a->s\,hift\_action\_new\left[\,a->s\,hift\_action\_new\left[\,a->s\,hift\_action\_new\left[\,a->s\,hift\_action\_new\left[\,a->s\,hift\_action\_new\left[\,a->s\,hift\_action\_new\left[\,a->s\,hift\_action\_new\left[\,a->s\,hift\_action\_new\left[\,a->s\,hift\_action\_new\left[\,a->s\,hift\_action\_new\left[\,a->s\,hift\_action\_new\left[\,a->s\,hift\_action\_new\left[\,a->s\,hift\_action\_new\left[\,a->s\,hift\_action\_new\left[\,a->s\,hift\_action\_new\left[\,a->s\,hift\_action\_new\left[\,a->s\,hift\_action\_new\left[\,a->s\,hift\_action\_new\left[\,a->s\,hift\_action\_new\left[\,a->s\,hift\_action\_new\left[\,a->s\,hift
                                                                         } else {
```

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```
new_action_map[{old_to_new[i], token}] = clr_action_map[{i, token}];
                                 }
                         }
                }
        return new_action_map;
}
map<pair<int, string>, int> lalr_goto_map(map<pair<int, string>, int>& clr_goto_map) {
        auto grouping = lalr_grouping();
        map<int , int > old_to_new;
        for (auto g : grouping) for (auto i : g.second) old_to_new[i] = g.first;
        map<pair<int, string>, int> new_goto_map;
        for (int i = 0; i < productions.size(); i++)
                 for (int j = 0; j \ll grouping.size(); <math>j++)
                         new\_goto\_map[\{j\,,\,\,str\_productions[i][0]\}] \ = \ -1;
        for (auto kp : clr_goto_map) {
                 int os = kp.first.first;
                 if (old_to_new.find(os) == old_to_new.end() || old_to_new.find(kp.second) == old_to_new.end())
                 int ns = old_to_new[os];
                int ngs = old_to_new[kp.second];
                 string nt = kp.first.second;
                 if (new_goto_map.find({ns, nt}) != new_goto_map.end()) {
                         new\_goto\_map[{ns, nt}] = ngs;
                }
        return new_goto_map;
void print_items() const {
        int i = 0;
        for (auto items : item_set) {
                cout << "State I" << i << endl;</pre>
                 for (auto item : items) {
                         cout << "[" << item << "]" << endl;
                cout << "----" << endl << endl;
                 i++;
        }
void print_parse_table(map<pair<int, Token>, Action*> action_map, map<pair<int, string>, int> goto_map) {
        cout << "state \t+\t*\t(\t)\tid\t$\tE\tT\tF\n";
        for (int i = 0; i < item_set.size(); i++) {
                 if (action_map.find({i, Token::EOI}) == action_map.end()) return;
                cout \ll i \ll "\t";
                 for (auto token : {Token::PLUS, Token::MULT, Token::BRACKET_OPEN, Token::BRACKET_CLOSE, Token::I
                         switch(action_map[{i, token}]->type) {
                                 case Action::Shift: {
                                          ShiftAction * a = reinterpret_cast < ShiftAction *> (action_map[{i, token}]);
                                          cout << "s" << a->shift_state << "\t";</pre>
                                          break;
                                 case Action::Reduce: {
                                          ReduceAction* a = reinterpret_cast<ReduceAction*>(action_map[{i, token}]
                                          cout << "r" << a->production_id << "\t";</pre>
                                          break;
                                 case Action::Accept: {
                                          cout << "acc\t";</pre>
                                          break;
                                 case Action::Error: {
                                          cout << "err\t";</pre>
                                          break;
                                 }
                         }
```

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709 710

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760

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764

```
766
                                                             }
767
                                                             for (auto nt : \{"E", "T", "F"\}) {
768
769
                                                                               if (goto_map[{i, nt}] != -1) {
                                                                                               cout << \;goto\_map\,[\,\{\,i\,,\ nt\,\}\,] \;<< \;"\,\backslash\,t\,"\,;
770
771
                                                                              } else {
772
                                                                                               cout << "\t";</pre>
773
774
                                                             }
                                                             cout << "\n";
775
776
                                            }
777
778
                           map<string , Token> token_map = {
779
                                            {"+", Token::PLUS}, {"*", Token::MULT}, {"id", Token::ID}, {"(", Token::BRACKET_OPEN}, {")", Token::BRACKET_OPEN}
780
781
                           }:
782
                   vector<pair<int, string>> goto_history;
783
                           vector<pair<int, string>, int>> existing_goto_history;
784
                   set<string> non_terminals;
785
                   vector<vector<Item>> item_set;
786
                   vector<production> productions;
787
          };
788
          int main() {
789
                            Grammar \ grammar(\{\{"E'", "E"\}, \{"E", "E+T"\}, \{"E", "T"\}, \{"T", "T*F"\}, \{"T", "F"\}, \{"F", "(E)"\}, \{"F", "id"\}\}); \\ Grammar \ grammar(\{\{"E'", "E"\}, \{"E", "E+T"\}, \{"T", "T*F"\}, \{"T", "F"\}, \{"F", "GE)"\}, \{"F", "id"\}\}); \\ Grammar \ grammar(\{\{"E'", "E"\}, \{"E", "E+T"\}, \{"E", "T"\}, \{"T", "T*F"\}, \{"T", "F"\}, \{"F", "GE)"\}, \{"F", "id"\}\}); \\ Grammar \ grammar(\{\{"E'", "E"\}, \{"E", "E+T"\}, \{"E", "T"\}, \{"T", "T*F"\}, \{"T", "F"\}, \{"F", "GE)"\}, \{"F", "GE)"\}, \{"F", "GE)"\}, \\ Grammar \ grammar(\{\{"E', "E", "E", "GE), "GE), [T, GE], [T, G
790
791
792
                           cout << "First of non-terminals: " << endl;</pre>
793
                           for (auto nt : {"E", "T", "F"}) {
794
                                            auto first_set = grammar.first(nt);
                                            cout << "FIRST(" << nt << ") = {";
795
                                            for (auto f : first_set)
796
                                                             cout << f << ", ";
797
798
                                            cout << "}\n";
799
800
                           cout << endl;
801
                           cout << "Generated LR(1) Items: " << endl;</pre>
802
803
                           grammar.print_items();
804
805
                           map<pair<int, Token>, Action*> action_map = grammar.action_map();
806
                           map<pair<int, string>, int> goto_map = grammar.goto_map();
807
                           cout << "CLR Parse table :" << endl;</pre>
808
809
                           grammar.print_parse_table(action_map, goto_map);
810
811
                           auto tmp = grammar.lalr_grouping();
                           cout << endl << "LALR Groupings from CLR Items: " << endl;</pre>
812
813
                           for (auto p : tmp) {
                                            cout << p.first << " = ";
814
815
                                            for (auto pt : p.second)
816
                                                             cout << pt << " ";
817
                                            cout << endl;
818
819
                           cout << endl;
820
821
                           map<pair < int , Token > , Action *> lalr_action_map = grammar.lalr_action_map (action_map);
822
                           map<pair<int, string>, int> lalr_goto_map = grammar.lalr_goto_map(goto_map);
823
                           cout << "LALR Parse table:" << endl;</pre>
824
825
                           grammar.print_parse_table(lalr_action_map, lalr_goto_map);
826
                           cout << endl;</pre>
827
828
                            // Create parser
829
                           Parser parser(lalr_action_map, lalr_goto_map);
830
                            string input;
                           cout << "Enter string to parse:";
831
832
                            getline(cin, input);
833
                           parser.parse(input);
834 }
```

```
$ g++ gen_lalr_main.cpp
$ ./a.out
First of non-terminals:
FIRST(E) = \{(, id, \}
FIRST(T) = \{(, id, \}
FIRST(F) = \{(, id, \}
Generated LR(1) Items:
State IO
[[E' -> .E , $]]
[[E -> .E+T , $/+]]
[[E -> .T , $/+]]
[[T -> .T*F , $/*/+]]
[[T -> .F , $/*/+]]
[[F -> .(E) , $/*/+]]
[[F -> .id , $/*/+]]
-----
State I1
[[E' -> E. , $]]
[[E -> E.+T , $/+]]
-----
State I2
[[E -> T., $/+]]
[[T \rightarrow T.*F , $/*/+]]
-----
State I3
[[T \rightarrow F. , \$/*/+]]
-----
State I4
[[F -> (.E) , $/*/+]]
[[E -> .E+T , )/+]]
[[E -> .T , )/+]]
[[T -> .T*F , )/*/+]]
[[T -> .F , )/*/+]]
[[F -> .(E) , )/*/+]]
[[F -> .id , )/*/+]]
State I5
[[F \rightarrow id., \$/*/+]]
-----
State I6
[[E \rightarrow E+.T , $/+]]
[[T -> .T*F , $/*/+]]
[[T -> .F , $/*/+]]
[[F -> .(E) , $/*/+]]
[[F -> .id , $/*/+]]
-----
State I7
[[T -> T*.F , $/*/+]]
```

```
[[F -> .(E) , $/*/+]]
[[F -> .id , $/*/+]]
-----
State I8
[[F -> (E.) , $/*/+]]
[[E -> E.+T , )/+]]
-----
State I9
[[E -> T. , )/+]]
[[T -> T.*F , )/*/+]]
-----
State I10
[[T -> F., )/*/+]]
-----
State I11
[[F -> (.E) , )/*/+]]
[[E -> .E+T , )/+]]
[[E -> .T , )/+]]
[[T -> .T*F , )/*/+]]
[[T -> .F , )/*/+]]
[[F -> .(E) , )/*/+]]
[[F -> .id , )/*/+]]
-----
State I12
[[F \rightarrow id., )/*/+]]
-----
State I13
[[E -> E+T. , $/+]]
[[T \rightarrow T.*F , $/*/+]]
-----
State I14
[[T -> T*F. , $/*/+]]
-----
State I15
[[F \rightarrow (E)., $/*/+]]
-----
State I16
[[E -> E+.T , )/+]]
[[T -> .T*F , )/*/+]]
[[T -> .F , )/*/+]]
[[F -> .(E) , )/*/+]]
[[F -> .id , )/*/+]]
-----
State I17
[[T -> T*.F , )/*/+]]
[[F -> .(E) , )/*/+]]
[[F -> .id , )/*/+]]
-----
```

```
[[E -> E.+T , )/+]]
-----
State I19
[[E -> E+T., )/+]]
[[T -> T.*F , )/*/+]]
-----
State I20
[[T -> T*F. , )/*/+]]
-----
State I21
[[F -> (E)., )/*/+]]
-----
CLR Parse table :
state
         +
                           (
                                   )
                                            id
                                                     $
                                                              Ε
                                                                      Τ
                                                                               F
                                                                       2
                                                                               3
0
                          s4
                                            s5
                                                     err
                                                              1
         err
                  err
                                   err
1
         s6
                  err
                          err
                                   err
                                            err
                                                     acc
2
         r2
                  s7
                           err
                                   err
                                            err
                                                     r2
3
         r4
                 r4
                                                     r4
                          err
                                   err
                                            err
4
         err
                          s11
                                   err
                                            s12
                                                     err
                                                              8
                                                                      9
                                                                               10
                 err
5
         r6
                 r6
                          err
                                   err
                                            err
                                                     r6
6
                          s4
                                   err
                                            s5
                                                                       13
                                                                               3
         err
                  err
                                                     err
7
         err
                 err
                          s4
                                   err
                                            s5
                                                     err
                                                                               14
8
         s16
                                   s15
                  err
                          err
                                            err
                                                     err
9
         r2
                  s17
                                   r2
                          err
                                            err
                                                     err
10
        r4
                 r4
                                   r4
                          err
                                            err
                                                     err
                                                              18
                                                                      9
                                                                               10
11
         err
                 err
                          s11
                                   err
                                            s12
                                                     err
12
        r6
                 r6
                                   r6
                          err
                                            err
                                                     err
13
         r1
                 s7
                          err
                                   err
                                            err
                                                     r1
14
         r3
                 r3
                                                     r3
                          err
                                   err
                                            err
15
        r5
                 r5
                          err
                                   err
                                            err
                                                     r5
                                                                       19
                                                                               10
16
                          s11
                                   err
                                            s12
         err
                 err
                                                     err
17
                                                                               20
         err
                 err
                          s11
                                   err
                                            s12
                                                     err
18
         s16
                 err
                          err
                                   s21
                                            err
                                                     err
19
         r1
                 s17
                          err
                                   r1
                                            err
                                                     err
20
         r3
                 r3
                                   r3
                          err
                                            err
                                                     err
        r5
21
                 r5
                                   r5
                          err
                                                     err
                                            err
```

LALR Groupings from CLR Items:

0 = 0

State I18

[[F -> (E.) ,)/*/+]]

1 = 1

2 = 2 9

3 = 3 10

4 = 4 11

5 = 5 12

6 = 6 16

 $7 = 7 \ 17$ $8 = 8 \ 18$

9 = 13 19

LALR 1	Parse tal	ble:							
state	+	*	()	id	\$	E	T	F
0	err	err	s4	err	s5	err	1	2	3
1	s6	err	err	err	err	acc			
2	r2	s7	err	r2	err	r2			
3	r4	r4	err	r4	err	r4			
4	err	err	s4	err	s5	err	8	2	3
5	r6	r6	err	r6	err	r6			
6	err	err	s4	err	s5	err		9	3
7	err	err	s4	err	s5	err			10
8	s6	err	err	s11	err	err			
9	r1	s7	err	r1	err	r1			
10	r3	r3	err	r3	err	r3			
11	r5	r5	err	r5	err	r5			

<pre>Enter string to parse :id + id * (id + id)</pre>							
Stack	Current Token	Input	Action				
[0]	-	id + id * (id + id) \$	-				
[05]	<id></id>	+ id * (id + id) \$	Shift to 5				
[03]	<plus></plus>	id * (id + id) \$	Reduce by F -> id				
[02]	<plus></plus>	id * (id + id) \$	Reduce by T -> F				
[0 1]	<plus></plus>	id * (id + id) \$	Reduce by E -> T				
[016]	<plus></plus>	id * (id + id) \$	Shift to 6				
[0165]	<id></id>	* (id + id) \$	Shift to 5				
[0163]	<mult></mult>	(id + id) \$	Reduce by F -> id				
[0169]	<mult></mult>	(id + id) \$	Reduce by T -> F				
[01697]	<mult></mult>	(id + id) \$	Shift to 7				
[016974]	<bracket_open></bracket_open>	id + id) \$	Shift to 4				
[0169745]	<id></id>	+ id) \$	Shift to 5				
[0169743]	<plus></plus>	id) \$	Reduce by F -> id				
[0 1 6 9 7 4 2]	<plus></plus>	id) \$	Reduce by T -> F				
[0169748]	<plus></plus>	id) \$	Reduce by E -> T				
[01697486]	<plus></plus>	id) \$	Shift to 6				
[016974865]	<id></id>) \$	Shift to 5				
[016974863]	<bracket_close></bracket_close>	\$	Reduce by F -> id				
[016974869]	<bracket_close></bracket_close>	\$	Reduce by T -> F				
[0169748]	<bracket_close></bracket_close>	\$	Reduce by E -> E+T				
[0 1 6 9 7 4 8 11]	<bracket_close></bracket_close>	\$	Shift to 11				
[0 1 6 9 7 10]	<\$>	\$	Reduce by $F \rightarrow (E)$				
[0169]	<\$>	\$	Reduce by T -> T*F				
[0 1]	<\$>	\$	Reduce by $E \rightarrow E+T$				
[0 1]	<\$>	\$	Accepted				