1) Find the rank and nullity of the following matrices using pivot element algorithm.

a) $\begin{pmatrix} 2 & 3 \\ 4 & 7 \end{pmatrix}$

b) $\begin{pmatrix} 1 & 0 & -1 \\ 1 & 1 & -3 \\ 0 & -1 & 2 \end{pmatrix}$

c) $\begin{pmatrix} 1 & 0 & 0 & 1 \\ -4 & 3 & 1 & 0 \\ -3 & 2 & 0 & 0 \end{pmatrix}$

d) $\begin{pmatrix} 1 & 4 & 5 & -1 & 0 \\ 0 & 1 & 0 & 4 & -1 \\ -1 & 0 & 1 & 0 & -1 \\ -1 & 1 & 1 & 4 & -2 \end{pmatrix}$

2) Use Gauss-Jordan method to find the row reduced echelon form of the following matrices.

a) $A = \begin{pmatrix} 1 & -2 & 3 & 1 & 2 \\ 1 & 1 & 4 & -1 & 3 \\ 2 & 5 & 9 & -2 & 8 \end{pmatrix}$

b) $B = \begin{pmatrix} 1 & 2 & -1 & 3 \\ 1 & 3 & 1 & 5 \\ 3 & 8 & 4 & 17 \end{pmatrix}$

3) State true or false the following statement and justify your answer.

Let A and B be two $m \times n$ matrices and suppose that there exists an invertible matrix Q such that $A = BQ$ then matrices A and B have same rank.