Unit 7: Differential Equations

7.1 - Modeling Situations with Differential Equations

- rate is a derivative
- when something directly varies something else, it means that $y = kx$
- when something indirectly varies something else, it means that $y = \frac{k}{x}$

📢 - This unit is about recognizing equations from word problems

---

7.2 - Verifying Solutions for Differential Equations

- questions ask you to solve a second or third derivative and then add or subtract the original function to get an answer
  - just don't mess up the derivatives when you take multiple??

📢 - A solution of a differential equation can be checked by substituting the function and its derivatives into the original differential equation

---

7.3 - Sketching Slope Fields

- the derivative value is the slope of the curve which can be represented graphically
  - if $\frac{dy}{dx} = 0$, then the graphical depiction is just a horizontal line
  - if $\frac{dy}{dx} = \text{undefined}$, then the graphical depiction is a vertical line
  - if the slope is positive, we will have an increasing line
  - if the slope is negative, we will have a decreasing line
- given a graph, draw a line (from one of the options above) at every coordinate point to plot the slope field
  - (it'll look like an electric field? where all of the lines are slowly turning and facing towards one point idk)
- interpret a slope field by either describing or sketching a solution curve
  - describe the boundaries for all points with lines of a certain type (positive, negative, etc)
  - when given a slope field and a point, draw the solution for that point by following the direction of the slope field lines, and make sure to go all the way to the ends of the graph

📢 - A slope field looks like an electric field
- Each coordinate point on the slope field has a corresponding line representing the slope at that point
- Eliminate answer choices on MCQ by identifying vertical or horizontal asymptotes

---

7.4 - Reasoning Using Slope Fields

- the slope field represents the derivative equation when the correct value is plugged in
  - sometimes the question will ask you to find the option that "could be a solution to the differential equation with the initial condition [condition]" so you should plug in the initial condition value and draw the line
- analyze the parent functions graphically to get an idea of the derivative or possible answer choices

- Look for the general solution to a slope field

7.6 - Finding General Solutions Using Separation of Variables

- solve for the general solution for a given differential equation
  1. separate the variables so that y is on one side and x is on the other side
  2. integrate both sides and add the $+C$ on the x side
  3. solve for y

$\frac{dy}{dx} = y\sin x$
$\frac{dy}{dx \cdot y} = \sin x$
$\frac{dy}{y} = dx \cdot \sin x$
$\int \frac{dy}{y} = \int dx \cdot \sin x$
$\ln|y| = -\cos x + C$
$y = e^{-\cos x + C}$

$\frac{dy}{dx} = \frac{3x^2+2}{5\cos y}$
$(5\cos y)dy = (3x^2+2)dx$
$\int(5\cos y)dy = \int(3x^2+2)dx$
$5\sin y = x^3 + 2x + C$
$\sin y = \frac{x^3+2x}{5} + C_1$
$y = \sin^{-1}(\frac{x^3+2x}{5} + C_2)$

$\frac{dy}{dx} = \frac{x+1}{y}$
$y \cdot dy = (x+1)dx$
$\int y \cdot dy = \int(x+1)dx$
$\frac{y^2}{2} = \frac{x^2}{2} + x + C$
$y^2 = x^2 + 2x + C$
$y = \sqrt{x^2 + 2x + C}$

- The general solution is the equation of y before it was derived and is the parent solution

7.7 - Finding Particular Solutions Using Initial Conditions and Separation of Variables

- the particular solution is the solution where C is found
  - the problem will usually give a set of coordinates that work for the particular solution, so that you can plug it into the general solution to solve for C

$(0, -2)$
$\frac{dy}{dx} = \frac{x+1}{y}$
$y \cdot dy = (x+1)dx$
$\int y \cdot dy = \int(x+1)dx$
$\frac{y^2}{2} = \frac{x^2}{2} + x + C$
$\frac{(-2)^2}{2} = \frac{0^2}{2} + 0 + C$
$2 = 0 + 0 + C$
$2 = C$
$\frac{y^2}{2} = \frac{x^2}{2} + x + 2$
$y^2 = x^2 + 2x + 4$
$-2 = -\sqrt{0^2 + 2(0) + 4}$
$y = -\sqrt{x^2 + 2x + 4}$

$(\frac{\pi}{2}, 1)$
$\frac{dy}{d\theta} = 4y^2\cos(2\theta)$
$\frac{1}{y^2} \cdot dy = 4\cos(2\theta)d\theta$
$y^{-2}dy = 4\cos(2\theta)d\theta$
$\int y^{-2}dy = \int 4\cos(2\theta)d\theta$
$\frac{-1}{y} = 2\sin(2\theta) + C$
$\frac{-1}{1} = 2\sin(2\frac{\pi}{2}) + C$
$-1 = 2\sin(\pi) + C$
$-1 = C$
$\frac{-1}{y} = 2\sin(2\theta) - 1$
$y = \frac{-1}{2\sin(2\theta) - 1}$

$(2, 0)$
$\frac{dy}{dx} = \frac{y-1}{x^2}$
$\frac{1}{y-1}dy = \frac{1}{x^2}dx$
$\int \frac{1}{y-1}dy = \int \frac{1}{x^2}dx$
$\ln|y-1| = -\frac{1}{x} + C$
$\ln|0-1| = -\frac{1}{2} + C$
$\ln(1) = -\frac{1}{2} + C$
$\frac{1}{2} = C$
$\ln|y-1| = -\frac{1}{x} + \frac{1}{2}$
$\ln|y-1| = -\frac{1}{x} + \frac{1}{2}$
$y - 1 = \pm e^{-\frac{1}{x}+\frac{1}{2}}$
$y = 1 - e^{-\frac{1}{x}+\frac{1}{2}}$

- A particular solution is a solution that applies to a specific set of coordinates
- To get a particular solution, solve for C first and then solve for y

7.8 - Exponential Models with Differential Equations

- show how $\frac{dy}{dx} = k \cdot y$
  - the sign $\pm$ will be determined by the initial condition

$\frac{dy}{dx} = k \cdot y$

$\frac{1}{y}dy = k \cdot dx$
$\int \frac{1}{y}dy = \int k \cdot dx$
$\ln|y| = kx + C$
$|y| = e^{kx+C}$
$|y| = Ce^{kx}$
$y = \pm Ce^{kx}$

📢 - The general solution of an equation $\frac{dy}{dx} = ky$ is $y = Ce^{kx}$