Echelon Form

13 January 2022 13:59

A matrix is said to be in Echelon from if

- All nows consisting of zeroes are at bottom
- The pivot (leading non-zero entry in non-zero row) is strictly right to pivot in any non-zero sow above it

Gaussian-Elimination Method to reduce any matrix to echelon form.

Ussian-Elimination Method to reclude any matrix to economic to reclude any matrix to economic to
$$\frac{1}{2} - 2 = 1$$
 $A = \begin{bmatrix} 2 & -2 & 2 & 1 \\ 3 & 6 & 0 & 1 \\ 1 & -7 & 10 & 2 \end{bmatrix}$
 $R_2 - R_2 + \frac{3}{2}R_2$
 $R_3 - \frac{1}{2}R_1$
 $R_3 - \frac{1}{2}R_2$
 $R_3 - \frac{1}{2}R_1$
 $R_3 - \frac{1}{2}R_1$
 $R_3 - \frac{1}{2}R_2$
 $R_$

-f(A) = 3 = # ob pivots

Rank of a matrix.

Notation of rank & A

lut A be any matrix, rank of onatrix A (P(A)) is defined as the total number of pivots in echelon form of matrix A

$$A = \begin{bmatrix} 2 & 2 & -1 & 6 & 4 \\ 4 & 4 & 1 & 10 & 13 \\ 8 & 8 & -1 & 26 & 23 \end{bmatrix}$$
To find: Rank of matrix $A \left(f(A) \right)$

By Gaussian-Elimination onethod, we find the echelon form of matrix of A.

$$\begin{bmatrix}
2 & 2 & -1 & 6 & 4 \\
4 & 4 & 1 & 10 & 13 \\
8 & 8 & -1 & 26 & 23
\end{bmatrix}
\xrightarrow{R_2 \to R_2 - 2R_Y}
\begin{bmatrix}
2 & 2 & -1 & 6 & 4 \\
0 & 0 & 3 & -2 & 5 \\
\hline{\$}
\xrightarrow{\$}$$

Rank of matrix
$$A = P(A) = \# G$$
 pivots in echelon from of matrix $A = 3$

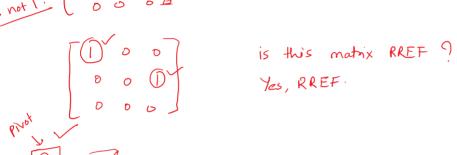
Row Reduced Echelon Form(RREF)

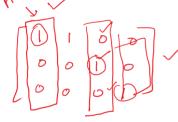
is said to be in now reduced echelon form (rref) | now canonical form, if

- The moting is in e dielon form
- Pivot element in each now must be equal to 1.
- The pivot in each sow must be the only non-zero number in it's column.

5 1 0 zero now is
0 0 0 1 not abottom
0 0 1 80. it is not in education from

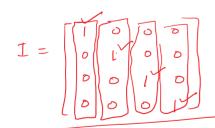
Pivot dement 0 0 0 0 pero row at bottom. Echelon form Not RREF.





Is this matrix in echelon form ? Yes, in echelon form

Is this matrix in RREF 9 Yes, in RREF.



is this in echelon form?

is this in met?

Yes, RREF. V

Crauss - Jordan Method.

Any matrix can be reduced to it's rref.

 $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ det(A) = 4-6 = -2 \neq 0. so A is invertible [1 2] - echelon form of A $R_2 \rightarrow R_2 - 3R_1$

$$\begin{bmatrix} 1 & 2 \\ 0 & \overline{2} \end{bmatrix} \longrightarrow \text{echelon form.}$$

$$\begin{bmatrix} R_2 \rightarrow R_2(-\frac{1}{2}) \\ 0 & \overline{2} \end{bmatrix} \longrightarrow \text{is this in oref ?}$$

$$\begin{bmatrix} R_1 \rightarrow R_1 - 2R_2 \\ 0 & \overline{1} \end{bmatrix} \longrightarrow \text{RREF.}$$

Question: The ref of every invertible matrix is identity on whix 99

$$A \xrightarrow{e_1} e_1(A) \xrightarrow{e_2} e_2(e_1(A)) \xrightarrow{\qquad \qquad } --- \xrightarrow{\qquad e_n} R$$

$$J$$
Invertible
$$matrix \qquad A \sim R$$

$$e_1 \rightarrow E_1 = e_1(I)$$
 $e_2 \rightarrow E_2 = e_2(I)$
 $e_1 \rightarrow E_2 = e_1(I)$
 $e_2 \rightarrow E_3 = e_1(I)$

luts say Ex be the corresponding elementary matrix of ex (K=1,2,--n)

$$R = \underbrace{E_n E_{n+1} E_{n-2} - E_3 E_2 E_1}_{a} A$$

Us product of invertible matrices

=> R is invertible being product of invertible matrices.

Also, Ris in ref.

that means, R has to be identity matrix.

-> Every invertible matrix has reef equals to identity matrix

Any invertible matrix is row equivalent to the identity matrix.

was used to prove

that every invertible matrix can be won'then an product of elementary matrices

Proof.

Suppose A is inventible matrix, Also assume that R be the roof of matrix A.

ie we have applied a finite sequence of elementary operations on A to get matrix R.

$$A \xrightarrow{e_1} g(A) \xrightarrow{\varrho_2} g(A) \longrightarrow --- \xrightarrow{e_n} R$$

$$R = \underbrace{E_{n} E_{n-1} - \cdots E_{3} E_{2} E_{1} A}_{\text{prodvd of invartible}} \quad \text{where} \quad E_{k} = e_{k}(I)$$

$$f_{0} = \underbrace{E_{n} E_{n-1} - \cdots E_{3} E_{2} E_{1} A}_{\text{prodvd of invartible}} \quad \text{where} \quad E_{k} = e_{k}(I)$$

80, R is rref(A) and invertible that means, R = I.

$$I = E_n E_{n+} - \dots E_3 E_2 E_1 A$$

 $A \sim I$ ie. A is now equivalent to identity motion. as R = I is obtained by a finite sequence of elementary speration on A.

- 1) (It A be any matrix, R be the row reduced exhelon from of matrix A. P(A) = P(R)
- 2) Rank of matrix = Total number of pivots in echelon form of matrix A

 and any now can have atmost one pivot, similarly any column can have atmost
 one pivot

Let A be man matrix

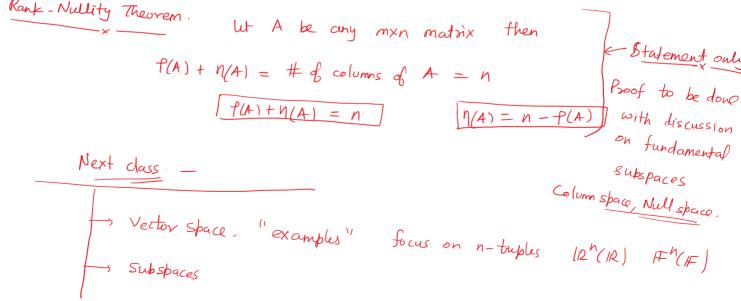
$$f(A) \leq m$$
 $\Rightarrow \qquad f(A) \leq min \{m, n\}$

$$(X)$$
 (X) (X)

Lemma Page 5

P(A)
$$\leq$$
 9 to 11 12

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\begin{align*}
\begin{alig



For any queries related to subject, please send an email to: kapilchaudhary@gujaratuniversity.ac.in