BTP REPORT

Research Article

A Mathematical Analysis of the Card Game of Betweenies through Kelly's Criterion

The card game of "In-Betweenies" or "In-Between" is also known under many alternative names including "Between the Sheets" , "Red Dog" , "Acey Deucey" , and "Yablon."

Rules:

The player is dealt two cards in the beginning of the game. The player then bets an amount, say bi, for each of the cards, say xi, which are strictly between the previously dealt two cards. The betted amount ranges from 0 to the pot amount (where each contributes a fixed amount to participate in a round). If the player wins the bet bi, he wins an amount bi*ti back including the original amount, where ti÷1 is the odd placed on xi; xi here is any possible outcome.

Kelly's Criterion:

KC is famous for suggesting an optimal betting strategy which, at the same time, eliminates the possibility of the gambler getting ruined. It suggests a betting strategy, based on which a player can expect an exponential increase of his wealth.

- **Classical Approach:**

Total initial wealth of the player is assumed to be w. The bets placed on xi's are bi's. Assume further that the odds placed on xi are ti÷1, whereby it is meant that, if a player places a bet bi on xi, and xi is indeed the outcome, he will receive a wealth of ti*bi back including the original bet; otherwise the bet is lost.

To determine various bi's, this approach focuses on maximizing the expected wealth of the player.

Hence the expected wealth after the game is:

$$w' = \sum_{i=1}^{n} p_i \left(w - \sum_{k=1}^{n} b_k + t_i b_i \right) = w - \sum_{k=1}^{n} b_k + \sum_{i=1}^{n} p_i t_i b_i.$$

We observe that,

$$\frac{\partial w'}{\partial b_i} = p_i t_i - 1.$$

Case I: pi*ti less than or equal to 1 for some i

It gives , $\frac{\partial w'}{\partial b_i}$ less than or equal to 0. Hence decrease in bet bi increases the expected wealth of the player. Hence, optimal bet under the given case can be taken to be 0.

Case II: pi*ti strictly greater than 1

It gives , $\frac{\partial w'}{\partial b_i}$ strictly greater than 0. Hence increase in bet bi increases the expected wealth of the player. Hence, optimal bet under the given case can be taken to be w.

Gambler's Ruin:

This strategy also poses a great risk to the player if the number of rounds in which he participates increases to large number. Let pi < 1 be the probability of the player not getting ruined in one round. Then (pi)^m is the probability of the player not getting ruined in m rounds. Clearly, lim(pi)^m = 0. Hence, the player eventually gets ruined. This is known as Gambler's ruin.

- **A new approach:**

The new approach suggests maximizing the expected log return of the game.

$$l = \sum_{i=1}^{n} p_i \ln\left(\frac{w - \sum_{k=1}^{n} b_k + t_i b_i}{w}\right).$$

$$l = \sum_{i=1}^{n} p_i \ln\left(\frac{t_i b_i}{w}\right) \text{ given that } w = \sum_{k=1}^{n} b_k.$$

To achieve this, we use Lagrange multipliers and maximize:

$$l'(\lambda) = \sum_{i=1}^{n} p_i \ln\left(\frac{t_i b_i}{w}\right) - \lambda\left(\sum_{k=1}^{n} b_k - w\right) : 0 = \frac{\partial l'}{\partial b_j} = \frac{p_j}{b_j} - \lambda \Longleftrightarrow b_j = p_j w.$$

In other words, the bets are proportional to the probabilities. In that case,

$$l = \sum_{i=1}^{n} p_i \ln(t_i p_i) = \sum_{i=1}^{n} p_i \ln\left(\frac{p_i}{1/t_i}\right)$$

In Information Theory this quantity is known as Kullback-Leibler distance D_{KL} (P||T).